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BEGINNING ALGEBRA STUDENT’S CONCEPTUAL UNDERSTANDING OF VARIABLES IN THE CONTEXT OF SOLVING EQUATIONS: A CONCEPTUAL CHANGE INQUIRY.

by

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DISSERTATION

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Approved By:

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Advisor Date

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DEDICATION

I dedicate my dissertation work to my family, my friends, and my Lord and Savior Jesus Christ. Special feelings of gratitude to my loving daughter Daylon, without you I just might have finished this three years sooner! But no, really, your mere existence motivates me in everything that I do. You are my biggest cheerleader and my greatest inspiration. Your constant love and support has carried me on days when I could not even stand on my own. To my husband thank you for everything, including praying for me and your encouragement. Believe it or not, it kept me going. My sister, Shaniece who proofread this dissertation so many times that she probably has it memorized. To my mom, thanks for being strict and causing me to not always be the coolest kid at school, but definitely one of the most focused. Thanks for always encouraging all of your girls to be life-long learners. We had an awesome example.

I also dedicate this dissertation to the many people who have supported me and prayed for me throughout this process. Two special friends who have went through this process with me, Olu and Carey-Ann, thank you both. Our conversations and study sessions pushed me through. I can’t forget Ms. Nakia P., who was always sincerely supportive and lifted my spirits every time I heard her voice. You never know how powerful a kind word is to someone else.
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CHAPTER 1 INTRODUCTION

1.1 Background

The National Council of Teachers of Mathematics’ (NCTM) *Principles and Standards for School Mathematics* (2000) outline the essential components of a quality school mathematics program. The *Principles and Standards* call for and present a common foundation of mathematics to be learned by all students. It acknowledges the importance of a carefully organized system for assessing students’ learning and a program’s effectiveness. Six principles outlined in the document include equity, curriculum, teaching, learning, assessment, and technology. This study will focus on the curriculum and learning principles. A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well-articulated across the grades. Students must learn mathematics with understanding, actively building new knowledge from experience and previous knowledge.

The *Principles and Standards* (2000) also outline ten essential standards all students should learn to be successful in mathematics. Among these standards is algebra. Algebra is considered a gatekeeper for higher learning in science, technology, engineering, mathematics (STEM) and related careers (Asquith, Stephens, Knuth, & Alibi, 2007). Precluding students from entering into STEM fields are conceptual difficulties in elementary algebra that have led to impediments in understanding complex algebraic concepts (Woodward & Howard, 1994). Because our students are not grasping a concrete conceptual understanding of algebraic topics, they are falling further and further behind in mathematics. “In response to students’ inadequate understandings of and preparation in algebra, as well as in recognition of algebra’s role as a gatekeeper to future educational and employment opportunities, many in the mathematics education community have called for algebra reform” (Asquith et al., 2007, p. 250). Usiskin (1995) states that lack of algebraic knowledge limits student’s opportunities in much of the same way that lack of reading, writing, and arithmetic would. To further drive the point home of how important algebraic
knowledge and understanding is, “many states and districts have adopted ‘algebra-for-all’ policies” (Chazan, 2000, p. 59) in order to ensure that a college education is attainable for all students. According to the NCTM Curriculum and Evaluation Standards for School Mathematics (1989),

The 9-12 standards call for a shift in emphasis from a curriculum dominated by memorization of isolated facts and procedures, by proficiency with paper-and-pencil skills, to one that emphasizes conceptual understandings, multiple representations and connections, mathematical modeling, and mathematical problem solving (p. 125).

As an educator, I can say that difficulties in mathematics are common among students. I am a practicing secondary mathematics educator. I see advanced mathematics students struggle every day, because they never clearly conceptually understood the basic algebraic principles and objectives that they find themselves needing in higher mathematics and sciences. Many times this prevents them from pursuing STEM related majors in college. As a researcher and an educator, I would like to help alleviate some of these difficulties. One reason for these difficulties is that as the mathematics curriculum progresses, it becomes exponentially more abstract. The curriculum’s use of abstract symbols and more abstract concepts, makes it increasingly more difficult for the students to conceptualize the mathematics. Mathematics educational literature suggests that misconceptions for many students may endure for years. As King (1994) points out, proficiency in mathematics for all students was one of the goals outlined in the Goals 2000: Educate America Act. To quote Thorne (1989), “it is time for a major reevaluation of the content of the algebra curriculum and of the instructional strategies that are used in teaching algebra (p. 11).”

1.2 Problem Statement

Conceptual difficulty in algebra is primarily posed by symbolic and abstract representation of knowledge. Students’ move from calculating with numbers in their mathematics classes to solving equations with unknowns represented by alphabets poses a persistent threat to their understanding of algebra. If difficulties are not caught in the early stages of algebraic learning, a continual, inauthentic
understanding of the mathematics will allow students to use improper algorithms or repair strategies, eventually resulting in entrenched and deep-seated misconceptions, which may endure for years (Woodward & Howard, 1994). Thorne (1989) asserts that “students should see algebra as an aid for thinking rather than a bag of tricks” (p.12). Instruction that encourages and promotes the understanding of concepts, as opposed to the memorization and practice of algorithms, deters the many difficulties that teachers have witnessed of students in algebra. “Much research on students’ understanding of algebra has documented difficulties and misconceptions” (Asquith et al., 2007, p.249). Why so many students have difficulties with mathematics and algebra in particular has been published since the 1980s (Baroudi, 2006; MacGregor & Stacey, 1997; Mestre, 1989; Rosnick, 1981).

“Purposes for algebra are determined by, or are related to, different conceptions of algebra, which correlates with the difference in relative importance given to various uses of variables” (Usiskin, 1988, p. 11). Conceptual understanding of variables (MacGregor & Stacey, 1997; Woodbury, 2000) and solving equations (Woodbury, 2000) are the basic building blocks of algebra and yet they cause difficulty. Driscoll (1982) tells us that the difficulties in algebra that look to be engrained and the most challenging to amend are students’ understanding of what an equation is and what a variable is. More recently, studies suggest that the focus on computations instead of conceptual understanding makes the learning of algebra difficult (Baroudi, 2006; Wu, 1999). Further arguments for students’ difficulties include the fact that variables tend to be presented in textbooks modestly and usually in ways that are potentially misleading (Rosnick, 1982). Furthermore, teachers may think that students come fresh to algebra, not considering that they already have ideas about the uses of letters and other signs in familiar contexts and are actually incorporating such understandings (MacGregor & Stacey, 1997). Understanding the concept of a variable requires students to be aware of the fact that letters are used in mathematics as references to other things and it also requires students to realize that a letter represents a number (Russell, O’Dwyer & Miranda, 2009). Although students use variables to represent quantitative relationships prior to having
an algebra course, most are insecure about the appropriate use of variables (Swafford & Langrall, 2000). Variables are central, and the several ways they are used in algebra highlight the complexity of the concept and may partially explain the difficulty experienced by the learner (Coxford, 1988, p. viii). Teachers need to pay more attention to the concept of variables. To achieve the success that we crave in our Algebra 1 students, they (the students) need to become more comfortable with variables in numerical contexts before they begin a formal study of content and approach that need with greater attention (Leitzel, 1989, 29). One of the objectives in high school algebra is that students will be able to solve equations. Due to the difficulties that students have with variables, it is often impossible for students to be able to solve equations. When students are able to manipulate the terms in order to solve an equation, they often have no clue what the solution to the equation means. To some students, the traditional algebra course is seen as “meaningful manipulation” (Chazan, 59, 2000). Many studies will demonstrate that there does not appear to be a simple elucidation or a quick resolution to the difficulties students encounter in relation to variables and solving equations (Rosnick, 1981). The literature, however, describes some possible sources and methods for dealing with students’ difficulties (Kieran, 1991; Tirosh, Even, & Robinson 1998; Wagner, 1983).

Amongst the NCTM (2000) standards for school mathematics is representation. The representation standard states that mathematics programs should enable students to use variables to select, apply, and translate amongst the various branches of mathematics to solve problems. In order to ensure this, students need more practice using variables and they need more concrete real-world examples that add to their knowledge-base of variables.

Although there are several studies on students’ difficulties with variables and solving equations (Asquith et al, 2007; Kieran, 1991; MacGregor & Stacey, 1997; NCTM, 2000; Rosnick, 1981; Rosnick, 1982; Swafford & Langrall, 2000), there are few studies juxtaposing the two concepts. This study responds to the call of previous studies for additional research that discusses the misconceptions related to the
concept of a variable (Baroudi, 2006; Rosnick, 1981), and investigates the effect of innovative curricula on enhancing students’ abilities to use variables and generalize functional relationships (Swafford & Langrall, 2000).

More and more the stakes are being raised for our students to be exposed to an increased amount of mathematics (Swafford & Langrall, 2000). Misconceptions that are acquired early in algebra will follow students throughout higher mathematics preventing them from fully conceptualizing the learning. For that reason, this study is intended to tackle the issue of why and how students’ attain these misconceptions and it will attempt to answer the question, how can teachers help students gain a greater conceptual understanding of variables in juxtaposition with solving equations?

1.3 Objectives and Questions

The purpose of this classroom-based study is to design a curricular intervention for a beginning Algebra I course and document, analyze, and interpret conceptual changes beginning algebra students will experience. The study will also critically analyze the mediation process and the interpretive discourse that results in conceptual changes. My aim is to take an empirical approach to learning, and to clarify the actions of students when they engage in mediation activities and the nature of the learning that they attain (Saljo, 1997, 188). Thus, one objective is outlined in this study. Within this objective, five research questions are stated.

To qualitatively document, analyze, and interpret beginning Algebra I students’ changes in understanding of variables and solving equations.

1. What are beginning Algebra 1 students’ prior understandings of variables?

2. What are beginning Algebra 1 students’ prior understandings of solving equations?

3. What are the various reason’s for these students’ understandings?

4. What is the discourse or mediation activity that leads to students’ understanding?
What are the beginning Algebra I students post-conceptions about variables and solving equations?

Rationale for Objective and Question Set: It is important to understand students’ pre-intervention conceptual understandings of the concept of variable and solving equations. It is just as important to understand and document their post-intervention conceptual understandings and to document and measure conceptual change based upon the curriculum and the classroom discourse. Knowing the various reasons for students’ understandings allows me to use these reasons and experiences in the mediation activities and in the classroom discourse. Documenting the mediation activities that lead to students’ authentic value-added understanding is vital to the current research and knowledge pool in mathematics education.

1.4 Significance of the Study

This study will address the issues that many algebra students’ have with the concept of variable, the concept of solving equations, and the juxtaposition of the two. This study is significant for teachers (both in-service and pre-service), curriculum developers, students, and the mathematics education community. There have been multiple studies on variables and many studies on solving equations (Asquith et al, 2007; MacGregor & Stacey, 1997; Swafford & Langrall, 2000.) However, there is not a significant amount of literature or studies of variables and solving equations in juxtaposition with each other. This study will produce a meticulous and detailed literature review that examines why students are having difficulties with these concepts and how to alleviate them. This analysis will provide a source that can assist teachers in detecting and correcting incorrect conceptual understandings when dealing with variables and solving equations. Documenting, interpreting, and analyzing the process of conceptual change learning while designing and measuring the achievement of the supplemented curriculum will help to enable curriculum developers to design similar curriculums. It will also give the mathematics
education community and educational psychologists insight into how students gain their conceptions and how we can intrude misconceptions. Most importantly, if the study turns out to have a positive, significant effect on student achievement, it can be used as a model for how to guide instruction on not only the conceptual research in the study, but all algebraic concepts.

Previous studies have called for additional research investigating the effect of innovative curricula on enhancing students’ abilities to use variables and generalize functional relationships (Swafford & Langrall, 2000; Baroudi, 2006.) Other researchers have asked for a study that discusses the misconceptions related to the concept of a variable (Baroudi, 2006.) Rosnick (1981) implied in his study that students needed to develop a better understanding of the basic concept of variable. This research will contribute to the existing field of knowledge by helping to formulate the direction of future curriculum reform in mathematics, specifically algebra.

**1.5 Overview of the Study**

Based on the empirical research, Chapter 1 has argued for documenting, analyzing, and interpreting students’ conceptions of variables and solving equations in algebra in order to characterize conceptual changes. The first chapter also outlines the research objective and questions.

Chapter 2 at the onset will provide a detailed account of current research on students’ conceptual understanding of the concept of variable and solving equations in algebra. Then it will delineate phenomenography as the theoretical framework that will guide the research questions. It will also outline conceptual change learning theory as the frame of the study. This chapter will also establish a link between research and the study at hand to demonstrate a need for the study.

