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# Making Real-World Connections In High School Mathematics: The Effectiveness Of A Professional Development Program In Changing Teachers’ Knowledge, Beliefs, And Practices

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# MAKING REAL-WORLD CONNECTIONS IN HIGH SCHOOL MATHEMATICS: THE EFFECTIVENESS OF A PROFESSIONAL DEVELOPMENT PROGRAM IN CHANGING TEACHERS' KNOWLEDGE, BELIEFS, AND PRACTICES

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## DEDICATION

To my family whose unwavering support and confidence inspired me to persevere.

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# CHAPTER 1: INTRODUCTION

## Background of the Problem

After 12 years of formal instruction, many American students graduate high school with a poor understanding of what mathematics really is and how it is used to make decisions and solve problems in the world outside of school (Richland, Stigler, & Holyoak, 2012; Schoenfeld, 1988). Broadening the scope of the problem to include not just individual students but the wider society, Niss (1994) made the following observation when describing what he referred to as the relevance paradox.

> it is a striking fact that although the social significance of mathematics seems to be ever *increasing* in scope and density, the place, role and function of mathematics are largely *invisible* to – and unrecognized by – the general public, decision makers and politicians. (p. 371)

Multiple factors contribute to this situation. The quality and quantity of mathematics instruction play a role (Boaler, 2002). The curriculum, both the intended curriculum and the implemented curriculum, have an impact on students' beliefs about mathematics (Bishop, 1991; Wilkins & Ma, 2003). Assessment practices also contribute to how students experience their mathematical education, as they influence both what and how mathematics is taught (Johnson, 2007; Paris et al, 1991). These factors and others have an impact on how students come to view mathematics, its place in their lives, and whether they choose to study it beyond what is compulsory. Teachers also are a significant factor in students' education (Rivkin, 2005).

Students' experiences with mathematics which form their own beliefs and knowledge about the field are mediated by their teachers. Before leading their own classrooms, teachers were influenced by the individuals who taught them. Oftentimes, prior to pursuing a career in education, secondary mathematics educators were students who enjoyed school mathematics as it was taught by their mathematics teachers. They continued to study mathematics at the university level. At university they were taught by professors or graduate student teaching assistants. These individuals enjoyed and excelled at mathematics to such an extent that they chose to pursue advanced degrees in mathematics. Graduate courses taught in the mathematics departments of universities are often more concerned with definitions, theorems, and proofs than with practical applications of mathematics. This being the case, it is a small fraction of university students who are motivated to continue participating in formal mathematical education for such a long time. For example, in 2011 there were nearly 21.6 million students in postsecondary institutions in the US (Snyder & Dillow, 2015). In 2014 there were 1,924 doctoral degrees awarded in mathematics in the US, and of those only 236 were in applied mathematics (Vélez et al, 2015). There is attrition from mathematics education at every level of schooling, as students decide to forego further study of a subject that becomes increasingly abstract and esoteric. College professors, like all educators, report many influences on their teaching, but how they learned and were taught did inform how they conducted their classes (Oleson & Hora, 2013). When aspiring secondary teachers graduated college and started teaching their own mathematics classes, they often did so in the same ways they were taught (Ball, 1996; Makonye, 2014) and, thus, perpetuated the cycle of school mathematics being taught in ways that appeal and make sense to only a narrow range of learners.

### *The Importance of Teaching the Relevance of Mathematics*

Teaching mathematics in ways that appeal to only a narrow range of learners may have been sufficient in a bygone era. It is no longer the case that only college-intending students are expected to be exposed to algebra in high school (National Commission on Mathematics and Science Teaching for the 21st Century, 2000). The College Board (2000) has a long record of recommending that all students complete an algebra course by the end of ninth grade. As of May, 2019, 44 states and the District of Columbia required students to take either three or four years of mathematics classes to earn a general high school diploma (Achieve, 2019). Teaching more mathematics to more students than ever before is a "major and demanding responsibility that has been mandated without a plan for how this new mandate can be achieved" (Chazan, 2008, p. 23). High school mathematics teachers may not be adequately prepared for the challenge of teaching more advanced mathematics topics to students who have typically struggled in mathematics (Chazan, 2008). An unintended consequence of increased high school graduation requirements has been an increase in the drop-out rate (Plunk et al, 2014). With a mandate to teach advanced mathematics to all students, there is a need for teachers to develop alternative methods that appeal to a much wider range of learners.

A perceived lack of relevance has been shown to be a reason why students stop participating in mathematics education (Brown et al, 2008). Demonstrating that mathematics is relevant to the lives of their students may be a task secondary mathematics teachers are not equipped to handle, as such information did not play a large role in their formal mathematical training. According to Gwendolyn Lloyd (2002), "Perhaps the greatest obstacle for teachers is a lack of personal familiarity with mathematical problem-solving and sense-making – processes that most have never experienced themselves, as students or teachers" (p. 149). If teachers are to motivate their students to continue in their mathematics education, the teachers may need to make a stronger case for the utility and relevance of mathematics. Such a change in practice may be difficult to implement, as mathematics teachers are the social products of the very system they need to reform. Hamilton (2007) juxtaposed rates of change in schools with those outside school when he stated, "Given that mathematics classrooms have proven highly resistant to change for decades and society and workplaces are changing rapidly, it is not surprising school mathematics and mathematics needed beyond school are diverging rapidly" (p. 5).

David Cohen and colleagues (2007) stated the following observation when considering how change occurs in schools.

> The policy makers who define problems and devise remedies are rarely the ultimate problem solvers. They depend on the very people and organizations that have or are the problem to solve it. At the same time, those that have or are the problem depend on policy makers or others for some of the resources—ideas, incentives, money, and more—that may enable a solution. (Cohen et al, 2007, p. 522)

Cohen and colleagues (2007) further observed that teachers' knowledge mediated the relationships between policy and practice. It follows from this relationship that the greater the divide between the proposed educational reform and conventional practice, the greater the need for professional development among teachers. Without additional training, teachers will be hard pressed to impart an appreciation for the real-world applicability of mathematics if they lack knowledge and experience with such connections between academic mathematics and mathematics as it is used in the world outside of school.

Standardized testing of students' mathematical knowledge gained greater prominence since the passage of the No Child Left Behind Act (Guilfoyle, 2006). When leaders want to understand and improve the performance of their organizations, they measure what they value. There is little agreement on what is truly valued in mathematics education (Schoenfeld, 2004). Although authoritative voices espouse the view that conceptual understanding and the ability to successfully apply those concepts to a wide range of problem situations should be the focus of K-12 mathematics education (National Research Council [NRC], 1989; NRC, 1990; National Council of Teachers of Mathematics [NCTM], 1989; NCTM, 2000), measuring those outcomes is difficult and expensive. Instead the high-stakes mathematics tests used in the United States for the past three decades have tended to focus on facts and isolated skills (Au, 2007; Cobb & Jackson, 2011). Due to federal and state accountability laws, student performance on these tests have a range of implications for schools and teachers. In lieu of measuring what is valued, many secondary mathematics educators decided to value what is measured (Shepard, 2002). Within the past decade an effort was made to improve assessment practices, partly motivated by Race to the Top grants from the Obama administration. Two multi-state consortia were established to assess college- and career-readiness standards in mathematics and English. Initially 45 states joined these consortia. Political and public backlash against these tests resulted in fewer than a third of those states remaining in the consortia (O'Keefe & Lewis, 2019).

The policy shifts that resulted from the No Child Left Behind Act (NCLB) pushed states and districts to rely on standardized testing to hold teachers accountable for students' learning outcomes (Schoen & Fusarelli, 2008). Deciding to teach to the test, therefore, was reasonable, even though the test may be invalid due to construct underrepresentation. School administrators, often judged by the performance of their students and the consequent ranking of their schools (Steen, 2001), are also susceptible to focusing more on test preparation than authentic learning. Upon observing the tension this creates in administrators, Schoen and Fusarelli (2008) remarked that "Many conscientious school leaders are trying to be simultaneously responsive to calls for innovation, critical thinking skills, adaptability, and creativity (21st-century skills) yet still meet the demands and adequate yearly progress (AYP) testing targets of NCLB" (p. 182). Even when administrators understand their mathematics programs need reform, they often adopt piecemeal changes that do not get at the heart of the problem (Spillane, 2000).

A possible consequence of the pressures exerted on secondary mathematics classrooms that keeps current curricula looking similar to curricula of past generations is the inadequate number of students motivated to pursue science, technology, engineering, and mathematics (STEM) careers (Atkinson & Mayo, 2010; NRC, 1989; NRC 1990). Students' interest in and motivation to continue studying mathematics wanes when they experience a curriculum that portrays mathematics as a static set of facts and skills. Kloosterman (2002) noted that "For motivation to be most effective, it needs to be aimed toward learning the mathematics that will be needed in the present and the future rather than the mathematics that was used in the past" (p. 252). A less tangible but related outcome of an uninspiring mathematics education is the social acceptability of innumeracy (Paulos, 1988; Fleener et al, 2002). Proclamations such as, "I've never been good at math," are often said in social situations without shame, regret, or recognition that such a belief could limit career options.

A dismissive attitude toward mathematics achievement can be even more detrimental to learning when parents express it to their children. This occurs more often in the US than in other countries (Chazan, 2008) and is reflective of a societal attitude that success in mathematics is based on ability instead of effort. Carol Dweck (2006) framed such attributions as "mindset" and contended that parents and teachers can undermine children and limit their achievement by sending them messages that their success depends on fixed ability rather than growth-oriented messages that promote perseverance in the face of obstacles. Jo Boaler (2016) extended Dweck's line of research to mathematics education. In her research, she "realized more strongly than ever before that many people have been traumatized by math… but the evidence I collected showed that the trauma is fuelled by incorrect beliefs about mathematics and intelligence" (Boaler, 2016, x). Teachers' beliefs about and knowledge of mathematics as a practical and powerful human endeavor can enable them to portray mathematics in ways that are more accessible and inspiring to more students.

### ***Contextualizing School Mathematics***

There is a need to address the underlying issue that school mathematics does not seem relevant to most of the students who study it (NRC, 1989; NRC 1990). One way to address this is to provide mathematics instruction that demonstrates how mathematics can be used as a tool to model the real world in order to solve problems and make better decisions. The eight Standards for Mathematical Practice (SMP) from the Common Core State Standards for Mathematics (CCSSM) stated that "Mathematically proficient students… continually ask themselves 'Does this make sense?'" (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGACBP & CCSSO], 2010, p. 6). The SMP Reason Abstractly and Quantitatively emphasizes students' ability to relate the mathematics they learn to the contexts of problem situations, as well as their ability to relate problem contexts to mathematical concepts. This bi-directional process of contextualization and decontextualization is a feature of the CCSSM position on mathematical problem solving in real-world situations.

> Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. (NGACBP & CCSSO, 2010, p. 6)

The CCSSM also emphasizes mathematical modeling. There are 61 content standards identified as modeling standards. Rather than being seen as isolated modeling standards, these are intended to be integrated into other topics. One of the SMPs is dedicated to Modeling with Mathematics. It states

> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace… Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. (NGACBP & CCSSO, 2010, p. 7)

Embracing this vision of routinely engaging students in genuine mathematical modeling activities could help students see the relevance of mathematics. It is not at all clear, however, whether teachers are prepared to teach school mathematics in this way. Doerr (2007) suggested that "one reason for the limited use of applications and modelling at the primary and secondary levels of schooling is the lack of knowledge by those who are expected to teach mathematics through applications and modelling" (p. 69).

The mathematics textbook publishing industry provides teachers with their primary teaching resources. Although textbooks do not constitute curricula, secondary mathematics teachers tend to rely heavily on textbooks (Reys et al, 2002; Rezat, 2012). Traditional mass-adopted mathematics textbook series have underemphasized true mathematical modeling in favor of drill and practice with rote skills and techniques (Schoenfeld, 2007; Sood & Jitendra, 2007; Usiskin, 2001). The story problems included in many of the sample sets within these textbooks bear little resemblance to the ways mathematics is actually used by professionals (K. R. Chelst, personal communication, May 1, 2015). The problems that are posed to students in mathematics classrooms typically have a right answer, which the teacher knows. Professionals who utilize mathematics frequently develop mathematical models of real-world systems. Thus the end result of authentic mathematical modeling work is not the one right answer to a problem, but "sharable, manipulatable, modifiable, and reusable conceptual tools (e.g., models) for constructing, describing, explaining, manipulating, predicting, or controlling mathematically significant systems" (Lesh & Doerr, 2003, p. 3). Students who experience mathematics education as a years-long series of homework assignments to practice the symbolic manipulations of solving equations and occasional contrived story problems can come to believe that mathematics is an uninspiring, tedious, and irrelevant discipline (Nardi & Stewart, 2003).

***When Will I Ever Use This?***

In 2007 work began on a National Science Foundation-funded curriculum development project (DRL-0733137). The project was a collaboration between educators, engineers, and mathematicians at three universities that sought to improve both students' abilities to model with mathematics and their attitude towards mathematics (Keene et al, 2011). A curriculum for a senior-level mathematics course was developed to teach mathematical modeling tools from the fields of operations research and industrial engineering. The curriculum was made up of one semester each of algebraic and probabilistic modeling techniques. A professional development program to train teachers to implement the curriculum was also developed as part of the project (Chelst et al, 2008).

The curriculum and supporting professional development program eventually came to be called "When Will I Ever Use This?" (WWIEUT). The curriculum features a variety of decision-making tools all centered on teaching students how to model with mathematics. Mathematical modeling was not emphasized in undergraduate mathematics teacher education programs (Doerr, 2007). Neither did it play a prominent role in the majority of high school mathematics curricula. To address the need for teachers to learn new content knowledge and pedagogical knowledge, a professional development program was created to train teachers to effectively implement the curricular materials. These workshops were the focus of the present study. Their structure will be described below.

**Problem Statement**

It is hypothesized that secondary mathematics teachers in the United States may not be able to adapt to the changes that adhering to the CCSSM necessitates, such as the SMP Model with Mathematics. Mathematically modeling real-world systems is a way of using mathematics with which most teachers do not have experience or training. It entails content knowledge and pedagogical knowledge as well as a disposition toward seeing mathematics as a practical tool for creatively formulating and solving problems that arise in non-academic settings. Without professional development to provide teachers with meaningful learning experiences where they can see not only how a broad range of professionals use mathematics to solve complex problems and make better decisions but also how to design and deliver relevant and engaging instruction, teachers will remain ill-equipped to meet the challenges of implementing the SMPs with fidelity. Professional development programs that aim to address these issues need to be studied and evaluated to ensure they are achieving their goals. The present study was an effort to examine the efficacy of a professional development workshop designed to address a perceived lack of knowledge of algebraic applications and modeling on the part of secondary mathematics teachers in the United States.

***Conceptual Framework***

The WWIEUT professional development workshops were designed to address three relevant concerns for secondary mathematics teachers: (a) their knowledge of how mathematics is used in the world outside of school, (b) their beliefs about mathematics as a modeling tool that is relevant to a broad range of careers, and (c) their instructional practices related to including mathematical modeling of real-world systems in their courses. It was hypothesized that participation in a WWIEUT workshop would increase teachers' knowledge of how to model with mathematics, enhance their beliefs about the relevance of mathematics beyond the classroom, and increase the amount of time they devote to mathematical modeling activities in their instruction.

The conceptual framework for the present study was based on related models from Desimone (2009), Boston (2013), and Karabenick and Conley (2011). Desmimone proposed a model and recommended its use for any empirical study seeking to make claims that professional learning activities caused specific teacher or student outcomes. The theory of action at the heart of Desimone's (2009) model is outlined below.

1. Teachers experience effective professional development.
2. The professional development increases teachers' knowledge and skills and/or changes their attitudes and beliefs.
3. Teachers use their new knowledge and skills, attitudes, and beliefs to improve the content of their instruction or their approach to pedagogy, or both.
4. The instructional changes foster increased student learning. (p. 184)

An important aspect of Desimone's model was that the core features of the professional development have a direct impact on teacher knowledge and beliefs. The increased knowledge and changed beliefs could then have an impact on instruction, which in turn resulted in improved student learning. In addition to a conceptual framework for studying the effects of professional development on teachers and students, Desimone identified five core features of high-quality professional development programs. These core features will be discussed later in this section.

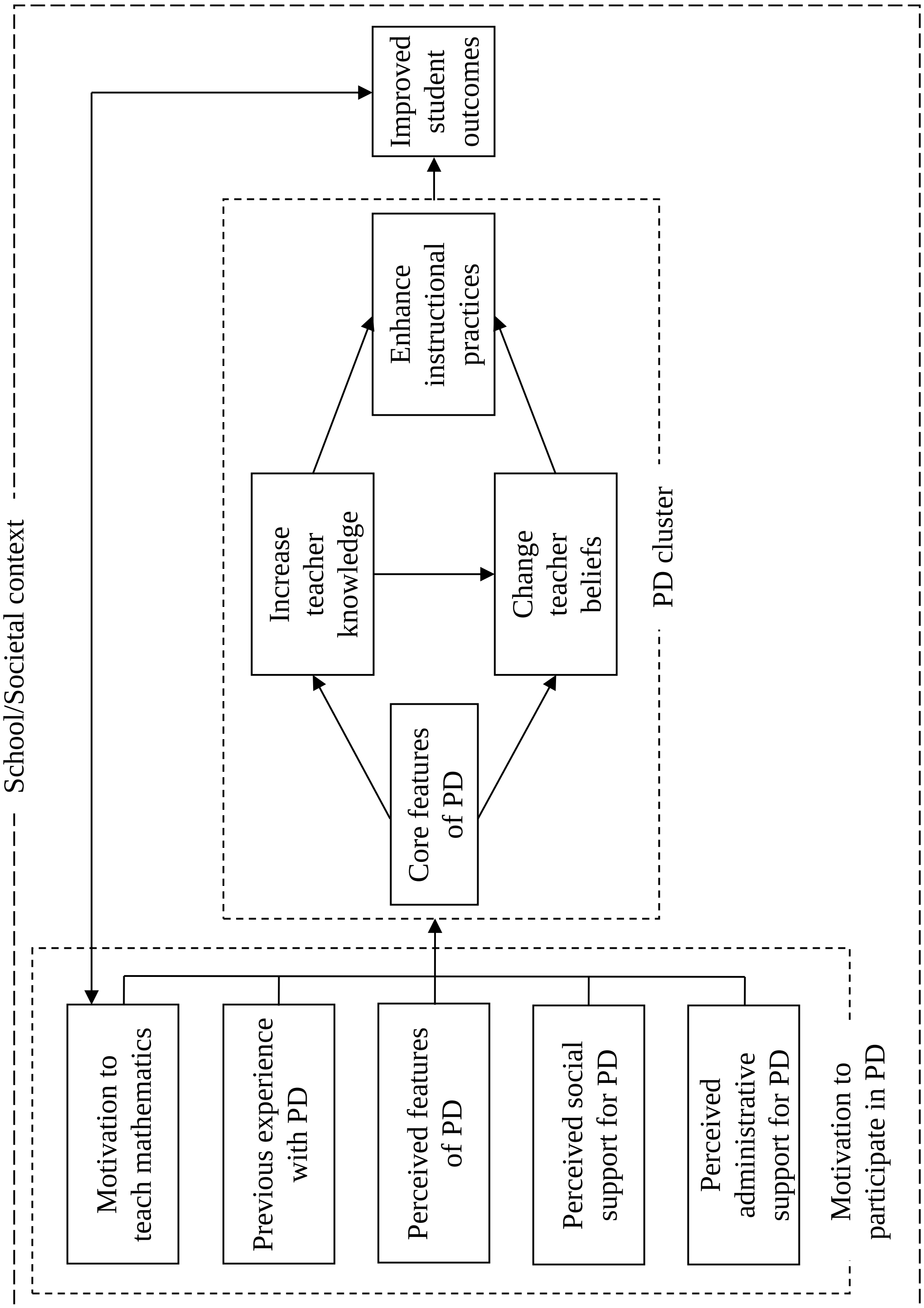
Boston's (2013) study of a professional development program designed to enhance secondary mathematics teachers' knowledge of cognitively challenging mathematical tasks relied on a conceptual model with four components. In her framework, teacher participation in a professional development workshop would produce enhancements in teacher knowledge which would influence and be influenced by enhancements in instructional practice. The instructional practice enhancements would influence and be influenced by improvements in student learning outcomes.

Karabenick and Conley (2011) developed a model for studying the effectiveness of teachers' professional development programs. Central to their framework was teachers' motivation to participate in professional learning activities. Teachers' motivation to participate in professional development activities was multifaceted and played a critical role in not just whether teachers chose to participate in professional development but also how willing they were to change their practice based on what they learned. Karabenick and Conley also acknowledged the importance of the school and societal context in which teachers operated in determining teachers' motivation to participate in professional development.

The conceptual framework for this study is shown in Figure 1. This broad framework includes aspects of teacher motivation and student outcomes that will not be addressed in this study. The research questions and hypotheses for the present study relate only to the components within the professional development (PD) cluster of the framework. Issues related to motivation to participate in professional development were a limitation of the study, as selection bias may have explained some of the observed differences between treatment and control groups. Investigating student outcomes was beyond the scope of the present study.

**Figure 1**

*Conceptual Framework for Changes in Teachers' Knowledge, Beliefs, and Practice (Boston, 2013; Desimone 2009; Karabenick & Conley, 2011)*



The PD cluster includes the learning opportunities teachers experience as they participate in a WWIEUT workshop. Desimone (2009) argued that there was empirical consensus for five core features of high-quality professional development. These core features were content focus, active learning, coherence, duration, and collective participation. The WWIEUT workshops; which were intended to bring about the hypothesized changes in teachers' knowledge, beliefs, and practice; included several of these core features in their design.

The content of the WWIEUT workshops was narrowly focused on algebraic modeling. Teachers were shown how concepts and skills developed in standard curricula could be applied to engage in the mathematical modeling process with real-world problem situations that arise in business, industry, public policy, and everyday life. Workshop activities focused on providing tools for teachers to help their students develop quantitative reasoning skills through problem formulation and solution interpretation. Workshop facilitators varied their instruction and frequently cycled between lectures about the modeling process and how it was used in the world outside of school, hands-on collaborative activities where teachers engaged in formulating models and interpreting the outputs of their models, and discussions of pedagogical issues that arose when teaching students to model with algebra. The impetus for developing the WWIEUT curricular materials and accompanying professional development workshops were the widespread adoption of the CCSSM and states' increased graduation requirements in mathematics. As such, the goals of the WWIEUT workshops, which were centered on helping students see mathematics as a powerful and practical tool that is widely used outside of school, were intended to be coherent with nationwide reform efforts in mathematics education.

The duration of the WWIEUT workshops ranged from one to four days of intensive training. Efforts were made to create a community of practice for workshop participants across the country. The online platforms that were meant to provide a means of communication and collaboration were not widely used by teachers, however. Beyond these underutilized online platforms, the WWIEUT workshops did not provide meaningful follow-up opportunities for teachers to continue to develop the knowledge, beliefs, and practice that were the goals of the workshops. The final core feature described by Desimone (2009) was collective participation. For the WWIEUT workshops to be most effective, all the teachers in a school's mathematics department should participate. This was not the case, however. Teachers elected to participate out of an interest in learning about algebraic modeling of real-world systems. It was most common for one or two mathematics teachers from a school to attend a workshop.

To provide context for the WWIEUT workshop's structure, this paragraph will outline the typical activities for teachers and facilitators. (See Appendix A for a sample agenda for a one-day workshop.) Facilitators generally began the session with an introduction to operations research and mathematical analytics as they were commonly used in business and industry. In this didactic portion of the workshop, teachers learned how professionals routinely used algebra to model real-world systems and improve decision-making. Most of the remainder of the workshop was devoted to teachers actively learning about the WWIEUT curriculum. Activities varied from discussions of both the algebraic modeling as well as the problem contexts to interacting with the text and technological tools to conversations about pedagogical issues. In a typical segment, teachers were given a brief introduction to a problem context in a whole group setting. This was followed by small groups of teachers collaboratively engaging with the text to develop a mathematical model, find a solution, interpret that solution, and then analyze the solution through a series of hypothetical changes in the problem's parameters. During this time when content knowledge was developed, facilitators moved from group to group to monitor progress and answer questions. When the large group reconvened, facilitators described common misconceptions and errors that high school students might encounter. Questions of pedagogy were the focus of this debriefing stage to provide teachers with knowledge for teaching. This cycle repeated five times in a typical day. Another common feature of the workshops was a presentation from a local operations research professional. This presentation gave teachers a firsthand account of how algebraic modeling was actually applied in modern workplaces. Teachers who attended the workshops were motivated volunteers. As a part of the workshop, teachers were actively engaged in the curriculum and eagerly asked insightful questions about how to effectively implement the curriculum with high school students.

