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An optimization of on-line monitoring of simple linear and polynomial quality functions

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AN OPTIMIZATION OF ON-LINE MONITORING OF SIMPLE LINEAR AND POLYNOMIAL QUALITY FUNCTIONS

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DEDICATION

To the soul of my mother

To my father, my sisters and my brother

To my wife, Rwaida

and

To my child, Cedra

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My great thanks must go to my supervisor, Professor Kai Yang who was abundantly helpful and offered invaluable support.

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CHAPTER 1 INTRODUCTION TO PROFILE MONITORING

This chapter provides an introduction on profile monitoring topic and some of existing profiling techniques considered in this research. Following the profile monitoring background is a summary of prior work (literature review) upon which the contributions of this thesis are built.

1.1 Background

We define profile monitoring as a relatively new trend in quality control applications used where the data of the process or the product follow a certain profile at each time interval. The main idea for profile monitoring is to model the quality profile (i.e., simple linear, polynomial or nonlinear, etc) and then monitor the fitted profiles over time to check if these profiles have been changed due to assignable causes. Corrective action is needed if process or product parameters are changed.

Over the last decade, several profiling techniques have been developed and examined in terms of their effectiveness in detecting deviations in process parameters when the quality might be explained by a simple, multiple linear models or much more sophisticated models such as nonlinear regressions models.

In fact, the use of profile monitoring techniques in statistical control applications (SPC) studies has been extended to include both phases I and II. In phase I, practitioners are mainly interested in analyzing a historical data to examine the statistical stability of process parameters and estimate their nominal values. In phase II, on-line data is used to detect any anticipated changes in the nominal values estimated in phase I.

1.2 Measuring the Performance of Profile Monitoring Techniques

As mentioned above and under profile monitoring framework, Phase II methods focus in reducing the effort of detecting changes in parameters of quality model. The performance of these techniques is usually measured and evaluated by using the parameters of the run-length distribution.

Figure 1 depicts how average run-length (ARL) might be estimated by using the probability that the statistic of the current profile falls outside the control limits. Figure 2 shows the situation where special causes has shifted the process parameter from θ_0 to θ_1

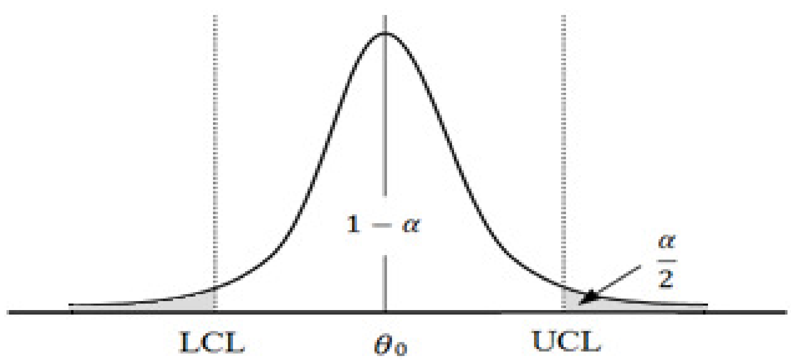

Fig. 1 Probability of falling out of control limits under θ_0

Then,

$$ARL = \frac{1}{\alpha} \tag{1}$$

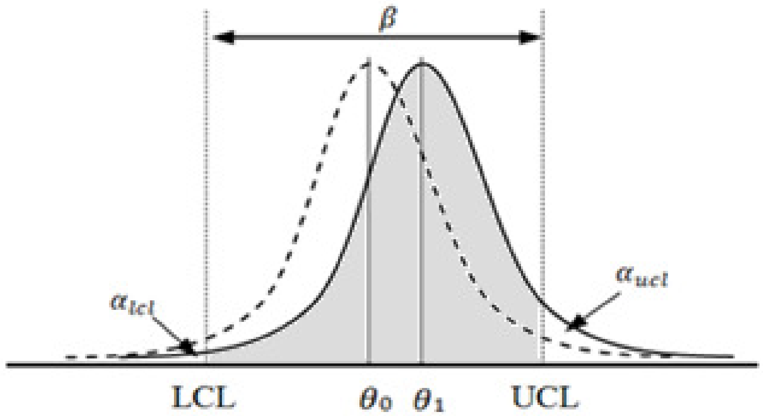

Fig.2 Probability of falling out of control limits under θ=θ_1

In this situation ($\theta_0 \rightarrow \theta_1$), the ARL can be estimated as follows:

$$ARL = \frac{1}{\alpha_{ucl} + \alpha_{lcl}} = \frac{1}{(1-\beta)_{\theta_1}} \qquad (2)$$

where β is the probability of falling between the control limits when the process runs at θ_1 .

1.3 Examples in Profile Monitoring

This section demonstrate some of profile monitoring examples used to examine the effectiveness of some of profiling techniques in detecting any changes in quality models.

Mestek et al. (1994) considered the photometric determination of Fe3+ with sulfosalicylic acid and examined the stability of the calibration curves. Stover and Brill (1998) considered the linear calibration of the multilevel ion chromatography and showed how to determine the response stability and the most proper calibration frequency. Kang and Albin (2000) presented another two different examples. One of these is an illustrative example where the relationship between the amount of dissolved sweetener aspartame and the level of temperature is better to be explained by a non-linear model. In the other one, they used a simple linear model to describe the semiconductor manufacturing process.

When it comes to the polynomial profiles, Montgomery (2005) used a second-order polynomial profile to describe the relationship between the automobile engine speed and the torque produced by an engine. The problem of violating homogeneity in data-rich environment is considered by Wang et al. (2005); charting schemes based on the quantile-quantile (Q-Q) plot in addition to profiling techniques are suggested to decrease the speed of detection. An example form a mobile phone assembly is used and tested. Another use of polynomial profiles in an industry can be found in Amiri et al. (2009). Researchers intersecting in finding more examples and comprehensive reviews of profile monitoring should read Woodall et al. (2004) and Woodall (2007).

1.4 Prior Related Work in Profile Monitoring

1.4.1 Use of simple linear regression: The results of the literature survey have shown that the majority of the previous work considered the simple linear profile framework gave a considerable attention to phase II analysis.

A. Phase I Approaches: Mestek et al. (1994) suggested the use of a Hotelling T^2 control scheme to examine the linear calibration curves in the photometric determination of Fe3+ with sulfosalicylic acid. Stover and Brill (1998) proposed and examined two Phase I methods for determining instrument response stability. The first approach is a Hotelling's T^2 control chart. The second is a univariate chart based on the first principal component analysis (PCA) corresponding to the vector of estimators. These two methods were applied to multilevel ion chromatography. Kang and Albin (2000) used the Hotelling- T^2 charting techniques for monitoring process parameters changes when the quality is described by a simple linear relationship. This method is similar to T^2 method suggested by Stover and Brill (1998), but Kang and Albin (2000) used different estimators for the variance covariance matrix. Mahmoud and Woodall (2004) suggested and tested a phase I method using global F-test to monitor the model parameters in conjunction with a univariate control chart for monitoring changes in standard deviation (σ). In their work, they examined the effectiveness of the F-test technique by comparing its statistical performance with several of existing profiling methods. Again, Mahmoud et al. (2007) developed another phase I method called change point approach based on the segmented regression technique for checking the stability of the model parameters.

B. Phase II Approaches: Kang and Albin (2000) suggested two phase II methods for monitoring simple linear profiles. The first method is a multivariate Hotelling T^2 chart based on successive vectors of the estimators of the intercept and slope. In the second method, they considered used an exponential weighted moving average control chart. They suggested the use of $EWMA$ to monitor the average deviation and R control chart to monitor variation of the deviation. This method is referred to as $EWMA/R$ method. Kim et al. (2003) suggested a new Phase II method recommended coding the X-values to remove the correlation between regression estimators and then they used two separate $EWMA$ control charts for monitoring these estimators. For monitoring a process standard deviation, they recommended the use of the $EWMA$ control charts developed by Crowder and Hamilton (1992). A phase II comparative study between the Croarkin and Varner (1982) control chart (National Institute of Standard Technology ($NIST$) method) and the combined approach (KMW method) proposed by Kim et al. (2003) was performed by Gupta et al. (2006). In this work, they replaced the three $EWMA$ control charts of KMW method by three univariate control charts. The $NIST$ method is described in the $NIST/SEMATECH$ e-Handbook of Statistical Methods, which is available online at http://www.itl.nist.gov/div898/handbook/. Noorossana and Amiri (2007) investigated the effectiveness of using a combination of *MCUSUM* proposed by Healy (1987) and χ^2 control chart for monitoring the regression parameters of simple linear profiles. Zou et al. (2006) suggested a Phase II technique, based on likelihood ratio statistic, to monitor simple linear quality profiles. Seyed, et al. (2007) proposed a control chart based on the generalized linear test to monitor parameters of the linear profiles and an R- control chart to monitor the variance; they refer to this combination as GLT/R chart. Zou et al. (2007-b) considered the case when the process parameters are unknown or Phase I samples are not large enough for proper estimation of simple linear function parameters; this approach is referred to as a self starting approach. Mahmoud et al. (2009) proposed and investigated the statistical performance of simple linear approaches when only two observations are used. In this study, they proposed an $EWMA$ control chart based on average squared deviations with two $EWMA$ control charts to monitor changes in the regression parameters of the simple linear quality profiles. The performance of T^2, $EWMA/R$ and $EWMA3$ methods under drift shift is investigated by Saghaei et al. (2009-a). The speed of detecting changes in regression parameters of simple linear profiles was also investigated by using a cumulative sum statistic; the results were presented in Saghaei et al. (2009-b). An approach using a single chart integrating likelihood ratio statistic with the $EWMA$ chart for monitoring linear profiles was developed by Zhang J. et al. (2009). For further performance improvement, they added the variable sampling interval (VSI) feature to the suggested technique. A new approach proposed by Zhu and Lin (2010) to monitor changes in the slope of the simple linear functions.

Another recent contribution by Noorossana et al. (2010) explored the performance of three control chart schemes when several correlated characteristics might be modeled as a set of linear functions of one independent variable. They referred to this situation as multivariate simple linear profiles structure. Li and Wang (2010) used an exponentially weighted moving average control chart using variable sampling intervals for monitoring simple linear profiles ($VSI - EWMA3$). In their study, the performance of this strategy is invetigated by using a real set of data.

1.4.2 Effect of violating normality and correlation assumptions: One of the essentials assumptions in the monitoring of the simple linear profiles is that the error is independent and identically normally distributed. Some other authors have showed interest in studying the impact of violating this basic assumption. For example, Noorossana et al. (2004) studied the effect of violating normality assumption of the error terms on the performance of $EWMA/R$ method.

Auto-correlated errors are usually within profile monitoring. Noorossana et al. (2007) explored the effect of ignoring autocorrelation of the error terms within profiles. Jensen et al. (2008) considered the correlation structure between linear profiles and investigated the effectives of a new technique accounting this issue. The results of studying the effect of the first order autocorrelation between linear profiles can be found in Noorossana et al. (2008).

Soleimani et al. (2009) presented an analytical study to investigate how the speed of catching changes in the regression parameters of simple linear profiles is influenced by within profiles autocorrelation. Qiu and Wang (2010) investigated the situation when nonparametric profiles are correlated.

1.4.3 Other profile modeling approaches: Sometimes, the process quality is better to be described by more sophisticated models such as polynomial, or multiple linear profiles rather than the simple linear profiles. In this section some of these models are presented.

A. Polynomial and multiple regressions: Zou et al. (2007-a) proposed a technique that integrates the multivariate exponentially weighted moving average control chart (MEWMA) with the GLR test based on nonparametric regression. Three different phase I methods for monitoring polynomial profiles were examined by Kazemzadeh et al. (2008-a); they also provided an approach based on likelihood ratio test to identify the shift location. Kazemzadeh et al. (2008-b) studied the performance of some profiling methods for detecting outliers in phase I of polynomial profiles. Kusiak et al. (2008) developed three curves, one by the least squares method and the other by maximum likelihood estimation method. They used the least square (parametric) model and non-parametric models for on-line monitoring of the power curve. Mahmoud (2008) introduced a phase I approach for monitoring multiple linear regression profiles. Kazemzadeh et al. (2009-b) considered the second-order polynomial profiles and introduced a new technique based on the idea of transforming the polynomial model to the orthogonal form. Then, the three regression parameters will be independent and one can use three individual $EWMA$ control charts to monitor them in conjunction with another $EWMA$ chart for monitoring the residuals ($\mathrm{EWMA4}$ method). Kazemzadeh et al. (2007) and (2009-a) considered polynomial profiles, and the autocorrelation between profiles is modeled as a first order-autoregressive. Zhang H. et al. (2009) developed a method that deals with the profiles as vectors in high-dimension space. He applied a χ^2 charting technique to explore and identify the outliers.

B. Parametric nonlinear regression models: Ding et al. (2006) presented a phase I technique for monitoring nonlinear quality functions (profiles). The suggested policy consists of two components: 1- Data-reduction component; 2- Data-separation technique. Williams et al. (2007-a) suggested and investigated the use of the T^2 chart for monitoring the coefficients of nonlinear regression models. The results of investigating the effect of correlation on nonlinear quality profiles using nonlinear mixed models can be found in Jensen and Birch (2007). Again, Williams et al. (2007-b) used the nonlinear regression method of Williams et al. (2007-a) to monitor dose-response quality profiles; in their study, they utilized a four-parameter logistic regression model to represent these profiles. Moguerza et al. (2007) explored and examined the monitoring of the fitted curves instead of monitoring the parameters.

C. Use of wavelets: Here we introduce the Wavelets as another method to be used for present quality profiles when simple models mentioned before are not enough to represent the pattern of the profile. Several authors, considered the use of the Wavelets to introduce methods for monitoring variability and changes in process quality. For instance, Reis and Saraiva (2006) utilized wavelets based technique to represent the surface of a paper. In their work Zhou et al. (2006) investigated a Monitoring System for Cycle-based Waveform Signals. Another contribution introduced by Jeong et al. (2006); they considered the uses of wavelet for complicated functional data. Chicken et al. (2009) developed and tested a semi parametric wavelet method for monitoring changes in nonlinear quality functions.

CHAPTER 2 ADAPTIVE SAMPLING SIZES (VSS) AND ADAPTIVE SAMPLING INTERVALS T^2 SCHEMES

Most of the traditional statistical process control applications describe the quality by the probability distribution of a univariate quality characteristic or by the multivariate probability distribution of a set of quality features. Sometimes, the quality of a process is better to be explained by a functional relationship between a quality response variable and one or more independent variables. Literatures usually refer to this type of monitoring techniques by *profiling techniques*.

Literature survey has shown that several Phase II charting techniques have been developed, and their ability to detect changes in simple linear and polynomial profiles is examined. Phase II charting methods assume that the values of quality function parameters such as intercept, slope and variance parameters are known or estimated in phase I.

2.1 Multivariate approach (T^2 method)

This method has been extensively used when the quality is described by multivariate distribution of a set of quality characteristics. Kang and Albin (2000) suggested and examined the use of this method when the quality function is described by a simple linear relationship. Some literatures refer to this method as a Multivariate Approach.