Chapter 3 will critically analyze the algebra curricular intervention based on the following questions:

1. What educational purposes should I seek to attain through the curriculum?

2. What educational experiences can I provide that are likely to attain these purposes?
3. How can these educational experiences be effectively organized?

4. How can I determine whether these purposes are being attained (Tyler, 1949, p. 1)?

The context for the study, rationale for the methodology, the population, and the sample will all be discussed in Chapter 3. This chapter will then introduce phenomenography and discourse analysis as the analytical frameworks for characterizing conceptual change. Chapter 3 will also discuss a tool for subject matter-based discourse analysis. Next, ethical considerations will be discussed in Chapter 3. This chapter will also address validity and reliability issues within the study.

Chapter 4 will discuss the interpretation of beginning Algebra 1 students’ prior and post-intervention understanding of the concept of variables and solving equations. It will highlight various sources of students’ reasoning that give birth to the conceptions. It will track the conceptual changes while students construct deep understanding of the concept of variables and solving equations, as well as the juxtaposition of the two together. It will discuss the factors that led to the students’ understanding of the mathematics. This chapter will develop answers to the research question that address the turning point that leads to the students’ understanding.

Chapter 4 will also present a discussion of the mediation process and characterize a critical analysis of the interpretative discourse that leads to student learning. This chapter will also identify and document factors that facilitate and/or impede the implementation of the curriculum.

Chapter 4 will reveal if the supplementary intervention/mediation activities, as well as the classroom discourse that helped to heighten the achievement rate of students in algebra, in particular in regards to the concept of variable and solving equations and the juxtaposition of the two.

Chapter 5 will conclude the dissertation with a research summary. It will state and answer each research question. The chapter will provide an interpretation of all the data. It will highlight and discuss issues with the research based on evidence presented in Chapter 4. Implications for conceptual change learning, algebra curriculum, pedagogy, learning, and phenomenography as an assessment tool for
learning will be discussed. This chapter will also discuss the study’s implications on curriculum, instruction, research, and education policy. Chapter 5 will conclude with recommendations for further research.
CHAPTER 2 REVIEW OF LITERATURE

2.1 Introduction

The purpose of this research is to document, interpret, and analyze beginning Algebra I students' pre-, evolving, and post-understandings of the concepts of variables and of solving equations. This chapter constitutes the literature review of relevant conceptual change and empirical studies that frame the study at hand, which involves the documentation, interpretation, and analysis of beginning Algebra I students' pre- and post-instructional conceptions of variables, solving equations, and juxtaposition of the two. A type of conceptual change theory known as the variation learning theory or phenomenography (Marton & Booth, 1997; Marton & Tsui, 2004) will be discussed in detail to demonstrate how this theory of learning applies to mathematical learning. To justify phenomenography as a framework for mathematical learning in the Algebra class, at the outset, this chapter will critically review studies on mathematical learning, especially those that are based on conceptual change theory. This chapter will identify and discuss theories, vocabulary, and vital variables that are relevant to the study.

There are very few empirical studies on students' conceptual understanding of variables and solving equations using these same particular theoretical frameworks. In this chapter the researcher will critically analyze past studies and attempt to make connections between past research and this study.

2.2 Theoretical Frameworks

The theoretical frameworks for this study are:

a. Conceptual Change Inquiry Learning, and

b. Variation Theory of Learning (Phenomenography).

2.2.1 Conceptual Change Inquiry Learning

Marton and Booth (1997) imply that everyone learns differently. They state that some people have learned to do things better and some people have learned to do things worse. If this is true, then
this study will help to find the best way to help students gain a conceptual understanding and to learn algebra. In that sense, we can call this study a study of learning and conceptual understanding. Without making any assumptions, as other researchers have (Marton & Booth, 1997, p.15), such as: some students are smarter than others; some students are better motivated; and some students work harder; or “students do not feel the need to know algebra (Chazan, 2000, p.2),” this study will aid in finding better ways to help students to better conceptualize the two basic algebra concepts of variables and solving equations.

A plethora of research and many studies have been done on students’ understanding of algebra (Asquith et al, 2007; Baroudi, 2006; Driscoll, 1982; Chazan, 2000; MacGregor & Stacey, 1997; Mestre, 1989; Rosnick, 1981; Woodbury, 2000; Woodward et al, 1994; Wu, 1999). These studies document the many difficulties and misconceptions that students’ have (Asquith et al., 2007). Because of their minimal understanding of both teachers and students, most algebra students simply memorize algebraic rules and arithmetic procedures and they ultimately come to believe that these rules and procedures represent the fundamental nature of algebra (Chazan, 2000; Kieran, 1992). Not truly having a concrete understanding on the basic algebraic concepts prevents students from later success in higher mathematics, because they do not encompass that much needed conceptualization in their mathematical toolbox.

The concept of variable and solving equations is a common discussion point in mathematics education. Researchers have used a variety of methods, mostly computer-based, to help instructors to either teach the concepts more efficiently, or to identify and/or categorize the common misconceptions in order to dispose of them before they pose a problem for students (Asquith et al, 2007; Baroudi, 2006; Rosnick, 1981; Russell et al., 2009; Woodward et al., 1994). This is due to the fact that technology can provide students with access to multiple approaches (Heid & Blume, 2008, 62). The concepts that will be the focus of this study are as Wagner puts it, “easy to use, but hard to understand” (Wagner, 1983, 474.) Rajano (1996) did an analysis study based on mathematical thinking in which, the students in the study
were given a pre-test and a post-test. The students were also individually interviewed. Rajano collected the data in the study using audio and video recordings of the individual interviews. Printouts of computer work (done in a spreadsheet environment) were collected. Handwritten paper and pencil work, and notes on observed teaching sessions are both examples of data collected in the study. The results of the study suggest that incorporating problem solving into the teaching of symbolic algebra is more meaningful and value-added for the student. It should be used to introduce variable Algebra, not as a concluding factor added in after the fact as is often done. The study also suggests that researchers consider the processes of change that intervene in the students’ passage from arithmetic to algebraic thought. That is, researchers should take note of where the conceptual change from arithmetic thinking to algebraic thinking starts to take place in a student. This is where this study will serve as an extension of that ideal. It is important to know where in the curriculum a student starts to change his or her way of thinking, and why the change occurs at that point. If researchers can isolate that point in the curriculum and figure out why students change their thinking at this point, then maybe we can duplicate such a critical point in other parts of the curriculum. This study will pick up where Rajano (1996) left off and use problem solving to introduce variable Algebra to determine whether doing so will increase conceptual understanding of the concept of variable and solving equations.

Mitchell and Miller (1995) also support this view that mathematics has to be meaningful, relevant, and real for students to be able to truly conceptualize it. They contend that mathematics has to be an enjoyable extension of the children’s lives. There is an abundance of research that backs this up, Schoenfeld and Arcavi (1988) state that mathematical meaning is often determined by context rather than by formal arithmetic rules. This study will attempt to use story-problems that are meaningful and relevant to the students to introduce the two concepts.

Booth (1988) conducted a study in the United Kingdom from 1980 to 1983 on the Strategies and Errors in Secondary Mathematics, the SESM project. The students that participated in the study were in
grades eight to ten (ages thirteen to sixteen). Despite differences in age and experience in algebra, similar errors appeared at each grade level. Interviews with the students who were making errors showed that many of the errors could be traced back to four specific categories, one of which is the meaning of letters and variables. The findings of this study imply that many students at the beginning stages of algebra have trouble going from arithmetic to algebra and using variables. Students often assume that different variables have different numerical values. The findings of this study also imply that these misconceptions gained at the beginning stages of algebra are carried over into other areas of mathematics.

Kieran (1988) preludes this study with research of her own. In her study she states that algebra is often called “generalized arithmetic” (p. 91). This implies that the arithmetic operations are generalized to expressions involving variables. However, this is not necessarily true. The operation signs in an equation are not necessarily the operations to be used in solving the equation. In fact, as Kieran points out, a major difference between arithmetic and algebra is this distinction between the operations that are used in the process of solving algebraic equations and the given operations of these equations. Kieran conducted a three-month teaching experiment which attempted to answer the question of how a student views equations and equation solving in the initial period of learning algebra. It was concluded that beginning algebra students tend to use one of two different approaches to solving equations when variables are involved.

The two approaches were called the “arithmetic approach” and the “algebraic approach.” Using the arithmetic approach, students focused on the given operations when trying to solve equations. The majority of the students who used this approach to solve equations viewed the variable as an unknown number. Thus, this group of students viewed the variable as part of the numerical relationship of the equation. When given an equation to solve, students in the arithmetic group used the given operations, substituting different values for the variable until they found a value that made the equation true.
Students using the algebraic approach focused on inverse operations. “This approach is characterized by the equation-solving procedure of “transposing terms to the other side (p. 92).” The students in the algebra group seemed to think that the variable only had meaning once its associated value was found. This created a problem for the students when dealing with algebraic expressions, as opposed to equations because there are no inverse operations to apply. When solving equations this group used the inverses of the given operations and solved by transposing terms to the opposite side of the equal sign.

Kierans’ study proved that student’s concepts can be reconstructed. In her study, the researchers were successful in reconstructing some of the students’ views of variables in an equation to include the concept of variables as numbers within the given sequence of operations. This type of mathematical learning takes time and these algebraic concepts should begin being reinforced starting in elementary school and continued through the study of Algebra.

Marton and others (2004) state that “learning is always the acquired knowledge of something (p. 5).” In other words, learning is the progression of not being capable of something to becoming capable of that something as a result of having certain experiences (Marton et al, 2004, p.5). It is believed that learning is the result of the relationship between what the student is taught and the students’ pre-existing conceptions (Poster, Strike, Hewson, & Gertzog, 1982). Conceptual change involves the actions taking place that causes the “pre-instructional conceptual structures of the learner to be fundamentally restructured in order to allow understanding of the intended knowledge” (Duit, 2003, p. 673), in this case the acquisition of the concept of variable and solving equations. Conceptual change takes the learner from what Skemp (1976) would call an instrumental understanding to a relational understanding. For example, a study about the ideas regarding symbolism that students bring with them to algebra class (MacGregor, 1997), outlined some of the causes of students’ common misunderstandings about the concept of variable and other symbols used in algebra. The conclusion of the study was that students had pre-instructional
interpretations about the concept of variable. These interpretations were often incorrect and needed to be restructured to allow for true understanding of the concepts. These pre-interpretations can only be restructured through clearly planned instructional techniques that change the previous thoughts about the concepts. Using conceptual change inquiry learning to frame the study, this study will use best practices of mediation activities to aid the students in conceptualizing the concepts that will be studied. This section will briefly review the literature on instructing for conceptual change.

Another study on students’ misconceptions in mathematics (Mestre, 1989) tells how students enter the mathematics classroom with previously constructed theories. Most of the time, these “theories” are incomplete or completely wrong. In fact, Mestre (1989) found that students who overcome a misconception after ordinary instruction often return to it only a short time later. This is why it is vital for instructors to employ conceptual change learning. Conceptual change learning helps the learner to completely restructure her or his thinking about the concept.

Lochhead and Mestre (1988) outlined a three step technique for conceptual change. The technique uses a line of questioning for qualitative, quantitative, and conceptual understanding. The three-step technique used in the study forced the students to participate actively in resolving the conflict by supplanting the misconception with appropriate conceptual understanding (Lochhead et al., 1988, 133). Each step asked questions to resolve a previous, misconception while probing for new knowledge. This line of questioning continues until the line of reasoning is exhausted. This inductive approach opens the classroom for discourse. This classroom discourse among the students, leads them to restructure their thinking and subsequently to understand the intended knowledge.

The use of teaching strategies associated with the conceptual change model was reported in a study by Smith, Blakeslee, & Anderson (1993). This study examined thirteen 7th-grade teachers’ instructional strategies. The results of the study implied that the instructional materials aided more in the acceptance of the conceptual change, rather than the actual teaching. The results of the study clearly
indicated that the conceptual change learning strategies improved student achievement. They showed that the use of conceptual change strategies by teachers was associated with higher student performance on tests that were designed to access conceptual change learning.

The conceptual change theoretical framework has been used many times in the science education research. It has not been popular in mathematics education research in the past. However, Vamvakoussi and Vosnайдou (2004) conducted research to show that the conceptual change approach was relevant in mathematics education. One of the findings of their research was that in order to use the conceptual change approach in mathematics, there does not need to be a change in mathematical theories, just a new development of mathematical concepts.

