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Teachers' Reflection On Their Beliefs And Question-Asking Practices During Mathematics Instruction

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TEACHERS' REFLECTION ON THEIR BELIEFS AND QUESTION-ASKING PRACTICES DURING MATHEMATICS INSTRUCTION

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DEDICATION

This work is dedicated to everyone in my village who has inspired me, pushed me, supported me, and made it possible for me to pursue and achieve this goal.

I thank my parents, Kenneth and Eve Davis, for always making me feel smart and always believing in my ability to conquer any task before me. Thank you for instilling high academic standards and a persistent work ethic in me. Your excitement about reaching this moment was often the nudge I needed to work a few more hours.

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CHAPTER 1 INTRODUCTION

Background

Mathematics teachers have the responsibility of facilitating students' critical thinking skills and guiding them in developing the mathematical practices described in the Common Core State Standards (National Governors Association, 2010). The rigorous standards, coupled with the sophisticated mathematical practices, require students' educational experiences to be rich and meaningful. Thus, a teacher's daily instructional practices are critical. Many factors contribute to the complex nature of teaching mathematics. Teachers bring a complicated web of knowledge, including pedagogical and content knowledge, along with their individual perceptions, assumptions, and expectations to their teaching environment (Nespor, 1987). Each of these factors, and more, are interwoven in a very sophisticated web of teachers' knowledge and ability and affects their instructional decisions and practices. One very powerful force inherent in mathematics teachers' instructional decisions and practices is their beliefs about teaching and learning mathematics.

Beliefs are powerful and complex mental structures that drive action (Richardson, 1996), affect behavior (Ernest, 1989; Fang, 1996; Thompson, 1992), and frame the cognitive approach to a task (Nespor, 1987). Some beliefs are held so strongly that they are considered to be synonymous with personal knowledge (Kagan, 1992). Beliefs play an essential role in our daily actions and are critical components in the study of individuals' decision-making processes, as they are "stronger predictors of behavior than knowledge" (Pajares, 1992, p. 311). The role of beliefs on behavior has piqued the interest of education researchers who seek to understand the instructional decisions made by teachers. However, understanding teachers' beliefs requires an understanding of how their beliefs about teaching and learning are developed.

Teachers begin to develop beliefs about mathematics, teaching, and learning during their years as grade school students through what Lortie (1975) called their “apprenticeship of observation.” As students, they experienced the delivery of mathematics curriculum and witnessed several instances of teaching as displayed by their teachers. These early experiences affect the internalization of subsequent experiences. Often, the intensity of teachers’ beliefs is the result of years of holding particular beliefs and filtering all subsequent experiences through those beliefs (Philipp, 2007). Due to the profound effect of beliefs on behavior, teachers’ beliefs about mathematics teaching and mathematics learning have been studied in order to make connections between beliefs and instructional practices. One instructional practice of interest is question-asking.

Questioning is a widely used instructional tool; one that may be overlooked due to its prevalent use by teachers during instruction. Questions have a tremendous impact on students’ learning experience. Weiland, Hudson, and Amador (2014) stated, "The opportunities children have to learn are directly impacted by the questions they are asked" (p. 332). In mathematics education, questions can allow students to demonstrate retention of facts and procedures, encourage thinking, and assist in students’ development of understanding by prompting students to describe, explain, explore, justify, and investigate. Teachers also benefit from questioning because students’ responses uncover what they are thinking and inform teachers of their educational needs. Knowledge of students’ educational needs steers instruction. Kazemi and Franke (2004) discovered shifts in teachers’ instructional trajectories upon learning of their students’ mathematical thinking. The impact of questioning on students' learning experiences makes it imperative for teachers to consider the role of their own beliefs on the questions they ask during mathematics instruction. Recognizing the role beliefs play in their questioning contributes to an understanding of the interactions their students will have with mathematics under their tutelage. One way to become aware of this effect is through reflection.

Problem Statement

Reflection is a tool that brings particular aspects of inquiry to the forefront. Educators use reflection to focus on specific nuances of their teaching practice. In doing so, they are able to concentrate on developing professionally in a way that will benefit students. Reflecting on one's practice is an essential characteristic of a professional practitioner, whom Schön (1983) described as "a specialist who encounters certain types of situations again and again" (p. 60). Mathematics teachers are specialists, in that they often deliver the same content multiple times a day and year after year. Teaching the same content regularly can lend itself to instructional practices that become so repetitive that they seem second-nature. Asking questions is a repetitive instructional practice that, due to its extensive use, may go without analysis or critique. Teachers may not think, reflectively, about the questions they ask nor how their questioning mirrors their beliefs about how mathematics should be learned and taught. However, Schön proposed that a teacher who reflects on his or her own practice could bring to light some of the deeply-rooted and repetitive actions that he or she engages in during practice.

The reflective process promotes what Feiman-Nemser (2001) believed to be one of the central tasks of teacher induction: developing a professional identity. Reflecting on teaching and learning "is the best way to develop teachers' professional way of thinking" (Slavik as cited in Ticha & Hospesova, 2006, p. 133) and is a more effective method for professional knowledge retention than other forms of learning (Mustafa, 2005). The knowledge gained from reflection is most valuable when it is used to improve practice (Jansen & Spitzer, 2009). Reflection allows a practitioner to be open to unique approaches and understandings. By reflecting and discussing their beliefs on teaching and learning, educators construct their professional identity.

Although reflection contributes to professional development, many teachers do not formally or proactively engage in the process, especially regarding the questions they ask during mathematics instruction. The process of reflection can be long, arduous, and difficult to carry out if it has not been modeled. Additionally, when teachers observe or analyze their teaching, they tend to reflect by listing or describing aspects of the teaching or moments within the lesson, without recognizing or accounting for the connections between their actions and the resulting learning experiences of students (Jansen & Spitzer, 2009). Reflection requires more than a recount of teaching, but also an interpretation of teaching in relation to teachers' assumptions, perceptions, and instructional goals.

Research Objectives and Questions

The purpose of this study is to engage in a cooperative reflective process with teachers in order to study their beliefs about mathematics teaching and learning in relation to the questions they ask during mathematics instruction. The following research questions framed this study:

1. What are teachers' question-asking practices during mathematics instruction?

   1a. What reasons do teachers provide for their question-asking practices during mathematics instruction?

2. What relationship exists, if any, between teachers' beliefs about mathematics teaching and learning and the reasons they provide for their question-asking practices during mathematics instruction?

3. What impact does reflection on question-asking practices have on teachers' thinking about the mathematics questions they ask during instruction?

Definitions of Terms

The following operational terms are used in this study:

Beliefs are teachers' assumptions about students, teaching, learning, and subject matter (Kagan, 1992).

Questions are requests for information or action (Kawanaka & Stigler, 1999).

Mathematics Questions are questions (spoken or written) that contain mathematics content and that elicit responses from students in the form of information or action.

High-Level Questions are questions that require students to reason, describe, explain, justify, analyze, or investigate (Booth, Lange, Koedinger, & Newton, 2013; Crespo & Sinclair, 2008; Sullivan & Clark, 1992; Weiland et al., 2014).

Low-Level Questions are questions that are leading or that require students to name, state, memorize, or recall. These types of questions also allow students to provide yes/no, technical, factual, or simple responses (Aizikovitsh-Udi & Star, 2011; Kawanaka & Stigler, 1999; Sahin & Kulm, 2008).

Mathematics Instruction is the delivery of mathematical content through presentation, practice, feedback, and assessment. (Stepich, Chyung, & Smith-Hobbs, 2009).

Question-Asking Practice is the customary usage of questioning during mathematics instruction.

Reflection is a teacher's systematic approach for inquiring into particular aspects of his or her own teaching. The approach involves a teacher collecting data on some aspect of teaching, analyzing the data, making connections to his or her own experiences and perceptions, and making future instructional decisions based on their analysis. (Friel, Hart, Schultz, Najee-ulla, & Nash, 1992; Richards & Lockhart, 1994).

Significance of the Study

This study contributes to investigations into the relationship between teachers' beliefs and their instructional practices. However, a significant characteristic of this study is inviting teachers to engage in the reflective process, allowing them to act as reflective practitioners taking a critical look at the questions they ask students during mathematics instruction. By reflecting on the specific practice of question-asking, the pervasiveness of beliefs that inform this widely used instructional practice can be better understood. Teachers' authentic analysis of their question-asking practices in relation to their beliefs about teaching and learning mathematics creates an awareness of their own experiences and helps them discover links to their actions. With an awareness of the impact of their beliefs and question-asking on students' learning experiences, teachers can use this study as a model for being thoughtful and reflective about their questioning practices, thus increasing their instructional effectiveness. This study gave teachers a voice in the research. By reflecting on their own beliefs and actions, teachers acted as researchers of their own practice (Schön, 1983) and played a contributing role in research conducted in their classrooms.

CHAPTER 2 LITERATURE REVIEW

The following review of literature summarizes teachers' beliefs about mathematics teaching and learning, the use of questioning in mathematics classrooms, and teachers' use of reflection as a method to evaluate and improve their teaching. Teachers' beliefs about the nature of mathematics, mathematics teaching, and mathematics learning is described, followed by relationships between teachers' beliefs and instruction. The prevalence of questioning in mathematics classrooms is explored by first considering the historical nature of using questioning to acquire knowledge and the impact education reform has had on the goal of questioning in mathematics instruction. Descriptions of low-level and high-level questions are followed by the significance of question-asking in the classroom. The pervasive use of the word "problems" to describe mathematics questions is discussed and a distinction is made between questions that promote problem-solving skills and questions that allow students to exercise procedural skills. The next section of this chapter focuses on reflection. A general description of reflection leads to a discussion of reflection for teaching. Ways to engage in teacher reflection is followed by the benefits and barriers of teacher reflection. This chapter culminates with a depiction of the conceptual framework that influenced the present study.

Teachers' Beliefs

Teachers' beliefs are comprised of their assumptions about students, teaching, learning, and subject matter (Kagan, 1992). These beliefs are developed during their time as students (Calderhead & Robson, 1991; Lortie, 1975) and during their time as teachers (Nespor, 1987). At an early age, teachers experience teaching and learning in ways that frame how they come to comprehend subsequent experiences with teaching and learning (Nespor, 1987). During their time as educators, beliefs are developed based on their experiences with teaching and reflection on those experiences (Ambrose, 2004). Teachers' experiences as students and as teachers have had a more powerful impact on their beliefs than their time in teacher education programs (Richardson, 1996). Experiences with teaching can overshadow teacher education experiences and knowledge of learning theory.

Many researchers have studied teachers’ beliefs in relation to their classroom behavior (Ambrose, 2004; Beswick, 2012; Cross, 2009; Francis, 2015; Raymond, 1997; Springs, 1999; Stipek, Givvin, Salmon, & MacGyvers, 2001; Sztajn, 2003; Thompson, 1984; Zakaria & Maat, 2012). Teaching is a complex profession that requires teachers to consider content, pedagogy, and learning theory (among other factors) when they create learning environments. Thus, when studying or reporting on teachers’ views, beliefs, or conceptions of the components of their profession, using the phrase “teachers’ beliefs” is too vague (Pajares, 1992). Instead, researchers should indicate the specific beliefs under consideration by stating, “*Teachers’ beliefs about….*” The following discusses teachers’ beliefs about the nature of mathematics, teaching mathematics, and learning mathematics.

Teachers' beliefs about the nature of mathematics. Teachers have wide ranging beliefs about the nature of mathematics. Their beliefs pertain to the processes, rules, and meanings of mathematics as a discipline and as a school subject (Beswick 2012; Thompson, 1992). Ernest (1989) categorized teachers' beliefs into three categories; *Instrumentalist*, *Platonist*, and *Problem-Solving*. An Instrumentalist view perceives mathematics as a static set of rules, facts, and procedures to be learned and applied in order to complete a task. The Platonist view of mathematics focuses on understanding concepts and seeing mathematics as something that is discovered, not created. The Problem-Solving view considers mathematics to be dynamic - a process in which ideas are created and ever-changing. There are no final or correct answers because mathematics is always open for revision.

Beswick (2012) highlighted differences between the mathematical practices that mathematicians employ and the mathematics taught in school. For instance, the discipline of mathematics consists of knowledge being created for the purpose of contributing to general knowledge. The problems under investigation are chosen by the mathematician and are investigated for as long as necessary. In contrast, mathematics as a school subject consists of studying preexisting knowledge for the purpose of personal achievement. The problems under investigation are selected by the teacher and given a fixed amount of time to complete. During her case study of secondary teachers' beliefs about mathematics as a discipline and as a school subject, Beswick (2012) found that more experienced teachers had differing views of mathematics as a discipline compared to their views of mathematics as a school subject. Less experienced teachers had similar views about the two types of mathematics. Teachers' beliefs about the nature of mathematics influence their beliefs about teaching and their beliefs about learning mathematics (Ernest, 1989; Thompson, 1992).

Teachers' beliefs about mathematics teaching. Teachers' beliefs about mathematics impact their practice (Ernest, 1989; Fang, 1996). In addition to categorizing three views of the nature of mathematics, Ernest (1989) also noted the impact of these beliefs on teaching and learning. Teachers holding an Instrumentalist view of mathematics act as instructors who strictly use the text. Their instructional goal is mastery shown by correct performance. Teachers holding a Platonist view act as explainers. They modify the text and supplement with additional activities. Their instructional goal is conceptual understanding. Teachers holding the Problem- Solving view act as facilitators. They construct the curriculum with the instructional goal of developing problem posers and problem solvers.

Teachers' beliefs about mathematics teaching impacts their instructional approaches. Nisbet and Warren (2000) performed a quantitative study with 398 primary school teachers and found that the number of teachers with a contemporary view of teaching, described as a constructivist approach, increased as grade level increased. Thompson (1984) described two orientations of mathematics teaching; calculational orientation and conceptual orientation. Teachers with a calculational orientation teach mathematics as the application of rules and procedures to obtain numerical results. Exercises are approached with a focus on getting the correct numerical answer. Teachers with a conceptual orientation provide activities that allow students to be intellectually engaged. They approach problem-solving by identifying the important concepts within the problem.

Ernest (1989) described models of teaching that correspond with the varying views of the nature of mathematics mentioned above. The models of teaching address teachers' instructional goals as well as the teaching actions and classroom activities that comprise their teaching style. Problem-solving, problem-posing, conceptual understanding, and mastery of skills and facts are some of the academic outcomes that motivate the way mathematics is taught. Differences in the selection of learning tasks, use of curriculum materials, and treatment of students' errors are some of the instructional decisions that distinguish the different models of teaching. Ernest's (1989) simplified models of teaching include:

- the pure investigational, problem posing, and solving model,
- the conceptual understanding enriched with problem solving model,
- the conceptual understanding model,

- the mastery of skills and facts with conceptual understanding model,
- the mastery of skills and facts model, and
- the day to day survival model.

Models of teaching that produce these outcomes are greatly influenced by teachers' Problem-Solving, Platonist, and Instrumentalist beliefs about the nature of mathematics.

Teachers' beliefs about mathematics learning. Teachers' beliefs about learning influence the classroom environment. Processes for learning are compatible with teachers' views about mathematics and mathematics teaching (Ernest, 1989). For instance, in an Instrumentalist classroom, students are compliant and focused on mastery of skills. In Platonist classrooms, students are receivers of knowledge. In Problem-Solving classrooms, students explore and construct knowledge autonomously. Evidence of understanding varies in these classrooms. Skemp (1978) acknowledged that there are two meanings of *understanding* mathematics: relational and instrumental understanding. A relational understanding of mathematics means a student understands procedures and concepts. A student with this type of understanding knows "both what to do and why" (Skemp, 1978, p. 20). This is related to Ernest's (1989) description of a Platonist view of mathematics. An instrumental understanding of mathematics means a student knows mathematical rules and has the ability to use them. Skemp (1978) describes this type of understanding as "rules without reasons" (p. 20).

In addition to describing models of teaching mathematics, Ernest (1989) also depicted models of learning mathematics, which involve teachers' perceptions of the mental processes, behaviors, and activities that contribute to learning mathematics. Exploration, autonomy, constructing understanding, mastery of skills, and compliant behavior are some of the views of learning that inform a student's role in the mathematics classroom. Differences in how knowledge is acquired and the role of the learner distinguish the different models of learning. Ernest's (1989) simplified models of learning include:

- child's exploration and autonomous pursuit of own interests model,
- child's constructed understanding and interest driven model,
- child's constructed understanding driven model,
- child's mastery of skills model,
- child's linear progress through curricular scheme model, and
- child's compliant behavior model.

Teachers' beliefs about learning mathematics impact students' experiences with the subject matter and influences students' future interaction with mathematics (Ernest, 1989). Table 1 displays a comparison of the models of teaching and learning associated with descriptions of Ernest's categories of teacher's beliefs about the nature of mathematics. Two additional categories are included (Instrumentalist/Platonist and Platonist/Problem-Solving) to account for the combination of models of teaching and learning for the specified categories. For instance, the Instrumentalist/Platonist model of teaching combines mastery of skills and facts from the Instrumentalist model of teaching with conceptual understanding from the Platonist model of teaching. Based on Table 1, Ernest associated teachers' roles and instructional goals as well as students' behaviors and learning expectations with different beliefs teachers hold about the nature of mathematics.

Table 1

Models of Teaching and Learning

Nature of mathematics	Model of teaching mathematics	Model of learning mathematics
Instrumentalist Mathematics is a static set of rules, facts and procedures to be learned and applied in order to complete a task.	Teachers act as instructors who strictly use the textbook. Instructional goal is mastery of skills and facts shown by correct performance.	Students are compliant and focused on mastery of skills
Instrumentalist/ Platonist	Mastery of skills and facts with conceptual understanding	Mastery of skills and a linear progression through the curriculum.
Platonist Mathematics focuses on understanding concepts and seeing mathematics as something that is discovered, not created.	Teachers act as explainers who modify the textbook and supplement with additional activities. Instructional goal is conceptual understanding.	Students are receivers of knowledge and they construct their own understanding.
Platonist/ Problem-Solving	Conceptual understanding enriched with problems solving	Students' constructed understanding and interest driven
Problem-Solving Mathematics is dynamic- a process in which ideas are created and ever changing. There is no final answer or correct answers because mathematics is always open for revision.	Teachers act as facilitators who construct the curriculum with pure investigation, problem solving and problem posing. Instructional goal is to develop problem posers and problem solvers.	Students explore and construct knowledge through autonomous pursuit of their own interests.

How are beliefs determined? Determining teachers' beliefs is a difficult task. Pajares (1992) stated, "Belief does not lend itself easily to empirical investigation" (p. 308). This is due to the difficulty people have stating or describing their beliefs (Pajares, 1992). Rokeach (1968) and

Pajares (1992) agree that beliefs must be inferred by what people do, say, and intend. Thus, the existence of particular beliefs is determined by the researcher's interpretation of the teacher's actions, lesson plans, and responses to questions.

Teachers' beliefs and instruction. Teachers' beliefs about the nature of mathematics, mathematics teaching, and mathematics learning have been studied in order to make connections between beliefs and instruction. Researchers have found that beliefs drive action (Richardson, 1996) and affect behavior (Beck, Czerniak, & Lumpe, 2000; Ernest, 1989; Fang, 1996; Giorgi, Roberts, Estepp, Conner, & Stripling, 2013, Thompson 1992). However, studies that have considered classroom practice in relation to beliefs have found no direct relationship between beliefs and practice (Thompson, 1984; Zheng, 2013). Both consistencies and inconsistencies have been shown in teachers' practices.

Consistency. Some studies have shown that there is consistency between teachers' espoused beliefs and their classroom practices (Stipek et al., 2001; Zakaria & Maat, 2012). Stipek et al. surveyed 24 teachers to determine their beliefs about mathematics and teaching. The surveys were compared to subsequent observations of the teachers in the study. They found consistency between teachers' espoused beliefs about mathematics and teaching and the observed classroom practices. Zakaria and Maat (2012) administered the Mathematics Beliefs Questionnaire and Teachers Teaching Practice Questionnaire to 51 teachers to discover that teachers' self-proclaimed beliefs about mathematics were consistent with their self-proclaimed practices. In general, studies have shown consistency between teachers' beliefs about mathematics and their classroom practice rather than consistency between their beliefs about teaching and their classroom practice (Raymond, 1997; Thompson, 1984).

Inconsistency. Many studies have uncovered perceived inconsistencies between teachers' espoused or inferred beliefs and their practices in the classroom. These inconsistencies have been attributed to contextual factors in schools (Francis, 2015; Nespor, 1987; Raymond, 1997), students' needs (Sztajn, 2003; Thompson, 1984), and the research methods employed (Fang, 1996; Francis, 2015; Thompson, 1992). Raymond's (1997) case study of inconsistencies between beliefs and practices highlighted factors that contribute to perceived inconsistencies. Classroom management, time constraints, standardized testing, and limited resources were among the factors that contributed to inconsistencies. Thompson (1984) discovered that perceptions about students supersede beliefs about teaching. This notion was confirmed with a case study of two elementary teachers by Sztajn (2003). It was found that teachers' beliefs about students' needs affect their teaching practices. One teacher (Teresa) taught third-grade students with a low socioeconomic background, and the other teacher (Julie) taught fourth-grade students with a high socioeconomic background. Both teachers shared beliefs in a problem-solving approach to teaching and learning mathematics. However, differences in instruction and the content covered were observed among the two teachers. Teresa had determined that because her students came from broken homes that lacked structure, discipline, and organization, her students would benefit from an environment in which they learned to follow rules, behave in class, and become responsible citizens. As a result, she did not teach in accordance with the mathematics reform recommendations pertaining to collaboration, problem-solving, and developing conceptual understanding. Her students did not experience the higher-order thinking and problem-solving experiences of other students. Instead she focused on the basics with emphasis on drills and memorizing steps and procedures for performing calculations.

On the other hand, Julie's students had different needs than Teresa's students. Julie's students began the school year with knowledge of basic arithmetic. They also came from homes that prepared them for school academically and behaviorally. Thus, Julie did not believe that she needed to spend instructional time on behavior management, structure, and basic skills. Julie believed that her students needed to be prepared to think and face challenges. Therefore, she used her instructional time to teach more sophisticated content through projects which were more process-oriented than result-oriented. Julie's teaching style reinforced responsibility, persistence, and collaboration. Julie's classroom reflected that of a progressive educator.

Due to the complex nature of determining beliefs, inconsistencies can also be attributed to the methods used to determine teachers' beliefs. Researchers could erroneously infer beliefs from surveys, interviews or observations of teaching. They may also misinterpret an espoused (or inferred) belief as being central or core to the teachers' belief system. Rokeach (1968) claimed that beliefs are not equally important and that people hold beliefs with differing intensities. Beliefs form clusters containing different core beliefs. This allows for the existence of seemingly conflicting beliefs to coexist (Cross, 2009; Thompson, 1992). This means the beliefs inferred by the researcher or espoused by the teacher may not be the same beliefs acted upon during a lesson. Since beliefs are contextual (Francis, 2015; Nespor, 1987; Raymond, 1997), a teacher's instructional decision, in the moment, may reflect a belief that is more central than their general beliefs about mathematics, teaching, and learning. Researchers' misinterpretations can lead to an observed disconnect between what a teacher says they believe and how they perform in the classroom. In response to the perceived inconsistencies found in research studies, Francis (2015) and Philipp (2007) assume that teachers are not intentionally inconsistent, instead they behave in a manner consistent with their instructional goals. To that end, Francis (2015) and Philipp (2007) suggest that researchers assume no contradictions and if inconsistencies appear, then researchers should view them as an opportunity to learn more about the teacher's perspectives.

Questioning in Mathematics Classrooms

Question-asking is a widely used instructional tool; especially in mathematics classrooms. Vogler (2008) pointed out, "Teachers ask about 300-400 questions per day and as many as 120 questions per hour" (p. 1). When used as a tool for learning, questions have a powerful impact because they support student participation, drive instruction and are useful for helping teachers determine students' understanding of mathematical concepts. Using question-asking to acquire knowledge has long-standing roots, dating back to the days of Socrates, that continue to influence many teachers today. Both educators and researchers have continuously been intrigued by the types of questions that yield true understanding and knowledge. One questioning strategy that has been critical in promoting conceptual understanding is the Socratic Method.

Socratic method. Asking questions has been considered a pivotal practice in constructing knowledge for centuries. In his description of the historic Socratic debates, Meyer (1980) explained the purpose of questioning as an opportunity to provide an occasion for those being questioned to assert themselves as masters. Socrates' emphasis on questions and answers has had far-reaching implications in education (Cotton, 2001; Elder & Paul, 1998; Rud, 1997). Today, mathematics educators employ various interpretations of the Socratic Method; which has also been called Socratic pedagogy, Socratic teaching or Socratic questioning. Elder and Paul (1998) made clear that the use of the word "Socratic" refers more so to a systematic use of questioning which includes questioning students' responses and seeking to understand the underlying foundations for students' responses. The Socratic Method has pedagogical strengths as it uses questioning to lead students to specific knowledge. Cotton (2001) claimed, "using questions and answers to challenge assumptions, expose contradictions, and lead to new knowledge and wisdom is an undeniably powerful teaching approach" (p. 1). The Socratic Method not only allows teachers to learn about their students' mathematical thinking, it also gives students an opportunity to clarify their knowledge by explaining what they know.

Influence of education reform. Question-asking has consistently been a part of mathematics education reform. Throughout the 20th century, the goals of mathematics education have fluctuated between emphasizing basic skills and developing problem-solving skills. There have also been fluctuations between content-centered curriculum and student-centered curriculum (Klein, 2003). These fluctuations were due, in part, to the changing opinions of what students needed in order to be productive American citizens and prepared for the demands of a globally competitive future. As standards changed, so did the associated pedagogical strategies teachers were expected to employ. Questioning is one strategy that varied based on the emphasis of instruction.

The School Mathematics Study Group (SMSG) initiated the "New Math" era of the late 1950s and 1960s, which was born out of the disagreements about instruction focused on skills versus instruction aimed at understanding (Klein, 2003). During this era, there was a focus on inquiry and discovery (Sloan & Pate, 1966). Mathematics researchers noticed differences between the traditional questioning approaches and the newer approaches recommended by the SMSG. When Sloan and Pate (1966) compared the teacher-pupil interactions between traditional methods of teaching and the methods recommended by the SMSG, they found that teachers who utilized the SMSG methods used more divergent questions than teachers using traditional methods. The education reform that the nation experienced was reflected in the questioning styles used in mathematics classrooms.

The "New Math" era also saw the development of several question-classification systems (Gall, 1970). Bloom's Taxonomy was one of the classification schemes that emerged and still receives much attention in American education. Originally, Bloom's Taxonomy defined six hierarchal categories in the cognitive domain: knowledge, comprehension, application, analysis, synthesis and evaluation. The taxonomy was often used to measure the cognitive demand of instruction and assessments. Krathwohl (2002) stated:

> One of the most frequent uses of the original Taxonomy has been to classify curricular objectives and test items in order to show the breadth, or lack of breadth, of the objectives and the items across the spectrum of categories. (p. 213)

Bloom's Taxonomy has influenced teaching methods, particularly questioning, in the mathematics classroom. In 1991, the National Council of Teachers of Mathematics (NCTM) developed professional standards which included a description of the teacher's role in discourse as "posing questions and tasks that elicit, engage, and challenge each student's thinking", "asking students to clarify and justify their ideas orally and in writing", and "regularly following students' statements with, 'Why?' or by asking them to explain" (p. 35). More recently, the Common Core State Standards (CCSS) for Mathematical Practices (2010) outline eight mathematical practices in which students should engage. Among them are the ability to explain, justify, reason, and analyze. These practices are supported when teachers ask students questions to elicit their thinking. Both the NCTM professional standards for teachers and the CCSS mathematical practices for students encompass several of the educational objectives within Bloom's Taxonomy (Krathwohl, 2002).

Reform efforts have impacted the use of questioning in mathematics classrooms in that attempts to prepare students for the rigorous demands of the future have resulted in a major focus on the mathematical practices promoted by the high-level categories of Bloom's Taxonomy. Since

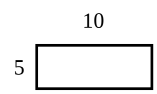
1991, national standards have recommended asking questions to develop mathematical practices of high cognitive demand. In order to comply with these recommendations, it is important to ask, "What is a question?" and "What is the purpose for asking certain questions?"

What is a question? A key component of teachers' instructional strategies is questioning. Questions are used to request information or action (Kawanaka & Stigler, 1999). With varying forms and purposes, questions have been attributed to provoking thinking and learning. Kawanaka and Stigler (1999) conducted a study of patterns of discourse in which the type, frequency, and placement of teachers' utterances during mathematics lessons were examined. The utterances intended to elicit responses or invoke a performance from students were called questions. Additionally, the utterances used for elicitation were categorized as either yes/no (Is 14 a prime number?), name/state, (What is the formula for the area of a rectangle?) or describe/explain (Why is a common denominator needed to add fractions?) to describe the cognitive demand necessary to respond. Although all utterances were not structured as interrogative statements, their inherent request for students to provide information or to perform an action deemed them as questions.

Question classifications. The types of questions teachers ask have been the focus of many studies (Aizikovitsh-Udi & Star, 2011; Franke, Webb, Chan, Freund & Battey, 2009; Sahin & Kulm, 2008). Generally, questions are classified as either low-level or high-level; depending on the level of cognitive demand required to answer the question. Many studies that have considered the frequency and the type of questions asked during a typical mathematics lesson show that teachers generally ask more low-level questions than high-level questions (Hus & Abersek, 2011; Kawanaka & Stigler, 1999; Kosko, Rougee & Herbst, 2014; Ni, Li, Li & Zou 2011; Sloan & Pate, 1966). Still, the types of questions asked depend on the teacher's instructional goal either in the moment or planned in advance. Both low-level and high-level questions have critical roles in mathematics education.

Low-level questions. Low-level questions have been labeled differently throughout research studies but generally refer to questions that are leading or that require students to name, state, memorize, or recall. These types of questions also allow students to provide yes/no, technical, factual, or simple responses (Aizikovitsh-Udi & Star, 2011; Kawanaka & Stigler, 1999; Sahin & Kulm, 2008). These characteristics of questions require low levels of cognitive demand from students. The tasks associated with low-level questions are related to the educational objectives within the cognitive domain, *Knowledge*; which is the lowest hierarchal category of Bloom's Taxonomy (Krathwohl, 2002). Examples of low-level mathematics questions include:

- *Given 73+58, what is the sum?*
- *What is the perimeter of this rectangle?*



Generally, low-level questions are not designed to elicit or build on student thinking. Instead they allow students to exhibit their knowledge of content and procedures. In a case study of two teachers in which one used questioning techniques and the other did not, Aizikovitsh-Udi and Star (2011) labeled the "conserving teacher" as the one who did not use questioning strategies and maintained traditional patterns of teaching by lecturing and asking questions focused on mathematical content. This teacher asked low-level questions that were labeled as "technical" because they elicited a mathematical answer without an explanation. Leading (or guiding) questions share the same classification. Although leading questions are used to assist students in arriving at desirable answers (Sahin & Kulm, 2008), Franke, et al. (2009) noted, "leading questions did not provide opportunities for students to build on their own understanding" (p. 390). Leading questions are low-level questions because they are not centered on the students' thought processes. The path to understanding is dictated by the teacher as opposed to the student.

As Kosko et al. (2014) pointed out, "Literature has consistently shown that instead of asking questions that probe student understanding, teachers generally ask leading or recall-oriented questions" (p. 459). These type of questions, alone, do not provide students with support in explaining, justifying, analyzing, and reasoning about mathematics. International studies reveal similar findings. In a comparison of the types of questions asked by U.S., Japanese, and German teachers, Kawanaka and Stigler (1999) found that U.S. teachers asked more yes/no questions than Japanese and German teachers. However, the most frequently asked questions, by teachers in each of the three countries, were ones that asked students to name or state. Chinese and Slovenian teachers have been found to ask more low-level questions as well (Hus & Abersek, 2011; Ni, et al. 2011). Although low-level mathematics questions do not engage students in thinking deeply about mathematics, these kinds of questions do provide opportunities for students to develop and maintain fluency with facts and procedures.

High-level questions. High-level questions have also been labeled differently throughout research studies. For instance, high-level questions have been described as good, effective, competent, and open, as well as questions that require students to describe, explain, justify, or investigate (Booth, Lange, Koedinger, & Newton, 2013; Crespo & Sinclair, 2008; Sullivan & Clark, 1992; Weiland et al., 2014). High-level questions allow students the opportunity to develop the mathematical practices of reasoning, justifying, and analyzing. Examples of high-level mathematics questions include:

- *Why should you line up the decimals when adding numbers?*
- *Draw a shape with a perimeter of 34 inches.*

Although low-level questions prevail, studies show that teachers do ask high-level questions in mathematics classrooms.

Aizikovitsh-Udi and Star (2011) claim that the "leveraging teacher" uses higher-order questions to understand how students think and uses students' thinking to guide their path to understanding. Weiland et al. (2014) analyzed the questions that preservice teachers asked students during formative assessment interviews. They found that the number of competent questions; that is, questions that “cause the student to justify thinking, explore reasoning, or create a cognitive conflict to reorganize thinking” (p. 340), increased over the course of the semester as preservice teachers began to interact with students and analyze their thinking. Similarly, Crespo (2003) analyzed the questions preservice teachers asked students by way of writing letters. It was discovered that preservice teachers began to ask more challenging questions once they were aware of their students’ thinking. High-level questions encourage thinking (Sahin & Kulm, 2008; Wegerif, 2011) and give teachers information about students' comprehension (Barlow & Cates, 2006; Franke et al., 2009). When focused on mathematical concepts, high-level questions contribute to cognitively guided instruction by eliciting and developing students’ mathematical thinking (Carpenter, Fennema, Franke, Levi, & Empson, 2000).

It is important to note that the term high-level is relative; depending on the current ability level of the student being questioned. Asking a student to explain their work or their thinking does not, necessarily, evoke the use of that student's higher-order cognitive processes. Kawanaka and Stigler (1999) reminded that, "Not all the questions that request descriptions or explanations can be called higher-order questions. Higher-order questions should be the ones that elicit higher-order thinking" (pp. 277-278). Similarly, asking questions that are challenging or demanding does not guarantee that they should be classified as high-level questions. The cognitive level of a question should not be confused with or equated to its difficulty level (Brophy & Good, 1984). Stein and Smith (1998) described mathematical tasks that could be approached in several ways; each way requiring a different level of cognitive demand from students. Higher-level approaches were those that asked students to build connections to mathematical meanings or to explore relationships among various representations.

The use of low-level and high-level questions. During one mathematics lesson, teachers may ask both low-level and high-level questions as they are both necessary for mathematics teaching and learning. The reasons for using either level of questioning generally pertain to students' ability and teachers' instructional goals (Delice, Aydin & Çevik, 2013; Gall, 1970; Kawanaka & Stigler, 1999; Sahin & Kulm, 2008). Delice et al. (2013) used a series of interviews and questionnaires with teachers in state schools, private schools, and teachers from courses that prepare students for college entrance exams to learn why teachers asked certain types of questions. It was discovered that the questions some teachers asked depended on whether the students were engaged in classwork, homework, or an exam. When determining examination questions, most teachers considered the classification of the questions. The development of critical thinking was the most frequent consideration when assigning homework questions and previous knowledge connection was considered most when developing classwork questions. Sahin and Kulm (2008) also interviewed teachers regarding the reasons for asking probing, guiding, and factual questions. When probing student thinking, teachers admitted to asking "why" questions to encourage students to explain and justify their thinking as well as to learn whether their students understood the content. On the other hand, when students experienced difficulties, guiding questions were asked to help them explain and lead them to the appropriate understanding. To check for understanding of properties of concepts and algorithms, factual questions were asked. In a meta-analysis of research linking teacher behavior to student achievement, Brophy and Good (1984) suggested:

> When teaching complex cognitive content or when trying to stimulate students to generalize from, evaluate, or apply their learning, teachers will need to raise questions that few students can answer correctly (as well as questions that have no single correct answer). (p. 117)

Many teachers have very intentional purposes for asking particular types of questions. Some purposes benefit the students while others serve as means of formative assessment for teachers. Franke et al. (2009) suggested that, "Finding the balance in the types of questions and when to ask them can make a large difference in how students continue to participate" (p. 381). Ultimately, it is the marriage of low-level and high-level questions that provide students with a rich educational experience.