The hypothesized teacher outcomes were tested using a sample of secondary mathematics teachers from high schools across the country. This mixed-methods study made a pair of comparisons on the knowledge, belief, and practice outcomes between groups of teachers. An online questionnaire was administered to obtain quantitative data from teachers. A set of comparisons was made between teachers who participated in a WWIEUT workshop and teachers who were unaware the workshops existed. A second set of comparisons was made by investigating a cohort of teachers both before and after they participated in a WWIEUT workshop. These comparisons were used to determine the efficacy of the professional development workshops in bringing about changes in teachers' knowledge, beliefs, and practice related to the real-world applicability of algebra. In the qualitative phase of this research, semi-structured interviews were conducted to corroborate the quantitative findings and also help the researcher understand how and why changes in their knowledge, beliefs, and practice occurred or did not occur.

***Operational Definitions***

The present study investigated the relationship between teachers' participation in a WWIEUT professional development workshop and three constructs which served as dependent variables: knowledge, beliefs, and practice. These constructs are complex and multi-faceted, and this study narrowly focused on a certain aspect of each. The research literature includes various conceptions of these terms (e.g., Hart, 1989; Hoy et al, 2006; McLeod & McLeod, 2002; Murphy & Mason, 2006; Philipp, 2007; Thompson, 1992). The particular knowledge, beliefs, and features of practice that served as the foci of this study are defined in this section.

Teachers' knowledge has been assessed for more than two centuries (Hill, Sleep, Lewis, & Ball, 2007). Teachers possess other types of knowledge that are relevant to instruction. Shulman (1986) describes content knowledge as "the amount and organization of knowledge per se in the mind of the teacher" (p. 9). Hiebert and Carpenter (1992) addressed the organization of knowledge when they defined understanding "in terms of the way information is represented and structured" (p. 67). In this light, ideas are understood if they are connected to an internal network. Goals of the WWIEUT professional development workshop included creating new knowledge of applications of algebra and connecting this knowledge to teachers' existing networks of algebraic understanding. Based on the conceptual framework in Figure 1, it is only through linking the new knowledge, both of content and pedagogy, to existing knowledge and thus changing teachers' practice that the ultimate goal of improving students' experiences in mathematics classes can be achieved.

Shulman (1986) contrasts knowledge of content and general knowledge of pedagogy with pedagogical content knowledge (PCK), which he describes as "the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (p. 9). Mishra and Koehler (2006) further explain what knowledge qualifies as PCK.

> This knowledge includes knowing what teaching approaches fit the content, and likewise, knowing how elements of the content can be arranged for better teaching. This knowledge is different from the knowledge of a disciplinary expert and also from the general pedagogical knowledge shared by teachers across disciplines. PCK is concerned with the representation and formulation of concepts, pedagogical techniques, knowledge of what makes concepts difficult or easy to learn, knowledge of students' prior knowledge, and theories of epistemology. It also involves knowledge of teaching strategies that incorporate appropriate conceptual representations in order to address learner difficulties and misconceptions and foster meaningful understanding. (p. 1027)

Ball and colleagues (2008) elaborated on Shulman's work on PCK. Their model was composed of six domains of mathematical knowledge for teaching. Common content knowledge, specialized content knowledge, and horizon content knowledge were all classified as subject matter knowledge. Knowledge of content and teaching, knowledge of content and students, and knowledge of content and curriculum were classified as pedagogical content knowledge. Several of these domains included knowledge that was unique to the teaching endeavor. Teachers needed to know mathematics in different ways than other professionals in order to respond to situations that arose in the classroom and plan effective instruction. The authors noted that knowing mathematics was necessary but not sufficient for effectively teaching mathematics to children. Furthermore, they criticized subject matter courses in teacher preparation programs for being "academic in both the best and worst sense of the word, scholarly and irrelevant, either way remote from classroom teaching" (Ball et al, 2008, p. 404).

Ernest (1989b) proposed "knowledge of other subject matter" as a component of mathematics teachers' knowledge.

> Knowledge of other subject matter has an important contribution to make to the teaching of mathematics. For it provides a stock of knowledge of uses and applications of mathematics, which can serve two purposes. First of all, the uses of mathematics such as networks in geography, graphs in economics, probability in genetics, formulas in physical science, and so on, provide justification and motivation for studying some of the content of mathematics by showing children its relevance. Secondly, it provides a range of models, images, and analogies for mathematical concepts. (p. 17)

One aspect of knowledge of other subject matter is how mathematics is productively put to use in the world outside of school to solve problems and make decisions. This will be referred to as teachers' knowledge of real-world mathematics.

The core features of the WWIEUT workshops were designed to impart content knowledge, pedagogical content knowledge, and knowledge of real-world mathematics. Goals for content knowledge included raising teachers' awareness of the myriad ways algebra can be used to model real-world systems and the wide range of careers to which algebra is relevant. For pedagogical content knowledge, instructional practices were modeled as were ways to anticipate and respond to common problems experienced by students. The WWIEUT curricular materials, which were the foundation for the workshops, presented all skills, concepts, and problems in richly contextualized and realistic scenarios. The primary focus of the workshops was to increase teachers' knowledge of real-world mathematics. Every problem and technique was discussed both in terms of the algebra involved as well as their real-world implications.

Knowledge of real-world mathematics was also the type of knowledge of interest in the present study. The degree to which participants possessed knowledge of real-world mathematics was inferred from their responses to relevant questionnaire items. From these responses, a scale score was created to assess participants' assessments of how much they knew about the broad applicability of algebra to a wide range of careers that are relevant to a majority of their students.

The second construct considered in the present study was teachers' belief about the relevance of mathematics, specifically as a component of their students' future educational goals and professional careers. Teachers hold many beliefs about mathematics, teaching, learning, their students, and the wider world. These beliefs inform the environments they create in their classrooms (Beswick, 2007; Stipek, Givvin et al, 2001). Consequently, teachers' beliefs have a significant impact on how students experience school mathematics. Philipp (2007) noted, "While students are learning mathematics, they are also learning lessons about what mathematics is, what value it has, how it is learned, who should learn it, and what engagement in mathematical reasoning entails" (p. 257). After observing that *beliefs* was a term not consistently defined or uniformly used in the literature, Philipp proposed the following definition.

> Psychologically held understandings, premises, or propositions about the world that are thought to be true… Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction and are not consensual. (Philipp, 2007, p. 259)

Teachers' beliefs about the relevance of algebra and its broad applicability to a wide range of careers, as described above, was operationalized in this study through their responses to questionnaire items explicitly aimed at assessing this belief as well as items where this belief was implicated. As with the knowledge construct, a scale score was created from participants' responses to assess these beliefs.

Before introducing the next definition, it should be noted that there is debate in various research communities about the distinction between knowledge and beliefs (e.g., Alexander & Dochy, 1995; Southerland et al, 2001). There appears to be some degree of consensus around the notion that knowledge is more factual and subject to external verification. This is in contrast to beliefs, which are more subjective and do not require verification. Even so, some researchers contend that beliefs and knowledge are overlapping constructs (e.g., Hoy et al, 2006; Pehkonen & Pietilä, 2003).

The final construct under consideration in the present study was teacher practice. Although classroom instructional practices are directly observable, this study assessed the nature and frequency of certain mathematical tasks and explanations based on teacher self-report of their algebra instruction. A goal of the study was to determine the effect of the WWIEUT professional development workshop on teachers' use of real-world applications of mathematics, which included their use of mathematical modeling tasks during instruction. What activities qualified as real-world applications and mathematical modeling tasks were identified by their relation to the definition of mathematical literacy set forth by Organisation for Economic Co-operation and Development (OECD) in its framework for the Programme for International Student Assessment (PISA), which defined mathematical literacy as

> An individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. (OECD, 2019a, pp. 14, 15)

Word problems in mathematics textbooks were frequently teachers' primary source of real-world connections, which made them the principal vehicle for developing students' mathematical literacy. Although such problems can give students opportunities to decontextualize a real-world situation to mathematize it and then recontextualize it to evaluate and interpret their solution, word problems can also be "pure mathematical tasks 'dressed up' in a real-world context that for their solution merely require that the students 'undress' these tasks and solve them" (Palm, 2009, p. 3).

De Corte and colleagues (2000) observed that the practice and culture in which students learn to solve word problems was hindered by “The impoverished and stereotyped diet of standard word problems occurring in mathematics lessons, textbooks, and tests, which can almost always be modeled and solved by carrying out one or more operations with the given numbers” (p. 69). An aim of the WWIEUT professional development workshop was to increase teachers’ knowledge of how mathematics is applied in the world outside of school. A hypothesized consequence of achieving this goal was that teachers’ ability to discern what constituted an authentic application of algebra in the world outside of school would be enhanced. To measure the effectiveness of the professional development workshop in this regard, the meaning of “real-world applications” was whatever the teacher believed it to be at the time of completing the questionnaire.

Modeling with mathematics has a narrower definition than word problems generally. Lesh and Doerr (2003) described models generally and mathematical models specifically.

> *Models* are conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behavior of other system(s)—perhaps so that the other system can be manipulated or predicted intelligently. A *mathematical* model focuses on structural characteristics (rather than, for example, physical or musical characteristics) of the relevant system. (p. 10)

This description emphasized the conceptual nature of models as well as their ability to be represented externally. It also clearly stated that models are often created for the purpose of better understanding the behavior of a system, which could result in the modeler’s ability to control or predict this behavior. Developing these abilities requires an instructional focus on mathematical literacy and quantitative reasoning. Lesh and Doerr’s description of mathematical models can be extended by the theory of modeling put forth by Confrey and Maloney (2007).

> Mathematical modelling is the process of encountering an indeterminate situation, problematizing it, and bringing inquiry, reasoning, and mathematical structures to bear to transform the situation. The modelling produces an outcome – a model – which is a description or a representation of the situation, drawn from the mathematical disciplines, in relation to the person's experience, which itself has changed through the modelling process. (p. 60)

This theory, which drew on a Deweyian ideal of inquiry, highlighted the transition from an indeterminate situation to a determinate one, a change that is brought about through engaging in the mathematical modeling process. The CCSSM defined modeling more succinctly as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (NGACBP & CSSO, 2010, p. 72). An aim of the WWIEUT professional development workshop was to develop teachers' knowledge of mathematical modeling and how to facilitate their students' engagement in it. Part of this development was making it clear to the teachers what mathematical models were and how to formulate and solve them and to interpret their outputs. A measure of the program's effectiveness in changing teacher practice was an increase in the proportion of instructional time teachers reported devoting to activities related to mathematical modeling.

Even as the role of mathematical modeling is being elevated by standards writers, international assessments, curriculum developers, and some in the research community, it is not clear whether it is a process of using mathematics that is well understood by many mathematics teachers. Mathematical modeling is the primary vehicle by which the WWIEUT professional development workshop addressed its aims. In light of teachers' inexact personal definitions of mathematical modeling, this study did not explicitly ask participants about mathematical modeling to assess participants' use of authentic real-world contexts. Instead the term "real-world applications" was employed. It was possibly problematic that the definition of "real-world applications" was a moving target, changing as teachers' knowledge and beliefs about the relevance of mathematics changed. To address this concern, the third research question focused on the proportion of instructional time dedicated to these problems as opposed to the quality or authenticity of the real-world contexts. The justification for this line of reasoning was the theory that as teachers know more about how mathematics is used to model systems and make decisions in the world outside of school and believe more strongly that mathematics is relevant to their students, both the nature and frequency of the real-world applications they include in their instruction would be enhanced.

**Purpose of the Study**

The purpose of the present study was to assess several aspects of high school mathematics teachers' knowledge of and beliefs about the use of algebra outside of school as well as their related instructional practice.  Knowing more about teachers' knowledge of how algebra is used in non-academic settings will contribute to the understanding of teacher knowledge by filling in gaps in the research literature around two areas: knowledge of the algebra used by professionals in a range of careers and knowledge of the process by which algebra is used as a modeling tool outside of school to solve practical problems and make better decisions. Shedding light on a pair of related teacher beliefs will aid in determining the impact that the professional development workshop: the relevance of algebra to students' lives and whether current instructional practices are consistent with that belief.

The focus of the research questions was on algebra in particular and not mathematics in general for two reasons. First and foremost, algebra topics make up the lion's share of the high school mathematics curriculum. The traditional high school course sequence of Algebra 1, Geometry, and Algebra 2 demonstrates this, as does an investigation of content standards. Many standards are explicitly about algebra, and many others are related to functions or other topics that could broadly be considered to be “pre-calculus.” These standards have a basis in algebra. The other reason for focusing on algebra is that less emphasized areas of high school mathematics, such as geometry, probability, and statistics, have more apparent relevance to life outside of school than does algebra. Because algebra is the predominant type of mathematics taught in high schools and its real-world applications are less obvious, it was a logical object of investigation in this study. The research questions were:

1. What are the differences in secondary mathematics teachers’ knowledge of how applicable algebra is to a wide range of careers that are relevant to the majority of their students between participants and nonparticipants in a WWIEUT professional development workshop?
2. What are the differences in secondary mathematics teachers’ beliefs about the broad applicability of algebra to a wide range of careers that are relevant to the majority of their students between participants and nonparticipants in a WWIEUT professional development workshop?
3. What is the difference in the amount of time spent on real-world applications of algebra by secondary mathematics teachers between participants and nonparticipants in a WWIEUT professional development workshop?

Essentially, the aim of the study was to determine how effective the WWIEUT workshops were at changing teachers’ (a) knowledge of how algebra is used as a modeling and decision-making tool by professionals in business, industry, and government as well as individuals; (b) beliefs about algebra as a tool for modeling real-world systems and evaluating a range of possible solutions, rather than as a means for finding the one right answer to highly structured problems; and (c) instructional focus from abstract procedural fluency to quantitative reasoning/mathematical literacy in meaningful contexts.

The ultimate goal of any professional development program is improved student outcomes. Indeed, in 2000 the National Commission on Mathematics and Science Teaching for the 21st Century asserted ‘‘The most direct route to improving mathematics and science achievement for all students is better mathematics and science teaching. In other words, better teaching is the lever for change (p. 18).’’ Student learning drives teachers’ instructional decisions and plays a pivotal role in teachers’ classroom practices (Guskey, 2002). The present study, however, focused on teacher outcomes instead of student outcomes. Despite over a thousand teachers across the country participating in WWIEUT workshops since 2008, relatively few schools have adopted and implemented the WWIEUT curricular materials. Assessing student outcomes from such a small sample of classrooms spread across the country was not feasible for the present study.

**Assumptions**

In both the qualitative and quantitative components of this study, the researcher assumed that all participants were honest in their responses. Even if the teachers did not intend to mislead, all data was self-reported and subject to recall bias. It was also assumed that all workshops were equivalent. Workshops varied in their duration. Content and delivery also evolved over time. It was assumed that the control group was representative of the population of high school mathematics teachers in the US. The statistical procedures and tests used had their own assumptions about the data. These assumptions will be discussed in detail in Chapter 4.

## Limitations

The present study had many limitations. Due to time and budgetary constraints, the qualitative portion of this study did not have reasonable access to a large nationally representative sample of high school mathematics teachers. Furthermore, it was not possible to randomly assign teachers to treatment groups, which precluded the use of experimental methods. Teachers who participated in WWIEUT workshops were self-selected, and the researcher did not understand the mechanism of the selection process, so selection bias was possible. There was a low response rate to the survey portion of the research as well as to invitations to be interviewed, thus opening the door to non-response bias.

As a writer of the curriculum, a frequent facilitator of the professional development workshops, and a teacher using the curricular materials with classrooms of high school students, the researcher had a strong interest in the WWIEUT curriculum and professional development workshops. The researcher took measures to safeguard against researcher bias and develop trustworthiness. It was possible that interview respondents did not speak candidly. They may have been unable to articulate their specific knowledge and beliefs. Finally, this study did not examine any student outcomes. The ultimate measure of the effectiveness of a professional development workshop for teachers is its impact on student learning, but assessment of such these distal outcomes was beyond the scope of this study.

## Summary

In the current information age and transition to a service economy, high school mathematics students in the US are expected to learn and use mathematics in ways that emphasize its applicability in the world outside of school (Steen, 2001). That students should learn how to put mathematics to use in productive and meaningful ways is embodied in current curriculum standards. It also represents a substantial shift in how school mathematics has traditionally been taught in the US. High school mathematics teachers may not have the experience or training to successfully manage this transition, as many of them do not possess, and hence cannot impart, this type of contextualized and practical knowledge. Although there have been standards that underscore the importance of mathematical problem solving and quantitative reasoning for more than three decades, it is only recently that attempts at assessing these standards were developed and implemented on a large scale in the United States. Assessment drives curriculum in this age of accountability, and there is a need to better understand how frequently teachers make real-world connections in their mathematics instruction. Even more fundamental than that may be understanding what teachers believe constitutes a real-world application of mathematics. Teachers' beliefs about the nature of mathematics and how it is used affect how and what they teach. If high school mathematics teachers are not currently able to teach their students how to use mathematics to model problems that occur outside of school, the question of how professional development can be deployed to remedy the situation becomes much more important.

# CHAPTER 2: LITERATURE REVIEW

The present study aimed to assess the impact of a professional development workshop on secondary mathematics teachers' knowledge, beliefs, and practices regarding the applicability of algebra to the world outside of school. To situate this study, the literature review synthesized research from several related fields: teacher knowledge, teacher beliefs, instructional practice, and professional development. The literature was examined in reference to the professional development workshops that supported teachers' implementation of instructional materials from the "When Will I Ever Use This?" (WWIEUT) (Edwards & Chelst, 2015) curriculum development project, which was the object of the study. The review exposed a gap in our understanding of teachers' knowledge of real-world mathematics. Likewise, when, how, and why teachers include real-world connections in their mathematics instruction were questions the extant literature did not adequately answer. This study aimed to address these gaps.

## Teacher Knowledge

Hill and colleagues (2007) described the history of research on mathematics teachers' knowledge for the *Second Handbook of Research on Mathematics Teaching and Learning*. After assessing the state of the field, they recommended assessment developers measure not just mathematics content knowledge but also mathematical knowledge for teaching. This entailed what others referred to as pedagogical content knowledge. Shulman (1986) claimed this type of knowledge includes "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others" (pp. 6-7). Moving beyond simply measuring teachers' mathematical content knowledge was needed in order to broaden the focus of research to include measuring mathematical knowledge for teaching, but there appeared to be a gap in Hill and colleagues' framework. No mention was made about the knowledge teachers need to relate high school mathematics instruction to how mathematics is used in a wide range of careers that would be relevant to many students. They recommended that researchers investigate how mathematical knowledge for teaching intersected other domains of knowledge for teaching, including students' affect. They went on to suggest that "Mathematical knowledge for teaching may also act upon (or be acted upon by) teachers' ability to motivate students to learn" (Hill et al, 2007, p. 152). Including real-world applications in their mathematics instruction is one way teachers could increase their students' motivation to learn (Middleton & Spanias, 1999; NRC, 2004; Smith & Morgan, 2016; Zbiek & Conner, 2006). However, very little is known about teachers' ability to make authentic real-world connections within their mathematics curricula.

Being able to apply school mathematics to real-world problems is seen as an important goal in modern American classrooms (Smith & Morgan, 2015). Taking a context-based approach has short- and long-term benefits, including increasing students' motivation to engage in learning activities and emphasizing the potential of using algebra in other fields (Tabach & Friedlander, 2008). The Common Core State Standards for Mathematics (CCSSM) include 27 (of 160) of content standards in grades three through eight that contain the term "real-world" (National Governors Association Center for Best Practices [NGACBP] & Council of Chief State School Officers [CCSSO], 2010). In the CCSSM's high school content standards, there are 14 (of 114) content standards that emphasize connections between mathematics and the context of the problem at hand. In addition to the content standards, one of the eight Standards for Mathematical Practice

(SMP) is Model with Mathematics, the description of which included the statement "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (NGACBP & CCSSO, 2010, p. 7). The SMP are seen by many as the cornerstone of all the CCSSM. In a study of nearly 2,000 postsecondary instructors, 71% of respondents from all disciplines rated the SMP as applicable to their field of study (Conley et al, 2011).

The National Council of Teachers of Mathematics (NCTM) recommended that "the curriculum should offer experiences that allow students to see that mathematics has powerful uses in modeling and predicting real-world phenomena" (NCTM, 2000, pp. 15-16). The College Board's SAT college entrance examination was redesigned for 2016 to make changes in how it assessed students' college readiness. The College Board (2014) claimed "these changes are firmly grounded in evidence about what is needed for all students to be ready for and to succeed in college and workforce training programs" (p. 3), which resulted in an emphasis on problem solving in real-world contexts in the mathematics portion of the examination as well as cross-test items that assessed students' ability to use quantitative reasoning and analysis skills in science and humanities reading passages. Both national curriculum standards and large-scale standardized assessments have expressed their value for real-world applications of mathematics, but we do not have evidence that the instructional capacity of mathematics teachers has kept pace with these changes or the recommendations from mathematics education professional organizations.

Regardless of whether teachers have the knowledge and motivation to teach real-world problems, there is compelling evidence from international assessments that high school students in the United States do not perform well in applied mathematical problem solving situations. The

Programme for International Student Assessment (PISA) claimed to assess “the extent to which students near the end of compulsory education have acquired the key knowledge and skills that are essential for full participation in modern societies” (OECD, 2019a, p. 3). An analysis of the data from the 2018 administration of the PISA test showed that 15-year-olds in the US performed significantly lower than peers in reference countries in several areas (OECD, 2019b). Two of the pertinent relative weaknesses were establishing a mathematical model of a given real-world situation and genuinely interpreting real-world aspects of problems.

The PISA test defined mathematical proficiency in a way that was consistent with the SMP. Instead of focusing on memorization of basic facts and skillful symbolic manipulations, for PISA “mathematics proficiency means the capacity of individuals to formulate, employ and interpret mathematics in a variety of contexts” (OECD, 2013, p. 14). A pair of poignant observations about the US students’ performance is stated below.

> It seems that the U.S. students have particular strengths in cognitively less-demanding mathematical skills and abilities, such as extracting single values from diagrams or handling well-structured formulae. And they have particular weaknesses in demanding skills and abilities, such as taking real world situations seriously, transferring them into mathematical terms and interpreting mathematical aspects in real world problems. These are tasks where the well-known superficial classroom strategy “Don’t care about the context, just extract the numbers from the text and do some obvious operations” is bound to fail. (OECD, 2013, p. 74)

This led to the recommendation that American mathematics teachers emphasize more cognitively demanding activities, like those found in mathematical modeling problems. Bringing this change about while not sacrificing procedural fluency has been seen as an insurmountable challenge by some mathematics teachers (Davis, 1986).

Although the research and mathematics education reform communities have focused on real-world applications of school mathematics curricula over the past several decades, relatively little attention has been paid to teachers' knowledge of how to implement the related standards with integrity. The Conference Board of the Mathematical Sciences (CBMS) (2001), an umbrella organization consisting of 19 professional societies, all of which have a primary objective of increasing or diffusing knowledge in the mathematical sciences, published *The Mathematical Education of Teachers* as a resource for all those involved in the education of mathematics teachers. The report outlined general recommendations to improve education in areas of mathematics curriculum and instruction for prospective teachers. However, none of these recommendations addressed the issue of improving prospective teachers' knowledge of how to apply mathematics in the world outside of school.

When making recommendations for the undergraduate mathematics education of future high school teachers, there were only a few instances where applications of mathematics were mentioned. One specific statement regarding the statistics education of future high school mathematics teachers was noteworthy because there was not a corresponding statement pertaining to algebra: "Because statistics is first and foremost about using data to inform thinking about real-world situations, it is critical that prospective teachers have realistic problem-solving experiences with statistics" (CBMS, 2001, p. 44). Algebra occupies a much more prominent place in high school curricula, but educating prospective teachers on its applicability to real-world situations was not explicitly recommended.

