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LQ-Moments for Regional Flood Frequency Analysis: A Case Study for the North-Bank Region of the Brahmaputra River, India

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LQ-Moments for Regional Flood Frequency Analysis: A Case Study for the North-Bank Region of the Brahmaputra River, India

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The LQ-moment proposed by Mudholkar, et al. (1998) is used for regional flood frequency analysis of the North-Bank region of the river Brahmaputra, India. Five probability distributions are used for the LQ-moment: generalized extreme value (GEV), generalized logistic (GLO) and generalized Pareto (GPA), lognormal (LN3) and Pearson Type III (PE3). The same regional frequency analysis procedure proposed by Hosking (1990) for the L-moment is used for the LQ-moment. Based on the LQ-moment ratio diagram and $\left|Z_i^{dist}\right|$ -statistic criteria, the PE3 distribution is identified as the robust distribution for the study area. For estimation of floods of various return periods for both gauged and ungauged catchments of the study area, regional flood frequency relationships are developed using the LQ-moment based PE3 distribution.

Key words: Regional flood frequency analysis, PE3 distribution, LQ-moment ratio diagram.

Introduction

Hosking (1990) introduced the concept of L-moment parameter estimation methods for regional frequency analysis. The performance of a particular model depends on the accuracy of the estimation of the parameters. Many parameter estimation methods are described in statistical literature. The unbiased estimation of parameters depends mainly on the parameter estimation method used and the data availability. Regional frequency analysis overcomes the difficulties arising from at-site frequency analysis. In many countries, the L-moments procedure for regional flood frequency analysis has been used and various researches are ongoing. In India, L-moments based regional flood frequency analysis was conducted by Paradia, et al. (1998) and Kumar et al. (1999, 2003 and 2005) to develop a flood frequency relationship for both gauged and ungauged catchments for different regions. Additionally, some recent application of regional flood frequency analysis include: Atiem and Harmancioglu (2006), Modarres (2007), Saf (2008) and Hussain, et al. (2008).

Kumar, et.al. (2005) used L-moments to develop a regional flood frequency relationship for both gauged and ungauged catchments of the North Brahmaputra region of India. Mudholkar, et al. (1998) introduced the concept of LQ-moment analogs of L-moments of Hosking (1990). LQ-moments are linear functions of the medians, trimeans, or Gastwirth's location estimators of the distributions of certain order statistics and reduce to weighted averages for certain population quantiles. LQ-moments are often easier to evaluate and estimate than L-moments and, in general, behave similarly to the L-moments when the latter exist. (Modhulkar, et al., 1998). Modhulkar, et al. (1998) used an LQ-moment in the context of generalized extreme value distribution for flood frequency analysis of the river Blackstote and Feather. Zin Wan, et al. (2008) used LQ-moments to determine the best fitting probability distribution for annual maximum rainfall in Peninsular Malaysia.

Various studies have found that LQ-moments are widely used to study at-site flood

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frequency analysis and at-site rainfall frequency analysis in different countries of the world. But in the case of flood frequency analysis, data availability is difficult for estimating floods for desired return periods. Therefore, this study uses regional frequency analysis as an alternative to at-site frequency analysis based on LQ-moments. The linear quantile estimator as a sample quantile estimator and trimean functional as quick estimator are also used in this study of regional flood frequency analysis.

Five probability distributions that are generally used for regional flood frequency analysis by using L-moments are used in this study: generalized extreme value (GEV), generalized Pareto (GPA), generalized normal (GNO), generalized logistic (GLO) and Pearson Type III (PE3). This study employs the LQ-moment as a parameter estimation method for regional flood frequency analysis of nine sites in the North-Bank region of the Brahmaputra River in India. The same procedure for regional frequency analysis for L-moments proposed by Hosking (1990) is used for LQ-moment. The relationship between LQ-skewness and LQ-kurtosis has been developed for each of the probability distributions used for this study.