Significance of questioning. Questions have benefits for both those who pose questions and those who respond to questions. By posing questions, the poser gains new information based on the response to their question. Thus, posing questions is important for teachers to gain knowledge; particularly about student understanding (Barlow & Cates, 2006; Franke et al., 2009). Franke et al. (2009) found that teachers who participated in a professional development focused on using students' mathematical thinking to guide instruction were more successful at using questions to assist in their students' learning. By asking probing questions, teachers gained knowledge of their students' thinking and were able to use that knowledge to support and scaffold their path to developing correct explanations of mathematical procedures. Asking students to pose a question also sheds light on what students are thinking (Barlow & Cates, 2006; Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008). By asking students to pose questions about the material being studied, teachers gain information about the aspects of the content on which the student is focused (Piccolo et al., 2008), and they are able to determine the level of their students' understanding (Barlow & Cates, 2006). While questions are important for helping teachers determine their students' needs, questioning also allows students to demonstrate their current understanding.

The types of questions asked by teachers are contextual (Brophy & Good, 1984; Cazden & Beck, 2003; Krussel, Edwards, & Springer, 2004; Mason, 2000; Piccolo et al., 2008; Wegerif, 2011). They pertain to the instructional goals that teachers either plan in advance or determine in action. Teachers set instructional goals to produce desirable learning outcomes, which are indications of what teachers would like their students to achieve and demonstrate by the end of the lesson or activity. When students attempt to respond to questions, they are required to think about the content of the question. By responding, the student is encouraged to think (Sahin & Kulm, 2008; Wegerif, 2011), and is able to demonstrate their knowledge of facts and procedures (Booth et al., 2013; Franke et al., 2009; Meyer, 1980; Sahin & Kulm, 2008).

Encouraging thinking and understanding. Questions are vital for provoking thinking. Wegerif's (2011) theory of thinking and learning how to think asserts that thinking begins when children are required to explain themselves to another person. Sahin and Kulm (2008) conducted a case study of two teachers to investigate the type, frequency, and place of questions. Specifically, probing questions were described as those that promote deeper thinking. While tracing the paths of questions that led to evidence of student understanding, Piccolo et al. (2008) found that a series of how, guiding, and probing questions led to students showing evidence of understanding. Franke et al. (2009) described the questioning used by teachers to assist students with arriving at correct and complete explanations regarding the meaning of the equal sign. They found that a probing sequence of specific questions helped to guide students in developing correct explanations after having previously given an incorrect or incomplete explanation.

Demonstrating knowledge of facts and procedures. Questions are important to the person being questioned. Weiland et al. (2014) suggested that learning experiences are impacted by the questions students are asked. By responding to a question, one is able to demonstrate knowledge or mastery of the content in question (Meyer, 1980). Teachers ask students to demonstrate their knowledge in several ways. A study conducted by Booth et al. (2013) used fully worked-out examples to encourage students to explain both correct and incorrect examples of solving equations. The researchers found that asking students to explain both correct and incorrect worked examples was effective in improving students' procedural knowledge and problem-solving approaches. Sahin and Kulm (2008) studied the type, frequency and intention of the questions asked by two sixth-grade teachers. The type of question used most frequently was factual questions. These are questions that request specific facts or definitions, the answer to an exercise, or the next step in a procedure (Sahin & Kulm, 2008). Both teachers in the study confirmed that their intention for asking factual questions was to request specific definitions or steps.

Problem-solving or exercising? Questions posed to students affect their interaction with mathematics; even when the questions are posed as problems. Although most mathematical tasks assigned to students are casually referred to as "problems", it is important to differentiate problem-solving from exercising. Problem-solving is a term that has subjective meanings among teachers due to the ambiguity of the word "problem". On one hand, "problems" are viewed as exercises that help students develop procedural fluency and involve, "practicing a procedure (e.g., an algorithm) or rehearsing specific facts or concepts (e.g., multiplication facts or definitions), to build proficiency and quickly obtain a correct answer" (Rickard, 2015, p. 2). Low-level questions can be used to engage students in mathematical exercises. On the other hand, "problems" are perceived as new tasks that compel students to: analyze given information, employ reasoning to determine an effective solution strategy, justify procedures, and explain their thinking, among other practices (National Council of Supervisors of Mathematics (NCSM), 1989; Polya, 1957). The latter kind of "problems" rely on high-level questioning that force students to apply their conceptual understanding and procedural skills to the context of a specific scenario. Adding to the inherent vagueness is the reality that what presents a problem for one student may not present a problem for another. For instance, comparing $\frac{7}{8}$ to $\frac{6}{7}$ may be a problem for a student who has yet to learn the process of using common denominators to compare fractions. A student's current knowledge and mathematical experience could be factors in determining if teachers deem a task as a problem or an exercise.

To eliminate the ambiguity of a task being labelled as a problem or an exercise, one can consider the work of researchers and mathematics councils that have provided characteristics describing problem-solving activities. George Polya (1957) depicted problem-solving in mathematics as a four-phase process of strategy implementation including: 1) understand the problem, 2) make a plan, 3) carry out the plan, and 4) look back (pp. 5-6). NCSM also views problem-solving as a process involving the implementation of strategies and mathematical practices. In its position paper on essential mathematics of the 21st century, problem-solving was listed as one of the twelve components of essential mathematics. NCSM (1989) stated:

> Problem-solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. Students should see alternate solutions to problems; they should experience problems with more than a single solution (p. 471).

Teachers can cultivate problem-solving skills by providing problem-solving experiences and facilitating students' thinking processes. Rickard (2015) conducted a case study to analyze one teacher's (Bob) evolution from instruction focused on mathematical exercises to instruction with an emphasis on problem-solving strategies and skills. Rickard (2015) highlighted ways which Bob supported students in developing problem-solving skills such as teaching specific problem-solving strategies including: guess and check, make a systematic list, make a model, solve the problem in multiple steps, draw a diagram, and look for patterns. More importantly, Bob provided opportunities for students to practice each of the strategies, separately. According to Rickard, Bob felt that providing and modeling solution processes for problems requiring multiple strategies would refine students' problem-solving skills:

> Bob expressed that by modeling specific strategies for his students and then providing them problems where they would need to select and apply one or more problem solving strategies they would build their skills as problem solvers and move beyond routine word problems (Rickard, 2015, p. 7).

Cultivating problem-solving skills entails not only providing students problems requiring them to reason about solution methods, but also asking questions that encourage students to "refine their thinking and justify their ideas" (Rickard, 2015, p. 10).

Reflection

There are several descriptions of what reflection is or of the reflective process. These descriptions entail 1) an analysis (during or after) of one's own behavior or experience and 2) a concurrent analysis of one's beliefs and assumptions about the behavior or experience under investigation. This process is conducted for the purpose of improving or gaining a better understanding of one's behavior or experience (Friel et al., 1992; Richards & Lockhart, 1994;

Varela, 1999). Reflection occurs at various levels (Van Manen, 1977) and with varying focal points of interest. In education, reflection is regarded as an approach that leads to acquiring and improving professional knowledge (Brown & Coles, 2012; Roy, 1998; Ticha & Hospesova, 2006).

Schön (1983) claimed that we possess knowledge that we may not be able to articulate or describe, yet our knowledge is implicit in our actions. In other words, our knowledge is in our action. Reflection on our action and on our knowledge is useful when confronted with spontaneous or unique situations that present an element of surprise to which we must respond. In trying to make sense of a new situation, reflection on our current understanding allows us to reconsider and recompose our understandings to apply to future actions. In this sense, we think about what we are doing while we are doing it. This process, known as "reflection-in-action" is imperative for dealing with divergent situations of uncertainty and instability (Schön, 1983).

While some reflection occurs during action, much reflection occurs after action. Schön (1983) calls this process "reflection-on-action." Reflecting on past experiences guides future behavior. García, Sánchez, and Escudero (2007) claimed, “the notion of reflection-on-action is related to how teachers interpret past classroom events - 'interpretative processes'- for the purpose of defining future actions" (p. 2). Reflection as a deliberate ensuing analysis of one's actions is a method for acquiring professional knowledge and developing a deeper understanding of one's practice (Lucas, 1991). During reflection-on-action, reflection occurs on three levels: technical, practical, and critical (Van Manen, 1977). Technical reflection focuses on attaining particular goals. Practical reflection is concerned with analyzing actions in relation to individual experiences and perceptions. Critical reflection focuses on the worth of knowledge and the social conditions that impact actions. Reflecting, at all levels, has the effect of determining future actions. Thus, reflection is considered a critical component of professional development for teachers.

Reflection for teaching. In education, reflection (or reflective thinking) is considered an approach to teaching (Jansen & Spitzer, 2009; Kramarski, 2010; Richards & Lockhart, 1994). Whether it occurs before, during, or after an action, reflection is an intentional and systematic process with goals of reaching a deeper understanding of a particular phenomenon in order to improve on it. Richards and Lockhart (1994) describe the reflective approach to teaching as an investigation in which teachers analyze data about teaching and learning, explore their own beliefs about subject matter content as well as their beliefs about teaching and learning, examine their teaching practices, and use the information obtained as a basis for critical reflection about teaching. This process is supported by Kramarski's (2010) notion that reflection is an instructional support that develops the “process” view of teaching and learning. Thinking about teaching, reflectively, allows teachers to learn from their own teaching experiences and use the knowledge to improve their teaching (Jansen & Spitzer, 2009). Analyzing their own teaching and recognizing connections between teaching and learning are key components of teacher reflection. Davis (2006) stated, "Productive reflection—the kind of reflection that is likely to promote teacher learning— requires these connections and this analysis" (p. 283).

Schön (1983) described the necessity of reflection by practitioners such as teachers. As specialists in their field, teachers gain a great deal of experience in working with students, navigating curriculum, employing pedagogy, and creating classroom environments. For some teachers, this abundant experience contributes to day-to-day actions that are repetitive and less spontaneous. That is, they become specialized in their mode of teaching. However, Schön suggested that this kind of automatic behavior can have a negative effect on teachers' specialization. He stated, “In the individual, a high degree of specialization can lead to a parochial narrowness of vision” (Schön, 1983, p. 60). Schön continued by suggesting, “as a practice becomes more repetitive and routine…the practitioner may miss important opportunities to think about what he is doing" (p. 61). Reflection is a way to uncover and analyze perceptions associated with the repetitive nature of teachers' practice and to overcome what Schön calls "over-learning".

How is reflection done? Due to the many descriptions of reflection, there are different ways in which people engage in the process. At the forefront of any reflective process is an inquiry about a particular phenomenon. In education, a teacher may want to develop a deeper understanding of specific pedagogical methods or particular classroom dynamics in an effort to improve upon these characteristics. Whether reflection is undertaken individually or within a group, having a specific focus is helpful in directing attention to a particular phenomenon of teaching or learning (Friel et al., 1992). Data such as audio or video recorded lessons, journals, observations, lesson reports, and surveys on the selected phenomenon should be collected by educators for analysis (Mustafa, 2005; Richards & Lockhart, 1994). To assist with the critical reflection process, Richards and Lockhart (1994) unpacked some important dimensions of teaching foreign language and provided questions for teachers to consider in each dimension. Table 2 shows the dimensions of teaching and examples of reflection questions Richards and Lockhart suggest teachers ask themselves as they reflect on each dimension.

Table 2

Teaching Dimensions and Examples of Reflection Questions

Dimension	Examples of reflection questions
Classroom Investigation	1) Did you depart from you lesson plan? If so, why? Did the change make things better or worse? 2) Would you teach the lesson differently if you taught it again? 3) Were students challenged by the lesson?
Exploring Teachers' Beliefs	1) In what ways does your personality influence the way you teach?

	2) How would you define effective teaching? 3) What kinds of students do best in your class?
Focus on the Learner	1) How would you characterize the cognitive style you favor in learning a foreign language? 2) How does it influence the kind of activities you prefer?
Teacher Decision Making	1) What are my alternative plans if problems arise with some aspect of the lesson? 2) How can I get the students' attention? 3) Was this lesson successful? Why or why not?
The Role of the Teacher	1) What aspects of classroom behavior or interaction do I encourage or discourage? 2) How were misunderstandings dealt with when they arose?
The Structure of a Language Lesson	1) Do you think a lesson that has fairly rapid pacing is necessarily better than one that does not? Why or why not?

Not only are teachers encouraged to reflect on their prior lessons, plans for future lessons, and instructional decisions, they are also prompted to explore their beliefs. Teachers should become aware of their beliefs and seek to understand how their practice tells what they believe about particular aspects of teaching (Mustafa, 2005; Senger, 1998).

Varela (1999) described an approach to reflection as "deliberate analysis". He claimed, "after acting spontaneously," a person can "reconstruct the intelligent awareness that justifies the action" (p. 32). Soliciting deliberate analysis provokes reflection (Brown & Coles, 2012) and contributes to continued learning (Varela, 1999). This is similar to both Schön's idea of reflection-on-action and to stimulated recall methods of research in which teachers review video or audio recordings of their lesson to provide data on their thought processes and decision-making (Calderhead, 1981; Meade & McMeniman, 1992). One necessary component of deliberate analysis, and reflection in general, is action. Csikszentmihalyi (1990) imparted the important relationship between action and reflection by stating, "Activity and reflection should complement and support each other. Action by itself is blind, reflection impotent" (as cited in Brown & Coles, 2012, p. 223). Reflecting on action and using the knowledge gained from reflection to affect future action is part of the learning process (Friel et al., 1992). Future actions include the many decisions teachers make regarding subsequent lessons. Reflection guides teachers in making planning decisions (made before delivering a lesson), interactive decisions (made as a result of reflecting-in-action) and evaluative decisions (made as a result of reflecting-on-action) (Richards & Lockhart, 1994).

Reflecting in groups has been considered a more beneficial experience than individual reflection (Kramarski, 2010; Mustafa, 2005). Reflecting within a group or with a partner has the potential to uncover more than just personal activity (Mustafa, 2005). Understandings, beliefs, and connections are revealed when reflecting with others. During a study with elementary school mathematics teachers, Kramarski found that self-questioning support strengthens pre-service teachers' pedagogical ability in the context of problem-solving. Self-questioning was a reflection strategy in which teachers addressed pre-determined, self-guided questions before, during, and after the solution process. These questions focused on comprehension, making connections, teaching and learning strategies, and reflecting on their experience while solving the problems. The self-questioning strategy encouraged teachers to "reflect on their goals, their understanding, making links, and restructuring ideas" (Kramarski, 2010, p. 148). Each of these components are in line with what Davis (2006) calls productive reflection. This directed-support method of reflection was more effective in improving pedagogical knowledge than the reflective discourse method, which involved teachers sharing their thinking with peers. Kramarski (2010) suggested, "direct support may act as a 'more able other,' prodding students to consider issues they may not have considered otherwise" (p. 149).

Why is reflection important? Reflection contributes to professional knowledge (Chamoso, Cáceres & Azcárate, 2012; García et al., 2007; Mustafa, 2005; Ticha & Hospesova, 2006) and affects practice (Brown & Coles, 2012, Friel et al., 1992; Jansen & Spitzer, 2009; Kramarski, 2010; Roy, 1998). Slavik (2004) stated, "It is possible to treat reflection connected with interpretation of teaching and learning situations as the best way to develop the teachers' professional way of thinking and to present practical didactical theory" (as cited in Ticha & Hospesova, 2006, p. 133). Teachers who engage in reflection are able to learn about the impact of their perceptions and their practice on student learning and use that knowledge to determine subsequent practice. This process contributes to teachers' professional development. Because teachers are encouraged to continuously engage in professional development that will enhance their practice and their knowledge, they can use reflection as a process to develop both.

Professional knowledge. Reflection has been considered a more effective method for professional knowledge retention than other forms of learning. Teachers' experiences, and the associated analysis of their experiences, resonate and remain with them more than the knowledge gained from other avenues of learning (Mustafa, 2005). Thus, a significant source of knowledge is oneself. Ticha and Hospesova (2006) noted four kinds of teacher's competence: 1) pedagogical competence, 2) subject-didactic competence, 3) pedagogical-organizational competence, and 4) competence in qualified pedagogical (self-) reflection. The fourth competence pertains to teachers' ways of thinking and dealing with students. The authors view qualified pedagogical reflection as a component of teachers' professionalism. This type of reflection allows actions to move from being intuitive to being justified. Reflection is also important because it allows teachers to focus on particular aspects of teaching in order to gain more understanding of the nuances associated with it. Chamoso et al. (2012) supported this idea by stating, "Reflection is important in making sense of the complexities of teaching" (p. 155).

Practice. Teachers who engage in reflection are thoughtful about their work; they act as researchers (Schön, 1983) when trying to make connections between their beliefs and their practice as well as connections between their practice and student performance. The reflective process is optimized when it is followed by action. Jansen and Spitzer (2009) describe "reflective thinking" as a practice in which teachers continuously learn from their actions and use their knowledge to improve their practice. This process is crucial for improving mathematics education (Roy, 1998). Regular reflection can result in a more developed practice and higher standards of performance (Mustafa, 2005). When they reflect on their instruction, teachers are better able to make connections between their planning and students' needs (Kramarski, 2010). During her study of two fourth grade teachers, Senger (1998) used video reflection and theory reflection to elicit teachers' beliefs and compare their beliefs to their actual practice. Senger found that not only were both methods of reflection effective for understanding teachers' beliefs, but also the amount of involvement the teachers had in the reflection segments of the study encouraged them to think about their beliefs in relation to their practice. It is critical that teachers have this awareness. Brown and Coles (2102) stated, "Over time, if we are able to become aware through our experiences, we literally come to see more linked to our actions" (p. 221). Although reflection is used to increase professional knowledge and gain a deeper understanding of teachers' beliefs and practices, there are some roadblocks that inhibit teachers from reflecting with consistency.

Barriers to reflection. Despite the documented benefits of reflection (Chamoso et al., 2012; Kramarski, 2010; Schön, 1983) some teachers have difficulty with the process. Reflection takes time, confidence, and skill to accomplish. One roadblock to reflection is the tendency to evaluate rather than interpret one's own teaching. Reflection requires teachers to interpret teaching in order to learn from it. Jansen and Spitzer (2009) shared one challenge to reflection by stating:

> Another reason why analysis is challenging is due to a potential temptation to evaluate rather than interpret teaching … When they evaluate teaching, teachers judge what was good, bad, or could have been done differently, and teachers' evaluations are often based on their beliefs rather than evidence of students' thinking. When interpreting their teaching, teachers make inferences that are connected to evidence of students' thinking and pose hypotheses regarding how an instructional strategy influenced students' learning. (pp. 135-136)

Because of the tendency to evaluate teaching, teachers may benefit from the support of a partner or a group, to tend to aspects other than personal behavior, while reflecting on their practice and interpreting their own teaching. (Kramarski, 2010; Mustafa, 2005).

In addition to being time consuming (Wilson & Berne, 1999) and requiring much effort (Ticha & Hospesova, 2006), most teachers experience difficulty with reflection because they simply are not accustomed to thinking about their teaching reflectively. This could be due to the number of additional professional skills involved in the process such as noticing, reasoning, analysis, questioning, change, and affective components (Mustafa, 2005). Even with this special subset of skills, reflecting can be difficult due to what may be revealed. Revealing the reality about one's teaching (to themselves or to another person) to be scrutinized can produce anxiety.

Conceptual Framework

The literature summarized above contributed to framing this study. Figure 1 embodies the conceptual framework of the present study by illustrating the role of reflection in understanding the connection between beliefs and questioning. The present study was framed by an overarching theory that one's beliefs influence their actions. This is represented in the top row of Figure 1.

Belief systems are deeply rooted mental constructions that Harvey (1986) defined as “a set of conceptual representations which signify to its holder a reality or given state of affairs of sufficient validity, truth and/or trustworthiness to warrant reliance upon it as a guide to personal thought and action” (p. 660). Along with driving actions (Richardson, 1996), beliefs also have "observable behavioral consequences" (Rokeach, 1968). Hence, one’s actions can be examined and provide insight into their beliefs. When trying to understand the impetus of one’s actions, it is important to be aware of their beliefs related to the action of interest.

An implication of this overarching theory begets another theory focused on education: teachers’ beliefs about teaching and learning shape their instructional behavior (Richardson, 1996; Springs, 1999; Thompson, 1984). This is shown on the second row of Figure 1. Teachers’ beliefs about teaching and learning impact such practices as the “selection of content and emphasis, styles of teaching, and modes of learning” (Ernest, 1989, p. 20). Even with their complexities, beliefs are of interest to education researchers because of their connection to instructional practices and their role in understanding the decision-making processes of educators (Beck, et al., 2000; Wilcox-Herzog & Ward ,2004). Observing instructional behavior might inform the observer of teachers’ beliefs about teaching and learning.

A narrow focus on the implications of the theories above led to the scope of the present study. Beliefs about teaching and learning mathematics are a subset of beliefs about teaching and learning. Presumably, these beliefs also impact a subset of instructional behavior such as question-asking practices. Reflection adds another component to the link between beliefs and question-asking practices. Reflection, as a deliberate analysis of question-asking practices, channels teachers’ beliefs as they justify their question-asking practices. The third row of Figure 1 belongs to the cyclic process of reflecting on one's question-asking in relation to their beliefs about teaching and learning mathematics (shown with bidirectional arrows).

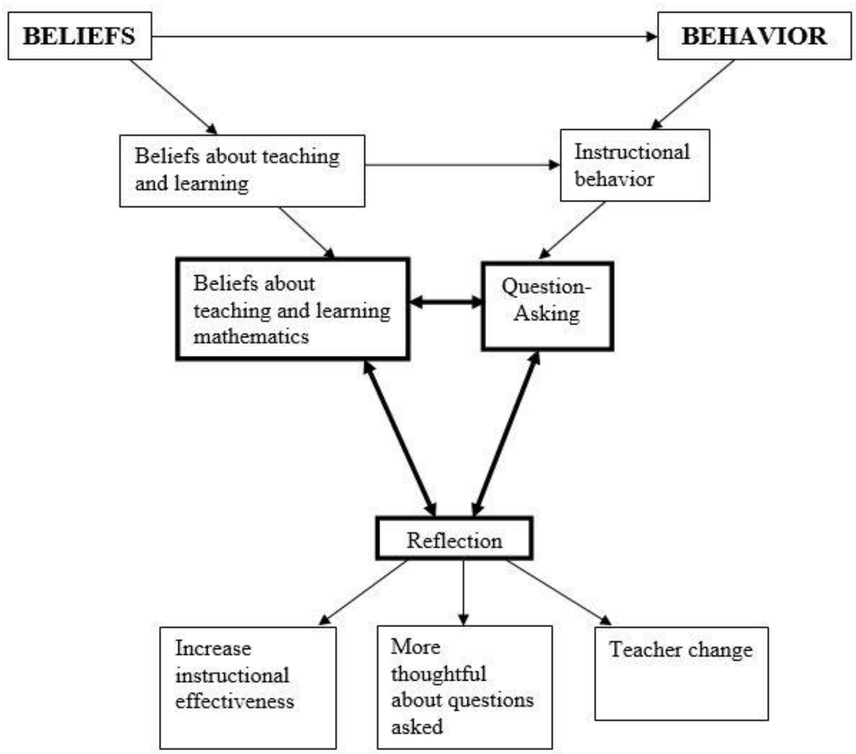


Figure 1. Conceptual framework for the process of reflection on beliefs and questioning practices. The reflective process has implications for teaching including increasing instructional effectiveness (Jansen & Spitzer, 2009; Roy, 1998); making teachers more thoughtful about the questions they ask during instruction (Mustafa, 2005); and causing teacher change (Richardson, 1996) as shown in the bottom row. These outcomes of reflection are critical to teachers' professional development and to students' educational experience.

In his framework for effective instruction, Marzano (2007) named questioning as one of the aspects of the critical-input experience; which describes ways in which teachers facilitate the processing of new content. Marzano claimed that students' comprehension is enhanced when they are asked questions that force them to elaborate on a topic. The Marzano Teacher Evaluation

Model (2013), which measures teacher effectiveness, includes domains focused on questioning and reflection on teaching. Reflecting on question-asking practices can also affect teacher change; shifts in a teacher's usual practice. As stated above, reflection occurs on three levels: technical, practical, and critical (Van Manen, 1977). Cheung and Wong (2017) found that teachers are more likely to change their practice depending on the level of reflection they were engaged. They specified their finding by stating, "The higher the level of reflection teachers have, the more motivated the teachers [are] to explore new teaching practices not only for the learning needs of students in [the] classroom but also for the society outside the classroom (p. 1135). Reflection-on-action (specifically question-asking practices) causes teachers to pause and consider their repetitive routines. By doing so, they become more thoughtful about the actual questions they have asked and plan to ask. Recognizing the influence of beliefs on questioning is significant because students' learning experiences are impacted by the questions they are asked (Weiland, et al., 2014). Consequently, teachers' beliefs have an inherent impact on students' learning experiences. The substantial relationship between questioning and student learning requires teachers to reflect on the influence of their beliefs on their questioning practices to become more aware of the effect of their beliefs on students' learning experiences.

Conclusion

Understanding the relationship between beliefs and practice helps to understand teachers' decision-making processes, particularly when they are deciding what questions to ask students during mathematics lessons. Researchers who study teachers' questioning practices often consider the type and frequency of questions asked during lessons or they analyze the effect of an imposed intervention on teachers' questioning. Consideration of teachers' beliefs about teaching and learning mathematics was not a significant focus of the studies. Similarly, studies and literature addressing teachers' beliefs in relation to their practice have not focused on the particular practice of question-asking in mathematics classrooms. Reflection is a way to bridge the gaps seen in the literature. This study encouraged teachers to consider their beliefs about mathematics teaching and learning while analyzing the questions they ask during mathematics instruction. By reflecting on the specific practice of question-asking, the pervasiveness of beliefs that inform this widely used instructional practice can be better understood by researchers.

CHAPTER 3 METHODOLOGY

The following chapter describes the methods used to answer the following research questions:

1. What are teachers' question-asking practices during mathematics instruction?

   1a. What reasons do teachers provide for their question-asking practices during mathematics instruction?

2. What relationship exists, if any, between teachers' beliefs about mathematics teaching and learning and the reasons they provide for their question-asking practices during mathematics instruction?

3. What impact does reflection on question-asking practices have on teachers' thinking about the mathematics questions they ask during instruction?

Varela's (1999) concept of deliberate analysis framed the methods used to obtain data on participants' reflections on their question-asking practices. Deliberate analysis consists of using a posterior analysis of one's actions to "reconstruct the intelligent awareness that justifies the action" (p. 32). Intelligent awareness is the knowledge gained from previous experiences that is used to affect new situations (Varela, 1999). Teachers' beliefs about teaching and learning mathematics become part of their intelligent awareness, which affects the common instructional practice of questioning. The deliberate analysis of the questions asked during mathematics instruction provoked teacher reflection. One strategy for engaging teachers in deliberate analysis was to ask them questions (Mustafa, 2005) about their questioning practices in relation to their beliefs about mathematics teaching and learning; this led to critical reflection and helped direct attention to a specific aspect of teaching so that focus remained on a particular topic.

This study employed an explanatory case study approach, which is used to "explain something or to find a cause-and-effect relationship" (Savin-Baden & Major, 2013, p. 156). This approach was used to explain teachers' question-asking practices in relation to inferred and espoused beliefs about teaching and learning mathematics. Four case study participants completed a survey from which the researcher inferred their beliefs about mathematics teaching and learning. Participants then took part in an open-ended interview which allowed them to espouse additional beliefs about mathematics teaching and learning as well as clarify inferences made by the researcher. Ensuing classroom observations were conducted to determine the mathematics questions participants ask during instruction. During reflection meetings, the researcher and participants reflected on the participants' questioning practices in relation to inferred and espoused beliefs about mathematics teaching and mathematics learning. To conclude, participants completed a questionnaire focused on making connections between beliefs, questioning practices, and reflection.

Participant Selection

Middle school and high school administrators were contacted for assistance with recruiting participants. A convenience sampling of middle and high school mathematics teachers was used to recruit volunteers for this study. These grade levels were chosen so that the participants would be engaged in teaching mathematics throughout the school day, unlike most elementary mathematics teachers. Teachers at the middle and high school grade levels have more extensive experience with mathematics teaching and learning on a daily basis and thus were able to provide different insight on the survey regarding beliefs about mathematics teaching and learning than elementary teachers. The identity of all participants and their students was concealed. Pseudonyms are used to refer to the participants and no students are mentioned in the context of the study. The following four middle school and high school mathematics teachers who agreed to the informed consent participated in this study. Tafari has taught mathematics for 6 years to students in grades 5-8 in public and private school settings. Eban has taught mathematics for 11 years to students in grades K- 8 and Algebra and Pre-Calculus to high school students in charter and private school settings. Oliver has taught computers for 7 years to students in grades 4-12 and Algebra and Geometry to high school students in charter, alternative, and Christian school settings. Adisa has taught mathematics and science for 2 years to middle school students and Algebra1, Algebra 2, Geometry, and Pre-Calculus to high school students, for 11 years. She has taught in charter, public and private school settings.

Data Collection

Beliefs inventory survey. The Beliefs Inventory Survey was used to infer teachers' beliefs about mathematics teaching and mathematics learning. Prior to classroom observations, the survey was emailed to participants on March 31, 2017 (Tafari), April 6, 2017 (Eban and Oliver), and April 27, 2017 (Adisa). Participants emailed the completed survey to the researcher within 48 hours of receiving it. Appendix A shows the survey participants were asked to complete. The 27 items on the Beliefs Inventory Survey were a compilation of questions adapted from:

- Hart (2002),
- The International Association for the Evaluation of Educational Achievement (IEA) Third International Mathematics and Science Study (TIMSS) Teacher Questionnaires for Mathematics: 4th Grade Population 1 Section 2 (1995), 8th Grade Population 2 Section A (1995),
- The International Association for the Evaluation of Educational Achievement (IEA) Third International Mathematics and Science Study (TIMSS) Teacher Questionnaires for Mathematics: 8th Grade (2003), and
- The IEA Trends in Mathematics and Science Study (TIMSS) Teacher Questionnaires for Mathematics: 8th Grade (2003) and 8th Grade (2007).

Hart's (2002) Mathematics Beliefs Instrument was used to determine the beliefs of pre-service teachers before and after participating in an urban alternative teacher preparation program. The items on Hart's survey assessed the congruence between pre-service teachers' beliefs and the recommendations put forth by the National Council of Teachers of Mathematics Curriculum and Evaluation Standards of 1989. The TIMSS Teacher Questionnaires for mathematics collect information pertaining to teachers' instructional practices and their attitudes toward teaching mathematics. These instruments were selected because of their inclusion of items pertaining to both teaching and learning mathematics. They contain a mixture of positively and negatively-worded questions that command focus and introspection.

Ernest (1989) claimed, "beliefs about mathematics are reflected in teachers' models of the teaching and learning of mathematics, and hence in their practices" (p. 22). Thus, the context of each survey item applies to one of two categories - *How Mathematics Should be Learned* (e.g. "Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.") or *How Mathematics Should be Taught* (e.g. "Good mathematics teachers should show students lots of different ways to look at the same question."). Appendix B displays the Beliefs Inventory Survey separated into categories (*How Mathematics Should be Learned* and *How Mathematics Should be Taught*) as well as the sources of each survey item.

The researcher coded responses to the survey items as Problem-Solving (PS), Platonist/Problem-Solving (P/PS), Platonist (P), Instrumentalist/Platonist (I/P), or Instrumentalist (I) based on Ernest's (1989) models of teaching and learning mathematics shown in Chapter 2 (see Table 1).

Pre-observation interview. A Pre-Observation Interview was conducted with each participant to review the results of the Beliefs Inventory Survey and to gather additional data on their beliefs about teaching and learning mathematics. The Pre-Observation Interview (see Appendix C) allowed participants to expound on their perceptions of teaching and learning as well as the perceived roles of students and teachers in the mathematics classroom. The interview contained 7 open-ended questions that asked participants to expound on their definitions of teaching and learning, beliefs about teaching and learning mathematics, and teacher and student roles in the classroom.

Classroom observations. The researcher observed 8 full mathematics lessons of each participant. The focus of the observations was *mathematics questions*. These are questions (spoken or written) that contain mathematics content and that elicit responses from students in the form of information or action. For instance, a mathematics question could be, "Which method for solving systems of equations can be used in this example?", "What are the first three multiples of 7?", or "Find the median of the following numbers: $\frac{5}{3}, \frac{2}{5}, \frac{3}{2}, \frac{5}{9}, \frac{4}{7}$." Classroom observations were audio recorded to ensure the collection of all verbal mathematics questions. Worksheets were also collected to record printed mathematics questions. The researcher “purposefully transcribed” audio recordings of each lesson by highlighting participants’ mathematics questions in preparation for reflection meetings.

Reflection meetings. Reflection meetings followed classroom observations. During these meetings, the researcher and participants used the Reflection Meeting guide (see Appendix D) to reflect on specific mathematics questions (chosen by the researcher) asked during previous classroom observation(s). During these meetings, participants were asked why they asked certain mathematics questions. They were also asked whether their response aligned with their beliefs about mathematics teaching and learning. Reasons for asking mathematics questions were considered in relation to inferred and espoused beliefs of the participants. Additionally, details about the next class to be observed and the content to be taught during the observations were discussed. Reflection Meetings were audio recorded and transcribed to ensure accurate descriptions of the participants' reflective thinking.

End-of-study questionnaire. The End-of-Study Questionnaire (see Appendix E) gave participants the opportunity to react to their experience as a participant in this study. The focus of the questionnaire was for participants to make connections among beliefs, questioning practices, and reflection. The End-of-Study Questionnaire was administered after the last Reflection Meeting. The questionnaire was emailed to participants. Upon completion of the questionnaire, participants emailed their responses to the researcher for analysis.

Timeline. Table 3 provides dates for data collection activities.

Table 3

Timeline of Data Collection Activities

	Time for each participant			
Activity	Tafari	Eban	Oliver	Adisa
Beliefs Inventory Survey	April 1, 2017	April 7, 2017	April 6, 2017	April 20, 2017
Pre-Observation Interview	April 4, 2017	April 25, 2017	April 25, 2017	May 1, 2017
Classroom Observation 1	April 10, 2017	April 26, 2017	April 26, 2017	May 1, 2017
Classroom Observation 2	April 25, 2017	May 2, 2017	May 2, 2017	May 1, 2017
Classroom Observation 3	April 27, 2017	May 4, 2017	May 3, 2017	May 3, 2017
Classroom Observation 4	May 5, 2017	May 10, 2017	May 11, 2017	May 3, 2017
Classroom Observation 5	May 9, 2017	May 10, 2017	May 11, 2017	May 8, 2017
Classroom Observation 6	May 11, 2017	May 10, 2017	May 23, 2017	May 8, 2017
Classroom Observation 7	May 15, 2017	May 23, 2017	May 23, 2017	May 9, 2017
Classroom Observation 8	May 16, 2017	May 23, 2017	May 23, 2017	May 10, 2017
Reflection Meeting 1	April 12, 2017	April 28, 2017	April 27, 2017	May 2, 2017
Reflection Meeting 2	May 1, 2017	May 5, 2017	May 11, 2017	May 4, 2017
Reflection Meeting 3	May 10, 2017	May 15, 2017	May 15, 2017	May 9, 2017

Reflection Meeting 4	May 22, 2017	May 25, 2017	May 24, 2017	May 11, 2017
End-of-Study Questionnaire	May 26, 2017	May 29, 2017	May 25, 2017	May 24, 2017

Trustworthiness

Pilot study. A pilot study of the original 39-item version of the Beliefs Inventory Survey was conducted with five mathematics teachers. Twelve survey items were deleted due to the disconnection between the survey item and Ernest's (1989) models of teaching and learning. Initially, the researcher attempted a holistic analysis of the survey responses by drafting a narrative profile of each participant based on their responses. However, this approach did not provide a clear alignment with the participants' responses and Ernest's (1989) models of teaching and learning. After deleting 12 survey items, a second version of the survey was administered. The researcher considered using weighted averages to generalize participants' beliefs. For instance, after assigning points to each type of response (Problem-Solving (PS) - 3 points, Platonist/Problem-Solving (P/PS) - 2.5 points, Platonist (P) - 2 points, Instrumentalist/Platonist (I/P) - 1.5 points, and Instrumentalist (I) - 1 point) a mean score would have been calculated and applied to the range of scores shown in Table 4.

Table 4

Mean Scores Associated with Classification of Beliefs

Classification	Mean
Problem-Solving	2.61 – 3.00
Platonist/Problem-Solving	2.21 – 2.60
Platonist	1.81 – 2.20
Instrumentalist/Platonist	1.41 – 1.80
Instrumentalist	1.00 – 1.40

However, the researcher decided to focus on the mode of responses for each survey category to capture and report on the frequency of each participant's response while acknowledging responses in other classifications. A description of this process is described in the Data Analysis section.