Vamvakoussi and Vosnайдou (2004) investigated the development of students’ ideas about rational numbers using the conceptual change approach. They used 16 ninth grade students from the same class, each of whom was familiar with rational number properties. Though the process was long, in the end, the students had a greater, deeper, and higher level of understanding of rational numbers.

Chazan (2000) did a study on his attempt to engage lower-track Algebra I students in the study of algebra. He wanted to help students to be able to see algebra in the world around them. In his study, Chazan took notice that the algebra textbooks available to him as a teacher were not useful resources for helping students to learn and understand algebra in a more real-world applicable manner. His study pointed out the relationship between the teacher’s understanding and the outcome of the students’ understanding of the subject matter.

Each of the studies described in this section outlined common misconceptions students often have about the material to be learned. They each used techniques that highlighted contradictions in the students’ understanding or misunderstanding. During this inquiry process, students become dissatisfied with their existing conceptions and reconstruct their understanding of the concept, leading to a conceptual change of their understanding. Although all of the studies acknowledge that this process is
long and time consuming, it can lead to meaningful classroom mathematical discourse, and it could save the time spent re-teaching more difficult concepts that follow.

2.2.2 Variation Theory Learning: Phenomenography

Using Variation Theory of Learning, a form of conceptual change, this study is designed to understand ninth grade Algebra I students’ pre-, evolving, and post-understandings of the concept of variable and solving equations. Marton (1981) introduced this version of conceptual change. He argues that knowledge derives from people’s experience of various aspects of the world. He named this new Variation Theory of Learning, Phenomenography. Figure 1, is a graphical representation of Martons argument (Mann, n.d.).

![Figure 1-Focus of Phenomenographic Research](image)
The Effects of Common Knowledge Construction Model Sequence of Lessons on Science Achievement and Relational Conceptual Change (Ebenezer et al, 2010) is a very good example of a phenomenographic research study. In this article Ebenezer and others explain that Variation Theory of Learning is a subset of conceptual change theory.

As I began to read literature on Variation Theory of Learning and Phenomenography, I asked myself “What is the motivation behind using the phenomenographic research approach?” The simplest way to explain it is by comparing two students. If two students were in the same mathematics class, and therefore had to take the exact same test, they would have the same problem, and they have the same motivation for doing well on the test. In short, all things compared, both students should get the exact same score on the test. However, the students do quite differently. One fails the story problem section of the test, and the other passes. This tells us that although the two students were equally prepared, they experienced the problems very differently. Therefore, they answered the story problems very differently. Hence, in order for the mathematics teacher to understand why these two students received very different grades, he or she has to understand the way in which the students experienced the story problems. This is the motivation for phenomenographic research. It explores the various ways that student’s experience the learning, allowing the researchers understand what life experience contribute to conceptual understanding. Purdie and Hattie (2002), remind us that “phenomenography is not concerned with evaluations of conceptions-their ‘rightness’ or ‘wrongness’; it is just as interested in mistaken conceptions of reality” (p. 19). Keeping in mind that one of the frameworks of this study is Variation Theory of Learning, the objective of the study is to explore different kinds of understanding and the various ways students come to understand solving equations and the concept of variable (Marton & Booth, 1997). The objective is not necessary to find out how individual students understand them. This research looks at learning from a phenomenographical perspective. This means that my aim is to “study and describe variations in people’s experiences” (Marton & Booth, 1997, p. 110) of learning the concepts
of variable and solving equations. In all the studies that I reviewed for this research, the focus was on the variation of ways that the subjects of the studies experience things, in particular learning.

Marton and Booth (1997) tell us that phenomenography acts as a catalyst or a philosophical approach to identifying and mediating certain types of research questions. It is particularly focused on questions that pertain to learning and conceptual understanding in an educational setting. It is mostly found in studies in the core subjects of language arts and science. This study will add to the small library of research using Variation Learning Theory in the core subject of mathematics.

Learning is associated with a change in judgment. This type of learning involves a change in the portion of the phenomenon of which the learner has a central understanding (Pang & Marton, 2005, p. 162). Judgment is based on experience. The best judgment is based on a variation of experiences. Therefore, the instructor should give students a variety of learning experiences for one learning objective, so that students can make a judgment or “discern” the learning (Marton & Tsui, 2004). Pang (2005) gives a great example of this, by pointing out that one could not discern ethnicity if there was only one ethnic group on Earth.

As stated above, learning is associated with a change in judgment. However, when asked, students have very different conceptions of what learning is. Purdie and Hattie (2002) developed an instrument designed to assess student’s conceptions of learning. Using a phenomenographic approach, they developed an instrument from qualitative data obtained from high school students. They identified six different conceptions of learning.

Vikstroms (2008) conducted a study using variation learning as the framework. The study investigated the various ways that students consider a particular concept. In particular, they investigated the ways in which 7 to 12 year olds understand cellular respiration and photosynthesis against the background of their teachers’ teaching. By focusing on the various ways that students experience learning, the teachers in this study were able to develop their pedagogical skills in a much more practical way than
if the study had focused on the activities used to teach the concepts. The study has implications for the relationship between teachers’ competence in the subject matter, teachers’ pedagogical styles, and students’ learning.

Pang et al (2005) conducted a study in which the object of study was how to make use of the variation of learning theory to design learning environments that create specific patterns of variation and invariance, and which thereby bring about student learning. The results and findings of this study provided support for the use of variation theory of learning in this study.

Lochheads’ (1988) study, though not stated as a phenomenographic study but more of a conceptual change study as stated earlier in this chapter, used a phenomenographic approach to mending misconceptions that beginning algebra students have. Their research used student interviews and student work as the primary methods of data collection. A three-step questioning approach was utilized where they first question for qualitative understanding, then quantitative understanding, and finally they question for conceptual understanding. Through this process it is easy for the interviewer as well as the student being interviewed to point out misconceptions and reach resolutions. They also use a classification system to identify different classes of problems that students tend to have difficulties with.

Similar to Lochheads’ work, the Chazan (2000) study was a conceptual change study as stated earlier in this chapter, but in my opinion, used a phenomenographic approach to help students become intellectually engaged in their studies. His research focused on engaging students in the study of algebra by the teachers relearning it themselves and learning to see it in the world that students experience (p. xii). This study is different as it looks at and focuses on the conceptual change of both the students and the teachers. It discusses and analyzes experiences of both the teachers and the students. This study was very different from the rest of the studies that were reviewed in that its results focused more on the teacher rather than the student. It yielded implications that challenge the community of secondary
mathematics teachers. Mathematics teachers can enormously benefit from a medium for the discussion of ideas and issues in curriculum, teaching, and learning.

A conception of mathematics questionnaire was designed and developed in the study entitled “University Mathematics Student’ Conceptions of Mathematics.” Crawford, Gordon, Nicholas, and Prosser, (1998b) intended for this questionnaire to provide some insight into the nature of students’ conceptions of the subject matter they were studying in mathematics. A disjointed and fragmented conception was found to be associated with a surface approach and an interconnected and cohesive conception was found to be associated with a more profound and problem-based approach to studying mathematics.

This particular study is relevant because while there has been substantial research into student’s prior knowledge of subject matter, and how that relates to their approaches to study and subsequent learning outcomes, there has been little or no research into students’ prior understanding of the nature of mathematics (Crawford et al, 1998b). This research describes an approach to investigating this issue. The study shows that there is a direct correlation between the manner in which students approach their studies and students’ prior conceptions.

One study investigated three aspects of learning mathematics: intention, approach, and outcome (Reid, Wood, Smith, & Petocz, 2005). This study also used a phemenographic approach. In this study students also began by answering questions about their experiences with learning mathematics. The researchers continued to ask probing questions until the line of questioning exhausted itself. The researchers analyzed the transcripts of the interviews and divided their findings into three consistent themes found throughout all of the interviews: (1) intention, (2) approach, and (3) outcome.

This study had a lot of implications for mathematics educators. One of the suggestions is that teachers develop learning materials that try to engage students at a broader level of mathematical thinking and learning. Another suggestion is for teachers to offer a variety of learning situations and
environments in order to encourage students to think more conceptually broader or narrower. By asking reflective questions to go along with their lessons, teachers can give students the opportunity to think about and discuss their own learning. The study itself and the findings of the study both implied the vitality of mathematical discourse.

Another study focuses on how students’ prior understanding of the nature of mathematics relates to their perceptions of learning contexts, approaches to study and subsequent outcomes (Crawford, Gordon, Nicholas, & Prosser, 1998(a)). The researchers looked at the variation in the way mathematics students conceive of the nature of the subject matter they are studying, and how their conceptions of the subject matter relate to their other experiences of studying and to the outcomes of their studies. The Crawford and others study suggests that students enter a learning situation with a vast storehouse of former experiences and understandings, and that these are in continuous interaction with both their perceptions of the context and approaches while studying the subject and their post experiences and understandings. In short, it is suggested that the past, present, and future awareness and learning activity are in continuous interaction with each other.

In all the studies that I reviewed for this research, the focus was on the variation of ways that students experience things, in particular learning. The studies reviewed in this section all investigated the various ways that students consider a particular concept. Each of the studies used audio-recorded interviews as their main data collection method, and they also all use a classification system to identify different classes of misconceptions that students have. There are not many studies on this topic using variation learning theory as a theoretical framework. This study is needed for that very reason. This topic has a dimension that has not previously been tapped. This study, as opposed to others, explores the nature of learning or more specifically learning mathematics. It focuses on conceptual understanding of two specific concepts, variables and solving equations.

2.3 Empirical Findings
Earlier in this chapter, conceptual change was defined as the actions taking place that cause the “pre-instructional conceptual structures of the learner to be fundamentally restructured in order to allow understanding of the intended knowledge” (Duit, 2003, p. 673). While Wheeler (1996b) did not directly state that the use of conceptual change theory was the best way to conduct a study on the concept of variables and solving equations, he states that “if the mathematical developments that took place in history are trustworthy guides to the development of mathematical instruction, then it seems clear that the introduction of algebra should follow the problem solving approach and focus on the solution of equations” (Wheeler, 1996b, 147). By the same token, Chazan (2000) implies that using a more functions-based approach to Algebra I can serve as a catalyst to students’ gaining a true conceptual understanding. The results and empirical findings of the research and mathematical developments that both Chazan and Wheeler talk about provide support for the use of conceptual change theory as a framework in this study. They also provide sufficient evidence that there is a need for more research on the concept of solving equations and thus provided evidence of the need of this study.

2.4 Chapter Summary

In conclusion, this chapter discussed in detail Conceptual Change, Inquiry Learning, and Variation Theory of Learning as the theoretical frameworks that will guide the study. The need for the use of these frameworks in the study was justified in this chapter. It summarized a review of the empirical literature explored for this study. The limitations of the literature that was reviewed were discussed and linked back to the present study. The chapter established a link between the previously completed research on the topic and this study. The guiding theoretical frameworks were explained and justified. The literature review is current and focused on work most relevant to this study.
CHAPTER 3 METHODOLOGY

3.1 Introduction

The objective of this study was to focus on the achievement and development of beginning Algebra I students’ conceptual understanding of variables and solving equations through curriculum implementation and classroom mathematical discourse. The study first explored students’ pre-intervention knowledge of variables and solving equations. Students then engaged in inquiry-based mediation activities designed to spark mathematical discourse. These mediation activities were aligned with the mathematics content standards outlined in the Michigan High School Content Expectations (HSCEs), the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (2000), and the Common Core State Standards (2010). In order to identify how and why students’ develop a conceptual change regarding variables and solving equations, the researcher traced the roots of students’ experiences. Following the intervention, the researcher will explored the students’ post-intervention understandings of the content.

The study investigated the effects of a conceptual change, inquiry-based learning sequence on students’ understanding of variables and solving equations. Additionally, the study tracked students’ prior- and post-intervention understanding of variables and solving equations.

This chapter also details phenomenography as an analytical tool. Based on the conceptual change inquiry framework, the chapter also describes the context of the study. It provides a justification for the enhancements to the curriculum that will be developed for the study. This chapter details the methods and procedures of data collection and analysis that were used to investigate the research questions stated Chapter 1. Ethical considerations as well as reliability and validity issues that are relevant to the study are also outlined and discussed in this chapter.
3.2 Methodological Frameworks

This study will make use of two analytical tools. These tools are as follows:

1. Phenomenography for describing conceptual changes.
2. Discourse Analysis for tracking student learning and the sourcing of their understanding, as well as for studying the character of their conversation.