This approach assumes that the process outgoing quality variable Y is a random variable, and it has a simple linear functional relationship with process input X; that is

$$y_{ij} = A_0 + A_1 x_i + \varepsilon_{ij} \tag{3}$$

where A_0 and A_1 are the nominal values of process parameters, intercept and slope, and $\varepsilon_{ij} \sim N(0, \sigma^2)$.

The multivariate method is based on the use of vectors of the estimators of intercept and slope to monitor the deviations in the linear quality profile. The process starts by collecting a set of observations of size n and then calculates the Hotelling's statistic as follows:

$$T_j^2 = \left(Z_j - U\right)^T \Sigma^{-1}\left(Z_j - U\right) \tag{4}$$

where $U = (A_0, A_1)^T$ is the vector of the target values of the intercept and the slope, and $Z_j = \left(a_{0j}\, , a_{1j}\right)^T$ is the vector of the estimated values of the process parameters. The variance–covariance matrix (Σ) is as follows :

$$\Sigma = \begin{pmatrix} \sigma_0^2 & \sigma_{01}^2 \\ \sigma_{01}^2 & \sigma_1^2 \end{pmatrix} \tag{5}$$

The joint probability density function of $Z_j \sim N(U, \Sigma)$ is:

$$f\left(Z_j\right) = \frac{1}{\sqrt{(2\pi)^2 \det(\Sigma)}} exp\left(-\frac{1}{2}(\left(Z_j - U\right)^T \Sigma^{-1} \left(Z_j - U\right)\right) \tag{6}$$

Kang and Albin (2000) used the least square method (LSM) to estimate the unknown values of the regression parameters such that $a_{0j} = \bar{Y} - a_{1j}\bar{x}$ and $a_{1j} = S_{xy(j)}\, S_{xx}^{-1}$. The variances of these parameters can be estimated by using $\sigma_0^2 = (\sigma^2 n^{-1} + \bar{x}^2 \sigma^2\, S_{xx}^{-1})$ and $\sigma_1^2 = \sigma^2\, S_{xx}^{-1}$.

However, the covariance of a_{0j} and a_{1j} is calculated using $\sigma_{01}^2 = -\sigma^2 \bar{x}\, S_{xx}^{-1}$. At the state of the statistical control, no assignable cause is present, the Hotelling statistics behaves as a central $\chi^2_{v,\alpha}$ distribution with v =2 degrees of freedom (see Kang and Albin (2000)). Based on that the following control limit is used:

$$cl = \chi^2_{2,\alpha} \tag{7}$$

The following are the decision rules used to judge about the process stability.

1. If $0 \le T_j^2 < CL$; the process is under control, continue
2. If $T_j^2 \ge CL$; the process is out-of-control; corrective action is required.

Example 2.1

In this example, we assume that the relationship between the response and the explanatory variable is described by $y_{ij} = 4 + 3x_i + \varepsilon_{ij}$; where ε_{ij} is normally distributed with mean=μ and variance=σ^2.

The design parameters are as follows:

1- The sample Size (n)= 4

2- The values of $X = \{\, 1\ 2\ 3\ 4\}$

3- The false alarm rate (1/α) =200, then the control limits (CL)=10.60

Table 1 and Figure 3 show the result of running 10 phase II profiles using the Hotelling T^2 chart.

Table 1 The results of running ten profiles using T^2 chart (Example 2.1)

j	Y_{1j}	Y_{2j}	Y_{3j}	Y_{4j}	a_o	a_1	T^2	Decision
1	7.18	10.10	11.61	15.60	4.43	2.68	0.191	In-Control
2	6.72	9.60	13.06	15.49	3.77	2.98	0.063	In-Control
3	8.59	9.50	12.87	15.32	5.68	2.36	2.845	In-Control
4	6.59	10.17	12.52	14.98	4.19	2.75	0.050	In-Control
5	6.52	8.98	14.86	15.85	3.08	3.39	0.849	In-Control
6	8.54	9.66	12.38	14.36	6.20	2.02	4.848	In-Control
7	6.85	9.95	11.71	17.54	3.06	3.38	0.888	In-Control
8	5.98	11.86	12.85	15.95	3.93	3.09	0.007	In-Control
9	5.36	8.06	13.92	13.95	2.41	3.16	2.759	In-Control
10	9.89	8.34	15.02	14.62	6.75	2.09	7.686	In-Control

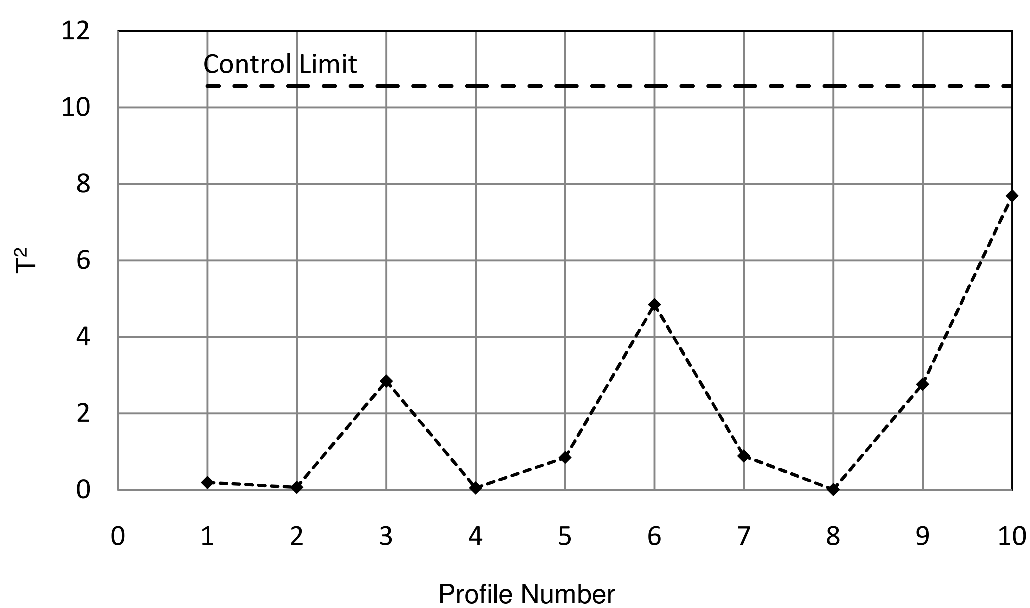

Fig. 3 Graphical presentation of Hotelling T^2 chart in Example 2.1

2.2 Performance Measures of T^2 Method

It is worthy to mention that, two types of performance measures are usually used for examining the statistical effectiveness of charting techniques. These two types are:

1- The initial-state performance measures: This type assumes that shift occurs at the beginning of the monitoring stage.
2- The steady state performance measures: This type assumes that the shift in the process parameters will appear at a future.

In fact, the average number of samples collected until the off-target signal detected (ARL) is the most common measure to assess the statistical performance of a Phase II T^2 method. In the case when more than one sampling interval are used (VSI), the expected value of the time from the beginning of the monitoring stage until the chart signals (ATS) is the recommended performance measure.

$$ARL = \frac{1}{P(T^2 > CL)} = \frac{1}{\alpha} \tag{8}$$

$$ATS = ARL * Sampling\ Interval \tag{9}$$

Usually, when a new profiling technique is developed, its statistical performance and effectiveness is evaluated by comparing its performance with its counterparts or some of the existing methods. This process requires matching the performance measures of the compared methods at the state of the statistical control, no changes in the model parameters, and then investigating and comparing their performance at the off-target conditions. In this research, the same procedure will be used to evaluate the performance of any proposed scheme. Additionally, when the quality of a product or a process is described by the multivariate probability distribution of quality characteristics, literature review shows that the charting techniques using variable design settings during the online monitoring stage have been widely and extensively utilized to enhance the statistical efficiency of many of the charting techniques. One of the objectives of this research is integrating the three known adaptive scenarios; VSS, VSI and VSSI with the multivariate approach (T^2 method), proposed by Kang and Albin (2000), and evaluating its statistical ability in monitoring changes in regression parameters of simple linear quality profiles.

2.3 Variable Sampling Size Scheme (VSS-T^2)

This VSS-T^2 scheme uses two sampling sizes, such that $n_1 < n_2$, and one warning limit (WL). The mechanism of the suggested scheme is described by Figure 4. Here, it is important to recommend that the first sample is taken using the large size (n_2); such step might helpful in detecting and capturing changes due to improper initial process setting. In practical situation, one more thing I would like to be considered here is that the warning limit is selected such that we do not lose the advantage of enlarging the sample size when there is an indication of process change. The following example is used to explain the mechanism of the suggested scheme.

Example 2.2

This example uses the same regression model described in example 2.1. The design parameters are:

1- The sampling sizes (n_1, n_2)= 3 and 5

2- The warning limit = 7.85 and the control limit = 10.60

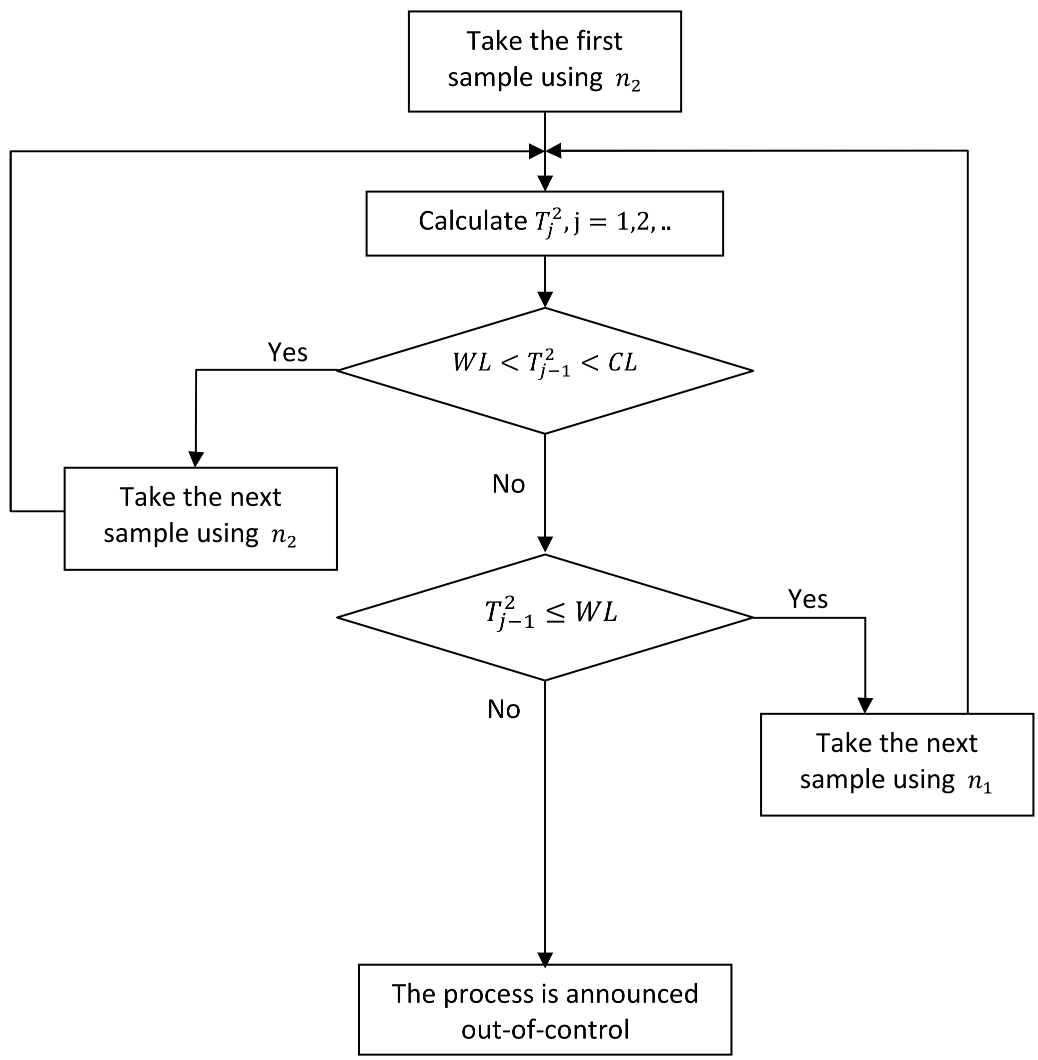

Fig. 4 The mechanism of adaptive sampling sizes approach

Table 2 The mechanism of adaptive sampling sizes Hotelling T^2 chart (Example 2.2)

Profile Number (j)	T^2	Decision
1	5.32	In-Control, use n_1
2	4.43	In-Control, use n_1
3	7.92	In-Control, use n_2
4	6.32	In-Control, use n_1
5	8.98	In-Control, use n_2
6	5.76	In-Control, use n_1
7	4.59	In-Control, use n_1
8	9.41	In-Control, use n_2
9	10.87	Out-of-Control

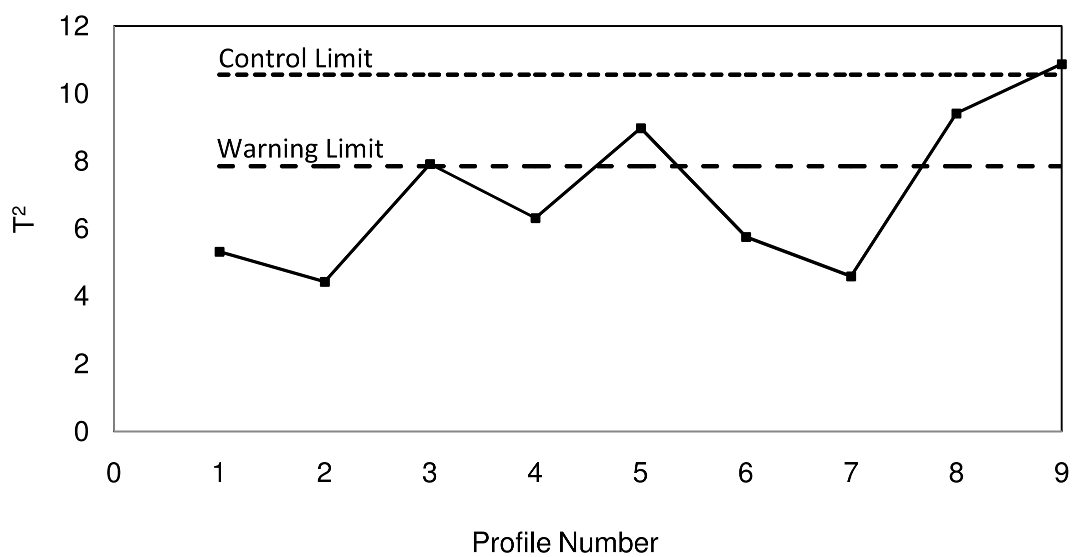

Fig. 5 Graphical presentation of adaptive sampling size Hotelling T^2 chart

The ARL as a performance measure of the suggested approach will be estimated and compared with the traditional T^2 control chart using fixed design setting. All compared schemes will be designed such that they have the same in-control average run length. In order to compare the adaptive and the traditional schemes under the same conditions, we select the warning limit such that $n_0 = n_1\, p_1 + n_2 p_2$; where p_1 and p_1 are the area under the warning limit and the area between the warning limit and the control limit, respectively.