Review of qualitative research tools. After conducting the pilot study, the Beliefs Inventory Survey, Pre-Observation Interview, and Reflection Meeting were reviewed by a panel of six experts to determine face validity and content validity. The panel consisted of three mathematics education researchers, two qualitative researchers, and one quantitative researcher. Each of the eight recommendations for refining the qualitative research tools were followed. Examples of the recommendations made on all three instruments are described below.

Reconsidering the assigned classifications of certain responses on the Beliefs Inventory Survey was suggested. For instance, survey item 4 states, "The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in isolation." A response of agreement to this statement is classified as "Instrumentalist" and a response of disagreement was classified as "Platonist." The suggestion was made to reclassify the disagreement response as "Platonist/Problem-Solving" as a teacher with beliefs associated with a Problem-Solving model of teaching or learning would also disagree with survey item number 4. Additional recommendations on the Beliefs Inventory Survey included removing the capitalization on the words “ABSORB INFORMATION” in survey item 10 due to the possible interpretation of the capitalization as providing value instead of emphasis. Lastly, it was recommended that the classifications (I, I/P, P, P/PS, and PS) associated with each survey item response demonstrate a continuum of opinions instead of having two different responses with the same classification. Initially, survey item 19 had the following classifications associated with each response: True (I/P), More True than False (P), More False than True (PS), and False (PS). Currently the More False than True option is classified as P/PS.

A review of the Pre-Observation Interview yielded suggestions for adding clarifying questions in case participants do not understand the original interview questions. For example, interview question 5 originally asked, " What role are students expected to assume in your classroom?" Upon revision, the following alternate phrasing was added to the interview protocol, "In other words, what role do students play in your classroom? What are they expected to do? Do they have specific academic responsibilities?" Revisions to questions 2 and 3 resulted from a recommendation to ask participants what they “believe” are the best ways to teach/learn mathematics instead of asking what are the best ways to teach/learn mathematics. An additional question was added to the Pre-Observation Interview due to a recommendation to allow participants the opportunity to add any additional information that was not address in the previous interview questions.

Reflection Meetings were designed to reflect on the mathematics questions asked during instruction. Due to the large number of questions expected to be asked during instruction, recommendations were made to specify how the questions for discussion would be selected. The researcher decided to focus on *mathematics questions*, which are defined as questions that contain mathematics content and that elicit responses from students in the form of information or action. The researcher randomly selected the mathematics questions chosen for reflection during Reflection Meetings.

The combination of survey questions, interview questions, observations, and reflection meetings provided triangulation for inferring teachers’ beliefs. Member checks with participants were held to validate the inferences made from the Beliefs Inventory Survey. The researcher collaborated with a disinterested peer to reach 80% agreement on the coding used for Pre-Observation Interviews. The resulting codes were used to develop individual participant profiles and to compare and contrast participants’ definitions and espoused beliefs about mathematics teaching and learning and roles of teachers and students.

Data Management

The researcher managed an electronic file consisting of each participant's Beliefs Inventory Survey and associated participant profile, notes from classroom observations, audio recordings, notes and transcriptions of the Pre-Observation Interview, Reflection Meetings, and the End-of-Study Questionnaire. The files were kept on the researcher’s personal password-protected data storing devices and updated after each observation and interview.

Data Analysis

Data gathered from the Beliefs Inventory Survey, Pre-Observation Interview, Reflection Meetings, and End-of-Study Questionnaire was used as evidence for addressing the research questions.

Each question on the Beliefs Inventory Survey provided multiple choices for responses. Each response was classified as Instrumentalist, Instrumentalist/Platonist, Platonist, Platonist/Problem-Solving, or Problem-Solving. The Beliefs Inventory Survey was scored based on the classification of responses found in Appendix F which shows each question of the survey and the corresponding classification for each response. A descriptive analysis of the survey was conducted by calculating the mode response for each category (*How Mathematics Should be Learned* and *How Mathematics Should be Taught*) as well as for the entire survey. The mode response was used to infer participants' beliefs about models of mathematics teaching and learning, overall, and to generalize their beliefs based on the two categories mentioned above. In instances when a participant’s response data was bimodal, they were described as having beliefs in both classifications. For instance, if a participant has 5 responses classified as Platonist and 5 responses classified as Platonist/Problem-Solving, they were described as having beliefs in both Platonist and

Platonist/Problem-Solving models of teaching and learning. Participants were later made aware of the classification of their beliefs based on their survey responses.

The Pre-Observation Interview was transcribed by an outside source. A combination of inductive and deductive content analysis (Elo & Kyngäs, 2007) was used to perform thematic coding (Boyatzis, 1998). Classroom observations were audio recorded to analyze the types of mathematics questions asked during instruction. Deductive content analysis was used to categorize mathematics questions as high-level or low-level, based on Bloom's Taxonomy. Low-level questions were at the Knowledge level while high-level questions were at the Comprehension level and above as shown in Figure 2.

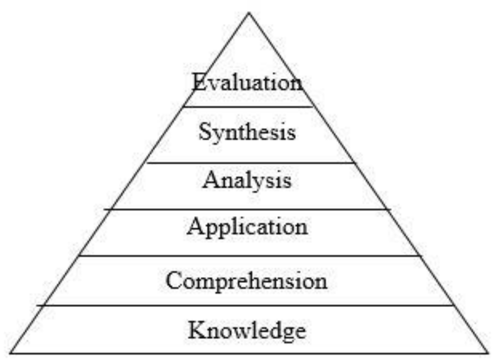


Figure 2. Original Bloom's Taxonomy.

Inductive content analysis was used to code the question types based on the type of response elicited from students. For instance, asking a student to tell why it is necessary to find a common denominator when adding fractions with unlike denominators was coded as a JUSTIFY mathematics question because it prompts a student to tell why a particular step or procedure was used. The same mathematics question would be classified as high-level due to the conceptual comprehension necessary to respond as shown on level two of Bloom's Taxonomy. Appendix G displays a description of each mathematics question code, the classification of the mathematics question, and an example for each code.

Table 5 shows the data used to address each research question of the present study.

Table 5

Data Supporting Research Questions

Research question	Supporting data
Research Question 1	Classroom Observations
Research Question 1a	Reflection Meetings
Research Question 2	Beliefs Inventory Survey
	Pre-Observation Interview
	Reflection Meetings
Research Question 3	Reflection Meetings
	End-of-Study Questionnaire

To address Research Question 1, What are teachers' question-asking practices during mathematics instruction?, the researcher used data from classroom observations to code and calculate the types and levels of questions asked. Research Question 1a, What reasons do teachers provide for their question-asking practices during mathematics instruction?, was addressed by focusing on Reflection Meetings to look for themes in the reasoning participants provided for asking particular mathematics questions. Research Question 2, What relationship exists, if any, between teachers' beliefs about mathematics teaching and learning and the reasons they provide for their question-asking practices during mathematics instruction?, relied on data collected from the Beliefs Inventory Survey and the Pre-Observation Interview to infer teachers' beliefs. Data from Reflection Meetings was used to seek connections between reasons for asking questions to teachers' beliefs. To address Research Question 3, What impact does reflection on question-asking practices have on teachers' thinking about the mathematics questions they ask during instruction?, data from Research Meetings were used to determine participants perceptions of alignment between their beliefs and question-asking practices. The End-of-Study Questionnaire was also analyzed for participants' accounts of the impact of reflection on their question-asking practices. Each data source was also used to create a narrative participant profile that describes each participant's beliefs, question-asking practices, reflections on questioning practices, and impacts of engaging in the reflective process.

CHAPTER 4 RESULTS

The following is a description of each participant's beliefs about mathematics teaching and learning, their questioning practices during mathematics instruction, and their reflections on their own beliefs and questioning practices. Data from the Beliefs Inventory Survey, Pre-Observation Interviews, Classroom Observations, Reflection Meetings, and the End-of-Study Questionnaire were used to develop each participant's profile.

Tafari

"*Confidence, risk-taking, and perseverance are my milk stool, the three legs that hold everything up.*" (Reflection Meeting 2)

Tafari is a middle school teacher who has taught mathematics for six years in public and private school settings. Dialogue was a prominent feature of Tafari's teaching as she used both high-level and low-level questions to promote mathematical thinking and to foster student behaviors she felt were pivotal for learning and practicing mathematics. Tafari sustained an environment of inquiry by asking several mathematics questions during instruction. During eight classroom observations, Tafari asked a total of 667 mathematics questions comprised of 204 (31%) high-level mathematics questions and 463 (69%) low-level mathematics questions. Tafari's question-asking practices were influenced by her desire to develop problem-solvers, build student confidence, and encourage perseverance. Figure 3 shows factors that influenced Tafari's question-asking practices during observed lessons. The center circle represents the core of Tafari's instructional goals; developing students into problem-solvers. The circles surrounding the core indicate that Tafari promoted problem-solving skills by asking questions that: 1) helped students apply their knowledge to real-world contexts; 2) allowed multiple approaches to mathematical tasks; 3) encouraged students to recognize that mathematics concepts build on one another, and 4) engaged students in problem-solving thought processes. The surrounding circles describe behaviors that contribute to problem-solving while the circles on the left of Figure 3 describe dispositions Tafari believes are necessary for becoming problem solvers. Tafari believes academic self-confidence and perseverance are critical dispositions of problem-solvers. Thus, she made concerted efforts to build student confidence by managing student stress related to learning mathematics and pushing students to take risks. Also, Tafari urged students to exercise resilience as they persevered through challenging mathematical tasks.

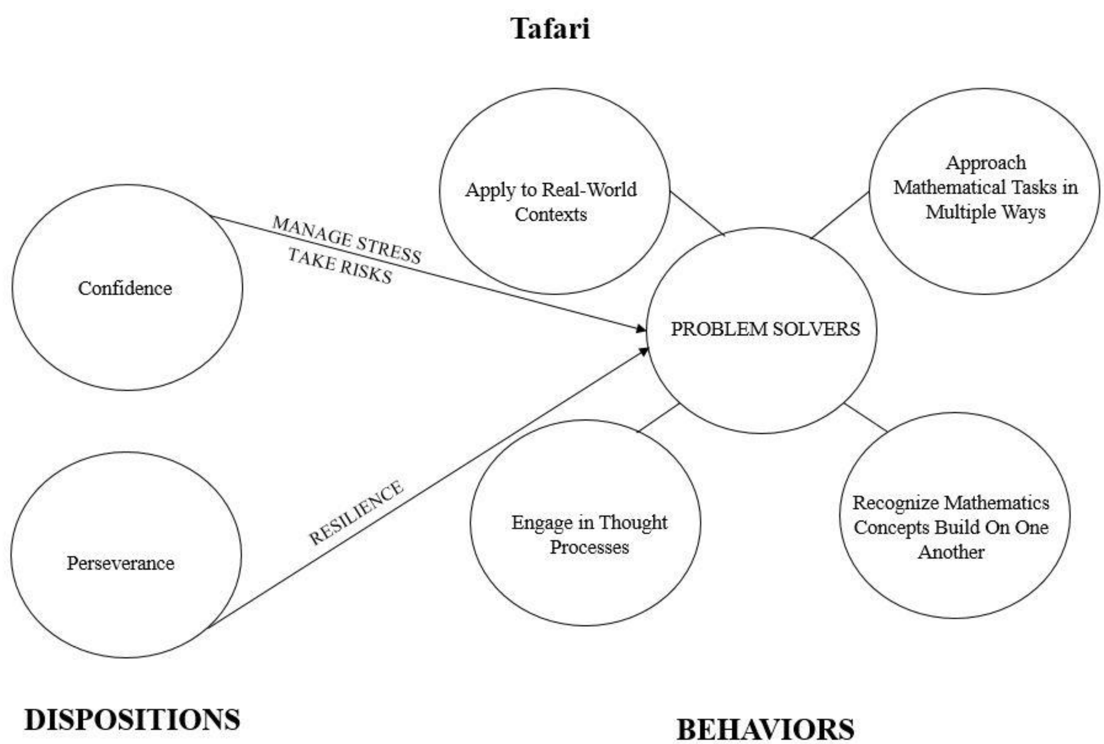


Figure 3. Framework for factors impacting questioning practices – Tafari.

Influences from each of the behaviors and dispositions mentioned above were evident in Tafari's description of her beliefs, in her questioning practices, and during her reflection meetings. Data supporting Tafari's efforts to develop problem-solvers, student confidence, and perseverance was collected from each data source. Dates of the pre-observation interview and four reflection meetings as well as dates and topics of the eight classroom observations are listed in Table 6.

Table 6

Dates of Pre-Observation Interview, Observations, and Reflection Meetings – Tafari

Observation	Topic	Date
	Pre-Observation Interview	April 4, 2017
Observation 1	Surface Area	April 10, 2017
	Reflection Meeting 1	April 12, 2017
Observation 2	Integers	April 25, 2017
Observation 3	Coordinate Plane	April 27, 2017
	Reflection Meeting 2	May 1, 2017
Observation 4	Adding Integers	May 5, 2017
Observation 5	Subtracting Integers	May 9, 2017
	Reflection Meeting 3	May 10, 2017
Observation 6	Multiplying Integers	May 11, 2017
Observation 7	Probability	May 15, 2017
Observation 8	Multiplying Binomials	May 16, 2017
	Reflection Meeting 4	May 22, 2017

Developing problem solvers. Developing problem-solving skills was a major impetus for Tafari's questioning practices. Tafari frequently used questioning practices to engage students in discourse that promoted understanding of mathematical concepts. Her efforts corresponded to the beliefs she shared on the Beliefs Inventory Survey. Figure 4 displays the percentage of times Tafari provided a response to the Beliefs Inventory Survey that aligned with each belief classification. Tafari's beliefs about mathematics teaching are concentrated around the Platonist

(27%) and Platonist/Problem-Solving (33%) classifications. This suggests that Tafari believes mathematics teaching should focus on conceptual understanding with problem-solving enrichment. For instance, Tafari's response (*somewhat agree*) to item 7 which stated, "A demonstration of good reasoning should be regarded even more than students' ability to find correct answers," was classified as Platonist. Tafari's response to item 21, "Students should *often* explain the reasoning behind an idea", was classified as Platonist/Problem-Solving. Tafari's beliefs about mathematics learning are concentrated toward the Platonist/Problem-Solving (42%) and Problem-Solving (42%) classifications which implies that she believes mathematics learning should consist of students' constructed knowledge and should be driven by student interests. For instance, Tafari's response (*somewhat disagree*) to item 20 which stated, "The best way to do well in math is to memorize all the formulas," was classified as Platonist/Problem-Solving. Additionally, Tafari's disagreement with item 11, "Mathematics should be learned as sets of algorithms or rules that cover all possibilities", was classified as Problem-Solving.

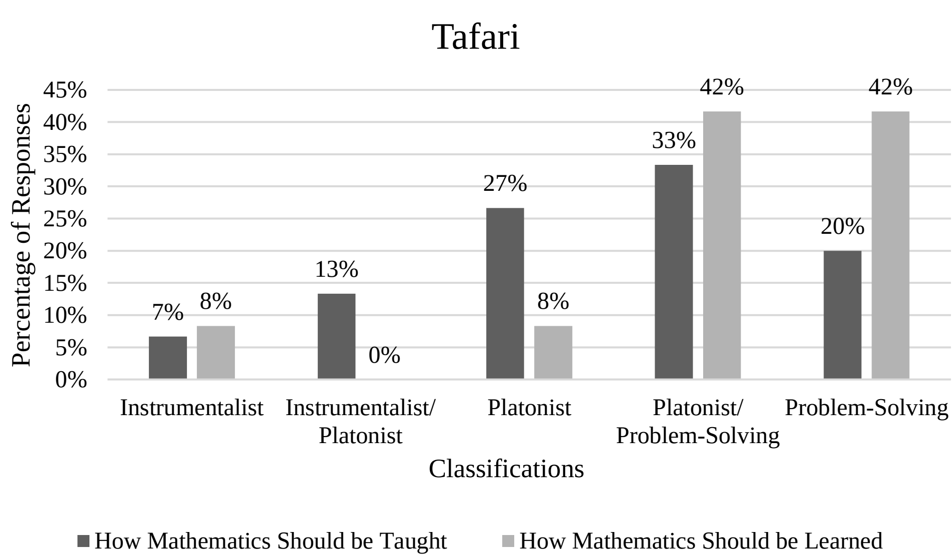


Figure 4. Percentage of survey responses in each classification – Tafari.

To help her students develop problem-solving skills, Tafari asked a combination of high-level and low-level questions to engage students in the lessons and to bring about conceptual understanding and procedural knowledge (see Table 7). Tafari asked 17 different types of high-level questions. As stated in Chapter 2, high-level mathematics questions allow students the opportunity to develop the mathematical practices of reasoning, justifying, and analyzing by requiring students to explain, describe, or investigate. Tafari asked Classify (13.2%) questions most frequently; particularly during Observation 1 when she often asked students to name shapes based on their physical qualities. Justify (11.3%), Explain (10.3%), and Clarify (9.8%) questions were also asked frequently as Tafari asked questions that engaged students in dialogue during instruction. Below are examples of high-level mathematics questions Tafari asked during observed lessons:

- "What makes this a prism?" (Classify)
- "Why did we subtract here [$6 + -5$]?" (Justify)
- "Tell me how that [spinner] is probability." (Explain)
- "What do we notice about the absolute value of opposite numbers?" (Generalize)
- "How many combinations, with two dice, do I have with a total less than six?" (Reason)

High-level mathematics questions were asked most frequently during Observation 6; a lesson on multiplying integers. During this lesson, Tafari repeatedly asked students to generalize about or justify the sign of the product of integers.

Low-level mathematics questions refer to questions that are leading or that require students to name, state, memorize, or recall. These types of questions also allow students to provide yes/no, technical, factual, or simple responses. Of the five different types of low-level mathematics questions asked, Fact (59%) and Procedure (27%) questions were asked by Tafari substantially more often than other types of low-level questions. Low-level mathematics questions were asked most frequently in Observation 8 (Multiplying Binomials, May 16, 2017) in which 56 of the 66 mathematics questions asked were Fact (55%) and Procedure (30%) questions. This lesson focused on the process for multiplying binomials. To that end, the Fact questions often asked students to provide the products obtained when multiplying terms and the Procedure questions reinforced the binomial multiplication algorithm. For example, while multiplying $(p+2)(3-q)$ Tafari asked a series of Fact and Procedure questions which included:

- "What's the first thing I need to do here?" (Procedure)
- "What is p times 3?" (Fact)
- "p times $-q$?" (Fact)
- "Now what?" (Procedure)
- "2 times 3 is?" (Fact)
- "2 times $-q$?" (Fact)
- "Now what do I do?" (Procedure)

The examples above, of the high-level mathematics questions Tafari asked, promote conceptual understanding of mathematics because students were asked to use their knowledge of mathematical ideas to address the questions. The examples above, of the low-level mathematics questions Tafari asked, demonstrate the instrumental use of mathematics because students used their fact and procedural knowledge to perform an algorithm. Table 7 displays the type and frequency of high-level and low-level mathematics questions Tafari asked during classroom observations. The table shows that although Tafari asked more types of high-level mathematics questions, she asked low-level mathematics questions most frequently.

Table 7

Frequency and Percentage of High-Level and Low-Level Mathematics Questions – Tafari

High-level			Low-level		
Question type	Frequency	% of high-level questions	Question type	Frequency	% of low-level questions
Classify	27	13.2%	Fact	274	59.2 %
Justify	23	11.3%	Procedure	126	27.2%
Explain	21	10.3%	Agree/Disagree	31	6.7%
Clarify	20	9.8%	Vocabulary	28	6.0%
Generalize	18	8.8%	Recall	4	0.9%
Reason	18	8.8%			
Multiple Representations	13	6.4%			
Definition_Open	12	5.9%			
Contrast	10	4.9%			
Compare	7	3.4%			
Predict	7	3.4%			
Recognize a Pattern	7	3.4%			
Analyze	6	2.9%			
Student Questions	5	2.5%			
Understand the Reasoning of Others	5	2.5%			
Make a Connection	4	2.0%			
Make Sense	1	0.5%			

Note. The *Agree/Disagree* type of low-level mathematics questions include: Agree/Disagree with a Classification, Agree/Disagree with a Definition, Agree/Disagree with a Fact, Agree/Disagree with a Prediction, Agree/Disagree with a Procedure, Agree/Disagree with Reasoning, and Agree/Disagree with Vocabulary.

To further contribute to the development of problem solvers, Tafari wanted her students to experience mathematics in real world contexts, realize that problems can be approached in multiple ways, and recognize that concepts build on one another. Students who understand the relationship between concepts can be creative in their approach to mathematical tasks, especially tasks with real world implications. Additionally, Tafari aimed to use her questioning practices to help her students manage sophisticated mathematical procedures and to model thought processes she wanted her students to engage in when completing mathematical tasks on their own.

Real world context. While discussing her role as a mathematics teacher, Tafari explained her obligation to help students realize that mathematics has purposes outside of the classroom. She stated:

> A lot of the kids are like, 'Well I'm not going to be a lawyer I'm never going to do this.' I want to make it relevant to them in whatever we're doing and show that it might carry over to something else outside of math (Pre-Observation Interview).

According to the Beliefs Inventory Survey item 2, Tafari agreed that teachers' modeling of real-world problems is essential to teaching mathematics. She added, "When students have a real-world context to math it 1) makes the skill seem less 'scary' and 2) highlights the importance/relevance of mathematics in everyday life" (Open-ended response to Beliefs Inventory Survey item 2, April 1, 2017).

During Reflection Meeting 1, Tafari reflected on the mathematics questions she asked students during Observation 1. During a brief discussion about the difference between area and volume, a student handed Tafari a small cylinder-shaped can which Tafari held high in the air and asked the following questions:

- "Anybody know what shape this would be?" (Classify)

- "It's going to hold a certain amount of stuff, right? Do we agree that that would be volume?" (Agree/Disagree with a Definition)
- "Why is it not area?" (Definition_Open)
- "What is different about this than area?" (Definition_Open)
- "That means this is a cylinder and not a what?" (Classify)

Tafari claimed she asked if anyone knew what the shape would be because, "It gives them [students] something that actually exists. It's not just a drawing on a paper. It's not just a bunch of words" (Reflection Meeting 1). Tafari felt this question aligned with her beliefs stating, "Anytime I can put some type of a real-world context, it helps them not only to know that this is useful, but why it is important that I invest my time in this." (Reflection Meeting 1). To provide a real-world context for multiplying binomials, Tafari presented a high-level mathematics question by asking students to find the area of the walkway in Figure 5.

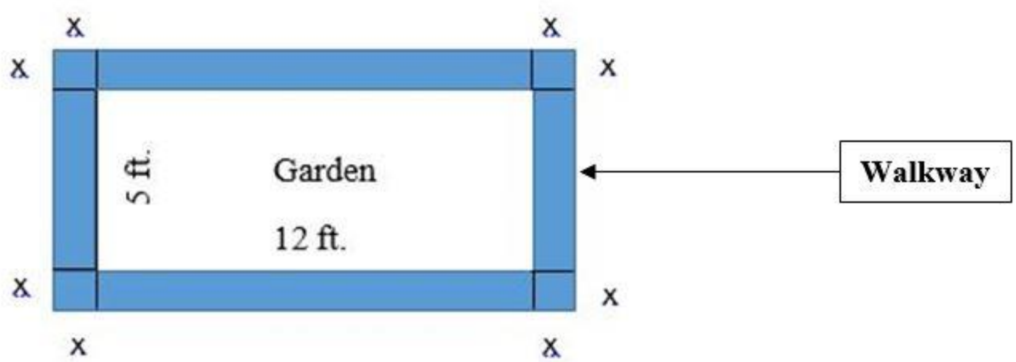


Figure 5. Area of the walkway problem.

This high-level mathematics question encouraged students to use reasoning as they evaluated the situation and determined a problem solving strategy. After a series of low-level questions such as:

- "What shape is this?" (Fact),
- "What's the width?" (Fact),
- What's the area of the small rectangle?" (Fact),

the class came to see that they could multiply the binomials, 5+2x and 12+2x, as part of the process to find the area of the walkway, which reinforced the overall concept of the lesson - multiplying binomials. During Reflection Meeting 4, Tafari further described her belief that students should apply mathematics to their lives. While reflecting on her beliefs about how mathematics should be taught, she stated, "I think using it is the biggest key. Ultimately, at the end of the day, we want the kids to be able to be out in the world using it [mathematics]" (Reflection Meeting 4).

Approaching mathematical tasks in multiple ways. While describing the best ways to teach mathematics, Tafari criticized the way she experienced mathematics as a student by stating, "I'm so mad I was taught that there was only one way, that there's just one right answer, just one way to get there" (Pre-Observation Interview). However, her teaching experience has led her to a different view of practicing mathematics. Now, Tafari wants her students to know that, "with math, there's not just one way" (Pre-Observation Interview). According to the Beliefs Inventory Survey item 22, Tafari believes students should often decide on their own procedures for solving complex problems. She disagreed that students should be encouraged to justify their solutions or thinking in a single way (Beliefs Inventory Survey item 3). Tafari clarified her disagreement by stating, "Children should be encouraged to use multiple ways to come to a result and explain in numerous ways, when appropriate" (Open-ended response to Beliefs Inventory Survey item 3). In addition to encouraging students to use and explain multiple strategies, Tafari also perceives teachers' use of multiple approaches to mathematical tasks as an eminent teaching practice. On item 17 of the Beliefs Inventory Survey, Tafari disagreed that mathematics problems can be done correctly in only one way. Additionally, she agreed that good mathematics teachers should show students lots of different ways to look at the same question and that more than one representation should be used in teaching a mathematical concept (Beliefs Inventory Survey items 16 and 8).

During Observation 5, students were learning different ways to view subtraction of integers. Students volunteered to walk along a number line to show the procedure for performing $-3-8$ on a number line. Next, Tafari showed the students how to rewrite $-3-8$ as $-3+\ -8$ while emphasizing the equality of the two expressions. Tafari then asked, “Looking at the two [$-3-8$ and $-3+-8$], which one do you think you’ll have the most success at doing correctly?” After students selected one of the two expressions, Tafari commented:

> There’s not just one way to come to a right answer. This problem is written in subtraction, you can completely do the subtraction and get the right answer. But I can also do the subtraction problem as addition as a second way that I can do that, and still get the right answer. So, my goal is to give you as many tools as possible for you to work with; whichever makes you comfortable.

Tafari not only presented multiple ways to approach subtraction of integers, she also encouraged students to exercise freedom in choosing the method that they feel most confident using. While reflecting on why she asked students which expression they would have the most success at doing correctly, Tafari stated, “I wanted them to see that although they looked different, and they are different, the process of what’s going on mathematically, is the same… they’re both going to get you to the same place” (Reflection Meeting 3). She confirmed that the discussion with her students aligned with her beliefs about teaching and learning mathematics by stating:

> I really think that math needs to be taught that way. That it can’t just be just one set of rules, it can’t just be it’s this way or no way. Or even just a this or that. I really think that it needs to be more fluid (Reflection Meeting 3).

Tafari wanted to impress upon the students that they have the freedom to choose a mathematical approach that was comfortable for them. Tafari believed that the more comfortable students felt, the more likely they would be successful when practicing mathematics. Thus, if students were comfortable practicing mathematics, they would become more confident mathematics students.

Mathematical concepts build on one another. While discussing her role as a mathematics teacher, Tafari stated that her job is to help students recall what they already know, take it to a deeper level, and, at times, assign new vocabulary to what they know. (Pre-Observation Interview). Tafari often reminded her students that mathematics concepts build on one another and that her students already possessed the fundamental knowledge necessary to explore new concepts. Accordingly, most of Tafari's lessons began with a question to introduce the topic. The introductory question was generally a low-level mathematics question that asked students to recall and state previous knowledge by providing a definition, fact or vocabulary term. For instance, Tafari began each observed lesson by asking students what they already knew about the topic to be discussed. Introductory questions included:

- Observation 1: "Surface area, what in that do we recognize already?" (Vocabulary), "What is area?" (Fact)
- Observation 2: "Who in here can tell me what they remember about integers?" (Recall)
- Observation 3: "Search your memory for what you remember about plane." (Recall)
- Observation 4: "Who can use their own words to tell me what absolute value means?" (Fact)
- Observation 5: "What are integers?" (Fact)
- Observation 6: "What are the four main operations?" (Fact)
- Observation 7: "What is probability?" (Fact)

The introductory questions above helped students recall information that would be relevant for the subsequent lesson. Being a problem-solver involves the ability to recognize relationships among mathematical ideas as well as the capacity to think critically. Tafari helped students think about concepts and procedures by modeling necessary thought processes.

Modeling thought processes. Tafari phrased many of her mathematics questions in first person while leading her students through a sophisticated mathematical procedure and while modeling thought processes she wanted her students to engage in when completing mathematical tasks on their own. For instance, to help students find the area of the walkway in Figure 5 above, Tafari attempted to make the task more manageable by asking:

- "*Do I* have enough information to find the area of the walkway?" (Reason)
- "*Do I* know how to find the area of the big rectangle?" (Reason)
- "*Do I* know how to find the area of the smaller one?" (Reason)
- "*What can I do* to find the area of this whole part on the outside, taking the middle part out?" (Procedure)

During Observation 1, Tafari used questioning to model the thought process for finding the surface area of various 3-dimensional figures. While discussing the procedure for finding the surface area of a square pyramid, Tafari asked the following series of low-level questions:

- "*Could I* find the surface area of a pyramid?" (Definition_Open)
- "*What do I need to do*, then, to find the surface area of the square pyramid?" (Procedure)
- "How many areas *do I* need to find?" (Fact)
- "*How do I* find the area of this triangle?" (Procedure)

Upon reflection on the questions above, Tafari stated:

> These are the questions they would be asking themselves—well hopefully be asking themselves while they're doing the problem. I'm trying to model for them too what it

> would look like, what it would sound like as they're solving these problems by themselves by doing it together (Reflection Meeting 1).

The mathematics questions above helped students navigate the processes for finding the area of a walkway bordering a garden and for calculating surface areas of 3 dimensional figures by helping them conceptualize the problems as a collection of smaller tasks that they already knew how to complete and by presenting the questions they should ask themselves when completing tasks on their own.

To develop problem-solving skills such as reasoning, analyzing, and critical thinking, Tafari exposed her students to scenarios they could encounter in the world and encouraged them to think critically, yet systematically, about an appropriate approach to a task. While considering approaches to tasks, students needed to rely on their current knowledge to face new tasks and ultimately build new knowledge. However, Tafari believes there is more to learning mathematics than solving problems. Tafari feels that having the confidence to try, even if the attempt may be erroneous, is pivotal in students' experience with mathematics. Accordingly, Tafari also used questioning to build student confidence.

Building student confidence. Building student confidence by avoiding stressful learning experiences and pushing students to take risks was considered when Tafari taught mathematics. Tafari often referred to student confidence when describing best ways to teach mathematics and when describing her role as a mathematics teacher. While discussing her teaching philosophy, Tafari stated, "I know the kids come to me already with their mind sets and I tell them from the beginning… those of you who think you are not good at math I'm going to show you that you are" (Pre-Observation Interview). Tafari acknowledges that some students come to her class with negative predispositions about their mathematical ability. For those students who lack confidence in their ability to do mathematics, she intends to show them that they are good mathematics students.

Tafari feels it is important for students to experience mathematics with as little stress as possible. One way she attempts to manage her students' stress is by rewarding effort more than correct answers. When determining final grades, Tafari places more emphasis on homework performance than on test performance. Because she recognizes that some students experience anxiety when taking tests, she allows students to correct the test questions they answered incorrectly and resubmit the test to receive partial credit on the corrected questions.

Tafari also used questioning to manage students' stress while teaching. During Observation 1, Tafari held up a rectangular prism and asked, "How do you find the area of this rectangle?" while pointing to one of the faces on the prism. Amid reflection on this lesson, Tafari revealed that she asked that question because she wanted students to feel that finding surface area is a manageable process that incorporates their current knowledge of finding areas of 2-dimensional figures. Tafari stated:

> I want them to see that it's more manageable, but too, also to help build that confidence in them that, 'Oh, yeah. Yeah. I know how to do a rectangle. That's easy.' I get them to start to feel a little bit on the inside like, 'Oh, this is manageable. Nothing about multiplying two sides together is hard for me.' So, when they're doing the whole thing, at least hopefully in the back of their mind like, 'I can do this. I just have to do one thing, then the next, then the next' (Reflection Meeting 1).

During Reflection Meeting 2, Tafari reflected on why she asked a rapid series of Fact and Procedure questions during Observation 3. Tafari stated that she not only wanted the process for plotting points to be instinctual for her students, but she also wanted them to feel more comfortable using the coordinate plane. For instance, while practicing the procedures for plotting points on the coordinate plane, Tafari asked:

- "7, is that positive or negative?" (Fact)
- "So am I going to go to the left or the right?" (Procedure)
- "How many places do I go to the right?" (Procedure)
- "Am I done?" (Procedure)
- "This 4 is what coordinate?" (Fact)
- "Is this 4 positive or negative?" (Fact)
- "So, should I go up or down?" (Procedure)
- "How many should I go up?" (Procedure)

Tafari claimed that the series of questions above did not align with her beliefs because the questions focused on mastery of a skill. However, in this instance, she only considered the types of responses her questions elicited rather than the intended outcome from the questions she asked. While the Fact and Procedure questions she asked were coded as low-level questions because they required students to state a fact or tell the next step in a procedure, Tafari used them to create an experience she felt was important for her students' learning.

Encouraging fearless risk taking was another way Tafari tried to build student confidence. She stated, "The best way to go about teaching math is to definitely get that environment where kids can feel like they can be risk takers…and just get this kind of confidence built up" (Pre-Observation Interview). She does not want the fear of being wrong to impede her students from trying. While practicing in class, Tafari wants her students to have the confidence to dive in and play with the concepts; knowing that she is a resource if or when assistance is needed. In addition to being creative thinkers and applying what they learn during practice, Tafari wants her students to take on challenges and feel confident experimenting with mathematical ideas. In Reflection Meeting 3, Tafari stated that she asked, “Does anybody disagree that all numbers are integers?” (Agree or Disagree with a Classification) because she wanted to encourage participation from students who previously had not been verbally engaged. Prior to asking that question, she noticed that some students were merely nodding in agreement with a consistently vocal student who claimed all numbers were integers. She wanted to allow students the opportunity to verbalize their disagreement – even if their opinion was not popular. Disagreeing with the majority opinion and being comfortable giving an incorrect response corresponds to Tafari’s beliefs that students should be confident risk-takers.

Perseverance. Tafari tried to appeal to her students with low mathematics confidence by affirming that making errors is an opportunity to exercise resilience. She stated:

> By the time they [students] get to me, they’ve already decided that either they’re good at it [math] or they’re not. [I] try to break down that idea that I’m not good at it and I’ll never be good at it…getting the kids to feel like it’s okay to try and fail as long as they know they can try again” (Pre-Observation Interview).

Tafari wanted her students to persevere whenever they encountered obstacles with mathematics learning. Exerting effort and learning from mistakes were ways students were expected to exhibit perseverance. Effort is the main consideration when Tafari scores homework; she wants her students to see the value in trying to complete the assignment. When discussing her regard for grading homework, Tafari stated, “The work that I give them to do is graded ‘did you try?’ You can have everything wrong, but you’ll get an A because you have to show me your work. Show me step by step” (Pre-Observation Interview).

Learning from mistakes was an essential lesson in resilience which Tafari tried to convey to her students. Tafari wanted her students to feel comfortable making mistakes and to be resilient in recovering from them. When asked about the best ways to learn mathematics, Tafari expressed one of the best ways by stating, “I think you have to make mistakes. We talk a lot in my class about how you learn more from a mistake than from a success” (Pre-Observation Interview). During Observation 1, Tafari asked a student to define *area*. After asking who agreed or disagreed with the student’s response, Tafari noticed that a student disagreed with the response, so she asked, “Why do you disagree?” Upon reflecting on why she asked the latter question, Tafari stated, “I tell them [students] all the time, we learn more from a mistake than we do from a success. So, when something is wrong, it’s okay...I want them to feel comfortable being wrong in front of each other” (Reflection Meeting 1). However, it is not just making a mistake, but finding and correcting mistakes that foster learning. As mentioned above, Tafari allows students to correct their tests to receive additional points toward their score to manage the stress associated with test-taking. During this process, students not only have to correct their answers, but they also must cite the section in the textbook where they found information on the related topic (Pre-Observation Interview). This process reinforced concepts or procedures they had yet to master.

