Journal of Modern Applied Statistical Methods

Volume 16 | Issue 2 Article 26

12-2017

A Remark for the Admissibility of Rao's U-test

Z. D. Bai
Northeast Normal University, Changchun, Jilin, China

C. R. Rao
Pennsylvania State University, State College, PA, crr1@psu.edu

M. T. Tsai
Academia Sinica, Nankang, Taipei, Taiwan, mttsai@stat.sinica.edu.tw

Recommended Citation
Bai, Z. D., Rao, C. R., & Tsai, M. T. (2017). A remark for the admissibility of Rao's U-Test. Journal of Modern Applied Statistical Methods, 16(2), 486-488. doi: 10.22237/jmasm/1509495960

A Remark for the Admissibility of Rao's U-test

Cover Page Footnote
The first author was partly supported by Grants from NSFC 11571067

Journal of Modern Applied Statistical Methods
November 2017, Vol. 16, No. 2, 486-488.
doi: 10.22237/jmasm/1509495960

ISSN 1538 − 9472

A Remark for the Admissibility of Rao's U-test

Z. D. Bai
Northeast Normal University
Changchun, Jilin, China

C. R. Rao
Pennsylvania State University
State College, PA

M. T. Tsai
Academia Sinica
Nankang, Taipei, Taiwan

Let $\{\mathbf{X}_i; 1 \le i \le n\}$ be independent and identically distributed random vectors (i.i.d.r.v.) with a p-variate normal distribution with mean vector $\boldsymbol{\theta}$ and dispersion matrix $\boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma}$ is assumed to be positive definite (p.d.). Partition $\boldsymbol{\theta}$ and $\boldsymbol{\Sigma}$ as

$$\boldsymbol{\theta} = \begin{pmatrix} \boldsymbol{\theta}_1 \\ \boldsymbol{\theta}_2 \end{pmatrix} \text{ and } \boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{bmatrix},$$

where $\boldsymbol{\theta}_1 : p_1 \times 1$, $\boldsymbol{\theta}_2 : p_2 \times 1$, $\boldsymbol{\Sigma}_{11} : p_1 \times p_1$, $\boldsymbol{\Sigma}_{22} : p_2 \times p_2$, $p_1 + p_2 = p$, $0 < p_2 < p$. The problem of interest is to test

$$\begin{aligned} &H_0 : \boldsymbol{\theta}_1 = \mathbf{0},\ \boldsymbol{\theta}_2 = \mathbf{0},\ \boldsymbol{\Sigma} \text{ unspecified} \\ &\qquad\text{versus} \\ &H_1 : \boldsymbol{\theta}_1 \ne \mathbf{0},\ \boldsymbol{\theta}_2 = \mathbf{0},\ \boldsymbol{\Sigma} \text{ unspecified.} \end{aligned} \tag{1}$$

For every n (≥ 2), let

$$\bar{\mathbf{X}} = n^{-1} \sum_{i=1}^{n} \mathbf{X}_i \text{ and } \mathbf{S} = \sum_{i=1}^{n} \left(\mathbf{X}_i - \bar{\mathbf{X}}\right)\left(\mathbf{X}_i - \bar{\mathbf{X}}\right)',$$

and express Hotelling's T^2-statistic as

$$T^2 = n(n-1)\bar{\mathbf{X}}'\mathbf{S}^{-1}\bar{\mathbf{X}}.$$

Dr. Bai is faculty in the School of Mathematics and Statistics. Email at baizd@nenu.edu.cn. Prof. Rao, Sc.D. (Cantab), FRS, is the Eberly Professor Emeritus of Statistics and the Director of Center for Multivariate Analysis at Penn State, and Research Professor in Biostatistics at the University at Buffalo. Email him at crr1@psu.edu. Dr. Tsai is a research fellow in the Institute of Statistical Science. Email at mttsai@stat.sinica.edu.tw.

Partition $\bar{\mathbf{X}}$ and $\mathbf{S}$ the same as $\boldsymbol{\theta}$ and $\boldsymbol{\Sigma}$, respectively, and define

$$\bar{\mathbf{X}}_{1:2} = \bar{\mathbf{X}}_1 - \mathbf{S}_{12}\mathbf{S}_{22}^{-1}\bar{\mathbf{X}}_2,$$
$$\mathbf{S}_{11:2} = \mathbf{S}_{11} - \mathbf{S}_{12}\mathbf{S}_{22}^{-1}\mathbf{S}_{21}.$$

For the problem (1), Rao (1946, 1949) proposed two test statistics which are of the forms

$$W = \frac{n(n-1)\bar{\mathbf{X}}_{1:2}'\mathbf{S}_{11:2}^{-1}\bar{\mathbf{X}}_{1:2}}{1+n(n-1)\bar{\mathbf{X}}_2'\mathbf{S}_{22}^{-1}\bar{\mathbf{X}}_2}$$

and

$$U = n(n-1)\bar{\mathbf{X}}_{1:2}'\mathbf{S}_{11:2}^{-1}\bar{\mathbf{X}}_{1:2}$$

respectively.