CBMS released *The Mathematical Education of Teachers II* in 2012. The widespread adoption of the CCSSM across the country and the need for undergraduate teacher education to adapt to provide preservice teachers with learning experiences that would enable them to teach the SMP to their students significantly influenced this document. The authors noted that

> doing mathematics in ways consistent with mathematical practice is likely to be a new, and perhaps, alien experience for many teachers. However, such experiences are necessary for teachers if their students are to achieve the Common Core State Standards for Mathematical Practice. (CBMS, 2012, p. 11)

The coursework recommended for prospective high school mathematics teachers included 13 courses that should be taken by all mathematics majors, only two of which focused on applications of mathematics: *statistics and probability* and *modeling*. The authors used the term “real-world” in describing these two courses but did not employ it anywhere else in their recommendations for preparing high school mathematics teachers. The modeling course was a new addition from the first edition of recommendations and cites the SMP Model with Mathematics as a reason why “prospective teachers should have experience modeling rich real-world problems” (CBMS, 2012, p. 60).

The Committee for Economic Development of The Conference Board (CED) is a nonprofit, nonpartisan, business-led, public policy organization that researches and analyzes critical economic issues in the United States and proposes policy solutions to these issues. One of its seven core principles is ensuring the United States has a globally competitive workforce. In 2003 the CED made recommendations about how to improve the quality of mathematics and science education to ensure the quality and size of the technical labor force, a scientifically literate citizenry, and increased diversity in the technical labor force. One of the recommendations addressed teacher knowledge. It included content knowledge, pedagogical content knowledge, and notably knowledge of real-world mathematics.

> Colleges and universities that educate future and current teachers must ensure that their courses of study emphasize acquisition of content knowledge, an understanding of the place of that knowledge in society, as well as the pedagogical training to deliver that knowledge to students of all backgrounds and abilities. (CED, 2003, p. 36)

In its report, *Foundations for Success*, a task group of the National Mathematics Advisory Panel (NMAP) (2008a) conducted a review of research studies on instructional practices in the United States. The only research the task group considered were studies that randomized or equated groups. The report synthesized this narrow range of mathematics education research to assess whether engaging with real-world problems improved students' mathematical performance generally or their performance on other real-world problems specifically. Of greater concern to this study, however, were the outcomes not addressed in the report. The effect that including real-world problems in mathematics curricula and instruction have on affective variables such as motivation was not investigated. Indeed, the task group that investigated instructional practices systematically excluded motivation as an outcome. Neither was teacher knowledge of real-world mathematics assessed. The instructional practices task group made the following note.

> Others have raised concerns about the adequacy of teachers' knowledge of the nonmathematical contexts—in which some of these problems are embedded—to assess the reasonableness of the problem's assumptions, and about the efficiency of using elaborate "real-world" problems in covering mathematics content. (NMAP, 2008b, p. 6).

No literature the task group reviewed addressed the question of whether teachers possessed the requisite knowledge to effectively teach real-world problems (Gersten et al, 2008).

In their analysis of NMAP's *Foundations for Success*, Borko and Whitcomb (2008) criticized the task force's narrow focus on teacher content knowledge. Borko and Whitcomb acknowledged the common sense notion that teachers must know the material they teach but maintained teachers needed to know more than mathematical content. The domain of knowledge they highlighted as conspicuously absent from the report's analysis was pedagogical content knowledge. Borko and Whitcomb called on the work of Shulman (1986) and Ball and colleagues (2008), discussed earlier, to support their concerns about focusing too narrowly on teachers' knowledge of mathematical content to the exclusion of other forms of professional knowledge for teaching. The present research project aimed to make a contribution to begin filling in the gap in the literature on teachers' knowledge of how algebra is applied by a wide range of professionals outside of academic settings.

**Teacher Beliefs**

Teachers' beliefs about the nature of mathematics mediate the choices they make when planning and delivering instruction as well as the ways students experience mathematics learning (Ernest, 1989a; Thom, 1973; Thompson, 1984). Ernest (1989a) identified three personal philosophies of mathematics commonly observed in teachers. These philosophies could also be considered to be teachers' personal views of the nature of mathematics. The three philosophies were referred to as the instrumentalist view, the Platonist view, and the problem solving view. Each philosophy was associated not just with an understanding of the field of mathematics but also how teachers understood their role, the role of the student, and the use of curricular materials in mathematics education. Each of the three philosophies will be discussed in the following paragraphs.

According to Ernest (1989a), the instrumentalist view held that mathematics is a collection of unrelated but useful facts, rules, and skills. These facts, rules, and skills have utility and can be put to use to serve as means to a variety of external ends. When mathematics is seen in this light, the role of the teacher was the instructor, who ensured facts are learned, rules were followed, and skills were mastered. To achieve these goals, teachers strictly adhered to their curricular materials. Students who learned from teachers with this view of mathematics must have exhibited compliant behavior and demonstrated mastery of skills in order to be successful.

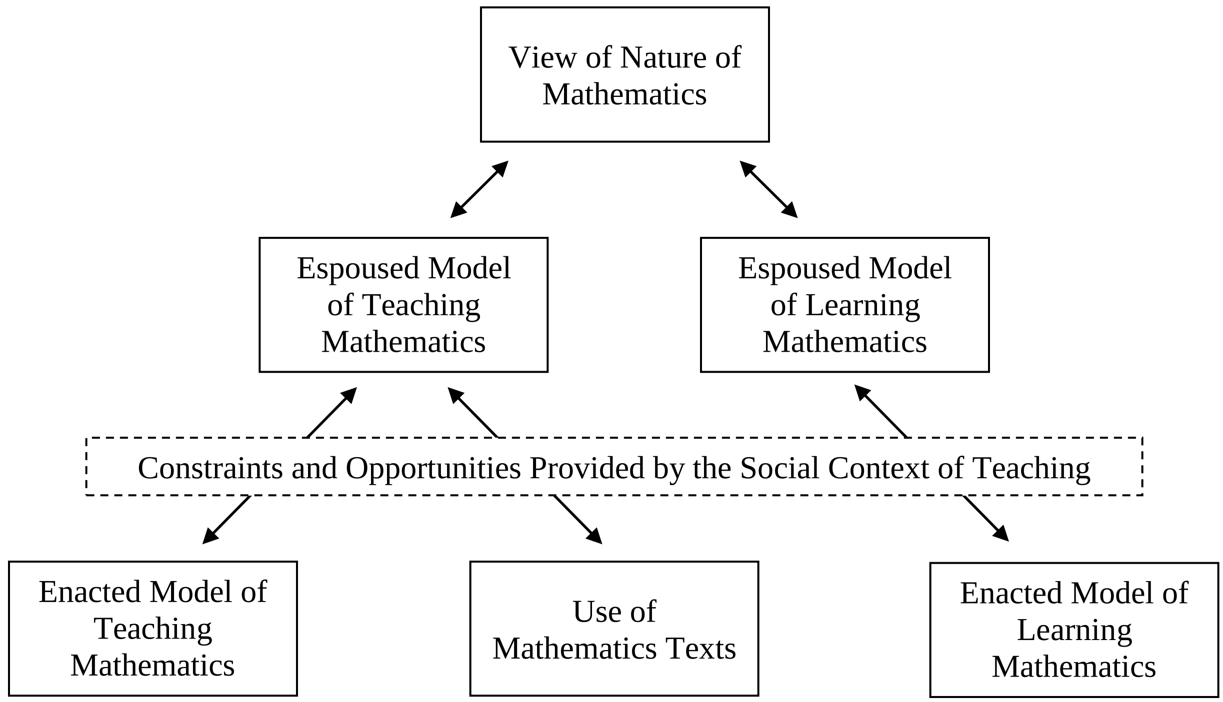
A teacher approaching mathematics from the Platonist view conceived of mathematics as a static and unified body of knowledge that was known with certainty (Ernest, 1989a). Adherents of this view of mathematics saw the discipline as one that was discovered rather than invented. Teachers with a Platonist perspective of mathematics saw themselves as the explainer of mathematical concepts. Their objectives were to impart unified knowledge and conceptual understanding. With these goals in mind, teachers modified their curricular materials to enrich students' experience with supplementary problems and activities. To succeed in such an environment, the role of the student was to receive the mathematical truths the teacher explained.

The third personal philosophy or view of the nature of mathematics described by Ernest (1989a) was the problem solving view. According to this perspective, mathematics was a dynamic discipline that was in a state of constant change. Mathematics was seen as a creation of human ingenuity. Conceived of this way, mathematical work was a process of inquiry, and its results were not a finished product but were open to interpretation and revision. To teach from this perspective, teachers took the role of the facilitator to help students become more proficient at posing and solving problems. Curricular materials were constructed at the local level to meet these goals. In this type of setting, students were expected to actively construct mathematical understandings in order to explore and pursue their own interests.

Research has found that teachers' professed beliefs about mathematics did not always align with their instructional practices (e.g., Ernest, 1989a; Thompson, 1984; Wilson & Cooney, 2002). Ernest (1989a) referred to this phenomenon as the espoused-enacted distinction. The model Ernest put forth to illustrate the relationships between teachers' beliefs and their instructional practice is shown in Figure 2.

**Figure 2**

*Relationships between Beliefs, and Their Impact on Practice (Ernest, 1989a)*




The present study investigated both teacher beliefs and teacher practices as dependent variables. The model presented above provided valuable perspective as it described the complex ways these constructs influenced one another. This model also highlighted the role played by contextual factors in mediating the interplay of espoused beliefs and enacted instructional practices.

**Classroom Practices**

If teachers are to provide instruction that demonstrates how mathematics is used as a tool in the world outside of school, they must possess knowledge of how mathematics can be applied to an extensive array of real-world situations. No research could be found to support the claim that high school mathematics teachers know how mathematics is used by professionals to model problems and make decisions in a wide range of careers. There was some evidence to the contrary, however. Popovic and Lederman (2015) conducted a study of secondary mathematics teachers that examined how teachers made connections between school mathematics and its applications in the real world.

> To be able to incorporate real-world connections and modeling into teaching, secondary mathematics teachers should be able to make such connections themselves. The results of this study clearly indicate that the teachers will not make such connections on their own. (p. 139)

Popovic and Lederman went on to recommend that teacher education programs include coursework on modeling real-world phenomena to give teachers opportunities to apply their mathematical content knowledge to solving problems in the real world. In addition to this content knowledge, Doerr (2007) described four characteristics of pedagogical content knowledge that are necessary for teachers to teach modeling and wondered at the source of this knowledge.

> (1) to be able to listen for anticipated ambiguities, (2) to offer useful representations of student ideas, (3) to hear unexpected approaches, and (4) to support students in making connections to other representations. How teachers acquire this knowledge, both in their preparation programs and in practice, remains an open question for researchers. (p. 77)

Many commonly used high school mathematics textbooks in the US do not include many truly real-world problems, and the “real-world” problems they do pose occasionally contain errors and ambiguities (NMAP, 2008). Leikin and Levav-Waynberg (2007) investigated teachers’ use of and beliefs about what they referred to as connecting tasks (CTs) to examine the gap between recommended best practices and actual classroom practices. A CT was defined as “one that may be attributed to different topics or to different concepts within a topic of the mathematics curriculum, and therefore may be solved in different ways” (Leikin & Levav-Waynberg, 2007, p. 350), and is a reasonable proxy for “real-world” problems, as many of the CTs investigated were of an applied nature. Even with recommendations from reform-oriented researchers to utilize CTs in school mathematics curricula, Leikin and Levav-Waynberg found that teachers were not enthusiastic about incorporating them into their classroom practices. Teachers in the study did not report finding value in investing time in CTs. This was the case despite the recommendations promoting CTs as vehicles for broad learning associated with solving problems in different ways and thus making connections between various mathematical concepts. This finding was relevant to the present study as it showed how professional development that attempts to increase teachers' knowledge of the benefits of a curricular innovation may not change teachers' espoused beliefs about mathematics or their enacted instructional practices.

High school mathematics teachers do not operate in a vacuum, and investigations of their teaching practices need to take that reality into account (Karabenick & Conley, 2011). Teachers act as part of a community that has concerns based on its particular context (Aguirre & Speer, 1999). These local contexts can involve an articulated curriculum that spans multiple years of students' experience, socially constructed interdependency among colleagues, the student population served, the level of support from school administration and parents, and multiple layers of accountability measures. These factors can lead teachers to make instructional decisions that are inconsistent with their beliefs about mathematics education (Cross, 2009; Ernest 1989a). This phenomena can result in an emphasis on procedural fluency and rote memorization at the expense of conceptual understanding and problem solving. Shifting to an instructional model that focuses on mathematical modeling of authentically real-world problems is far less likely when there is local pressure to concentrate on standardized test preparation or an absence of perceived administrative or collegial support for instructional innovation.

Although secondary mathematics teachers may be ill equipped and unmotivated to invest time and energy to develop and effectively implement rich tasks involving real-world applications of mathematics, such as mathematical modeling tasks, students who engage in these tasks find them motivating. Zbiek and Conner (2006) noted three different types of motivation are enhanced when students become mathematical modelers.

> The first type of motivation is confirmation that real world situations appeal to (some) learners… A second type of motivation is simply motivation to (or to continue to) study mathematics in general… Motivation to learn new mathematics, the third type of motivation, emerges when a student modeler embraces a purpose that, in that modeler's opinion, is not sufficiently met by the mathematics the modeler knows, and the modeler actively seeks understanding of the needed mathematics and thus is motivated to add a new piece of knowledge or new connections among known pieces of knowledge. (pp. 105-106)

Disengagement and disinterest in mathematics classes are frequently reported by high school students (e.g., Brown et al, 2008; Nardi & Stewart, 2003). Middleton and Spanias (1999) suggested that when students found mathematics interesting, they were more likely to engage in mathematical activity out of intrinsic motivation. On the other hand, when students did not find mathematics interesting, they avoided engagement in mathematical activity without even evaluating whether it is cognitively stimulating.

Teaching students how to put the mathematics they learn in school to some productive use is a clear goal of modern American mathematics curriculum standards (CCSSM, 2010), and its value has been demonstrated by attempts to measure it on large-scale standardized tests (College Board, 2014; OECD, 2019a). Much of a high school mathematics teacher's time and energy are devoted to addressing the triad of curriculum, instruction, and assessment. While authoritative voices in secondary education have promoted the importance of making mathematics education relevant to students' lives beyond the walls of the classroom through curriculum standards and accountability measures, little attention has been paid to the support teachers need to shift their instructional practices to bring about this paradigm shift. An important aspect of this shift is the nature of the product of mathematical work.

**The Process of Using Mathematics**

In school mathematics, students typically seek the one right answer to a well posed problem, an answer which the teacher already knows. When mathematics is deployed to model real-world phenomena to solve a problem or aid in making a decision or forecast, there is not one right answer. Moreover, in the real world of business, industry, finance, or public policy, if the answer were already known, the question would not be asked. The substantive challenges of using mathematics outside the classroom are often in asking the right questions, identifying and formulating the problem, finding useful data, making reasonable assumptions, and interpreting the results of a model. Lesh and Zawojewski (2007) made the following contention.

> Although traditional topics of school mathematics courses are clearly fundamental in the work of engineers, architects, and business administrators, the method in which the mathematics is deployed and often combined and recreated for the situation is not in the formal "mathematical" fashion learned in school. In fact, one might say that the traditional topics serve as good descriptions of the work, but when used to interpret and understand new and dynamic situations, the mathematics generated and deployed by the users is more complex, situated, and multidisciplinary than the conventional topic descriptions imply. (p. 781)

The quantitative reasoning that is necessary for mathematical modeling requires creativity, analytical thinking, sense-making skills, and reflection. Polynomial long division and finding the foci of a hyperbola are skills through which secondary mathematics teachers can comfortably lead a classroom full of adolescents, but they are not skills that are relevant to most students' lives outside of their mathematics classrooms, nor will they be relevant to many careers. These activities do not give students an accurate portrayal of what mathematics is or how it is used by professionals, in either content or practice. Neither do they inspire a majority of students to continue studying mathematics (Boaler, 2002; Brown et al, 2008; Nardi & Stewart, 2003), even as more students are being compelled to take more mathematics classes to graduate from high school (Achieve, 2009) and our economy is demanding more technical knowledge and skills from workers (Niss, 1999).

Even when mathematics content is taught well, many students take away from their experiences that mathematics is a rule-following discipline whose knowledge is arbitrary and incoherent (Schoenfeld, 1988). Teaching mathematics from an instrumentalist view can lead students to see it as a collection of disconnected skills in symbolic manipulation that reinforces the notion that mathematics is about procedures for finding the one right answer to well posed problems. However, "nearly all people who are engaged in solving problems in their local contexts are able to develop mathematical concepts and conceptual tools for problem situations that are powerful and reusable" (Lesh & Zawojewski, 2007, p. 787). This suggests that mathematical practices at school differ substantially from mathematical practices in the workplace (Hoyles et al, 2010). Instruction must keep pace with new curriculum standards and assessments if their goals are to be realized. Teachers need knowledge of how mathematics will be used by their students in their future education, professional career, and civic life to make appropriate instructional choices about when, how, and why to make real-world connections.

**Professional Development**

The object of the present study was a professional development program that aimed to increase teacher's knowledge, enhance their beliefs, and improve their practice where real-world connections to algebra were concerned. There is a large body of knowledge on professional development programs for teachers. In the following paragraphs, an effort will be made to describe the landscape of this literature and relate it to the present study. The focus of the review was to gain insight into how professional development programs should be evaluated. Three different models were explored. Borko (2004) outlined an approach to studying professional development programs. Desimone (2009) developed a conceptual framework for studying the effects of professional development that included a set of core features of the professional development programs. Kennedy (2016) characterized professional development programs according to their theories of action rather than design features. The WWIEUT professional development workshops were assessed using each model.

In an effort to map what was known about the impact of professional development programs on teacher learning and to provide direction for how to extend that knowledge, Hilda Borko (2004) reviewed research and proposed a framework for analyzing the effectiveness of professional development programs. She conceptualized professional development as a system consisting of four key elements: the professional development program, the teachers who participate as learners in the system, the facilitator who guides the teachers, and the context in which the professional development occurs. In this model, the teachers and facilitator interact with each other and with the professional development program itself. These interactions are framed by and occur within the contextual setting of the professional development program. Borko recommended programs of research be developed to study the impact of professional development activities. These research programs should proceed in stages, with each new phase of research building on the findings of the last. The first phase of research is an existence proof of effective professional development, and the present study falls into that category of research. Borko's model will be discussed further in Chapter 3.

In 2009 Laura Desimone examined the research on the impact of teachers' professional development experiences. The purpose of her review was to propose a conceptual framework for future impact studies of professional development programs for teachers, and in so doing she reported "a consensus about at least some of the characteristics of professional development that are critical to increasing teacher knowledge and skills and improving their practice" (Desimone, 2009, p. 183). She noted five features of effective professional development that could form the core of her conceptual framework for future scholarly research in this arena: (a) a content focus for teacher learning; (b) opportunities for teachers to engage in active learning; (c) coherence between teacher learning and existing teacher knowledge and beliefs as well as between pertinent reform agendas and the goals of the professional development workshop; (d) the duration of the professional development activities, with the number of contact hours and the span of time over which the professional development activities are distributed both being relevant; and (e) the collective participation of a cohort of teachers from the same group. The original development of the WWIEUT workshops predated Desimone's review, but the workshops relied on several of the aforementioned best practices nonetheless.

The WWIEUT professional development workshops were designed both to show to high school teachers the broad relevance of algebra and to help them learn how to teach in ways that reveal that relevance and applicability to their students (K. R. Chelst & T. G. Edwards, personal communication, June 15, 2010). Penuel and colleagues (2007) found that "helping teachers to prepare for their classroom practice yields results that are most directly translatable to practice" (p. 928). Workshops were thus designed to encourage teachers to engage in the same kinds of active learning their students would experience and also explicitly attend to pedagogical issues raised by these sorts of learning experiences. The coherence of the workshops with broad-based recommendations to more thoroughly address the SMPs was achieved by focusing on active learning of relevant mathematics content, presenting it in ways that were grounded in sound learning theory, and making connections to the SMP (Desimone, 2009; Fisher & Frey, 2015; Garet et al, 2001; Hill et al, 2005). Other aspects of the workshops, however, were not consistent with these best practices. For instance, professional development is more effective if a school's entire mathematics department collectively participates in learning activities that endure over long periods of time (Wilson, 2009).

In another review of the professional development research literature, Mary M. Kennedy (2016) criticized the way researchers sort professional development programs based on their visible features and suggested that focusing on these design features may not reliably predict program success. As a basis for her criticism, she acknowledged the lack of a consensus on an overarching theory of teaching or teacher learning. Due to the variety of theories about teaching and how teaching can be improved, Kennedy suggested characterizing professional development programs, not by their core features, but by their underlying theories of action.

The two aspects of these theories of action were the teaching problem the professional development aims to address and the andragogy employed by facilitators to help teachers enact new ideas. Kennedy identified four persistent teaching problems: portraying curriculum content, containing student behavior, enlisting student participating, and exposing student thinking. Kennedy contended that successful teaching requires teachers to resolve all four of these problems.

> Teachers cannot be said to be teaching unless students are learning and students cannot learn unless teachers portray content in a way that is comprehensible to naïve thinkers, enlist student participation in the lessons, contain distracting behavior among students, and

> expose student thinking so that they can adjust their lessons accordingly. (Kennedy, 2016, p. 954)

The second aspect of a professional development's theory of action related to the way it facilitates enactment of the program's aims in teachers' existing system of practices. Kennedy identified four methods. *Prescription* relies on telling or demonstrating what facilitators believe is the best way to for teachers to address a teaching problem. The method referred to as *strategy* begins by defining a learning goal and then providing an assortment of teaching strategies that will help teachers achieve the goal. The *insight* goal relies on facilitators raising provocative questions that cause teachers to reflect on and reexamine their knowledge, beliefs, or practice. The final method identified by Kennedy (2016) was termed *body of knowledge*, by which she meant "knowledge that is organized into a coherent body of interrelated concepts and principles and that can be summarized in books, diagrams, and lectures" (p. 956). These four methods were presented as lying on a continuum of teacher discretion in how the new ideas were to be implemented.

In this framework, the theory of action of the WWIEUT professional development workshops could be seen as attempting to address all four teaching problems. A majority of the time in the workshops was devoted to portraying curricular content. Using algebra to model real-world systems was new territory for most teachers, so workshops were designed to help teachers understand the modeling process well enough to share it with their students. Pedagogy was frequently discussed and modeled during the workshops. The WWIEUT curricular materials require much more reading and discussion than typically occurs in secondary mathematics classrooms. This caused many teachers to express concerns about off-task behaviors and how to handle them. Most of the classroom management concerns raised by teachers could be resolved by giving students many opportunities to engage in meaningful activities related to modeling relevant problems. As these real-world problems were all presented as model-eliciting activities, teachers were taught how to ask purposeful questions that would prompt students to explain assumptions and decisions they made when developing models. Common misconceptions and modeling errors were discussed in order to enable teachers to more effectively lead students through a process of troubleshooting their models and to be able to gain insight into students' thinking based on their models. The methods relied on by facilitators to enable teachers to enact the goals of the WWIEUT workshops did not cleanly fit into just one of Kennedy's characterizations. The predominant method was strategy. There were occasions when prescription and insight were used as well.

**Conclusion**

The study presented here aimed to assess the effectiveness of a professional development workshop at changing secondary mathematics teachers' knowledge, beliefs, and practices regarding the relevance of algebra to their students' lives beyond formal schooling. A great deal of scholarly research has been conducted on the knowledge, beliefs, and practice of secondary mathematics teachers as well as the interplay between these constructs. This study endeavored to add to this literature by addressing a gap in our understanding of what teachers know and believe about how algebra is used in the world outside of school and how these constructs inform instructional practice. These ideas were investigated through a study of the effectiveness of the WWIEUT professional development workshops.

# CHAPTER 3: METHODOLOGY

## Procedures

The following sections outlines the design, sampling plan, data gathering methods, and data analysis of this mixed methods study of the impact of the When Will I Ever Use This? (WWIEUT) professional development workshop on high school mathematics teachers' beliefs, knowledge, and practices related to the relevance of algebra to the world outside of school. The results of the study addressed the following research questions.