During Observation 4, students practiced the correct way to read the expression $|x+y|$. Tafari called on a student (Student A) who was reluctant to respond, but Tafari encouraged her to try anyway by saying, “It’s OK, just try.” Still reluctant, the student said, “No.” and Tafari responded, “You don’t have to say no, you have to try. It’s OK to be wrong, you’re just learning. You have to be comfortable with being wrong because otherwise you won’t learn.” The following exchange proceeded:

Student A: “Parentheses x plus y”

Tafari: “Very close right? Student B, tell her what she missed.

Student B: “The absolute value of x plus y”

Tafari: (to student A) “So what did you say? You said parentheses x plus y. What did Student B say?”

Student A: “The absolute value of x plus y”

Tafari went on to provide formal instruction on the proper way to read an expression containing absolute value brackets. Upon reflecting on the teaching decision to have Student A compare the two readings, Tafari stated that she wanted to stress the importance of communication with precise terminology and to reinforce that you learn from mistakes. She stated, “I definitely keep this close to heart, that you learn more from a mistake. If you had that wrong, and you understand why it was wrong, you’re less likely to make that mistake again in the future” (Reflection Meeting 3). Instead of assuming Student A would decipher the difference between the two readings and adjust her thinking on her own, Tafari used questioning to force Student A to recognize her own error and to learn from another student in the class.

Tafari wanted to support and impart on students the necessity of confidence and perseverance while learning and practicing mathematics. She wanted her students to take risks confidently and be resilient in response to errors, misconceptions, or low grades. Tafari described her philosophy on the traits students need to practice mathematics by stating, “Confidence, risk-taking and perseverance are my milk stool, the three legs that hold everything up”.

Reflections on Questioning Practices. Tafari’s deliberate analysis of the questions she asked during instruction incorporated her beliefs about her students and about mathematics teaching and learning. Based on data collected from four reflection meetings, Tafari agreed 61.5% of the time that her questioning practices aligned with her beliefs. Table 8 shows the percentage of times

Tafari felt her beliefs were in alignment with the high-level and low-level mathematics questions asked during instruction. This table shows that Tafari felt her high-level mathematics questions always aligned with her beliefs, while her low-level mathematics questions were often not aligned with her beliefs. The instance in which Tafari felt her low-level question was aligned with her beliefs occurred during Observation 3. While discussing numbers on the left of zero on the x-axis, the students stated that the numbers get smaller as you move toward the left. Tafari then asked, "What's getting smaller about them?" Although this question was coded as a Fact question, Tafari claimed that students tend to have a misconception about the values of negative numbers. She believed her question aligned with her beliefs as she was trying to reinforce conceptual understanding of the relative value of negative numbers.

Table 8

Percentage of Times Beliefs Aligned with Questioning Practices - Tafari

	High-level mathematics questions	Low-level mathematics questions
Aligned	100%	17%
Not Aligned	0%	83%

There were times which Tafari felt the low-level questions she asked did not align with her beliefs. Tafari felt some Fact, Procedure and Vocabulary questions such as:

- "What's half of 2?" (Fact),
- "If I'm at the origin, do I have to go to the left for my x?" (Procedure),
- "What's another math word for when I add two numbers that are the same?" (Vocabulary),

did not align with her beliefs because they were, "not so deep thinking" (Reflection Meeting 2). During Reflection Meeting 4, Tafari reflected on Observation 8 which consisted of 60 low-level questions out of a total of 66 mathematics questions. When asked why she asked more low-level questions in this lesson than other lessons, she referred to her students and her own experience with the topic. Tafari stated that the students were low [performing] and she presumed that her students were anxious about the end of the school year, thus their attention span was also low. She also mentioned that the students were very social and would use the time given to think about a question to socialize instead. Therefore, to keep her students focused on the lesson, she stated, "[I] forced them to give me answers instead of giving them time to think" (Reflection Meeting 4). Asking Fact and Procedure questions coincided with her intentions as they tend to have short responses that can be provided quickly to keep students engaged. Tafari also referred to her own experience with teaching polynomials by admitting this was her first year teaching polynomials to students who were not in her accelerated class and she didn't know a better way to teach the topic. Although Tafari did not believe the questions she asked during this lesson aligned with her beliefs, she stated that students must be able to "crank out the numbers" when asked to perform operations. Although many of the questions in that lesson were coded as Fact or Procedure questions, the reasons for asking them pertained to practicing classroom management and her lack of experience teaching the topic to students on a slower pace.

Although Tafari believed the questioning practices discussed during reflection meetings generally supported her beliefs about mathematics teaching and learning, the percentage of high-level and low-level mathematics questions fluctuated between reflection meetings. Figure 6 shows the percentage of high-level and low-level mathematics questions Tafari asked between reflection meetings. The first observation after each reflection meeting (Observation 2, Observation 4, and Observation 6) showed an increase in the percentage of high-level mathematics questions (which Tafari believed aligned with her beliefs about mathematics teaching and learning) and a decrease in the percentage of low-level mathematics questions (which Tafari believed did not align with her beliefs). However, during the subsequent observations, prior to the next reflection meeting (Observation 3, Observation 5, and Observation 7 and 8), the trend reversed as the percentage of high-level mathematics questions decreased and the percentage of low-level mathematics questions increased. The reflection meetings did not have a noticeable effect on the percentage of high-level and low-level questions asked during instruction. Overall, 31% of Tafari's questions were high-level and 69% were low-level; which is closely related to her baseline percentages as determined by Observation 1.

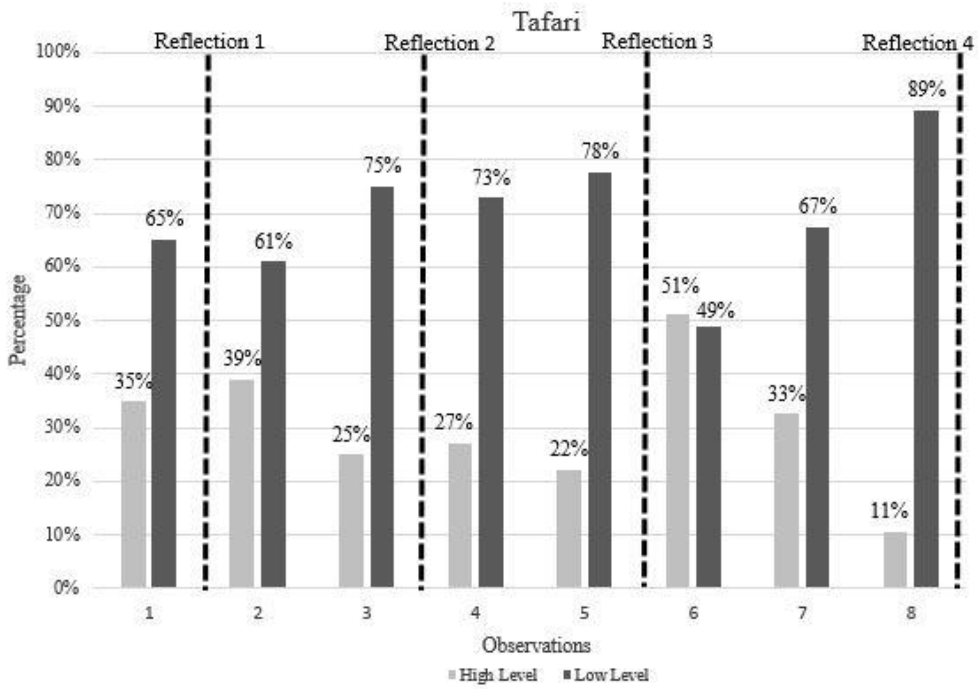


Figure 6. Questioning practices between reflection meetings – Tafari.

The End-of Study Questionnaire revealed the impact of reflection on Tafari's questioning practices. Tafari claimed that the reflection meetings affected her perception of her questioning practices. She stated that her questioning practices, particularly the phrasing of her questions, are influenced by the content she teaches, her beliefs about mathematics, and her students. Although her intention was to allow students to discover and become problem-solvers, she admitted that her questions were also dictated by the content she teaches regardless of alignment to her beliefs. Tafari stated:

> I tend to skew towards having students be problem-solvers and discoverers so many of my questions give students room to find the mathematical reasoning on their own. But the content also dictated the types of questions I was asking, regardless if those questions aligned with my beliefs (End-of-Study Questionnaire, May 26, 2017).

Tafari continued to explain the incongruence between her questioning practices and her beliefs by claiming, “I do not think of math as purely a system of rules and steps to perform. But sometimes my questions are only about this style because of the math I'm teaching (such as how to multiply polynomials)” (End-of-Study Questionnaire, May 26, 2017).

Additionally, Tafari stated that participating in reflection meetings provoked her to think about her questions both before and after asking them:

> At the beginning, I didn't put a lot of thought into the specific questions I was asking. After meeting and discussing my questions, I found myself reflecting on my questions in the moments following asking them and adjusting my approach within the lesson.

Tafari used reflection-in-action to consider the questions she planned to ask and to offer a variety of questioning styles, “I found that while I was teaching, I was analyzing the questions I was about to ask even in the thick of the lesson. I was much more cognizant of what my expectations for the students were and modified my questions accordingly. I also found myself trying to vary my questioning style more” (End-of-Study Questionnaire, May 26, 2017).

Eban

"Math is a process and you go through step one, step two, step three. Even though the numbers may change, the variables may change, you still do the same thing" (Reflection Meeting 1).

Eban has taught mathematics for eleven years to students in grades K – 12 in both private and charter school settings. Eban perceives mathematics as a mechanical process focused on the correct performance of procedures. Emulation was a key feature in Eban's classes as she modeled mathematical procedures and required students to practice the demonstrated procedures independently or with other students. Figure 7 displays Eban's views on the nature of mathematics, mathematics teaching, and evidence of mathematics learning.

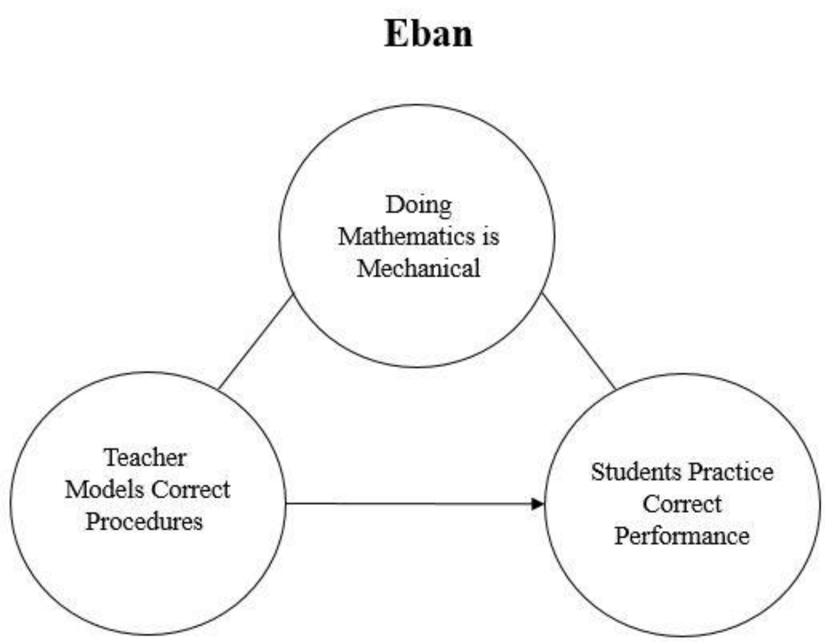


Figure 7. Framework for factors impacting questioning practices – Eban.

The top circle indicates Eban's mechanical view of the study of mathematics. Her view of mathematics impacted her model of teaching which was centered on modeling the correct performance of procedures; as shown on the circle of the left. Eban's mechanical perception of mathematics also influenced her views on evidence of mathematics learning. Eban believes learning occurs when students are able to correctly perform mathematical procedures, as the circle on the right shows. The instrumentalist style of teaching and learning prevalent during observations of Eban's classroom teaching was incongruent with the beliefs she shared during the Beliefs Inventory Survey.

Figure 8 displays percentages of Eban's responses to the Beliefs Inventory Survey that aligned with each belief classification. Eban's beliefs about mathematics teaching aligned with both Platonist (27%) and Problem-Solving (27%) classifications. That is, Eban believes teachers should act as explainers with an instructional goal of conceptual understanding as well as facilitators of student investigation, problem-solving and problem-posing.

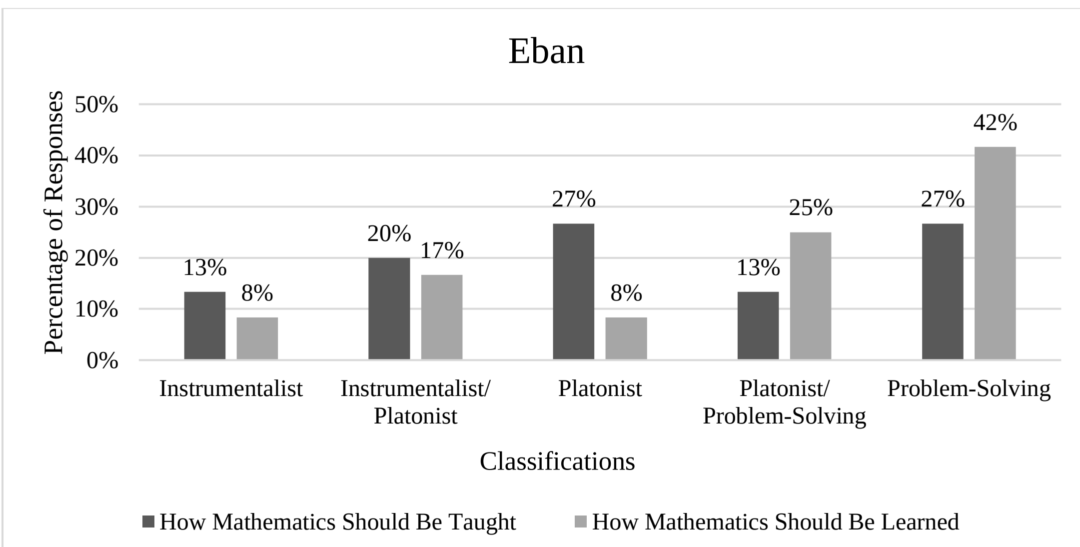


Figure 8. Percentage of survey responses in each classification – Eban.

On the Beliefs Inventory Survey, Eban's responses to item 21 (Students should *sometimes* explain the reasoning behind an idea) and item 22 (Students should *sometimes* decide on their own procedures for solving complex problems) were classified as Platonist. Her disagreement with item 1 which stated, "Problem solving should be a separate, distinct part of the mathematics curriculum," was classified as Problem-Solving. Eban's beliefs about mathematics learning coincide with a Problem-Solving model of learning (42%). In other words, Eban believes students learn mathematics by exploring and constructing knowledge through autonomous pursuit of their own interests. Eban's agreement with item 12, (Solving mathematics problems often involves hypothesizing, estimating, testing, and modifying findings), was classified as Problem-Solving. Additional data on Eban's beliefs about teaching and learning mathematics were collected during the Pre-Observation Interview, Classroom Observations, and Reflection Meetings. The dates, topics and grade levels of each observation, as well as the dates of the Pre-Observation Interview and each Reflection Meeting are listed in Table 9.

Eban's beliefs about mathematics teaching and learning coincide with teaching models that encourage creative thinking and an understanding of underlying mathematical concepts. However, Eban's use of questioning was more aligned with Instrumentalist models of teaching and learning. Instrumentalist models of teaching consist of teachers who act as instructors who strictly use the text. Their instructional goal is mastery of skills and facts shown by correct performance. Instrumentalist models of learning include students who are compliant and focused on mastery of skills. During the Pre-Observation Interview, Eban claimed her role as a mathematics teacher was to model, ask questions, and assess student performance, while her students' roles included practicing, taking notes, and being respectful and quiet during lessons.

Eban believes that mathematics is mechanical, can be taught by demonstration, and is learned by exhibiting correct performance. Correspondingly, Eban defined teaching by stating:

> I model, I ask questions, and I allow them to practice. I go through the processes of first finding out what they know, showing them the lesson, doing some independent practice, allowing them to do homework, and going over that process until they get it. So that's where the learning shows" (Pre-Observation Interview).

Table 9

Dates of Pre-Observation Interview, Observations, and Reflection Meetings - Eban

Observation	Topic	Date
	Pre-Observation Interview	April 25, 2017
Observation 1	Solutions of Quadratic Equations	April 26, 2017
	Reflection Meeting 1	April 28, 2017
Observation 2	Multiplying Polynomials	May 2, 2017
Observation 3	Multiplying Binomials	May 4, 2017
	Reflection Meeting 2	May 5, 2017
Observation 4	Factoring Quadratic Expressions	May 10, 2017
Observation 5	Graphing Quadratic Equations	May 10, 2017
Observation 6	Solving Systems of Equations	May 10, 2017
	Reflection Meeting 3	May 15, 2017
Observation 7	Factoring with Difference of Squares	May 23, 2017
Observation 8	Quadratic Formula	May 23, 2017
	Reflection Meeting 4	May 25, 2017

Evidence of Eban's adherence to this process is present in data collected during Classroom Observations and Reflection Meetings. Eban reinforced the mechanics of performing mathematics by using questions as she modelled mathematical procedures and led students through practice exercises.

Mechanical performance of mathematics. Several of Eban's mathematics questions focused on the mechanics of performing mathematical tasks. Eban believes, "Math is a pattern, and there's procedures, and steps-by-steps to get to the final answer" (Reflection Meeting 3). Accordingly, many of her mathematics questions were procedural, which were coded as low-level questions. Table 10 displays the type and frequency of high-level and low-level mathematics questions Eban asked during classroom observations. During eight classroom observations, Eban asked a total of 487 mathematics questions comprised of 428 (88%) low-level mathematics questions and 59 (12%) high-level mathematics questions.

Of the 5 different types of low-level mathematics questions asked, Procedure (56.8%) and Fact (39%) questions accounted for 96% of the low-level mathematics questions. Low-level mathematics questions were asked most often during Observation 3 (in which 35 of the 37 mathematics questions asked were Procedure (84%) and Fact (11%) questions). Procedure questions were used to practice the FOIL method for multiplying binomials. The warmup exercise asked students to find the product of $(x+7)(x+2)$. To demonstrate the FOIL method for multiplying binomials, Eban asked as series of Procedure and Fact questions:

- "Where should the F's go?" (Procedure)
- "F times F would give me what?" (Fact)
- "O times O would give me what?" (Fact)
- "What do I do with the two middle terms?" (Procedure)

These questions led students through the mechanical process of multiplying binomials. Eban often used one or two-word questions to reinforce mathematical procedures as well. Observation 6 contained 60 Procedure questions, many of which were one or two-word questions. Given the context of the exchange, the phrases were considered as mathematics questions. For instance, while the class was solving $\begin{cases} y = -3x+4 \\ y = x^2 - 4x + 2 \end{cases}$, Eban asked a series of Procedure and Fact questions

Table 10

Frequency and Percentage of High-Level and Low-Level Mathematics Questions – Eban

High-level			Low-level		
Question type	Frequency	% of high-level questions	Question type	Frequency	% of low-level questions
Compare	18	30.5%	Procedure	243	56.8%
Justify	10	16.9%	Fact	167	39.0%
Definition_Open	7	11.9%	Vocabulary	12	2.8%
Classify	7	11.9%	Recall	4	0.9%
Analyze	5	8.5%	Agree/Disagree	2	0.5%
Make Sense	4	6.8%			
Contrast	3	5.1%			
Make a Connection	2	3.4%			
Predict	2	3.4%			
Explain	1	1.7%			

Note. The *Agree/Disagree* type of low-level mathematics questions include: Agree/Disagree with a Classification, Agree/Disagree with a Definition, Agree/Disagree with a Fact, Agree/Disagree with a Prediction, Agree/Disagree with a Procedure, Agree/Disagree with Reasoning, and Agree/Disagree with Vocabulary.

to help students arrive at $x = -1$. Eban continued to lead the students to solve for y by engaging students in the following exchange:

Eban: "So what's the next step?" (Procedure)

Student: "Do -3 times -1."

Eban: "Which is?" (Fact)

Student: "3."

Eban: "And then?" (Procedure)

Student: "Plus 4."

Eban: "And that's?" (Fact)

Student: "7."

Eban: "And then what am I going to do?" (Procedure)

Reinforcing the mechanics of performing mathematics was the main use of Eban's questioning practices; which is incongruent to beliefs inferred from the Beliefs Inventory Survey. According to survey item 26, Eban believes that to be good at mathematics at school, it is very important for students to be able to think creatively. Yet, during Reflection Meeting 1, Eban explained her reasoning for asking numerous Procedure questions (47%) during Observation 1 by stating, "I tell them all the time, math is a process and you go through step one, step two, step three. Even though the numbers may change, the variables may change, you still do the same thing." Additionally, Eban believes questions based on recall of facts and procedures should *always* be included on mathematics tests or examinations and questions requiring explanations or justifications should be included *sometimes* (Beliefs Inventory Survey items 24 and 25). These responses were classified as Instrumentalist and Instrumentalist/Platonist, respectively.

Eban's beliefs about the instrumental use of mathematics align with her intention to prepare students to be successful on standardized examinations such as the ACT. The ACT does not require students to explain or justify mathematical concepts or procedures, but students are expected to employ appropriate conceptual knowledge and procedural skills to complete exercises and solve problems. During Reflection Meeting 4, Eban reflected on her reason for asking, "Who can tell me one way we have already solved quadratics?" She admitted that she wanted her students to recognize that they have learned multiple methods for solving quadratic equations and that they were about to learn another method. She wanted her students to choose and use their preferred method for solving quadratics. Eban felt this question aligned with her beliefs about mathematics learning because not only should students choose a preferred method, but when taking standardized tests, they should select the method they can perform the quickest:

> I'm trying to get them to figure out the fastest method to get the answer. If they're on the ACT and they've got 60 questions in 60 minutes, you don't have the time to hit the caret [key], so use the *x* squared [key] unless it's raised to a higher power than 2. When you're given a standardized test or any kind of test, you want to do the quickest way—or your preferred method sometimes too (Reflection Meeting 4).

Completing mathematical tasks was not the only driving force behind Eban's questioning practices. She occasionally asked high-level mathematics questions while leading students through mathematical processes.

Although Eban asked several low-level questions, she also asked 10 different types of high-level mathematics questions. Some examples included:

- "What type of equation is $y = -3x + 4$?" (Classify)
- "Explain what you did." (Explain)
- "What's different about this equation [y $= 3x^2 + 12x - 8$] than the one we worked with yesterday?" (Compare)
- "What should our parabola do?" (Make a Connection)
- "Why did you put a 0 here?" (Justify)
- "What type of term is $3x^2$?" (Definition_Open)

High-level questions were asked most often during Observation 5. During this lesson, students were asked to graph the quadratic equation $y = 3x^2 + 12x - 8$. After completing a table of *x* and *y* values, a student volunteered to write the table and draw the graph on the board. The student incorrectly graphed the quadratic as a parabola that opened downward based on the table of *x* and *y* values shown in Table 11. Upon noticing the error in the student's work, Eban used questioning to help the class realize the mistake in the calculations which affected the graph. To reinforce the concept of graphing quadratics, Eban referred to the standard quadratic form ($ax^2 + bx + c = 0$) to remind students of the relationship between the standard equation and the direction on the resulting parabola. Eban asked:

- "What's our 'a' here?" (Compare)
- "So, what should our parabola do?" (Make a Connection)
- "So, what happened here?" (Analyze)
- "So, where is the mistake?" (Analyze)

Table 11

Table of x and y Values for $y = 3x^2 + 12x - 8$

x-value	y-value
-2	-20
-1	-17
0	-8
1	-17
2	-20

At this point, Eban had drawn her students' attention to the table of x and y values and challenged them to analyze the table to find and correct the errors. Eban's instrumentalist perception of the nature of mathematics was also evident in the routines shown during classroom observations. Eban deemed it important that students, first, view a demonstration of a mathematical process, then imitate the process during practice.

Modeling. Eban believes that modeling the correct performance of mathematical procedures is a critical component of teaching mathematics because students will benefit from seeing mathematics performed prior to performing it themselves. Eban *agreed* with the Beliefs Inventory Survey item 19 which stated that students have to be taught the correct procedure to solve most mathematics problems. Her response to item 19 was classified as Instrumentalist/Platonist, whereas her mode responses for how mathematics should be learned were classified as Problem-Solving. Although 17% of her responses to the Beliefs Inventory Survey were classified as Instrumentalist/Platonist and 42% were classified as Problem-Solving, Eban's questioning practices aligned with the belief expressed in item 19.

Correspondingly, Eban believes teaching consists of providing a model example of a procedure and learning occurs when a student can replicate the modeled procedure, even when the components of the task change. During the Pre-Observation Interview, Eban professed that her students were visual, thus one of the best ways to teach mathematics was to let students watch her go through the steps for arriving at a solution to a problem. Eban feels modeling is one of her roles as a mathematics teacher, "I think as I model and ask questions I'm showing them the things that they need to be able to do in the process of solving the equation or whatever we're doing". Yet, Eban felt it is false that mathematics problems can be done correctly in only one way and it is true that good mathematics teachers should show students lots of different ways to look at the same question (Beliefs Inventory Survey items 16 and 17). Her responses to these survey items were not congruent to her use of questioning during instruction.

During Observation 3, Eban asked her students to copy a model of the "square of the binomial pattern" from their textbook. After the students copied the example, Eban reiterated by demonstrating the process on the board; emphasizing that the students were to follow the pattern described in the textbook. Using questioning, Eban reinforced the procedural skills she wanted students to mimic by reinforcing the pattern for the next example, $(2x+5)^2$. Eban asked the following series of questions:

- "What am I going to write? (Procedure)
- "And then?" (Procedure)
- "What's going to be next to that 2?" (Procedure)
- "And then?" (Procedure)
- "Then?" (Procedure)

During independent practice, students were asked to write the "sum and difference pattern" for multiplying the following binomials:

- $(r+3)(r-3)$ (Procedure)
- $(4x+y)(4x-y)$ (Procedure).

The pattern was provided in the textbook and students were to apply the model to the multiplication exercises above. When a student asked if they should simplify their answers, Eban responded, "Don't simplify, just show me that you can follow the pattern".

Note-taking was the method Eban encouraged students to use to chronicle the models they observed in class. During the same observation mentioned above, a student asked if he could do Example 1A, $(x+4)^2$, on the board. Eban responded, "No, I do the examples…your notes will be wrong…you don't know what I'm going to tell you to do." When discussing her reasons for insisting that students refer to their notes when practicing mathematical procedures, Eban stated:

> I want them to realize that math is a pattern. So even if the variables change or the numbers change, 'I have an example that's set up this same way and if I just look at the steps of how she did it on the board, or I did it for independent practice, I should be able to use those

> same steps to figure out the answer. I have an example to look at'…The model should help them learn it. That's the goal (Reflection Meeting 3).

Eban also allowed students to use their notes during tests so they could follow the steps in the examples from their notes to answer test questions. During the same reflection meeting, Eban explained the rationale for allowing students to use their notes and homework assignments during tests:

> I do allow them to use their notes on the test. I do allow them to use their old homework. If you can look at this problem and say, 'Hmm, I have this problem in my notes that looks just like it, then I should be able [to do it] even if I don't remember exactly how to do it.' I can go back and look at it, and if I really have the notes, then I can look at it and say, 'Okay, how did I go from this step to this step, to this step?' (Reflection Meeting 3).

Providing demonstrations (from the textbook or in person) was a necessary component of Eban's teaching as she believed students needed to see a process before they were able to perform it. Eban believed that learning occurred when a student was able to perform procedures correctly. Thus, practice was also a major component of Eban's lessons.

Performance and practice. Eban believes learning is based on student performance. When asked to define learning, Eban stated:

> Learning can be shown in understanding. So, it can be how they perform as far as performing the operations, knowing the vocabulary, the steps and applying it to real world situations. So, what they can do, what they can't do and how they use it in the real world" (Pre-Observation Interview).

Thus, practice was the impetus for many of Eban's questions due to her belief that the best way to learn mathematics is to practice (Pre-Observation Interview). Hence, 47% of the total mathematics questions asked were Procedure questions. Eban provided several opportunities for students to practice procedural skills. Each observed lesson began with a warmup which consisted of 1 to 3 exercises. Eban used warmup exercises to assess students' understanding of previously studied concepts (Reflection Meeting 1). The warmup questions for each observation were:

- Observation 1: Evaluate the expressions. *1)* $-\sqrt{49}$, *2)* $\sqrt{200}$, *3)* $\pm\sqrt{121}$ (Procedure)
- Observation 2: Simplify $-2(9a-b)$. (Procedure)
- Observation 3: Find the product. $(x+7)(x+2)$ (Procedure)
- Observation 4: Use FOIL to find the product. *1)* $(x+6)(x+4)$, *2)* $(2y+3)(y+5)$ (Procedure)
- Observation 5: Evaluate the expression. *1)* $x^2-2,$ when $x=3$, *2)* $2x^2+9,$ when $x=2$ (Procedure)
- Observation 6: Solve $\begin{cases} y=3-2x \\ y=x+9 \end{cases}$. (Procedure)
- Observation 7: Find the product-FOIL. *1)* $(m+2)(m-2)$, *2)* $(2y-3)^2$ (Procedure)
- Observation 8: Evaluate the expression for the given value of *x*. *1)* $15-(-x)+9$ when $x=-2$, *2)* $14-x+3$ when $x=8$ (Procedure)

The warmup exercises were a precursor to the day's lesson as they required students to use skills that would be used later in the lesson. Following the warmup exercises, students completed additional practice exercises that were written on the board and exercises from the textbook. Students were then assigned homework problems to receive additional practice at home.

During Observation 2, Eban noticed her students struggled when multiplying $3x^2(2x^3-x^2+4x-3)$. Therefore, she asked the students to try another example, $2x^3(x^3+3x^2-2x+5)$. While reflecting on her reason for asking students to try another example, Eban stated, "When I saw they really wasn't getting it, I wanted them to try another. I said, 'Okay, let's try another one just to make sure that they get more practice on it before I do the next example'" (Reflection Meeting 2). This coincides with her response to item 9 of the Beliefs Inventory Survey which she *somewhat agreed* that if students are having difficulty, an effective approach is to give them more practice by themselves during class. This response was classified as Instrumentalist/Platonist.

Practicing during assignments was a role Eban expected her students to assume (Pre-Observation Interview). Thus, Eban frequently asked Procedure questions to guide students through and remind students of procedures previously demonstrated. The lesson delivered during Observation 7 consisted of students factoring several binomial expressions using difference of squares. Eban asked several procedure questions to assist students as they practiced the process. For instance, to help students factor $y^2 - 9$, Eban asked:

- "What do you think I'm going to put first in each parentheses?" (Procedure)
- "What's going to be in the first parenthesis after the y?" (Procedure)
- "What's going to be in the second parenthesis after the y?" (Procedure)
- "Put in your calculator the square root of 9." (Procedure)

A similar series of questions were asked as students practiced factoring additional binomial expressions. Students continued to practice until the class ended-which was the routine for each observed lesson. The cycle of teacher/textbook modeling and student practice permeated the observed lessons and supported Eban's perception of mathematics as a static and mechanical process that adheres to procedural patterns.

Reflections on questioning practices. Eban's deliberate analysis of her questioning practices revealed her perceptions about teaching and learning mathematics. Based on the data collected during four Reflection Meetings, Eban agreed 91% of the time that her questioning practices aligned with her beliefs. Table 12 shows the percentage of times Eban felt her beliefs were in alignment with the high-level and low-level mathematics questions asked during instruction. This table shows that Eban felt all the high-level mathematics questions she asked aligned with her beliefs as well as many of the low-level mathematics questions.

Table 12

Percentage of Times Beliefs Aligned with Questioning Practices – Eban

	High-level mathematics questions	Low-level mathematics questions
Aligned	100%	86%
Not Aligned	0%	14%

During Reflection Meeting 3, Eban discussed her role as a facilitator as described in the Problem-Solving model of teaching, which states that teachers act as facilitators who construct the curriculum with pure investigation, problem-solving and problem-posing. Their instructional goal is to develop problem posers and problem solvers. She shared her belief that asking both high-level and low-level mathematics questions was necessary to develop problem-solvers:

> Problem-solving requires you to maybe explain and justify, even investigate. If you're solving something, you have to look at it and figure something out, so that's why I would say more high-level, but I still think it's both. But before you can even solve it, you need to recall. You need to be able to state, memorize certain things in order to do it. So, it's a combination of both.

Eban believed that many of the low-level mathematics questions she asked did align with her beliefs which were classified as Platonist and Problem-Solving. During Reflection Meeting 3, Eban reflected on why she asked Procedure and Fact mathematics questions during Observations

4, 5, and 6. Although Eban did not assign story problems during the observed lessons, she felt the Procedure and Fact mathematics questions she asked helped students to solve other types of problems, "Even though I don't do as many story problems that I would like to, I still think I try to get them to solve the problem." Eban's use of the word "problem" appeared to include both exercises (such as the warm up and practices given during class) and story problems.

During Reflection Meeting 4, Eban reflected on why she asked students to complete a variety of examples of factoring using the difference of squares method. For instance, Eban asked students to factor:

- $y^2 - 9$ (Procedure),
- $64c^2 - 16$ (Procedure), and
- $x^2 - 81y^2$ (Procedure).

Eban stated that she wanted students to recognize when to use the difference of squares method regardless of whether a term has a coefficient or not. Eban felt these questions aligned with her beliefs that performing mathematics is a procedural pattern, "Like I said previously, I think it's a pattern. I want them to realize that the numbers and the situation may change, you still have to take the same pattern, or the steps to solve it." Eban's mathematics questions were more congruent to the beliefs espoused during the Pre-Observation Interview than beliefs inferred from the Beliefs Inventory Survey.

Table 12 above also shows that Eban felt one of her low-level questions did not align with her beliefs. This instance occurred during Observation 2 when Eban demonstrated the FOIL method by multiplying $(x+4)(2x-1)$. Eban asked several Fact and Procedure questions to lead students to the expression $2x^2 - x + 8x - 4$. After informing students that the middle terms can be combined, she posed the following question to a student, "Using your calculator, what's $-1+8$?"

Upon reflecting on why she asked the student to use his calculator to find the sum of -1 and 8, Eban admitted to asking certain students, who have weak multiplication skills, to use their calculator to ensure getting the correct answer. She claimed the question did not align with her beliefs because it focused more on mastery of skills and facts than conceptual understanding (Reflection Meeting 2).

Eban's belief that the questioning practices discussed during reflection meetings greatly supported her beliefs about mathematics teaching and learning is shown in the minor fluctuations in percentages of high-level and low-level mathematics questions between reflection meetings. Figure 9 shows the percentage of high-level and low-level mathematics questions Eban asked between reflection meetings.

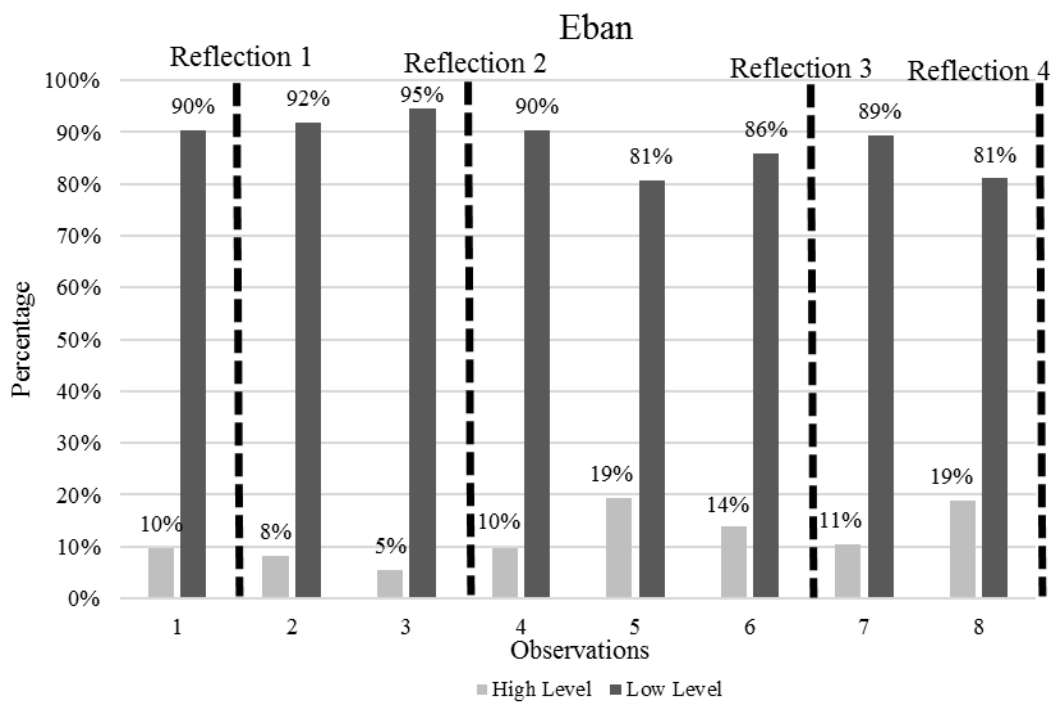


Figure 9. Questioning practices between reflection meetings- Eban.