LQ-Moments

Let $X_1, X_2, ..., X_n$ be a sample from a continuous distribution function $F_X(\cdot)$ with quantile function $Q_X(u) = F_X^{-1}(u)$. If $X_{1:n} \le X_{2:n} \le .... \le X_{n:n}$ denote the order statistics, then the r^th LQ-moments ζ_r of X proposed by Mudholkar, et al. (1998) are given by

$$\zeta_r = r^{-1}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} \tau_{p,\alpha}(X_{r-k:r}), \quad r = 1,\ 2,\ ... \tag{1}$$

where $0 \le \alpha \le 1/2,\ 0 \le p \le 1/2$, and

$$\tau_{p,\alpha}(X_{r-k:r}) = pQ_{X_{r-k:r}}(\alpha) + (1-2p)Q_{X_{r-k:r}}(1/2) + pQ_{X_{r-k:r}}(1-\alpha). \tag{2}$$

The linear combination $\tau_{p,\alpha}$ is a quick measure of the location of the sampling distribution of order statistic $X_{r-k:r}$. With appropriate combinations of α and p, estimators for $\tau_{p,\alpha}(\cdot)$ can be found which are functions of commonly used estimators such as median, trimean and Gastwirth. This study considers the trimean-based estimator, defined as:

$$Q_{X_{r-k:r}}\left(\frac{1}{4}\right)/4 + Q_{X_{r-k:r}}\left(\frac{1}{2}\right)/2 + Q_{X_{r-k:r}}\left(\frac{3}{4}\right)/4.$$

The first four LQ-moments of the random variable X are given by:

$$\zeta_1 = \tau_{p,\alpha}(X),$$

$$\zeta_2 = \frac{1}{2}[\tau_{p,\alpha}(X_{2:2}) - \tau_{p,\alpha}(X_{1:2})],$$

$$\zeta_3 = \frac{1}{3}[\tau_{p,\alpha}(X_{3:3}) - 2\tau_{p,\alpha}(X_{2:3}) + \tau_{p,\alpha}(X_{1:3})],$$

$$\zeta_4 = \frac{1}{4}\begin{bmatrix}\tau_{p,\alpha}(X_{4:4}) - 3\tau_{p,\alpha}(X_{3:4}) \\ +3\tau_{p,\alpha}(X_{2:4}) - \tau_{p,\alpha}(X_{1:4})\end{bmatrix}.$$

The LQ-CV, LQ-skewness and LQ-kurtosis are defined by

$$\eta = \varsigma_2 / \varsigma_1,\ \eta_3 = \zeta_3 / \zeta_2$$

and

$$\eta_4 = \zeta_4 / \zeta_2.$$

If $Q_X(.) = F_X^{-1}(.)$ is the quantile function of the random variable X then the quick location measure (2) defined by Modhulkar, et al. (1998) is

$$\tau_{p,\alpha}(X_{r-k:r}) = pQ_X[B^{-1}_{r-k:r}(\alpha)] + (1-2p)Q_X[B^{-1}_{r-k:r}(1/2)] + pQ_X[B^{-1}_{r-k:r}(1-\alpha)]$$

where $B^{-1}_{r-k:r}(\alpha)$ denotes the corresponding α^{th} quantile of a beta random variable with parameters $r-k$ and $k+1$.

Sample Estimates of LQ-Moments

Modhular, et al. (1998) defines sample estimates of LQ-moments as follows. Let $X_{1:n} \le X_{2:n} \le \ldots \le X_{n:n}$ denote the sample order statistics then the quantile estimator of $Q_X(u)$ is given by

$$\hat{Q}_X(u) = (1-\varepsilon)X_{[n'u]:n} + \varepsilon X_{[n'u]+1:n},$$

where $\varepsilon = n'u - [n'u]$ and $n' = n+1$. Thus for samples of size n, the r$^{\text{th}}$ sample LQ-moment is given by

$$\hat{\zeta}_r = r^{-1}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} \hat{\tau}_{p,\alpha}(X_{r-k:r}),$$

where $\hat{\tau}_{p,\alpha}(X_{r-k:r})$, the quick estimator of the location for the distribution of $X_{r-k:r}$ in a random sample of size r.