2.3.1 Extending the ARL approximation: Here I will briefly describe how the Markov Chain Principles have been used by several authors such as Aparisi (1996), and Aparisi and Haro (2001), to evaluate the performance of adaptive T^2 control chart and then I show how this method might be extended to the profile monitoring framework.

In their work, they defined three states and suggested the transition probability matrix described by 10:

1. State 1 represents $0 \leq T_j^2 < WL$
2. State 2 represents $WL < T_j^2 < CL$
3. State 3 represents $T_j^2 \geq CL$ (absorbing state)

$$P_{All} = \begin{pmatrix} p_{11}^{\delta} & p_{12}^{\delta} & p_{13}^{\delta} \\ p_{21}^{\delta} & p_{22}^{\delta} & p_{23}^{\delta} \\ 0 & 0 & 1 \end{pmatrix} \tag{10}$$

where p_{ij}^{δ} is the probability of moving from state *i* to state *j;* see Figure 6. Second, they used the following ARL approximation; note that the symbols might be different:

$$ARL_{\delta} = P_0{}^{'}(I - P_{\delta})^{-1}1 \tag{11}$$

where $P_0{}' = (P_1, P_2) \quad = \left(\frac{P\left(T_j^2 \le WL\right)}{P\left(T_j^2 < CL\right)}, \frac{P\left(WL < T_j^2 < CL\right)}{P\left(T_j^2 < CL\right)}\right)$; where $P_1 + P_2 = 1$, I is the identity matrix, P_δ is the P_{All} matrix without the probabilities of the absorbing state.

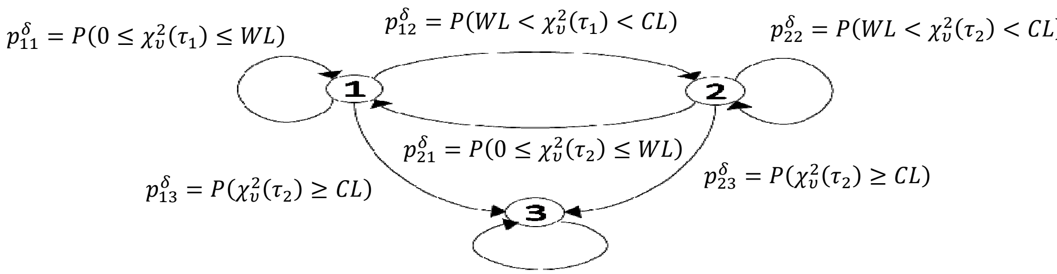

Fig. 6 The probability transition diagram

Kang and Albin (2000) reported that, under the off-target conditions, the Hotelling T^2 statistic follows a non-central $\chi^2_{v,\alpha}$ distribution; this distribution has a non-centrality parameter (τ) as follows:

$$\tau = (\lambda + \beta\bar{x})^2 n + \beta^2 S_{xx} \tag{12}$$

Since we extend the ARL approximation to the profile monitoring framework, and we are using variable sampling sizes, the Equation 11 might be rewritten in terms of the non-centrality parameter as follows:

$$ARL_\delta = \left(\frac{P(\chi_v^2 < WL)}{P(\chi_v^2 < CL)}, \frac{P(WL < \chi_v^2 < CL)}{P(\chi_v^2 < CL)}\right)\left(\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \begin{pmatrix}P(0 \le \chi_v^2(\tau_1) < WL) & P(WL < \chi_v^2(\tau_1) < CL)\\ P(0 \le \chi_v^2(\tau_2) < WL) & P(WL < \chi_v^2(\tau_2) < CL)\end{pmatrix}^{-1}\right)\begin{pmatrix}1\\ 1\end{pmatrix} \tag{13}$$

2.3.2 VSS-T^2 scheme with no specified range of explanatory variable: In this part of the study the VSS-T^2 scheme is investigated and compared with the traditional chart when the independent variable values are not bounded by a certain range and it takes values from 1 to n. For instance, if n=5, then X={ 1 2 3 4 5}. Here the two types of shift in regression parameters, intercept and slope, will be considered. In the situation where the deviation in the intercept is the only shift of interest, Tables 3, 4, 5 and 6 show the ARL values the adaptive and the fixed schemes at four different values of n_0. The next four tables illustrate that the adaptive scheme performs better than the traditional one over all the levels of changes.

Table 3 VSS-T^2 versus FSR-T^2 when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$, $ARL_{\delta=0}$=200 n_0=5 and unspecified range of X values

No.	Sampling Sizes	Shift in Intercept, λ						
		0.15	0.30	0.45	0.60	0.75	0.90	1.0
1	(5,5)	152.45	82.76	41.38	21.21	11.54	6.75	4.92
2	(4,6)	152.14	80.96	38.72	18.79	9.78	5.62	4.12
3	(3,7)	151.71	78.61	35.67	16.40	8.28	4.76	3.54

Table 4 VSS-T^2 versus FSR-T^2 when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$, $ARL_{\delta=0}$=200, n_0=6 and unspecified range of X values

No.	Sampling Sizes	Shift in Intercept, λ						
		0.15	0.30	0.45	0.60	0.75	0.90	1.0
1	(6,6)	145.18	72.61	34.01	16.68	8.85	5.14	3.76
2	(5,7)	144.84	70.93	31.81	14.86	7.63	4.41	3.27
3	(4,8)	144.39	68.84	29.38	13.11	6.60	3.86	2.92
4	(3,9)	143.83	66.43	26.92	11.55	5.78	3.45	2.67

Table 5 VSS-T^2 versus FSR-T^2 when intercept shifts from $A_0 \to A_0 + \lambda\sigma_0$, $ARL_{\delta=0}$=200, n_0=7 and unspecified range of X values

No.	Sampling Sizes	Shift in Intercept, λ						
		0.15	0.30	0.45	0.60	0.75	0.90	1.0
1	(7,7)	138.46	64.29	28.48	13.48	7.04	4.09	3.02
2	(6,8)	138.10	62.73	26.64	12.09	6.17	3.61	2.71
3	(5,9)	137.65	60.86	24.67	10.77	5.44	3.24	2.49
4	(4,10)	137.08	58.75	22.70	9.60	4.85	2.96	2.33

Table 6 VSS-T^2 versus FSR-T^2 when intercept shifts from $A_0 \to A_0 + \lambda\sigma_0$, $ARL_{\delta=0}$=200, n_0=8 and unspecified range of X values

No.	Sampling Sizes	Shift in Intercept, λ						
		0.15	0.30	0.45	0.60	0.75	0.90	1.0
1	(8,8)	132.23	57.37	24.22	11.15	5.76	3.37	2.51
2	(7,9)	131.87	55.93	22.67	10.06	5.13	3.04	2.32
3	(6,10)	131.40	54.25	21.05	9.04	4.60	2.79	2.17
4	(5,11)	130.84	52.38	19.44	8.14	4.17	2.60	2.07

For more understanding of the performance of the VSS-T^2 control chart, the next section is dedicated for studying the effect of the distance between the sampling sizes when the process is under a shift in the intercept only.

Table 7 The ARL versus $(n_2 - n_1)$ when intercept shifts by $\lambda\sigma_0$, n_0=5, and unspecified range of X

No.	Sampling Sizes	Distance $(n_2 - n_1)$	λ=0.45, β=0
1	(5,5)	0	41.38
2	(4,6)	2	38.72
3	(3,7)	4	35.67

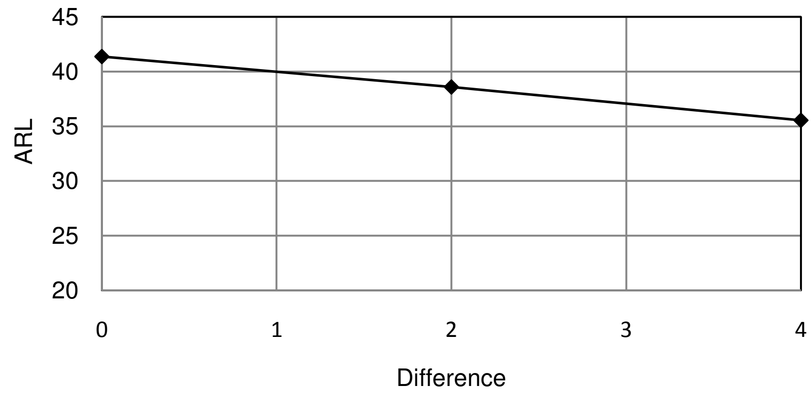

Fig. 7 The ARL values of VS-T^2 versus the distance between the sampling sizes when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$ and n_0=5

Table 8 The ARL versus $(n_2 - n_1)$ when intercept shifts by $\lambda\sigma_0$, n_0=6, and unspecified range of X

No.	Sampling Sizes	Distance $(n_2 - n_1)$	λ=0.45, β=0
1	(6,6)	0	34.01
2	(5,7)	2	31.81
3	(4,8)	4	29.38
4	(3,9)	6	26.92

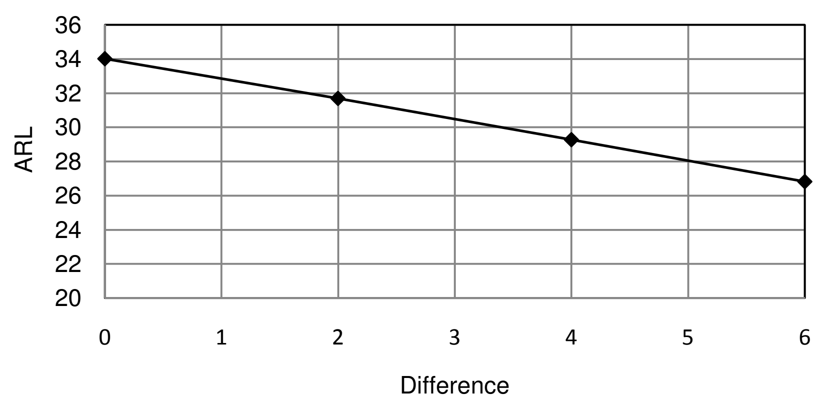

Fig. 8 The ARL values of VS-T^2 versus the distance between the sampling sizes when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$ and n_0=6

Table 9 The ARL versus $(n_2 - n_1)$ when intercept shifts by $\lambda\sigma_0$, n_0=7, and unspecified range of X

No.	Sampling Sizes	Distance $(n_2 - n_1)$	λ=0.45, β=0
1	(7,7)	0	28.48
2	(6,8)	2	26.64
3	(5,9)	4	24.67
4	(4,10)	6	22.70

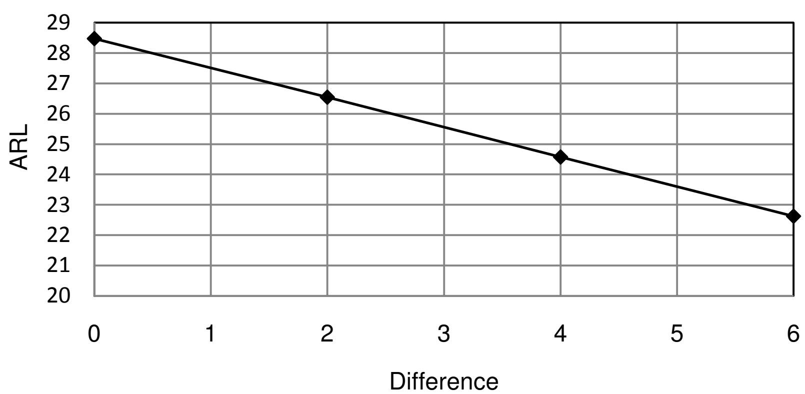

Fig. 9 The ARL values of VS-T^2 versus the distance between the sampling sizes when intercept shifts from $A_0 \to A_0 + \lambda\sigma_0$ and n_0=7

Table 10 The ARL versus $(n_2 - n_1)$ when intercept shifts by $\lambda\sigma_0$, n_0=8, and unspecified range of X

No.	Sampling Sizes	Distance $(n_2 - n_1)$	λ=0.45, β=0
1	(8,8)	0	24.22
2	(7,9)	2	22.67
3	(6,10)	4	21.05
4	(5,11)	6	19.44

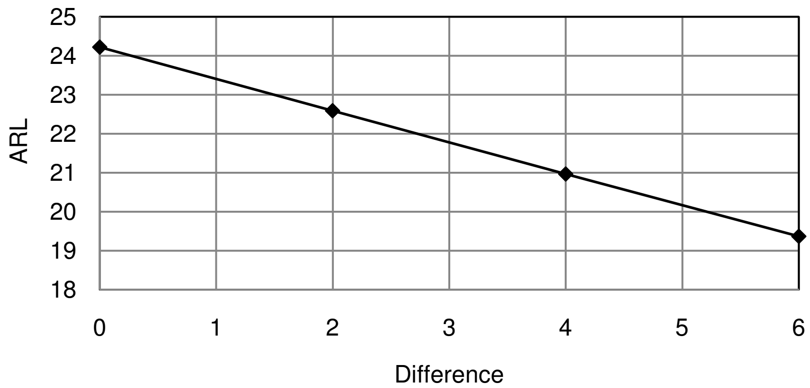

Fig. 10 The ARL values of VS-T^2 versus the distance between the sampling sizes when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$ and n_0=8

It can be seen from Figures 7, 8, 9 and 10 that the performance of the adaptive scheme improves as the difference between the two sampling sizes increases.

However, if the slope shifts from $A_0 \rightarrow A_0 + \beta\sigma_0$ then τ is described as follows:

$$\tau = (\beta\bar{x})^2 n + \beta^2 S_{xx} \qquad (14)$$

It can be easily noticed that the value of the non-centrality parameter, described by 14, is affected not only by the sample size but also the average and the sum of squares of the explanatory variable.

Based on that, improving the performance of the VSS-T^2 control chart requires considering the effect of location of the independent variable; this suggestion will be considered in chapter 3.

In most of the application of the profile monitoring, the values of the X are usually distributed over a certain range. In the next section we will examine the VSS-T^2 when these values are equally spaced over a range from 1 to 6.

2.3.3 VSS-T^2 scheme with specified range of explanatory variable: In this part of the study the VSS-T^2 scheme will be compared with the traditional chart when the independent variable values are bounded and two points are set at the edges. For instance, if n=5 and the range is from 1 to 6, then the set of X values is equal to {1 2.25 3.5 4.75 6}.

From Equation 12, it can be easily conclude that, if the intercept changes from $A_0 \rightarrow A_0 + \lambda\sigma_0$, then the new value of τ is

$$\tau = n\lambda^2 \tag{15}$$

Equation 15 shows that the location of the independent variable has no effect on the value of the non-centrality parameter then the results will not differ from the results of the case when the values of X are unbounded (See tables 3-6).