Low-level questions were asked most frequently during observations 2, 3, 4, and 7. These were classes with students in Period 1, which Eban claimed were her weakest students:

> They came to us very low. The behavior issues get in the way a lot of times too. They're a little bit more immature. Some of them, who could be more independent, get distracted. A lot of them don't do homework and their skill level is really low. They don't practice a lot, so even if they got it, maybe, in class, by the next day, they have totally forgotten. They don't take notebooks [home]. They forget (Reflection Meeting 3).

The observations with the greatest percentage of high-level questions, Observations 5 and 8, were lessons delivered to Period 2, which Eban described as a stronger group of students than Period 1. Overall, 12% of Eban's questions were high-level and 88% were low-level; which is closely related to her baseline percentages as determined by Observation 1.

The End-of Study Questionnaire revealed Eban's perception of her questioning practices. Eban believed her questioning practices supported her belief that learning is exhibited in correct performance:

> I believe that students show their understanding in their performance of solving equations/problems. I ask students questions to try to get them to think and rely on their prior knowledge of skills to guide and assist them in the process. I also ask students to follow the procedures given from their notes. Asking them what step they are on and what step may be needed to completely answer/solve the problem (End-of-Study Questionnaire, May 29, 2017).

Eban also shared that the questions she asks are not always congruent to the intention of the question:

> During the reflection I realized how some of the questions that I often ask are low-level thinking questions according to Bloom's Taxonomy but that I am trying to get my students to

> analyze or think which makes them better problem-solvers which in turn is actual high-level thinking (End-of-Study Questionnaire, May 29, 2017).

While she intended to ask questions to elicit high-level thinking, she expected her students to apply that high-level thinking to mathematical procedures, “I try to plan certain questions while teaching my lesson which causes my students to apply or sometimes examine why they have to do certain steps in getting to the solution of the problem” (End-of-Study Questionnaire, May 29, 2017).

Oliver

“I've always been a problem-solving teacher because I used to teach computers too. That's why I tend to lean this way - very minimalistic instruction” (Pre-Observation Interview).

Oliver is a high school mathematics teacher who has taught computers and mathematics for 11 years to charter and private school students in grades 4-12. Oliver’s style of teaching consisted of providing little instruction as he preferred students to work independently on mathematical tasks. He wanted his students to employ problem-solving skills, such as synthesizing and transferring mathematical concepts and skills, to new tasks. Thus, Oliver provided opportunities for students to work on challenging problems and exercises. When students struggled with assigned tasks, he occasionally used questioning to demonstrate critical thinking processes and to lead students to correct solutions. Oliver’s instruction was influenced by the factors shown in Figure 10. The top circle signifies Oliver’s instructional objective; students being able to perform new tasks. This objective coincides with his perception of learning as being able to do something you were not able to do before (Pre-Observation Interview). The bottom circles indicate ways in which Oliver supported students' efforts to perform new tasks. He challenged students with critical thinking exercises and led them to correct answers when they struggled with challenging tasks or procedural exercises. Oliver led students to correct solutions by modeling problem-solving thinking processes and demonstrating correct solution processes; which is shown on the bottom circle. Oliver varied his method of question delivery based on his perception of students' needs as they worked on exercises and based on the amount of involvement he wanted to have while students' practiced mathematics; which is indicated on the path between Perform New Tasks and Lead Students.

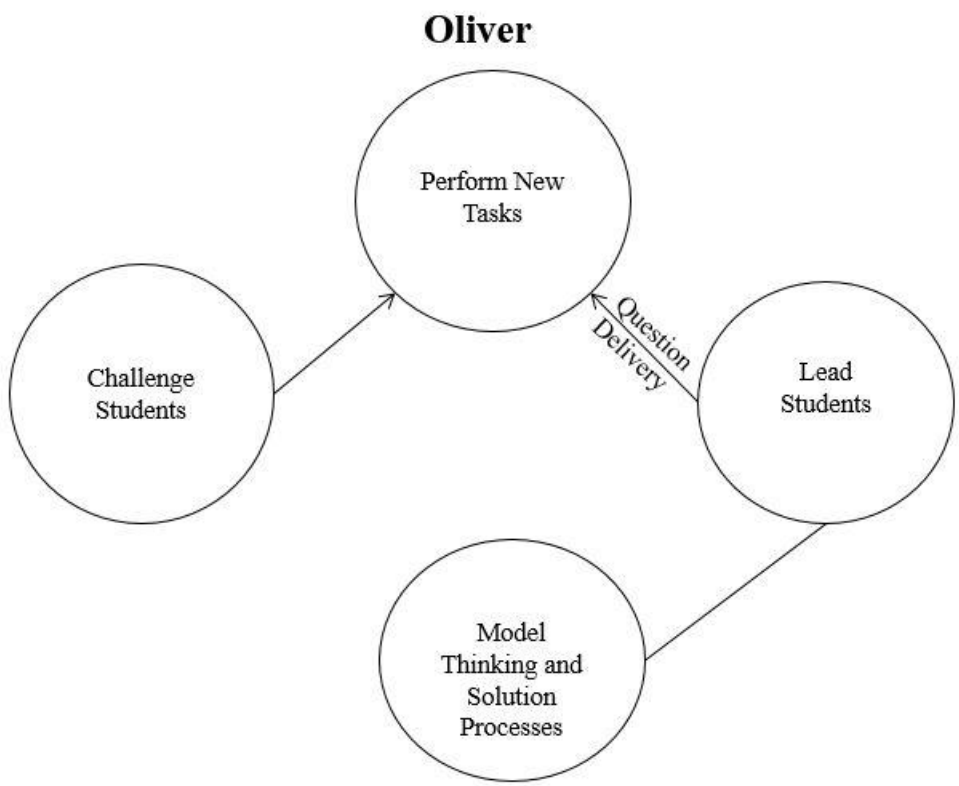


Figure 10. Framework for factors impacting questioning practices- Oliver.

The Beliefs Inventory Survey suggests that Oliver's beliefs about mathematics education align with Problem-Solving models of teaching and learning. Figure 11 displays the percentage of times Oliver provided a response to the Beliefs Inventory Survey that aligned with each belief classification. Oliver's beliefs about mathematics teaching were classified as Problem-Solving (47%) which holds that teachers act as facilitators who construct curriculum with pure investigation, problem-solving and problem-posing. For instance, Oliver *disagreed* that problem solving should be a separate, distinct part of the mathematics curriculum (item 2) and he felt students should *always* explain the reasoning behind an idea (item 21). Oliver's beliefs about mathematics learning were also classified as Problem-Solving (67%) which claims that students learn by exploration and construction of knowledge through autonomous pursuit of their own interests. On the Beliefs Inventory Survey, Oliver *disagreed* that mathematics should be learned as sets of algorithms or rules that cover all possibilities (item 11) and *agreed* that to be good at mathematics at school, students should understand how mathematics is used in the real world (item 14).

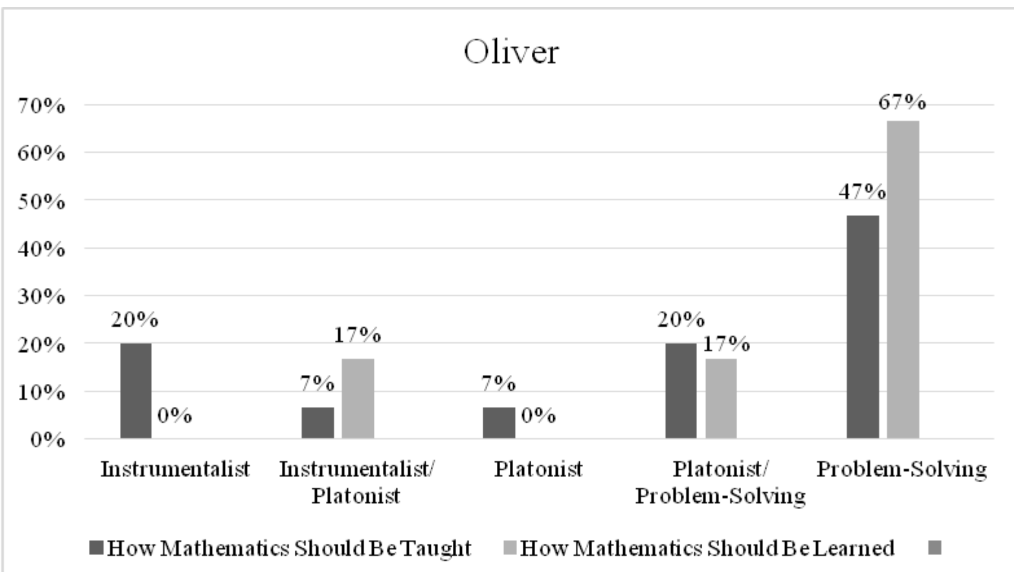


Figure 11. Percentage of survey responses in each classification – Oliver.

Additional data regarding Oliver's beliefs and question-asking practices was collected while reflecting on his beliefs and during classroom observations. Table 13 lists dates, topics, grade levels, and courses for each classroom observation, as well as the dates of the Pre-Observation Interview and Reflection Meetings.

Oliver's belief that mathematics teaching and learning should focus on problem-solving techniques was not evident in his use of questioning. Although he used a combination of high-level and low-level mathematics questions to provide opportunities for students to practice critical thinking and procedural skills, he neither instilled nor fostered any particular problem-solving strategies for students to rely on when approached with challenging mathematical tasks.

Table 13

Dates of Pre-Observation Interview, Observations, and Reflection Meetings - Oliver

Observation	Topic	Date	Grade level	Course
	Pre-Observation Interview	April 25, 2017		
Observation 1	Classify Quadrilaterals	April 26, 2017	10	Geometry
	Reflection Meeting 1	April 27, 2017		
Observation 2	Fractions and Least Common Multiple	May 2, 2017	11	Algebra 2
Observation 3	Graphing Quadrilaterals	May 3, 2017	10	Geometry
	Reflection Meeting 2	May 11, 2017		
Observation 4	Translations and Matrices	May 11, 2017	10	Geometry
Observation 5	Chapter 5 Review	May 11, 2017	11	Algebra 2
	Reflection Meeting 3	May 15, 2017		
Observation 6	Functions	May 23, 2017	11	Algebra 2
Observation 7	Reflections, Rotations, Symmetry	May 23, 2017	10	Geometry
Observation 8	Reflections, Rotations, Symmetry	May 23, 2017	10	Geometry
	Reflection Meeting 4	May 24, 2017		

Alternatively, he asked high-level and low-level mathematics questions to present students with problems and exercises and to lead them to correct solutions. Table 14 displays the type and frequency of high-level and low-level mathematics questions Oliver asked during classroom observations. During this study, Oliver asked a total of 145 mathematics questions which consisted of 44 (30%) high-level mathematics questions and 101 (70%) low-level mathematics questions. Oliver asked 10 different types of high-level mathematics questions that encouraged critical thinking. Some examples include:

Table 14

Frequency and Percentage of High-Level and Low-Level Mathematics Questions – Oliver

High-level			Low-level		
Question type	Frequency	% of high-level questions	Question type	Frequency	% of low-level questions
Reason	13	29.5%	Procedure	49	48.5%
Explain	10	22.7%	Fact	43	42.6%
Classify	5	11.4%	Agree/Disagree	4	4%
Make Sense	4	9.1%	Vocabulary	3	3%
Generalize	3	6.8%	Estimate	1	1%
Justify	3	6.8%	Recall	1	1%
Compare	2	4.5%			
Definition_Open	2	4.5%			
Analyze	1	2.3%			
Make a Connection	1	2.3%			

Note. The *Agree/Disagree* type of low-level mathematics questions include: Agree/Disagree with a Classification, Agree/Disagree with a Definition, Agree/Disagree with a Fact, Agree/Disagree with a Prediction, Agree/Disagree with a Procedure, Agree/Disagree with Reasoning, and Agree/Disagree with Vocabulary.

- Is the inverse of $f(x) = x^2$ a function? (Reason)
- “Why do you think that?” (Explain)
- “What’s the useful information in this story problem?” (Make Sense)

- "How can you verify that?" (Justify)

High-level mathematics questions were asked most frequently during Observation 1 in which seven of the eight mathematics questions asked were high-level. During this lesson, students were often asked to classify certain quadrilaterals and to explain their reasoning. For instance, students were asked to refer to the quadrilateral in Figure 12 and respond to the following statement, "Is enough information given in the diagram to show that Quadrilateral PQRS is an isosceles trapezoid? Explain" (Reason, Explain).

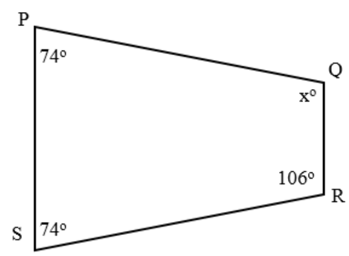


Figure 12. Trapezoid PQRS.

No high-level mathematics questions were asked during Observation 2, which was an Algebra 2 lesson. This lesson was dedicated to students practicing procedures for adding, subtracting, and dividing fractions as well as finding the least common multiple of two expressions.

To reinforce procedural skills and lead students during mathematical tasks, Oliver asked 6 different types of low-level mathematics question (See Table 14). Procedure (48.5%) and Fact (42.6%) questions were asked more often than other types of low-level questions or tasks. Some examples include:

- "Draw Triangle RST with vertices R (2, 2), S (5, 2), T (3, 5)." (Procedure)
- "Reflect image B over the line." (Procedure)
- "How many degrees are in a quadrilateral?" (Fact)
- "What does this apostrophe mean?" (Fact)

Although the first and second examples above are not interrogative statements, they fit the description of mathematics questions defined in Chapter 3 as questions (spoken or written) that contain mathematics content and that elicit responses from students in the form of information or action. Low-level questions were asked most frequently during Observation 2 in which 100% of the 13 mathematics questions asked were coded as low-level. As stated above, the lesson for Observation 2 was dedicated to practicing procedures. Correspondingly, each mathematics question was either a Procedure (77%) question:

- "Add $\frac{7}{20}+\frac{9}{20}$." (Procedure);
- "Find the least common multiple of 20 and 45." (Procedure);
- "Perform the indicated operation. $\frac{7}{12x}+\frac{5}{12x}$" (Procedure);

or a Fact (23%) question:

- "7 plus 5 is what?" (Fact),
- "What two numbers multiply to give you 4 but add to give you -4?" (Fact).

As Figure 10 above indicates, Oliver wanted his students to be able to perform new tasks and supported their efforts by asking questions that challenged them and led them to correct solutions. Many of the high-level questions were used to challenge students while the low-level questions were often used to allow students to practice procedural skills or to lead them through a mathematical task.

Challenging students. Oliver described himself as a problem-solving teacher who provides little instruction but wants his students to apply what they have learned. He believes students should *always* work on problems for which there is no immediately obvious method of solution (Beliefs Inventory Survey item 23). During Reflection Meeting 1, Oliver stated:

> I've always been a problem-solving teacher because I used to teach computers too. That's why I tend to lean this way - very minimalistic instruction. Computers is more an application, doing or applying it to a specific item. I see math working the same way. Because in the real world you're never [told] just, 'Oh, here's a problem that you have to solve,' and number, number, solve. It's more application-based.

Likewise, Oliver believes it is *more false than true* that students have to be taught the correct procedure to solve most mathematics problems (Beliefs Inventory Survey item 19). To help students hone their problem-solving skills, Oliver asked questions that forced students to synthesize and apply both new and prior knowledge due to his belief that being able to perform a new task is an indication of learning:

> Learning is, basically when you incorporate a new thing into what you do. For example, if you don't know how to snap and then you keep on practicing and then you're actually able to do it, that's when learning has taken place. It's when you're able to do something you weren't able to do before (Pre-Observation Interview).

Oliver challenged students in Observations 7 and 8 by giving them a handout that contained several high-level mathematics questions such as:

- "Does every figure have a line of symmetry?" (Reason)
- "What did you do to justify that the lines you constructed were, in fact, lines of symmetry?" (Justify)

Although Oliver assigned few problems from the handout, he chose to give the handout to the students because:

> It also gives them more of a chance to think on it instead of just, all right, you just need to do a whole bunch of problems. I wanted them to actually think on the content more. This

curriculum actually tends to line up more with my problem-solving methods than the previous curriculum where it was more problem volume and more just surface-level questions. This actually helps them to dig deeper a little bit too (Reflection Meeting 4).

During Observation 1, seven of the eight high-level mathematics questions asked required students to rely on and apply their previous knowledge of the classification of quadrilaterals. Students were asked questions such as:

- "Quadrilateral ABCD has at least one pair of opposite angles congruent. What types of quadrilaterals meet this condition?" (Classify), and
- "What is the most specific name for quadrilateral ABCD? (Classify). **(**See Figure 13)

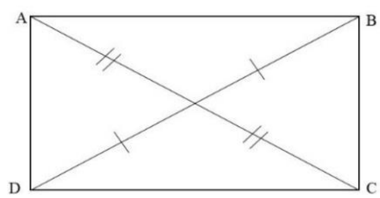


Figure 13. Quadrilateral ABCD.

Upon reflecting on why he posted the questions above, Oliver revealed that he expected his students to use recently learned concepts to address the questions in this lesson, "They just had to remember the information, then synthesize it into that question because they had to know a little bit about each [type of quadrilateral]" (Reflection Meeting 1). Similarly, during Observation 6, Oliver asked:

- "Is the inverse of a function always a function? Why or why not?" (Explain)
- "Describe the cases in which the inverse of a function is also a function." (Generalize)

Again, students had to refer to previously studied concepts to provide explanations and generalizations about inverse functions. Allowing them to grapple with mathematical tasks was a way Oliver used to challenge students and help hone their emerging critical thinking skills.

Oliver's reaction to struggling students corresponds to his response to item 9 on the Beliefs Inventory Survey which he *disagreed* that giving students more practice by themselves during class was an effective approach for students having difficulty. However, Oliver did not use student struggle as an opportunity to redirect students to use problem-solving strategies. Instead, when students needed assistance, Oliver used questioning to model ways in which he would encounter the problem; which does not allow student to devise their own plan to address the task.

Leading students. Oliver used questioning to lead students when they struggled with completing mathematical tasks by modeling problem-solving thinking processes and demonstrating correct solution processes. Oliver's method of delivering questions was influenced by his perception of students' needs during the lesson. During Observation 5, Oliver first challenged his students by asking them to address the following questions:

> A new computer game costs $90,000 to develop. Once completed, individual games can be produced for $0.60 each. The first 100 are given away as samples. Write and graph a function C(x) for the average cost of each game that is sold. How many games must be sold for the average cost to be less than $1? (Reason, Procedure)

Oliver allowed students to collaborate before facilitating the solution process. He modeled problem-solving techniques by asking questions he wanted his students to eventually ask themselves when faced with solving problems in the future:

- "How much do those 100 games cost?" (Make Sense)
- "How many samples are they making?" (Fact)
- "How much does it cost just to make those samples?" (Make Sense)
- "How do you figure out how much games cost?" (Procedure)

While reflecting on the leading questions he asked to help students with this problem, Oliver stated he asked, “What’s the useful information in this story problem?” (Reason) because, “It’s a step I would personally take to solve that problem, and I want to model how I would go about solving a story problem” (Reflection Meeting 3). Additionally, he asked, “What are you trying to find?” (Make Sense) to make sure students attended to the specific question presented in the problem. Oliver felt these questions aligned with his beliefs about modeling problem-solving processes:

> Well, those questions actually line up more with my problem-solving, because I’ve been trying to model the problem-solving skills I want them to adhere to. So, when they go to another problem, they’re like, ‘Oh, Oliver did this. Maybe I should do this.’ Those particular questions, I think, align up with problem-solving skills (Reflection Meeting 3).

He also expressed that he wanted more of his lessons to contain questions that cause students to think critically:

> I feel like there should be more questions like I had in this lesson right here. I would love to purposely do more questions like that, and I personally need to be more deliberate about that. Questions like, ‘Explain to me in your own words,’ ‘What question are we trying to answer here?” get them to think about the problem. Have them do the work as opposed to me solving it for them. (Reflection Meeting 3).

Asking a combination of high-level and low-level mathematics questions was one way Oliver was able to demonstrate problem solving thinking. He also provided students step-by-step solutions to problems he assigned. Oliver often asked students to copy a problem from the board and the corresponding solution to the problem, simultaneously. For instance, during Observation 1, Oliver posted five exercises on the board along with the solution. Students were asked to copy the problem and solution in their notes before moving on to the next problem. Observation 2 was similar in that step-by-step solutions were posted along with three of the exercises and Observation 3 consisted of one solution being posted at the same time as the problem. While reflecting on the reason for supplying the problems and solutions simultaneously, Oliver stated:

> I wanted them to have a quality example for their notes so that they could reflect upon that when they did another problem like it. Normally, it's the first time that they had been exposed to that kind of a problem, so normally I give them a quality example in order to help them with further problems like it (Reflection Meeting 3).

He felt offering the solution with the question aligned with his beliefs because it allowed him to model problem-solving thinking and processes he hoped students would apply on subsequent problems (Reflection Meeting 3). However, leading by providing solutions is contrary to a facilitator's role in cultivating students' problem-solving skills. While modeling or leading, Oliver varied his method of question delivery based on the amount of instruction he wanted to provide during each lesson.

Question delivery. Oliver's question delivery was influenced by his inclination to use technology and his perception of students' needs. Oliver believes there are advantages to using technology in the classroom:

> I love using technology in my classroom. I like to incorporate those kinds of resources because I feel it engages students at times. I am trying to move away from general PowerPoints but sometimes they are needed in order to have that structure for the particular type of lesson (Pre-Observation Interview).

Hence, 48% of the mathematics questions asked during the first three observations were presented on PowerPoint slides. In addition to verbal questions, each of the remaining observations included mathematics questions printed on worksheets. In fact, of the 145 mathematics questions asked during this study, 51 (35%) were presented on slides or worksheets and 94 (65%) were asked verbally. Table 15 shows the frequency and percentage of printed and verbal mathematics questions asked during eight classroom observations.

Table 15

Frequency and Percentage of Printed and Verbal Mathematics Questions

	Frequency		Percentage	
Observation	Printed	Verbal	Printed	Verbal
1	7	1	87.5%	12.5%
2	9	4	69.2%	30.8%
3	7	20	25.9%	74.1%
4	4	15	21%	79%
5	4	19	17.4%	82.6%
6	7	21	25%	75%
7	10	6	62.5%	37.5%
8	2	9	18.2%	81.8%

Observations 1, 2, and 7 contained more printed questions than verbal questions. During these observations, Oliver allowed students to copy questions and solutions from the board (Observation 1), practice procedural skills with multiple exercises (Observation 2), and spend time working on challenging problems printed on a handout (Observation 7). Oliver's involvement during these observations correspond to his comment above about providing minimalist instruction and also supports the perception of his role as a passive resource more than a teacher (Pre-Observation Interview). Oliver asked one verbal question during Observation 1 and increased his number of verbal questions as the study progressed. However, his involvement was much different during

Observations 3, 4, 5, 6, and 8 as he became more engaged (as shown in the frequency of verbal questions) due to his facilitating and leading more often. Oliver's involvement during Observations 3, 4, 5, 6, and 8 align with his goals to address students' needs by leading and modeling with questioning.

Oliver's discernment with determining when to provide minimal instruction and when to engage students with verbal questions was shared during Reflection Meeting 2. Oliver discussed why he asked more verbal questions with the group of students in Observation 3 than the students in Observation 2 by stating:

> With Period 2[Observation 2], it was a lesson they tended to know a little bit better. It took a lot less prompting on my part for them to get it. They were able to get it because they had prior knowledge of it. Whereas for Period 3[Observation 3], Geometry is more visual. For some of them it's something they don't think about. It's a different way of thinking. There tends to be a lot more prompting to help them develop critical thinking skills about those specific kinds of problems. I'm trying to more model, okay. Here's the questions you should be asking yourself, but obviously you're not, so I'm going to ask them for you, whereas here [Period 2] I'm like, okay. You know how to do this. On parts that you're kind of iffy on I'll ask those questions (Reflection Meeting 2).

As mentioned above, all the mathematics questions during Observation 2 were low-level due to the lesson focusing on procedurals skills. For instance, nine of the questions were presented on PowerPoint slides and included:

- "Add $\frac{5}{18}+\frac{31}{72}$." (Procedure);

- "Simplify $\dfrac{\frac{1}{2}-\frac{2}{5}}{\frac{1}{3}+\frac{3}{4}}$." (Procedure);

- "Perform the indicated operation. $\dfrac{2}{3x^2}+\dfrac{1}{3x^2}$" (Procedure);

- "Perform the indicated operation. $\dfrac{4x}{x-2}-\dfrac{x}{x-2}$" (Procedure);

- "Perform the indicated operation. $\dfrac{4x}{x^2+1}+\dfrac{2}{x^2+1}$" (Procedure);

- "Find the least common multiple of $4x^2-16$ and $6x^2-24x+24$." (Procedure);

while 4 questions were asked verbally, including:

  - "Can we reduce that?" (Fact),
  - "What's $4x$ minus x?" (Fact).

Oliver felt the Fact questions above did not align with his beliefs because he used Instrumentalist methods to guide students instead of letting them struggle with the exercises (Reflection Meeting 2).

Alternatively, during Observation 3, Oliver asked students to use Figure 14 below to work on the following question, "Write a rule for the translation of ΔABC to $\Delta A'B'C'$. Then verify that the translation is an isometry" (Procedure, Justify).

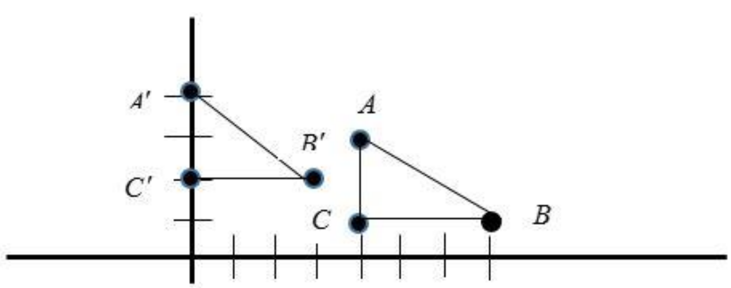


Figure 14. Translation of ΔABC to $\Delta A'B'C'$.

After allowing some time for students to work on the problem, Oliver began to ask leading questions to help students arrive at the desired rule and determine if the translation is an isometry:

- "This is our pre-image. Where is A going?" (Fact)
- "How far is it going to the left?" (Fact)
- "How far is it going up?" (Fact)
- "Does that rule (4 left, up 1) work for C and B?" (Procedure)
- "How wide is this [ΔABC] triangle?" (Fact)
- "How wide is this [$\Delta A'B'C'$] triangle?" (Fact)
- "Are they both right triangles?" (Fact)

Oliver claimed he asked the series of leading questions because he wanted to reinforce procedures and address common misconceptions (Reflection Meeting 2). Oliver felt it was necessary to ask more questions during Observation 3 to help students answer the given problem. Whereas in Observation 2, he only asked 4 verbal questions because the students did not require as much assistance completing the assigned exercises. During these observations, Oliver recognized his students' needs for additional clarification and guidance with question-posing; which he addressed with verbal questioning. However, he would like his students to become more independent when problem-solving:

> Well, I eventually want to get to the point where I don't have to ask them questions. They can ask the questions themselves. I want them to be the ones that start asking the questions- which goes towards the problem solving (Reflection Meeting 2).

Since many of the questions Oliver asked, verbally, were low-level leading questions, Oliver associated asking questions to asking low-level questions. Oliver preferred to monitor students as they worked rather than ask questions to facilitate problem-solving or redirect students with misconceptions.

Oliver expressed different reasons for the differences in printed and verbal questions during Observations 7 and 8. The students of both observations received the same worksheet; yet, the data in Table 15 indicate stark differences in the percentage of printed and verbal mathematics questions asked during these lessons. Students were told to complete more of the problems from the worksheet in Observation 7 than in Observation 8. More mathematics questions were asked during Observation 7 (16) than Observation 8 (11). However, a majority of the mathematics questions presented to students during Observation 7 were printed (62.5%) whereas the majority of the mathematics questions presented during Observation 8 were verbal (81.8%). Oliver allowed the students of Observation 7 to work more independently than their counterparts in Observation 8. While reflecting on why he took different approaches with the two groups of students, Oliver provided the following comments about Observation 8:

> I felt like I was dealing with more classroom management issues than I should've been doing. That's one reason why I asked fewer questions. Another reason I was asking fewer questions was honestly I was more overwhelmed with just trying to make sure students were on task and trying to apply the work than I should've been. I need to still probably ask the questions and try to refocus them more, but for some reason I chose not to (Reflection Meeting 4).

Classroom management contributed to the differences seen between Observation 7 and Observation 8. Still, throughout the study, Oliver made use of different question types and modes of delivery to challenge his students, model problem-solving techniques and lead them through procedural exercises.

Reflections on questioning practices. A deliberate analysis of his questioning practices supported Oliver's beliefs in problem-solving models of teaching and learning. Based on data collected from four reflection meetings, Oliver agreed 71% of the time that his questioning practices aligned with his beliefs. Table 16 shows the percentage of times Oliver felt his beliefs were in alignment with the high-level and low-level mathematics questions asked during instruction. Oliver believed that his high-level mathematics questions always aligned with his beliefs. For instance, during Observation 1, Oliver posted, "Quadrilateral ABCD has at least one pair of opposite angles congruent. What types of quadrilaterals meet this condition?" This question was coded as a high-level question because students were asked to classify Quadrilateral ABCD. Oliver claimed the question aligned with his beliefs because he thought it presented a challenge to the students since they had just returned to school from a break.

Table 16

Percentage of Times Beliefs Aligned with Questioning Practices – Oliver

	High-level mathematics questions	Low-level mathematics questions
Aligned	100%	50%
Not Aligned	0%	50%

Oliver felt half of the low-level questions discussed during Reflection Meetings aligned with his beliefs. During Observation 4, 16 of the 19 mathematics questions were Fact and Procedure questions. He asked the low-level questions because he wanted students to properly perform procedures for image translations. Oliver was conflicted when considering alignment to his beliefs:

> It does and it doesn't, because me asking the procedural questions doesn't allow for self-discovery. But at the same time, if I don't ask the procedural questions, then I'm not

necessarily going to get the material for that particular lesson, either. I mean, the ultimate goal, I want them to learn the content. I wish they'd learn it this way, but sometimes I have to go back to some more instrumentalist goals, unfortunately (Reflection Meeting 3).

Although Oliver believes in Problem-Solving models of teaching and learning there are times when he feels Instrumental instruction is necessary. Generally, Oliver felt his questions aligned when he was modeling problem-solving techniques and challenging students to think (Reflection Meetings 1, 3, and 4). He felt questions that were leading did not allow students to struggle and engage in self-discovery and thus, did not align with his beliefs (Reflection Meetings 2 and 3).

Figure 15 shows the percentage of high-level and low-level mathematics questions Oliver asked between Reflection Meetings. As stated in Table 13 above, Oliver taught Geometry during Observations 1, 3, 4, 7, and 8. The percentage of high-level mathematics questions (which intend to encourage students to think critically) decreased significantly from Observation 1 to Observations 3 and 4. The following times Oliver was observed teaching Geometry (Observations 7 and 8), the percentage of high-level mathematics questions spiked upward.

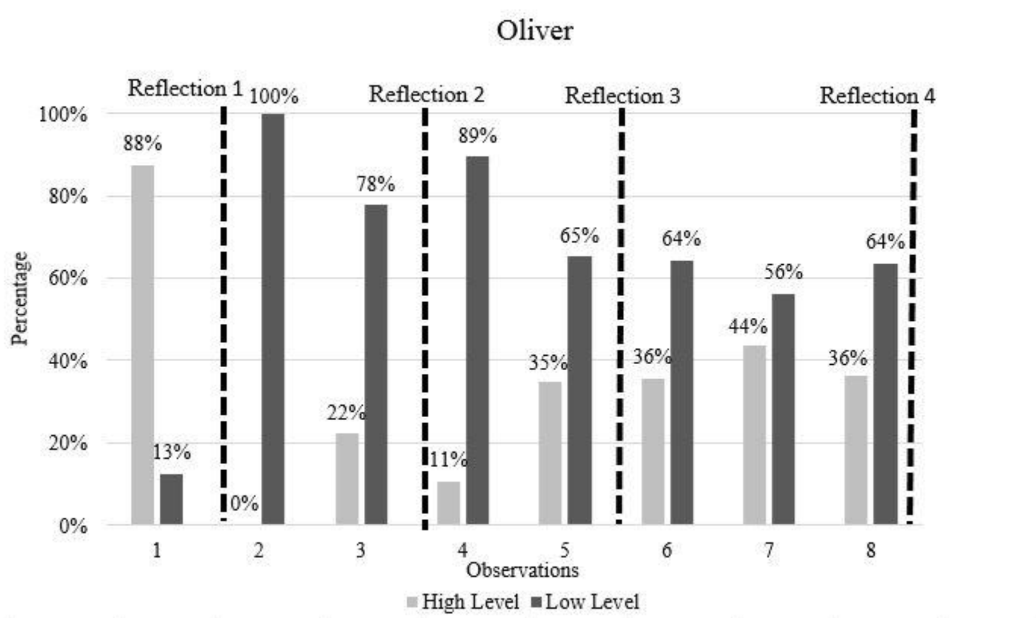


Figure 15. Questioning practices between reflection meetings – Oliver.

Compared to the data in Table 15, the decrease in high-level question percentages from Observation 1 to Observations 3 and 4 correlates with the decrease in the percentage of printed questions delivered to students. Alternatively, Algebra 2 was taught during Observations 2, 5, and 6. There were no high-level mathematics questions asked during Observation 2, yet during the next Algebra 2 lessons (Observations 5 and 6), the percentage of high-level mathematics questions rose significantly. The increase in high-level mathematics questions asked to Algebra 2 students correlate with the increase in the percentage of verbal questions as indicated in Table 15. The Geometry lessons contained both a greater percentage of printed questions and high-level mathematics questions than the Algebra 2 lessons; indicating that the majority of Oliver's high-level questions were retrieved from outside resources and delivered in print form on handouts or PowerPoint slides.

Oliver's responses to the End-of-Study Questionnaire confirmed that he believed his questioning practices aligned with his beliefs about mathematics teaching and learning. He claimed his beliefs are apparent in his lesson planning, "My beliefs affect my practices by controlling my lesson plans and planning in general" (End-of-Study Questionnaire, May 25, 2017). Oliver noticed that the types of questions he asked shifted during this study, "I asked more leading questions near the beginning of this experience, but then I asked more thinking questions near the end." Oliver's shift in questioning practices indicate a transition toward his problem-solving beliefs about teaching and learning as the percentage of high-level mathematics questions increased. He also noticed a shift in his thinking, "This experience did make me reflect on the questions I asked during instruction and being more considerate of the students' needs." This consideration was shown as Oliver used increasingly more verbal questions throughout the study to model and lead students when necessary. Oliver' participation in this study adheres to one of the roles he declared as a mathematics teacher, "I'm always trying to learn more to make it better for the students" (Pre-Observation Interview).