3.2.1 Phenomenography

One often wonders why two students in the same class who are equally prepared and motivated do not score the same on a test. One response would be that they experienced the problems very differently. Therefore, they answered the problems very differently. Therefore, in order to understand the differences in scores, the teacher must understand the ways in which the students experienced the problems. Such scenarios are the motivation for phenomenographic research. Keeping in mind that the objective of the study was to explore different kinds of understanding and the various ways that students understand solving equations and the concept of variable (Marton, 1997), the objective was not necessarily to find out how individual students understand them. This research looks at learning from a phenomenographical perspective, meaning that one of my aims was to “study and describe variations in people’s experiences” (Marton and Booth, 1997, p110) of learning the concepts of variable and solving equations, along with finding out what types of activities and experiences lead to understanding.

Hasselgren tells us that there are five recognizable context-types of phenomenography research:

1. Experimental;
2. Discursive;
3. Naturalistic;
4. Hermeneutic; and
This study borrows the Marton and Booth (1997) version of phenomenography, which is an example of phenomenological, hermeneutic phenomenography (Saljo, 1997, 1975). Orgill discusses how phenomenographic conclusions can aid instructors in understanding how students gain an understanding. Phenomenography allows the researcher to categorize the variety of different ways in which people understand and experience the same thing (as cited in Cousin, 2009). Algebra is usually associated with numbers and functions on numbers. However, Mason (1996) says that there are various ways in which symbolic expression of generality can be studied. The reason for emphasizing expression of generality in number patterns is only to provide experiences which highlight the process. My objective was to study and describe variations in students’ experiences (Marton & Booth, 1997, p. 110) as they change their understanding of variables and solving equations. Phenomenography as an analytical tool aligns with the following NCTM (2000) statement; “Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge” (p. 16). To achieve this educational goal in mathematics, phenomenography is the best analytical tool, because this study sought to understand how students learn and gain an authentic, concrete understanding of variables and solving equations through a conceptual change inquiry. It sought to comprehend what experiences and prior knowledge students used to gain a new understanding. Furthermore, NCTM (2000) states that “effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well” (pg. 16). Phenomenographic studies take place through the interaction between students’, their teacher, and the content (Entwistle, 1997).

Phenomenography is “research which aims at description, analysis, and understanding of experiences; that research which is directed towards experiential description” (Marton, 1981, p. 180). “The aim of phenomenography is to take students’ differing experiences and understandings and characterize them” (Ashworth, & Lucas, 2000, p. 297). In phenomenography, it is the students’ conceptions, understandings, and experiences that should be revealed, not the expectations of the
researcher. Hasselgren (1997), states that the better the analysis of the students’ experiences are, the better we are able to understand learning, and teaching. Most phenomenographic studies thus far have been done in higher education, not K-12 (Entwistle, 1997). “Phenomenography is an experimental perspective of conceptual change, a view of conceptions of phenomena as relational (i.e. describing relations between the conceptualizing individual and the conceptualized phenomena)” (Ebenezer, 2009, no page number available.)

Using phenomenography, this study was designed to understand students’ conceptions of variables and solving equations and their relational conceptual change. I consider this study to be a contribution to a phenomenography of mathematics education (Marton & Booth, 1997).

3.2.2 Discourse Analysis

Vikstrom (2008) states that according to variation theory, the definition of learning is to become able to understand in a new way. A new relationship is then produced from this understanding between the learner and the concept that is being learned. In this theory, language is the central tool for building relationships between the learner and the concept that is being learned.

With that in mind, this study also sought to understand what educational strategies guide meaningful discourse, which leads to conceptual understanding. Mason (1996) suggests that the key to student understanding is to get students to engage in not only mathematical thinking, but mathematical speaking and listening. I believe Mason is trying to say that real mathematical discourse is the key to conceptual understanding of solving algebraic equations with variables. Therefore, this study also borrowed Gee’s (1999) version of discourse analysis. “Discourse analysis is analysis of spoken and written language as it is used to enact social and cultural perspectives and identities” (Gee, 1999, p. 2). In short it is the linguistic breakdown of a continuing course of communication. My objective was to study, analyze, and describe how the classroom discourse between students and other students, and students and teacher is used to build new conceptual knowledge of variables and solving equations.
Driscoll’s (1982) work supports using discourse analysis as a methodological framework for this type of conceptual change inquiry. He implies that teaching and learning mathematics is realized through human interaction, through the listening and speaking of the mathematics.

Gee (2005) explains that a discourse analysis is based on the details of speech that are considered relevant in the situation and relevant to the arguments that the discourse analysis is making. With the recent advances in technology and the availability of more modern complex recording and computer transcribing equipment, it is possible to get incredibly detailed records of speech that would not be possible to record otherwise. These details include, but are not limited to pauses and changes in pitch of the voice, as well as the speed at which the speaker is talking. In order to capture and analyze the classroom discourse, the class will be observed and recorded. The recordings were transcribed. The transcripts were used as tools of inquiry for the study, as well as research and analytical tools. Therefore, the transcript is a theoretical entity (Gee, 2005, 106). It is not a separate part of the study that stands alone; it is a part of the analysis. Any speech data can be transcribed in more or less detailed ways such that we get a continuum of possible transcripts ranging from very detailed and narrow transcripts, to much less detailed and broad transcripts. For the purposes of this study, the transcripts will be very narrow and detailed. This will be done purposefully, so as not to miss any detail.

3.3 Context of Study

The context of the study was a middle school in Southeastern Michigan. The school had a population of 716 students during the 2013-2014 school year. The ethnic make-up of the school was 77% African-American, 16% Caucasian, 3% Hispanic, and 4% other. The participants of the study were 31 beginning algebra students. The researcher provided the teacher with mediation activities that supplemented and complemented the existing curriculum. The researcher also provided the teacher with guiding questions to launch and guide the mathematical discourse.
Michigan students were particularly selected based upon apparent need. Research and data analysis indicates that Michigan students are not on pace to catch up with their peers in top-performing states (Higgins, 2013). The National Assessment of Educational Progress (NAEP) exam is a national exam given to a representative sample of students in each state. The scores of the NAEP exam from 2003 to 2011 found that Michigan students ranked 41st out of 50 states in overall improvement in eighth grade math (Higgins, 2013). Furthermore, the Michigan ACT Profile Report indicated that 46 percent of students graduating in 2012 nationally hit the benchmark score, while only 36 percent of Michigan 2012 graduates did. These percentages are based upon the ACT Mathematics Benchmark score of 22, which is an indicator or whether students are ready for college level algebra. The benchmark score is the minimum score needed on an ACT subject test to indicate a 50 percent chance of obtaining a B or higher or about a 75 percent chance of obtaining a C or higher in the corresponding credit-bearing college course.

The state percentage of students meeting the benchmarks has been consistently lower than the national percentage in years past. Higgins (2013) quoted Robert Floden, the co-directed of the Education Policy Center at Michigan State University, when he stated that “What’s a concern is that the whole country is doing better and Michigan isn’t keeping pace.”

3.3.1 The Researcher

The researcher is a doctoral student at a large, research-intensive university in Michigan majoring in Curriculum and Instruction with a concentration in Mathematics Education and a minor cognate in Educational Leadership. During the time of the study, she was a secondary mathematics teacher in a small school district, and has taught Algebra One for twelve years.

3.3.2 The Students

The students who were involved in the study were beginning Algebra I students. They ranged in age between 12 and 15. There were 31 students engaged in this study. In previous years, personal
communications with students has suggested that the majority of the students in this grade, at this level, in this district, have not been very successful in mathematics throughout their years in middle school (middle school is 6th, 7th, and 8th grade). They appeared to have very little motivation for trying to excel in their mathematics classes.

3.4 Curriculum

“In a coherent curriculum, mathematical ideas are linked to and build on one another so that students' understanding and knowledge deepens and their ability to apply mathematics expands” (NCTM, 2000, p. 15). The curriculum used was the textbook that the teacher was already using and familiar with. The researcher and teacher will amended that curriculum and provide additional instructional strategies to enhance the learning.

Tyler (1945) proposed a set of questions that should be asked when writing, viewing, analyzing, and/or interpreting a curriculum. Those questions are as follows:

1. What educational purposes should the curriculum writer(s) seek to attain?
2. What educational experiences can be provided that are likely to attain these purposes?
3. How can these educational experiences be effectively organized?
4. How can we determine whether these purposes are being attained?

I answered these questions as a guide I developed the supplementary mediation instruction activities for the curriculum.

This study focused on beginning Algebra I students. These students were transitioning from arithmetic to algebra. Wheeler (1996) implies that algebra is an expansion of arithmetic. He argues that algebra completes arithmetic. Therefore, since arithmetic is the parent of algebra, it is expected that beginning algebra study relates back to arithmetic. However, students often need help relating the exact numbers and definite answers in arithmetic to the generalized, abstract symbolic system that is Algebra. Herein may lie the root of the conceptual misunderstandings that originate in students.
3.5 Ethical Considerations

In order to adequately protect the rights of human participants in scholarly university research, an application was submitted to and approved by the University Human Investigations Committee (HIC) before the study began. In addition to following all of the policies and procedures outlined in the University Human Subjects Protection Program and the HIC, a meeting took place with the teacher and her students at which I explained the study in detail. The school administration was given the same information.

I personally explained the purpose of the study to each participant. Along with the consent form was the district’s and University’s audio-taping consent form for parents/guardians to sign. All other ethical considerations not previously discussed in this paper were addressed through the University HIC.

Participation in the study was voluntary. Consent forms will be given to the parents and the teacher. Before the study began, signed consent forms will be returned and collected from each research participant.

3.6 Data Collection and Analysis

Objective: To document, analyze, and interpret beginning Algebra I students’ changes in understanding of variables and solving equations. This objective specifically asked five questions:

a. What are beginning Algebra I students’ prior understandings of variables?

b. What are beginning Algebra I students’ prior understanding of solving equations?

c. What are the various reasons for these students’ understandings?

d. What discourse or mediation activity leads to students’ understanding?

e. What are the beginning Algebra I students’ post-conceptions about variables and solving equations?

To meet this objective, students participated in open mathematical discourse. These conversations were audio taped. The teacher led a whole class discussion developed as a focus group
interview. The teacher was given launch questions to begin and guide the conversation. The students were questioned until the line of reasoning was exhausted. The recordings were used to gather data on students’ previous knowledge and understandings about variables, solving equations, and the juxtaposition of the two. Besides the following questions, students’ were asked to represent story-problems using variables. Here are some of the questions that were posed:

“What words would you use to describe variables?”

“What is a variable?”

“Tell the definition of variable in your own words.”

“How did you come up with that definition?”

“What is an equation and what does it mean to solve an equation?”

“Do all equations need to be solved?”

“How do you go about solving equations?”

“Write an equation, and solve it for me.”

During the course of the study, students were also asked to write an equation using variables. The following is an example used in the study:

“Write an equation, using the variable $S$ and $T$ to represent the following statement: “At this school there are 25 times as many students as there are teachers.” Use $S$ for the number of students and $T$ for the number of teachers.” (Rosnick, 1981).

“Crunk Sounds Media sells all their CDs for $12.99 each. Write an equation that represents the total cost of a given number of CD’s.”

At the end of the study, students were asked to participate in a similar line of questioning. These recordings were also audio-taped and compared to the first and other recordings. They were specifically analyzed to uncover change in conceptual understanding. Besides the audio recording, the classroom
discourse was also captured in the researcher’s field notes, and any class work turned in by the students was analyzed.

Question (a), (b), and (c) ask “What are beginning Algebra I students’ prior understanding of variables and solving equations?” These questions were answered by the analysis of the 1st interview and the analysis of the discussion during the bubble map activity. Using phenomenography helped the researcher to gain understanding of how the students attained those understandings.

Question (d) and (e) were answered from the analysis of the interviews and the classroom discussion during the mediation activities. The answers on the mediation activities also gave insight into conceptual understanding. Also, phenomenography was used to specify what discourse or mediation activities lead to student understanding.

Conceptual change involves:

- Making students aware of their own thinking
- Having students share and discuss their ideas and their reasoning
- Having students confront their existing ideas through collaboration
- Students summarizing, discussing, and incorporating new information to accommodate a revised concept
- Students applying and making connections between their old understanding and their new understanding of the same idea (“The Conceptual Change Model,” n.d.).