Tsai (2003) generalized the Stein (1956) approach to show that Rao's U-test is admissible for the problem (1), which seems contradictory to Marden and Perlman (1980) who proved that Rao's U-test is inadmissible for the problem

$$\begin{gathered} H_0 : \boldsymbol{\theta}_1 = \mathbf{0},\ \boldsymbol{\Sigma} \text{ unspecified} \\ \text{versus} \\ H_1 : \boldsymbol{\theta}_1 \neq \mathbf{0},\ \boldsymbol{\Sigma} \text{ unspecified.} \end{gathered} \tag{2}$$

There is no contradiction between the two results, because the parameter spaces for the two problems are different. Rao's parameter space is $\Theta_R = \{(\boldsymbol{\theta}_1;\mathbf{0};\boldsymbol{\Sigma})\}$ and MP's is $\Theta_{MP} = \{(\boldsymbol{\theta}_1;\boldsymbol{\theta}_2;\boldsymbol{\Sigma})\}$. Rao's parameter space is smaller than MP's. It is possible that a test is admissible for a smaller parameter space while inadmissible for a larger parameter space. There may exist a test ϕ, such that $\beta_\phi(\boldsymbol{\theta}) \leq \beta_U(\boldsymbol{\theta})$, for all $\boldsymbol{\theta} \in \Theta_R$, $\beta_U(\boldsymbol{\theta}) \leq \beta_\phi(\boldsymbol{\theta})$, for all $\boldsymbol{\theta} \in \Theta_{MP}\backslash\Theta_R$, and $\beta_U(\boldsymbol{\theta}) < \beta_\phi(\boldsymbol{\theta})$, for some $\boldsymbol{\theta} \in \Theta_{MP}\backslash\Theta_R$, where $\beta_\phi(\boldsymbol{\theta})$ denotes the power of the test ϕ at the parameter $\boldsymbol{\theta}$.

Marden and Perlman (1980) proved the admissibility of Hotelling's T^2-test for the problem (2), while Tsai (2003) proved the inadmissibility of Hotelling's T^2-test for the problem (1). The inadmissibility result of Tsai (2003) does not contradict to the admissibility result of Marden and Perlman (1980). If ψ is a test controlling T^2 for the problem (1), that is, $\beta_{T^2}(\boldsymbol{\theta}) \leq \beta_\psi(\boldsymbol{\theta})$, for all $\boldsymbol{\theta} \in \Theta_R$ and $\beta_{T^2}(\boldsymbol{\theta}) < \beta_\psi(\boldsymbol{\theta})$, for some $\boldsymbol{\theta} \in \Theta_R$, we must have $\beta_{T^2}(\boldsymbol{\theta}) > \beta_\psi(\boldsymbol{\theta})$, for some $\boldsymbol{\theta} \in \Theta_{MP} \backslash \Theta_R$. Hence, it does not control T^2 for the problem (2).

Although the results of Tsai (2003) and Marden and Perlman (1980) are not contradictory, the results of Tsai (2003) were obtained for an accurate model of Rao.

Acknowledgments

The first author was partly supported by Grants from NSFC 11571067 and 11471140.

References

Marden, J. & Perlman, M. D. (1980). Invariant tests for means with covariates. *The Annals of Statistics*, *8*(1), 25-63. doi: 10.1214/aos/1176344890

Rao, C. R. (1946). Tests with discriminant functions in multivariate analysis. *Sankhya, 7*, 407-413.

Rao, C. R. (1949). On some problems arising out of discrimination with multiple characters. *Sankhya, 9*, 343-366.

Stein, C. (1956). The admissibility of Hotelling's T2-test. *The Annals of Mathematical Statistics, 27*(3), 616-623. doi: 10.1214/aoms/1177728171

Tsai, M. T. (2003). *Admissibility of invariant tests for means with covariates* (Technical Report No. 2003-05). Nankang, Taiwan: Institute of Statistical Science, Academia Sinica.