Adisa

"*When you approach anything new, you look at it and you analyze it, and say, 'Well, how does this compare to what I've done before? How is this new? What are the rules and the scheme I can use to apply it to solve the things I need it to do?*" (Pre-Observation Interview)

Adisa teaches mathematics to students in middle school and high school. She has been teaching for 13 years in grades 6-12 in public, charter, and private schools. Data of this study show that Adisa placed great significance on students being able to make connections within mathematical concepts as well as between mathematics and real-world applications in order to complete mathematical tasks. Figure 16 describes how Adisa's beliefs were manifested during instruction and during reflection on her beliefs and question-asking practices. Making connections was pivotal during Adisa's observed lessons, as indicated in the first row of the figure. The connectors to the second row describe the methods Adisa believes will encourage students to recognize relationships between ideas and to draw on current knowledge. For instance, Adisa encouraged students to make connections within mathematical concepts by analyzing new information and comparing it to their previous knowledge. Also, Adisa wanted students to be able to connect mathematics to real-world scenarios by applying their knowledge in practical contexts. According to the Beliefs Inventory Survey, Adisa *agreed* that to be good at mathematics at school, students should understand how mathematics is used in the real world (item 14). The bottom row indicates the purpose for making connections within mathematical concepts and to real-world scenarios; to transfer knowledge when performing mathematical tasks. Adisa wanted her students to be able to draw on their knowledge to complete both practice exercises and critical thinking

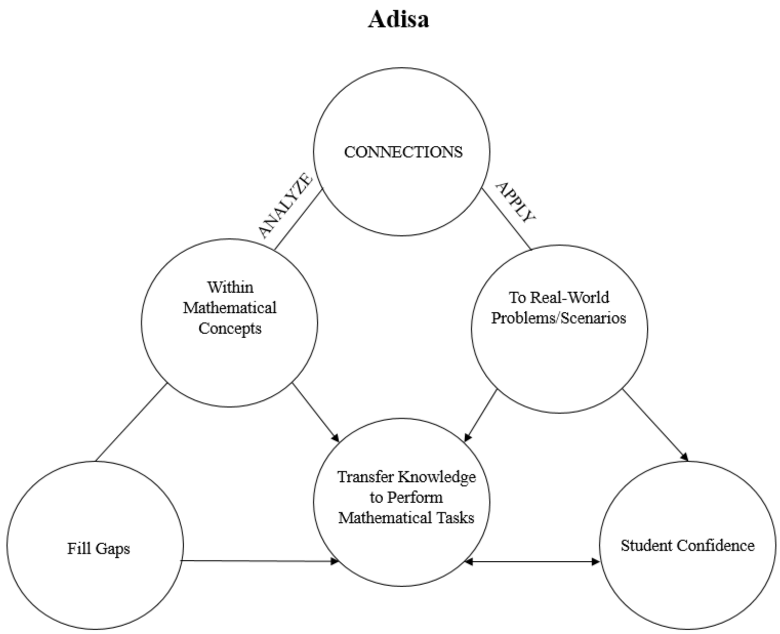


Figure 16. Framework for factors impacting questioning practices- Adisa.

exercises. The bottom row of Figure 16 also showcases Adisa's acknowledgement of the role of student confidence in performing mathematical tasks and the necessity of filling gaps in fundamental knowledge when students have difficulty recognizing connections or completing exercises. To that end, Adisa helped her middle school and high school students build on current knowledge by asking questions that encouraged them to analyze new information, perform mathematical tasks, and reinforce foundational skills. Although Adisa prefers to provide instruction that promotes problem-solving skills, she finds that a strong foundation is necessary to make connections among mathematical concepts and to apply mathematics to real-world contexts.

Adisa expressed additional beliefs on the Beliefs Inventory Survey. Figure 17 shows the percentage of times Adisa provided a response to the Beliefs Inventory Survey that aligned with each belief classification. According to the Beliefs Inventory Survey, Adisa does not believe mathematics should be learned instrumentally (0%), meaning students should not only focus on mastery of skills. Adisa believes mathematics education should follow problem-solving models of teaching and learning. Her beliefs about mathematics teaching were classified as Problem-Solving (47%), meaning teachers act as facilitators who construct curriculum with pure investigation, problem-solving and problem-posing. For instance, she *agreed* that students should always work on problems for which there is no immediately obvious method of solution (item 23) and *disagreed* that problem-solving should be a separate, distinct part of the mathematics curriculum (item 1).

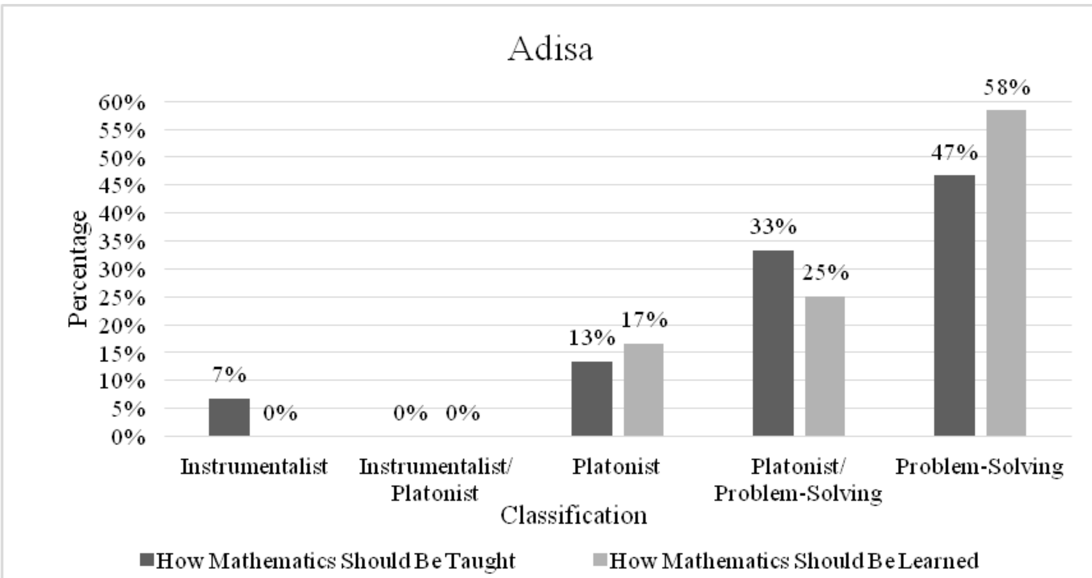


Figure 17. Percentage of survey responses in each classification – Adisa.

Adisa's beliefs about mathematics learning were also classified as Problem-Solving (58%) which means she believes students learn by exploration and construction of knowledge through autonomous pursuit of their own interests. Adisa *agreed* that solving mathematics problems often involves hypothesizing, estimating, testing, and modifying findings (item 12).

Several of Adisa's beliefs about teaching and learning mathematics were also classified as Platonist/Problem-Solving. For instance, Adisa *disagreed* that the mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in isolation (item 4) and that students should o*ften* decide on their own procedure for solving complex problems (item 22). She also believes that in order to be good at mathematics at school, it is *somewhat important* for students to think creatively (item 26). Data from the Pre-Observation Interview, Classroom Observations, and Reflection Meetings established Adisa's emphasis on making connections necessary for completing mathematical tasks (see Table 17).

Table 17

Dates of Pre-Observation Interview, Observations, and Reflection Meetings - Adisa

Observation	Topic	Date	Grade level
	Pre-Observation Interview	May 1, 2017	
Observation 1	Evaluate Logarithms	May 1, 2017	High School
Observation 2	Evaluate Logarithms	May 1, 2017	High School
	Reflection Meeting 1	May 2, 2017	
Observation 3	Linear Modeling	May 3, 2017	Middle School
Observation 4	Linear Modeling	May 3, 2017	Middle School
	Reflection Meeting 2	May 4,2017	
Observation 5	Logarithm Properties	May 8, 2017	High School
Observation 6	Graph Lines in Slope-Intercept Form	May 8, 2017	Middle School
	Reflection Meeting 3	May 9, 2017	
Observation 7	Graph Lines in Slope-Intercept Form	May 9, 2017	Middle School
Observation 8	Graph Lines in Slope Intercept Form	May 10, 2017	Middle School
	Reflection Meeting 4	May 11, 2017	

Adisa's use of high-level and low-level questioning scaffolded learning and allowed students to practice problem-solving and procedural skills. Table 18 displays the type and frequency of high-level and low-level mathematics questions she asked during eight classroom observations.

Table 18

Frequency and Percentage of High-Level and Low-Level Mathematics Questions – Adisa

High-level			Low-level		
Question type	Frequency	% of high-level questions	Question type	Frequency	% of low-level questions
Explain	32	20.9%	Procedure	140	50.0%
Analyze	31	20.3%	Fact	124	44.3%
Justify	26	17.0%	Agree/Disagree	8	2.9%
Compare	24	15.7%	Recall	4	1.4%
Reason	12	7.8%	Vocabulary	3	1.1%
Predict	8	5.2%	Estimate	1	0.4%
Contrast	5	3.3%			
Multiple Representation	4	2.6%			
Clarify	3	2.0%			
Make a Connection	3	2.0%			
Make Sense	2	1.3%			
Generalize	1	0.7%			
Student Questions	1	0.7%			
Use the Reasoning of Others	1	0.7%			

Note. The *Agree/Disagree* type of low-level mathematics questions include: Agree/Disagree with a Classification, Agree/Disagree with a Definition, Agree/Disagree with a Fact, Agree/Disagree with a Prediction, Agree/Disagree with a Procedure, Agree/Disagree with Reasoning, and Agree/Disagree with Vocabulary.

Adisa asked a total of 433 mathematics questions comprised of 153 (35%) high-level and 280 (65%) low-level mathematics questions. Adisa's engineering background influenced her teaching as she sought to impress upon her students how prevalent mathematics is in the world and in the study of other sciences. Accordingly, 66% of Adisa's high-level mathematics questions asked students to Explain, Justify, Reason, or Analyze. Adisa feels it is important to ask "Why" and "How" questions to become aware of students' thinking as they apply concepts to mathematical exercises and real-world contextual problems, "That's one of the best parts of doing these kinds of things-it's a vision into their brains. That's why I guess I ask the questions a lot. How are we going to do that? How do you find an average?" (Reflection Meeting 2). Likewise, Adisa asked 14 different types of high-level mathematics questions including:

- "How are you thinking you're going to find slope, or rate of change?" (Explain)
- "What do you notice when you look at the product property?" (Analyze)
- "Why add [to both sides]?" (Justify)
- "What does the logarithm [equation] have in common with the exponential equation?" (Compare)

Generally, Adisa asked more low-level questions than high-level questions; except during Observation 6. During this lesson, students were asked to graph eight linear equations given on a handout. Adisa used high-level questioning to make sure students recognized the function of *m* and *b* in the linear equation, $y = mx + b$ by asking them to justify the placement of their initial point and the moves they followed to arrive at the second point necessary to graph a line. For instance, when graphing $y = \frac{5}{4}x + 2$, Adisa asked:

- "Why did they go up 5?" (Justify)
- "What in the equation told them to do that?" (Justify)
- "Why didn't they go left?" (Justify)

These questions supported understanding of the use of slope when locating points on a line.

Adisa asked six different types of low-level mathematics questions with Procedure (50%) and Fact (44.3%) questions being asked most often. Low-level mathematics questions were asked most frequently during Observation 7 in which 30 of the 32 mathematics questions were low-level. The middle school students who participated in Observation 7 were asked to graph five of the linear equations printed on a handout. Prior to graphing, Adisa used questioning to review some critical facts and procedures regarding graphs:

- "What axis is the horizontal axis?" (Fact)
- "When we're looking for the number representing the slope, it's always next to who?" (Fact)
- "To graph it $\left[y = \frac{1}{4}x - 1\right]$, where are we starting?" (Procedure)
- "There are 4 slopes that we saw on there, do we remember them?" (Recall)

The questions above relied on students being able to remember facts and procedures because they were asked to restate given information and to provide a step of a procedure. Although Adisa asked a greater variety of high-level mathematics questions, most of her questions were concentrated on Procedures and Facts (60%). The combination of high-level and low-level mathematics questions contribute to Adisa's belief that the best way to learn mathematics is to analyze new information, compare to current knowledge, and learn the procedures or rules to apply to new tasks (Pre-Observation Interview). Thus, her use of questioning consistently forced students to make connections between prior and new knowledge. She aided in this process by encouraging students to analyze new information with the goal of using the new information to perform unfamiliar mathematical tasks. Adisa believes performing mathematical tasks and students' academic confidence are interconnected. Thus, when students were not able to recognize connections among concepts or between concepts and procedures, Adisa reinforced mathematics skills and facts.

Making connections. Ideally, Adisa would like students to connect mathematics to other disciplines by applying mathematics to real-world scenarios. As mentioned above, she *agreed* that to be good at mathematics at school, students should understand how mathematics is used in the real world (Beliefs Inventory Survey item 14). Adisa believes one of her roles as a mathematics teacher is to provide real world context for the mathematics topics she teaches. While discussing the significance of studying logarithmic functions, Adisa stated:

> Where would we use this? Some of my kids have chemistry - pH. Some of my kids are studying waves. I said the Richter Scale is logarithmic. You just go through it and say here's where this works into life, this is what it is and why you need it, or this is how you would solve things. (Pre-Observation Interview).

Although Adisa believes students should connect mathematics to real-world scenarios by applying mathematics in practical contexts, she encouraged students to identify connections within mathematical concepts in order to perform mathematics tasks. Adisa believes the best way to teach mathematics is to invoke and engage students' prior knowledge to make connections to new information (Pre-Observation Interview). She used questioning to prompt students to make such connections and to help them recognize the relationship between mathematical concepts and their procedural applications. While discussing her perception of the best way to teach mathematics, Adisa stated:

> Starting with what we know, and then pushing them towards the fact that it's just, what have we done all along? It's not new. When you're teaching it's kind of; here's where we've been,

> here's the new stuff, here's how it's similar and different than where we've been (Pre-Observation Interview).

Adisa wants students to compare their current knowledge to the new material presented and recognize relationships between ideas. While teaching high school students how to rewrite logarithmic equations as exponential equations during Observation 1, Adisa referred to the statement, "$\log_b y = x$ if and only if $b^x = y$." To help her students see connections between the two equations, Adisa asked:

- "How do I rearrange the exponential equation to get the logarithmic version of it?" (Procedure)
- "What do you see?" (Analyze)
- "Where do I put the "b" from this equation in the log equation?" (Procedure)
- "Do you see any letters in common with the two?" (Compare)
- "In this exponential equation, where is the 'x'?" (Fact)
- "Is it a base or an exponent?" (Vocabulary)

The series of questions above guided students through a process of recognizing connections between logarithmic and exponential equations.

Encouraging students to make connections was such a vital element of Adisa's teaching that she stressed the importance of making connections during each reflection meeting. During Reflection Meeting 1, questions such as:

- "What is 0.25 the same as?" (Fact) and
- "How do I rearrange the exponential equation to get the logarithmic version of it?" (Procedure)

were asked to help students make connections between different numerical representations and different arithmetic structures to help guide the process of evaluating logarithms. During Reflection Meeting 2, Adisa reflected on the reason for the difference in the volume of questions asked during Observations 1 and 2 (166), which was a lesson on evaluating logarithms, and Observations 3 and 4 (96), which was an exploratory lesson on linear modeling. Adisa believes that when trying to help students see connections, she must engage them with questions, “I see the connection but making them see requires that you involve them so they’re with you” (Reflection Meeting 2). Therefore, the frequency of mathematics questions was greater in Observations 1 and 2 due to encouraging high school students to participate in the process of making connections. During Reflection Meeting 3, Adisa reflected on the following questions:

- “How do I undo power logs?” (Procedure), and
- “Why is 3 over 1 the same as 3?” (Justify)

Similar to Reflection Meeting 1, Adisa claimed she asked these questions because she wanted her middle school students to recognize relationships between numerical representations and arithmetic structures to develop fluency with procedural skills; such as knowing how to employ “rise over run” when the slope is written as a whole number. Reflection Meeting 4 focused more so on students’ difficulty making connections resulting in lessons that focused on mechanical procedures. The following exercises were assigned because students struggled with making the necessary connections to study linear equations more critically.

- Sketch the graph of each line: (Procedure)

a) $y = \frac{1}{4}x - 1$ b) $y = x + 1$ c) $y = 3x - 3$ d) $y = 4$ e) $x = 5$

Although Adisa felt these questions did not align with her beliefs because they seemed basic and mechanical, she felt that before they could make the important connection between data and graphing, they first had to master the skill of graphing linear equations:

> For me, this feels really elementary. They don't make that connection. I hate mechanics. For me, it's not in my beliefs. It doesn't make me happy to teach it this way. But understanding connections between data and graphing is so important for successful moving forward that I feel uncomfortable leaving them in a place where they haven't mastered the skill (Reflection Meeting 4).

Adisa helped her students make connections within mathematical concepts by prompting them to analyze new information and compare it to their current knowledge. To assist with this, Adisa used open-ended analysis questions to prompt students to inspect new information.

Analyze. Adisa believes that an initial analysis of the components of a new mathematical task is an essential technique for problem-solving, “I think that one of the first things I tell any kid approaching anything you don't know is that you analyze it. You study it (Reflection Meeting 4). Adisa often asked, “What do you see?” or “What do you notice?” as a way to encourage students to analyze given information using their existing knowledge. Not only did she use these questions to invoke students’ ability to notice, but she also used the questions to engage all students:

> I think one thing it does is it takes it out of my hands of just saying, ‘Here’s what’s there, and here’s what you do with it,’ and it puts it back into a place where a kid could—I mean, every kid in the room, high, low, lost, can ultimately see something. It gives them a place to start and to jump into the problem and then hopefully will lead towards the connection that gets them to the things they already know (Reflection Meeting 1).

Most times, Adisa used open-ended questions, such as “What do you see?” and “What do you notice?”, to encourage students to analyze. Other times, she was more specific about where she wanted students to focus their attention. During Observations 1 and 2, Adisa asked students to focus on the relationship between $\log_4 1 = 0$ and $4^0 = 1$ by asking:

- “What do you notice about this one?” (Analyze)
- “What do you notice about the right side of the equal sign?” (Analyze)

The same was seen during Observation 5 when Adisa used questions to help students recognize the components of logarithmic equations exhibiting the Product Property. For instance, Adisa gave students a handout on which the following was printed: “Product Property: $\log_b mn = \log_b m + \log_b n$”. To force students to analyze the equation and notice similarities between the expressions on both sides of the equal sign, Adisa asked:

- “What do you notice when you look at the product property?” (Analyze)
- “What do you notice about the bases of all of these [terms]?” (Analyze)

Analyzing new information is necessary for understanding relationships among mathematical concepts. Therefore, when Adisa recognized a lack in fundamental knowledge necessary to make such connections, she reinforced facts and skills to ensure students would have the confidence to pursue mathematical tasks.

Reinforcement and building confidence. A deliberate analysis of Adisa’s questioning practices supported her beliefs about teaching and learning mathematics and revealed another closely held belief - students need to master fundamental skills in order to experience success and build confidence. This underlying belief sometimes held precedence over her espoused beliefs regarding problem-solving models of teaching and learning, “I think a kid who has swag[ger] will take a swing at something. There's some confidence lacking that you have to fill in. Yeah, I do think sometimes it overrides wanting it to be this inquiry, solving kind of situation" (Reflection Meeting 4). As much as Adisa enjoyed her students analyzing, making connections and applying mathematics, she also had to attend to their need to master certain foundational facts and skills. For example, Adisa included questions such as:

- "Anything to the 0 [power] is?" (Fact),
- "How do we write that [$3 \times 3 \times 3 \times 3$] as a shortcut?" (Fact),
- For $9x + 3y = 26$, "How do I move this [9x] to the other side?" (Procedure),

in her lessons on logarithms and the slope-intercept form of linear equations to reinforce facts about exponents and the process for solving equations. She also believes mathematics tests should *sometimes* include questions based on recall of facts and procedures (Beliefs Inventory Survey, item 24). However, teaching mechanics and algorithms conflicted with her preference for Problem-Solving models of teaching and was a task she did not like to perform. This sentiment was also expressed on the Beliefs Inventory Survey when she *disagreed* that mathematics should be taught as sets of algorithms or rules that cover all possibilities (item 11).

Not only did Adisa dislike teaching mechanically, she also found it difficult to reach students who experienced gaps in their foundational knowledge. Adisa shared her difficulty with being able to employ problem-solving models of teaching with students who lacked basic arithmetic skills:

> I'm starting to understand it's a lack of fundamental skills in some things. You sometimes have to live in that low-end mechanics, or even maybe the middle, in order to be able to help a kid be able to do problem-solving on an inquiry basis. I want them to be inquiry-based. I want them to figure it out, but when you don't have the language, or you don't have that ability, it's really hard for me to support (Reflection Meeting 4).

Nevertheless, she wanted her students to develop number sense as a way to build confidence and experience success in mathematics. Adisa believes students with confidence will be less afraid to try mathematical tasks and will be able to engage in inquiry-based learning (Reflection Meeting 4). Thus, she assigned daily exercises to help improve her students' numeracy (Reflection Meeting 3) even though the action contradicts her response on the Beliefs Inventory Survey in which she *disagreed* that giving students, who are having difficulty, more practice by themselves during class is an effective approach (item 9). Failure was not an experience she wanted her students to endure, "Letting kids fail is the biggest nightmare of my teaching career. You want every kid in your room to be able to do this" (Reflection Meeting 4). This sentiment was evident as Adisa encouraged students to transfer their knowledge to mathematical tasks.

Transferring knowledge. Utilizing mathematical concepts and skills to complete exercises and story problems was the goal of Adisa's observed lessons. Applying mathematics was engrained in Adisa due to her engineering background, "For me, my background is in engineering, and so if it doesn't apply it's really hard for me to justify. The abstract mathematics, abstract algebra, that was tougher, because I couldn't see an application" (Pre-Observation Interview). Appropriately, Adisa believes the best way to learn mathematics is to apply it:

> When you approach anything new you look at it and you analyze it and say, 'Well, how's this compare to what I've done before? How is this new? What are the rules and the scheme I can use to apply it to solve the things I need it to do?' (Pre-Observation Interview).

Using new knowledge to perform mathematical tasks was evident in the observed lessons which Adisa asked students to apply procedural skills to assigned exercises. During Observation 5, students learned properties of logarithms that they were to use to complete exercises on a handout.

After analyzing the product, quotient, and power properties, Adisa asked questions that encouraged students to apply the properties:

- "How can I use the properties at the top of your paper to solve that problem $\left[\log_3 \frac{6}{5}\right]$ without typing it in the calculator?" (Procedure)
- "How are you going to break this $[\log_3 36]$ up?" (Procedure)
- "How do I apply the rule then?" (Procedure)
- "How do I know when I've expanded far enough?" (Reason)

The first three "How" questions above were coded as Procedure questions due to the elicitation of students' step-by-step processes for utilizing properties of logarithms. Adisa did not feel the questions above aligned with her beliefs because although students were asked to apply new concepts to mathematical tasks, she felt the tasks were mechanical (Reflection Meeting 3).

The lessons delivered during Observations 3 and 4 were more closely aligned to her beliefs about applying mathematics. During these observations, middle school students were asked to predict the number of marbles that would make the water level in a graduated cylinder rise from 80mL to 100mL. After recording their predictions, the students performed an experiment by placing marbles in the cylinder and recording the water level. The students were then asked to analyze their data table and determine the rate of change, independent and dependent variables, domain, range and a linear model of the experiment. After placing six marbles in the cylinder, the students had the data shown in Table 19. Adisa used questioning to help students think about how to determine the rate of change in the water levels:

- "What do you notice about the data and what's happening?" (Analyze)
- "What is your slope going to be?" (Reason)

- “What are you thinking of doing for calculating average change?” (Explain)
- “What do you think is causing that [changes in water level] to be different?” (Explain)

Table 19

Number of Marbles and Water Level

Number of marbles (x)	Water level (mL) (y)
0	80
1	81
2	82
3	84
4	86
5	87
6	88

Determining the method for finding the rate of change aligned with Adisa’s belief that in mathematics you can be creative and discover things by yourself (Beliefs Inventory Survey item 18). She appreciated the authentic nature of this problem because the data did not make the solution obvious, “I think the one thing for me is the numbers in math problems written by a math book, they’re always perfect. Ours weren’t perfect. I was so happy they weren’t perfect because that’s life. That’s reality” (Reflection Meeting 2). This lesson also coincided with her belief that solving mathematics problems often involves hypothesizing, estimating, testing, and modifying findings (Beliefs Inventory Survey item 12).

Recognizing relationships between mathematical concepts, structures, and processes was encouraged in each observed lesson. Adisa assisted in this process by helping students analyze new information and providing opportunities to utilize the new information. Yet, all students were not always able to recognize such connections, nor were they always able to use mathematical concepts, structures, or processes correctly. While Adisa believes learning mathematics should include discovery and real-world applications (Beliefs Inventory Survey items 14 and 18), she recognized her students’ need to master procedural skills; which compelled her to try to strengthen students' understanding of fundamental facts and processes during instruction. With that in mind, Adisa attempted to augment students' mathematics knowledge by using questioning techniques that sometimes did not align with her beliefs.

Reflections on questioning practices. Based on data collected from four reflection meetings, Adisa agreed 50% of the time that her questioning practices aligned with her beliefs. Table 20 shows the percentage of times Adisa felt her beliefs were in alignment with the high-level and low-level mathematics questions she asked during instruction. This table shows that Adisa felt a majority of her high-level questions did align with her beliefs, while a majority of the low-level questions did not align.

Table 20

Percentage of Times Beliefs Aligned with Questioning Practices - Adisa

	High-level mathematics questions	Low-level mathematics questions
Aligned	60%	40%
Not Aligned	40%	60%

For instance, during Observation 1, students were asked to evaluate $\log_{\frac{1}{6}} 36$. Each exercise prior to this example contained a whole number base which prompted Adisa to ask, "What's wrong with this problem now?" This question was coded as a Contrast (high-level) question because students were asked to recognize the difference between this example and the previous examples containing whole number bases. However, Adisa felt the question did not align with her beliefs. She felt the question was leading and fed students processes and algorithms (Reflection Meeting 1). Yet, she felt the low-level question, "It [32] is even. What does it break down to?" did align with her belief because she helped a student complete $\log_{32} 2$ by scaffolding the process. Adisa led the student to the correct answer by reminding him that the answer had to be a fraction. To assist the student with determining the fractional exponent, she asked, "What does it break down to?" to help the student realize that $32 = 2^5$. She followed by relating $32 = 2^5$ to $\sqrt[5]{32}$. Adisa explained, "I know that if you get it written in the same base, then exponent and logarithm connections become easy" (Reflection Meeting 1). Generally, she felt her mathematics questions aligned when she was encouraging students to hone their problem-solving skills or make connections (Reflection Meetings 1 and 2) and they did not align when she was promoting mechanical procedures (Reflection Meetings 1, 3, and 4).

Figure 18 shows the percentage of high-level and low-level mathematics questions Adisa asked between reflection meetings. Overall, 35% of Adisa's mathematics questions were high-level and 65% were low-level; which indicates a slight increase in the percentage of high-level and a slight decrease in the percent of low-level questions compared to the baseline percentages determined by Observation 1.

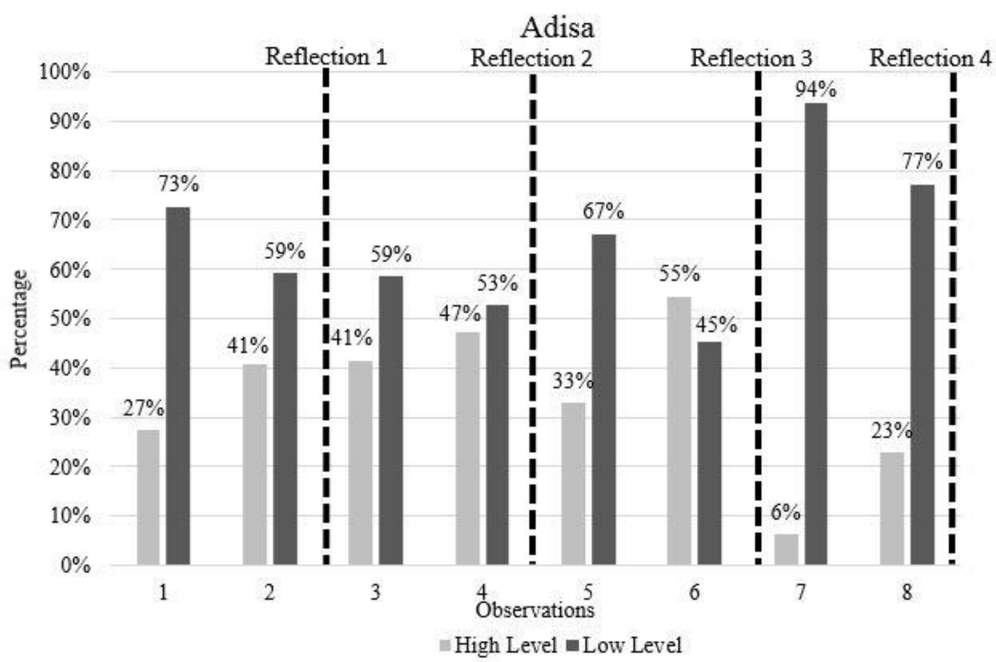


Figure 18. Questioning practices between reflection meetings – Adisa.

As listed in Table 17 above, Observations 1, 2, and 5 were lessons taught to high-school students and Observations 3, 4, 6, 7, and 8 were lessons taught to middle school students. The average percentage of high-level mathematics questions asked to high-school students was 33% compared to the average of 38% for middle-school students. Observation 7 (a middle-school lesson) appears to be an outlier as Adisa asked several low-level mathematics questions to prepare students to complete a handout which allowed them to practice the procedures for graphing linear equations.

During the End-of-Study Questionnaire, Adisa reiterated her belief that question-posing is a critical part of the learning environment and she would like to provide an environment that supports inquiry, question-posing, and critical thinking:

> I feel like one of the beliefs I have is that student questioning should be a part of the class and that my questions need to encourage them to inquire about math and hopefully to encourage them to think independently and persistently (End-of-Study Questionnaire, May 25, 2017).

She feels that an environment, where the teacher and students ask and answer questions, breeds confident and successful students who will participate in lessons. Adisa admitted that she doesn't often reflect on her questioning practices as she is frequently occupied with the multiple duties of teaching. However, this experience has encouraged her to investigate questioning further, "The experience has encouraged me to want to go observe others through that lens of questioning. I can see that there is a possible professional learning community that encourages this type of reflection" (End-of-Study Questionnaire, May 25, 2017). Adisa stated that she will continue to ask, "What do you see?" because she feels it engages students. She also felt that reflection allowed her to notice that she would like to develop high-level mathematics questions, "The questions I ask are not incredibly deep. I would like to develop a set of questions that are at a higher thinking level" (End-of-Study Questionnaire, May 25, 2017).

Cross-Sectional Analysis

An analysis of the four case studies revealed pervasive themes related to differences in their beliefs about their teaching methods, ideas about student learning, questioning practices, and reflection on questioning practices. Data collected from the Beliefs Inventory Survey, Pre-Observation Interview, Classroom Observations, Reflection Meetings, and the End-of-Study Questionnaire produced the following themes: different uses of questioning to engage students, perceived incongruencies, and reflecting on reflection. These themes are detailed below.

Use of questioning to engage students. Student engagement was influenced by the questions participants asked while teaching and while assisting students as they worked on exercises. Students were invited to engage in lessons at low and high levels of cognition as demonstrated in the types of questions asked by participants. Additionally, each participant required students to draw on new knowledge to complete mathematical tasks. But modes of engaging students differed based on participants' perceptions of what knowledge students were expected to use to complete tasks. Lastly, assigning "problems" was a common method for engaging students. However, the level of engagement differs when students are problem-solving versus solving problems. Each of the methods of engaging students are described below.

Question types. Table 21 displays the frequency and percentage of high-level and low-level mathematics questions asked by all participants. The table indicates that participants mainly engaged students by asking questions that elicited factual (48.4%) or procedural (43.2%) responses among the low-level questions. These two types of questions accounted for 65% of the total questions asked by all participants. This means a significant amount of student engagement

Table 21

Frequency and Percentage of High-Level and Low-Level Mathematics Questions – All Participants

High-level			Low-level		
Question type	Frequency	% of high-level questions	Question type	Frequency	% of low-level questions
Explain	64	13.9%	Fact	616	48.4%
Justify	62	13.5%	Procedure	550	43.2%
Compare	51	11.1%	Vocabulary	46	3.6%
Analyze	43	9.3%	Agree/Disagree	45	3.5%
Reason	43	9.3%	Recall	13	1.0%
Classify	39	8.5%	Estimate	2	0.2%
Clarify	23	5.0%			
Generalize	22	4.8%			
Definition_Open	21	4.6%			
Contrast	18	3.9%			
Multiple Representations	17	3.7%			
Predict	17	3.7%			
Make Sense	11	2.4%			
Make a Connection	10	2.2%			
Recall a Pattern	7	1.5%			
Student Questions	6	1.3%			
Use the Reasoning of Others	6	1.3%			

Note. The *Agree/Disagree* type of low-level mathematics questions include: Agree/Disagree with a Classification, Agree/Disagree with a Definition, Agree/Disagree with a Fact, Agree/Disagree with a Prediction, Agree/Disagree with a Procedure, Agree/Disagree with Reasoning, and Agree/Disagree with Vocabulary.

involved stating a previously learned mathematics fact or completing practice exercises to rehearse newly learned procedures. Following these methods of student engagement were high-level questions that required students to explain their thinking (13.9%), justify solution processes (13.5%), and compare mathematical ideas (11.1%). These top 3 types of high-level questions amount to 10% of the total questions asked by all participants. Students were not often engaged in sharing their thinking about mathematical ideas or about their particular mathematical process. Nor were they often engaged in comparing concepts learned in class. Distinct differences in the ways participants used questioning to engage students surfaced during classroom observations.

Eban asked questions while demonstrating procedures and when reinforcing procedures as students practiced. While showing students how to multiply polynomials, Eban used the example, $3x^2(2x^3 - x^2 + 4x - 3)$, to demonstrate the process by asking questions such as:

- "What's 3 times 2?" (Fact)
- "If I have 2 x's and 3 more x's, how many x's do I have?" (Fact)
- "What am I going to multiply $3x^2$ by?" (Procedure)

Mathematics questions like the examples above demanded simple responses and reflected students' mastery of facts and skills. Students engaged in the lesson by providing quick and short responses to a series of fact and procedure questions.

Each of Oliver's classroom observations consisted of students working on exercises for the duration of the class period. Several of Oliver's mathematics questions were delivered in printed form on a screen (Observations 1, 2, and 3) or on a handout (Observations 4, 5, 6, 7, and 8). Thus, students engaged in the lesson by responding to the mathematics questions Oliver presented and by responding to mathematics questions Oliver asked to guide students through difficult exercises. For instance, Oliver presented the mathematics question shown in Figure 19 during Observation 4. After letting students work on the problem for approximately 20 minutes, Oliver asked the following questions to assist his students with completing the exercise:

- "What's the coordinates for point A?" (Fact)
- "What's the coordinates for point A prime?" (Fact)
- "7 plus what gives us -1?" (Fact)
- "8 plus what gives us 0?" (Fact)

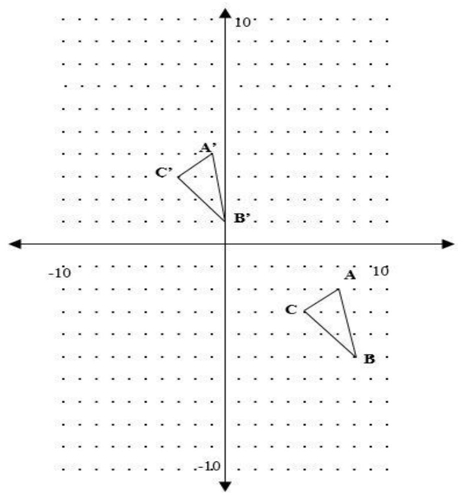


Figure 19. Translation problem.

Adisa's questioning focused mainly on helping students understand connections between concepts, procedures, and mathematical structures. She prompted students to recognize the connections within the structures of the Product Property, Quotient Property, and Power Property by asking questions such as:

- "What do you notice about the bases of all of these [properties]?" (Compare)
- "When we had exponents, and were multiplying, and had the same base, we added. But when we divided what did we do?" (Procedure); "What do you notice about this [Quotient Property] then?" (Analyze)
- "What's the change from this [$log_b m^n$] to this [$n\ log_b m$]?" (Contrast)

Adisa later asked students to use what they noticed about the properties to complete practice exercises. Students engaged in the lessons by responding to questions that sparked analysis and discussion and questions that prompted them to transfer their knowledge of these connections to mathematical tasks.