Table 11 VSS-T^2 versus FSR-T^2 when slope shifts from $A_1 \rightarrow A_1 + \beta\sigma_0$, $ARL_{\delta=0}$=200, n_0=5 and specified range of X values

No.	Sampling Sizes	Shift in slope, β						
		0.03	0.06	0.09	0.12	0.15	0.18	0.20
1	(5,5)	168.28	110.03	64.91	37.66	22.31	13.71	10.14
2	(4,6)	168.14	108.98	62.82	35.20	20.04	11.90	8.66
3	(3,7)	167.87	107.47	60.15	32.37	17.72	10.23	7.39

Table 12 VSS-T^2 versus FSR-T^2 when slope shifts from $A_1 \rightarrow A_1 + \beta\sigma_0$, $ARL_{\delta=0}$=200, n_0=6 and specified range of X values

No.	Sampling Sizes	Shift in slope, β						
		0.03	0.06	0.09	0.12	0.15	0.18	0.20
1	(6,6)	163.36	100.68	56.19	31.22	17.92	10.78	7.91
2	(5,7)	163.22	99.66	54.32	29.17	16.16	9.46	6.87
3	(4,8)	163.00	98.30	52.06	26.92	14.41	8.27	5.98
4	(3,9)	162.67	96.61	49.51	24.64	12.81	7.27	5.27

Table 13 VSS-T^2 versus FSR-T^2 when slope shifts from $A_1 \rightarrow A_1 + \beta\sigma_0$, $ARL_{\delta=0}$=200, n_0=7 and specified range of X values

No.	Sampling Sizes	Shift in slope, β						
		0.03	0.06	0.09	0.12	0.15	0.18	0.20
1	(7,7)	158.65	92.51	49.14	26.30	14.71	8.72	6.37
2	(6,8)	158.50	91.51	47.46	24.58	13.32	7.73	5.62
3	(5,9)	158.29	90.25	45.50	22.76	11.98	6.86	4.99
4	(4,10)	158.01	88.74	43.36	20.93	10.75	6.12	4.48

Table 14 VSS-T^2 versus FSR-T^2 when slope shifts from $A_1 \rightarrow A_1 + \beta\sigma_0$, $ARL_{\delta=0}$=200, n_0=8 and specified range of X values

No.	Sampling Sizes	Shift in slope, β						
		0.03	0.06	0.09	0.12	0.15	0.18	0.20
1	(8,8)	154.15	85.33	43.36	22.48	12.31	7.22	5.27
2	(7,9)	153.99	84.36	41.85	21.02	11.20	6.47	4.72
3	(6,10)	153.78	83.17	40.13	19.51	10.15	5.81	4.26
4	(5,11)	153.52	81.78	38.28	18.02	9.19	5.26	3.88

The results in Tables 11-14 show the ARL comparisons of the two sampling scheme when the slope shifts from its nominal value A_1 to $A_1 + \beta\sigma_0$. In this study four different sampling sizes are used and the in-control ARL is set equal to 200. The values of X are equally spaced between 1 and 6. Like the case when the intercept shifts from the nominal values, the adaptive scheme performs better than the FSR-T^2 one at all levels of shift in slope.

2.3.4 Optimizing the Design of the Adaptive Scheme (VSS-T^2): Finding the optimal settings of the adaptive approach might be formulated as an optimization problem and solved by using one of the optimization techniques, such as the Genetic Approach. The following is the mathematical model:

Min $ARL\ (\nu, \delta, WL, n_1, n_2)$ $$(16)$$

Subject to:

$$n_0 = n_1\, p_1 + n_2 p_2 \tag{17}$$

$$n_1 < n_0 < n_2 \tag{18}$$

$$0 < WL < CL \tag{19}$$

$$2 \leq n_1 < n_2 \tag{20}$$

$$n_1, n_2 \in Z^+$$

Table 15 illustrates the estimated value of the ARLs at $n_0 = 5$ for different adaptive strategies when the deviation occurs in intercept by amount of $\lambda\sigma$. When we look at the results in Table 19 we easily conclude that, for any shift there are a set of adaptive sampling sizes strategies outperforming the control chart using fixed settings during the online monitoring of a simple quality profiles.

Table 15 ARLs comparison when intercept shifts from A_0 to $A_0 + \lambda\sigma$ (in-control ARL=200, $n_0 = 5$)

Plan	n_1	n_2	λ						
			0.15	0.30	0.45	0.60	0.75	0.90	1.0
Fixed	5	5	152.45	82.76	41.38	21.21	11.54	6.75	4.92
Adaptive	4	8	151.65	78.48	35.18	15.74	7.74	4.41	3.29
	3	8	151.32	76.86	33.47	14.75	7.29	4.22	3.19
	2	7	151.45	77.77	34.92	16.00	8.11	4.69	3.50
	3	6	151.92	80.32	38.10	18.41	9.60	5.53	4.06

Similarly, Table 16 shows the ARLs value for the set of adaptive strategies under the shift in the slope (β ≠0). For each possible value of shift size there are a number of possible adaptive strategies that performs better than their traditional fixed counterpart (FRS-T^2).

Table 16 ARLs comparison when slope shifts from A_1 to $A_1 + \beta\sigma$ (in-control ARL=200, $n_0 = 5$)

Plan	n_1	n_2	β						
			0.03	0.06	0.09	0.12	0.15	0.18	0.20
Fixed	5	5	168.28	110.03	64.91	37.66	22.31	13.71	10.14
Adaptive	4	8	167.89	107.51	59.99	31.91	17.13	9.68	6.91
	3	8	167.66	106.35	58.21	30.34	16.10	9.10	6.54
	2	7	172.44	117.40	70.47	40.47	23.33	13.91	10.13
	3	6	167.97	108.47	62.16	34.63	19.67	11.67	8.51

Based on Eq. 14 and when the process is under a shift in the slope the ARL value of the adaptive approach might be significantly improved if the investigator neglected the condition of equally spacing the X values. Table 17 and 18 shows a comparison between three adaptive sampling plans using the same sampling sizes, but they have different value of X. Table 18 illustrate the difference in ARL values when we run these two sampling plans at different values of shift in the slope.

Table 17 ARLs comparison when slope shifts from A_1 to $A_1 + \beta\sigma$ (in-control ARL=200, $n_0 = 5, n_1 = 4$ and $n_2 = 6$

Type	n_1	n_2	Sample 1				Sample 2					
			x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4	x_5	x_6
Equally spaced	4	6	1	2.67	4.33	6	1	2	3	4	5	6
Unequally Spaced	4	6	1	3.45	3.87	6	1	2.30	3.60	5.51	5.76	6
	4	6	1	2.75	5.39	6	1	3.21	4.90	5.34	5.90	6

Table 18 ARLs comparison when slope shifts from A_1 to $A_1 + \beta\sigma$ (in-control ARL=200, $n_0 = 5, n_1 = 4$ and $n_2 = 6$

Type	β						
	0.03	0.06	0.09	0.12	0.15	0.18	0.20
Equally spaced	168.04	108.79	62.65	35.09	19.98	11.86	8.64
Unequally Spaced	162.76	97.96	51.86	26.85	14.40	8.28	5.99
	158.44	90.36	45.47	22.67	11.90	6.81	4.96

2.4 Variable Sampling Interval Scheme (VSI- T^2)

In this section a T^2 chart changing the sampling intervals during the monitoring of simple linear profiles is examined, see Figure 11. Here we refer to this scheme as (VSI-T^2).

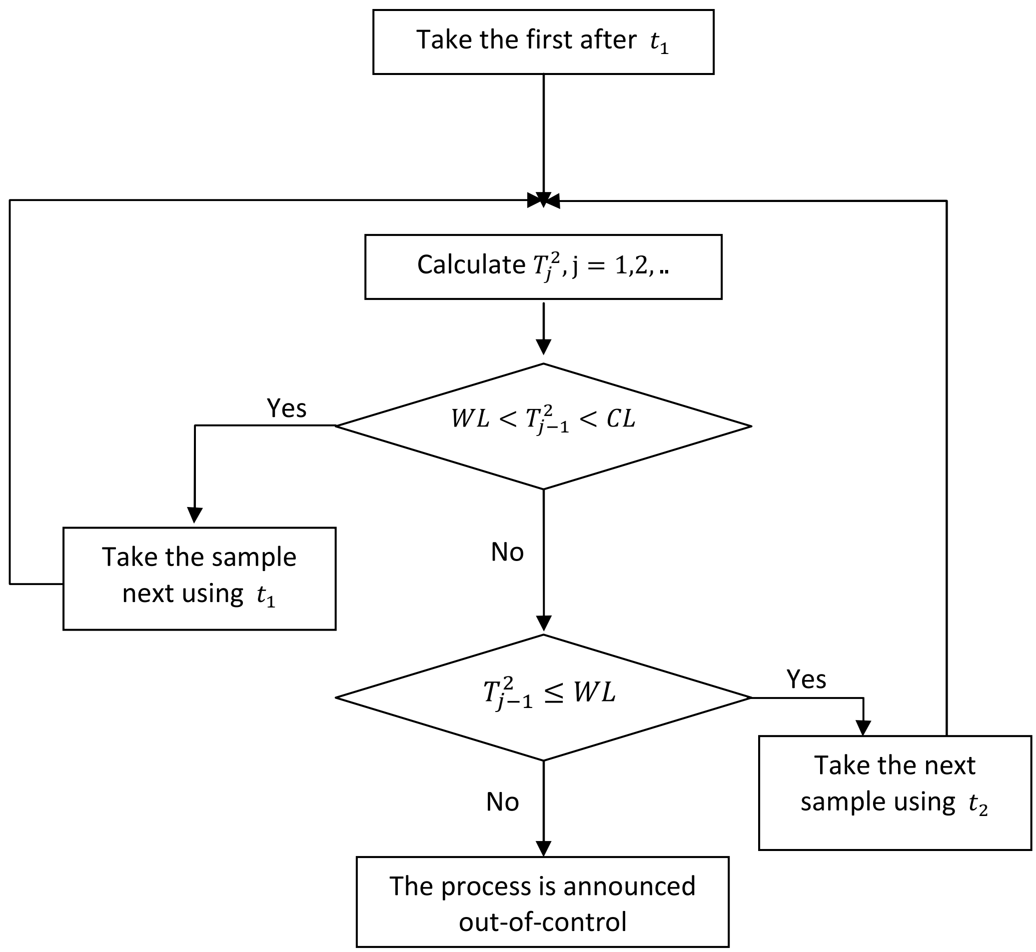

Fig. 11 The mechanism of adaptive sampling intervals approach (VSI-T^2)

2.4.1 Extending the ATS approximation: In this section we extend the ATS approximation used by Aparisi (1996), and Aparisi and Haro (2001), to the profile monitoring frame work and use it to evaluate the performance of the VSI-T^2 control chart. The time matrix is added to the ARL approximation shown in 11. And since we use only one sample size, there will be one non-centrality parameter. Then

$$ATS_{\delta} = \left(\frac{P(\chi_v^2 < WL)}{P(\chi_v^2 < CL)}, \frac{P(WL < \chi_v^2 < CL)}{P(\chi_v^2 < CL)}\right)\left(\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \begin{pmatrix}P(0 \le \chi_v^2(\tau) < WL) & P(WL < \chi_v^2(\tau) < CL)\\ P(0 \le \chi_v^2(\tau) < WL) & P(WL < \chi_v^2(\tau) < CL)\end{pmatrix}^{-1}\right)\begin{pmatrix}t_2\\ t_1\end{pmatrix} \quad (21)$$

2.4.2 Evaluating the performance of VSI-T^2Scheme: When the process is under a shift in the intercept, different FSR-T^2 control charts are tested versus the adaptive scheme (VSI-T^2). The results of this comparative study are presented in Tables 19-22. Note that all the control schemes using variable sampling intervals (t_1, t_2) are symmetric around the fixed sampling interval (t_0).

Table 19 ATS comparison when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$, n_0=5, CL=10.60 and $ATS_{\delta=0}$=150

No.	Sampling Interval	Shift in intercept, λ						
		0.15	0.30	0.45	0.60	0.75	0.90	1.0
1	0.75	114.34	62.07	31.04	15.91	8.65	5.06	3.69
2	(0.5,1)	113.13	59.32	28.20	13.67	7.08	4.02	2.91
3	(0.25,1.25)	111.92	56.57	25.36	11.43	5.51	2.98	2.14

Table 20 ATS comparison when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$, n_0=5, CL=10.60 and $ATS_{\delta=0}$=200

No.	Sampling Interval	Shift in intercept, λ						
		0.15	0.30	0.45	0.60	0.75	0.90	1.0
1	1	152.45	82.76	41.38	21.21	11.54	6.75	4.92
2	(0.75,1.25)	151.24	80.01	38.55	18.97	9.97	5.71	4.15
3	(0.5,1.5)	150.03	77.26	35.71	16.73	8.39	4.66	3.37
4	(0.25,1.75)	148.82	74.51	32.87	14.49	6.82	3.62	2.59

Table 21 ATS comparison when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$, n_0=5, CL=10.60 and $ATS_{\delta=0}$=300

No.	Sampling Interval	Shift in intercept, λ						
		0.15	0.30	0.45	0.60	0.75	0.90	1.0
1	1.5	228.67	124.14	62.07	31.81	17.31	10.12	7.39
2	(1.25,1.75)	227.47	121.39	59.24	29.57	15.74	9.08	6.61
3	(1,2)	226.26	118.64	56.40	27.33	14.16	8.04	5.83
4	(0.75,2.25)	225.05	115.89	53.56	25.09	12.59	7.00	5.05

Table 22 ATS comparison when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$, n_0=5, CL=10.60 and $ATS_{\delta=0}$=400

No.	Sampling Interval	Shift in intercept, λ						
		0.15	0.30	0.45	0.60	0.75	0.90	1.0
1	2	304.90	165.52	82.76	42.41	23.08	13.50	9.85
2	(1.75,2.25)	303.69	162.77	79.93	40.17	21.50	12.46	9.07
3	(1.50,2.50)	302.48	160.02	77.09	37.93	19.93	11.41	8.29
4	(1.25,2.75)	301.27	157.27	74.25	35.69	18.36	10.37	7.51

As it was expected, if the user decreases the sampling interval whenever the there is an indication of changes in the intercept of the simple linear quality function, then detection time will be decreased.

Now, the case when the slope has shifted by the amount of $\beta\sigma_0$ will be considered and the performance of the adaptive scheme is subjected to the test against the traditional scheme. The design parameters of the compared schemes are set such that they have the same ATS at the in-control state. Tables 23-26 illustrate the advantage of the adaptive sampling interval scheme over the traditional chart in detecting changes in the slope of the simple linear profiles.