Each of the five research questions corresponds to an element in the conceptual change model. Using this theory as a theoretical framework and using discourse analysis and phenomenography as methodological frameworks ensured the significance, reliability, and validity of this study.
3.7 Data Analysis

This was a phenomenographic study. There are many ways to analyze qualitative data. The study documented and interpreted students’ pre-instruction understandings of the concept of variables and solving equations by reviewing all the written data, and analyzing the classroom discourse between students and instructor, as well as the discourse between students. The classroom discourse and the teacher lead whole class discussion were transcribed. After reviewing the transcriptions, variations in relation to how students appeared to understand the concepts were documented and put into categories. These categories are what Cousin (2009) calls “categories of description.” The study attempted to identify 3-5 categories. Next, the study documented, using qualitative methods, the discourse of the Algebra I class during instructional mediation activities, and the study attempted to discern students’ evolving conceptualizations during mediation. Finally, the study documented, qualitatively, students’ post-intervention understandings of the concept of variables and solving equations.

3.8 Reliability and Validity Issues

During this study, I utilized a range of qualitative techniques to guarantee the integrity of information, data, and data analysis. These techniques include numerous observations of the students for an extended period of time, recorded classroom conversations, and continuous communication with the teacher.

The present study presents one possible threat to external validity. First, the sample consisted of only one Algebra I class, in one school district. It is possible that the study would have led to different results in a different class or district.

Finally, the study presented one possible threat to internal validity. The researcher specifically developed the supplementary mediation activities used for the experimental classroom in this study.

Trustworthiness is very important in a study of this nature. To confirm the trustworthiness of the study, another researcher checked the interpretations of the researcher. Also, selected quotes have been
displayed in the study to demonstrate their fit with the categories of descriptions that were generated to show plausibility (Cousin, 2009).

3.9 Chapter Summary

In conclusion, this chapter discussed in detail how to use phenomenography and discourse analysis were used to document, analyze, and interpret beginning Algebra I students’ understandings of the concept of variable and solving equations and the juxtaposition of the two. The phenomenographic framework was discussed and explained. The context of the study was thoroughly described. The Algebra I curriculum that will be used was described. Also in this chapter, the development of the instructional and material additions to the curriculum for the enhancement of the learning was described. The research methods and procedures were outlined for each objective. Ethical considerations were discussed. Finally, reliability and validity issues that are significant to qualitative research and investigations and issues that are specific to this study were discussed. The confirmation of trustworthiness was also discussed within this chapter. This chapter has set the stage for the reporting of Chapter 4, the results of the actual study that was conducted.
CHAPTER 4 RESULTS

Why and how students’ attain misconceptions in Algebra were investigated in this study. The purpose was to identify reasons for students’ prior misconceptions, and provide ways that teachers may help students gain a greater understanding of variables in juxtaposition with solving equations. The results describe different insights as to the past and present understandings of variables and solving equations. These results also give light to what influences students’ performances in their mathematics class.

After observing and audio recording the participants for eight hours, the recordings were transcribed and analyzed for common themes. The data were triangulated several ways for verification and reliability (Denin & Lincoln, 1994, p.214-215). There were a variety of data sources in the study such as audio recordings, observations and field notes, student work, and responses to semi-structured group interviews. Two theoretical frameworks were used in the study: Conceptual Change Inquiry Learning and Phenomenography. All of the information from the data was sorted and coded into the following themes and categories:

- Vocabulary
- Student Enthusiasm
- Teacher Enthusiasm
- Real-Life Situations
- Group-Work
- Classwork
- Hands-On
- Defining the Variable
- Types or Purposes of Variables
- Classroom (Mathematical) Discourse
Some of the themes were logically connected to other themes. For example, after reviewing the data again, it followed that student enthusiasm was a product of teacher enthusiasm, thus, student enthusiasm was eliminated as a theme because it was teacher influenced. The themes were reviewed and paired down several times (See Figure 2). After beginning with an initial plethora of common themes four major common themes emerged from the data: (a) teacher influence, (b) real-world value/defining the variable, (c) classroom discourse, and (d) time on task. The data rendered a great deal of evidence for three of the four themes. The teacher influence theme was observed to have such an impact on the classroom culture, and it helped in the classroom discourse in such a massive way that it became the fourth theme.

4.1 Teacher and Influence

The teacher influences the students through expectations, interactions, relationships, and her level of enthusiasm. Based upon observations, students seemed to be more observant, more willing to
try, and more keen to have academic discourse using correct vocabulary when they were motivated by
the teacher. Each time that the researcher observed the classroom, the class worked from the moment
they entered until the bell rang. The teacher compliments the students on a job well done. She encourages
the students to do better and work harder, not just verbally, but through pedagogical techniques such as,
wait time, group work, monitoring, and sharing. These techniques, whether intentional or not, motivated
the students to persist in their studies. For example, when this teacher expects for the students to work
individually before sharing with the class, she would instruct the class to work on the task quietly for a
designated number of minutes. When their time was up the students were all able to share something
with the class. Everyone participated because they knew that they were expected to share after their
individual work time was completed. The teacher used this technique on a regular, almost daily basis. The
teacher also used this time to get the students excited about sharing their work with each other. On the
very first day of the study, the teacher was observed walking around during the time when students were
working with a partner and asking them “wanna volunteer” in an exciting enthusiastic voice. It was
observed that the students took on the attitude of the teacher and were excited to share what they had
on their papers when it was time to share with the whole class. The teacher also monitored the students’
work constantly. She used a microphone which allowed her to walk around the room in order to provide
instant feedback to students. The microphone allowed the students and the researcher to adequately
hear directions and lectures. It also allowed her to get the students attention faster and waste less class
time. The teacher, through her techniques and her attitudes and beliefs about learning had cultivated a
classroom that is conducive to discourse. In addition, every class provided opportunities for students to
share their findings with each other. Through observations, this not only allowed the students to be proud
of their work, but also to throw ideas off of each other to expand their understanding of concepts.
I also observed that the class, as a whole, fed off the teacher. For example, on Day 5 of the classroom observations, the teacher started off by saying, “we have a fun activity planned for today.” The students were excited and ready to dive in before they even knew what the assignment was.

The following excerpt is a snap-shot of how the teacher influences the students through expectations, interactions, relationships, and her level of enthusiasm.

**Teacher:** This is some good metacognition, thinking about your own thinking....How do you know what you know? Let’s share a couple.....(Teacher walking around, paraphrasing what students are saying)

**Student 1:** Prior knowledge

**Student 2:** You use this almost everyday

**Student 3:** From practice and watching others

**Student 4:** Everyday life

**Teacher:** Making connections with other things....Couple of compliments today

- When it was individual work, I liked how people were working
- When it was partners time to share, I like how I heard math topics being talked about
- When it was a whole group time I enjoyed all the volunteers. That’s going to make our math class better...

The analysis of this excerpt of classroom discourse articulates how the teacher communicates her expectations. The quickness of their responses tells that the teachers’ enthusiasm had transferred to the students. The teacher complimented the students and told them “this is some good metacognition.” When the students walked in the classroom, the teacher was playing a “math song.” The class started off on a good note, listening to the song and singing along. The teacher told them that they were going to do something fun (as she did almost every day that was observed), and then she complimented them on their hard work, as shown in the excerpt above, throughout the lesson. This was a motivator for the students.

Analysis of the whole class semi-structured interview implied that most of the students had attained their notions of variables and solving equations because “a teacher told them” in a previous class. The students related their previous knowledge, whether correct or incorrect, to tasks or information given by a teacher in a previous class. For example, the following figure is a student answer when asked how they came up with the pre conceptions of solving equations.
Another thing that the teacher did that influenced the students was that she almost always modeled good work first and periodically as the students revised their work. This allowed the students to be active in the process of refining their work. During one assignment, a student requested the example several times over the course of three days. This student was observed working independently, and then asking to see the example, and then working independently again, refining his/her own work along the way. This is what Berger (2003, p.85) calls tribute work. He explains that tribute work is the work of a student who built on, borrowed ideas from, or imitated the work of an example. The example could be from a teacher, a current student, or a former student. The majority of the time spent observing provided examples that support Berger’s construct on tribute work. Tribute work proved to be an unexpected vital part of students’ conceptual understanding. It improved the quality of the student work and it increased their level of thinking.

4.2 Real World Value/Defining the Variable

Mathematics that the students believed will add value to their lives in the real-world and/or mathematics that meant more to students than just problems being worked on a paper seemed to have a greater value to students. Figure 4 displays a student stating, in their own words, how using real-world examples had a greater value. When students did problems out of the book or completed problems on the board, they simply used an algorithm to solve the problem. Although they may have been successful
at solving the problem, they did not ask questions. The algorithm did not generate any classroom discourse, and the students did not really seem to get a concrete understanding of the concepts. The students seemed to gain a personal connection to the mathematics when two things took place: 1.) they had a real world story problem, and 2.) they had to define the variable. For example, the students were given an assignment that required them to make up a story. The students seemed to be very excited about the assignment. They were given several days to complete the task in its entirety. On the last day that was allotted for the assignment, the students were observed making comments amongst themselves about how they felt smart, and what they accomplished during the course of completing the assignment. Two students in particular were heard saying “I feel like I’m in Math League,” and “I made up two equations!” Another example was when the students were discussing lead poisoning. They were engaged and attentive. They really made a personally connection with the topic prior to doing the mathematics, which lead to a personal connection to the mathematics.

The classroom discourse was much more reflective and in-depth and meaningful when the students had to define a variable in the context of a real world problem. One of the times that this was observed was on the last day of observations and the students were given three question sets. In two of the final three questions the students had to define the variable (in one of the questions it was defined for them) and then write an equation using the variable. Defining the variables seemed to be the most meaningful and value-added part of the lesson, because after they knew what the variables were, they easily could write equations and expressions to match the problem, and then they were able to solve the equations.

![Figure 4-Value of Real World Problems](image-url)
Inserting real world problems that required the students to define the variable addresses Chazan’s (2000, p.2) statement that “students do not feel the need to know algebra.” These types of problems gave students perceptible examples of why they needed to learn algebra, and how it could possibly add value to their lives. The students were involved in the discussions and many times connected to the topic on a personal level. These types of problems step away from allowing the student to simply memorize algebraic rules and arithmetic procedures (Kieran, 1992). They force the student to use other tactics to display that they have a concrete conceptual understanding of the content.

The supplemental curriculum materials provided during the study all added real-world value to the mathematics. Through these assignments, students were engaged. They asked more guiding questions, and interacted with each other. They discussed their findings and as a result of the curriculum they were able to explain their work using correct mathematical vocabulary. There is a plethora of data to support this claim. For example, the students were given a worksheet with formula problems on it. The first four problems dealt with the area of a rectangle. The students were given the formula for the area of a rectangle \( A = lw \). They were then asked to solve for each variable when given the other two. The worksheet itself was very simple. However, the discussion that took place amongst the students and their teacher was very complex. They talked about what area means. They discussed that area is defined in units squared and what that means. They discussed isolating the variable. The remaining problems involved different formulas and the students had to define the variables and solve for each variable in terms of the others.

At the beginning of the study the students were given a bubble map to help guide a discussion about what their understandings of variables and solving equations were, as well as where, how, and/or whom they had gained these understandings from. On Day 8 when the teacher gave the students two problems and asked the difference between the two, the students explained with correct vocabulary that one was an equation and one was an expression. They further went on to state the role of variables in
expressions as opposed to the role of variables in equations. The students stated that “in the first one (an expression) $x$ can be anything; in the second one (an equation) $x$ has to be something.” These, along with other examples, show that engaging the students with problems that they perceive as valuable or engaging sparks discourse that promotes correct use of vocabulary and conceptual understanding. In this case, the students easily took a concept that was simple and that they were familiar with, like an equal sign, and tied it in with new knowledge, like to be able to explain the difference between an equation and an expression. The fact that they were able to express new knowledge with correct vocabulary aided in the students being willing to engage in discussion and share ideas with each other.

The class had an assignment that required them to create a book. They were given various topics, such as basketball, shopping, and growing up. They had to relate variables and solving equations to these real life topics and make them into a story. Some students found this very hard to do in the beginning. However, after relating it to a real-life situation, and getting a bit more guidance, they were able to not only successfully complete the assignment, but also gain a better understanding of linear equations, and the concept of variables. One student in particular did a book on the temperature change during the course of a day. Since it was a real-life situation, the student found it easier to make a table that related to the story. In a real-life situation, the students were able to input numbers and chart the outputs in a table. After making the table, the student was able to see a linear pattern in the data. For example, if the situation stated that a person drinks two cups an hour, the students were able to say in one hour the person drank two cups, after two hours the person drank four cups, after three hours, the person drank six cups, and so on. Through a table, students were able to see that every time the input increased by one, the output increased by two. Students could now envision and have a discussion about someone sitting and drinking two cups of coffee (or whatever the drink) an hour. The students would be able to understand that the number of hours that they inputted would replace (or substitute for) the variable. Another student struggled with creating the very same book, but a book about basketball. The teacher asked the
student some guiding questions to help get him/her back on track. As a result of the questioning, the student began to explain that each shot was worth two points. The student was then able to define the variable as the number of shots, later being able to correctly write an equation involving $2x$. After defining the variable, the student was able to persevere through the exercise by talking and reasoning his/her way through it. Many of the students struggled until they had a real-life situation and talked through it with their teacher and/or their classmates.