Tafari engaged students in dialogue about mathematics concepts and mathematical tasks by asking questions and encouraging students to ask questions as well. Students revisited their previous knowledge about the area of polygons before exploring the new topic of surface area during Observation 1. Tafari provoked discussion by asking questions such as:

- “Jerry said area is length times width. Is he always correct though; is it always length times width?” (Generalize)
- “How is volume different than area?” (Contrast)
- “Any questions about surface area before we move on?” (Student Questions)

Tafari frequently used the word “I” in her questions to demonstrate thinking processes that students were to transfer to exercises. For instance, Tafari helped students determine the signs of the (x, y) coordinates of points located in Quadrant 3 by asking:

- “If I am at the origin, do I have to go to the left for my *x*?” (Procedure)
- “Do I have to go down for my *y*?” (Procedure)

Tafari explained her deliberate use of the word “I”, stating:

> Yes, that was intentional. I have found that when I ask them "you", many think that means someone else. But when I ask "I", more students tend to start thinking about the question for themselves. That simple pronoun choice throws them off just enough that it gets them thinking. I want them to put themselves into the situation, so when they hear "I", they begin thinking about themselves (personal communication, November 27, 2017).

The questioning practices participants used to engage students provided the knowledge students were expected to use when working on mathematical tasks. The apparent differences in the methods used to engage students were also present in the ways participants expected students to draw on their new knowledge while completing mathematical tasks.

Drawing on new knowledge to complete mathematical tasks. Participants shared the belief that the best way to learn mathematics is to apply concepts and procedures to critical thinking exercises, practice exercises, and real-world scenarios. Each participant encouraged students to connect previous or new mathematical knowledge to assigned tasks. However, participants held different views on what was to be connected or transferred to mathematical tasks. For instance, Tafari asked questions that allowed students to use the knowledge they gained from classroom discussions and lectures to assigned exercises. During Observation 7, Tafari asked questions that led students through the process of determining outcomes of rolling 2 dice. She began by asking, "If I roll 2 dice, the first die could be… and the second die could be…?" (Fact). As students provided possibilities, Tafari constructed the table of outcomes shown in Figure 20.

Dice

1	1
1	2
1	3
1	4
1	5
1	6

2	1
2	2
2	3
2	4
2	5
2	6

Figure 20. Tables of outcomes for rolling 2 dice.

Next, Tafari asked:

- "How many outcomes are there when the first die is 1?" (Fact)
- "How many outcomes are there when the first die is 2?" (Fact)
- "How many outcomes do you think I might have when the first die is 3?" (Predict)

- "How many possible outcomes are there when there are 2 dice?" (Predict)

These questions led students to the following mathematical task Tafari asked them to complete, "If I wanted to know the probability or rolling 2 dice and getting a total less than 6, how many total outcomes are there?" (Reason)

Adisa asked questions to assist students with recognizing connections between mathematical structures; specifically, between previously studied structures and structures students were asked to manipulate in mathematical tasks. During Observation 6, she encouraged students to compare $y = \frac{5}{4}x + 2$ to $y = mx + b$ by asking:

- "How do you know [that 'y' is alone in this equation]?" (Explain)
- "What is 'm'?" (Compare)
- "What is it next to?" (Fact)
- "What is 'b'?" (Compare)
- "Do you see where they plotted 2 on the graph? It's on which axis?" (Fact)

After students practiced comparing the standard slope-intercept form to specific linear equations, they were asked to complete a worksheet that asked them to sketch lines and identify the slopes and y-intercepts.

In Eban's and Oliver's classrooms, students were expected to connect observations of correctly-performed mathematical procedures to practice exercises assigned during class. Eban insisted students see correct examples printed in the book or performed by her. One instance occurred during Observation 1 when Eban told students to copy Example 1a (Solve $3x^2 = 27$) from the textbook. When a student attempted to ask a question about solving the equation, the following exchange occurred:

Eban: "Should *you* do the example?"

Student: "No, but…"

Eban: "Am I going to answer your question?"

This exchange reinforced Eban's insistence that students wait for her demonstration before attempting a practice exercise. Once she began demonstrating the process for solving the equation, she asked the same student to tell her the first step. When the student answered incorrectly, Eban responded, "So this is why you should wait for me, right?" After demonstrating three more examples, Eban assigned practice problems that students were to complete by emulating the procedures they had just observed.

Oliver also expected students to transfer demonstrated procedures to practice exercises. As stated above, Oliver often posted questions and solutions simultaneously and instructed students to copy both in their notes. He explained why he used this method to deliver instruction by stating, "Normally, it's the first time that they have been exposed to that kind of a problem, so normally I give them a quality example in order to help them with further problems like it" (Reflection Meeting 3). Oliver admitted that while his method related to Instrumentalist models of teaching and learning, the method aligned with his beliefs because, "I'm just giving the answer to a problem to model what they should be doing" (Reflection Meeting 3). Although one of Oliver's questioning practices was to provide solutions, without allowing students time to address the questions he posed, he assumed students would use the solutions as a guide when responding to similar mathematics questions in the future. Each participant wanted their students to be able to complete practice and critical thinking exercises. During Reflection Meetings, it became evident that there were differences between preparing students to solve problems and preparing students to engage in problem-solving.

Problem-solving versus solving problems. While mathematical tasks (also called "problems") were assigned by each participant, it became apparent that solving "problems" is not the same as "problem-solving." Reflection Meetings revealed the dual use of the word "problem" when referring to both practice exercises and critical thinking exercises. However, participants' perceptions of teaching and learning mathematics influenced their multiple interpretations of what constitutes a "problem-solving" experience for students. Tafari perceived problem-solving as a type of capability or approach to a task. When discussing her emphasis on asking students to look for patterns associated with multiplying several integer factors, Tafari stated:

> I definitely think the more time the kids have to come to their own understanding, the more problem-solving capabilities they end up [with]…you're trying to reconcile the stuff in front of you with something that makes sense…relying on what you already know. That's what problem-solving is; you have something in front of you, and you want it to make more sense, or you want to organize it (Reflection Meeting 4).

Oliver viewed problem-solving as any type of exercise that challenges students' current knowledge or ability and involves minimum teacher involvement. As stated above, Oliver presented the question, "Quadrilateral ABCD has a least one pair of opposite angles congruent. What types of quadrilaterals meet this condition?" during Observation 1. During Reflection Meeting 1, Oliver explained that asking students this question was a "problem-solving" experience because:

> For them, especially after the long break that they had, it was also a problem-solving question because a lot of them have a hard time remembering what they were thinking of. It's also problem-solving because they thought they knew it, but they weren't paying attention to all of the information that was in the picture.

During the same reflection meeting, Oliver described himself as a "problem-solving teacher" who leans toward "very minimalistic instruction".

Eban believes problem-solving pertains to real-world scenarios in the form of story problems; which she expressed while reflecting on the low-level questions she asked during Observation 1, "I mean it's problem-solving, but it's not problem-solving to what I like to think of as real world, like the story problems" (Reflection Meeting 1). Eban also believes problem-solving means recognizing when a mathematical task invites multiple approaches. For instance, she began Observation 8, a lesson on the Quadratic Formula, by asking, "Who can tell me one way we have already solved quadratics?" When asked if wanting students to realize that there are multiple methods for solving a problem aligned with her beliefs, she responded, "For problem-solving, I would say yes." Eban's specific response supports her belief that it is false that mathematics problems can be done correctly in only one way (Beliefs Inventory Survey item 17). Although problem-solving was not explicitly discussed during interviews or Reflection Meetings with Adisa, she frequently referenced her desire to expose students to real-world applications of the mathematics her students learn:

> For me, my role is to facilitate it. Some of my kids are studying waves. You just go through it and say, 'Here's where this works into life. This is what it is and why you need it,' or "This is how you would solve things," that's, for me, the facilitation aspect (Pre-Observation Interview).

Among the participants, differences in perceptions of teaching and learning mathematics attributed to differences in their use of questioning. Additionally, perceived incongruencies surfaced within individual beliefs and practices that raise questions about the correlation between questioning practices and beliefs.

Perceived incongruencies. As stated in Chapter 2, beliefs drive action, affect behavior, and frame the cognitive approach to a task. Due to the relationship between beliefs and actions, any disaccord between espoused beliefs and actions are assumed to be "perceived incongruencies" as participants may hold concurrent beliefs that are unfamiliar to the researcher. Thus, the beliefs described below were espoused by participants or inferred by the researcher. The following describes perceived incongruencies between beliefs and questioning practices.

Presumably, the best ways to teach mathematics would coincide with the best ways to learn mathematics. In other words, teaching methods should be based on learning theories and philosophies. According to Figure 21, which displays aggregate responses to the Beliefs Inventory Survey, participants believe models of teaching and learning mathematics should resemble Platonist/Problem-Solving and Problem-Solving characteristics. This is according to the figure which indicates 60% of the responses (regarding mathematics teaching) and 79.2% of responses (regarding mathematics learning) align with these classifications.

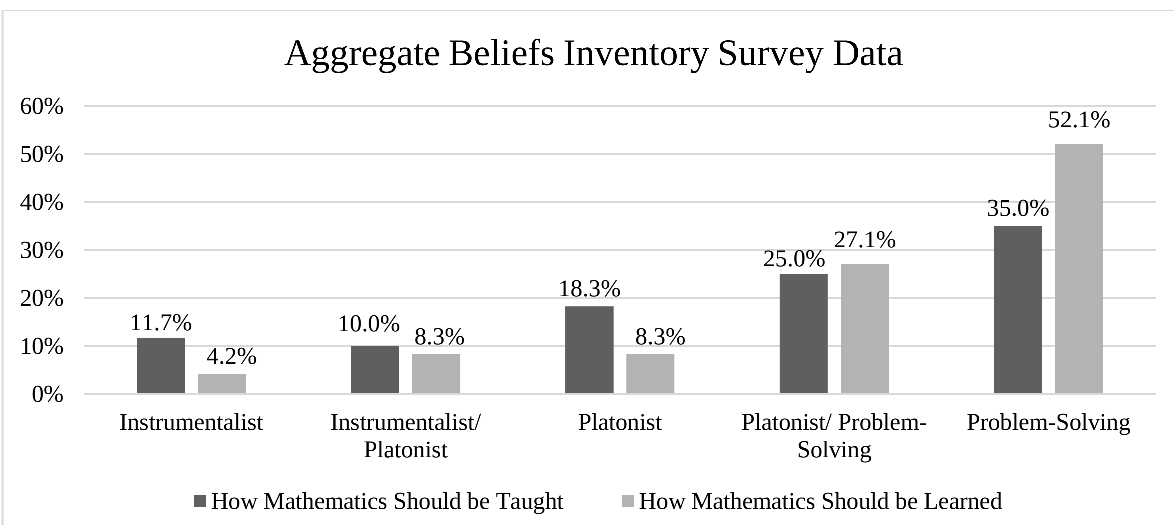


Figure 21. Aggregate response data from Beliefs Inventory Survey.

Each participant believes that in mathematics, you can be creative and discover things by yourself (item 18) and that to be good at mathematics, it is very important for students to be able to provide reasons to support their solutions (item 27). These beliefs coincide with high-level questioning that gives students the opportunity to explain, justify, and investigate. However, during classroom observations, the majority of participants' questions were more closely related to Instrumentalist and Instrumentalist/Platonist models as they frequently asked students to state facts and follow procedures (see Table 21). Details surrounding the perceived incongruencies between espoused or inferred beliefs and questioning practices were provided by participants as they engaged in the reflection process.

Beliefs and questioning practices. Due to the influence teachers' questions have on students' interaction with mathematics, teaching and learning mathematics require both high-level and low-level questioning. This idea is supported by data shown in Figure 21 above. The figure shows that participants believe aspects of each model of teaching and learning should occur in the classroom. Still, 43% of the overall responses on the Beliefs Inventory Survey were classified as Problem-Solving (35% for how mathematics should be taught, 52.1% for how mathematics should be learned). Problem-solving models of teaching and learning encourage high levels of cognitive demand and involve student inquiry, discovery, critical thinking, and exploration. While it is unrealistic to expect mathematics teaching and learning to always take this form, it is worth noting when teachers' questioning practices allow (or not) opportunities for students to experience learning in this fashion.

Since participants believe teaching and learning mathematics should resemble Platonist/Problem-Solving and Problem-Solving models, one may not expect every question to be high-level but, might assume most lessons would contain a substantial proportion of questions that provoke high levels of cognitive demand. A perceived incongruency exists between beliefs shared on the Beliefs Inventory Survey and participants' questioning practices; which indicate that low-level questions were asked more than twice as often as high-level questions (with the exception of Adisa). Table 22 compares the percentage of low-level and high-level mathematics questions asked by each participant as well as the ratio of low-level questions to high-level questions. Adisa, Tafari, and Oliver asked low-level questions at similar rates, whereas Eban asked more than 7 times as many low-level questions as high-level ones.

Table 22

Comparison of Participants' Ratio of Low-Level to High-Level Questions

	Percentage of low-level questions	Percentage of high-level questions	Ratio of low-level to high-level questions
Adisa	65%	35%	1.83
Tafari	69%	31%	2.27
Oliver	70%	30%	2.30
Eban	88%	12%	7.25

Assuming teachers act in accordance with their beliefs, perceived incongruencies between espoused beliefs and questioning practices warrant clarification. Therefore, low-level mathematics questions were analyzed to determine factors that drove participants' use of low-level questions as they, presumably, contradict Problem-Solving methods of teaching and learning. Table 23 displays the classifications of models of teaching and learning mathematics associated with each participant based on responses to the Beliefs Inventory Survey. Also displayed is the percentage of low-level mathematics questions that participants claimed were in alignment with their beliefs. According to Table 23, Tafari's beliefs about teaching and learning mathematics were classified as Platonist/Problem-Solving and Problem-Solving; which indicates teaching for

Table 23

Beliefs Classifications and Percentage of Low-Level Questions Aligned to Beliefs

	How mathematics should be taught	How mathematics should be learned	% of low-level mathematics questions aligned with beliefs
Tafari	Platonist/Problem Solving	Platonist/Problem-Solving and Problem-Solving	17%
Adisa	Problem-Solving	Problem-Solving	40%
Oliver	Problem-Solving	Problem-Solving	50%
Eban	Platonist/Problem-Solving	Problem-Solving	86%

conceptual understanding and learning that is student-constructed. Reflecting on the low-level questions she asked, Tafari claimed only 17% of the low-level mathematics questions discussed during reflection meetings aligned with the beliefs inferred from the survey (83% did not align). Tafari recognized and accepted that many of her low-level questions did not align with her espoused beliefs. On the other hand, Eban's Beliefs Inventory Survey produced similar classifications as Tafari's survey. However, reflecting on the low-level mathematics questions she asked, Eban claimed 86% of the low-level mathematics questions aligned with the beliefs inferred from her survey. Her claim indicates that she did not recognize the incongruencies between the frequent use of low-level questioning and Problem-Solving models of teaching and learning.

Although participants' questioning practices appear to be incongruent to the beliefs inferred from the Beliefs Inventory Survey, their questioning practices do align with concurrently held beliefs participants revealed in Pre-Observation Interviews and Reflection Meetings. An analysis of the reasons given for asking low-level mathematics questions showed that participants asked low-level mathematics questions when reviewing facts or procedures, assessing, combating common misconceptions or because students were classified as low-performing. Frequent use of low-level questioning (in relation to high-level questioning) that leads students through a process or reviews factual content does not support Problem-Solving models of teaching and learning which include open-ended questioning and student inquiry and investigation as classified in the Beliefs Inventory Survey. However, participants were able to rationalize the perceived incongruencies by providing context and revealing additional beliefs (not expressed during the Beliefs Inventory Survey) that played a role in asking low-level questions.

For instance, Tafari admitted she asked some low-level mathematics questions to make mathematics seem "manageable" and to increase student participation during lessons. She explained that she asked Procedure questions while delivering instruction on finding surface areas because, "[I] wanted them to see, yes, there's a lot of faces on this thing, it's one easy piece at a time" (Reflection Meeting 1). These reasons contribute to her belief that students need to exhibit confidence, risk-taking, and perseverance to learn mathematics. Adisa realized that in addition to wanting students to apply mathematics to real-world scenarios, she also wants to increase student confidence and build the foundational skills of her low-performing students. While explaining why she lives in "procedure land" despite her propensity to conduct lessons with inquiry and discovery, Adisa stated, "I see the kids who suffer or struggle. Watching them feel like they can't do this makes my heart just break. I tell them all the time, 'If you can follow a procedure…then you can do math" (Reflection Meeting 4). Thus, the low-level mathematics questions she asked were used to build procedural fluency and knowledge of facts to support her students' needs. She believes this level of questioning will also prepare her students to engage in problem-solving on an inquiry basis, which is one of her instructional goals (Reflection Meeting 4).

Oliver used low-level questions to review material; which follows his description of learning as practicing an action until you are able to do it successfully (Pre-Observation Interview). While reflecting on low-level questions asked during Observation 4, Oliver revealed that the entire lesson was designed to review previously learned concepts, “It was all supposed to be review from when they had a sub[stitute]” (Reflection Meeting 3). Oliver’s claim that the entire lesson was a review provides insight into why 17 of the 19 total mathematics questions were low-level. The focus of Eban’s low-level questions was to encourage mastery of facts and procedures and cater to low-performing students. She frequently described mathematics as a process to be learned and applied to practice exercises. When asked why she asked a series of Fact questions during Observation 1, Eban stated, “You need to know the facts. Whether it be the vocab[ulary] that they need to know of just related to procedures, but they need to know the facts” (Reflection Meeting 1). The participants’ perspectives shared above support the idea that beliefs are contextual and that actions may actually correspond to beliefs reserved for particular contexts. These contexts, as well as other impacts on teaching become evident as teachers reflect on their practices.

Reflecting on reflection. Participants reflected on more than their question-asking practices, they also took a metacognitive look at the act of reflecting. The End-of-Study Questionnaire allowed participants to reflect on the experience of reflecting on their questioning in relation to their beliefs about mathematics teaching and learning. While completing the questionnaire, participants expressed different impacts of performing a deliberate analysis of their questioning practices. When asked to, “Describe your experience with reflecting on the questions you asked during mathematics instruction” (End-of-Study Questionnaire), various reactions were shared. Adisa expressed a desire to continue conducting research on questioning by analyzing the questioning practices of other teachers, “The experience has encouraged me to want to go observe others through that lens of questioning.” Tafari mentioned how she developed, professionally, by thinking about her questioning practices during instruction:

> I learned a lot about myself as a teacher through this dialog. At the beginning, I didn't put a lot of thought into the specific questions I was asking. While reflecting, it was eye-opening to talk about 'why' I was asking the particular questions in my lesson.

Oliver described his experience by sharing what he observed about his own questioning practices throughout the study, "I asked more leading questions near the beginning of this experience, but then I asked more thinking questions near the end". Eban found it interesting to learn the impact of her beliefs on her questioning practices, "Because teaching is sometimes second nature to me, I do things without the realization of what I do…I found it interesting to learn how my beliefs affected the way I teach." The theme throughout Eban's Pre-Observation interview was that teaching requires teacher modeling and learning results from student practice. Her beliefs about mathematics teaching and learning were reflected in her classroom instruction as she modeled correct procedures and expected students to practice modeled procedures. The impacts of reflecting on questioning practices (as described above) are important for teacher development. The desire to observe other teachers, learning about your own professional identity, noticing changes in your instruction, and recognizing links between your beliefs and questioning practices all contribute to the development of a teacher's professional knowledge and can be achieved by reflecting on specific aspects of teaching.

All in all, the various uses of questioning to engage students during instruction and the ways participants wanted students to draw on and transfer knowledge to mathematical tasks followed from their individual perceptions of teaching and learning mathematics. Questions that preceded and guided students through practice exercises were more common than questions that elicited critical thinking such as analyzing, justifying, and reasoning. Through reflection, participants were able to contemplate the beliefs that informed their questioning practices and decide if their use of questioning aligned with their ideal perceptions of engaging students in mathematics learning. While each believed that teaching and learning mathematics should encompass characteristics of Problem-Solving models, they exhibited Instrumentalist models of teaching which involves asking more low-level than high-level mathematics questions (Hus & Abersek, 2011; Kawanaka & Stigler, 1999; Kosko et al., 2014; Ni, et al., 2011; Sloan & Pate, 1966). However, experience with reflecting on their questioning practices affected them in very different, yet very important ways. This study allowed teachers to not only focus on student learning (related to question-asking practices), but also on their own learning as practitioners.

CHAPTER 5 DISCUSSION

This chapter summarizes findings of the present study and addresses the research questions presented in Chapter 1. Additionally, connections to current literature are made to the findings of this study along with implications for future teaching and learning. Recommendations for future research are provided based on additional inquiries of the researcher and on apparent gaps in current research. Finally, limitations of the present study are described.

The following research questions guided this study:

1. What are teachers' question-asking practices during mathematics instruction?

   1a. What reasons do teachers provide for their question-asking practices during mathematics instruction?

2. What relationship exists, if any, between teachers' beliefs about mathematics teaching and learning and the reasons they provide for their question-asking practices during mathematics instruction?

3. What impact does reflection on question-asking practices have on teachers' thinking about the mathematics questions they ask during instruction?

Research Question 1 – What are Teachers' Question-Asking Practices During Mathematics Instruction?

Classroom observation data revealed that participants in this study asked more low-level mathematics questions than high-level mathematics questions. In the present study, 73.4% of mathematics questions were low-level and 26.6% were high-level. This finding supports findings of previous studies indicating that teachers tend to ask more low-level questions (Boaler & Brodie, 2004) and that 50% - 80% of teachers' questions are low-level (Delice, et al., 2013). Although low-level questions prevailed, the present study showed participants asked a greater variety of high-level questions than low-level questions. This could be due to participants indicating that they wanted their students to understand concepts and to use problem-solving strategies. Thus, providing a greater variety of opportunities to experience or practice problem-solving methods.

Participants of the present study wanted their students to be able to practice problem-solving strategies and to correctly complete mathematical exercises. Their question-asking practices reflected these contextual instructional goals as many of the high-level mathematics questions were Justify, Analyze and Reason questions while many of the low-level questions were Procedure questions. Differences among their practices could be due to differences in what each participant believed were necessary prerequisites to successfully perform mathematical tasks. Tafari believed that certain dispositions (perseverance, confidence, and risk-taking) were necessary to successfully complete tasks. She asked the most Agree/Disagree questions; which allowed students to safely take a position, share an opinion, and be the subject of critique by their classmates. Eban viewed the practice of mathematics as a mechanical process to be modeled and replicated. Successful mathematics students were those who performed procedures correctly. Consequently, Eban asked the most Procedure questions; which emphasize the rules and steps to be followed to find the answer to mathematical exercises.

Oliver believed students would be successful at completing tasks when they assumed certain roles in the classroom. While working collaboratively, Oliver believed students should assume dual roles of receivers and dispensers of knowledge within their group. Students could successfully complete mathematical tasks when they worked and learned from each other. Correspondingly, Oliver asked the fewest questions and used leading questions when students struggled with mathematical tasks. Adisa believed it was important for students to recognize connections among mathematical concepts when attempting new tasks. Appropriately. She asked the most Analyze and Compare questions often asking students, “What do you notice”? This question prompted students to connect new ideas to previously studied content.

Overall, the percentages of low-level and high-level questions asked during the present study are consistent with traditional mathematics instruction. However, individual question-asking practices related to participants’ conceptions of what necessitates the successful completion of mathematical tasks. Student dispositions, replication, collaboration, and recognizing connections were perceived as essential for problem-solving and completing mathematical exercises. Thus, question-asking practices supported these endeavors.

Research Question 1a - What Reasons do Teachers Provide for Their Question-Asking Practices During Mathematics Instruction?

Based on data from Reflection Meetings, several reasons were provided to justify participants’ questioning practices. The most frequently given reasons for asking questions were: reviewing content; addressing a common misconception; and reinforcing procedures. Reviewing content was the reason given for asking both low-level and high-level mathematics questions. Low-level questions were asked to review facts, procedures, and definitions such as finding the surface area of a 3-dimensional shape, performing transformations, and computing logarithms. High-level questions were asked to explore characteristics of algebraic structures and of geometric figures when students engaged in activities such as comparing structures of quadratic equations and classifying quadrilaterals. Based on data from Reflection Meetings, participants did not indicate that the questions intended for review were in preparation for assessments. Rather, they were used to scaffold as students learned new skills to complete exercises related to the objective of the lesson.

Each participant used questioning to address anticipated student misconceptions. Preempting and tackling common misconceptions were reasons for asking questions that solidify facts and procedures such as finding the area of a rectangle, subtracting like terms, remembering $a^0 = 1$ (a $\neq$0), and recognizing that the numerator of a slope indicates the vertical part of the shift from one point to another point on a line. Although these concepts were taught prior to students' current grade level, they were concepts inherent in the mathematical exercises associated with the current lesson. Participants acknowledged that these concepts were difficult for students to retain, thus they used questioning intended to overcome these misconceptions.

Reinforcing procedures was the intention of questions that focused on graphing ordered pairs, determining the measure of missing angles, and manipulating properties of exponents. As these examples (and examples in the two preceding paragraphs) indicate, the most frequently given reasons for asking questions were to support procedural fluency. This coincides with classroom observation data which showed that Procedure questions accounted for a significant amount of overall questions asked by these participants. This implies that knowledge and usage of procedures was a prevalent aspect of the instruction observed during the present study.

The reasons mentioned above are heavily focused on student retention of facts and skills, which was the objective of most observed lessons. Rarely were participants' reasons for questioning to serve the purpose of discerning student thinking or to inform instructional decisions. Informing instructional decisions is one of the reasons Wiliam (2011) suggested for asking questions during instruction when he stated, “I suggest there are only two good reasons to ask questions in class: to cause thinking and to provide information for the teacher about what to do next" (p. 79). Instructional decisions are often based on student responses to questions. Further, questions that help teachers know what students are thinking allows teaches to design lessons relevant to students' needs (Kazemi & Franke, 2004).

Data from Reflection Meetings also indicated differences in reasons for asking low-level and high-level mathematics questions. Table 24 displays the six most frequently given reasons for asking low-level and high-level mathematics questions during instruction.

Table 24

Reasons for Asking Low-Level and High-Level Mathematics Questions

Reason for asking low-level mathematics questions	Reason for asking high-level mathematics questions
Reinforce Procedures	Encourage Critical Thinking
Review Content	Review Content
Address a Common Misconception	Address a Common Misconception
Make a Connection	Make a Connection
Build Confidence	Develop Conceptual Understanding
Increase Participation	Provide Real World Context

The three most frequent reasons for asking low-level mathematics questions were centered around procedural fluency, retention, and misconceptions, as described in the sections immediately above. The last two reasons refer student dispositions and behaviors. Asking low-level questions was sometimes intended to build students' academic confidence by making content seem accessible and mathematical tasks seem manageable. Additionally, low-level questions were asked as a tactic to receive more responses from more students during instruction. These questions required quick and simple responses from students allowing more students an opportunity to respond. Many of the reasons for asking high-level questions align with higher-order student thinking. Critical thinking, conceptual understanding, making connections among mathematical concepts and to real-world contexts allow opportunities for rich engagement in lessons. As Table 24 indicates, reviewing content, addressing a common misconception, and making a connection were reasons for asking both low-level and high-level questions. This is because both levels of questioning can be effective for helping students reinforce, solidify, and analyze mathematical content.

Research Question 2 - What Relationship Exists, If Any, Between Teachers' Beliefs About Mathematics Teaching and Learning and the Reasons They Provide for Their Question-Asking Practices During Mathematics Instruction?

Based on the data, a positive relationship exists between participants' reasons for asking questions and their espoused beliefs about mathematics teaching and learning. The complex structure of one's beliefs and the format of the Beliefs Inventory Survey and Pre-Observation Interview did not allow participants to exhaustively disclose their beliefs about mathematics teaching and learning. Additional beliefs became apparent during Reflection Meetings as participants shared instructional goals and desired student behaviors. This triangulation of data supports Rokeach's (1968) and Pajares' (1992) notion that beliefs can only be inferred by what people do, say, and intend. Repeated reasons were considered as persistent intentions and thus related to one's beliefs about mathematics teaching and learning.

Tafari repeatedly mentioned building student confidence as a reason for asking questions. As stated in her profile in Chapter 4, she expressed her intent to show students that they are good at mathematics. Thus, Tafari asked questions that she felt would invite differences of opinion and reduce stress. Eban repeatedly mentioned review of content and reinforcing procedures as reasons for asking questions. These reasons align with beliefs she shared during the Pre-Observation Interview when she described the best way to teach mathematics as showing students steps to completing exercises, then asking them to practice those steps independently or collaboratively and when she agreed that mathematics should be taught as a collection of concepts, skills and algorithms on the Beliefs Inventory Survey.

Review of content was repeatedly mentioned by Oliver as well. Although Oliver did not suggest that frequent review of content was a pivotal component of instruction during the Pre-Observation Interview, he did agree (on the Beliefs Inventory Survey) that learning mathematics is a process which involves repeated reinforcement. Many of Adisa's reasons for asking questions centered around students making connections among mathematics concepts and making connections between mathematics and real-world scenarios. During the Pre-Observation Interview, she expressed the importance of making such connections as a crucial aspect of the learning process. Her reasons were further supported by her open response to the Beliefs Inventory Survey item 1 when she stated, "Students are more effective at understanding abstract concepts if they see problems that require connections between procedures and 'real-life.'"

The relationship between beliefs and reasons for asking questions supports the findings of Wilcox-Herzog and Ward (2004) who concluded that beliefs are predictive of intentions and when teachers espouse their beliefs, they are more likely to intend to act accordingly. In the present study, beliefs were more aligned to reasons for questioning than to the level (low or high) of questions asked. Reasons for asking questions refer to a teacher's perception of which questions would bring about desired student behaviors and dispositions; which is informed by their beliefs about teaching and learning. Whereas the classification of questions as low-level or high-level is based on the cognitive demand necessary for a student to respond. Teachers' knowledge of the various levels of cognition solicited from different question types does not imply that their question-asking practices will comply with that knowledge. As stated in Chapter 1, beliefs are stronger predictors of behavior than knowledge (Pajares, 1992). Thus, instructional decisions are more so based on teachers' perceptions than the objective classification of question types. Due to both low-level and high-level questions being effective for achieving teachers' intentions, it is difficult to map a belief to a specific question type. Regardless of one's beliefs about mathematics teaching and learning, mathematics instruction not only should include review of content and practice of procedures but also opportunities for students to reason about how to apply concepts and knowledge to new tasks. A combination of low-level and high-level mathematics questions provide a comprehensive mathematics learning experience.

Research Question 3- What Impact Does Reflection on Question-Asking Practices Have on Teachers' Thinking About the Mathematics Questions They Ask During Instruction?

The reflective process impacted participants noticing linkages and disconnections between their question-asking practices and their beliefs about mathematics teaching and learning. What participants discovered about their beliefs and question-asking practices influenced future instructional decisions. It is important for teachers to experience the reflective process in order to determine how their beliefs influence students' experience with mathematics. Linkages between beliefs and questioning were connected to static thinking and question-asking practices while disconnections prompted changes in thinking and questioning.

Engaging in reflection on one's question-asking practices requires an exploration of one's beliefs about teaching and learning and an examination of their own teaching practices (Richards & Lockhart, 1994). This process began with participants becoming aware of their own beliefs about mathematics teaching and learning. Completing the Beliefs Inventory Survey and providing responses to the Pre-Observation Interview forced participants to consider and verbalize their beliefs about mathematics teaching and learning. Inviting participants to reflect on their question-asking practices (by asking why they asked questions and if their question-asking practices aligned to their beliefs) brought their beliefs to the forefront of their professional knowledge. Awareness of their beliefs informed their decision on whether an alignment existed between their beliefs and their question-asking practices. Metacognition played a role in determining linkages and disconnections as participants used a posterior analysis of the thinking that directed their questioning decisions during instruction.

Analysis of their question-asking practices revealed that participants (with the exception of one person) generally believe their question-asking practices are in alignment with their beliefs more often than they are unaligned. This means that participants recognized linkages between their beliefs and question-asking strategies more often than the recognized disconnections. As stated in Chapter 4, the percentages of times participants felt their questioning aligned with their beliefs were: Eban (91%); Oliver (71%); Tafari (61%); and Adisa (50%). The percentage of times participants felt their questioning aligned with their beliefs could be related to how they described the effects of the reflective experience on the ways they think about their questioning during instruction. At the culmination of the present study, Eban, who expressed linkages most often, did not suggest making changes to her current question-asking practices during the End-of-Study Questionnaire. Oliver, Tafari, and Adisa, who expressed linkages less often, either recognized changes in their thinking and questioning or expressed a desire to continue reflective work. Based on this data, participants who felt their practice often aligned to their beliefs, are less likely to alter, or aspire to alter, their question-asking practices.

Connections to Existing Literature

The findings of this study highlight teachers' question-asking practices and the implications of teachers' reflection on their beliefs and their question-asking practices. The type and frequency of mathematics questions asked in the present study corroborate with findings of previous studies indicating the prevalent use of low-level questions during mathematics instruction (Boaler & Brodie, 2004; Delice, et al., 2013; Hus & Abersek, 2011; Kawanaka & Stigler, 1999; Kosko, et al., 2014; Ni, et al., 2011). The percentage of low-level mathematics questions asked during the present study are in the 50% - 80% range described by Delice, et al. (2013). Studies connecting teachers' questions to their conceptions of what is necessary to successfully complete mathematics tasks is scarce.

Several reasons for asking questions emerged from the data of the present study. A similar finding occurred in a study by Brown and Edmonson (1989) who polled 36 teachers of English, science, mathematics, second languages, history, and geography to determine their reasons for asking questions during instruction. Brown and Edmonson used teachers' responses to develop a classification system. Although the present study only focused on mathematics teachers, the most common reasons that emerged from the data of the present study (review of content, addressing common misconceptions, reinforcing procedures) are similar to the reasons Brown and Edmonson classified. In their nine-item schema, the most common reasons given for asking questions were: 1) encouraging thought, understanding of ideas, phenomena, procedures and values; 2) checking understanding, knowledge and skills; 3) gaining attention to task; 4) review, revision, recall, reinforcement of recently learned point, reminder of earlier procedures; and 5) classroom management, draw attention to teacher or text. Their findings indicate that review and checking knowledge and skills are also prevalent reasons for asking questions in subjects other than mathematics.

The present study found a relationship between teachers' reasons for asking questions and their beliefs about mathematics teaching and learning. Literature regarding the relationship between teachers' beliefs and their reasons for question-asking is also scarce. However, education researchers have studied the relationship between teachers' beliefs and their intentions (Beck et al., 2000; Wilcox-Herzog &Ward, 2004) - describing intention as the desire to engage in a particular behavior. Wilcox-Herzog and Ward (2004) found that teachers' beliefs about the importance of varying types of teacher-child interactions and their intentions for behaviors with children were significantly positively correlated. They also concluded that beliefs are predictive of intentions. This conclusion is part of Ajzen's (1991) seminal Theory of Planned Behavior which contends that the strength of one's salient belief about a behavior multiplied by the evaluation of the (positively or negatively valued) outcome of the behavior is directly related to one's attitude toward the behavior. One's attitude toward a behavior influences their intentions which, in turn, influence their behavior. The findings of these studies are similar to the findings of the present study in determining a relationship between one's beliefs and their intentions to or reasons for engaging in a particular behavior.

Another finding of this study indicated that, in most cases, teachers' reflection on their question-asking practices impacted their subsequent instructional decisions. This finding contributes to Brown and Coles (2012) notion of the two most important outcomes of reflection; learning and affecting future actions. Most participants of the present study noticed a shift in their question-asking intentions during instruction and while planning instruction. In a similar study, Senger (1998) used data from two forms of reflection, Video Reflection and Theory Reflection, to infer beliefs underlying mathematics' teachers' actions. Senger shared three implications following from her results:

> Collaborative and reflective dialogue (1) help teachers to discover beliefs that affect their practice. In the process, (2) teachers also change practices deemed not in alignment with

> their true beliefs. (3) The self-reflective nature of the process provides an opportunity for personalized professional development. (p. 37)

The second implication particularly applies to the different impacts the reflective process had on participants of the present study. An undesirable alignment between beliefs and question-asking practices could have acted as the catalyst for teachers to make adjustments to their practice, whereas a desirable alignment between beliefs and question-asking practices may have been seen as confirmation of effective teaching necessitating no adjustments to practice.