Table 23 ATS comparison when slope shifts from $A_1 \rightarrow A_1 + \beta\sigma_0$, n_0=5, CL=10.60 and $ATS_{\delta=0}$=150

No.	Sampling Interval	Shift in slope, β						
		0.03	0.06	0.09	0.12	0.15	0.18	0.20
1	0.75	126.21	82.53	48.68	28.25	16.73	10.28	7.61
2	(0.5,1)	125.46	80.25	45.76	25.47	14.44	8.52	6.17
3	(0.25,1.25)	124.71	77.98	42.85	22.70	12.15	6.76	4.73

Table 24 ATS comparison when slope shifts from $A_1 \rightarrow A_1 + \beta\sigma_0$ n_0=5, CL=10.60 and $ATS_{\delta=0}$=200

No.	Sampling Interval	Shift in slope, β						
		0.03	0.06	0.09	0.12	0.15	0.18	0.20
1	1	168.28	110.03	64.91	37.66	22.31	13.71	10.14
2	(0.75,1.25)	167.53	107.76	61.99	34.89	20.02	11.95	8.71
3	(0.5,1.5)	166.78	105.49	59.07	32.11	17.72	10.19	7.27
4	(0.25,1.75)	166.03	103.22	56.16	29.34	15.43	8.43	5.83

Table 25 ATS comparison when slope shifts from $A_1 \rightarrow A_1 + \beta\sigma_0$, n_0=5, CL=10.60 and $ATS_{\delta=0}$=300

No.	Sampling Interval	Shift in slope, β						
		0.03	0.06	0.09	0.12	0.15	0.18	0.20
1	1.5	252.41	165.05	97.36	56.49	33.47	20.56	15.21
2	(1.25,1.75)	251.67	162.78	94.44	53.72	31.17	18.80	13.78
3	(1,2)	250.92	160.51	91.52	50.94	28.88	17.04	12.34
4	(0.75,2.25)	250.17	158.23	88.61	48.17	26.59	15.28	10.90

Table 26 ATS comparison when slope shifts from $A_1 \rightarrow A_1 + \beta\sigma_0$, n_0=5, CL=10.60 and $ATS_{\delta=0}$=400

No.	Sampling Interval	Shift in slope, β						
		0.03	0.06	0.09	0.12	0.15	0.18	0.20
1	2	336.55	220.07	129.81	75.33	44.63	27.41	20.29
2	(1.75,2.25)	335.80	217.79	126.89	72.55	42.33	25.65	18.85
3	(1.50,2.50)	335.06	215.52	123.98	69.77	40.04	23.90	17.41
4	(1.25,2.75)	334.31	213.25	121.06	67.00	37.74	22.14	15.97

Table 27 studies the effect of the distance between the sampling intervals on the power of the VSI-T^2 control chart. Table 27 and Figures 12 and 13 show that, the power of the VSI-T^2 scheme increases as the distance between the sampling intervals increases.

Table 27 Studying the effect of distance between sampling intervals, n_0=5, CL=10.60 and $ATS_{\delta=0}$=200

No.	Sampling Intervals	Distance $(t_2 - t_1)$	λ =0.45, β=0	β=0.12, λ=0
1	(1,1)	0	41.38	37.66
2	(0.75,1.25)	0.5	38.55	34.89
3	(0.5,1.5)	1	35.71	32.11
4	(0.25,1.75)	1.5	32.87	29.34

However, so far all the variable sampling intervals used here are symmetric around the fixed interval.

Like the case of the VSS- T^2, here we introduce an optimization model to be solved using the Genetic Approach to find the optimal settings of the VSI- T^2.

Min $ATS\ (\nu, \delta, WL, n_1, n_2)$ (22)

Subject to:

$t_0 = t_2\, p_1 + t_1 p_2$ (23)

$t_1 < t_0 < t_2$ (24)

$0 < WL < CL$ (25)

$t_1, t_2\ > 0$

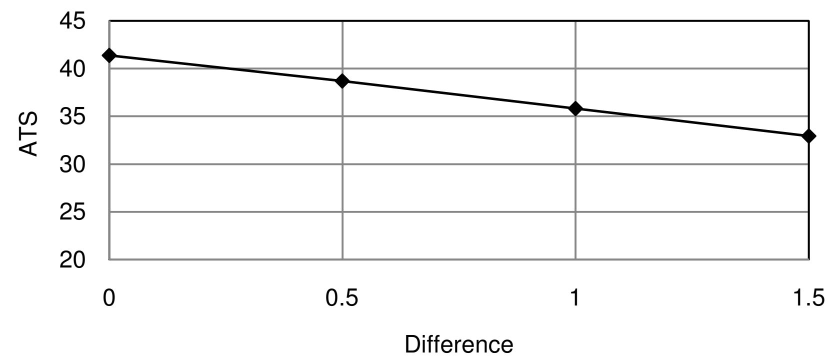

Fig. 12 The ATS versus the distance between the sampling intervals (λ =0.45, β=0)

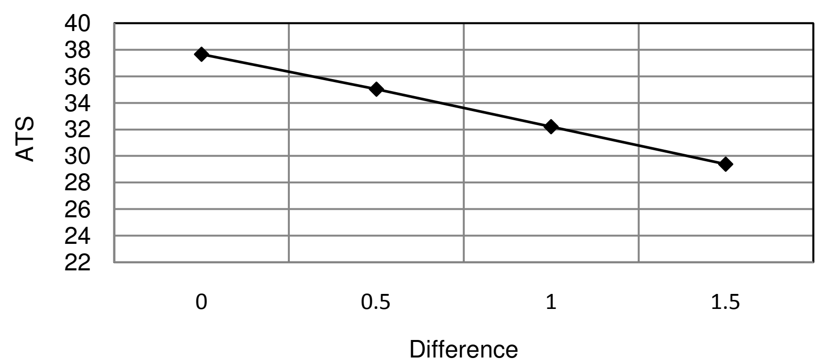

Fig. 13 The ATS versus the distance between the sampling intervals (β=0.12, λ=0)

The statistical performance of the set of the optimized adaptive plans, reported in Table 28, shows that the VSI-T^2 is capable to beat the traditional scheme at all levels of changes in the intercept of simple linear regression model. Similarly, the same adaptive strategies are examined when the process is under a shift in the slope ($\beta \neq 0$), see Table 29.

Table 28 ATSs comparison when intercept shifts from A_0 to $A_0 + \lambda\sigma$ (in-control ATS=400, $n_0 = 5$)

Plan	t_1	t_2	λ						
			0.15	0.30	0.45	0.60	0.75	0.90	1.0
Fixed	2	2	304.90	165.52	82.76	42.41	23.08	13.50	9.85
Adaptive	0.32	2.30	301.73	156.75	72.88	33.90	16.53	8.76	6.11
	0.25	2.15	303.36	159.99	76.14	36.41	18.24	9.83	6.87
	0.29	2.20	302.73	158.73	74.86	35.41	17.55	9.39	6.55

Table 29 ATSs comparison when slope shifts from A_1 to $A_1 + \beta\sigma$ (in-control ATS=400, $n_0 = 5$)

Plan	t_1	t_2	β						
			0.03	0.06	0.09	0.12	0.15	0.18	0.20
Fixed	2	2	336.55	220.07	129.81	75.33	44.63	27.41	20.29
Adaptive	0.32	2.30	334.94	213.17	120.20	65.54	35.96	20.29	14.18
	0.25	2.15	336.07	215.92	123.60	68.72	38.54	22.22	15.72
	0.29	2.20	335.63	214.85	122.27	67.47	37.52	21.44	15.10

The results illustrated in Table 29 show the advantage of the adaptive scheme over the traditional T^2 control chart at all levels of shift. Note that these adaptive strategies are optimized in terms of the ATS. As we previously mentioned, when the process is under a shift in the slope, the power of the T^2 control chart is effected by the location of the independent variable.

The following two Tables examine the effect of the location of *X* on the statistical performance of the VSI-T^2 profiling scheme under the two different scenarios of changes in the regression parameters. In this study we will examine one VSI-T^2 control chart under three different sets of *X*.

Table 30 ATS comparison when slope shifts from A_1 to $A_1 + \beta\sigma$ (in-control ATS=400, $n_0 = 5,\ t_1 = 0.32$ and $t_2 = 2.30$

Type	X-Values				
	x_1	x_2	x_3	x_4	x_5
Equally spaced	1	2.25	3.5	4.75	6
Unequally Spaced	1	2.30	3.60	5.51	6
	1	3.21	4.90	5.34	6

Table 31 ATSs comparison when slope shifts from A_1 to $A_1 + \beta\sigma$ (in-control ATS=400, $n_0 = 5, t_1 = 0.32$ and $t_2 = 2.30$

Type	β						
	0.03	0.06	0.09	0.12	0.15	0.18	0.20
Equally spaced	334.94	213.17	120.20	65.54	35.96	20.29	14.18
Unequally Spaced	328.49	200.84	108.79	57.31	30.59	16.93	11.76
	318.41	182.97	93.35	46.77	24.02	12.99	8.98

The significant improvement achieved by reallocating the *X* values clearly shows the effect of the position of the independent variable on the power of the VSI-T^2 scheme. Such recommendation should be considered in the optimization model described by 22-25. For increasing the applicability level, each adaptive plan presented in Table 31 has two points assigned at the edges of the range of the explanatory variable.

CHAPTER 3 VARIABLE SAMPLING INTERVAL AND SAMPLING SIZES SCHEME (VSSI- T^2) FOR MONITORING SIMPLE LINEAR PROFILES

As a result of the comprehensive comparison conducted in chapter 2, we suggest integrating both of the schemes to capture changes in intercept and slope of the simple linear function. This chart will be referred to as a VSSI-T^2 control chart.

As in the situation where the quality is described by the probability distribution, the VSSI-T^2 scheme uses two sample sizes (n_1, n_2), two time intervals (t_1, t_2) and one warning limit (WL).

The mechanism of this scheme is shown in Eq. 26 and Figure 14; see Aparisi and Haro (2001).

$$(n_{profile\ j}, t_{profile\ j}) = \begin{cases} (n_1, t_2),\ if\ \ 0 \leq T_{j-1}^2 < WL \\ (n_2, t_1), if\ \ WL < T_{j-1}^2 < CL \end{cases} \tag{26}$$

However, to investigate the effectiveness of this chart, its performance should be matched with its counterparts at the state of the statistical control and then we calculate these measures at the off-target state. Note that the sampling sizes and intervals of the adaptive scheme should be selected such that:

$$n_1 \leq\ n_0 \leq n_2 \tag{27}$$

$$t_1 \leq\ t_0\ \leq t_2 \tag{28}$$

where n_0 , t_o are the fixed sampling size and sampling interval of the traditional T^2 control chart.

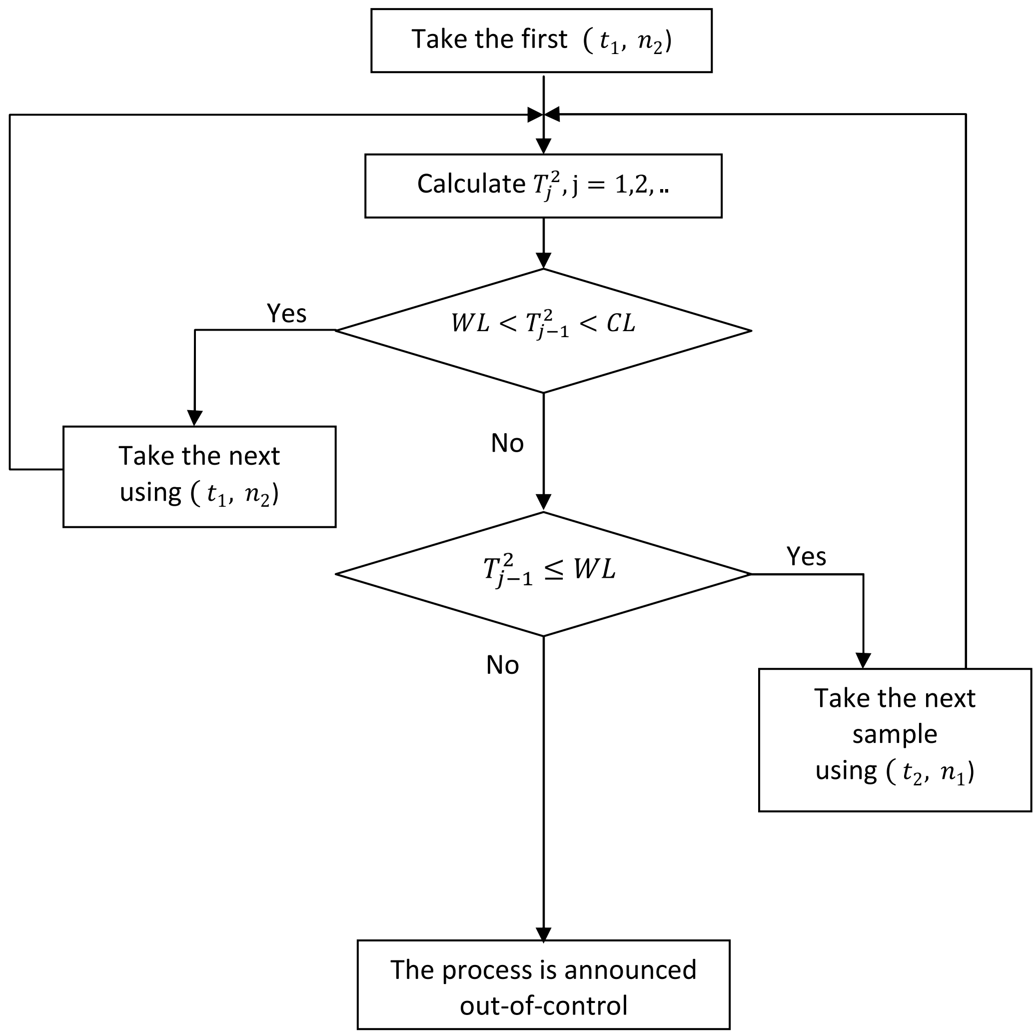

Fig. 14 The mechanism of VSSI-T^2 scheme

3.1 Extending the ATS Approximation for VSSI-T^2

However, since we extend these approximations to the VSSI-T^2, the ATS approximation can be written as follows:

$$ATS_{\delta} = \left(\frac{P(\chi_{v}^{2} < WL)}{P(\chi_{v}^{2} < CL)}, \frac{P(WL < \chi_{v}^{2} < CL)}{P(\chi_{v}^{2} < CL)}\right)\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} P(0 \le \chi_{v}^{2}(\tau_1) < WL) & P(WL < \chi_{v}^{2}(\tau_1) < CL) \\ P(0 \le \chi_{v}^{2}(\tau_2) < WL) & P(WL < \chi_{v}^{2}(\tau_2) < CL) \end{pmatrix}^{-1}\right)\begin{pmatrix} t_2 \\ t_1 \end{pmatrix} \quad (29)$$

where τ_1 and τ_2 are the non-centrality parameters of the two sampling intervals, and v is the degree of freedom ($v = 2$).

If the sampling interval is fixed, then $\tau_1 = \tau_2$. Here we develop a mathematical model to determine the optimal design parameters of the VSSI-T^2 chart such that the both compared schemes have the same $ATS_{\delta=0}$, that is:

Min $ATS\ (\nu, \delta, WL, n_1, n_2, t_1, t_2)$ (30)

Subject to:

$n_0 = n_1\, p_1 + n_2 p_2$ (31)

$t_0 = t_2\, p_1 + t_1 p_2$ (32)

$n_1 \le n_0 \le n_2$ (33)

$t_1 \le t_0 \le t_2$ (34)

$0 < WL < CL$ (35)

$n_1, n_2\, \epsilon\, Z^{+}$

where δ is the amount of deviation from the target values of the model parameters

The procedure to solve the above model might be described as follows:

1- Find two values for the sampling sizes
2- Solve Eq. 31 and find the values of $p_1$and p_2
3- Find a value for t_1 or t_2; consider Eq. 34
4- Sub in Eq. 32 and determine the other value (t_1 or t_2)

3.2 Setting the Parameters of the Optimization Technique

The genetic approach (GA) is a well-known stochastic optimization technique utilized for optimizing design settings of many charting methods (e.g. $\bar{X}$ and Hotelling T^2) (see He et al. (2002), Chen (2004), He and Grigoryan (2005), Chen and Hsieh (2007), and Chou et al (2008). In this research, the genetic algorithm is used to find the optimal design parameters of VSSI-T^2scheme that minimizes ATS.