1. What are the differences in secondary mathematics teachers' knowledge of how applicable algebra is to a wide range of careers that are relevant to the majority of their students between participants and nonparticipants in a WWIEUT professional development workshop?
2. What are the differences in secondary mathematics teachers' beliefs about the broad applicability of algebra to a wide range of careers that are relevant to the majority of their students between participants and nonparticipants in a WWIEUT professional development workshop?
3. What is the difference in the amount of time spent on real-world applications of algebra by secondary mathematics teachers between participants and nonparticipants in a WWIEUT professional development workshop?

### *Design*

This study utilized mixed methods to address the research questions. It was a quantitatively driven design that also had a qualitative component (QUAN-qual). The first phase of the research was conducted through an online survey. The second phase of the research was conducted using telephone interviews with individual informants. Mixing quantitative and qualitative research enabled the researcher to “collect multiple data using different strategies, approaches, and methods in such a way that the resulting mixture or combination is likely to result in complementary strengths and nonoverlapping weaknesses” (Johnson & Onwuegbuzie, 2004, p. 18). The quantitative component allowed for greater objectivity and a larger sample size. The qualitative component was included to obtain data that could not be acquired quantitatively, which increased the validity of the quantitative component (Morse, 2012). Statistical analysis of the survey data revealed whether or not teachers’ knowledge, beliefs, and practices were associated with participation in a WWIEUT professional development workshop but could not shed light on why or how those changes, if any, occurred. Content analysis of the interview data enabled the researcher to develop a richer understanding of the reasons behind the results obtained through the statistical analysis of the survey data.

The aim of the study was to assess the effectiveness of the WWIEUT professional development workshops. To this end, it was helpful to locate this study within a framework of research on professional development for teachers. This study of a professional development workshop is what Hilda Borko (2004) referred to as Phase 1 research. Borko’s model organized the programs of study of the various elements of professional development workshops for teachers into three phases. The goal of Phase 1 studies is “to provide evidence that a professional development workshop can have a positive impact on teacher learning” (Borko, 2004, p. 5). Phases 2 and 3 have more sophisticated goals and build on insights gained from studies in previous phases. The WWIEUT professional development workshop has yet to be taken to scale. It has not been rigorously studied since its first iteration with the original cohort of teachers nearly a decade ago

(Keene et al, 2011). Since that time the curricular materials, the professional development program, and the facilitators have changed substantially. These factors require this study to be situated in Phase 1 of Borko's model. Indeed, the paucity of studies in the research literature on US high school mathematics teachers' knowledge of and beliefs about the relevance of the algebra they teach make this study somewhat exploratory in nature. The data collected can serve as baseline data for future studies.

***Sampling Procedures***

There were three groups of participants in the survey study. Group A was comprised of high school mathematics teachers who never attended a WWIEUT workshop and were unaware of the existence of the program. This sample was randomly selected from a database of high school mathematics teachers from across the United States. Participants in Group B were from the high school mathematics teachers who attended a WWIEUT workshop in the past. All former workshop attendees were invited to participate. Group C was comprised of high school mathematics teachers who would be attending a WWIEUT workshop in the near future. All participants of these workshops were invited to complete a pre-workshop survey. The population for Group A was all high school mathematics teachers in the US who were unaware of the WWIEUT professional development workshops. The population for Group B was all the teachers who attended WWIEUT workshops in the past. The population for Group C was all the teachers who were soon to be attending a WWIEUT workshop. The WWIEUT workshops have been conducted in cities across the country since 2009. All members of these three groups completed an online questionnaire. All members of Groups A and C completed the pre-workshop questionnaire. All members of Groups B and C completed the post-workshop questionnaire. Teachers were contacted via email and invited to complete the questionnaire. To reduce the non-response rate and guard against non-response bias, a financial incentive valued at $5 was provided for completion of the questionnaire.

Several teachers from Groups A, B, and C were selected to be participate in interviews. These teachers were selected to maximize the variation within the group of informants. This maximum variation sampling strategy was based on the logic that “Any common patterns that emerge from great variation are of particular interest and value in capturing the core experiences and central, shared dimensions of a setting or phenomenon” (Patton, 2015, p. 283). Having diverse representatives in the interview phase of this research also enabled the researcher to corroborate preliminary findings from the survey phase. The planned imbalance of the treatment and control groups in the interview sampling was primarily due to a desire to understand the knowledge, beliefs, and practices of teachers who attended a WWIEUT workshop. The survey research would show what differences were likely to exist between the treatment and control groups. In the interview research, the researcher wanted to gain more insight into the impact of the workshop and the characteristics of teachers who chose to attend the workshop.

The random sampling for the quantitative component of the study produced samples that were representative of their respective populations. The purposeful sampling of the qualitative component ensured a uniform distribution instead of oversampling near the mean and having scarce data at the ends of the distribution (Morse, 2012). The samples for interviews were small, because the qualitative component of the study is supplementary to the primary quantitative component. Interviews typically took between 15 and 20 minutes to conduct and teachers who participated were given a financial incentive valued at $20. The group information and the sizes of the samples are shown in Table 1.

**Table 1**

*Sample Sizes of Participant Groups for Phases of Research*

| Phase | Never (A) | Past (B) | Near future (C) |
|---|---|---|---|
| Pre-survey | 202 | 0 | 147 |
| Post-survey | 0 | 111 | 68 |
| Interview | 8 | 8 | 8 |

It should be noted that the samples of participants in the qualitative component of the study were subsets of the same samples of participants in the quantitative component. For example, the eight participants in Group B who participated in the interview were a subset of the 111 Group B participants who took the post-survey. It should also be noted that the 68 participants in Group C who took the post-survey were not a subset of the 147 Group C participants who took the pre-survey. All members of Group C were invited to complete both the pre- and post-test questionnaires. There were 58 participants who chose to complete both the pre-survey and the post-survey. When comparisons were made, post-test responses of Groups B and C combined to form the treatment group. Group A was the control group. The matched pairs of pre-test and post-test questionnaire responses from 58 members of Group C made up the pre-test and post-test groups in another set of comparisons.

The WWIEUT workshops were delivered in multiple states over the past 11 years. Most workshops were held in or near major population centers. The workshops attracted teachers from urban, suburban, and rural schools. Workshop participants included public, private, and parochial school teachers. The workshops shared many common elements but vary in others. The curriculum and instructional model have been consistent throughout the history of the workshops. Workshop participants were self-selected, motivated volunteers. A major source of variability in participants' experiences of the workshops was the length of the professional development workshop. There were two formats for the program: single-day and multiple-day. One-day workshops tended to be held during the school year, while three- and four-day workshops tended to be held during the summer months. Whatever the duration of the workshops, participants spent much of their time actively engaged in mathematical modeling tasks, as this feature of the WWIEUT professional development workshops has remained constant.

***Data Gathering Methods and Instruments***

In the first phase of this research, data were gathered via an online questionnaire. Participants were contacted through email and asked to submit responses through the Qualtrics platform. Questionnaire items were selected response, and only questionnaires that were completed in their entirety were included in the statistical analysis. Some questionnaire items produced nominal data, and others produced ordinal data. The questionnaire items were developed by the researcher expressly for the purposes of this study. (See Appendices B and C.)

Before developing the questionnaire, existing instruments were examined. Teachers' content knowledge has been measured for more than two centuries (Hill et al, 2007). The knowledge of interest in the present study relates to what teachers know about how algebra is used outside of school and who uses it. No literature could be found that reported results of examining this knowledge in teachers. Mathematics teachers' beliefs about mathematics has also been widely studied. The "Beliefs about Mathematics Scale," developed by Baydar (2000; as cited in Mert & Bulut, 2006); the "Students' Mathematics-Related Beliefs Questionnaire," developed by Op't Eynde and De Corte (2003); and the "Social and Geographic Dimensions of Mathematics

Education," developed by Lowrie and Jorgensen (2016) all contained items that assessed a range of beliefs about mathematics. Beliefs about the real-world applicability of algebra were not adequately assessed by any instrument discovered in the research literature. However, items from the aforementioned instruments informed the development of the questionnaire used in the present study. Questionnaire items meant to assess teachers' instructional practices related to making real-world connections in their algebra lessons asked teachers to reflect on their instructional choices and lesson planning.

In the second phase of this research, data were gathered via interviews. Interviews were conducted over the phone with the researcher as the interviewer. The format of the interviews was semi-structured. They were pragmatic in nature in order to obtain practical and useful insights (Patton, 2015). The interview guide was developed after preliminary analysis of the survey data. The development of the interview guide was also informed by informal and unstructured interviewing of mathematics teachers who had and had not participated in a WWIEUT professional development workshop. These experiences provided the researcher with a deeper understanding of the teachers' beliefs, knowledge, and practices related to the relevance of algebra and supported the development of meaningful and relevant semi-structured interview questions. (See Appendix D.) Interviews were audio-recorded and subsequently transcribed for content analysis using Atlas.ti.

**Setting**

Participants in the survey phase of the study drew on different experiences with the WWIEUT professional development workshops. Participants in Group A never participated in a WWIEUT workshop and completed an online questionnaire. Participants in Groups B and C all completed an online questionnaire after attending a workshop. Participants in Group C also questionnaire a pre-workshop questionnaire before experiencing the workshop. For the qualitative component of the study, several members from each group were selected to participate in telephone interviews. The setting for both phases of data collection were familiar to the participants. The setting for the WWIEUT workshops was novel to the participants.

This study assessed the effectiveness of a professional development workshop for high school mathematics teachers. The WWIEUT professional development workshops had several goals. The primary purpose of the workshops was to provide teachers with an alternative view of mathematics. The way mathematics is often taught in schools leaves students with little insight into what mathematics is or how it is used in the world outside of school (Boaler, 2016). The researcher hypothesized that secondary mathematics teachers often lack this insight as well. Participating in a WWIEUT workshop provided teachers with experiences that demonstrated the utility of algebra as a modeling and decision-making tool in a variety of contexts that were relevant to administrative careers in business, industry, and public policy, as well as in students' personal lives.

The workshops were intensive, with teachers consistently immersed in reading, working, and discussing the formulation of a mathematical model and interpretation of its results. While much of the teachers' time in the workshops was invested in active learning of mathematical modeling, attention was also paid to increasing teachers' pedagogical knowledge. Teaching students to model with algebra requires a different set of skills than does teaching students to execute algebraic manipulations efficiently. For example, mathematical models make assumptions about a real-world system in order to simplify it. These assumptions need to be discussed and understood for the outputs of the model to be compelling. During the WWIEUT workshops, many of the participating secondary mathematics teachers who participated reported not having much experience facilitating classroom discussions such as these.

## Quantitative Data Analysis

The first phase of data analysis was the statistical analysis of the quantitative survey data. The data obtained from the questionnaire responses were a mix of nominal and ordinal. All data from selected response items were exported from Qualtrics to the software SPSS Statistics for analysis. Only the ordinal data were analyzed for the present study.

### *Scales*

To reduce the dimensions of the data, scales were created using principal components analysis (PCA) for ordinal data. PCA is an exploratory tool that was developed to "reduce the dimensionality of a dataset, while preserving as much 'variability' (i.e. statistical information) as possible" (Jolliffe & Cadima, 2016, p. 10). The researcher hypothesized the existence of three components: teachers' knowledge about how algebra is used outside of school, teachers' beliefs about the relevance of algebra to the lives of their students, and teachers' practices related to making real-world connections during algebra instruction. Cronbach's alpha was calculated for each component to assess reliability and internal consistency. Valid components were achieved, and scaled scores were calculated for each participant on each component using regression.

### *Statistical Hypotheses*

The statistical analysis of the quantitative survey data tested the null hypotheses that no differences existed in the mean scores of the components between the groups being compared. An independent-samples *t*-test was used to compare the treatment and control groups for the knowledge component. Mann-Whitney *U* tests were used to compare the treatment and control groups for the beliefs and practice components. Paired-samples *t*-tests were used to compare mean scores from pre-test and post-test groups on all three components. The significance level for all tests was .05.

**Qualitative Data Analysis**

The second phase of data analysis was the content analysis of the qualitative interview data. Content analysis was defined by Smith (2000) as "a technique used to extract desired information from a body of material (usually verbal) by systematically and objectively identifying specified characteristics of the material" (p. 314). This characterization highlighted the fact that, as a qualitative research method, content analysis enables the researcher to extract specific information from text. Krippendorf (2013) extended this conceptualization by stating that "Content analysis is a research technique for making replicable and valid inferences from texts…to the contexts of their use" (p. 24). This definition's focus on the allowance for the researcher to draw inferences makes it apt for the purposes of this study.

All interviews were transcribed and imported into the qualitative data analysis software ATLAS.ti. Coding of these texts reduced the amount of information to a more manageable form of representation. Some of the codes used in this analysis were developed when the semi-structured interview protocol was developed, which occurred after preliminary statistical analysis of the survey data. The qualitative content analysis focused on the same research questions as the quantitative component. The role of the content analysis, however, was to clarify and explicate the between-group differences discovered in the statistical analysis of the survey data. The quantitative component of the study addressed questions of whether differences existed and assessed the magnitude of any such differences. The qualitative component addressed questions of how and why those differences came to be.

The researcher attempted to collect qualitative data from a stance of empathetic neutrality. Patton (2015) explained this seemingly contradictory approach by stating, “While empathy describes a stance toward the people we encounter in fieldwork, calling on us to communicate interest, caring, and understanding, neutrality suggests a stance toward their thoughts, emotions, and behaviors, a stance of being nonjudgmental” (p. 59). In an effort to establish trustworthiness, a more detailed account of the qualitative content analysis is the interview transcripts appears in the next chapter. A transparent account of the analytical process will be provided to give the reader means of evaluating the research for credibility, transferability, dependability and confirmability.

# CHAPTER 4: RESULTS

This mixed-methods study was quantitatively driven with a qualitative supplement. Results from the statistical analysis of the quantitative core are presented below, followed by results from the content analysis of the qualitative supplement. This chapter concludes with the synthesized results of the quantitative and qualitative strands.

## Statistical Analysis of Quantitative Data

The quantitative data analyzed in this study were obtained through online questionnaires administered to high school mathematics teachers across the US. The dimensions of these data were reduced using principal components analysis (PCA). Statistical analysis focused on comparing group means of the resulting three factor scores. Two sets of comparisons were made. The first set were between-subjects comparisons of the factor scores from teachers who participated in WWIEUT workshops to those from teachers who did not (i.e., treatment versus control). The second set were a within-subjects comparisons of factor scores from teachers prior to and after attending a WWIEUT workshop (i.e., pre-test versus post-test). The details of how these groups were constituted will be discussed below.

### *Sample*

An outline of the sampling plan was presented in Chapter 3. This plan had participants organized into three groups: (A) teachers who never attended a WWIEUT workshop, (B) teachers who attended a WWIEUT workshop in the past, and (C) teachers who would be attending a WWIEUT workshop in the near future. The sizes of the obtained national samples for the quantitative phase are shown in Table 2.

**Table 2**

*Sizes of Samples of Participant Groups for Quantitative Phase of Research*

| Phase | Never (A) | Past (B) | Near future (C) |
|---|---|---|---|
| Pre-test | 202 | 0 | 147 |
| Post-test | 0 | 111 | 68 |

For the treatment versus control group comparisons, the post-survey scores from teachers in Groups B and C (combined $n$ = 179) were compared to scores from teachers in Group A ($n$ = 202). For the pre-test versus post-test comparisons, only matched pairs of scores from teachers in Croup C ($n$ = 58) were used. The data used in these comparisons were obtained from these teachers' responses to online questionnaires that were then reduced by performing a PCA.

All participants were invited through email to complete the questionnaire online. Table 3 shows how many teachers were invited to participate and the response rate for each group. The overall questionnaire response rate for the study was 15.9%.

**Table 3**

*Number of Teachers Invited to Complete Questionnaire and Response Rates*

| | | | Near future (C) | |
|---|---|---|---|---|
| | Never (A) | Past (B) | Pre-test | Post-test |
| Invitations | 1,800 | 782 | 367 | 367 |
| Response rate | 11.2% | 14.2% | 40.1% | 18.5% |

### *Dimension Reduction*

PCA was run using data from a 40-item questionnaire that assessed characteristics of 470 high school mathematics teachers. Of these 40 items, 20 were included in the current analysis. The 20 retained items produced ordinal data and assessed variables relevant to the research questions. The suitability of PCA was assessed prior to analysis.

The researcher observed a gap in theoretical models and empirical findings in the research literature where US high school mathematics teachers' perceptions and actions regarding the role of algebra in the world outside of school was concerned. As such, the questionnaire instrument was designed by the researcher to assess knowledge, beliefs, and practice constructs in high school mathematics teachers related to the relevance and broad applicability of algebra. Table 4 shows the 20 questionnaire items relevant to the current analysis. It also indicates whether the ordinal data from the items was scored in its original order or if that order was reversed to maintain a common orientation so responses on multiple items could be combined into a single meaningful score. For example, Item 14 reads "I believe the algebra I teach will be used by _____ of my students in their careers," with answer options of 0-10%, 10-25%, 25-50%, 50-75%, and 75-100%, in that order. These answer options were converted to numeric (ordinal) values in SPSS Statistics. A lower percentage range resulted in a lower numeric value and represented a teacher having a weaker belief about algebra being relevant to students' eventual careers. In contrast, Item 34 reads "To succeed in their eventual careers, it is important for my students to understand algebra," with answer options strongly agree, agree, neutral, disagree, and strongly disagree, in that order. Again the answer options were converted to numeric (ordinal) values in SPSS Statistics. In this case, a lower numeric value corresponds to more agreement with the statement and represents a stronger belief that algebra is relevant to students' eventual careers. For all questionnaire items, the decision to retain the original order or reverse the order of scores was made so that smaller numeric values corresponded to lower levels of the dependent variables of knowledge, beliefs, and practice (i.e., less knowledge, weaker beliefs, and less instructional time) and higher numeric scores corresponded to higher levels. The last column of Table 4 shows the researcher's hypothesis for which construct would load highly on each item. Not all questionnaire items are accounted for in Table 4, because not all items were relevant to the three constructs and/or did not generate ordinal data. See Appendices A and B for the complete pre-survey and post-survey instruments.

**Table 4**

*Questionnaire Items Involved in Principal Components Analysis with Hypothesized Construct*

| Item number | Questionnaire item | Scoring | Hypothesized construct |
|---|---|---|---|
| 12 | I believe the algebra I teach is relevant to about _____ of my students' educational goals. | Original | Beliefs |
| 14 | I believe the algebra I teach will be used by _____ of my students in their careers. | Original | Beliefs |
| 16 | About how often does a question like, "When will I ever use this?" get asked in your algebra classes? | Original | Practice |
| 20 | During algebra instruction, I typically spend ____ on real-world applications. | Original | Practice |
| 21 | During algebra instruction, I would like to spend ____ on real-world applications. | Original | Beliefs |
| 22 | The amount of instructional time I devote to real-world applications of algebra has _____ over time. | Reverse | Practice |
| 25 | High school algebra courses would be _____ valuable to most students if more instructional time was devoted to real-world applications of algebra. | Reverse | Practice |
| 28 | Algebra as a discipline is relevant outside of school. | Reverse | Beliefs |

| Item number | Questionnaire item | Scoring | Hypothesized construct |
|---|---|---|---|
| 29 | I understand how working professionals use algebra as a decision making tool. | Reverse | Knowledge |
| 30 | I use algebra to help make important decisions outside of school. | Reverse | Knowledge |
| 31 | I know which professions utilize algebra and which do not. | Reverse | Knowledge |
| 32 | My classroom instruction demonstrates the usefulness of algebra outside of school. | Reverse | Practice |
| 33 | To succeed in their future education, it is important for my students to understand algebra. | Original | Beliefs |
| 34 | To succeed in their eventual careers, it is important for my students to understand algebra. | Reverse | Beliefs |
| 35 | The training I received prior to becoming a teacher provided me with enough knowledge about how algebra is used in the world outside of school. | Reverse | Knowledge |
| 36 | The professional development I've received since becoming a teacher has provided me with enough knowledge about how algebra is used in the world outside of school. | Reverse | Knowledge |

| Item number | Questionnaire item | Scoring | Hypothesized construct |
|---|---|---|---|
| 37 | This problem represents an application of mathematics akin to what my students may encounter in their future employment. “For a school concert the ratio of tickets sold to parents, children, and faculty is 25 : 20 : 3. If 240 tickets are sold, how many tickets are sold to each group?” | Original | Knowledge |
| 41 | This problem represents an application of mathematics akin to what my students may encounter in their future employment. “A young channel catfish weighs about 0.1 pound. During the next 8 weeks, its weight increases by about 23% each week. About how much will the catfish weigh after 4 weeks?” | Reverse | Knowledge |
| 42 | This problem represents an application of mathematics akin to what my students may encounter in their future employment. “A passenger train averaging 62 mi/h begins the 355-mi trip from Glenville to Lintown at 12:00 noon. A freight train traveling 48 mi/h leaves Lintown at 2:00 p.m. the same day and travels to Glenville on an adjacent track. At what time will the two trains pass each other?” | Original | Knowledge |

| Item number | Questionnaire item | Scoring | Hypothesized construct |
|---|---|---|---|
| 43 | This problem represents an application of mathematics akin to what my students may encounter in their future employment. "How much larger is the volume of a cube with dimensions ($x$ + 3) than the volume of a cube with dimensions ($x$ + 2)?" | Original | Knowledge |

The researcher conducted PCA using SPSS Statistics on data from 528 responses to the questionnaires. PCA was forced to retain three components to conform to the hypothesized model. Four iterations of PCA were required to obtain a final model.

The first iteration was run with all 20 questionnaire items shown in Table 4. Based on this analysis, questionnaire items 16 and 25 were discarded as ambivalent because no component loaded on these variables greater than .40. Item 30 was also discarded as irreconcilable because two different components loaded on it greater than .40 (S. S. Sawilowsky, personal communication, September 26, 2017).

The second iteration was run under the same parameters with the 17 remaining items. Four more questionnaire items were eliminated based on this analysis. The researcher hypothesized that questionnaire items 29, 31, 35, and 36 assessed the Knowledge construct but PCA showed the component representing the Practice construct loading greater than .40 on them. Items 29, 31, 35, and 36 were discarded as conceptually inconsistent with the Practice factor.

The third iteration of PCA was run under the same parameters with the 13 remaining items. One more questionnaire item was discarded based on this analysis. The researcher hypothesized questionnaire item 16 assessed the Belief construct but PCA showed the component representing the Practice construct loading greater than .40 on it. Item 16 was discarded.

The fourth iteration of PCA was run under the same parameters with the 12 remaining items and obtained a viable model. The resulting model had three components that each loaded on the remaining 12 questionnaire items as hypothesized. Inspection of the correlation matrix showed that all variables had at least one correlation coefficient greater than .30. The overall Kaiser-Meyer-Olkin (KMO) measure was .75 with individual KMO measures all greater than .60, classifications of “mediocre” to “meritorious” according to Kaiser (1974). Bartlett’s Test of Sphericity was statistically significant ($p < .0005$), indicating that the data were likely factorizable.

PCA was forced to retain three components: Component 1, Component 2, and Component 3. These explained 20.0%, 19.6%, and 15.1% of the total variance, respectively. A three-component solution met the interpretability criterion, so three components were retained.

The three-component solution explained 54.8% of the total variance. A varimax orthogonal rotation was employed to aid interpretability. The rotated solution exhibited “simple structure” (Thurstone, 1947). The interpretation of the data was consistent with the constructs the questionnaire was designed to measure with strong loadings of Knowledge items on Component 1, Beliefs items on Component 2, and Practice items on Component 3. Component loadings and communalities of the rotated solution are presented in Table 5.

**Table 5**

*Factor Loadings and Communalities for Principal Components Analysis with Varimax Rotation of Questionnaire Items*

| Questionnaire Item | Rotated Component Coefficients | | | |
|---|---|---|---|---|
| | Component 1 | Component 2 | Component 3 | Communalities |
| 37 | .779 | | | .616 |
| 42 | .772 | | | .614 |
| 41 | .729 | | | .537 |
| 43 | .705 | | | .525 |
| 14 | | .711 | | .540 |
| 34 | | .696 | | .556 |
| 28 | | .648 | | .504 |
| 33 | | .645 | | .433 |
| 12 | | .613 | | .384 |
| 20 | | | .832 | .701 |
| 22 | | | .727 | .529 |
| 32 | | | .668 | .632 |

*Note.* Factor loadings < .40 are suppressed.