Through observations and field notes, it was detected that many of the students who were the most confused at the beginning of the assignment about how to solve an equation and how to correctly use a variable were able to persevere and complete the assignment. Many of these students struggled at first with defining a variable. They wrote equations but did not know what their variables represented. In almost all of the cases, once the students knew what each part of the equation represented, they were able to define the variables, write an equation, solve the equation, and complete their book.

For example, a student was writing about doing chores to save money for a pair of basketball shoes. The student wrote the equation $144 = x - 48$. So the teacher asked the student “Where did the 144 come from? What does the $x$ represent? Where did the 48 come from?” Asking these questions helped the student put her equation into perspective, and it lead to a discussion of ‘what we have and what we are trying to get’. The student was able to revise her equation, and, with more discussion on defining the variable, the student continued to revise the equation, getting better and showing a gain in conceptual understanding with every revision.

In a separate assignment on Day 7, the students were given an assignment that dealt with lead poisoning. They began the lesson by watching a video on lead poisoning (Human Rights Watch, 2012). Before they watched the video, the teacher posed the questions, “Has anyone heard of lead poisoning before? Anyone care to share in what kind of context, or what they know about it, or anything at all?” This immediately sparked responses from almost every student in the class. Although their responses
were not mathematical by nature, they sparked interest, which carried through to the end of the lesson. Furthermore, after seeing the video, the students were aware of how this relates to the real-world, and they were very interested in the topic. Here is an excerpt from that day of instruction:

**Student 1:** Like back in the 70s and 80s or something like that, the schools and houses they used mostly lead paint, but now they have moved out a lot of that stuff, because people are exposed to lead.

**Student 2:** Also, I know that like before you can have a foster kid, or take care of a foster kid or adopt a kid, they have to check your windows and check to see if you have paint that has lead in it or not.

**Teacher:** Okay, so one of the things that they do for fostering and adopting children, is I guess they come to your home and make sure that it’s a safe environment and part of that might include lead testing.

**Student 3:** I heard, it was on a TV show, this kid, ever since he was five, he has been chewing on those little toys that were from China and they had lead in them, and when he was 20 years old he had a job, and he did something really bad, and he didn’t get arrested for it because the lead made him have brain damage.

**Teacher:** So that was his defense?

**Student 4:** Yeah!

**Teacher:** Was that like on 20/20 or was that on a fictional show. Was it on like 60 minutes on something.

This passage of classroom discourse depicts how the real-world situations can draw the students into the mathematics without them recognizing it. The teacher allowed the conversation to go into a few different directions, while still facilitating the conversation. She paraphrased after each student. This made the students feel heard and welcomed to share. After the discussion the students were eager to do the mathematics and figure out how many days of medicine a person would need to get the lead out of their system.

This assignment also required the students to define the variables in every problem. These problems were advanced; however, the students were able to solve them through defining the variable and putting the mathematics into a real-world context. They had to define the variable and tell how to measure each variable. For example, if the variable was \( t \), the students had to say that \( t \) represented time, and that they were measuring time in days. The students had to figure out how many days it would take until the amount of lead in a child’s system dropped below a certain level. The last question on this assignment asked the students to explain how their knowledge of variables helped them to solve the
problem. 21% of the students admitted that they guessed and checked. 79% of them made tables. 6% of the students used mathematical manipulations such as subtraction, because they saw a pattern and kept going with it. Twelve per cent of the students saw that they needed higher exponents to get the lead level lower. In all these examples the students had to know that the variable stood for time in days in order to be able to solve it. In fact one student wrote, “I know that ‘t’ equals time so some number had to replace ‘t.’” This shows that the student defined the variable and recognized what she/he was looking for.

Both examples given above required the students to define the variable. In most cases, once the students knew what the variable represented, they displayed a greater understanding as explained above in the book creation and the lead poisoning assignments. By the end of the study, it became second nature for the students to define the variable. During the last day of the study, the students were given this problem. “Crunck Sounds Media sells CDs for $12.99 each. Write an equation that represents that total cost.” The majority, if not all, of the students wrote the correct equation the first time and fairly quickly. They were able to define their variables and actually were able to make a table from their equation.

One thing that was observed in all of these examples, was that, when given a real-life situation, students were more prone to ask the teacher direct questions for clarification and help as opposed to saying, “I don’t get it.” Also, when students were given a real-life situation, more mathematical discourse was observed when students broke up into groups.

When students were asked about their previous knowledge of variables and solving equations, very few of them related their knowledge of these concepts to real-world situations. The students who did appeared to have a greater conceptual understanding than the students who did not. It seemed as if those students were more able to vocalize and articulate their understandings. For example, when students were asked about previous knowledge of variables, many gave responses like “control variable” used in science experiments. This led to a discussion about independent and dependent variables, which in turn led to students stating that variables can sometimes be changed or manipulated.
4.3 Classroom Discourse

The classroom discourse helped shape the ideas of the students. The more discourse the more concrete the understanding. The teacher used a concept that she called ‘think, pair, share.’ This is where she would give the students an individual problem or an activity. After having the students brainstorm (think) about the problem she would then pair them up with one or two other students to work on it together. The students would bounce answers and ideas off of each other until they came to a common solution. Finally, each group would share out to the class what their solution was and their reasoning behind it. In almost every observation opportunity the students helped each other by talking through the problems together via organized mathematical discourse. They took turns. They raised their hands. They repeated to each other for clarification. They used what others said to build upon their own knowledge. This was observed every single day of observation. At the beginning of each class the teacher went over the homework from the night before or a previous assessment. During this time she called students to the board to have them present their work. The students raised their hands if they felt confident in their work and wanted to share. If a student did something that helped another student, the other student would ask the teacher and/or the student to repeat and clarify for further understanding. Students would also add on to where the student may have left off in their explanation of their answer.

The students seemed to really enjoy sharing their ideas with each other. The students mostly stayed on task and the vast majority of them contributed to the classroom discourse regularly. Linguistically, it was very interesting to observe how the discourse in the class between teacher and student, and between student and student is conducted. The teacher had a culture of turn-taking already set up in the classroom. The students used a lot of “I” (individual work) and “we” (group work) statements. This indicated that they are taking ownership of their work. The students were not afraid to answer questions and share their ideas. They showed no fear of being wrong or being criticized by their peers.
The following excerpt comes from the first day of observations. The students were describing their previous understandings of variables.

**Student 1:** Like on a graph

**Teacher:** Okay so on a graph, a lot of times we set it up with an independent variable and a dependent variable. So I might even just put a little picture of a graph so I’ll remember what I’m talking about here

**Student 2:** It could change, you can change it

**Teacher:** It could vary, it could change...Ok so it changes...you can manipulate...Good so we see variables in tables, graphs, and equations.......(students gave a few more suggestions or answers)

I think we have a pretty good collective brain coming up with some good answers. Anything else? The question was” What is a variable?”...... Let me ask you this question. How did you come up with what a variable is? Cause all I did was give you a map and said “Tell me what a variable is.”

The analysis of this excerpt of classroom discourse tells us a great deal. It shows that the teacher is not the holder of knowledge or power in the classroom, but that this classroom has a culture of shared knowledge. The teacher is the facilitator, as opposed to the authority of knowledge or authority of discipline. The questions were given by the teacher; however, she posed them in a way that made the assignment seem more like a conversation than a mere worksheet that needed to be completed. The students were the spokesmen for the assignment. Their voices were heard and the teacher just paraphrased what they said, giving them conformation of receipt of their knowledge. The teacher used very affirmative language, for example when she says, “I think we have a pretty good collective brain coming up with some good answers.” I noticed that this type of language promoted classroom participation. In the excerpt, the teacher also sneaks in mathematical vocabulary as she paraphrases the students’ thoughts, “so it changes, you can manipulate.” She uses their words first, and then encourages alternative language based upon the audience.

There were some explicit things that the teacher did to set her classroom for a culture of sharing and involvement. She almost never told a student that an answer was wrong. She usually redirected them or asked the class what they thought of the student’s answer. She would kindly suggest a better answer.
Another tactic that she used was asking the students to explain everything. This usually resulted in the students finding their own mistakes. The students knew that in this classroom, being involved equaled learning. She often told the students, “that one is kind of hard, but you will know how to do it by the end of class today.”

Through classroom discourse analysis, the attitudes and dispositions of the students and the teacher are observable. This study has looked at the aspect of the students’ and the teacher’s conduct that is accomplished through language use. Language use is defined as the use of vocabulary and grammar and the sequential organization of the classroom discussions. As teachers and students talk to each other, conduct class, ask and answer questions, tell stories related to the subject matter, quote each other, hedge or assert, they are actively engaging in restructuring the established environment of their learning. The classroom discourse helped the researcher to investigate the phenomenon of classroom culture through conversation. This is important to the study, because classroom culture may or may not contribute to an increase in conceptual understanding. In this case, I believe it contributed to a more concrete conceptual understanding of the topics being studied as the following examples show.

Through discourse the teacher was able to aid the students in developing a more concrete use of mathematical vocabulary. She was able to turn the course of the class into a more student-centered classroom, where the teacher is merely a facilitator and where the students can have ownership of their learning. For example, on Day 3 of the classroom observations the teacher asked the class, “What was the big idea? Why are we reviewing these formulas? Why are we learning this?” The student answers sparked interesting discourse, because they gave the students an opportunity to express why they thought they were learning a topic and why it was important, as opposed to this is what we are doing and just do it. Some of the student responses were, “To give us more practice”, “To use formulas to isolate variables”, “To rewrite formulas for specific variables”, “To learn how to use formulas.” It was important that the
students knew what they were supposed to learn and why because the next activity depended upon whether they had mastered those skills.

On Day 4 of the classroom observations, the students were given a task that required them to use a TI-Nspire calculator. The students were very excited by the use of new technology. This assignment allowed the students to discuss every aspect of an equation. They discussed which variables changed; which did not change; why did they changed; and what happened if they were negative. During my observations, the students did not challenge or pay attention to these aspects of the mathematics when they were merely doing bookwork. The use of the calculators abetted the students’ discussion of variables and solving equations. They could see the variable being replaced. It no longer was an abstract idea, but a concrete model that was fluid and changeable.

4.4 Time on Task

Time on task seemed to make a difference with students’ understandings of the concepts being studied. The more time spent on the subject, the more understanding the students seemed to develop. Each task was an extension of the previous task, building upon knowledge that students had just attained. The teacher always started the class reviewing homework. Many of these problems were repetitive. However, taking the time to review them and having the students come to the board to do them as a class increased the confidence level of the students when it came to the particular concept being reviewed. Based upon the classroom discourse, the number of students volunteering, and the quality of student work, I inferred that the more problems reviewed, the more confidence the students gained. Berger (2003, pg.65) supports this theory when he states, “When they (students) begin to make discoveries that Impress their classmates, solve problems as part of the group, put together projects that are admired by others, produce work of real quality, a new self-image as a proud student will emerge.” In addition, on one of the days, during the last 2 minutes of class while students were cleaning up and putting their
supplies up students were observed making several statements like, “I feel like I’m in Math League,” and, excitedly, “I made up two equations!”

Because students knew that doing more examples would help them understand the concepts better, they freely volunteered when the teacher asked them which problem they wanted to go over, as observed on Day 2 of the observations. The students would all raise their hands and scream out the numbers to the problems that they wanted the teacher to go over. As the teacher began to do some examples the students had fewer and fewer questions. Also the more examples that the teacher reviewed, the more the students correctly answered the leading/guiding questions that the teacher posed as she reviewed the problems. Through this analysis, it appears that the students were gaining more confidence and also beginning to gain more understanding of what was being taught. In addition, on the last day of observations the students were asked:

1. What activities did we do in this class that helped you to better understand variables and/or solving equations?
2. How did those activities help you to gain a better understanding?

Nearly 35% of the students mentioned time on task in their response and being able to do examples with their teacher and their classmates. They enjoyed the activities and assignments for which they were given more time to complete and in which they were able to ask questions and receive help from each other and their teacher. Being able to discuss each problem or strategic problems before moving on seemed to be key in their understanding of the concepts. In fact, the very first of the eight practices of the Common Core Mathematical Practices is to “make sense of problems and persevere in solving them.” This entails giving students the appropriate time on task so that they may have time to work through the problem to gain conceptual understanding and finish the problem from beginning to the end with explanation.