The first four sample LQ-moments are given by

$$\hat{\zeta}_1 = \hat{\tau}_{p,\alpha}(X)$$
$$\hat{\zeta}_2 = \frac{1}{2}[\hat{\tau}_{p,\alpha}(X_{2:2}) - \hat{\tau}_{p,\alpha}(X_{1:2})]$$
$$\hat{\zeta}_3 = \frac{1}{3}\begin{bmatrix}\hat{\tau}_{p,\alpha}(X_{3:3}) - 2\hat{\tau}_{p,\alpha}(X_{2:3}) \\ +\hat{\tau}_{p,\alpha}(X_{1:3})\end{bmatrix}$$
$$\hat{\zeta}_4 = \frac{1}{4}\begin{bmatrix}\hat{\tau}_{p,\alpha}(X_{4:4}) - 3\hat{\tau}_{p,\alpha}(X_{3:4}) \\ +3\hat{\tau}_{p,\alpha}(X_{2:4}) - \hat{\tau}_{p,\alpha}(X_{1:4})\end{bmatrix},$$

where, the quick estimator $\hat{\tau}_{p,\alpha}(X_{r-k:r})$ of the location of the order statistic $X_{r-k:r}$ is given by

$$\begin{aligned}\hat{\tau}_{p,\alpha}(X_{r-k:r}) &= p\hat{Q}_{X_{r-k:r}}(\alpha) + (1-2p)\hat{Q}_{X_{r-k:r}}(1/2) + p\hat{Q}_{X_{r-k:r}}(1-\alpha) \\ &= p\hat{Q}_X[B^{-1}_{r-k:r}(\alpha)] + (1-2p)\hat{Q}_X[B^{-1}_{r-k:r}(1/2)] + p\hat{Q}_X[B^{-1}_{r-k:r}(1-\alpha)]\end{aligned}$$

$0 \le \alpha \le 1/2,\ 0 \le p \le 1/2, B^{-1}_{r-k:r}(\alpha)$, is the α^{th} quantile of beta random variable with parameters $r-k$ and $k+1$, and $\hat{Q}_X(.)$ denotes the linear interpolation estimator shown above.

Probability Distributions and Parameters Based on Trimean Function: Generalized Extreme Value Distribution (Modhulkar, et al., 1998)

The probability distribution function (PDF) for the generalized extreme value distribution is defined as:

$$f(x) = \frac{1}{\alpha}\left[1-k\left(\frac{x-\xi}{\alpha}\right)\right]^{\frac{1}{k}-1} \exp\left[-\left\{1-k\left(\frac{x-\xi}{\alpha}\right)\right\}^{\frac{1}{k}}\right].$$

Its quantile function is given by

$$Q(u) = \xi + \alpha Q_0(u)$$

where

$$\begin{aligned}Q_0(u) &= [1-(-\log u)^k]/k, \quad k \ne 0 \\ &= -\log(-\log u), \quad k = 0.\end{aligned}$$

The shape parameter k can be estimated with good accuracy by using the approximation equation

$$k = 0.2985 - 2.0234\eta_3 + 0.3732\eta_3^2 - 0.1429\eta_3^3 + 0.0449\eta_3^4.$$

The estimates of the parameters ξ and α are then given by:

$$\xi = \varsigma_1 - \alpha[Q_0(1/4)/4 + Q_0(1/2)/2 + Q_0(3/4)/4]$$

and

$$\alpha = 8\varsigma_2 / \begin{bmatrix} 2Q_0(0.707) - 2Q_0(0.293) \\ +Q_0(0.866) - Q_0(0.134) \end{bmatrix}.$$

Probability Distributions and Parameters Based on Trimean Function: Generalized Pareto Distribution

The PDF of the generalized Pareto distribution is:

$$f(x) = \frac{1}{\alpha}\left[1 - k\left(\frac{x-\xi}{\alpha}\right)\right]^{\frac{1}{k}-1}.$$

Its quantile function is given by

$$Q(u) = \xi + \alpha Q_0(u)$$

where

$$\begin{aligned} Q_0(u) &= [1-(1-u)^k]/k, \quad k \neq 0 \\ &= -\log(1-u), \qquad k = 0. \end{aligned}$$

The shape parameter k can be estimated with good accuracy by using the approximation equation

$$\begin{aligned} k = 0.9998 - 3.4965\eta_3 + 1.4681\eta_3^2 \\ - 0.6243\eta_3^3 + 0.1535\eta_3^4 \end{aligned}$$

The estimates of the parameters ξ and α are then given by

$$\xi = \varsigma_1 - \alpha[Q_0(1/4)/4 + Q_0(1/2)/2 + Q_0(3/4)/4]$$

$$\alpha = 8\varsigma_2 / \begin{bmatrix} 2Q_0(0.707) - 2Q_0(0.293) \\ +Q_0(0.866) - Q_0(0.134) \end{bmatrix}.$$