The questionnaire items used to form each component needed to have internal consistency, meaning they should all measure the same thing, and thus should be correlated with one another (Bland & Altman, 1997). Cronbach's alpha was calculated for each component as an index of reliability to represent the internal consistency of the items making up each component. Table 6 shows these coefficients.

**Table 6**

*Reliability of Components in Principal Components Analysis Model*

| Component | Construct | Number of Items | Cronbach's alpha |
|---|---|---|---|
| 1 | Knowledge | 4 | .74 |
| 2 | Beliefs | 5 | .70 |
| 3 | Practice | 3 | .67 |

To interpret the value of Cronbach's alpha, many authors rely on a rule-of-thumb that alpha must be at least .70 to be show an acceptable level of internal consistency, but there is little empirical evidence to support this position (Taber, 2018). Peterson's (1994) meta-analysis of 4,286 reported alpha coefficients from social and behavioral research published between 1960 and 1992 provided some context for interpreting values of Cronbach's alpha. Tables 7 and 8 locate the current study in the contexts of constructs assessed and number of items included in each construct, respectively.

**Table 7**

*Comparison of Cronbach's Alpha Values in Current Study to Relevant Research Literature by Construct Assessed*

| Current Study | | Peterson's meta-analysis[a] | | |
|---|---|---|---|---|
| Construct | $\alpha$ | Construct | Mean $\alpha$ | First quartile $\alpha$ |
| Knowledge | .74 | Cognition/Knowledge | .81 | .75 |
| Beliefs | .70 | Value/Belief | .70 | .63 |
| Practice | .67 | Reported behavior | .71 | .63 |

*Note*. $\alpha$ = Cronbach's alpha reliability coefficient.

[a]Adapted from "A meta-analysis of Cronbach's coefficient alpha," by R. A. Peterson, 1994, *Journal of Consumer Research*, *21*(2), 381-391.

**Table 8**

*Comparison of Cronbach's Alpha Values in Current Study to Relevant Research Literature by Number of Items onto which Each Construct Loads*

| Number of items | Current Study | | Peterson's meta-analysis[a] | |
|---|---|---|---|---|
| | Construct | $\alpha$ | Mean $\alpha$ | First quartile $\alpha$ |
| 3 | Practice | .67 | .73 | .65 |
| 4 | Knowledge | .74 | .76 | .68 |
| 5 | Beliefs | .70 | .78 | .71 |

*Note*. $\alpha$ = Cronbach's alpha reliability coefficient.

[a]Adapted from "A meta-analysis of Cronbach's coefficient alpha," by R. A. Peterson, 1994, *Journal of Consumer Research*, *21*(2), 381-391.

The data in Table 8 show the value of Cronbach's alpha for the Practice and Knowledge constructs in the current study were above the first quartile for their corresponding alpha values in related research. The alpha value for the Beliefs construct was only slightly below its corresponding first quartile alpha value. These comparisons put the reliability of the PCA model in the current study in the realm of acceptable behavioral research.

Subsequent to developing a viable PCA model for the survey data, a score for each component was created for each completed questionnaire using regression. These scores represent the values of the dependent variables for each participant. The factor score for Component 1 represented the teacher's knowledge of how applicable algebra is to a wide range of careers that are relevant to the majority of their students and was used to address Research Question 1. The factor score for Component 2 represented the teacher's beliefs about the broad applicability of algebra to a wide range of careers that are relevant to the majority of their students and was used to address Research Question 2. The factor score for Component 3 represented the amount of instructional time the teacher spends on real-world applications of algebra and was used to address Research Question 3.

***Treatment Versus Control***

The independent variable for one set of comparisons to investigate the research questions was workshop attendance (i.e., condition). Each of the three factor scores for each participant generated during the PCA were the values of the dependent variables. The planned method for determining whether statistically significant differences existed between workshop attendees ($n$ = 179) and non-attendees ($n$ = 202) was independent-samples $t$-tests. All statistical tests used an alpha level of .05. Prior to conducting these tests, the assumptions for the test were checked and some necessary changes were made to the data and the planned method.

The data on the dependent variables met several assumptions of an independent-samples *t*-test. A score for each factor for each participant was generated by regression during PCA, so each dependent variable was measured at the continuous level. The independent variable consisted of two categorical, independent groups. All observations were independent as there were different participants in each group. Several other assumptions were not met. The independent-samples *t*-test assumes data are normally distributed, have no significant outliers, and have homogeneity of variances. How the data in the current study fit these assumptions will be discussed one variable at a time.

The first variable to be examined was Knowledge. The Shapiro-Wilk test was used to determine if data were normally distributed (Razali & Wah, 2011). The data for the factor score corresponding to the Knowledge dependent variable were normally distributed for both the treatment and control groups ($p$ = .14 and $p$ = .08, respectively). Of the 381 cases, eight outliers were detected. In the set of eight outliers, three cases had low values (one control and two treatment) and five cases had high values (all control). An independent-samples *t*-test was conducted with and without outliers being included to determine their impact on the analysis. Both tests showed significant results. Osborne and Overbay (2011) found that removing outliers resulted in significant changes in statistics and enhanced the accuracy of estimates. Hence, the eight outliers in the factor scores for the dependent variable Knowledge were removed from the analysis. There was homogeneity of variances for Knowledge scores for treatment and control groups, as assessed by Levene's test for equality of variances ($p$ = .395).

An independent-samples *t*-test was run to determine if there were differences in Knowledge between teachers who attended a WWIEUT workshop (i.e., participants in Groups B and C post-test) and those who did not (i.e., participants in Group A). The null and alternative hypotheses for the test are below.

$H_0$: The population means for Knowledge of the two groups are equal.

$H_a$: The population means for Knowledge of the two groups are not equal.

Workshop attendees were more knowledgeable ($M$ = 0.16, $SD$ = 0.97) than were non-attendees ($M$ = –0.19, $SD$ = 0.89), a statistically significant difference with a small effect (Vacha-Haase & Thompson, 2004), $M$ = 0.35, 95% CI [–0.35, 0.10], $t(379) = –2.64$, $p < .0005$, $d$ = 0.271. There was a statistically significant difference between means for Knowledge ($p < .05$), and therefore, we can reject the null hypothesis and accept the alternative hypothesis.

Data for the Beliefs variable also violated assumptions of the *t*-test. The Shapiro-Wilk test showed the data for treatment and control groups were both non-normally distributed ($p = .00$ and $p = .03$, respectively). A total of five outliers were observed as well. All outliers were extremely low values. Three outliers belonged to the control group, and the remaining two belonged to the treatment group. The Beliefs data were Winsorized by replacing any score beyond two standard deviations from the mean with a score that was precisely two standard deviations from the mean in its respective direction. The resulting distribution was still non-normal. Eliminating the outliers altogether likewise produced a non-normal distribution. After removing the outliers, the distributions were relatively symmetrical (skewness = 0.02) and platykurtic (kurtosis = –0.49). Multiple transformations were attempted, but none successfully normalized the data. Due to this violation of the normality assumption of the independent-samples *t*-test, an independent-samples

Mann-Whitney *U* test was conducted for the Beliefs factor instead. The assumptions about the study design (i.e., continuous or ordinal dependent variable, dichotomous independent variable, and independence of observations) for the Mann-Whitney *U* test were the same as the *t*-test, and those assumptions were satisfied in the current study.

A Mann-Whitney *U* test was run to determine if there were differences in Beliefs factor scores between teachers who attended a WWIEUT workshop and those who did not. The null and alternative hypotheses for this test are stated below.

$H_0$: The distribution of scores for Beliefs for the two groups are equal.

$H_a$: The distribution of scores for Beliefs the two groups are not equal.

Distributions of the beliefs scores for attendees and non-attendees were similar, as assessed by visual inspection. Differences in the distributions of Beliefs scores were not statistically significant between attendees ($Mdn$ = –0.035) and non-attendees ($Mdn$ = 0.042), $U$ = 18,181, $z$ = 0.095, $p$ = .924, using an exact sampling distribution for $U$ (Dineen & Blakesley, 1973). There was not a statistically significant difference between means for Beliefs ($p > .05$), and therefore, we retain the null hypothesis.

Inspection of the histogram showing the distribution of the data for the Practice component revealed a bimodal distribution. No transformation would normalize those data, so an independent-samples Mann-Whitney *U* test was conducted for the Practice factor as well. This test was run to determine if there were differences in Practice factor scores between teachers who attended a WWIEUT workshop and those who did not. The null and alternative hypotheses for this test are stated below.

$H_0$: The distribution of scores for Practice for the two groups are equal.

$H_a$: The distribution of scores for Practice the two groups are not equal.

Distributions of the Practice scores for attendees and non-attendees were similar, as assessed by visual inspection. Distributions of Practice scores were not statistically significantly different between attendees (*Mdn* = 0.153) and non-attendees (*Mdn* = 0.213), $U$ = 19,154, $z$ = 1.002, $p$ = .316, using an exact sampling distribution for $U$ (Dineen & Blakesley, 1973). There was not a statistically significant difference between means for Practice ($p > .05$), and therefore, we retain the null hypothesis.

***Pre-test Versus Post-test***

The second set of comparisons to investigate the three research questions had time of testing as its independent variable. Each of the three factor scores for each participant's pre-test and post-test generated during the PCA were the dependent variables. The planned method for determining whether statistically significant differences existed between matched pre-test and post-test scores ($n$ = 58) was paired-samples $t$-tests. All statistical tests used an alpha level of .05. Prior to conducting these tests, the assumptions for the test were checked.

Each pre-test versus post-test comparison had one dependent variable that was measured at the continuous level. Each comparison also had an independent variable that consisted of two categorical, related groups. Inspection of box plots for each dependent variable revealed no outliers in the differences between the two related groups. The Shapiro-Wilk test was used to determine if data were normally distributed (Razali & Wah, 2011). The data for the differences of factor scores corresponding to the Knowledge, Beliefs, and Practice dependent variables were all normally distributed ($p$ = .26, $p$ = .93, and $p$ = .62, respectively). Thus the four assumptions of the paired-samples $t$-test were all satisfied.

A paired-samples *t*-test was run to determine if there were differences between teachers' pre-test and post-test scores for Knowledge. The null and alternative hypotheses for the test are below.

$H_0$: The population mean difference between the paired values of Knowledge is equal to zero.

$H_a$: The population mean difference between the paired values of Knowledge is not equal to zero.

Attending the WWIEUT workshop did not elicit a statistically significant difference in Knowledge factor scores, $M$ = .179, 95% CI [–0.034, 0.392], $t(57) = 1.686$, $p = .097$. There was not a statistically significant difference between means ($p > .05$), and therefore, we cannot reject the null hypothesis.

A paired-samples *t*-test was run to determine if there were differences between teachers' pre-test and post-test scores for Beliefs. The null and alternative hypotheses for the test are below.

$H_0$: The population mean difference between the paired values of Beliefs is equal to zero.

$H_a$: The population mean difference between the paired values of Beliefs is not equal to zero.

Attending the WWIEUT workshop did not elicit a statistically significant difference in Beliefs factor scores, $M$ = .072, 95% CI [–0.140, 0.283], $t(57) = 0.681$, $p = .499$. There was not a statistically significant difference between means ($p > .05$), and therefore, we cannot reject the null hypothesis.

A paired-samples *t*-test was run to determine if there were differences between teachers' pre-test and post-test scores for Practice. The null and alternative hypotheses for the test are below.

$H_0$: The population mean difference between the paired values of Practice is equal to zero.

$H_a$: The population mean difference between the paired values of Practice is not equal to zero.

Workshop attendees did focus more instruction on attending to real-world applications of algebra after the WWIEUT workshop ($M$ = 0.14, $SD$ = 1.00) than before ($M$ = –0.15, $SD$ = 0.89), a statistically significant difference with a small effect, $M$ = 0.285, 95% CI [0.044, 0.525], $t(57)$ = 2.373, $p$ = .021, $d$ = 0.311. There was a statistically significant difference between means ($p < .05$), and therefore, we can reject the null hypothesis and accept the alternative hypothesis.

### *Summary of Statistical Analysis of Quantitative Data*

Table 9 summarizes all statistical tests conducted on the quantitative data. Cohen's $d$ was calculated to show the effect size of each significant result. This study addresses a gap in the research literature, so there are few previous studies with which to compare and thus interpret these effect sizes. Given this paucity of points of reference, both results will be considered small, using Cohen's benchmarks (Vacha-Haase & Thompson, 2004).

**Table 9**

*Summary of All Statistical Tests of Quantitative Data*

| Independent variable | Dependent variable | Test | *p* | Null hypothesis decision | Cohen's *d* |
|---|---|---|---|---|---|
| Condition | Knowledge | Independent-samples *t*-test | .009 | Reject | 0.271 |
| | Beliefs | Mann-Whitney *U* test | .924 | Retain | |
| | Practice | Mann-Whitney *U* test | .316 | Retain | |
| Time of test | Knowledge | Paired-samples *t*-test | .097 | Retain | |
| | Beliefs | Paired-samples *t*-test | .499 | Retain | |
| | Practice | Paired-samples *t*-test | .021 | Reject | 0.311 |

The study detected a small significant effect on participants' knowledge of the applicability of algebra to a wide range of careers that are relevant to the majority of their students when comparing treatment and control conditions. The study also detected a small significant effect in the amount of time spent on real-world applications of algebra when comparing pre-test to post-test survey responses. The study did not detect any significant differences in participants' beliefs about the broad applicability of algebra to a wide range of careers that are relevant to the majority of their students for either independent variable.

**Content Analysis of Qualitative Data**

The qualitative data analyzed in this study were the transcripts of semi-structured interviews conducted over the phone with high school mathematics teachers across the US. (See Appendix D for the semi-structured interview guide.) The researcher conducted all interviews.

All informants agreed to be recorded, and the audio recordings of the interviews were transcribed by the researcher and an assistant to facilitate qualitative content analysis (QCA) of the text using the qualitative analytics software Atlas.ti. The purpose of the qualitative phase of the study was to corroborate the quantitative findings and gain insight into the reasons why the treatment and control groups of teachers were similar or varied in reference to the research questions. The aim of the QCA was to systematically describe the meaning of the interview transcripts (Schreier, 2012).

***Sample***

A maximum variation sample of 24 informants from the 470 teachers who participated in the quantitative phase of the study was selected to participate in the semi-structured interviews. The control group was comprised of eight informants who never attended a WWIEUT workshop. The treatment group had 16 informants who had attended a WWIEUT workshop. From there, three characteristics were used to create a diverse group: undergraduate major, teaching experience, and other professional experience. Table 10 gives a description of the sample.

**Table 10**

*Characteristics of Interview Informants*

| Pseudonym | Group | Major[a] | Teaching experience[b] | Other professional experience? |
|---|---|---|---|---|
| C1 | Control | Mathematics | Novice | Yes |
| C2 | Control | Mathematics | Novice | No |
| C3 | Control | Mathematics | Veteran | Yes |
| C4 | Control | Mathematics | Veteran | No |
| C5 | Control | Other | Novice | Yes |
| C6 | Control | Other | Novice | No |
| C7 | Control | Other | Veteran | Yes |
| C8 | Control | Other | Veteran | No |
| T1 | Treatment | Mathematics | Novice | Yes |
| T2 | Treatment | Mathematics | Novice | No |
| T3 | Treatment | Mathematics | Novice | Yes |
| T4 | Treatment | Mathematics | Novice | No |
| T5 | Treatment | Mathematics | Veteran | Yes |
| T6 | Treatment | Mathematics | Veteran | No |
| T7 | Treatment | Mathematics | Veteran | Yes |
| T8 | Treatment | Mathematics | Veteran | No |
| T9 | Treatment | Other | Novice | Yes |
| T10 | Treatment | Other | Novice | No |

| Pseudonym | Group | Major[a] | Teaching experience[b] | Other professional experience? |
| --- | --- | --- | --- | --- |
| T11 | Treatment | Other | Novice | Yes |
| T12 | Treatment | Other | Novice | No |
| T13 | Treatment | Other | Veteran | Yes |
| T14 | Treatment | Other | Veteran | No |
| T15 | Treatment | Other | Veteran | Yes |
| T16 | Treatment | Other | Veteran | No |

[a]Undergraduate mathematics major includes pure, applied, and computational mathematics.

[b]Teachers labeled novice had fewer than five years of teaching experience.

To obtain this sample of interview informants, 146 teachers were invited to participate. The response rate for the interview phase of the study was 16.4%.

***Coding***

The primary first cycle coding method was initial coding. This method was appropriate for the research questions, which were concerned with knowledge as well as understanding the impact of an intervention (Saldaña, 2016). Initial coding allowed the researcher to closely examine discrete parts of the data to compare them for similarities and differences. The coding process proceeded in stages.

Prior to coding the researcher read and reread interview transcripts. After becoming familiar with the data corpus, an initial codebook of a priori codes was developed. This codebook was pilot tested with three transcripts. Pilot testing resulted in augmenting the a priori codebook as new and unexpected understandings of teachers' knowledge, beliefs, and practices emerged from the interview transcripts. All 24 transcripts were then coded, and data-driven codes were added to the codebook throughout the coding process. For the first cycle of coding, there were 61 a priori codes and 62 codes that emerged from the data. At this point there were 573 coded segments of text to which 123 codes were applied. (See Appendix E for the QCA codebook.) Upon completion of the first cycle of coding, codes were reviewed, and some were clarified, subsumed, or eliminated. At the conclusion of this process, 97 codes remained. The decisions about refining codes were made to ensure codes accurately reflected the meaning intended by the informants and to more narrowly focus analysis on the research questions. These decisions were recorded in analytical memos. Table 11 shows the length of each interview transcript along with the number of coded segments of text and the number of unique codes used in the transcripts after the second cycle of coding.

**Table 11**

*Code Density of Interview Transcripts*

| Pseudonym | Word count | Coded text segments | Unique codes used |
|---|---|---|---|
| C1 | 2,024 | 26 | 24 |
| C2 | 3,356 | 23 | 21 |
| C3 | 2,513 | 22 | 17 |
| C4 | 2,077 | 24 | 22 |
| C5 | 1,907 | 20 | 16 |
| C6 | 3,695 | 19 | 18 |

| Pseudonym | Word count | Coded text segments | Unique codes used |
|---|---|---|---|
| C7 | 1,170 | 19 | 16 |
| C8 | 3,118 | 24 | 19 |
| T1 | 2,400 | 30 | 24 |
| T2 | 3,038 | 23 | 19 |
| T3 | 2,033 | 22 | 19 |
| T4 | 1,632 | 17 | 15 |
| T5 | 2,110 | 16 | 16 |
| T6 | 2,577 | 23 | 21 |
| T7 | 2,045 | 29 | 23 |
| T8 | 1,311 | 25 | 24 |
| T9 | 2,024 | 28 | 22 |
| T10 | 2,205 | 25 | 22 |
| T11 | 1,469 | 23 | 21 |
| T12 | 3,966 | 20 | 15 |
| T13 | 1,217 | 24 | 21 |
| T14 | 1,962 | 22 | 18 |
| T15 | 2,425 | 24 | 22 |
| T16 | 2,692 | 21 | 20 |

### *Analysis*

The researcher has nearly 20 years of experience in secondary mathematics education, and as such has some access to the “insider’s perspective.” The researcher was also a frequent facilitator of WWIEUT workshops, a member of the writing team that developed the WWIEUT curricular materials, and a teacher of operations research classes that utilized those materials. For this project, however, the researcher attempted to maintain some degree of detachment to analyze the qualitative data with more authenticity. Even so, the researcher’s identity, assumptions, beliefs, and judgments all could have played a role in the content analysis. To move beyond mere awareness of these biases, the researcher practiced reflexivity throughout the QCA. One method employed for safeguarding against any unconscious predispositions was writing identity memos, which were “statements addressing the questions of who I am, the beliefs I have that might impact on the work, and how I will account for my beliefs and assumptions during my study, are helpful ways of making tacit assumptions explicit” (Butler-Kisber, 2010, p. 19). This memo writing was helpful in interrogating the meaning ascribed to data, the inferences made, and conclusions drawn.

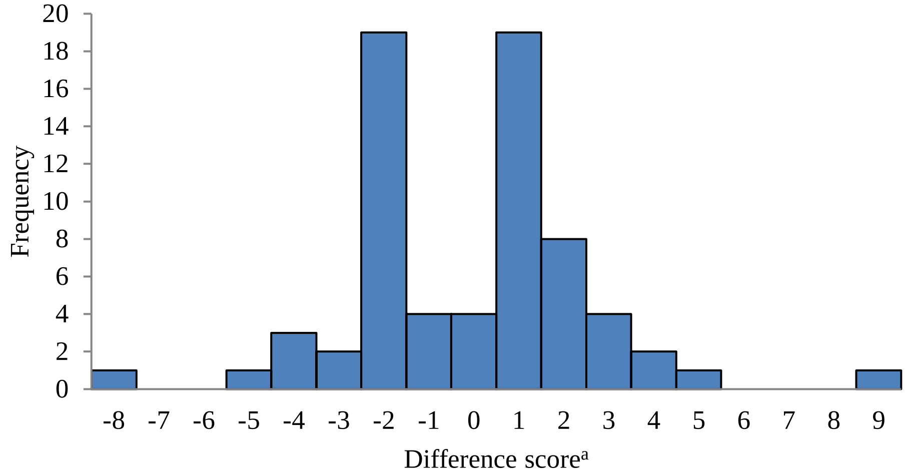
The etic meanings, or “outsider’s perspective,” were determined by the researcher through conceptual analysis and abstraction (Patton, 2015). Codes were analyzed and grouped into categories based on their researcher-determined meanings. These categories were then organized into themes that corresponded to the three research questions: knowledge, beliefs, and practice. The codes were grouped into 14 categories which were subsequently mapped to three broader themes. The themes were mutually exclusive but not collectively exhaustive. In other words, no category belonged to more than one theme, but not all categories were mapped to one of the three themes.

Results from the statistical analysis of the quantitative data were integrated with the analysis of the qualitative data to aid in triangulation. Categories of codes were examined to determine the value they could add to the investigation of the research questions generally and the quantitative results specifically. The categories that did not directly address one of the research questions were filtered out of the analysis. Analysis of the codes that were removed from further consideration was informative, but the light it shed did not illuminate answers to the research questions. After this filtering, 383 segments of text, 69 codes, 10 categories, and three themes remained. The first phase of analysis was focused on differences between teachers in the treatment and control groups, so codes that showed differences were highlighted.

To determine which codes showed meaningful differences between the treatment and control groups, a difference score was calculated for each code. To compensate for there being 16 treatment interviews and eight control interviews, the difference score was determined by subtracting twice the number of transcripts of interviews with teachers in the control group in which a code occurred from the number of transcripts of interviews with teachers in the treatment group in which the same code occurred. The mean of these difference scores across the 69 codes under consideration was a –0.06 with a standard deviation of 2.60. The histogram in Figure 3 shows the frequencies of each difference score across all 69 codes under consideration.

**Figure 3**

*Histogram Showing Frequencies of Difference Scores for All Analyzed Codes*




[a] Difference score = number of treatment transcripts in which code occurred minus twice the number of control transcripts in which the same code occurred

Any code with a difference score with a magnitude of at least four was considered as suggesting a meaningful difference between the treatment and control groups. There were nine codes that had relatively extreme difference scores, either less than or equal to negative four or greater than or equal to four. Two of these codes were in the knowledge theme, four were in the belief theme, and three were in the practice theme. Each code that suggested a difference between teachers in the control and treatment groups will be presented and analyzed below. The results will be shown beginning with the knowledge theme, then beliefs, and finally practice.

Table 12 shows the codes, categories, and difference scores for the four codes that suggested a difference between the teachers in the treatment and control groups related to their knowledge of how applicable algebra is to a wide range of careers that are relevant to the majority of their students.