In order to get a better quality of the genetic approach output influenced by the magnitude of its inputs, two Taguchi experiments are conducted to find the optimal settings of the GA inputs. These levels were determined and presented in Table 32, which were the same for the shift in the slope and intercept. These experiments were conducted with n_0 =6.

Table 32 Optimal Settings of GA parameters

Shift in	Population size	Mutation rate	Crossover probability
λ,β	100	0.1	0.5

3.3 Measuring the Performance of VSSI- T^2 Control Chart

In order to evaluate the performance of the VSSI-T^2 scheme in catching changes in the parameters of a simple linear profile, we compare its power with the FSR-T^2 chart for $n_0 = 6$ and $t_0 = 1$.

In this comparison, the values of the explanatory variable *X* are assigned at equal distance from 1 to 6. To increase the level of applicability we located two points at the edges of this range. Note that the ATS values for the FSR-T^2 scheme can easily be calculated by making $WL = UCL$, $n_0 = n_1 = n_2$ and $t_0 = t_1 = t_2$.

The ATS values at the in-control state are set to approximately 200. The comparison is done by introducing three adaptive sample sizes and sampling intervals plans and evaluating these plans at the two expected types of shifts in coefficients of a simple linear model.

Table 33 and Figure 15 illustrate the ATSs comparison between a traditional sampling plan and a set of adaptive strategies developed by solving the optimization model at n_0 =6 and t_0 =1. The comparative study shows that, at all levels of change in intercept, the adaptive strategies outperform the fixed sampling settings.

Here the approach followed to solve the optimization model considers the sampling efforts and generates a set of particular adaptive strategies that minimize the objective function ($Min\ ATS$) and maintain the sampling rate as low as possible.

Table 33 ATS comparison of VSSI- T^2 and FSR-T^2 when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$

No.	Sampling Intervals	Sampling Sizes	WL	Shift in intercept, λ						
				0.15	0.30	0.45	0.60	0.75	0.90	1.0
1	1	6	No	145.18	72.61	34.01	16.68	8.85	5.14	3.76
2	(0.25,1.80)	(5,7)	1.3383	140.97	61.39	22.89	8.61	3.73	2.07	1.61
3	(0.15,1.94)	(4,8)	1.3605	140.17	58.86	20.45	7.16	3.04	1.77	1.45
4	(0.05,1.98)	(3,9)	1.3678	139.03	56.03	18.11	5.98	2.57	1.61	1.37

The results of comparing the two schemes when the slope deviates by the amount of β are presented in Table 34 and Figure 16. As it can be seen the adaptive strategies show better ability in detecting the off-target conditions. Another finding is that, the adaptive approach VSSI-T^2 is more powerful in detecting changes in the slope than the intercept. However, it is quite expected that, like traditional multivariate charting applications, adaptive sampling sizes and sampling intervals schemes perform better than the traditional T^2 control charts, especially for small to moderate changes in the parameters of simple linear profiles. All the adaptive plans are optimized using the model presented in 36-41.

Table 34 ATS comparison of VSSI- T^2 and FSR-T^2 when slope shifts from $A_1 \rightarrow A_1 + \beta\sigma_0$

No.	Sampling Intervals	Sampling Sizes	WL	Shift in Slope, β						
				0.03	0.06	0.09	0.12	0.15	0.18	0.20
1	1	6	No	163.36	100.68	56.19	31.22	17.92	10.78	7.91
2	(0.25,1.80)	(5,7)	1.3383	161.33	91.64	44.43	20.46	9.56	4.83	3.28
3	(0.15,1.94)	(4,8)	1.3605	160.88	89.86	41.90	18.26	8.08	3.96	2.71
4	(0.05,1.98)	(3,9)	1.3678	160.03	87.54	39.05	16.08	6.79	3.30	2.30

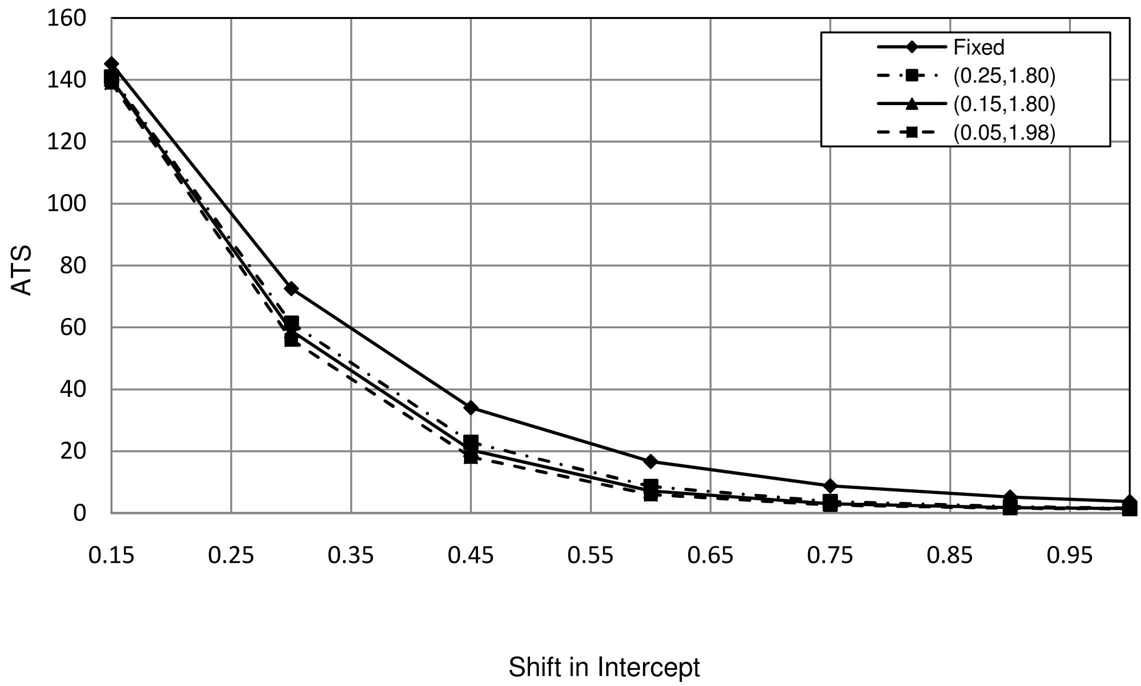

Fig. 15 ATS comparison of VSSI- T^2 and FSR-T^2 when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$

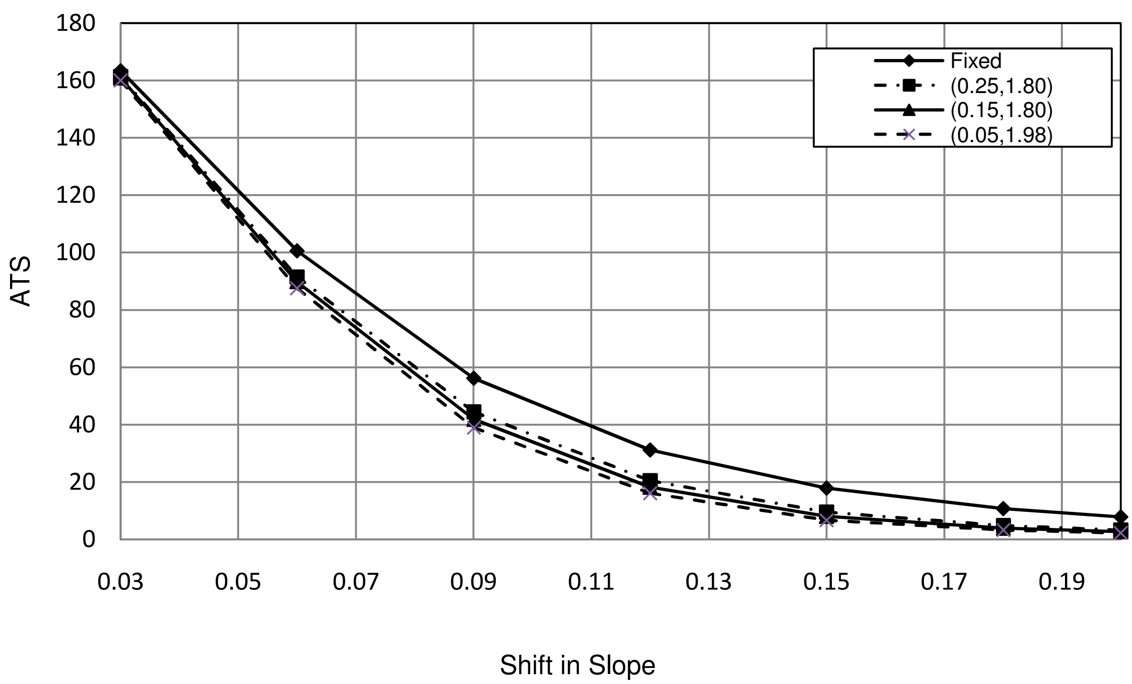

Fig. 16 ATS comparison of VSSI- T^2 and FSR-T^2 when slope shifts from $A_1 \rightarrow A_1 + \beta\sigma_0$

As an illustrative example, we evaluate the performance of the VSSI-T^2 control chart using a set of adaptive plans representing some of the results of solving the optimization model previously introduced in this chapter.

The ATS values of the traditional scheme are calculated by making $n_0 = n_1 = n_2$, $t_0 = t_1 = t_2$ and $0 \leq WL \leq CL$; see Table 35.

Table 35 ATSs values of FSR-T^2 when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$, and $ATS_{\delta=0}$=200

n_1	n_2	t_1	t_2	λ				
				0.3	0.6	0.9	1.2	1.5
5	5	1.0000	1.000	82.76	21.21	6.75	2.90	1.67
				β				
				0.03	0.06	0.09	0.12	0.15
				168.28	110.03	64.91	37.66	22.31

Table 36 ATSs comparison when intercept shifts from $A_0 \rightarrow A_0 + \lambda\sigma_0$, n_0=5, t_0=1, and $ATS_{\delta=0}$=200

Scheme Type	n_1	n_2	t_1	t_2	λ		
					0.3	0.9	1.5
VSSI	2	6	0.0521	3.962	67.05	2.02	1.07
	3	6	0.0517	2.9854	68.01	2.07	1.06
	2	8	0.0687	1.9572	65.55	1.97	1.15
	3	8	0.0705	1.6412	67.67	2.04	1.11
	4	8	0.0735	1.326	71.34	2.32	1.10
	2	10	0.0949	1.5585	63.86	2.03	1.23
	3	10	0.0844	1.3798	66.62	2.05	1.16
	4	10	0.087	1.1942	71.23	2.32	1.14

Table 37 ATSs comparison when intercept shifts from $A_1 \rightarrow A_1 + \beta\sigma$, n_0=5, t_0=1, and $ATS_{\delta=0}$=200

Scheme Type	n_1	n_2	t_1	t_2	β		
					0.03	0.09	0.15
VSSI	2	6	0.0521	3.962	163.95	48.21	10.15
	3	6	0.0517	2.9854	164.40	49.32	10.63
	2	8	0.0687	1.9572	164.29	46.28	8.92
	3	8	0.0705	1.6412	165.17	48.67	9.82
	4	8	0.0735	1.326	166.21	52.45	11.67
	2	10	0.0949	1.5585	164.47	44.40	8.02
	3	10	0.0844	1.3798	165.51	47.43	8.99
	4	10	0.087	1.1942	166.65	52.15	11.11

The ATS comparisons presented in Tables 36 and 37 show that the VSSI-T^2 is able to outperform the FSR- T^2 scheme whenever the optimal settings of the adaptive scheme are determined and used.

3.4 The Effect of Location of X-Values

When the slope of a simple linear model deviates from $A_1 \rightarrow A_1 + \beta\sigma$, the non-centrality parameter takes the value described in Eq. 14.

However, here I will test the hypothesis that the performance of the adaptive scheme (VSSI-T^2) can be much improved if the values of the explanatory variable are assigned such that the non-centrality parameter is maximized. Table 38 shows the design settings of two adaptive plans selected for a comparison. Here the values of the independent variable are set such that $\tau_2^1 < \tau_2^2 < \tau_2^3$; where τ_1^j and τ_2^j are the non-centrality parameter values of the j^{th} strategy. Note that τ_1^1=τ_1^2=τ_1^3.

As one may have noticed from Table 39 that, when the process is under a shift in the slope, the performance of the VSSI-T^2 is affected by the location of the independent variable and this power might be significantly enhanced if we considered the location of the *X* values in the optimization problem described by 30-35.

Table 38 The design setting of two adaptive plans to study the effect of location of *X*-values, n_1= {1.00, 6.00}

Strategy	WL	t_1	t_2	The second sample, n_2					
				x_1	x_2	x_3	x_4	x_5	x_6
Plan 1	0.5555	0.0521	3.962	1.00	2.00	3.00	4.00	5.00	6.00
Plan 2	0.5555	0.0521	3.962	1.00	3.00	3.33	4.09	5.96	6.00

Table 39 Studying the effect of location of *X*-values, $ATS_{\delta=0}$=200, n_0=6, and t_0=1

Strategy	β						
	0.03	0.06	0.09	0.12	0.15	0.18	0.20
Plan 1	163.95	97.02	48.21	22.22	10.15	4.91	3.24
Plan 2	158.34	86.59	39.32	16.69	7.21	3.48	2.38

3.5 Evaluating the Adaptive Schemes under Uncertainty of Process's Shift

Literature review shows that most of the existing profiling techniques have been examined under the assumption that the process changes is a step or a drift shift. In section, the statistical performance of the three adaptive T^2 schemes, discussed in chapter 2 and 3, are evaluated under the assumption that shift in the process takes a random value one profile to another according to some probability distributions.

Table 40 and 41 shows the results of comparing the three adaptive schemes versus the FSR-T^2 when the two expected types of shift behave as an exponentially distributed random variable ($\lambda \cong Exp(\mu_\lambda)$, $\beta \cong Exp(\mu_\beta)$)

Table 44 shows that the VSSI-T^2 scheme detects changes in A_0 faster than the other schemes. The results indicate that the difference in performance between the FSR-T^2 and the VSSI-T^2 is slightly significant.

Table 40 ATS comparison when the change in the intercept behaves as a random variable, n_0=5, n_1=3, n_2=6, t_0=1 and $ATS_{\delta=0}$=200

Scheme Name	t_1	t_2	μ_λ						
			0.15	0.30	0.45	0.60	0.75	0.9	1.0
FSR-T^2	1	1	152.59t_0	113.89t_0	89.44t_0	74.94t_0	64.20t_0	55.79t_0	51.08t_0
VSS-T^2	1	1	152.46t_0	112.68t_0	88.54t_0	73.64t_0	62.81t_0	54.80t_0	49.71t_0
VSI-T^2	0.5	1.5	151.33	110.49	86.93	71.86	61.64	52.36	48.50
VSSI-T^2	0.0521	2.9391	148.30	107.21	81.95	67.51	57.44	48.90	45.80

Table 41 ATS comparison when the change in the slope behaves as a random variable, n_0=5, n_1=3, n_2=6, t_0=1 and $ATS_{\delta=0}$=200

Scheme Name	t_1	t_2	μ_β						
			0.03	0.06	0.09	0.12	0.15	0.18	0.20
FSR-T^2	1	1	164.42t_0	128.29t_0	105.17t_0	88.50t_0	75.69t_0	67.24t_0	62.11t_0
VSS-T^2	1	1	163.85t_0	127.35t_0	102.97t_0	86.26t_0	74.74t_0	64.60t_0	60.33t_0
VSI-T^2	0.5	1.5	162.68	125.90	102.86	84.79	72.38	63.67	60.02
VSSI-T^2	0.052	2.939	159.95	121.64	96.85	79.94	68.82	60.35	55.79

As one may have noticed, the results in Table 45 shows the ability of the VSSI-T^2 scheme to be the appropriate chart to be selected for catching changes in the slope of a simple linear quality profiles when the value of this shift changes as an exponentially random.