Probability Distributions and Parameters Based on Trimean Function: Generalized Logistic Distributions

The PDF of the generalized logistic distribution is given by Rao and Hamed (2000) as

$$f(x) = \frac{1}{\alpha}\left[1 - k\left(\frac{x-\xi}{\alpha}\right)\right]^{\frac{1}{k}-1}\left[1 + \left\{1 - k\left(\frac{x-\xi}{\alpha}\right)\right\}^{\frac{1}{k}}\right]^{-2}.$$

Its quantile function is given by

$$Q(u) = \xi + \alpha Q_0(u)$$

where

$$\begin{aligned} Q_0(u) &= [1-\{(1-u)/u\}^k]/k, \qquad k \neq 0 \\ &= -\log\{(1-u)/u\}, \qquad k = 0. \end{aligned}$$

The shape parameter k can be estimated with good accuracy by using the approximation equation

$$k = -1.3328\eta_3 - 0.0286\eta_3^3 + 0.0166\eta_3^5.$$

The estimates of the parameters ξ and α are then given by

$$\xi = \varsigma_1 - \alpha[Q_0(1/4)/4 + Q_0(1/2)/2 + Q_0(3/4)/4]$$

and

$$\alpha = 8\varsigma_2 / \begin{bmatrix} 2Q_0(0.707) - 2Q_0(0.293) \\ +Q_0(0.866) - Q_0(0.134) \end{bmatrix}.$$

Probability Distributions and Parameters Based on Trimean Function: Generalized Lognormal Distribution

The PDF of the generalized lognormal distribution is

$$f(x) = \frac{\exp^{-\log\{1-k(x-\xi)/\alpha\}-\frac{1}{2}[-\frac{1}{k}\log\{1-k(x-\xi)/\alpha)]^2}}{\alpha\sqrt{2\pi}}.$$

Its cumulative distribution function is

$$F(x) = \Phi\left[\frac{\{\log(x-\varsigma)-\mu\}}{\sigma}\right]$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution, and its quantile function is given by

$$Q(u) = \varsigma + \exp(\mu)Q_0(u)$$

where

$$Q_0(u) = \exp[\sigma\Phi^{-1}(u)]$$

and $\Phi^{-1}(.)$ has a standard normal distribution with mean zero and unit variance. The σ can be approximated by the

$$\sigma = 2.3284\eta_3 - 0.0002\eta_3^2 + 0.1220\eta_3^3 + 0.0009\eta_3^4 - 0.0332\eta_3^5.$$

Estimates of the parameters ς and $\exp(\mu)$ are then given by

$$\varsigma = \varsigma_1 - \exp(\mu)[Q_0\left(\frac{1}{4}\right)/4 + Q_0\left(\frac{1}{2}\right)/2 + Q_0\left(\frac{3}{4}\right)/4]$$

$$\mu = 8\varsigma_2 / \begin{bmatrix} 2Q_0(0.707) - 2Q_0(0.293) \\ +Q_0(0.866) - Q_0(0.134) \end{bmatrix}$$

The parameters k, α and ξ can be obtained from the relation given below after determining the parameter values ς, μ and σ for the standard cumulative lognormal distribution. The relations between the parameter are $k = -\sigma,\ \alpha = \sigma e^{\mu}$ and $\xi = \varsigma + e^{\mu}$.

Probability Distributions and Parameters Based on Trimean Function: Pearson Type III Distribution (PE3)

The PDF of the Pearson Type III distribution is given by

$$f(x) = \frac{(x-\xi)^{\beta-1}e^{-(x-\xi)/\beta}}{\alpha^{\beta}\Gamma(\beta)}.$$

The cumulative distribution function is given as

$$F(x) = \frac{1}{\alpha^{\beta}\Gamma(\beta)}\int_{\xi}^{x}(x-\xi)^{\beta-1}e^{-(x-\xi)/\beta}dx,$$

and the quantile function can be given as

$$Q(u) = \xi + \alpha Q_0(u)$$

where

$$Q_0(u) = \beta\left[1 - \frac{1}{9\beta} + \Phi^{-1}(u)\sqrt{\frac{1}{9\beta}}\right]^3.$$

The location (μ), scale (σ) and shape (k) can be represented in terms of α, β and ξ as: $\beta = \frac{4}{k^2}$, $\alpha = \frac{1}{2}\sigma|k|$ and $\xi = \mu - 2\sigma / k$.