The students spent four days on an assignment that required them to make books. For this assignment each student was asked to make a book. The book could be made from any type of paper or
construction paper. They could use glue, staples, or whatever was at their disposal to assemble the book. Each student was given a topic at random. In the first part of the book, the students were supposed to establish a storyline. They were also supposed to identify a variable and identify different outcomes when the variable was changed. In the second part of the book, the students were supposed to solve for a constant variable. It took a lot of time, but 28 of the 31 students turned in a book that showed that they were able to accomplish the objective of the assignment. This was a significant accomplishment given that the majority of the students did not know where to begin when they first started the project. They asked questions each day and understood more and more. They asked questions pertaining to defining the variable, and setting up the equation before solving. By the time the students began creating their books they had already discussed their previous ideas and understandings of variables and solving equations. They had already solved multi-step equations and they had learned how to rewrite formulas for specific variables. They were given adequate time to process new knowledge with prior knowledge without being rushed. The students who asked questions that led to constant revisions and covered every objective of the assignment had the best books.

One student was observed asking the teacher for help. The student had an equation written, but was not quite sure if it was right. In fact it was incorrect, but in the end, with time and guidance, the student was able to figure that out. The teacher asked the student several questions to get the student to explain where the numbers in the equation came from. In the end this was the product that the student produced:

```
"Weekly Pay
Quafa started helping out around the house. Her dad said she can get a weekly pay. Quafa will receive $4 per chore she does without being told. After a course of several weeks Quafa has raised $48. How many weeks has she worked if she did 6 chores per week?

4(6x) = $48
6(4x) = $48
24x = $48
24x = $48
24
X= 2 weeks
```
Quafa has worked for 2 weeks and has made $48. She needs to make $144 to get these basketball shoes she really wants.

\[
6(4x) - 48 = 144
\]

\[
24x - 48 = 144
\]

\[
+ 48 + 48
\]

\[
24x = 192
\]

\[
24x = 192
\]

\[
24 \cdot 24
\]

\[
X = 8 \text{ weeks}
\]

After Quafa worked 8 weeks she finally earned $144 and she got the basketball shoes she really wanted. Thanks for all the help.”

The student even wrote, “Thanks for all the help,” as the last line of her story. She was thanking the teacher for helping and taking the time to answer all of her questions and giving her enough time to work on the problem.

There were many times where the teacher did not have the students turn in work until after they revised it a few times and had adequate time for practice and understanding. For example, on Day 8 the teacher told the students to take out their homework, but informed the class that the homework was not due yet. She spent a little time reviewing, explaining, and allowing the students to revise their work. After observing this process a number of times, I concluded that the process helped the students to be more successful with solving equations.

Allowing the students more time on tasks gave them ample opportunities to produce multiple drafts. Students need to understand how to achieve quality work. Quality work means rethinking and reworking their work until it is refined. They need to understand that they can keep building on their knowledge to gain more understanding. Each day through lecture and classroom discourse students’ understanding should increase, and therein lies the potential for them to revise their work in light of their new thinking. Revisions can even take place on a one-day assignment.

In summary, the findings show that helping students to gain a conceptual change involves curriculum interventions, various pedagogical techniques, and participation from students in classroom discourse. The following themes related to supporting students in gaining a conceptual change emanated
from the study’s findings: (a) teacher influence, (b) real-world value/defining the variable, (c) classroom discourse, and (d) time on task.
CHAPTER 5 DISCUSSION OF FINDINGS

The overall purpose of this classroom-based study was to design a curricular intervention for a beginning Algebra I course and document, analyze, and interpret conceptual changes beginning Algebra students experienced. Specifically, this study sought to address and analyze the issues that many algebra students have with the concept of variable, the concept of solving equations, and the juxtaposition of the two.

After analysis of the data and in reference to the original objective stated in Chapter 1, to document, analyze, and interpret beginning Algebra I students’ changes in understanding of variables and solving equations, possible implications will be presented in this Chapter followed by a discussion relevant to present day literature on this topic. The work represented in Chapter 4 identifies some of the reasons found in the study of why and how students attain misconceptions in Algebra, especially in relation to variables and solving equations.

5.1 Summary of Findings

Of the multiple findings, real-world value, defining the variable and classroom discourse were identified as the findings that mostly effect conceptual change and understanding. Using phenomenography (Marton & Booth, 1997, Marton & Tsui, 2004), particular interest was paid to how the students reacted to their learning, their teacher, and to the curriculum given to them. The researcher initiated a study of the classroom discourse. This analytical approach led to some very interesting outcomes. For example, using semi-structured group interviews, the teacher was able to recruit students to participate in mathematical discourse and create opportunities for students to explain their mathematical ideas and understandings. This took away the notion that Chazan introduced that “students do not feel the need to know algebra” (Chazan, 2000, p.2.) Through discourse, students were expressing the need to know algebra. They were talking about real-world situations that involved algebra, variables, and solving equations.
Overall, the findings of this study indicate that through the use of whole class semi-structured interviews, and very carefully picked curriculum activities, the students were able to embark in some meaningful mathematical discourse, which in return gave the mathematics real-world value for them. The discourse seemed to excite the teacher which in return excited the students, and the communication gave the teacher immediate feedback. As a result, the teacher was able to immediately interpret whether the students had a concrete understanding or not, which helped her to decide if they needed more time on task or not.

Overall, the findings indicated that taking time to discuss new mathematical concepts and allowing students to relate them to other things that they are more familiar with is very beneficial to student’s gaining a conceptual understanding of mathematical concepts. Through this process some of the NCTM Standards (1989, 2000) were highlighted, such as, the students were modeling with mathematics, they were problem solving, and they were communicating. They had begun to start to make connections and use multiple representations, but more observations would be needed in order to show a strong connection in those areas.

5.2 Research Questions Revisited

This study was guided by five research questions which were addressed separately:

What are beginning Algebra 1 students’ prior understandings of variables?

This study showed that students had a considerably narrowed understanding of variables. They knew that variables were a letter or a symbol and—without discussion or prompting—that was mostly all that they knew. The data indicated that the majority of their prior understanding was not so much incorrect as it was incomplete, and students felt as if their prior understanding of the concept of variables was correct. Many of the students defined variables as types of variables in science like “control variable,” which is an element of a study or an experiment that never changes and is compared to something that
does change. Data to support these findings were retrieved from classroom discourse, as well as student work, such as the bubble map shown here in Figure 5:

![Figure 5-Example of Variables Bubble Map](image)

**What are beginning Algebra 1 students’ prior understandings of solving equations?**

This study confirmed the notion that beginning algebra students had an extremely narrow concept of solving equations and the mathematical possibilities that are included in the concept. At the beginning of the study, students were asked to brainstorm about their prior understandings of solving equations. These exercises yielded understandings about the ways in which students thought that solving equations was the same as simplifying or that it was finding the missing variable. Although some
students had a reasonable idea of solving equations, it seemed as if they were still not sure or saying what they knew they were supposed to say but not really understanding their answers. For example in Figure 6, one student stated that solving equations sometimes used variables and that same student also stated that solving an equation is finding the value of the variable. If this is true then you need a variable all the time not just sometimes in order to solve an equation.

What are the various reasons for these students’ understandings?

Analysis of the whole class semi-structured interview yielded that the majority of the students acquired their prior understandings from a teacher in a previous mathematics and/or science course. They stated that they attained their prior understandings because “a teacher told them.” They also used their own independent thinking and derived their understanding by using prior knowledge of recognized vocabulary and developing new knowledge. They sometimes accomplished this by using context clues. In the case of “solving equations” most students felt that their prior understandings were gained from solving equations in elementary and middle school, prior to attending algebra class in 8th or 9th grade.
What is the discourse or mediation activity that leads to students’ understanding?

This study was looking for concrete understanding of the concepts instead of mere manipulation of mathematical elements. This was hard to measure, because these particular mathematical concepts are, as Wagner puts it, “easy to use, but hard to understand” (Wagner, 1983, p. 474.) Understanding was generated through any activity that sparked discourse and that students could redo over and over until completely correct. In all cases, when given an example of a real-life situation posed as “math problems” the students were more prone to engage in mathematical discourse. It is the discourse, especially in small groups, in the context of the problem that leads to the conceptual understanding along with repetitive modeling of correct mathematics and ample time on task. Figure 7 explains this in the students’ own words.

![Figure 7-Student Explaining How Classroom Discourse Helped]

Remembering, that is, the progression of not being capable of something to becoming capable of something as a result of having certain experiences (Marton et al., 2004, p. 5), the students were learning and restricting their understandings through mathematical discourse and problem-solving on a level that mattered to them. When the students were instructed to make a book where they could choose and write the story line, that experience led to conceptual understanding. This activity led many students from instrumental understanding to relational understanding (Skemp, 1976). The assignment was designed in such a way that the students had to explain their work and how they constructed the
equations. They had to define the variables. It was nearly impossible to “guess” and correctly complete the assignment.

The students themselves indicated that time on task and being able to do examples with their teacher and classmates helped them to better understand variables and/or solving equations. Analysis of student work, field notes, and audio recordings indicates pedagogical techniques that encourage student perseverance, activities and problems that have real-world value, defining the variable when solving equations, having a classroom culture of mathematical discourse, and providing ample time on task all led to student understanding.

What are the beginning Algebra I students post-conceptions about variables and solving equations?

At the conclusion of the study students were able to distinguish between an equation and an expression. They were able to articulate the difference in the role of the variable in each. More students recognized that variables play more than one role in mathematics than had in the beginning of the study. Students were beginning to see the connections between what they were learning in class and how they can use it in everyday life at home, in another class, or at work.

5.3 Limitations of the Study

One of the limitations of this study was that I did not have a control group. Being able to compare the class I observed to a class that used the regular curriculum only and had no remediation activities would have yielded more generalizable outcomes.
Time was also a limitation of this study. Algebra is a school year long class. Being able to observe the class for the whole school year or even tracking their progress through high school would have enabled me to see if having a more concrete understanding of these basic algebra concepts really made abundant impact on their success in more advanced mathematics courses.

5.4 Implications and Recommendations for Future Research

This study can benefit the research community, school districts, and educational policy makers. This study can assist the research community because it specifically examines how classroom mathematical discourse aids students in gaining conceptual understanding. There are not many studies that examine this phenomenon. This is one of the newer areas of focus in the mathematics education research. This study adds to the base of existing research on mathematical discourse in juxtaposition with mathematical conceptual understanding. In addition, other researchers will be able to utilize the data in this study to augment their own studies. While the findings in this study ignite discussion about curriculum and students’ understandings, they also serve as a catalyst for a more general discussion about meaningful mathematics and approaches to teacher education. It should be understood that these findings are only representative of this particular class, with this particular teacher. There is no guarantee that the success found in this class will be found in other classes. Some concerns were discovered that were not addressed which offer areas for future research.

The methodologies utilized were all qualitative. Future research is needed to examine the same subjects, but with quantitative methods.

This research study utilized one 8th grade Algebra I class in one school. This study could be extended to include multiple teachers, 8th and 9th grade classrooms, in different districts. Also, a multiple year study would beneficial to gain data on implications this study may have on higher mathematical courses.

5.5 Implications for Teachers
The data utilized in this study were taken from observations, field-notes, semi-structured group interviews, and student work. The classroom discourse allowed for reflections on ideas. Novice and expert teachers could benefit from the study’s findings by making decisions regarding ways to improve pedagogical practices, curricular decisions, and professional development. The results of this study agreed with Rajano (1996), suggesting that incorporating problem solving into the teaching of symbolic algebra is more meaningful and value-added for the student. The students gained a greater conceptual understanding more often when the curriculum involved real-life problem solving. This study used problem solving in every phase of learning, the beginning, the middle, and the end. Because of the success of the students, this is the recommended method. In short, this study has determined that using problem solving to introduce variable Algebra and going on to solving equations increases conceptual understanding of the concept of variables and solving equations.

When Booth (1988) conducted a study on the Strategies and Errors in Secondary Mathematics (SESM project), the study found that many students at the beginning stages of algebra have trouble going from arithmetic to algebra and using variables. Many of these issues did not carry over into this study, because we began the study with a discussion of previous understandings of variables and solving equations and the mathematical discourse of that discussion set the tone for inquiry and understandings. The students allowed the teacher and each other to aid in shaping their understandings and the students were not afraid to let everyone know what their understandings were. Thus the teacher could have a sense of what they were thinking and if they were having some misunderstandings. This allowed any misunderstandings that students had to be corrected before moving on to new concepts.