3.6 The effect of location on the performance of EWMA4 Method.

There have been several control charts methods developed to monitor polynomial profiles. In this section, we consider the case of second-order polynomial regression models (k=2) as another type of models used for characterizing the relationship between response and one explanatory variable. More specifically, we seek for improving the statistical performance of an existing profiling method which is referred to as EWMA4.

3.6.1 Orthogonal polynomial method (EWMA4) : This section briefly introduces the EWMA4 method proposed by Kazemzadeh et al. (2009). This method is mainly based on the use of orthogonal polynomial regression to monitor changes in the process parameters.

As it is expected, this method assumes that the process quality response Y is a random variable, and the polynomial function is the best fit of its relationship with the explanatory variable X; that is

$$y_{ij} = A_0 + A_1 x_i + A_2 x_i^2 + \ldots + A_k x_i^k + \varepsilon_{ij}\,; \quad i = 1,2,3,\ldots\ldots,n \quad j = 1,2,3,\ldots \qquad (36)$$

where $A_0, A_1, A_2, \ldots, A_k$ are the target values and $\varepsilon_{ij} \sim N(0, \sigma^2)$.

The transformed model is as follows:

$$y_{ij} = B_0P_0(x_i) + B_1P_1(x_i) + B_2P_2(x_i) + \dots + B_kP_k(x_i) + \varepsilon_{ij} \tag{37}$$

where the value B_o is the coefficient of the o^{th} order orthogonal polynomial $(P_o(x_i))$

Kazemzadeh et al. (2009) suggested and used Eq. 38 to calculate the least square estimators of $B_0, B_1, \dots, B_k$. In their paper, they mentioned that the three least square estimators are independently normally distributed such that $E(\hat{B}_{lj}) = B_l$ and $Var(\hat{B}_{lj}) = \sigma^2/\sum_{i=1}^{n} P_l^2(x_i)$; where j is the profile number., and l is the order orthogonal polynomial; where $l = 1,2,3, \dots \dots, k$.

$$\hat{B}_{lj} = \frac{\sum_{i=1}^{n} P_l(x_i)y_{ij}}{\sum_{i=1}^{n} P_l^2(x_i)} \;; \qquad l = 0,1,2, \dots, k \quad j = 1,2,3, \dots \tag{38}$$

Kazemzadeh et al. (2009) used three individual $EWMA$ control charts to monitor changes in these parameters. The $EWMA$ statistics and the control limits are calculated as follows:

$$EWMA_l(j) = \theta\hat{B}_{lj} + (1-\theta)EWMA_l(j-1) \tag{39}$$

$$CL_l = B_l \pm K_l\sqrt{\frac{\theta\,\sigma^2}{(2-\theta)\sum_{i=1}^{n} P_l^2(x_i)}} \;; j = 1,2,3, \dots \quad l = 0,1,2, \dots, k \tag{40}$$

The smoothing constant, $0 < \theta \leq 1$, and the multiplier, $k_l > 0$, are set such that a certain $\mathrm{ARL}_{\delta=0}$ is achieved.

Another one-sided $EWMA$ control chart is suggested to monitor the process variability; for further information, refer to Kazemzadeh et al. (2009). The $EWMA_v$ statistic and the control limit are as follows:

$$EWMA_v(j) = Max\{\theta(MSE_j - 1) + (1-\theta)EWMA_v(j-1), 0\} \quad (41)$$

$$UCL_v = K_v\sqrt{\frac{\theta\, Var(MSE_j)}{(2-\theta)}} \qquad j = 1,2,3,\dots\dots\dots \quad (42)$$

Kazemzadeh et al. (2009) reported that the change in the coefficients of the original form leads to larger change in the coefficients of the orthogonal polynomial form. Based on this fact, we suggest an improvement approach that optimizes the location of the independent variable; this optimization will maximize the amount of shift of the orthogonal model once the original parameter shifts. The result of that is a reduction in the average run length (ARL). The following section defines the model and the basic principles of this approach.

3.6.2 Model description: Let us assume that our response variable Y is described by a second order polynomial regression as follow:

$$y_{ij} = A_0 + A_1x_i + A_2x_i^2 + \varepsilon_{ij}\,; \qquad i = 1,2,3,\dots\dots, n \quad j = 1,2,3,\dots \quad (43)$$

We use $\vec{A} = (A_0, A_1, A_2)^T$ to describe the regression coefficients vector and $\hat{A} = (a_0, a_1, a_2)^T$ when we point to the *Least Square Estimators* vector. For a fixed design $X = (x_1, x_2, \dots, x_n)$, the least square estimators vector (unbiased estimator of $\vec{A}$) has a multivariate normal distribution with mean $\vec{A}$ and covariance matrix $\sigma^2(X^TX)^{-1}$). Then

$$\hat{A} \sim N(\vec{A}, \sigma^2(X^TX)^{-1})) \tag{44}$$

where X is the design matrix and it is only represented by $(K+1)$ vectors of $\vec{g}_j(x_i)$; $j = 0,1,..,K$. For the second order linear polynomial profiles $g_j(x_i)$ are as follows:

$$g_o(x_i) = 1$$

$$g_1(x_i) = x_i$$

$$g_2(x_i) = x_i^2$$

Then, the design matrix (X) can be written as:

$$X = \begin{bmatrix} 1 & x_1 & x_1^2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 \end{bmatrix} \tag{45}$$

The least squares vector $\hat{A}$ can be calculated using

$$\hat{A} = (X^TX)^{-1}X^Ty \tag{46}$$

where $y = (y_1, y_2, \ldots y_n)^T$ is the vector of observed sample. The expectation of $\hat{A}$ can be calculated as follow; see Korostelev (2010):

$$\mathbb{E}_\theta[\hat{A}] = \mathbb{E}_\theta[(X^TX)^{-1}X^Ty]=[(X^TX)^{-1}X^T]X\,\mathbb{E}_\theta[\vec{A}] = \vec{A} \tag{47}$$

Now, we need to look at the vector of the difference between least square estimators $\hat{A}$ and the coefficient of regression $\vec{A}$;

$$[\hat{A} - \vec{A}\,] \sim N(0, \sigma^2 (X^T X)^{-1})) \tag{48}$$

We can rewrite this vector as follows:

$$\begin{bmatrix} a_0 - A_0 \\ a_1 - A_1 \\ a_2 - A_2 \end{bmatrix} \sim N(0, \sigma^2 (X^T X)^{-1})) \tag{49}$$

The marginal distribution of a_l is $N \sim (A_l, \sigma^2 X_{ll})$; where $0 \leq \ l \leq k$.

3.6.3 Analyzing of orthogonal polynomial model: The EWMA4 method transforms the original model to orthogonal one having independent regression coefficients and then uses three separate EWMA control charts in addition to another EWMA chart to monitor the process variability. In their paper, Kazemzadeh et al. (2009) presented the relationship between the orthogonal polynomials and the regression coefficients of the original model as follows:

$$B_0 = A_0 + \ \bar{x} A_1 + \left[\bar{x}^2 + \left(\frac{n^2 - 1}{12} \right) d^2 \right] A_2 \tag{50}$$

$$B_1 = \frac{d}{\lambda_1} \ (A_1 + 2 A_2 \bar{x}) \tag{51}$$

$$B_2 = \frac{A_2 d^2}{\lambda_2} \tag{52}$$

The three scenarios of changes considered here are:

$$shift\ (d) = \begin{cases} A_0 + \lambda\sigma\ , \beta = \delta = 0 \\ A_1 + \beta\sigma\ , \lambda = \delta = 0 \\ A_2 + \delta\sigma\ , \lambda = \beta = 0 \end{cases} \quad (53)$$

If we insert the three scenarios of the shift into equations 50-52, a new form of the relationship between the original and the orthogonal parameter can be formatted. Table 46 summarizes this relationship

Table 42 Changes in original polynomial versus orthogonal polynomial models

Original	Shift	Change in Orthogonal Coefficients		
		B_0	B_1	B_2
A_0	$\lambda\sigma$	$\lambda\sigma$	No change	No change
A_1	$\beta\sigma$	$\bar{x}\beta\sigma$	$\frac{d\beta\sigma}{\lambda_1}$	No change
A_2	$\delta\sigma$	$\delta\sigma\left(\bar{x}^2 + \left(\frac{n^2-1}{12}\right)d^2\right)$	$\frac{d}{\lambda_1}2\bar{x}\delta\sigma$	$\frac{d^2\delta\sigma}{\lambda_2}$

The most important finding can be observed from Table 42 is that, the amount of changes in regression coefficients in orthogonal polynomial model are affected by some properties of the explanatory variable such as $\bar{x}$, n and d. Based on that, the amount of changes in the orthogonal model can be controlled by changing the values (locations) of the explanatory variable. Finding the optimal location of the X values might be formulated as an optimization problem and solved using one of the stochastic optimization techniques such as, the genetic approach. The following section is dedicated to describe the procedure of the improvement approach.

Here the MATLAB software is utilized to run the Genetic Approach. Again two Taguchi experiments were run to find the optimal inputs of the GA; which were the same settings in Table 36. The model is described as follows:

$$Min\ ARL \tag{54}$$

Subject to:

$$x_n \leq uu \tag{55}$$

$$x_1 \geq ll \tag{56}$$

$$x \in Z^{+n}$$

where ll and uu are the lower and upper limits of the practical range of X. The sample size might be considered as another constraint ($K + 1 < L \leq n$); where L is the number of locations. The left side ($K + 1$) is adjustable, and it depends on K and how the MSE_j is estimated.

3.6.4 Evaluating the power of the suggested approach: This section is dedicated to examine the ability of the suggested approach in enhancing the performance of EWMA4 technique. The four EWMA charts were designed such that the in-control ARL is equal to 200. Table 47 shows the settings of the design parameters of the EWMA4 method. Several techniques have been developed to find the orthogonal polynomial when the independent variable is unequally spaced; see Kendall (1959), Robson (1959), and Grandage (1958). This research uses the Robson’s method. The recommendations reported in an analytical and comparative research in using the polynomial orthogonal curves in Dental Healthcare Industry developed by Toby et al (2002), is the main reason behind the use of Robson's method; see Appendix A. In order to better show the effect of independent variable location, a simulation study with 40,000 runs was conducted to estimate ARL values under different shifts in A_1 and A_2. Table 43 and 44 show the settings and the ARL values of two types of strategies, respectively. The first type is designed with equally spaced of X-values and the second is sampling plans optimized in terms of the location of X. Here the following polynomial model is used:

$$y_{ij} = 4 + 3x_i + 2x_i^2 + \varepsilon_{ij}\,; \quad i = 1,2,3,\ldots\ldots,n \quad j = 1,2,3,\ldots \tag{57}$$

where $A_0 = 4, A_1 = 3,$ and $A_2 = 2$ are the coefficient of regressions, $\varepsilon_{ij} \sim N(0,\sigma^2)$, and $\sigma = 1$. When n=5 and the X values are equally distributed from 1 to 5, the orthogonal model might be described as follows:

$$y_{ij} = 34.99P_0(x_i) + 15.0P_1(x_i) + 2P_2(x_i) + \varepsilon_{ij}\,; \quad i = 1,2,3,\ldots,n \quad j = 1,2,3, \tag{58}$$

Table 44 and Figure 17 illustrate that, under the shift in the second parameter from A_1 to $A_1 + \beta\sigma$ and at all levels of shifts considered in this comparison, the optimized plan outperforms the regular one. As it is shown in the reduction row (ARL-regular – ARL-optimized), optimizing the location of X will lead to a significant reduction in sampling time and costs. This simulation study will be repeated in the case that the third parameter has shifted from A_2 to $A_2 + \delta\sigma$. The results are shown in Table 45 and graphically presented in Figure 18. Compared to regular plans, the optimized plans perform perfectly at all levels of shift in A_2.

Table 43 Orthogonal polynomials of regular and optimized plans

Plan Type		i = 1	2	3	4	5	B_0	B_1	B_2
Regular	$P_0(x_i)$	1	1	1	1	1	34.99	15	2
	$P_1(x_i)$	-2	-1	0	1	2			
	$P_2(x_i)$	2	-1	-2	-1	2			
Optimized	$P_0(x_i)$	1	1	1	1	1	26.20	1.66	0.157
	$P_1(x_i)$	-13	-8	7	7	7			
	$P_2(x_i)$	9	-12	1	1	1			

Table 44 ARL comparisons between regular and optimized plans under three levels of shift in second parameter from A_1 to $A_1 + \beta\sigma$ and n=5

Plan Type	Shift in A_1		
	0.02	0.05	0.08
Regular	168.71	79.9	37.27
Optimized	156.09	61.35	25.30
Reduction %	7.48	23.32	32.21

Table 45 ARL comparisons between regular and optimized plans under three levels of shift in second parameter from A_2 to $A_2 + \delta\sigma$ and n=5

Plan Type	Shift in A_2		
	0.004	0.02	0.035
Regular	177.32	36.49	12.51
Optimized-1	157.76	18.36	7.00
Reduction %	11.03	49.68	44.04

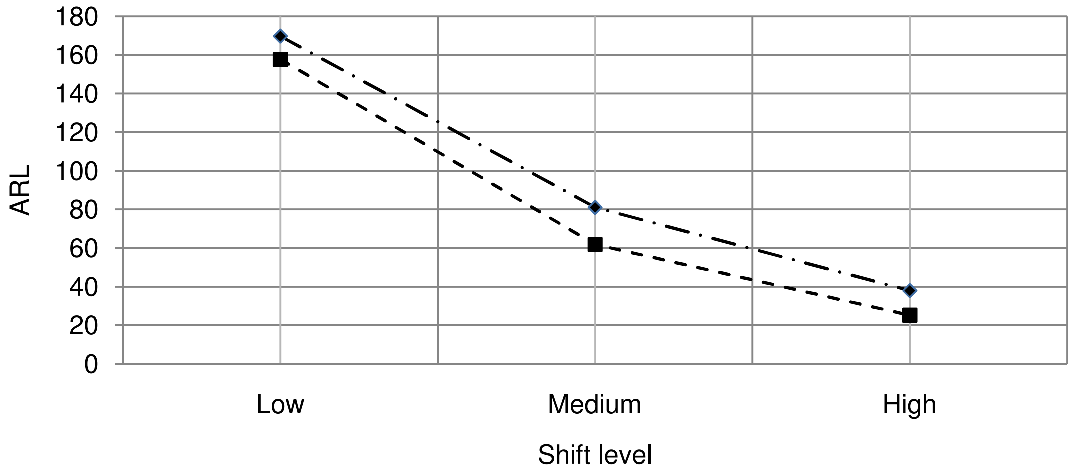

Fig. 17 ARL comparisons between regular and optimized plans under shift in second parameter from A_1 to $A_1 + \beta\sigma$ and n=5

The difficulty might face the user is the calculations needed to find the orthogonal polynomials when X values are unequally spaced. This difficulty increases if X values are not integer. In Appendix A, we described the Robson's method as one of the options available for the user.