The regression equation developed for estimating the shape parameter k in terms of LQ-skewness (η_3) is now given as

$$k = 6.9839\eta_3 + 0.0001\eta_3^2 - 6.6634\eta_3^3 - 0.0035\eta_3^4,$$

and the estimates of the parameters ξ and α are then given by

$$\xi = \varsigma_1 - \alpha[Q_0(1/4)/4 + Q_0(1/2)/2 + Q_0(3/4)/4]$$

and

$$\alpha = 8\varsigma_2 / \begin{bmatrix} 2Q_0(0.707) - 2Q_0(0.293) \\ +Q_0(0.866) - Q_0(0.134) \end{bmatrix}.$$

Relationship between LQ-Skewness and LQ-Kurtosis based on Trimean Functionals

The relationship between η_3 and η_4 are developed for the probability distribution used in this study are given as follows:

$$\eta_4^{GEV} = 0.1080 + 0.1130\eta_3 + 0.8178\eta_3^2 - 0.0314\eta_3^3 - 0.0103\eta_3^4 - 0.0015\eta_3^5 + 0.0069\eta_3^6 - 0.0037\eta_3^7$$

$$\eta_4^{GLO} = 0.1585 + 0.8190\eta_3^2 - 0.0117\eta_3^4 - 0.0045\eta_3^6$$

$$\eta_4^{GPA} = -0.0019 + 0.2228\eta_3 + 0.8606\eta_3^2 - 0.0618\eta_3^3 - 0.0590\eta_3^4 + 0.0501\eta_3^5 + 0.0059\eta_3^6 - 0.0160\eta_3^7$$

$$\eta_4^{LN3} = 0.1201 + 0.7934\eta_3^2 - 0.0001\eta_3^3 - 0.0064\eta_3^4 + 0.0005\eta_3^5 - 0.0059\eta_3^6$$

$$\eta_4^{PE3} = 0.1227 - 0.0007\eta_3 + 0.4179\eta_3^2 + 0.0019\eta_3^3 - 0.5133\eta_3^4$$

Methodology

Study Area and Data Availability

Regional flood frequency analysis is carried out for North Bank region of the Brahmaputra River of India. The Brahmaputra River basin extends over an area of 580,000 km^2 and lies in Tibet, Bhutan, India and Bangladesh. The drainage area of the basin lying in India is 194,413 km^2, which forms nearly 5.9% of the total geographical area of the country. The mean annual rainfall over the basin (excluding Tibet and Bhutan) is approximately 2,300 mm. Annual maximum peak flood data for nine stream flow gauging sites lying in the North Bank region of the Brahmaputra River and varying between 11-36 years in record length were used in this study.

Steps in Regional Flood Frequency Analysis

The steps involved in the regional flood frequency analysis by L-moments proposed by Hosking and Wallis (1997) are:

1. Screening of the data;
2. Formation of homogeneous region;
3. Selection of appropriate distribution;
4. Estimation of parameters of the probability distribution; and
5. Development of regional flood frequency relationship for gauged and ungauged catchments of the region.

This procedure has been applied for LQ-moment for the study area described.

Data Screening

Hosking and Wallis (1997) proposed a discordancy measure (D_i) based on L-moments, to recognize sites that are grossly discordant with the group as a whole. The discordancy measure (D_i) for the LQ-moment is defined as if there are N sites in the group. Let $u_i = [\eta^{(i)}\ \eta_3^{(i)}\ \eta_4^{(i)}]^T$ be a vector containing the sample LQ-moment ratios η, η_3 and η_4 for site i, and T denote transposition of a vector or matrix. Let

$$\bar{u} = N^{-1}\sum_{i=1}^{N} u_i ,$$

be the (unweighted) group average. The matrix of sums of squares and cross product is then defined as:

$$S = \sum_{i=1}^{N}(u_i - \bar{u})(u_i - \bar{u})^T ,$$

and the discordancy measure for site i is defined as:

$$D_i = \frac{1}{3}N(u_i - \bar{u})^T S^{-1}(u_i - \bar{u}).$$

Site i is declared to be discordant if D_i is greater than the critical value of the discordancy statistic D_i given in a tabular form for the L-moment by Hosking and Wallis (1997). Based on such discordancy measures for LQ-moment, no discordance site was found for this study region. The discordancy measure, site names, sample sizes and LQ-moments are shown in Table 1.