This study, however, did not agree with Kierans’ (1988) research. Kieran stated that when students focus on the “algebraic approach” it creates a problem when students deal with algebraic expressions instead of algebraic equations. Our study used supplementary material that we inserted into the existing curriculum. While the supplementary material used a more problem solving approach, the
existing curriculum was very dense in the “algebraic approach.” However, the students did not have a problem when they had to switch from expressions to equations. This tells us that a mixture of the two approaches may be useful helping students learn algebra concepts.

One thing that this study indicated that is new to the research base, is the notion that conceptual understanding is a direct result of time spent on a concept. It seems like a very elementary idea; however, I found no research directly relating the two. This study specifically found that students gain a greater understanding when they have more time to practice, along with more to exchange ideas, and more time to model correct techniques or ideas.

Finally, teacher education programs should take more time to teach future and current mathematics educators the art of orchestrating mathematical discourse. Designing lessons that spark constructive discourse is an art form and a skill that can be learned. This study also provides more evidence that student’s concepts can be reconstructed as the Kieran study eluded to (Kieran, 1988.)
5.6 Conclusion

The overall purpose of this classroom-based study was to design a curricular intervention for a beginning Algebra I course and document, analyze, and interpret conceptual changes beginning Algebra students experienced. Specifically, this study sought to address the issues that many algebra students’ have with the concept of variable, the concept of solving equations, and the juxtaposition of the two.

Because there are so many different variables involved, the questions raised in this discussion are difficult to address. Nonetheless, the situation remains hopeful. This inquiry has offered new insight into conceptual change. This qualitative research study explored beginning Algebra I students’ changes in understanding of variables and solving equations. The theoretical framework allowed the researcher to examine and understand the eighth grade Algebra I students’ pre-, evolving, and post-understandings of the concept of variable and solving equations.

According to the observations, classroom discourse, interviews, and work of the students, classroom discourse, curriculum materials and activities that have real-world value, and the amount of time on task impact conceptual understanding of variables and solving equations.

5.7 Chapter Summary

In Chapter 5 a brief summary was discussed and the findings were correlated to the evolution of the field of mathematics education and its impact on current ideas and practices. The results from an analysis of observations, semi-structured group interviews, field-notes, classroom mathematical discourse, and student work were presented in relation to the research question. Data was categorized into a discussion of this study, including a discussion of findings, implications for teachers, and implications for future research, conclusion, and summary. It was shown how this research has added to the existing research base and how it has extended the knowledge and research base of mathematics education.
APPENDIX A

Variable
For this assignment each student will be asked to make a book. The book can be physically made from any type of paper or construction paper. You can use glue, staples, or whatever is at your disposal to assemble the book. Each student will be given an index card at random. The index card will have a topic written on it. You will be given an opportunity to exchange or trade cards with your classmates. That will be the topic of that student’s book.

A plot or storyline must be established for each book. The story must identify a variable and identify different outcomes when the value of the variable changes. The story must also include solving for an unknown variable. Include an equation(s) in your book. Please have fun with this assignment and make your books creative and colorful.
The formula for the area of a rectangle is Area = Length times Width or A = lw.

1. If the length = 5 and the width = 6, what is the area? Show work.

2. If the area = 20, and the width = 4, what is the length? Show work.

3. If the area = 32, and the length = 8, what is the width? Show work.
4. Using the formula $A = lw$, how would you solve for the length if you only knew the area and the width? Explain in words. Then show work using the equation.

Now try with other formulas.
First identify each variable (if known), then solve for the variable that is specified.

5. $A = \frac{1}{2} bh$, for $h$.

6. $A = \pi r^2$, for $r$.

7. $10x + 4y = 3$, for $y$. 
8. \( Y = mx + b \), for \( b \).

9. \( A = bh \), for \( b \)

10. \( P = 2l + 2w \), for \( w \).
INTRODUCTION:

Toxic effects from exposure to lead plague many communities. Many children are at high risk due to environmental exposure. Some children have been poisoned by eating paint chips containing lead. Many children are at risk from lead-contaminated dust and soil. Tons of lead were deposited in areas with heavy traffic congestion prior to the use of unleaded fuel. This lead is still in the soil, and children absorb into their bodies about 40% of the lead in things that are eaten, but they absorb about 90% of inhaled lead particles. In children, high lead levels can cause mental deterioration and impair the child’s ability to learn.

It is recommended that children be treated for lead poisoning if tests show they have more than 45µg (micrograms) of lead for each tenth of a liter of blood. This would be about 0.9 mg of lead in the blood of a 60 lb child. From this it is clear that very small amounts of lead taken into a child’s body can harm the child. What further complicates the problem is that only about 1.5% of the lead is removed from the blood each day. This means that without treatment, it will take a long time for a high level of lead in a person’s blood to drop to an acceptable level.

Suppose a child has 1.0mg of lead in her blood. Assume this child’s body removes 1.5% of lead in the blood each day:

a. How do we write 1.5% as a decimal number (without a percent sign)?

b. How much lead would be in the child’s body after 1 day?

c. A function that would give the amount of lead in this child’s blood after a certain number of days, assuming no additional exposure is .985^t. What is t? What does t stand for?

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1 Adapted from [http://faculty.georgetown.edu/sandefur/handsonmath/downloads/pdf/lead.pdf](http://faculty.georgetown.edu/sandefur/handsonmath/downloads/pdf/lead.pdf)

We also showed the students a video on lead poisoning available at [https://www.youtube.com/watch?v=5N8e3XbnxG8](https://www.youtube.com/watch?v=5N8e3XbnxG8).
d. Assuming this child is removed from all sources of lead, how much lead is in her body after 1, 2, 3, and 4 days? Use 4-decimal-place accuracy.

e. How many days will it take until the amount of lead in this child’s system drops below 0.9 mg?

f. How did your knowledge of variables help you to solve this problem?
APPENDIX E

Open the TI-Nspire document *Variables_and_Expressions.tns*.

If the numbers that can be substituted for the symbol $x$ can vary, we call $x$ a *variable*. This activity lets you change the value for $x$ on a number line and see the effect on an algebraic expression involving $x$.

---

**Move to page 1.2.**

1. As you grab the point and move the arrow beneath the number line, what changes? What stays the same?

2. Wade says that when $x$ is negative, the value of $3(x) + -4$ is always negative. Explain why he is right or wrong.

3. a. Find a value of the variable $x$ that causes the expression $3(x) + -4$ to equal 17.

   b. Estimate a value of the variable $x$ that causes the expression $3(x) + -4$ to equal 15. Explain your reasoning.

4. Find a value for $x$ that will make the value of the expression $3(x) + -4$ equal to $-4$. 
5. a. If the value of $x$ is increased by 1, how does the value of the expression change?

   b. How is this change related to the expression?

6. a. Write an expression you think will increase by 5 when the value of $x$ is increased by 1.

   b. Give some examples to support your reasoning.

7. Write an expression that will not vary (change in value) when the value of $x$ is increased by 1. Explain your reasoning.
Observation Protocol Form and Field Notes

<table>
<thead>
<tr>
<th>Begin Time:</th>
<th>End Time:</th>
<th>Date:</th>
<th># Of Students</th>
</tr>
</thead>
</table>

Lesson Objective:

Which of the following occurred during observation?

<table>
<thead>
<tr>
<th></th>
<th>No</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson Objective was stated by student(s) during class.</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Main points are summarized by students at end of session.</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Were students using proper mathematical language?</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Were students engaged with each other?</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Did students ask relevant questions for clarification</td>
<td>☐</td>
<td>☐</td>
</tr>
</tbody>
</table>

Comments:

What statements did students make that suggest conceptual understanding?

Briefly describe the discourse?
Fieldnotes:
APPENDIX G

Interview Guide 1

(Goes Along with Bubble Map)

Teacher asks the questions to the class as a whole.
These are guiding questions that may lead to other questions.

1. What is a variable? high

2. How did you come up with your definition of what a variable is?

3. What is the significance of variables?

4. What is an equation?

5. What does it mean to solve an equation?

6. Do all equations need to be solved?

7. How are variables and equations related?

8. How do you know what you know about variables/solving equations?

9. Where exactly did you get the information that you know about Variables/solving equations?
Teacher will put students into groups of three or four and allow students to discuss answers within group before discusses with class as a whole. (Teacher may also write the questions on the board)

Question Set 1

1. In your groups, please write an equation, using the variables $S$ and $T$ to represent the following statement: “At this middle school there are 20 times as many students as there are teachers.” Use $S$ for the number of students and $T$ for the number of teachers.

2. What equations did you come up with.

3. Explain how you developed your equations.

4. How could you have tested your equations for correctness.

Question Set 2

1. In the equation $3 + x$, what is $x$?

2. Explain.

3. Does $3 + x = 4$?

Question Set 3

1. Crunk Sounds Media sells all their CDs for $12.99 each. Write an equation that represents the total cost of a given number of CD’s.

2. Define the variables in your equation.
Whole class interview with all students. Remember, these questions are used as a guide, but teacher may allow the conversation to deviate from the questions is necessary.

3. Variables can be defined (or used) in how many different ways?

4. Explain you last answer.

5. What does it mean to “solve an equation?”

6. Give an example of when you might need to solve an equation outside of school (in the real-world).

7. What activities did we do in this class that helped you to better understand variables and/or solving equations?

8. How did those activities help you to gain a better understanding?
APPENDIX J

Solving Equations “Flow Chart”

Each student will be given an equation. You must make two flow chart posters for the equation. The first flowchart should consist of numbers only. The second flow chart, placed directly under the first one, should explain the justification for each step of the first flow chart. When you are finished you should have a poster with two flow charts, one with numbers, one with words.
Solving Equations
REFERENCES


Routledge.


Educational Studies in Mathematics, 71, 263-278.*


http://aaee-scholar.pbworks.com/Research+Method++Phenomenography


*Instructional Science*, 10(2), 177-200.


Rojano, Y. (1996). The Role of Problems and Problem Solving in the Development of


Usiskin, Z. (1995). Why is Algebra Important to Learn: (Teachers, This One’s for You


ABSTRACT

BEGINNING ALGEBRA STUDENT’S CONCEPTUAL UNDERSTANDING OF VARIABLES IN THE CONTEXT OF SOLVING EQUATIONS: A CONCEPTUAL CHANGE INQUIRY.

by

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December 2015

Advisor: Dr. Thomas G. Edwards

Major: Curriculum and Instruction

Degree: Doctor of Education

This dissertation aims to illuminate novice algebra student’s conceptual understandings of variables, solving equations, and the juxtaposition of the two. The goal of this study is to design a curricular intervention for a beginning Algebra 1 course and document, analyze, and interpret conceptual changes beginning algebra students experience in their understanding of variables and solving equations. Previous literature in this field indicate that students are not grasping a concrete conceptual understanding of algebraic topics starting at the very basic building blocks of algebra. This study advances our understanding of how we can alleviate some students’ frustrations and misunderstandings of variables, solving equations, and the juxtaposition of the two. The research was conducted using Conceptual Change Inquiry Learning and Phenomenography as theoretical frameworks. This qualitative study used data from field observations and notes, audio recordings, student work, and responses from semi-structured group interviews to retrieved results. Findings from the research show that helping students to gain a conceptual change involves curriculum interventions, various pedagogical techniques, and participation from students in classroom discourse. The following themes related to supporting students in gaining a conceptual change themes emanated from the study’s findings: (a) teacher influence, (b) real-world value/defining the variable, (c) classroom discourse, and (d) time on
task. The results, implications for teacher education, and implications for future research are discussed.
When I graduated from Western Michigan University with a Bachelor’s Degree in Secondary Mathematics Education and began my career as an educator, I had a passion for teaching. During my student teaching, I grew to love teaching mathematics more than I thought I ever would. I knew the field of education was the path I was born to take. My experiences at Western Michigan University made me realize how teachers can shape a person’s life.

Early in my career as an educator, I realized that teachers are limited in the decision making process about Curriculum and Instruction and/or Educational Policies. This led me to pursue a Master’s Degree in Educational Leadership at Eastern Michigan University. As a life-long learner, I wasn’t satisfied and continued my education by pursuing my Educational Specialist Degree in Administration and General Supervision at Wayne State University. After graduating, I really began to consider leaving teaching and becoming an administrator so that I could really implement effective change in education. I continued my education by applying and being accepted into the doctoral program at Wayne State University in Curriculum and Instruction with an emphasis in K-12 Mathematics Education. I have since left teaching and am currently a high school principal at a small high school in Michigan.