**Table 12**
*Knowledge Codes Suggesting a Difference between Treatment and Control Groups*

| Code | Category | Difference Score[a] |
|---|---|---|
| Teacher tries to make connections to students' interests to help them see algebra as relevant | Algebra uses outside of school | –5 |
| Real-world connections in school algebra are niche or contrived and do not reflect how algebra is used in a broad range of careers | Algebra uses outside of school | 4 |

[a] Difference score = number of treatment transcripts in which code occurred minus twice the number of control transcripts in which the same code occurred

Before analyzing the two codes in Table 12, an explanation of why they were included in the Knowledge theme is in order. On the surface, the code "Teacher tries to make connections to students' interests to help them see algebra as relevant" appears to be related to the ways a teacher behaves in the classroom, which could justify its classification as a practice. The code was used primarily in teachers' responses to a question about how they respond when students ask, "When am I ever going to use this?" The interpretation here is that the tendency on the part of a teacher to make tenuous connections between sports or music, for example, and the high school algebra curriculum is a reflection of the teacher's lack of knowledge of real-world connections of algebra that would allow for more consequential responses. The code "Real-world connections in school algebra are niche or contrived and do not reflect how algebra is used in a broad range of careers" was nested in the Knowledge theme and not as a belief. As discussed earlier, knowledge and beliefs are overlapping constructs (e.g., Hoy et al, 2006; Pehkonen & Pietilä, 2003), so there was some ambiguity. In this case, the decision to classify the code as an indication of teacher knowledge was the inference that if a teacher had knowledge of how algebra is applied outside of academic settings, he or she would be more apt to recognize when story problems in curricular materials are not accurately reflecting of how algebra is actually used. These ideas will be developed further in Chapter 5.

The first code in Table 12 has a negative difference score. A negative difference score implies the code occurred in more (proportionately) transcripts of interviews with teachers in the control group than the treatment group. The second code has a positive difference score, which implies that code occurred more in more transcripts of interviews with teachers in the treatment group than the control group. The occurrences of each code in Table 11 will be described below. Each paragraph will focus on one code. The code will be stated, the number of transcripts in which it occurred in treatment and control groups given, and poignant quotations from transcripts shared. This pattern will repeat itself when analyzing codes for beliefs and practice as well.

The code "Teacher tries to make connections to students' interests to help them see algebra as relevant" occurred in three transcripts from interviews with teachers in the control group and in just one transcript from a teacher in the treatment group. The difference score of –5 for this code was calculated by subtracting twice the number of control transcripts in which it occurred (i.e., two times three) from the number of treatment transcripts in which it occurred (i.e., one). Below is how Teacher C6 described his approach to finding real-world connections to include in his algebra instruction.

> If I know that they're into cars, then you can talk about acceleration or you could, if you know that someone is into music, you could relate things to timing. Just different aspects, and I like to try to mix it up, so that each student feels like it's not just, because I can get very sports heavy.

Teacher T7 described her approach similarly.

> The ways that you do that though, it's about knowing your students, knowing what interests them, and trying to find ways to tie in pop culture and you know, current events–current music, pop culture references, and sporting events–things that are going on in their lives that they are reading about in their tweets and their timelines and stuff like that. Those are the great ways you can help make it relevant to them, but it's not always easy.

Teachers in the control group made references to students' personal interests to make algebra seem relevant at a higher rate than teachers in the treatment group.

There were six transcripts of interviews with teachers in the treatment group in which the code "Real-world connections in school algebra are niche or contrived and do not reflect how algebra is used in a broad range of careers." This code was used in just one transcript of interviews with control teachers. Teacher T10 described story problems in the algebra curriculum as "just so fake and so cheesy that they didn't work for me." Teacher T4 cited a lack of genuine relevance in real-world connections in algebra curricular materials as a reason students are disengaged during algebra lessons.

> A lot of the word problems or real-world problems don't really apply to the kids, or I feel like it's kind of arbitrary for what we kind of make-up sometimes for real-world problems. And the kids don't see where it actually applies.

Teacher T7 assessed the situation with the "real-world" problems in traditional algebra textbooks succinctly when she stated "these story problems are so lame though."

Table 13 shows the codes, categories, and difference scores for the five codes that suggested a difference between the teachers in the treatment and control groups related to their beliefs about the broad applicability of algebra to a wide range of careers that are relevant to the majority of their students. The first two codes in Table 12 belong to a category that addresses teachers' beliefs about algebra in particular. The final three codes belong to a category that has to do with teachers' beliefs about mathematics as a whole.

**Table 13**

*Beliefs Codes Suggesting a Difference between Treatment and Control Groups*

| Code | Category | Difference Score[a] |
|---|---|---|
| Algebra develops problem solving skills | Algebra develops thinking skills | –8 |
| Math is about problem solving | Math has utility | –4 |
| Students are disengaged in algebra lessons due to perceived irrelevance | School algebra is not relevant | 5 |
| Math is about understanding the world | Math has utility | 9 |

[a] Difference score = number of treatment transcripts in which code occurred minus twice the number of control transcripts in which the same code occurred

The code "Algebra develops problem solving skills" occurred in seven transcripts of interviews with teachers in the control group and in six transcripts interviews with treatment teachers. Just as before, the difference score was calculated by subtracting twice the number of transcripts of interviews with teachers in the control group in which a code was used from the number of transcripts of interviews of teachers in the treatment group in which the same code occurred. When asked why studying algebra is important for high school students, Teacher C3 said, "the problem solving skills that they obtain in an algebra class, to me, is the real benefit to taking algebra." Responding to the same question, Teacher T3 said, "Really what I'm focusing on there is problem solving, so being able to, in a novel situation, utilize skills and resources that they've learned to solve a problem." Teacher T16 claimed that algebra is taught for its "inherent problem solving skills and efficiency that [students] develop when doing the math problems."

The preceding code dealt with teachers' beliefs about algebra in particular. The next code had to do with teachers' beliefs about mathematics in general. The code "Math is about problem solving" occurred in four transcripts of interviews with teachers in the control group and another four transcripts of interviews with treatment teachers. Teacher C2, whose belief that mathematics is about critical thinking was reported earlier, also asserted that mathematics is problem solving.

> Mathematics is problem solving. It's not just numbers, and it's not just theory, and it's not, you know… Yes, it is very abstract, but it is very tangible. Because we can see it at work! So I look at it as, what is math? Math is problem solving.

Teacher T15 situated his belief that mathematics is about problem solving in the real world when he said, "I could probably define it as problem solving methods: numerically, graphically, algebraically. How to solve problems. How to represent real-world situations symbolically and numerically."

The code "Students are disengaged in algebra lessons due to perceived irrelevance" occurred in 13 transcripts of interviews with teachers in the treatment group and in four transcripts of interviews with control teachers. When discussing why her students are disengaged in her algebra lessons, Teacher C2 said, “We’re not talking to them in their own world. We’re talking to them as an academic mathematician… they don’t see how it relates to them.” Teacher T12 seemed to implicate curriculum and instruction when she said, “Because I think we approach using symbols and numbers instead of approaching it from real-world experiences and working from that real-world experience,” to explain why some of her students were disengaged during algebra lessons. A common thread through many occurrences of this code was expressed by Teacher C6: “I think the biggest thing that I’ve run into with disengagement is that they don’t feel like it ever is going to apply to their life.”

The code “Math is about understanding the world” occurred in nine transcripts of interviews with teachers in the treatment group but not in any transcripts of interviews with control teachers. When explaining his beliefs about the nature of mathematics, Teacher T3 said, “mathematics is a way of making sense of the world around us, and a way of making sense of the data around us.” Several teachers spoke about mathematics as a way to use quantitative reasoning to understand the world. Teacher T1’s explanation that “[mathematics] helps us quantify and understand certain realities of the world,” was typical of several other teachers’ expressed beliefs. Teacher T6 described how he explains the nature of mathematics to his students.

> People would look at the world around them, and they had science or whatever, but they needed a way to quantify or to explain what was going on. And a natural way to explain what was going on was to put some numbers to it… So mathematics is just describing what’s going on around you and trying to make some sense of it.

Table 14 shows the codes, categories, and difference scores for the three codes that suggested a difference between the teachers in the treatment and control groups related to the role of real-world applications of algebra in their instruction.

**Table 14**

*Practice Codes Suggesting a Difference between Treatment and Control Groups*

| Code | Category | Difference Score[a] |
|---|---|---|
| Instructional focus is primarily algebraic skills and/or concepts without context/application | Instruction does not prioritize algebra applications | –4 |
| Students frequently ask, "When will I ever use this?" | Instruction does not prioritize algebra applications | –4 |
| Algebra instruction frequently includes context/application | Instruction prioritizes algebra applications | 4 |

[a] Difference score = number of treatment transcripts in which code occurred minus twice the number of control transcripts in which the same code occurred

The first two codes in Table 14 were used to note when teachers described different ways that their instruction did not prioritize genuine applications of algebra in the world outside of school. The third and final code was used when teachers described how they emphasized real-world contexts or applications during algebra instruction.

The code "Instructional focus is primarily algebraic skills and/or concepts without context/application" occurred in six transcripts of interviews with teachers in the control group and in eight interviews with treatment teachers. This code was used to indicate when teachers said something that indicated that the primary goal of their algebra instruction was skill-based. Teachers were asked to describe a typical lesson, and Teacher C7 described an instructional model that was commonly heard in other interviews.

> It's reviewing some concepts, presenting a new concept, doing a little bit of working problems out on the board, assign them some problems to work out, and have some practice–make sure they understand, give them problems that are more difficult than the ones they worked out, so that they can figure out a little bit on their own and not just spoon feed them everything.

Teacher C5 included a focus on test preparation in the goals of his algebra lessons.

> Again where I was, trying to teach everything toward the test, I guess each day you would just have a new lesson in a unit where, I had my kids follow note packets or something that were filled out, and then they filled in the others along the way. So we might start off with just the basic, the numbers and symbols and how you would perform solving equivalent proportions or something like that for a missing variable.

Teacher C8 described a slightly different instructional model, but the goal remained proficiency in algebraic skills.

> If we were doing, let's say, solving quadratics… I would throw some problems up there, I would let them work individually and then talk to the others who want to share, so I give them a problem to do. They think about it, and then talk to their neighbor and see if they get the same answer. And then share with me. So we would talk about all the different ways you could solve it, and is there a way that is better than a different way? Does it depend on the problem? Does it depend on your personal preference? And then have like, kind of a discussion about what that means–what exactly does it mean to solve a quadratic equation? What does that look like on a graph? And then usually at the end I would give them some time to start working on their homework.

It is noteworthy that of the 24 informants interviewed, 14 of them described a typical algebra lesson as one that focused on symbolic manipulation. This included six of eight in the control group and eight of 16 in the treatment group. The frequency with which this code occurred in interview transcripts will be analyzed further when exploring commonly used codes.

The code “Students frequently ask, ‘When will I ever use this?’” occurred in three transcripts of interviews with teachers in the control group and in two interviews with treatment teachers. Teacher C2 stated a common sentiment concisely, acknowledging, “I’ve been asked that a lot.” After explaining that her typical response to students when they ask “When will I ever use this?” is to say, “You probably never will use it. That’s not why learning it is important,” Teacher C4 said it is a response she repeats often. Teacher T9 was reflective in her response.

> I sometimes can give them an answer, sometimes I can make up an answer, but often they’re not convinced. And that just makes me unhappy. But I don’t have the answers myself, so that makes me wonder: Are we teaching things that don’t need to be taught, or do I just not know enough of when this is useful and I should?

The code “Algebra instruction frequently includes context/application” occurred in eight transcripts of interviews with teachers in the treatment group and in two interviews with control teachers. Teacher T5 said, “So I was probably not a traditional teacher in the sense that, just teach by the book and teach tests and stuff like that. I was all about real-world activities,” when she recalled how she was not like her colleagues in the mathematics department. Teacher T6 explained the emphasis he puts on applications of algebra with his students.

> I want to put up posters in my class that say “No naked math!” but I think I’d get in trouble. But that’s what I call it in class. Like, why would I teach you all this stuff if we’re not going to apply it to something? Like, that would be a waste of everyone’s time and completely useless.

Teacher T9 said, “I might put a situation up on the overhead, and that’s all we work on today. How would you solve this situation? A person has a problem, how would you solve that problem?” to illustrate how she conducts a typical algebra lesson.

The second phase of qualitative content analysis focused on the sentiments that were most commonly expressed by teachers, irrespective of differences between teachers in treatment and control groups. This was an effort to understand what knowledge, beliefs, and practices related to the applicability of algebra were potentially common to high school mathematics teachers. Table 15 shows the six that occurred in at least half of all interview transcripts, regardless of treatment group.

**Table 15**

*Codes that Occurred in at Least 12 Transcripts*

| Code | Theme | Number of Transcripts |
|---|---|---|
| Algebra is relevant to particular careers | Knowledge | 20 |
| Students are disengaged in algebra lessons due to perceived irrelevance | Beliefs | 17 |
| Quantity of real-world applications in algebra instruction is inadequate | Practice | 16 |
| Instructional focus is primarily algebraic skills and/or concepts without context/application | Practice | 14 |
| Algebra develops logic, reasoning, and thinking skills | Beliefs | 13 |
| Algebra develops problem solving skills | Beliefs | 13 |

Each of the six codes in Table 15 will be explored below. Three of these codes, “Students are disengaged in algebra lessons due to perceived irrelevance,” “Instructional focus frequently includes context/application," and "Algebra develops problem solving skills," in addition to occurring frequently in interview transcripts also suggested a difference between treatment and control teachers. As such, they were discussed above. In the following paragraphs, these codes will be revisited to analyze their meaning as commonly expressed sentiments of high school mathematics teachers. The remaining three codes in Table 15 did not suggest a strong difference between teachers who attended a WWIEUT workshop and those who did not.

The code "Algebra is relevant to particular careers" occurred in 20 transcripts, seven of which were from interviews with teachers in the control group and 13 were from interviews with treatment teachers. "Algebra is relevant to particular careers" belonged to the "Careers that use algebra" category and was utilized when a teacher mentioned specific career paths that were likely to use algebra. The careers that were mentioned most frequently by teachers were STEM fields (27 times), business (12 times), and skilled trades (8 times). Several teachers conceded a belief that algebra is not relevant for most students. Teacher C3 stated, "the algebra, the math side of it, unless they're going into a math career, is not going to benefit them." Teacher C3 did not specify what he meant by a "math career." Teacher T14 made a similar observation when he said, "graphing a quadratic or factoring, I don't think those are things that they're probably going to use unless they're in maybe an engineering field or something." It was most common for teachers to name one or two particular careers that use algebra. Teacher T7 showed greater knowledge of a range of careers that use algebra: "we have some students who have an interest in the engineering and the STEM and the medical professions, but even the students who are interested in the trades. Students who are going to go into plumbing and construction."

The code “Students are disengaged in algebra lessons due to perceived irrelevance” was discussed previously in the context of possible differences between teachers in the control and treatment groups. It occurred in 17 transcripts, 13 of which were from interviews with teachers in the treatment group and four were from interviews with control teachers. During the semi-structured interviews, teachers were directly asked why they thought students are disengaged in algebra lessons. The most common reason cited by teachers for students’ disengagement with algebra was that the students saw it as irrelevant to their lives. Teacher C5 expressed his belief by stating the following.

> If there’s not a context around it—again like, “Where can I use this?” If they don’t know what it’s going to be used for, then it’s just something they’re going to memorize for a day or two, or something that goes completely over their head.

Teacher T5 expressed the same belief when she said, “Some of the kids are turned off by algebra or math just because they don’t see where they’re going to use it.” It was a commonly held belief on the part of high school mathematics teachers that students disengage with algebra because the students do not perceive algebra as relevant to their lives outside of school.

The code “Quantity of real-world applications in algebra instruction is inadequate” occurred in six transcripts of interviews with teachers in the control group and in 10 of the interviews with treatment teachers. This code belonged to the “Instruction does not prioritize algebra applications” category. Teacher C1 said, “I would say that definitely my quantity is pretty limited.” Teacher C5 answered with, “I personally would like to use more.” All interview informants were asked directly to comment on the quantity of the real-world applications they include in their algebra instruction. In responding to that question, 16 of the 24 informants stated that their instruction did not include an adequate amount of real-world applications of algebra. Teacher T1 said, “Quantity definitely low–too low.”

The code “Instructional focus is primarily algebraic skills and/or concepts without context/application” occurred in six transcripts of interviews with teachers in the control group and in eight of the interviews with treatment teachers. When asked to describe a typical algebra lesson, most informants described lessons that had procedural skills as their goals. Some teachers included a focus on helping students understand the conceptual underpinnings of algebraic procedures, but it was uncommon for teachers to use real-world connections to drive their algebra lessons. Teacher T8 described typical algebra instruction as “I would throw some problems up there, I would let them work individually and then talk to the others who want to share, so I give them a problem to do.” Teacher T14 described his typical algebra lessons along with his dismay about his own instructional practice.

> So in a typical day I’ll do a warm-up type activity, review homework, and do a lesson. So it’s really hard in that short timeframe to do a lot of tasks and other work. I’m not a fan of the format for that class at all, but that’s out of my control.

The code “Algebra develops logic, reasoning, and thinking skills” occurred in nine transcripts of interviews with teachers in the treatment group and in four interviews with control teachers. It belonged to the category “Algebra develops thinking skills.” Teacher C4 made the assertion that, “More important is that we teach you to be a better critical thinker and have logic skills, and we’re doing that through mathematics,” which was echoed in the transcripts of interviews with other teachers. Teacher C3 similarly expressed his belief that algebra is a vehicle for mental development when he said, “the study of algebra helps the student in a logical sense. [It] helps for their minds to develop logical thinking skills.” Teacher T16 asserted that algebra accomplishes this mental development differently than other disciplines, saying, “it actually teaches them thinking processes that they don’t learn quite the same in other areas.”

The code “Algebra develops problem solving skills” occurred in seven transcripts of interviews with teachers in the control group and in six of the interviews with treatment teachers. Like the code explored in the previous paragraph, this code belonged to the category “Algebra develops thinking skills,” which was part of the Beliefs theme. Teacher C2 expressed this belief by stating, “I feel the biggest thing students get from learning algebra is to develop their mind, in terms of processes and problem solving.” When answering a question about why students should study algebra in high school, Teacher T15 said, “I guess to learn some problem solving skills.” This and the previous code were both used when teachers expressed a belief that a primary reason algebra is taught in high school is to develop thinking skills in students. One or both of these codes occurred in 20 of the 24 interview transcripts.

Another noteworthy finding that does not neatly fit into the above structure has to do with teachers’ sources, or lack thereof, of knowledge about how algebra is used in the world outside of school. Only one teacher of the 24 interviewed cited their academic preparation when answering a question about how they learned about how algebra is used in the non-academic world. More common sources of this knowledge were acquaintances in technical fields, as was heard in Teacher C4’s response, “I know people who are in architecture and architectural engineering, things like that. They all use math all the time.” Teacher T14 cited a similar source of knowledge when he said “I think more just from talking with friends in different fields, and things that they might use more than an actual class in school.” Teacher T15 reflected poignantly that “So, no, my own education, even through college, did not, I think, give me a lot of quality where, what kind careers might this be used in." Commenting on her graduate level coursework, Teacher T1 said, "I don't ever come across in that any of what you mentioned, which is when would this be useful to our students. That never came up." It was rare to hear a teacher answer questions about how algebra is used outside of school with confidence and authority.

### *Summary of Content Analysis of Qualitative Data*

The qualitative component was included to obtain data that could not be acquired quantitatively. Analysis of these qualitative data enabled the researcher to develop a richer understanding of the reasons behind the results obtained through the statistical analysis of the survey data. In particular, the QCA of interview transcripts focused on comparing and contrasting the treatment and control groups of interview informants.

To provide context for integrating the results of the QCA to the statistical analysis of the survey data, this paragraph provides a brief summary of findings from the statistical analysis. (See Table 9.) A significant difference was found between the means of the treatment group and control groups factor scores for knowledge from the survey data. Another significant difference was found when the means of factor scores for practice were compared between pre-test to post-test survey responses from Group C participants. No significant differences were observed in the survey data related to the beliefs factor scores. Each of these results will be addressed in light of the QCA of the interview transcripts.

There were two codes in the Knowledge theme that suggested a difference between informants in the treatment and control groups. (See Table 12.) The code "Real-world connections are niche or contrived and do not reflect how algebra is used in broad range of careers" had a large and positive difference score. This may suggest that participants in the WWIEUT workshop (i.e., the treatment group) were more likely to possess knowledge about the broad applicability of algebra to a wide range of careers as well as how algebra is put to use in those careers, which made them more apt to characterize the attempts textbooks make to include "real-world" problems as inauthentic. The code "Teacher tries to make connections to students' interests to help them see algebra as relevant" had a large and negative difference score. This may suggest that teachers who did not attend a WWIEUT workshop had a more vague understanding of how algebra might be used in the world outside of school and were thus less able to make career-related connections and instead focused on more superficial connections to topics such as sports or music.

Three codes in the Practice theme that suggested a difference between informants in the treatment and control groups. (See Table 14.) The code "Instructional focus frequently includes context/application" had a large and positive difference score. This seemed to suggest that participants in the WWIEUT workshop made more of an effort to include contextualized applications of algebra during their instruction. The codes "Students frequently ask 'When will I ever use this?'" and "Instructional focus is primarily algebraic skills and/or concepts without context/application" had large and negative difference scores. This suggested that teachers who did not attend a WWIEUT workshop did not make as much of an effort to demonstrate algebra's utility in the world outside of school, instead they focused on teaching algebra primarily as a skill-based or procedurally driven discipline.

Whereas there were no significant differences found in comparisons of the beliefs construct in the quantitative phase of this research project, the two codes with the largest difference scores in the QCA of interview transcripts belonged to the beliefs theme. (See Table 13.) The code "Math is a way of understanding the world" was noteworthy because it never occurred in any transcript from interviews with informants in the control group. It occurred in nine transcripts of interviews with informants in the treatment group. This suggested that participants in the WWIEUT workshop were more likely to hold a belief that mathematics can be a tool to be applied and put to productive use outside of academic settings. The teachers in the treatment group seemed to conceptualize mathematics more broadly as a sense-making discipline rather than merely a problem-solving discipline. The codes “Algebra develops problem solving skills” and “Math is about problem solving” both had large and negative difference scores. This suggested that teachers who did not attend a WWIEUT workshop were more likely to hold beliefs that the subject they teach is a vehicle for developing mental skills but is not necessarily useful in and of itself.

# CHAPTER 5: DISCUSSION

In this chapter, the results of the study will be discussed and synthesized. A brief overview of the study will be provided followed by a summary of results. Each result will be interpreted and examined in the context of extant literature. The limitations of the study will be discussed and directions for future research will be explored.

The study presented here utilized mixed methods. The primary phase was quantitative and its method was survey research. The secondary phase was qualitative and its method was semi-structured interview research. The population of interest to the study was secondary mathematics teachers in the United States. A sample of 470 individuals from that population participated in the study. The study was undertaken to answer the following three research questions related to the "When Will I Ever Use This?" (WWIEUT) professional development workshops.

1. What are the differences in secondary mathematics teachers' knowledge of how applicable algebra is to a wide range of careers that are relevant to the majority of their students between participants and nonparticipants in a WWIEUT professional development workshop?
2. What are the differences in secondary mathematics teachers' beliefs about the broad applicability of algebra to a wide range of careers that are relevant to the majority of their students between participants and nonparticipants in a WWIEUT professional development workshop?
3. What is the difference in the amount of time spent on real-world applications of algebra by secondary mathematics teachers between participants and nonparticipants in a WWIEUT professional development workshop?

Versions of the WWIEUT professional development workshops began in 2008 and were still occurring throughout this research project. This presented the researcher the opportunity to conduct two types of tests with the quantitative data. In one set of tests, a sample of workshop alumni formed the treatment group which was compared to a control group of randomly selected secondary mathematics teachers from across the country. Because the workshops were ongoing, pre-test versus post-test comparisons were also conducted using data gathered from teachers prior to participating in a workshop and then again several months after the workshop.

Statistical analysis of the survey data included dimension reduction followed by statistical tests to infer whether significant differences existed between the means of the two groups being compared. Questionnaire items that specifically addressed one of the research questions were combined using principal components analysis (PCA). This resulted in each participant's questionnaire responses being distilled to three scores: one for their knowledge, one for their beliefs, and one for their practice related to the broad real-world applicability of algebra, as conceived in the research questions. With these scores as dependent variables, two sets of statistical tests were conducted. In the first set of tests, the independent variable was whether a teacher had attended a WWIEUT workshop. The control group was a nationally representative random sample of 202 teachers. All WWIEUT workshop attendees were invited to participate as part of the treatment group, and 179 of these teachers completed a questionnaire. In the second set of tests, which only included teachers who would be attending a WWIEUT workshop in the near future, the independent variable was the timing of questionnaire completion. For these tests, a sample of 58 teachers had their knowledge, beliefs, and practice assessed both before and after attending the workshop to ascertain whether any changes could be detected.