Regional Homogeneity

The procedure proposed by Hosking and Wallis (1997) for L-moments, with required modification for an LQ-moment was used to test for regional homogeneity. The regional average LQ-CV, LQ-skewness and LQ-kurtosis, weighted proportional to the sites' record length were calculated and, similar to the regional average mean considered as 1 in the L-moment method, the regional average first LQ-moment ratio is also considered as 1. For this the LQ-moments and the parameters of the Kappa distribution based on the Trimean function have been developed and fit the developed Kappa distribution to the regional average LQ-moment ratios for 500 simulations. The values of heterogeneity measure computed by carrying out 500 simulations using the Kappa distribution based on the data for the 9 sites are provided in Table 2. Based on the heterogeneity measure the 9 site study area was found to be homogeneous.

Table 1: North Brahmaputra Region Site Information, Sample Statistics and Discordancy Measures

Site No.	Site Name	Sample Size	Catchment Area (km^2)	$\hat{\zeta}_1$	LQ-CV	LQ-Skewness	LQ-Kurtosis	D_i
1	Monas	17	30,100	5965.56	0.1739	0.1437	0.1008	0.25
2	Nonai	11	148	91.32	0.2159	0.1580	0.2021	0.14
3	Borolia	15	310	194.22	0.2540	-0.0345	0.1656	0.88
4	Dhansin	21	530	1275.50	0.1715	0.2039	0.3430	1.74
5	Pachnoi	22	198	196.82	0.2930	0.2915	0.1767	2.06
6	Jiabharali	36	11,000	4015.77	0.2607	0.0856	0.0412	0.54
7	Subansiri	27	25,886	8498.75	0.1777	0.2060	0.1198	0.36
8	Beki	13	1,331	748.60	0.2957	-0.0512	0.1490	1.26
9	Sankush	12	9,799	1865.99	0.1418	0.0703	-0.0942	1.76

Table 2: Heterogeneity Measure Based on LQ-Moment

Site No.	Heterogeneity Measures	Values
1	Heterogeneity Measure H(1)	
	(a) Observed standard deviation of group LQ-CV	0.0522
	(b) Simulated mean of standard deviation of group LQ-CV	0.0457
	(c) Simulated standard deviation of standard deviation of group LQ-CV	0.0108
	(d) Standardized test value H(1)	0.6000
2	Heterogeneity Measure H(2)	
	(a) Observed average of LQ-CV/LQ-Skewness distance	0.1015
	(b) Simulated mean of average LQ-CV/LQ-Skewness distance	0.1363
	(c) Simulated standard deviation of average LQ-CV/LQ-Skewness distance	0.0327
	(d) Standardized test value H(2)	-1.0600
3	Heterogeneity Measure H(3)	
	(a) Observed average of LQ-Skewness/LQ-Kurtosis distance	0.1331
	(b) Simulated mean of average LQ-Skewness/LQ-Kurtosis distance	0.1953
	(c) Simulated standard deviation of average LQ-Skewness/LQ-Kurtosis distance	0.0403
	(d) Standardized test value H(3)	-1.5400

Goodness-of-Fit Measure: $\left|Z_i^{dist}\right|$ Statistic Criteria

The same $\left|Z_i^{dist}\right|$-statistic criteria for the L-moment proposed by Hosking and Wallis (1997) was used as the goodness-of-fit measure for the LQ-moment to select the best fit distribution for the study region. The Z_i^{dist} statistic for the various three parameter distributions is shown in Table 3.

Table 3: Z_i^{dist} Statistic for Various Distributions for the Study Area

Distribution	Z_i^{dist} -statistic
GEV	0.77
LN3	0.71
GLO	1.38
GPA	-0.87
PE3	0.64

It may be observed from Table 3 that $\left|Z_i^{dist}\right|$-statistic values of all the five distributions are less than the critical value 1.64. Further, the $\left|Z_i^{dist}\right|$-statistic is found to be the lowest for PE3 distribution than all other distribution used for this study. Thus, the $\left|Z_i^{dist}\right|$-statistic criteria for the LQ-moment identifies the PE3 distribution as the best fitting distribution for the study region.