Implications for Future Teaching and Learning

As reiterated above, it is teachers' beliefs, more so than their knowledge, that impacts their behavior. Education stakeholders, such as teacher educators, professional developers, education reformers, and school district administrators, need to recognize this fact and not only rely on teachers' pedagogical knowledge to inform their teaching decisions but also their beliefs. The same stakeholders also need to understand the connection between teachers' beliefs, intentions, and actions and support opportunities for teachers to collaboratively conduct deliberate analyses of their instructional practice-with the intention of modifying their practice to meet the demands of a prescribed curriculum or reform efforts.

Based on their personal experience as mathematics students and mathematics teachers, educators hold perceptions of the kinds of experiences students need in order to be successful mathematics students. The choices inherent in their question-asking practices may be influenced by their notion that students need to possess procedural fluency and knowledge of facts and conceptual understanding to be successful mathematics students. These notions and related questioning practices occurred in the present study. The questioning practices of the participants of this study represent the practices of a larger sample of teachers with similar views of students' mathematics experiences. However, researchers (Boaler & Brodie, 2004; Fredericks, 2010; Weiland, et al., 2014) have found that teacher's questions influence students' thinking about mathematics. Fredericks theorized:

> Students tend to read and think based on the kinds of questions they anticipate receiving from the teacher. If students are constantly bombarded with questions that require only low levels of intellectual involvement (or no involvement whatsoever), they will tend to think accordingly. Conversely, students who are given questions based on higher levels of thinking will tend to think more creatively and divergently. (p. 128)

As noted throughout the present study, questions affect students' experience with mathematics. If student thinking about mathematics is to change, the questions they experience need to reflect the change we, as teachers, want to see in students' thinking.

Recommendations for Future Research

The present study triggered additional questions to be addressed with future research. Since questioning is such a widely used instructional tool, some aspects of questioning tend to be overlooked due to its excessive usage. Two questions, regarding questioning, that are worthy of investigation are: 1) "Do teachers know how to use questioning to their students' advantage?" In other words, "Do teachers' questions achieve their intended effect?"; and 2) "Do teachers plan their questions before instruction?" Question 1 derives from instances during the present study when participants thought they were asking questions to trigger higher-order thinking, but later determined that their questions did not achieve that instructional goal – leading one to consider if members of the teaching profession take for granted that teachers know how to question effectively. Question 2 follows from Question 1 as planning the questions to be asked can be useful in asking purposeful questions which have teachers' intended effects.

Additional research on the topic of reflection would also benefit the teaching profession. Reflection has been characterized as a key component of professional development, but do teachers share that conception? Another question for future research is, “Do teachers’ preconceptions about engaging in a reflective process affect their reflective experience and the outcomes of reflection?” Although the participants of this study volunteered to engage in all aspects of the study, including reflection, their preconceptions of engaging in a reflective process were not solicited before collecting data. Some participants of the present study were more responsive than others, meaning they provided more detailed descriptions of their beliefs and justifications for the questioning than other participants. While the difference in responses could be an indication of differences in personality, it is worth investigating whether the differences could be attributed to a disposition toward engaging in the reflective process.

An inventory of one’s beliefs is critical to the reflective process, however the role of beliefs begs the question, “What role, if any, does teachers’ awareness of their beliefs about mathematics teaching and learning play when they are reflecting on their questioning practices?” The impact of reflection on future action also presents questions for future studies. An investigation into changing teachers’ questioning practices could address the question, “How does reflection on their question-asking practices and beliefs about mathematics teaching and learning influence teachers’ subsequent question-asking practices?”

Limitations

This study included the reflections and questioning practices of a small number of teachers. The conclusions drawn address the reflection experiences, beliefs, and questioning practices of these participants, only. Thus, generalizations about the relationship between beliefs and questioning practices cannot be made solely based on the results of this study. Due to beliefs being complex mental structures and difficult to determine explicitly, beliefs must be inferred based on what one says, does, and intends (Pajares, 1992). The researcher selected survey and interview questions that she believed would provide an adequate depiction of participants' beliefs. However, participants may hold additional relevant beliefs about mathematics teaching and learning that were not elicited from survey questions or evoked during Reflection Meetings. The researcher used data triangulation and member checks to obtain the most accurate description of participants' beliefs. The classifications of survey responses, chosen by the researcher, were based on her interpretation of Ernest's (1989) models of teaching and learning. Based on Ernest's descriptions, others may arrive at different conclusions about teachers' beliefs.

Response bias could have also played a role in participants' responses to the Beliefs Inventory Survey and the Pre-Observation Interview. Responses that are unintentionally untrue or based on the participant's perception of the "right" answer could have skewed the data related to participants' beliefs about mathematics teaching and learning. The number of participants and the number of observations were also limitations of the present study. More teachers could have been studied and more observations could have been conducted. Reflection on more of the mathematics questions asked during instruction also limited the data collected on the reasons for teachers' questioning practices.

Conclusion

The question-asking practices of the participants of this study are consistent with the practices of most teachers in that questioning was most often used to review and practice mathematical facts, concepts, and procedures. However, teachers' reflection on their beliefs and question-asking practices yields several outcomes. Regardless of teachers' pedagogical knowledge or the instructional methods required of them, teachers act on their intentions, which are informed by their beliefs about mathematics teaching and learning. Awareness of teachers' beliefs aids in the process of productive reflection making it possible to notice dissonance or accord between beliefs and questioning practices. Recognizing the relationship between one's beliefs and practices can be an indicator of teachers' intention to enhance their instruction or continue on the same course of action; which is a significant finding of the present study. Educators and other stakeholders must acknowledge the importance and necessity of teacher reflection on professional development and should also make efforts to ensure teachers engage in the reflective process regularly, throughout their teaching careers.

Studying the relationship between teachers' beliefs and question-asking practices requires more than a cursory observation and account of the low-level and high-level questions asked. A more inclusive approach requires an understanding of the reasons or intentions of teachers' questioning. Another significant finding of the present study indicates beliefs are more linked to the reasons for asking questions than the level of questions asked. This could be due to question levels being based on the cognitive demand required to respond rather than a teachers' instructional goal. Thus, making it difficult to associate beliefs about mathematics teaching and learning to a particular level or type of questioning.

APPENDIX A

BELIEFS INVENTORY SURVEY

No.	ITEM	Agree	Somewhat Agree	Somewhat Disagree	Disagree	COMMENTS
1	Problem solving should be a separate, distinct part of the mathematics curriculum.					
2	Teachers' modeling of real-world problems is essential to teaching mathematics.					
3	Children should be encouraged to justify their solutions, thinking, and conjectures in a single way.					
4	The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in isolation.					
5	Skill in computation should precede word problems.					
6	Mathematics should be taught as a collection of concepts, skills and algorithms.					
7	A demonstration of good reasoning should be regarded even more than students' ability to find correct answers.					
8	More than one representation (picture, concrete material, symbol set, etc.) should be used in teaching a mathematical topic.					
9	If students are having difficulty, an effective approach is to give them more practice by themselves during class.					
10	Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.					
11	Mathematics should be learned as sets of algorithms or rules that cover all possibilities.					

12	Solving mathematics problems often involves hypothesizing, estimating, testing, and modifying findings.					
13	Appropriate calculators should be available to all students at all times.					
14	To be good at mathematics at school, students should understand how mathematics is used in the real world.					
		True	**More True than False**	**More False than True**	**False**	**COMMENTS**
15	In mathematics something should be either right or wrong.					
16	Good mathematics teachers should show students lots of different ways to look at the same question.					
17	Math problems can be done correctly in only one way.					
18	In mathematics you can be creative and discover things by yourself.					
19	To solve most math problems you have to be taught the correct procedure.					
20	The best way to do well in math is to memorize all the formulas.					
		Never	**Sometimes**	**Often**	**Always**	**COMMENTS**
	In teaching mathematics, how often should students be asked to:					
21	Explain the reasoning behind an idea					
22	Decide on their own procedures for solving complex problems					
23	Work on problems for which there is no immediately obvious method of solution					

	How often should the following types of questions be included in your mathematics tests or examinations?					
24	Questions based on recall of facts and procedures					
25	Questions requiring explanations or justifications					
		Not Important	**Somewhat Important**	**Very Important**		**COMMENTS**
	To be good at mathematics at school, how important do you think it is for students to:					
26	be able to think creatively?					
27	be able to provide reasons to support their solutions?					

APPENDIX B

BELIEFS INVENTORY SURVEY ITEMS BY CATEGORY AND SOURCE

How Mathematics Should Be Learned

Item Number	Beliefs Inventory Survey Item	Source
10	Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.	Hart 2002
11	Mathematics should be learned as sets of algorithms or rules that cover all possibilities.	TIMSS 1995
12	Solving mathematics problems often involves hypothesizing, estimating, testing, and modifying findings.	TIMSS 2003
13	Appropriate calculators should be available to all students at all times	Hart 2002
14	To be good at mathematics at school, students should understand how mathematics is used in the real world.	TIMSS 1995
18	In mathematics you can be creative and discover things by yourself.	Hart 2002
19	To solve most math problems you have to be taught the correct procedure.	Hart 2002
20	The best way to do well in math is to memorize all the formulas.	Hart 2002
24	How often should you include questions based on recall of facts and procedures in your mathematics tests or examinations?	TIMSS 2007
25	How often should you include questions requiring explanations or justifications in your mathematics tests or examinations?	TIMSS 2007
26	To be good at mathematics at school, students should be able to think creatively.	TIMSS 1995
27	To be good at mathematics at school, students should be able to provide reasons to support their solutions.	TIMSS 1995

How Mathematics Should be Taught

Item Number	Beliefs Inventory Survey Item	Source
1	Problem solving should be a separate, distinct part of the mathematics curriculum.	Hart 2002
2	Teachers' modeling of real-world problems is essential to teaching mathematics.	TIMSS 2003
3	Children should be encouraged to justify their solutions, thinking, and conjectures in a single way.	Hart 2002
4	The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in isolation.	Hart 2002
5	Skill in computation should precede word problems.	Hart 2002
6	Mathematics should be taught as a collection of concepts, skills and algorithms.	Hart 2002
7	A demonstration of good reasoning should be regarded even more than students’ ability to find correct answers.	Hart 2002
8	More than one representation (picture, concrete material, symbol set, etc.) should be used in teaching a mathematical topic.	TIMSS 1995
9	If students are having difficulty, an effective approach is to give them more practice by themselves during class.	TIMSS 1995
15	In mathematics something is either right or it is wrong.	Hart 2002
16	Good mathematics teachers show students lots of different ways to look at the same question.	Hart 2002
17	Math problems can be done correctly in only one way.	Hart 2002
21	In teaching mathematics, how often should you usually ask students to explain the reasoning behind an idea?	TIMSS 1995
22	In teaching mathematics, how often should you usually ask students to decide on their own procedures for solving complex problems?	TIMSS 2003
23	In teaching mathematics, how often do you usually ask students to work on problems for which there is no immediately obvious method of solution?	TIMSS 1995

APPENDIX C

PRE-OBSERVATION INTERVIEW

"Hello, thank you for taking the time to complete the Beliefs Inventory Survey. This interview will allow me to learn more about your beliefs about mathematics teaching and learning."

1. How do you define learning?

2. How do you define teaching?

3. What do you believe are the best ways to learn mathematics?

4. What do you believe are the best ways to teach mathematics?

5. What role are students expected to assume in your classroom?
   (Alternate phrasing: In other words, what role do students play in your classroom? What are they expected to do? Do they have specific academic responsibilities?)

6. What is your role as a mathematics teacher?
   (Alternate phrasing: In other words, what is your job as a mathematics teacher involve? What are your responsibilities?)

7. Is there anything else you would like to share about teaching and learning in your classroom?

APPENDIX D

REFLECTION MEETING

"Thank you for allowing me to observe your classroom. I've selected some of the mathematics questions you asked during instruction. We will discuss the questions and talk about how they are related to your beliefs."

Mathematics Question 1: <First researcher-selected mathematics question from the classroom observation>

Researcher: "Why did you ask this question?
Participant Response: ____________________
Researcher: "How does your response relate to your beliefs?"
Participant Response: ____________________

Mathematics Question 2: <Second researcher-selected mathematics question from the classroom observation>

Researcher: "Why did you ask this question?
Participant Response: ____________________
Researcher: "How does your response relate to your beliefs?"
Participant Response: ____________________

Mathematics Question 3: <Third researcher-selected mathematics question from the classroom observation>

Researcher: "Why did you ask this question?
Participant Response: ____________________
Researcher: "How does your response relate to your beliefs?"
Participant Response: ____________________

Mathematics Question 4: <Fourth researcher-selected mathematics question from the classroom observation>

Researcher: "Why did you ask this question?
Participant Response: ____________________
Researcher: "How does your response relate to your beliefs?"
Participant Response: ____________________

APPENDIX E

END-OF-STUDY QUESTIONNAIRE

1. How do you feel your beliefs affect your questioning practices during mathematics instruction?

2. Describe your experience with reflecting on the questions you asked during instruction.

3. Did this experience affect your thinking about the questions you ask during instruction? Explain.

APPENDIX F

BELIEFS INVENTORY SURVEY WITH CLASSIFICATION OF RESPONSES

No.	ITEM	Agree	Somewhat Agree	Somewhat Disagree	Disagree	Comments
1	Problem solving should be a separate, distinct part of the mathematics curriculum.	I	I/P	P/PS	PS	
2	Teachers' modeling of real-world problems is essential to teaching mathematics.	I	I/P	P/PS	PS	
3	Children should be encouraged to justify their solutions, thinking, and conjectures in a single way.	I	I/P	P/PS	PS	
4	The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in isolation.	I	I/P	P	P/ PS	
5	Skill in computation should precede word problems.	I	I/P	P	P/ PS	
6	Mathematics should be taught as a collection of concepts, skills and algorithms.	I/P	P	P/PS	PS	
7	A demonstration of good reasoning should be regarded even more than students' ability to find correct answers.	P/PS	P	I/P	I	
8	More than one representation (picture, concrete material, symbol set, etc.) should be used in teaching a mathematical topic.	P/PS	P	I/P	I	
9	If students are having difficulty, an effective approach is to give them more practice by themselves during class.	I	I/P	P	PS	
10	Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.	I/P	P	P/PS	PS	
11	Mathematics should be learned as sets of algorithms or rules that cover all possibilities.	I	I/P	P/PS	PS	

12	Solving mathematics problems often involves hypothesizing, estimating, testing, and modifying findings.	PS	P/PS	I/P	I	
13	Appropriate calculators should be available to all students at all times	PS	PPS	I/P	I	
14	To be good at mathematics at school, students should understand how mathematics is used in the real world.	PS	PPS	I/P	I	
		True	**More True than False**	**More False than True**	**False**	**Comments**
15	In mathematics something should be either right or wrong.	I	I/P	P/PS	PS	
16	Good mathematics teachers should show students lots of different ways to look at the same question.	PS	P/PS	I/P	I	
17	Math problems can be done correctly in only one way.	I	I/P	P/PS	PS	
18	In mathematics you can be creative and discover things by yourself.	PS	P	I	I	
19	To solve most math problems you have to be taught the correct procedure.	I/P	P	P/PS	PS	
20	The best way to do well in math is to memorize all the formulas.	I/P	P	P/PS	PS	
		Never	**Sometimes**	**Often**	**Always**	**Comments**
	In teaching mathematics, how often should students be asked to:					
21	Explain the reasoning behind an idea	I	P	P/PS	PS	
22	Decide on their own procedures for solving complex problems	I	P	P/PS	PS	
23	Work on problems for which there is no immediately obvious method of solution	I	P	P/PS	PS	

	How often should the following types of questions be included in your mathematics tests or examinations?					
24	Questions based on recall of facts and procedures	PS	PPS	I/P	I	
25	Questions requiring explanations or justifications	I	I/P	P	P/PS	
		Not Important	**Somewhat Important**	**Very Important**		**Comments**
	To be good at mathematics at school, how important do you think it is for students to:					
26	be able to think creatively	I	P/PS	PS		
27	be able to provide reasons to support their solutions	I	P	PS		

APPENDIX G

MATHEMATICS QUESTION TYPES, DESCRIPTIONS, AND LEVELS

Mathematics Question Type	Description	Classification	Example
Analyze	Analyze given mathematical information	High-Level	What happened here? Where is the mistake?
Clarify	Eliminate ambiguity, confusion or misunderstanding	High-Level	Can you explain that to him using your own words?
Classify	Use knowledge of the characteristics or properties of numbers, shapes, or groups	High-Level	Is ½ an Integer?
Compare	Compare ideas, concepts, examples, formulas to specific examples	High-Level	Is -3-8 the same as -3+8?
Contrast	Contrast ideas, concepts, examples, formulas to specific examples	High-Level	What's difference between these equations and the other equations?
Definition - Open	Use the meaning of a term to answer a question	High-Level	Could I find the surface area of a pyramid?
Explain	Share student's thinking about or describe student's understanding of a concept or process	High-Level	What do you think is causing the changes in water level to be different?
Generalize	Make a general claim about a mathematical concept or phenomenon	High-Level	When I have [multiply] one positive and one negative, the answer will always be what?
Justify	Tell why a particular step or procedure was used	High-Level	Why add?
Make a Connection	Relate the current content of study to content previously studied	High-Level	We've heard that [coordinate] before. Does it make us think of anything?
Make Sense	Determine what is being asked or what unknown information must be found	High-Level	What am I solving for here?
Multiple Representations	Perform other methods to find a solution or use other	High-Level	The first problem I have here is 7x-3.

	representations of the same quantity		What's another way of thinking about 7x-3?
Predict	Make a prediction based on given information	High-Level	If I put this equation in my calculator, what would the graph look like?
Reason	Select a problem-solving strategy, draw conclusions, evaluate a situation, reflect	High-Level	If they're going in opposite directions, who's running a positive distance and who's running a negative distance?
Recognize a Pattern	Look for a pattern within the data	High-Level	What's the missing part of the pattern?
Student Questions	Pose a question about content	High-Level	What are your questions?
Understand the Reasoning of Others	Understand the reasoning of another student	High-Level	Does his method seem reasonable?
Agree or Disagree with Reasoning	Express agreement or disagreement with another student's reasoning	Low-Level	Thumbs up or down if you agree with her reason for choosing 7.
Agree or Disagree with a Classification	Express agreement or disagreement with a classification	Low-Level	Anybody disagree with that statement that all negative numbers are integers?
Agree or Disagree with a Definition	Express agreement or disagreement with a definition	Low-level	Do we agree that that [definition] would be volume?
Agree or Disagree with a Fact	Express agreement or disagreement with a fact	Low-level	Thumbs up or down if you think 6 −5 will be -1.
Agree or Disagree with a Procedure	Express agreement or disagreement with a procedure	Low-level	Did he do the right thing by adding outcomes?
Agree or Disagree with a Prediction	Express agreement or disagreement with a prediction	Low-level	Does anybody agree with his prediction of 12 marbles?
Agree or Disagree with Vocabulary	Express agreement or disagreement with a student's use of vocabulary	Low-level	Thumbs up if you agree, thumbs down if you disagree that this area is called the 3rd quadrant.
Estimate	Estimate the value of a number	Low-level	Estimate the value of the function to the nearest whole number.

Fact	Based on previously acquired knowledge, define, name, state, or provide a numerical value	Low-level	The |-6| is what?
Procedure	Provide the next step in a procedure, give the name of a procedure/process, tell how to arrive at a solution, determine if a step is valid	Low-level	I have to rewrite [-7−(-4)] as addition problem and I have to start off by doing what?
Recall	Refer to or recite information previously studied	Low-level	What were the other two methods we used?
Vocabulary	Provide mathematical vocabulary for a concept or fact	Low-level	What's another math word for when I add two numbers that are the same?

REFERENCES

Aizikovitsh-Udi, E., & Star, J. (2011). The skill of asking good questions in mathematics teaching. *Procedia-Social and Behavioral Sciences, 15*, 1354-1358.

Ajzen, I. (1991). The theory of planned behavior. *Organizational Behavior and Human Decision Processes, 50*(2), 179-211.

Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education*, *7*(2), 91-119.

Barlow, A. T., & Cates, J. M. (2006). The impact of problem posing on elementary teachers' beliefs about mathematics and mathematics teaching. *School Science and Mathematics, 106*(2), 64-73.

Beck, J., Czerniak, C. M., & Lumpe, A. T. (2000). An exploratory study of teachers' beliefs regarding the implementation of constructivism in their classrooms. *Journal of Science Teacher Education, 11*(4), 323-343.

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics, 79*(1), 127-147.

Boaler, J., & Brodie, K. (2004). The importance, nature, and impact of teacher questions. *Proceedings of the Twenty-Sixth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, 2*, 774-790.

Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2013). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction, 25*, 24-34.

Boyatzis, R. E. (1998). *Transforming qualitative information: Thematic analysis and code* *development*. Thousand Oaks, CA: Sage.

Brophy, J., & Good, T. L. (1984). Teacher behavior and student achievement. Occasional Paper No. 73.

Brown, L., & Coles, A. (2012). Developing "deliberate analysis" for learning mathematics and for mathematics teacher education: How the enactive approach to cognition frames reflection. *Educational Studies in Mathematics*, *80*(1-2), 217-231.

Brown, G. A., & Edmondson, R. (1989). Asking questions. *Classroom Teaching Skills,* 97-120.

Calderhead, J. (1981). Stimulated recall: A method for research on teaching. *British Journal of Educational Psychology*, *51*(2), 211-217.

Calderhead, J., & Robson, M. (1991). Images of teaching: Student teachers' early conceptions of classroom practice. *Teaching and Teacher Education*, *7*(1), 1-8.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2000). *Cognitively guided instruction: A research-based teacher professional development program for elementary school mathematics.* (Report No. NCISLA-RR-00-3). Retrieved from https://files.eric.ed.gov/fulltext/ED470472.pdf

Cazden, C. B., & Beck, S. W. (2003). Classroom discourse. *Handbook of Discourse Processes*, 165-197.

Chamoso, J. M., Cáceres, M. J., & Azcárate, P. (2012). Reflection on the teaching-learning process in the initial training of teachers. Characterization of the issues on which pre-service mathematics teachers reflect. *Teaching and Teacher Education*, *28*(2), 154-164.

Cheung, W. S., & Wong, J. L. N. (2017). Understanding reflection for teacher change in Hong Kong. *International Journal of Educational Management, 31*(7), 1135-1146.

Cotton, K. (2001). Classroom questioning. *School Improvement Research Series, 3.*

Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in preservice teachers' practices. *Educational Studies in Mathematics*, *52*(3), 243-270.

Crespo, S., & Sinclair, N. (2008). What makes a problem mathematically interesting? Inviting prospective teachers to pose better problems. *Journal of Mathematics Teacher Education*, *11*(5), 395-415.

Cross, D. I. (2009). Alignment, cohesion, and change: Examining mathematics teachers' belief structures and their influence on instructional practices. *Journal of Mathematics Teacher Education*, *12*(5), 325-346.

Davis, E. A. (2006). Characterizing productive reflection among preservice elementary teachers: Seeing what matters. *Teaching and Teacher Education*, *22*(3), 281-301.

Delice, A., Aydın, E., & Çevik, K. S. (2013). Mathematics teachers' use of questions: Is there a change of practice after the curriculum change? *Eurasia Journal of Mathematics, Science & Technology Education*, *9*(4), 417-427.

Elder, L., & Paul, R. (1998). The role of Socratic questioning in thinking, teaching, and learning. *The Clearing House*, *71*(5), 297-301.

Elo, S., & Kyngäs, H. (2008). The qualitative content analysis process. *Journal of Advanced Nursing*, *62*(1), 107-115.

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching*, *15*(1), 13-33.

Fang, Z. (1996). A review of research on teacher beliefs and practices. *Educational Research*, *38*(1), 47-65.

Feiman-Nemser, S. (2001). From preparation to practice: designing a continuum to strengthen and sustain teaching. *Teachers College Record*, *103*(6), 1013-1055.

Francis, D. I. C. (2015). Dispelling the notion of inconsistencies in teachers' mathematics beliefs and practices: A 3-year case study. *Journal of Mathematics Teacher Education*, *18*(2), 173-201.

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education*, *60*(4), 380-392.

Fredericks, A. D. (2010). *The teacher's handbook: Strategies for success*. Lanham, MD: R&L Education.

Friel, S. N., Hart, L. C., Schultz, K., Najee-ulla, D., Nash, L. (1992). Implementing the professional standards for teaching mathematics: The role of reflection in teaching. *Arithmetic Teacher*, *40*(1), 40-42.

Gall, M. D. (1970). The use of questions in teaching. *Review of Educational Research*, 707-721.

García, M., Sánchez, V., & Escudero, I. (2007). Learning through reflection in mathematics teacher education. *Educational Studies in Mathematics*, *64*(1), 1-17.

Giorgi, A. J., Roberts, T. G., Estepp, C. M., Conner, N. W., & Stripling, C. T. (2013). An investigation of teacher beliefs and actions. *North American Colleges and Teachers of Agriculture Journal*, *57*(3), 2-9.

Hart, L. C. (2002). Preservice teachers' beliefs and practice after participating in an integrated content/methods course. *School Science and Mathematics*, *102*(1), 4-14.

Harvey, O. J. (1986). Belief systems and attitudes toward the death penalty and other punishments. *Journal of Personality*, *54*(4), 659-675.

Hus, V., & Abersek, M.K. (2011). Questioning as a mediation tool for cognitive development in early science teaching. *Journal of Baltic Science Education*, 10*(1), 6-16.*

Jansen, A., & Spitzer, S. M. (2009). Prospective middle school mathematics teachers' reflective thinking skills: Descriptions of their students' thinking and interpretations of their teaching. *Journal of Mathematics Teacher Education*, *12*(2), 133-151.

Kagan, D. M. (1992). Implication of research on teacher belief. *Educational Psychologist*, *27*(1), 65-90.

Kawanaka, T., & Stigler, J. W. (1999). Teachers' use of questions in eighth-grade mathematics classrooms in Germany, Japan, and the United States. *Mathematical Thinking and Learning*, *1*(4), 255-278.

Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education*, *7*(3), 203–35.

Klein, D. (2003). A brief history of American K-12 mathematics education in the 20th century. *Mathematical Cognition*, 175-225.

Kosko, K. W., Rougee, A., & Herbst, P. (2014). What actions do teachers envision when asked to facilitate mathematical argumentation in the classroom? *Mathematics Education Research Journal*, *26*(3), 459-476.

Kramarski, B. (2010). Developing a pedagogical problem solving view for mathematics teachers with two reflection programs. *International Electronic Journal Environmental Education*, *2*(1).

Krathwohl, D. R. (2002). A revision of Bloom's taxonomy: An overview. *Theory Into Practice*, *41*(4), 212-218.

Krussel, L., Edwards, B., & Springer, G. T. (2004). The teacher's discourse moves: A framework for analyzing discourse in mathematics classrooms. *School Science and Mathematics*, *104*(7), 307-312.

Lortie, D. C. (1975). *Schoolteacher: A sociological study*. Chicago, IL: University of Chicago Press.

Lucas, P. (1991). Reflection, new practices, and the need for flexibility in supervising student teachers. *Journal of Further and Higher Education*, *15*(2), 84-93.

Marzano, R. (2013). Marzano teacher evaluation model. Learning Sciences International.

Marzano, R. J. (2007). *The art and science of teaching: A comprehensive framework for effective instruction*. Alexandria, VA: ASCD.

Mason, J. (2000). Asking mathematical questions mathematically. *International Journal of Mathematical Education in Science and Technology*, *31*(1), 97-111.

Meade, P. & McMeniman, M. (1992). Stimulated recall—An effective methodology for examining successful teaching in science. *The Australian Educational Researcher*, *19*(3), 1-18.

Meyer, M. (1980). Dialectic and questioning: Socrates and Plato. *American Philosophical Quarterly*, 281-289.

Mustafa, G. (2005). Reflection in teaching. *Learning and Teaching in Higher Education: Gulf Perspectives*, *2*(2).

National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics.* Reston, VA: National Council of Teachers of Mathematics. Retrieved from https://www.nctm.org/flipbooks/standards/professionalteaching/html5/index.html

National Council of Supervisors of Mathematics. (1989). Essential mathematics for the twenty-first century: The position of the National Council of Supervisors of Mathematics. *Mathematics Teacher*, *82*(6), 470-474.

National Governors Association Center for Best Practices & Council of Chief State School

Officers (2010). *Common core state standards for mathematics*. Washington, DC: Authors. Retrieved from: http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies*, *19*(4), 317-328.

Ni, Y. J., Li, Q., Li, X. Q., & Zou, J. (2011). China's new millennium curriculum reform in mathematics and its impact on classroom teaching and learning. *The Impact and Transformation of Education Policy in China*, 99-124.

Nisbet, S., & Warren, E. (2000). Primary school teachers' beliefs relating to mathematics, teaching and assessing mathematics and factors that influence these beliefs. *Mathematics Teacher Education and Development*, *2*(34-47).

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, *62*(3), 307-332.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F.K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning.* (pp. 257-315). Charlotte, NC: Information Age Publishing, 257-315.

Piccolo, D. L., Harbaugh, A. P., Carter, T. A., Capraro, M. M., & Capraro, R. M. (2008). Quality of instruction: Examining discourse in middle school mathematics instruction. *Journal of Advanced Academics*, *19*(3), 376-410.

Polya, G. (1957). *How to solve it: A new aspect of mathematical model* (2nd ed.). Garden City, NY: Princeton University Press.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education*, 550-576.

Richards, J. C., & Lockhart, C. (1994). *Reflective teaching in second language classrooms*. Cambridge, England: Cambridge University Press.

Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In J. Sikula, T. Buttery, E. Guyton & Association of Teacher Educators (Eds.), *Handbook of research on teacher education: A project of the association of teacher educators* (2nd ed.). (pp. 102-119). New York, NY: Macmillan Library Reference.

Rickard, A. (2005). Evolution of a teacher's problem solving instruction: A case study of aligning teaching practice with reform in middle school mathematics. *Research in Middle Level Education Online*, *29*(1), 1-15. doi: 10.1080/19404476.2005.11462024

Rokeach, M. (1968). *Beliefs, attitudes and values: A theory of organization and change*. San Francisco, CA: Jossey-Bass

Roy, P. (1998). Staff development that makes a difference. *Professional Development for Cooperative Learning: Issues and Approaches*, 79-99.

Rud, A. G. (1997). Use & abuse of Socrates in teaching. *Education Policy Analysis Archives*, *5*, 20.

Sahin, A., & Kulm, G. (2008). Sixth grade mathematics teachers' intentions and use of probing, guiding, and factual questions. *Journal of Mathematics Teacher Education*, *11*(3), 221-241.

Savin-Baden, M., & Major, C. H. (2013). *Qualitative research: The essential guide to theory and practice.* London, England: Routledge.

Schön, D. A. (1983). *The reflective practitioner: How professionals think in action*. New York, NY: Basic books.

Senger, E. S. (1998). Beyond classroom description: Methods of understanding reflection and beliefs in mathematics teaching. *Educational Research Quarterly*, *21*(3), 21.

Skemp, R. R. (1978). Relational understanding and instrumental understanding. *The Arithmetic Teacher*, *26*(3), 9-15.

Sloan, F. A., & Pate, R. T. (1966). Teacher-pupil interaction in two approaches to mathematics. *The Elementary School Journal*, 161-167.

Springs, J. A. (1999). *Ethnic-cultural influences on question-asking behaviors of African-American and European-American teachers*.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School*, *3*(4), 268-275.

Stepich, D., Chyung, S. Y., & Smith-Hobbs, A. (2009). Research on Cultural Factors in Global E-Learning. In P. Rogers, G. Berg, J. Boettcher, C. Howard, L. Justice, & K. Schenk (Eds.), *Encyclopedia of Distance Learning, Second Edition* (pp. 1758-1765). Hershey, PA: IGI Global. doi:10.4018/978-1-60566-198-8.ch259

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education*, *17*(2), 213-226.

Sullivan, P., & Clarke, D. (1992). Problem solving with conventional mathematics content: Responses of pupils to open mathematical tasks. *Mathematics Education Research Journal*, *4*(1), 42-60.

Sztajn, P. (2003). Adapting reform ideas in different mathematics classrooms: Beliefs beyond mathematics. *Journal of Mathematics Teacher Education*, 6(1), 53-75.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, *15*(2), 105-127.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws & National Council of Teachers of Mathematics (Eds.), *Handbook of research on mathematics teaching and learning*. (pp. 127-146). New York, NY: Macmillan.

Ticha, M., & Hospesova, A. (2006). Qualified pedagogical reflection as a way to improve mathematics education. *Journal of Mathematics Teacher Education*, *9*(2), 129-156.

TIMSS 1995 Assessment. International Association for the Evaluation of Educational Achievement (IEA). TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.

TIMSS 2003 Assessment. International Association for the Evaluation of Educational Achievement (IEA). TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.

TIMSS 2007 Assessment. International Association for the Evaluation of Educational Achievement (IEA). TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.

Van Manen, M. (1977). Linking ways of knowing with ways of being practical. *Curriculum Inquiry*, *6*(3), 205-228.

Varela, F. J. (1999). *Ethical know-how: Action, wisdom, and cognition*. Stanford, CA: Stanford University Press.

Vogler, K. E. (2008). Asking good questions. *Educational Leadership, 65*(9).

Wegerif, R. (2011). Towards a dialogic theory of how children learn to think. *Thinking Skills and Creativity*, *6*(3), 179-190.

Weiland, I. S., Hudson, R. A., & Amador, J. M. (2014). Preservice formative assessment interviews: The development of competent questioning. *International Journal of Science and Mathematics Education*, *12*(2), 329-352.

Wilcox-Herzog, A., & Ward, S. L. (2004). Measuring teachers' perceived interactions with children: A tool for assessing beliefs and intentions. *Early Childhood Research & Practice*, *6*(2).

Wiliam, D. (2011). *Embedded formative assessment*. Bloomington, IN: Solution Tree.

Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. *Review of Research in Education*, *24*, 173-209.

Zakaria, E., & Maat, S. M. (2012). Mathematics teachers' beliefs and teaching practices. *Journal of Mathematics & Statistics*, *8*(2), 191-194.

Zheng, H. (2013). The dynamic interactive relationship between Chinese secondary school EFL teachers' beliefs and practice. *The Language Learning Journal*, *41*(2), 192-204.

ABSTRACT

TEACHERS' REFLECTION ON THEIR BELIEFS AND QUESTION-ASKING PRACTICES DURING MATHEMATICS INSTRUCTION

by

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Teachers' daily instructional practices are a critical component in creating a rich and meaningful educational experience for students. Thus, factors that inform instructional practices are of particular importance and interest to education researchers and other stakeholders. Beliefs about teaching and learning are a known factor influencing teachers' instructional practices (Ernest, 1989). This study focused on a specific instructional practice, question-asking, which has a profound impact on students' experience with mathematics (Weiland, Hudson, and Amador (2014). Understanding the relationship between teachers' beliefs and practice helps to make sense of teachers' decision-making processes, particularly as they choose questions to ask students during mathematics lessons.

This study solicited teachers' beliefs about mathematics teaching and learning using qualitative tools (Beliefs Inventory Survey and Pre-Observation Interview) and classroom observations. The researcher engaged participants in a reflective process which deliberately focused on the mathematics questions they asked during instruction. Teachers were encouraged to reflect on their question-asking practices, in relation to their beliefs, during a series of reflection meetings occurring between classroom observations.

The findings of the present study indicate that whereas teachers ask more low-level questions than high-level questions, they ask a greater variety of high-level questions during mathematics instruction. The most frequently provided reasons for asking questions included review of content, addressing common misconceptions; and reinforcing procedures. According to the present study, teachers' beliefs uncovered during the reflection meetings were more aligned to the reasons for asking questions than the level (low or high) of questions asked. Another finding of the present study pertained to the potential effects of reflection on practice. When participants of this study felt their beliefs aligned with their practice, they were less likely to experience changes in thinking or question-asking practices.

AUTOBIOGRAPHICAL STATEMENT

Kaili Hardamon is a native of Detroit. She received a Bachelor of Science in Mathematics from the University of Michigan in 2000 and a Master of Arts in Elementary Education from the University of Phoenix in 2004. Kaili has taught mathematics in Detroit for 12 years to students in elementary, middle, and high school. She has also taught mathematics at Henry Ford Community College and Wayne State University. Currently, Kaili is a Mathematics Training and Support Coordinator for K-8 teachers in the Detroit Public Schools Community District.