The sample of teachers who participated in the interview portion of the study was a subset of the sample who participated in the survey portion. Eight teachers who did not attend a WWIEUT workshop and 16 teachers who did were selected in such a way as to maximize their variation on the key demographic variables of college major, teaching experience, and experience working in a profession other than teaching. Semi-structured interviews were conducted over the phone. Audio recordings of these interviews were transcribed and then coded. Qualitative content analysis (QCA) was employed to provide insights into the findings from the survey research. Both similarities and differences between teachers who did and did not attend a workshop were investigated.

**Summary of Findings**

In the quantitative phase of the research, two significant differences were found. When comparing teachers who did and did not attend a WWIEUT workshop, a difference in teachers' knowledge about the applicability to algebra to a wide range of careers was detected. This result was relevant to Research Question 1. When comparing questionnaire responses before attending a WWIEUT workshop with the same teachers' questionnaire responses after attending the workshop, a significant difference in teachers' instructional practices related to real-world applications of algebra was detected. This result was relevant to Research Question 3. A total of six statistical tests were conducted. The two discussed above were the only ones that met the statistical significance level of $\alpha = .05$.

In the qualitative phase of research, both similarities and differences were found between teachers who attended a WWIEUT workshop and those who did not. The analysis compared and contrasted teachers' knowledge, beliefs, and practices related to the relevance of algebra to their students' future careers. The findings from the QCA of the interview transcripts provided insights that helped make sense of the findings from the statistical analysis of the survey responses.

## Interpretation of Results

The two significant results from the quantitative phase of the research project will be discussed and contextualized. The Beliefs construct, which did not have a significant result associated with it, will also be discussed. Findings from the qualitative phase of the research project will be interspersed to illuminate the results from the survey data. General limitations of the study were discussed in Chapter 1. Other limitations of the study's results will be addressed as they arise in the following discussion.

### ***Knowledge***

A significant result of small effect was found when comparing high school mathematics teachers' knowledge of the broad applicability of algebra to their students' eventual careers. A significant difference was observed between the treatment and control groups but not between the pre-test and post-test groups. This result was echoed in the qualitative phase of the study. The interview informants who attended a WWIEUT workshop were more likely to criticize the triviality of real-world connections that are typically included in algebra curricular materials, perhaps indicating these teachers were better able to recognize authentic applications of algebra. Informants who did not attend a WWIEUT workshop were more likely to describe making superficial connections between topics in the algebra curriculum and students' extracurricular activities and interests instead of finding ways to relate algebra concepts and skills to a broad range of potential careers that would be applicable to their students.

It should be noted that the design of this study was such that claiming the WWIEUT workshop caused a change in teacher knowledge was not warranted. It was possible that teachers who possessed more knowledge about the ways professionals use algebra in the world outside of school were more likely to choose to participate in a WWIEUT workshop than those who possessed less of this knowledge. Teacher knowledge of how algebra is in the world outside of school may have influenced teachers to attend the workshop, and not the other way around. This possibility may be supported by the fact that the study did not discern a significant difference between Group C participants' pre-test and post-test assessment of knowledge. It was also the case that the present study detected a correlation between the Knowledge construct and attendance of a WWIEUT workshop. Confounding variables such as desire to make algebra more relevant to students or orientation toward reform agendas in mathematics education may also have been present and explain some of the variability. As such, no causal conclusions were warranted.

Data gathered in interviews showed similarities and differences in informants' knowledge of the applicability of algebra to careers within and between the treatment and control groups. The code that occurred in more transcripts than any other in the entire study was "Algebra is relevant to particular careers," which was utilized when an informant listed a small number of careers to which algebra is relevant. Informants most often cited specific careers in STEM or business-related fields. The code occurred frequently and at similar rates in transcripts of interviews with teachers from both treatment and control groups. This evidence suggested that it was common for secondary mathematics teachers in the US to have knowledge of only a few careers in which their students could potentially use algebra. Knowledge of a wide array of careers that utilize algebra, careers that are of interest to many more students, appeared to be less common among this population. If this knowledge, which was more frequently observed in interview informants who attended WWIEUT workshops, were shared with students, it could motivate more of them to continue studying mathematics beyond their required classes (Brown et al, 2008).

The paucity of research literature on mathematics teacher knowledge related to real-world applications of mathematics presented a challenge for contextualizing this result. Evidence exists in the literature to suggest teachers' attempts at incorporating real-world connections into their instruction are rare and superficial (Gainsburg, 2008). In the present study, this tendency to shortchange applications of algebra was described more frequently in interviews with teachers in the control group than with teachers in the treatment group. According to the conceptual framework presented in Figure 1, an increase in teacher knowledge may enhance instructional practice. Such a change in practice related to real-world connections was observed when comparing WWIEUT workshop attendees' pre-test and post-test survey data, suggesting there may have been an accompanying but undetected change in knowledge.

***Practice***

The other significant result from the statistical analysis of survey data, also of small effect, was found when comparing high school mathematics teachers' practice related to the amount of instructional time spent on real-world applications of algebra. A significant difference was observed between the pre-test and post-test groups but not between the treatment and control groups. The 58 Group C participants who completed the post-test questionnaire did so two to three months after participating in a WWIEUT workshop. There was a much longer gap between the workshop and the questionnaire for the 111 Group B participants. One plausible explanation for a significant result for pre-test versus post-test but not for treatment versus control could be that the experience of the workshop was fresher in the minds of the participants who chose to complete the post-test questionnaire. The workshop's effects on instructional practice may wane over time, especially as workshop attendees did not typically participate with a cohort of colleagues from their buildings and there was no follow-up workshop (Crowther & Cannon, 2002); hence no significant difference was seen between the treatment and control groups on the question of instructional time devoted to real-world applications of algebra.

The significant difference that was detected between pre-test and post-test questionnaire results on the Practice construct does not necessarily imply a causal relationship between participating in a WWIEUT workshop and a change in practice related to including real-world applications in algebra instruction. The two-tailed, paired-samples *t*-test detected that the degree to which participants included real-world applications in their instruction was significantly different before they attended a WWIEUT workshop than afterwards. Inspection of the means of the pre-test and post-test group's Practice scores did reveal a greater focus on real-world applications of algebra after participating in a WWIEUT workshop. Teachers who attended a WWIEUT workshop were only interviewed after attending the workshop, so no direct pre-test versus post-test comparisons could be made in the qualitative phase of the study. The interview data did suggest that teachers who attended a WWIEUT workshop were more likely to use applications and contextualized problems as a focus of their algebra instruction, whereas teachers who did not attend a WWIEUT workshop were more likely to focus their algebra instruction on skills and concepts.

In the QCA of the interview transcripts, three codes belonging to the Practice theme provided further insight into how teachers' participation in a WWIEUT workshop related to their use of real-world applications of algebra in their classroom instruction. The code "Instructional focus frequently includes context/application" occurred more frequently in interview transcripts of informants in the treatment group. It followed that these teachers' students were given a clearer picture of how algebra can be applied to a variety of situations outside of school. In contrast, the codes "Instructional focus is primarily algebraic skills and/or concepts without context/application" and "Students frequently ask 'When will I ever use this?'" occurred at higher rates in interview transcripts of informants in the control group. This is another piece of evidence that suggested teachers who did not attend a WWIEUT workshop did not as clearly demonstrate or communicate to their students the broad applicability of algebra to a wide range of careers. If the relevance of algebra were better understood by students, they would be less inclined to ask the question.

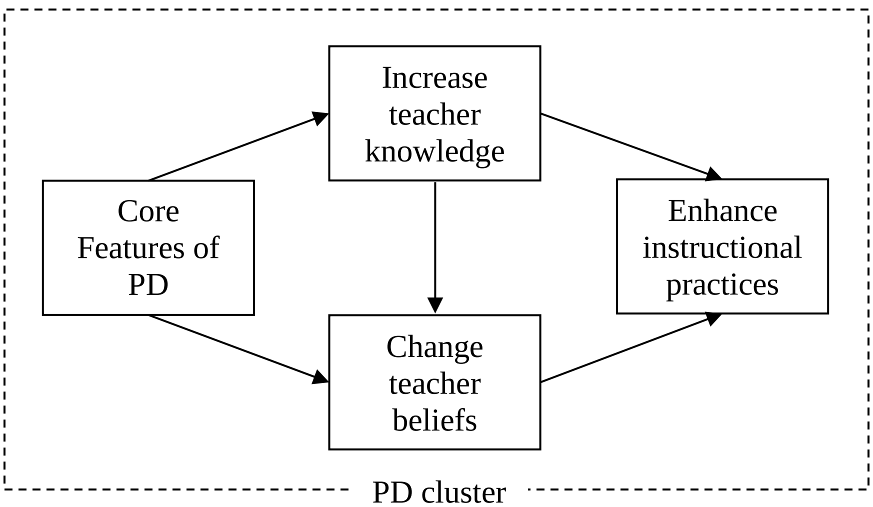
One code in the Practice theme occurred frequently and at similar rates in transcripts of interviews with informants in the treatment and control groups. The common occurrence of the code "Quantity of real-world applications in algebra instruction is inadequate," among both groups of informants suggested that teachers did not devote as much time as they would have liked to sharing with their students the ways algebra can be applied to the world outside of school. Teachers cited a variety of reasons for not including more relevant applications. Common reasons included having an overly ambitious and prescriptive curriculum that did not allow time for applications as well as a lack of quality real-world problems available in curricular materials. Several informants also noted their own lack of knowledge about how algebra was used outside of school.

### *Professional Development*

At its core, the present study was an examination of a professional development intervention for secondary mathematics teachers. The conceptual framework developed in Chapter 1 (see Figure 1) described a sequence of outcomes for professional development. The portion of that conceptual framework referred to as the PD Cluster is shown in Figure 4.

**Figure 4**

*PD Cluster of Conceptual Framework for Changes in Teachers' Knowledge, Beliefs, and Practice (Boston, 2013; Desimone 2009; Karabenick & Conley, 2011)*

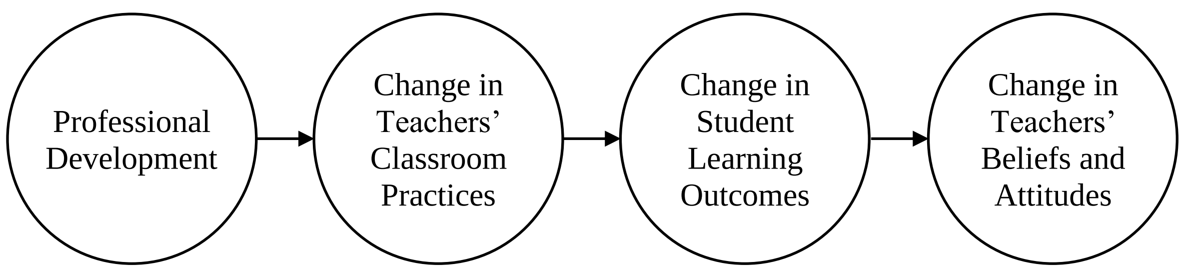



According to this model, the professional development activities would produce changes in teachers' knowledge and beliefs which would, in turn, bring about changes in teachers' instructional practices. In the pre-test versus post-test comparison in the present study, a significant result was detected in teachers' instructional practices, but no significant result was found in their knowledge or beliefs. On its surface, this finding seems contrary to the conceptual framework.

An alternative view of how professional development brings about teacher change was presented by Guskey (2002). In his model, shown in Figure 5, the professional development activities have an immediate impact on instructional practices. Only after teachers observe that these new practices have a positive effect on student learning outcomes do teachers' beliefs change.

**Figure 5**

*A model for teacher change (Guskey, 2002)*




Guskey acknowledged the relationship between the three desired outcomes shown above are to some degree reciprocal, but claimed the configuration shown in Figure 5 was the most likely to result in durable change. In the present study, the observed change in instructional practice without observed changes in beliefs was more consistent with Guskey's model than the one proposed based on the work of Boston (2013), Desimone (2009), and Karabenick and Conley (2011). Guskey's model neglected to account for the change in teachers' knowledge which would be required to produce the change in classroom practices. This new knowledge was presumably a result of the professional development, as teachers must learn new ways to conduct their classrooms. Teachers must also leave the professional development believing enough in the new instructional methods that they are not just willing to adopt the new practices but also abandon their existing ones (Kennedy, 2016).

The observed change in teachers' practice but not their knowledge or beliefs in the pre-test versus post-test comparisons was not sufficient evidence to abandon the model proposed in Chapter 1 in favor of Guskey's (2002) model. In the proposed model, the core features of the professional development drive the changes in teachers' knowledge and beliefs. As discussed in Chapters 1 and 2, not all of the core features of the WWIEUT workshops were optimized (e.g., duration and collective participation), which may explain the discrepancy in the findings. It may have also been the case that, due to the limitations of the present study, discussed in Chapter 1 and continued below, the WWIEUT workshop did produce a change in teachers' beliefs or knowledge that was not detected. In reference to Guskey's model, the presumption that the population of teachers studied here could implement changes in their instructional practices related to real-world connections of algebra without first learning about how algebra was used outside of school or having their beliefs about the relevance of algebra altered seems unlikely.

***Beliefs***

No significant differences between groups of participants, either treatment versus control or pre-test versus post-test, were detected by the statistical analysis of the survey data related to the Beliefs construct. Beliefs are stable and are thus more resistant to change (McLeod, 1989), which may help explain why the hypothesized change in beliefs was not observed. Results from the QCA of the interview data did provide some insights into secondary mathematics teachers' beliefs about algebra in particular and mathematics in general that will be discussed below.

One observed difference between interview informants in the treatment and control groups was related to their view of the nature of mathematics. Nine of the 16 informants in the treatment group stated a belief that mathematics is a way of understanding the world. None of the eight informants in the control group stated such a belief. In contrast, half of the informants in the control group expressed the belief that mathematics is about problem solving. This belief was stated by a quarter of informants in the treatment group. These two assertions, that mathematics is a way of understanding the world and that mathematics is about problem solving, were the two most frequently voiced characterizations of mathematics by interview informants. Both philosophies of mathematics could qualify for what Ernest (1989a) called the problem solving view. If this espoused view were put into practice with fidelity, teachers would take the role of facilitator as students are able to explore. The evidence from the QCA of the interview data related to practice, however, suggested that teachers in the control group were more like to espouse a role of explainer while students must be compliant as work to master skills.

During WWIEUT workshops, attendees continually interacted with complex situations based on real-world examples of how algebra is used to model problems in business, industry, and public policy as well as in people's personal lives. Workshop attendees engaged with these scenarios to model problems, explored possible solutions, and interpreted the results of modeling. They also heard from operations research professionals about how mathematics is used to model problems and make decisions to continually improve operations. Despite the quantitative phase of the present study failing to detect a significant difference in teachers' beliefs between the treatment and control groups, the fact that a majority of interview informants from the treatment group believed that mathematics is a way of understanding the world, a belief not stated by any informant in the control group, was noteworthy.

The research literature on mathematics teachers' beliefs is rich (e.g., Leder et al, 2002; Philipp, 2007; Thompson, 1992). What teachers believe about the applicability of mathematics outside of academic settings appeared to be neglected in this literature. Due to this gap in the literature, our understanding of what teachers believe about how and which professionals use mathematics to formulate models, make decisions, and solve problems in a wide range of fields is limited. These beliefs play a role in how teachers conduct their classrooms and the subsequent impressions that are made on students about the relevance of mathematics.

**Limitations**

In hindsight, some methodological flaws in the present study are apparent. Major concerns were the construct and content validity of the questionnaire that was the source of all quantitative data. The researcher created this instrument for the very specific purpose of gaining insight into the three research questions. Using an existing instrument, one with established validity and reliability, would be preferable. However, as this project attempted to fill in gaps in the relevant research literature, no instrument could be found that would serve the purposes of the study.

A second issue with the methodology was how the Practice construct was investigated. Both the survey and interview phases of the study relied exclusively on teacher self-report. Teachers often provide a too-optimistic view of their instructional practices (Wubbels et al, 1992). Kennedy (2016) observed "teachers can learn and espouse one idea, yet continue enacting a different idea, out of habit, without even noticing the contradiction" (p. 947). The present study could have been improved by including teacher observation in addition to the surveys and interviews. With a sample of teachers from across the country, this would not have been feasible, but the resulting data could have further illuminated the espoused-enacted distinction between beliefs and practices.

The final limitation to be discussed here is one of granularity. The results discussed in this chapter were general in the sense that they were about secondary mathematics teachers in the US. However, in addition to data about knowledge, beliefs, and practice, demographic data were also collected from participants in the quantitative phase of the study. Variables such as gender, years of teaching experience, type and setting of school, college major, and previous professional experience could have been controlled for to obtain finer grained results. Such a fine-grained analysis of the data was beyond the scope of this project.

**Recommendations for Future Research**

The present study was conducted partly to address a gap in the research literature on teacher knowledge. The paucity of research on teachers' knowledge of how mathematics is used in nonacademic settings should continue to be addressed. Many students have made it through their compulsory education in mathematics without ever being shown the powerful utility of the subject. A program of research, including developing valid and reliable instruments, aimed at assessing teacher knowledge about the applications of mathematics in a broad range of fields would inform mathematics teacher educators and professional development designers. If it were found that this particular type of knowledge is scarce among mathematics educators, learning opportunities could be developed to remedy the situation for inservice and preservice teachers. When teachers are more knowledgeable about how their students will likely be using mathematics in their future, lessons could be imbued with authentic connections to the real world and motivate students to learn the mathematics at hand. Seeing the practical use of a discipline they have been studying their entire educational careers could also inspire more students to persist in mathematics programs in order to learn more and become more proficient users of mathematics.

The study presented here had individual teachers as the unit of analysis. Future studies of related professional development activities should be based on a sociocultural conceptual framework and have classrooms as the unit of analysis. In this way, patterns in teacher-student and student-student interaction and participation could be better understood (Borko, 2004). For the WWIEUT workshops or other professional development projects with the goal of helping teachers design and deliver instruction centered on authentic applications of mathematics, program evaluation studies should include observations of classroom instruction. Combining data from observations of instructional practices with self-report data on beliefs from questionnaires and interviews, project developers could gain greater insight into the effectiveness of their professional development program and lead to refinements. Maintaining contact with workshop participants in an ongoing supportive relationship could also increase the likelihood of attaining desired practice-related outcomes, especially if teachers do not have a cohort of colleagues in their local contexts working toward the same goals.

To continue the program of research into the WWIEUT professional development workshops, research should continue into Phase 2 of Borko's (2004) model. The study presented here was situated it in Phase 1. It provided evidence that the workshop can have a positive impact on teacher learning and focused on interactions between the professional development program and the teachers. Impact studies of the WWIEUT workshops can now proceed to investigations of whether the program can be scaled up and implemented with integrity in different settings and by different facilitators.

**Conclusion**

The study presented in this dissertation was designed to add to the research literatures in the fields of teacher knowledge, beliefs, and practice as well as professional development evaluation. Of special interest was teacher knowledge of how algebra can be applied to real-world problems as well as teacher knowledge of who uses those applications in a professional capacity. In a time when our economy is shifting so rapidly, a shift away from manufacturing and toward service and information, there is a consensus among writers of curriculum standards and developers of large-scale assessments as well as professional organizations in the mathematics education community that there should be a complementary shift in school mathematics. Such a shift would allow students to learn the powerful utility of a discipline that is increasingly essential in the modern workforce. To move school mathematics curriculum and instruction in a direction that emphasizes the utility of algebra, teachers need to know about which and how professionals apply algebra to do their work.

Relying on mixed methods, this study found that participating in a WWIEUT professional development workshop was associated with a significant change in teachers' knowledge about how algebra is applied in the wider world, particularly as a modeling tool. Interviews revealed that teachers who participated in the workshops were less likely to rely on superficial connections between algebra and the real world (e.g., music and sports) as a way of demonstrating algebra's relevance to their students' lives. These teachers were also more likely to criticize the meagerness and inauthenticity of the way their algebra textbooks attempted to make real-world connections.

This study also detected a change in teachers' instructional practices related to applications of algebra before and after their participation in a WWIEUT workshop. Teachers who attended a workshop were more likely to describe a focus on contextualized problems in algebra whereas teachers who did not attend a workshop more commonly described their lessons as focused on practicing algebraic skills.

The quantitative phase of this study failed to detect any significant difference teachers' beliefs about the relevance of algebra to their students' lives, either in treatment versus control or pre-test versus post-test comparisons. The qualitative phase of the study, however, revealed a striking difference between treatment and control groups in how teachers described their personal philosophies of mathematics. Teachers who attended a WWIEUT workshop commonly expressed a belief that mathematics was a way of understanding the world, whereas teachers who did not attend a WWIEUT workshop more commonly expressed a belief that mathematics was about problem solving.

Taken together these results suggested that attending a WWIEUT workshop can be effective in helping teachers see how algebra can be put to productive use outside of school and giving them ways to share those insights with students. Consistent with Kloosterman's (2002) contention that students were more motivated to learn things that are or will be useful, teachers who attended WWIEUT workshops seemed more inclined to approach mathematics education with a broader purpose than merely trying to prepare their students for their next mathematics course. These teachers wanted their students to learn powerfully useful mathematics that prepared them to fully participate in our economy and society.

The literature reviewed in this study exposed a gap in our understanding of what teachers know and believe about how algebra can be used as a powerful tool in a wide variety of professions that might be relevant to a majority of their students. Findings in this study highlighted that teachers commonly believed algebra is relevant only to only a few careers. This study also exposed that secondary mathematics teachers in the US reported spending an inadequate amount of time addressing real-world applications of algebra. Instead their focus was predominately on skill-based instruction. These teachers believed their students were disengaged precisely because they saw algebra as irrelevant to their lives. This was possibly the case because teachers seemed to frequently present algebra as a means for mental development and not as a useful discipline in and of itself.

High school mathematics teachers are the social products of the very system they are being called upon to reform. To bring about the recommended changes, teachers will need support. Professional development programs can be powerful instruments for change. This research highlighted the need to better understand what teachers know and do not know about how their students will need to use mathematics when they venture out of the classroom and into a rapidly changing environment where mathematical literacy and quantitative reasoning, which can be leveraged to model with mathematics, are in increasingly high demand.

## APPENDIX A: SAMPLE WWIEUT WORKSHOP AGENDA

# Mathematical Modeling with Algebra

One Day Workshop: Ford Chicago Assembly Plant, March 21, 2018

Lead Instructors: Kenneth Chelst and Anna Barnett (Chicago Public Schools)

kchelst@wayne.edu and abarnett6@cps.edu

Wayne State University, Detroit, MI

| Start Time | **Wednesday, March 21, 2018 – High School Math Teachers** |
|---|---|
| 8:00 | BREAKFAST AVAILABLE IN CLASSROOM |
| 8:30 | Introduction to Operations Research and Mathematical Analytics |
| 8:45 | Multi-objective decisions (MAUT) Wireless Plan (Ch 1.1) |
| 10:10 | BREAK (20 minutes) |
| 10:30 | Lego And Systems of Linear inequalities |
| 11:00 | LP example Product Mix - Max Profits (Ch 2.2) |
| 11:40 | Ford Analytics and Manufacturing Management – Q&A |
| 12:10 | LUNCH (40 minutes) |
| 12:50 | Linear Programming with Excel Solver |
| 1:40 | Binary Decision Variables (Ch 6.1) |
| 2:20 | Closing Discussions |
| 2:30 | End |

## APPENDIX B: PRE-WORKSHOP QUESTIONNAIRE ITEMS

Demographic/background

1. Unique identifier

   All answers will be kept anonymous, as no personal identifiers will be placed on the surveys except a unique 5-digit code that you will create. Follow the instructions below on how to create your unique code.

   - First digit is the first letter of your last name.
   - Second and third two digits is the day of the month you were born.
   - Fourth digit denotes gender.
     - 1 = Male
     - 2 = Female
   - Last digit is how many siblings you have.
     - 0 = no other siblings
     - 1 = 1 sibling
     - 2 = 2 siblings, etc

   Example 5-digit code: Jane Smith was born on July 8th and has 3 siblings. Her code is:

   S 08 2 3

What is your unique 5-digit code?