Goodness-of-Fit Measure: LQ-Moment Ratio Diagram

The LQ-moment ratio diagram is another goodness-of-fit measure for identifying the best fitting distribution for the study region. The relationships, given above between η_3 and η_4 for the five distributions are used to draw the theoretical curves in the LQ-moment ratio diagram. It can be observed from the LQ-moment ratio diagram (see Figure 1) that the regional values of LQ-skewness ($\eta_3 = 0.1332$) and LQ-kurtosis ($\eta_4 = 0.1324$) lie closest to PE3 distribution, thus the $\left|Z_i^{dist}\right|$-statistic criteria as well as the LQ-moment ratio diagram show that the PE3 distribution is the best fitting distribution for the study region.

Parameters and Quantile Estimates for the Region

The regional parameters and quantiles for the various distributions are given in Tables 4 and 5 respectively.

Regional Flood Frequency Relationship Based on LQ-Moments: Gauged Catchments

The regional flood frequency relationship for gauged catchments was developed, by using the identified best fitting distribution for the study area. The PE3 distribution was identified as the best fitting distribution for the study region in LQ-moment; thus, the relationship was developed using the PE3 distribution. The cumulative density function of the three parameter PE3 distribution as parameterized by Hosking and Wallis (1997) is: If $\alpha = 4/\gamma^2$, $\beta = \frac{1}{2}\sigma|\gamma|$, and $\xi = \mu - 2\sigma/\gamma$, where μ, σ and γ are its location, scale and shape parameters, respectively, then

$$F(x) = G(\alpha, \frac{x-\xi}{\beta}) / \Gamma(\alpha), \text{ if } \gamma > 0$$

and

$$f(x) = 1 - G\left(\alpha, \frac{\xi - x}{\beta}\right) / \Gamma(\alpha), \text{ if } \gamma < 0.$$

When, $\gamma = 0$, it becomes a normal distribution with μ and σ. In each case this distribution has no explicit analytical inverse form. Floods of various return periods *T* may be computed by multiplying $\hat{\zeta}_1$ (the first LQ-moment) of a catchment by the corresponding values of growth factors of the PE3 distribution.

Figure 1: LQ-Moments Ratio Diagram for the North-Bank Region of the Brahmaputra River