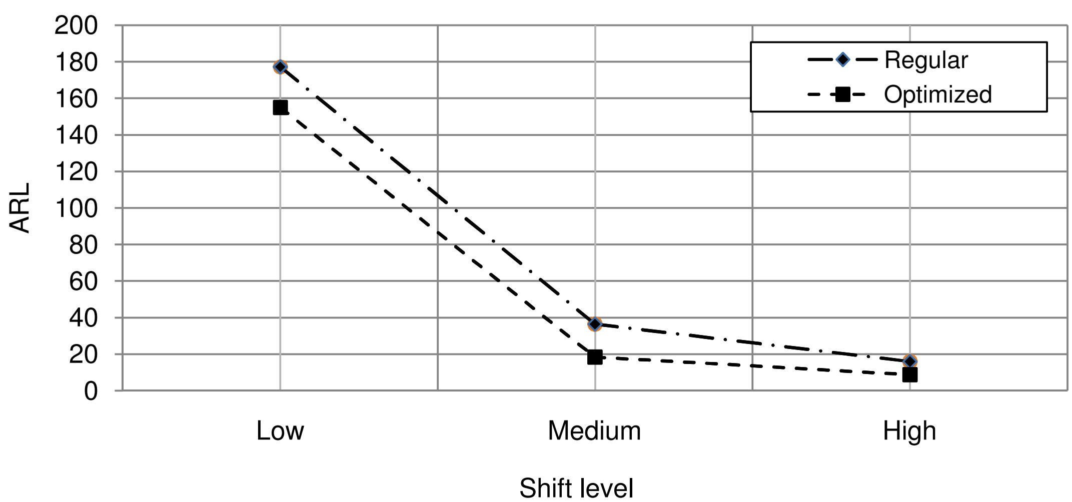

Fig. 18 ARL comparisons between regular and optimized plans under shift in second parameter from A_2 to $A_2 + \delta\sigma$ and n=5

One of the advantages of the Robson's method is the simplicity of coding on the computer. In the case that the X values are equally spaced, some of linear regression references such as Montgomery (2005) can be helpful. His book considers the orthogonal transformation and provided the orthogonal polynomials for a set of equally spaced X-values. So far, the effect of location optimization is examined using a simulation study and results are compared with non-optimized strategies in terms of average run length criterion. As we have seen, the results reveal the potentials of optimizing the location of X-values in improving the performance of EWMA4 method. Such methodology can be utilized to reduce the cost and effort of sampling and minimize the time that a process stays off-target. The following is another example of using the proposed approach. In this illustrative example, the sample is set equal to 10. An illustrative example of the regular EWMA4 method using a sample size of 10 can be found in Kazemzadeh et al. (2009).

Example 3.1

The following model is used

$$y_{ij} = 3 + 2x_i + 1x_i^2 + \varepsilon_{ij}\,; \quad i = 1,2,3,\ldots\ldots,n \quad j = 1,2,3,\ldots \tag{59}$$

Table 46 and 47 show the design parameters and orthogonal polynomials of the regular plan used in this example, respectively. These values can be found in Kazemzadeh et al. (2009) and some of references considering regression topic.

The results of the simulation study are listed in Tables 48 and 49. As one may have noticed that the optimized plans work better than the regular plan. Here we emphasize the importance of determining the optimal parameters of the GA in powerfully enhancing the performance of the EWMA4 method.

Table 46 The design parameters of EWMA4 method (in-control ARL=200 and n=5)

Parameter	Chart	Value
Smoothing factor (θ)	All	0.2
The multiplier (K_l)	$EWMA_l$	3.14
The multiplier (K_v)	$EWMA_v$	3.594

Table 47 Orthogonal polynomial and coefficients values of a regular plan when n=10

	i									
	1	2	3	4	5	6	7	8	9	10
$P_0(x_i)$	1	1	1	1	1	1	1	1	1	1
$P_1(x_i)$	-9	-7	-5	-3	-1	1	3	5	7	9
$P_2(x_i)$	6	2	-1	-3	-4	-4	-3	-1	2	6

Table 48 ARL comparisons between regular and optimized plans under shift in second parameter from A_1 to $A_1 + \beta\sigma$, and n=10

Shift	ARL		Independent Variable									
	Reg.	Opt.	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
0.01	144.82	118.41	2.00	3.00	7.00	7.00	7.00	9.00	10.0	10.0	10.0	10.0
		124.92	2.00	3.00	6.00	7.00	7.00	7.00	9.00	10.0	10.0	10.0
0.05	13.43	6.201	7.00	8.00	8.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0
		8.154	1.00	6.00	7.00	7.00	8.00	8.00	9.00	10.0	10.0	10.0
0.09	5.281	2.791	7.00	8.00	9.00	10.0	10.0	10.0	10.0	10.0	10.0	10.0
		2.862	7.00	7.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0	10.0

Table 49 ARL comparisons between regular and optimized plans under shift in third parameter from A_2 to $A_2 + \delta\sigma$, and n=10

Shift	ARL		Independent Variable									
	Reg.	Opt.	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
0.001	160.2	126.11	2.00	3.00	7.00	7.00	7.00	9.00	10.0	10.0	10.0	10.0
		131.43	1.00	2.00	3.00	7.00	7.00	9.00	10.0	10.0	10.0	10.0
0.005	21.76	6.991	7.00	8.00	8.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0
		10.51	1.00	6.00	7.00	7.00	8.00	8.00	9.00	10.0	10.0	10.0
0.009	7.792	2.942	7.00	8.00	9.00	10.0	10.0	10.0	10.0	10.0	10.0	10.0
		3.190	7.00	7.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0	10.0

Table 45 and 46 reveal that the optimized strategies outperform the regular plan at all types and suggested values of shift in the second and the third regression parameters. Generally, the simulation studies conducted in this section reveal the potentials of enhancing the performance of EWMA4 method by re-allocating the values of the explanatory variable. The suggested methodology can be effectively utilized to minimize the cost of sampling and then the time that a process stays on the off-target state.

3.7 Recommendations for Future Research

Under the linear quality profiles, we introduce three areas for future research and investigation.

I. Variable sampling size and control limits T^2 control chart for monitoring simple linear profiles: Here we suggest extending the VSSC idea from the case when the quality is described by the probability distribution to the of simple linear quality profiles case; see Chen and Hsieh (2007).

II. VSSI-EWMA/R and VSSI-EWMA3 method for monitoring simple linear profiles: Based on the results of adding the adaptive feature named VSSI, we recommend using the same feature with the EWMA/R method proposed by Kang and Albin (2000), and EWMA3 method proposed by Kim et al. (2003); see Appendix B.

III. Adaptive EWMA4 method: This suggestion is motivated by the significant improvement have been made in the power of the multivariate method introduced in chapter 1. There is a chance to integrate the three adaptive schemes, VSS, VSI and VSSI, with the EWMA4 and examine its performance versus the regular EWM4 method.

IV. EWMA4 using three observations per a set of data: Mahmoud (2010) proposed the use of two observations per sample to monitor simple linear profiles. He used an EWMA control chart based on the average squared deviation from the line of in-control state in conjunction with two EWMA control charts to monitor process parameters. In this section, we suggest extending this idea such that three observations per sample will be used to monitor the quadratic polynomial quality functions. Researcher might evaluate tall the charting techniques proposed by Mahmoud (2010).

V. Monitoring polynomial profiles under uncertainty in process's shift: Here we suggest evaluating the existing polynomial profiling techniques such as, T^2 method, Multivariate EWMA and EWMA4 methods under different pattern of shift. The following are examples of shift types:

1- Drift Shift (Linear)

2- Random shift with known probability distribution

APPENDIX A

A-1 Robson's method (Robson (1959)

In fact, the orthogonal polynomials for a set of equally spaced explanatory variable (X) are usually available in references covering the regression topic. When it comes to unequally spaced X-values, a specific technique is required. Robson (1959) considered this case and provided a simple method to calculate the values of these orthogonal polynomials. An illustrative example can be found in Robson (1959). To calculate the function $P_l(x_i)$, he suggested the following:

$$P_l(x_i) = \frac{1}{c_l}\left[x_i^l - \sum_{v=0}^{l-1} P_v(x_i) \sum_{i=1}^{n} x_i^l P_v(x_i)\right] \qquad (A-1)$$

where c_l represents the normalizing factor (constant) and can be calculated as follows:

$$c_l = \left\{\sum_{i=1}^{n}\left[x_i^l - \sum_{v=0}^{l-1} P_v(x_i) \sum_{i=1}^{n} x_i^l P_v(x_i)\right]^2\right\}^{\frac{1}{2}}, l = 0,1,2, \dots \dots, k \qquad (A-2)$$

APPENDIX B

B-1 Residuals Method (EWMA/R)

Kang and Albin (2000) proposed the use of the exponentially weighted moving average chart (EWMA) to monitor the average deviation from the in-control line. Additionally, they conjunctionally used an R chart with this $EWMA$ chart to control the variability about the regression line. The $EWMA$ chart uses the following statistic:

$$z_j = \theta \bar{e}_j + (1-\theta) z_{j-1} \qquad (B-1)$$

where θ is the smoothing parameter $(0 < \theta < 1)$.

The control limits for $EWMA$ chart are

$$CL_z = \pm L\sigma\sqrt{\theta\,(2-\theta)^{-1}n^{-1}} \qquad (\mathrm{B}-2)$$

where L is a constant to be selected to have a certain ARL at in-control state. For R chart, Kang and Albin (2000) suggested the use of $R_j = max_i(e_{ij}) - min_i(e_{ij})$; where $i = 1,2, \dots n$. The control limits for the range chart are:

$$CL_R = \sigma(d_2 \pm L d_3) \qquad (\mathrm{B}-3)$$

where d_2 and d_3 are constants that depend on the sample size. Further information about this method can be found in their paper.

B-2 Exponentially Weighted Moving Average Method (EWMA3)

Kim et al. (2003) suggested and used this method to monitor changes in simple linear quality profiles. The $EWMA3$ method suggests transforming X-

values to have zero average so that the regression parameters will be independent of each other. After that, they suggested the use of three separate $EWMA$ control charts for monitoring the slope, the intercept and the variance. The new linear model after coding X-values is $y_{ij} = B_0 + B_1 x_i^* + \varepsilon_{ij}$, where $i = 1,2, \dots, n$, $B_0 = A_0 + A_1\bar{x}$, $B_1 = A_1$, and $x_i^* = (x_i - \bar{x})$.

The $EWMA3$ method uses the following statistics and control limits for monitoring the intercept and slope.

$$EWMA_{Intercept}(j) = \theta b_{0j} + (1-\theta)EWMA_I(j-1) \qquad (B-4)$$

$$EWMA_{Slope}(j) = \theta b_{1j} + (1-\theta)EWMA_S(j-1) \qquad (B-5)$$

where θ is the smoothing parameter $(0 < \theta \leq 1)$, $EWMA_{Intercept}(0)$=B_0 and $EWMA_{Slope}(0)$=B_1. For control limits calculations, the following were proposed

$$CL_{Intercept} = B_0 \pm L_I \sigma \sqrt{\theta\,(2-\theta)^{-1} \mathrm{n}^{-1}} \qquad (B-6)$$

$$CL_{Slope} = B_1 \pm L_S \sigma \sqrt{\theta\,(2-\theta)^{-1} {\mathrm{S_{xx}}}^{-1}} \qquad (B-7)$$

The third chart is $EWMA$ chart based on the $EWMA$ chart of Crowder et al. (1992). The statistic and the control limit of this chart ($EWMA_v$) are:

$$EWMA_v(j) = Max\{\theta(MSE_j - 1) + (1-\theta)EWMA_v(j-1), 0\} \qquad (B-8)$$

$$CL = K_v \sqrt{\frac{\theta\, Var(MSE_j)}{(2-\theta)}} \quad ; \quad j = 1,2,3, \dots \qquad (B-9)$$

where $EWMA_v(0)= \ln(\sigma_0^2)$. If the reader is interested on this method, I recommend reading the original paper (Kim et al. (2003)).

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ABSTRACT

AN OPTIMIZATION OF ON-LINE MONITORING OF SIMPLE LINEAR AND POLYNOMIAL QUALITY FUNCTIONS

by

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Degree: Doctor of Philosophy

This research aims to introduce a number of contributions for enhancing the statistical performance of some of Phase II linear and polynomial profile monitoring techniques.

For linear profiles the idea of variable sampling sizes (VSS), variable sampling intervals (VSI) and variable sampling sizes and sampling intervals (VSSI) have been extended from multivariate control charts to the profile monitoring framework to enhance the power of the traditional Hotelling T^2 chart in detecting shifts in linear quality models. Finding the optimal settings of the proposed schemes has been formulated as an optimization problem solved by using a Genetic Approach (GA). Here the average time to signal (ATS) and the average run length (ARL) are regarded as the objective functions, and ATS and ARL approximations, based on Markov Chain Principals, are extended and modified to capture the special structure of the profile monitoring. The performances of the proposed control schemes are compared with their fixed

sampling counterparts for different step-shift and random-shift levels. The extensive comparison studies reveal the potentials of the proposed schemes in enhancing the performance of T^2 control chart when a process yields a simple linear profile.

For polynomial profiles, where the linear regression model is not sufficient, the relationship between the parameters of the original and orthogonal polynomial quality profiles is considered and utilized to enhance the power of the orthogonal polynomial method (EWMA4). The problem of finding the optimal set of values of the explanatory variable minimizing the average run length is described by a mathematical model and solved using the Genetic Approach. In the case that the shift in the second or the third parameter is the only shift of interest, the simulation results show a significant reduction in the mean of the run length distribution of the orthogonal polynomial method.

AUTOGRAPHICAL STATEMENT

I was born in Apr. 1, 1970 in Libya. After graduation from the secondary school (1988-1989), I was admitted to the Faculty of Engineering at Garyounis University in Libya. The undergraduate program at this school requires the student to choose a major field of study at the end of the second semester. In my second semester, the Faculty of Engineering had conducted a scientific exhibition in which each department at the school has to introduce its field of study to new students who have not made the choice of their major. In this Exhibition I have heard about the Industrial Engineering for the first time, which is relatively new engineering discipline where an engineer can see the big picture, use a lot of various skills to solve problems, and above all read and understand financial reports well as a technical report. It sounds like magic to me! That was the first thought that had come into my mind and it did not take me long to decide that I wanted to be among those magical engineers. During undergraduate study most of my interest was in the areas of applied statistics (statistical quality control, reliability and design of experiments). In 1999, I successfully completed my Master's degree requirements in Statistical Quality Control. In winter 2008, I joined the Industrial and System Engineering Department as Ph. D candidate. During the Ph. D preparation period, I have earned excellent research and teaching skills which will help me to have a distinctive academic profession.