2. Undergraduate major
    a. Mathematics – applied
    b. Mathematics – computational
    c. Mathematics – education
    d. Mathematics – pure
    e. Mathematics – statistics
    f. Business
    g. Engineering
    h. English
    i. Fine or performing arts
    j. Foreign language
    k. Science
    l. Social studies
    m. Other (please specify)

3. Undergraduate minor

    a. Mathematics – applied
    b. Mathematics – computational
    c. Mathematics – education
    d. Mathematics – pure
    e. Mathematics – statistics
    f. Business
    g. Engineering
    h. English
    i. Fine or performing arts
    j. Foreign language
    k. Science
    l. Social studies
    m. None (please specify)
    n. Other

4. Master’s degree

    a. Education – administration
    b. Education – counseling
    c. Education – curriculum and instruction
    d. Education – mathematics
    e. Education – technology

f. Business

g. Engineering

h. Mathematics

i. Science

j. None

k. Other (please specify)

5. Years teaching experience

    a. 0 to 4 years

    b. 5 to 9 years

    c. 10 to 14 years

    d. 15 to 19 years

    e. 20 to 24 years

    f. 25 to 29 years

    g. 30 or more years

6. Other professional experience that has informed your mathematics teaching (check all that apply)
    a. Accounting
    b. Banking
    c. Construction
    d. Engineering
    e. Insurance
    f. Management
    g. Sales
    h. Scientific research
    i. Tutoring
    j. None
    k. Other (please specify)

7. Levels of mathematics taught (check all that apply)

    a. Middle school
    b. Pre-algebra
    c. Algebra 1 or Integrated 1
    d. Geometry or Integrated 2
    e. Algebra 2 or Integrated 3
    f. Pre-calculus
    g. Calculus
    h. Statistics
    i. College

8. Which best describes the setting of your school?

    a. Rural
    b. Suburban
    c. Urban

9. Which best describes your school?

    a. Private – parochial
    b. Private – secular
    c. Public – charter
    d. Public – comprehensive
    e. Public – magnet

10. At the conclusion of this workshop, will you be the only teacher at your school to have participated in this training?
    a. Yes
    b. No

11. I am attending the WWIEUT workshop mostly because _____.
    a. I want to know more about how mathematics is used in the world outside of school
    b. I am looking for resources to support spending more instructional time on applied mathematics
    c. I am always looking for ways improve my teaching
    d. of a mandate from my department, building, or district
    e. of the continuing education credits offered
    f. of the stipend (if applicable)
    g. I did not attend the workshop.
    h. Other (please specify)

Intervention-related items

12. I believe the algebra I teach is relevant to about _____ of my students' educational goals.

    a. 0-10%
    b. 10-25%
    c. 25-50%
    d. 50-75%
    e. 75-100%

13. This belief is based mostly on _____.

    a. feedback from former students
    b. firsthand educational experience
    c. relevant data on current trends in skills valued by employers
    d. traditional values related to teaching academic mathematics
    e. other (please specify)

14. I believe the algebra I teach will be used by _____ of my students in their careers.

    a. 0-10%
    b. 10-25%
    c. 25-50%
    d. 50-75%
    e. 75-100%

15. This belief is based mostly on _____.

    a. feedback from former students
    b. firsthand work experience outside of education
    c. current trends in employment data
    d. traditional values related to teaching academic mathematics
    e. other (please specify) __________________

16. About how often does a question like, "When will I ever use this?" get asked in your algebra classes?

    a. Daily
    b. Weekly
    c. Monthly
    d. Practically never

17. When students ask about the relevance of algebra to their lives, my typical response _____.

    a. focuses on how my instruction will prepare them for future mathematics learning
    b. portrays algebra as useful to specific professions
    c. demonstrates the authentic and broad utility of algebra
    d. other (please specify) __________________

18. When students ask about the relevance of algebra to their lives outside of school, my typical response is mostly _____.

    a. based on hope and/or faith that it actually will be relevant
    b. based on a belief that math is all around us in our everyday lives
    c. affirmed by my knowledge of how professionals use mathematics

19. When students ask about the relevance of algebra to their lives outside of school, most of my students are _____ my usual response.

    a. convinced by
    b. ambivalent about
    c. cynical about

20. During algebra instruction, I typically spend ____ on real-world applications.

    a. less than 1 class period per month
    b. 1 to 2 class periods per month
    c. 3 to 4 class periods per month
    d. more than 4 class periods per month

21. During algebra instruction, I would like to spend ____ on real-world applications.

   a. less than 1 class period per month

   b. 1 to 2 class periods per month

   c. 3 to 4 class periods per month

   d. more than 4 class periods per month

22. The amount of instructional time I devote to real-world applications of algebra has _____ over time.

   a. increased

   b. remained nearly constant

   c. decreased

23. My knowledge of how algebra is used to model problems in the world outside of school is _____.

   a. less than adequate

   b. adequate

   c. more than adequate

24. My knowledge of how algebra is used to model problems in the world outside of school comes mostly from _____. (Choose up to 3.)

    a. my academic coursework in mathematics, science, or engineering
    b. my academic coursework in education
    c. professional development during my teaching career
    d. work experience outside of teaching
    e. personal research
    f. professional collaboration
    g. other (please specify)

25. I believe my knowledge of how algebra is used as a modeling tool by professionals is _____.

    a. insufficient
    b. sufficient, because such knowledge is not necessary for algebra instruction
    c. sufficient, because I know a great deal

26. High school algebra courses would be _____ valuable to most students if more instructional time was devoted to real-world applications of algebra.

    a. more
    b. equally
    c. less

27. I agree most with the statement that algebra is _____.

a. more important for my students' educational goals than for their future careers
b. more important for my students' future careers than for their educational goals
c. equally important for my students' educational goals and for their future careers

For the statements in items 28-36, rate your level of agreement with each statement.

28. Algebra as a discipline is relevant outside of school.

a. Strongly agree
b. Agree
c. Neutral
d. Disagree
e. Strongly disagree

29. I understand how working professionals use algebra as a decision making tool.

a. Strongly agree
b. Agree
c. Neutral
d. Disagree
e. Strongly disagree

30. I use algebra to help make important decisions outside of school.

a. Strongly agree
b. Agree
c. Neutral
d. Disagree
e. Strongly disagree

31. I know which professions utilize algebra and which do not.

a. Strongly agree
b. Agree
c. Neutral
d. Disagree
e. Strongly disagree

32. My classroom instruction demonstrates the usefulness of algebra outside of school.

a. Strongly agree
b. Agree
c. Neutral
d. Disagree
e. Strongly disagree

33. To succeed in their future education, it is important for my students to understand algebra.

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

34. To succeed in their eventual careers, it is important for my students to understand algebra.

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

35. The training I received prior to becoming a teacher provided me with enough knowledge about how algebra is used in the world outside of school.

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

36. The professional development I've received since becoming a teacher has provided me with enough knowledge about how algebra is used in the world outside of school.

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

For the story problems in items 37-40, rate your level of agreement with the statement:

> "This problem represents an application of mathematics akin to what my students may encounter in their future employment."

37. For a school concert the ratio of tickets sold to parents, children, and faculty is 25 : 20 : 3. If 240 tickets are sold, how many tickets are sold to each group?

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

38. A young channel catfish weighs about 0.1 pound. During the next 8 weeks, its weight increases by about 23% each week. About how much will the catfish weigh after 4 weeks?

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

39. A passenger train averaging 62 mi/h begins the 355-mi trip from Glenville to Lintown at 12:00 noon. A freight train traveling 48 mi/h leaves Lintown at 2:00 p.m. the same day and travels to Glenville on an adjacent track. At what time will the two trains pass each other?

a. Strongly agree
b. Agree
c. Neutral
d. Disagree
e. Strongly disagree

40. How much larger is the volume of a cube with dimensions $(x+3)$ than the volume of a cube with dimensions $(x+2)$?

a. Strongly agree
b. Agree
c. Neutral
d. Disagree
e. Strongly disagree

## APPENDIX C: POST-WORKSHOP QUESTIONNAIRE ITEMS

Demographic/background

1. Unique identifier

   All answers will be kept anonymous, as no personal identifiers will be placed on the surveys except a unique 5-digit code that you will create. Follow the instructions below on how to create your unique code.

   - First digit is the first letter of your last name.
   - Second and third two digits is the day of the month you were born.
   - Fourth digit denotes gender.
     - 1 = Male
     - 2 = Female
   - Last digit is how many siblings you have.
     - 0 = no other siblings
     - 1 = 1 sibling
     - 2 = 2 siblings, etc

   Example 5-digit code: Jane Smith was born on July 8th and has 3 siblings. Her code is:

   S 08 2 3

What is your unique 5-digit code?

Intervention-related items

2. I believe the algebra I teach is relevant to about _____ of my students' educational goals.
    a. 0-10%
    b. 10-25%
    c. 25-50%
    d. 50-75%
    e. 75-100%

3. This belief is based mostly on _____.
    a. feedback from former students
    b. firsthand educational experience
    c. relevant data on current trends in skills valued by employers
    d. traditional values related to teaching academic mathematics
    e. other (please specify)

4. I believe the algebra I teach will be used by _____ of my students in their careers.

    a. 0-10%

    b. 10-25%

    c. 25-50%

    d. 50-75%

    e. 75-100%

5. This belief is based mostly on _____.

    a. feedback from former students

    b. firsthand work experience outside of education

    c. current trends in employment data

    d. traditional values related to teaching academic mathematics

    e. other (please specify) ___________________

6. About how often does a question like, “When will I ever use this?” get asked in your algebra classes?

    a. Daily

    b. Weekly

    c. Monthly

    d. Practically never

7. When students ask about the relevance of algebra to their lives, my typical response _____.

   a. focuses on how my instruction will prepare them for future mathematics learning

   b. portrays algebra as useful to specific professions

   c. demonstrates the authentic and broad utility of algebra

   d. other (please specify) __________________

8. When students ask about the relevance of algebra to their lives outside of school, my typical response is mostly _____.

   a. based on hope and/or faith that it actually will be relevant

   b. based on a belief that math is all around us in our everyday lives

   c. affirmed by my knowledge of how professionals use mathematics

9. When students ask about the relevance of algebra to their lives outside of school, most of my students are _____ my usual response.

   a. convinced by

   b. ambivalent about

   c. cynical about

10. During algebra instruction, I typically spend ____ on real-world applications.

   a. less than 1 class period per month
   b. 1 to 2 class periods per month
   c. 3 to 4 class periods per month
   d. more than 4 class periods per month

11. During algebra instruction, I would like to spend ____ on real-world applications.

   a. less than 1 class period per month
   b. 1 to 2 class periods per month
   c. 3 to 4 class periods per month
   d. more than 4 class periods per month

12. The amount of instructional time I devote to real-world applications of algebra has _____ over time.

   a. increased
   b. remained nearly constant
   c. decreased

13. My knowledge of how algebra is used to model problems in the world outside of school is _____.

    a. less than adequate
    b. adequate
    c. more than adequate

14. My knowledge of how algebra is used to model problems in the world outside of school comes mostly from _____. (Choose up to 3.)

    a. my academic coursework in mathematics, science, or engineering
    b. my academic coursework in education
    c. professional development during my teaching career
    d. work experience outside of teaching
    e. personal research
    f. professional collaboration
    g. other (please specify)

15. I believe my knowledge of how algebra is used as a modeling tool by professionals is _____.

    a. insufficient
    b. sufficient, because such knowledge is not necessary for algebra instruction
    c. sufficient, because I know a great deal

16. High school algebra courses would be _____ valuable to most students if more instructional time was devoted to real-world applications of algebra.

    a. more
    b. equally
    c. less

17. I agree most with the statement that algebra is _____.

    a. more important for my students' educational goals than for their future careers
    b. more important for my students' future careers than for their educational goals
    c. equally important for my students' educational goals and for their future careers

For the statements in items 18-26, rate your level of agreement with each statement.

18. Algebra as a discipline is relevant outside of school.

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

19. I understand how working professionals use algebra as a decision making tool.

a. Strongly agree
b. Agree
c. Neutral
d. Disagree
e. Strongly disagree

20. I use algebra to help make important decisions outside of school.

a. Strongly agree
b. Agree
c. Neutral
d. Disagree
e. Strongly disagree

21. I know which professions utilize algebra and which do not.

a. Strongly agree
b. Agree
c. Neutral
d. Disagree
e. Strongly disagree

22. My classroom instruction demonstrates the usefulness of algebra outside of school.

   a. Strongly agree
   b. Agree
   c. Neutral
   d. Disagree
   e. Strongly disagree

23. To succeed in their future education, it is important for my students to understand algebra.

   a. Strongly agree
   b. Agree
   c. Neutral
   d. Disagree
   e. Strongly disagree

24. To succeed in their eventual careers, it is important for my students to understand algebra.

   a. Strongly agree
   b. Agree
   c. Neutral
   d. Disagree
   e. Strongly disagree

25. The training I received prior to becoming a teacher provided me with enough knowledge about how algebra is used in the world outside of school.

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

26. The professional development I've received since becoming a teacher has provided me with enough knowledge about how algebra is used in the world outside of school.

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

For the story problems in items 27-30, rate your level of agreement with the statement:

> "This problem represents an application of mathematics akin to what my students may encounter in their future employment."

27. For a school concert the ratio of tickets sold to parents, children, and faculty is 25 : 20 : 3. If 240 tickets are sold, how many tickets are sold to each group?

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

28. A young channel catfish weighs about 0.1 pound. During the next 8 weeks, its weight increases by about 23% each week. About how much will the catfish weigh after 4 weeks?

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

29. A passenger train averaging 62 mi/h begins the 355-mi trip from Glenville to Lintown at 12:00 noon. A freight train traveling 48 mi/h leaves Lintown at 2:00 p.m. the same day and travels to Glenville on an adjacent track. At what time will the two trains pass each other?

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

30. How much larger is the volume of a cube with dimensions $(x+3)$ than the volume of a cube with dimensions $(x+2)$?

    a. Strongly agree
    b. Agree
    c. Neutral
    d. Disagree
    e. Strongly disagree

31. Please indicate what materials from the When Will I Ever Use This? Workshop you have used in your classroom. (Check all that apply.)

    a. Multi-criteria decision making
    b. Linear programming – maximization
    c. Sensitivity analysis
    d. Linear programming – minimization
    e. Integer programming
    f. Binary programming
    g. Location problems
    h. Queueing theory
    i. Project management
    j. None

32. If you have used some of the curricular materials, please explain why you chose to use them.

33. If you have not used any of the curricular materials, please explain why you chose not to use them.

## APPENDIX D: SEMI-STRUCTURED INTERVIEW GUIDE

A. Knowledge

1. Why do you think it’s important for high school students to study algebra?

   Prompt: For other math classes? To get into college? For their career? For citizenship?

2. How do you usually respond when a student asks, “When am I ever going to use this?”

   Follow-up: How do students typically react to your response?

3. Can you tell me the types of careers that your students might pursue in which they would use algebra?

   Follow-up: Why did you choose those careers?

4. Could you describe how you learned about the ways math is used outside of school?

   Prompt: In undergraduate classes? Graduate classes? Other work experience? PD while you’ve been teaching? Things you’ve read?

   Follow-up: When did you learn that?

B. Belief

5. How would you describe what mathematics is?

   Prompt: What is it all about? Why do we teach it?

Follow-up: How is that related to why you became a math teacher?

6. Why do you think students might feel disengaged in algebra lessons?

7. What are your thoughts on the relevance of the algebra curriculum to the lives of your students?

    Follow-up: What makes you think that?

8. What's your understanding of how algebra is used outside of school?

    Follow-up: Do you think that's changed in the past 10-20 years?

C. Practice

9. Could you describe a typical algebra lesson?

10. Tell me about the last time you made a real-world connection during an algebra lesson.

11. Where do you find real-world applications to include in your algebra instruction?

12. How do you feel about the quality and quantity of the real-world applications you use in your algebra lessons?

## APPENDIX E: QUALITATIVE CONTENT ANALYSIS CODEBOOK

AP_Algebra curriculum is not relevant to students' lives

AP_Algebra curriculum is relevant to students' lives

AP_Algebra develops logic, reasoning, and thinking skills

AP_Algebra develops mental fitness

AP_Algebra develops problem solving skills

AP_Algebra is a building block for higher mathematics

AP_Algebra is important for personal financial decision-making

AP_Algebra is important to model the real world

AP_Algebra is not relevant to a wide range of careers

AP_Algebra is relevant to a broad range of careers

AP_Algebra is relevant to particular careers

AP_Algebra is useful in the world outside of school (Belief)

AP_Algebra skills are important as gatekeeper to higher education

AP_Algebraic techniques can be used in everyday life

AP_Algebraic techniques not used in everyday life

AP_Became a math teacher because of love of math

AP_Became a math teacher because of math ability

AP_Became a math teacher to help children learn

AP_Became teacher to help students be successful in careers

AP_Instructional focus frequently includes context/application

AP_Instructional focus is primarily algebraic skills and/or concepts without context/application

AP_Knowledge of how algebra is used in various careers comes from casual interaction with STEM professionals

AP_Knowledge of how algebra is used in various careers comes from casual interactions with other professionals by happenstance

AP_Knowledge of how algebra is used in various careers comes from college/univeristy coursework

AP_Knowledge of how algebra is used in various careers comes from prior professional experience

AP_Knowledge of how algebra is used in various careers comes from professional development

AP_Knowledge of math outside of school came from prior STEM professional experience

AP_Knowledge of math outside of school came from undergraduate math coursework

AP_Knowledge of which careers use algebra comes from casual interactions with other professionals by happenstance

AP_Knowledge of which careers use algebra comes from prior professional experience

AP_Math is a (technical/procedural) skill set

AP_Math is a language

AP_Math is about critical thinking

AP_Math is about logic

AP_Math is about patterns

AP_Math is about problem solving

AP_Math is relevant because it is part of formal education

AP_Prescriptive curriculum and limited time does not allow for real-world connections to be explored

AP_Quality of real-world applications in algebra instruction is adequate

AP_Quality of real-world applications in algebra instruction is inadequate

AP_Quantity of real-world applications in algebra instruction is adequate

AP_Quantity of real-world applications in algebra instruction is inadequate

AP_Real-world connections are niche or contrived and do not reflect how algebra is used in a broad range of careers

AP_Source of real world applications in instruction is colleagues

AP_Source of real world applications in instruction is prior professional experience

AP_Source of real world applications in instruction is professional development

AP_Source of real world applications in instruction is textbook

AP_Source of real world applications in instruction is the internet

AP_Students are disengaged in algebra due to issues in their personal lives interfering with schooling

AP_Students are disengaged in algebra due to lack of foundational skills and knowledge

AP_Students are disengaged in algebra lessons because lessons are rote/mimicry

AP_Students are disengaged in algebra lessons due to perceived irrelevance

AP_Students mostly believed teacher's response to WWIEUT

AP_Students mostly did not believe teacher's response to WWIEUT

AP_Teacher does not know what students will be asked to do in careers

AP_Teacher interperting "outside of school" as "everyday life" as opposed to "in eventual careers"

AP_Technological changes in the last 10 to 20 years make calculating easier

AP_The way algebra is taught is not how it is used outside of school

AP_The way algebra is used outside of school has changed in the last 10 to 20 years

AP_The way algebra is used outside of school has not changed in the last 10 to 20 years

AP_When asked about last time a real-world connection was taught in algebra, teacher cites a non-algebraic topic

D_Activities built around how teacher used math in previous professional experience

D_Algebra curriculum is not inherently relevant, but teachers can make connections

D_Algebra is important because it’s useful in the real world

D_Algebra is important to model decision making

D_Amount of real world connections in instruction has decreased

D_Applications of math are everywhere

D_Became a math teacher for job security

D_Became a math teacher to show students that math is useful

D_Blue collar jobs need pre-algebra skills (measurement, ratios, proportions, etc.)

D_Data analysis and visualizations are important for careers in government

D_Different students need different math curricula

D_Graduate coursework in education did not address how math is used outside of school

D_Instruction may include story problems at end of lesson

D_Knowledge of how algebra is used in various careers comes from construction experience

D_Knowledge of how algebra is used outside of school comes from personal life (e.g., personal finance)

D_Knowledge of how algebra is used to solve real-world problems came from operations research course

D_Knowledge of how math is used outside of school came from articles

D_Knowledge of how math is used outside of school came from science classes

D_Knowledge of how math is used outside of school came from word problems in high school textbooks

D_Knowledge of math outside of school from reports of former students

D_Last real-world connection in algebra was about finance

D_Math is about modeling

D_Math is about organization/bookkeeping

D_Math is about understanding the world

D_Math is beautiful and should be appreciated

D_Math is black and white (one right answer)

D_Math is important for number sense

D_Math is the study of numbers

D_Need to understand data analysis has changed in past 10 to 20 years

D_Other professionals report using geometry not algebra in careers

D_Real problems don't have one right answer

D_Real-world connection in instruction: optimization

D_Real-world connections in instruction drawn from local economy/context

D_Real-world example from instruction involved modeling

D_Relevant applications are difficult to find and are not part of curricular materials

D_Service jobs require arithmetic

D_Source of real world applications is current events

D_Students are disengaged in algebra because it is difficult

D_Students are disengaged in algebra because teachers lack CK and PCK

D_Students are disengaged in algebra because they cannot relate to math (no discussion)

D_Students are disengaged in algebra because they don't care and just want to graduate

D_Students are engaged in algebra lessons when teacher makes content relevant

D_Students disengaged in math due to low confidence

D_Students do not frequently ask WWIEUT

D_Students frequently ask WWIEUT

D_Students use informal algebra in their everyday lives

D_Students will appreciate the relevance of algebra curriculum when they apply it in a career

D_Teacher acknowledges never using mathematics outside of academics

D_Teacher describes how algebra is used in many careers

D_Teacher dissatisfied with his/her knowledge of how algebra is used in a wide range of careers

D_Teacher does not know which careers use algebra

D_Teacher has never considered nature of mathematics

D_Teacher makes real world connection to all topics covered in class

D_Teacher not sure how the use of algebra outside of school has changed in the past 10 to 20 years

D_Teacher tries to make connections to students' interest to help them see algebra as relevant

D_Teacher uses measurement to provide context for concepts (e.g., stairs and roofs for slope)

D_Teacher uses WWIEUT textbooks to answer WWIEUT question from students

D_Teacher's formal education did not provide knowledge about how math is used outside of school

D_Technological advances in last 10 to 20 years have enabled students to solve bigger problems more creatively

D_When answering WWIEUT?, teacher uses non-algebraic example

D_When asked about careers that use algebra, teacher cites non-algebraic examples

D_When asked about relevance of curriculum, teacher does not address relevance to life outside of school

Note: AP designates a piori codes and D designates data-driven codes

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## ABSTRACT

### MAKING REAL-WORLD CONNECTIONS IN HIGH SCHOOL MATHEMATICS: THE EFFECTIVENESS OF A PROFESSIONAL DEVELOPMENT PROGRAM IN CHANGING TEACHERS' KNOWLEDGE, BELIEFS, AND PRACTICES

by

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**December 2020**

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The study aimed to assess the impact of a professional development workshop at changing secondary mathematics teachers' knowledge, beliefs, and practices related to real-world applications of algebra. It also addressed gaps in the research literature related to teacher knowledge of how algebra is used by professionals in non-academic settings and their beliefs about the relevance of algebra to their students' lives. The observational study employed mixed methods. Principal components analysis was conducted on responses to an online questionnaire. Pre-test vs. post-test comparisons were made for workshop participants. Treatment vs. control comparisons were also made using a nationally representative random sample of secondary mathematics teachers as the control group. Results from the statistical analysis of the quantitative data showed the professional development program had a positive impact on teachers' knowledge and practice related to real-world connections of algebra. Results from qualitative content analysis of semi-structured interview transcripts with treatment and control teachers revealed that teachers have limited knowledge of how algebra is used in the world outside of school. Further research is needed

to better understand teacher knowledge about applications of the mathematics they teach, teacher beliefs about the relevance of their curricula, the impact these have on their instruction.

## AUTOBIOGRAPHICAL STATEMENT

**Education**

Wayne State University, Detroit, MI
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