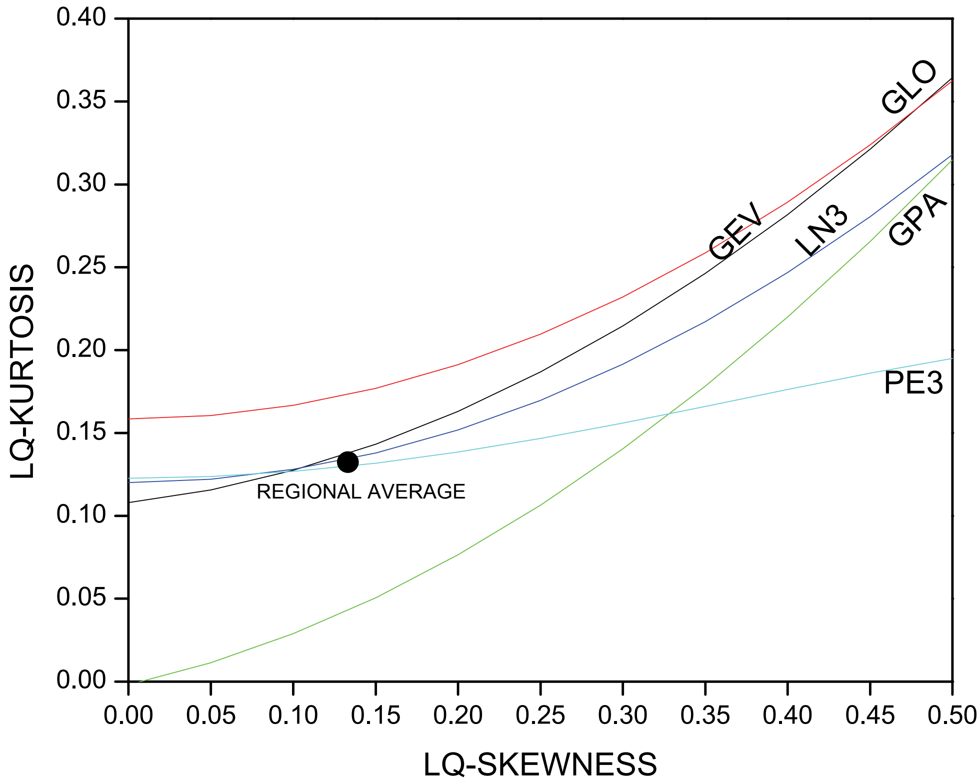

Table 4: Regional Parameters for Various Distributions Based on LQ-Moments

Distribution	Distribution Parameters		
GEV	ξ=0.857	α=0.353	k=0.035
GLO	ξ=0.987	α=0.244	k=-0.178
GPA	ξ=0.519	α=0.808	k=0.559
LN3	ξ=0.738	α=0.259	k=0.310
PE3	ξ=1.049	α=0.212	k=0.915

Table 5: Regional Quantile Estimation Based on LQ-Moments

Distribution	Return Periods (years)								
	2	5	10	25	50	100	200	500	1000
GEV	0.986	1.373	1.621	1.925	2.145	2.357	2.563	2.828	3.023
GLO	0.987	1.371	1.643	2.030	2.357	2.722	3.133	3.758	4.303
GPA	0.983	1.377	1.565	1.725	1.802	1.854	1.890	1.920	1.934
LN3	0.738	0.930	1.012	1.088	1.132	1.167	1.198	1.231	1.253
PE3	1.017	1.212	1.333	1.478	1.580	1.678	1.773	1.894	1.984

Regional Flood Frequency Relationship Based on LQ-Moments: Ungauged Catchments

In this case a relationship between the $\hat{\zeta}_1$ (the first LQ-moments) of gauged catchments in the region and their physiographic catchment characteristics is developed and is used to estimate first LQ-moments for an ungauged site. The relationship developed for the region in log domain using least squares approach based on the data of the study area is given as:

$$\hat{\zeta}_1 = 4.317 * (A)^{0.719} \quad (3)$$

where, A is the catchment area, in square kilometers (km^2) and is the $\hat{\zeta}_1$ first LQ-moments in meters per second (m^3/s). For equation (1), the correlation coefficient is $r = 0.947$. By coupling the regional flood frequency relationship for gauged catchment and the relationship between first LQ-moments and catchment area given by equation (1), the regional flood frequency relationship for ungauged catchments is obtained as:

$$Q_T = C_T A^{0.719} \quad (4)$$

where, Q_T is the flood estimate in m^3/s for return period T, A is the catchment area in km^2 and C_T is a regional coefficient. In Table 7 values of C_T are given for different return periods T for the study area.

Conclusion

The following conclusions can be drawn from the regional flood frequency analysis of the study area using LQ-moments:

1. In the initial screening step of the data the discordancy measure is used, the discordancy measure (Table 1) shows that data for the nine gauging sites of the study area are suitable for using regional flood frequency analysis.

2. For testing homogeneity of the region, the LQ-moment based heterogeneity measure was used, the LQ-moment based heterogeneity measure shows that the region is homogeneous.

3. The regional flood frequency analysis was performed using various frequency distributions: GLO, GEV, LN3, PE3 and GPA and KAP. The LQ-moment ratio diagram and $\left|Z_i^{dist}\right|$-statistic criteria (see Table 2) were used to identify best fitting distribution PE3 for the region.

4. The regional flood frequency relationship for gauged and ungauged catchments was developed for the region. The regional quantile estimates with different return periods *T* for the PE3, LN3, GPA, GLO and GEV distributions were calculated. To estimate floods of various return periods *T* for gauged catchments of the study area, the first LQ-moment of the catchment may be multiplied by corresponding values of the growth factors, computed using the PE3 distribution; however, more accurate results for ungauged sites can be obtained if more physiographic characteristics other than catchments area are available.

Table 7: Values of Regional Coefficient C_T

Return Periods (years)								
2	5	10	25	50	100	200	500	1000
PE3 Growth Factors								
4.390	5.232	5.755	6.381	6.821	7.244	7.654	8.176	8.565

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