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Forward and Backward Continuation Ratio Models for Ordinal Response Variables

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There are different types of continuation ratio (CR) models for ordinal response variables. The different model equations, corresponding parameterizations, and nonequivalent results are confusing. The purpose of this study is to introduce different types of forward and backward CR models, demonstrate how to implement these models using Stata, and compare the results using data from the Educational Longitudinal Study of 2002 (ELS:2002).

Keywords: Ordinal logistic regression, continuation ratio (CR) model, forward and backward CR models, Stata

Introduction

Ordinal response variables are often used in many research situations. There exist different types of statistical models to analyze ordinal data, such as the proportional odds (PO) and continuation ratio (CR) models. However, these two models have different focuses. The PO model (Agresti, 2007, 2010, 2013; Hilbe, 2009; Liu, 2009, 2016; Long, 1997; Long & Freese, 2014; McCullagh, 1980; McCullagh & Nelder, 1989; O'Connell, 2000, 2006; Powers & Xie, 2000) estimates the cumulative odds of being at or below a particular level of an ordinal response variable, or the inversed odds, the odds of being above that particular level. The effect of each predictor is assumed to be invariant across the ordinal responses. This is defined as the proportional odds assumption, or the parallel lines assumption of the PO model.

Unlike the PO model with the focus on the cumulative odds of grouped categories, the CR model estimates the conditional odds of being in a particular category, given that an individual has reached that category or above (Agresti,

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2010; Allison, 2012; Fienberg, 1980; Fullerton, 2009; Fullerton & Xu, 2016; Greenland, 1994; Liu et al., 2011; Liu, 2016; Long & Freese, 2006, 2014; O'Connell, 2006). Therefore, the CR model is also referred to as the stage approach (Fullerton, 2009), because it focuses on transitions of successive stages or proficiency levels and assumes that lower stages or proficiency levels are reached first. It is also referred to as the sequential model or sequential logit model (Buis, 2013; Liao, 1994; Long & Freese, 2014; Tutz, 1991, 2012), although they may be parameterized differently.

The CR model estimates the odds of being in a certain category versus being above that category. In terms of probability, unlike the cumulative probabilities in the PO model, the CR model estimates the conditional probability of being in a category given that an individual has been in or above that category (i.e., $P(Y = j \mid Y \geq j)$). It also estimates the conditional probability of being above a category given that a person has attained that particular category (i.e., $P(Y > j \mid Y \geq j)$ since these two conditional probabilities are complementary.

The most commonly used CR model is also called the forward CR model (Bender & Benner, 2000; O'Connell, 2006), because it compares a particular category to higher categories. For example, if an ordinal response variable has four categories from 1 to 4, then the forward CR model compares category 1 with categories 2, 3, and 4; category 2 with categories 3 and 4; and category 3 with category 4. The other version of the CR model estimates the odds of being in a particular category versus being below that category. This type of model is called the backward CR model (Bender & Benner, 2000) since the order of the comparisons of the ordinal categories is reversed. The backward CR model compares the odds of being in a particular category to the odds of being in lower categories. For example, the comparisons include category 2 versus 1; category 3 versus categories 1 and 2; and category 4 versus categories 1, 2, and 3.

Because both the odds and the inversed odds can be estimated in the CR model, each of the forward and backward CR models can also have two different versions when they are parameterized differently. The forward CR model estimates the odds of being in a particular category versus being above that category, or the inversed odds, the odds of being above a particular category versus being in that category. Conversely, the backward CR model estimates either the odds of being in a particular category versus being below that category or the odds of being below a category versus being in that category. The models for the inversed odds are referred to as the sub-models of the forward and backward CR models, respectively.

Although different types of CR models exist, they are all called CR models, with different model equations, corresponding parameterizations, and nonequivalent results. It is important to make a clear distinction among these CR models to be aware of their differences and interpret the results correctly.

Purpose of the Study

Therefore, the purpose of this study is to introduce different types of forward and backward CR models, demonstrate how to implement these models using Stata, and compare the results of these models. This will explicate the different parameterizations of the CR models, their applications, and the interpretation of the analysis results. The empirical data from the Educational Longitudinal Study of 2002 (ELS:2002) were used to demonstrate the procedures for the ordinal regression analyses.

Theoretical Framework

The Forward CR Model

The forward CR model estimates the odds of being in a particular category j relative to being above that category. The CR model can be expressed in the following form:

$$\ln\left(\frac{\mathrm{P}\left(Y=j\mid x_1,x_2,\ldots,x_p\right)}{\mathrm{P}\left(Y>j\mid x_1,x_2,\ldots,x_p\right)}\right)=\alpha_j+\left(-\beta_1X_1-\beta_2X_2-\ldots-\beta_pX_p\right), \tag{1}$$

where $\mathrm{P}(Y = j \mid x_1, x_2,\ldots, x_p)$ is the conditional probability of being in category j conditional on being in or above that category given a set of predictors, that is, $\mathrm{P}(Y = j \mid Y \geq j)$; $j = 1, 2,\ldots, J - 1$; α_j are the cut points; and $\beta_1, \beta_2,\ldots, \beta_p$ are the logit coefficients. This form is commonly seen in the literature for the CR model (Ananth & Kleinbaum, 1997; Armstrong & Sloan, 1989; Fienberg, 1980; Fullerton & Xu, 2016; Liu et al., 2011; Liu, 2016; Long & Freese, 2006) although not named the forward CR model. As with the PO models, the CR model also assumes that the logit coefficients for each predictor are the same across ordinal categories, so this model is also called the constrained CR model (Cole & Ananth, 2001). The CR model can also estimate the conditional probability of being above a category given that the individual has achieved that particular category since $\mathrm{P}(Y > j \mid Y \geq j)$ is the complementary form of $\mathrm{P}(Y = j \mid Y \geq j)$.

When estimating the conditional probability of being above a category given that an individual has attained that particular category, that is, $\mathrm{P}(Y > j \mid Y \geq j)$, the forward CR model can be expressed in this form by simply transforming equation (1):

$$\ln\left(\frac{\mathrm{P}\left(Y > j \mid x_1, x_2, \ldots, x_p\right)}{\mathrm{P}\left(Y = j \mid x_1, x_2, \ldots, x_p\right)}\right) = -\alpha_j + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p \tag{2}$$

When parameterized differently, this equation is commonly seen as a modified form in the literature (Allison, 2012; O'Connell, 2006, Long & Freese, 2014) as follows, where the negative sign before the cut points or intercepts is omitted and the sign before the coefficients remain unchanged. Please note that different software packages may use either of these two forms.

$$\ln\left(\frac{\mathrm{P}\left(Y > j \mid x_1, x_2, \ldots, x_p\right)}{\mathrm{P}\left(Y = j \mid x_1, x_2, \ldots, x_p\right)}\right) = \alpha_j + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p \tag{3}$$

The Backward CR Model

Unlike the forward CR model, the backward CR model estimates the odds of being in a certain category j relative to being below that category, which are different from the odds in the forward CR model. The backward CR model is expressed as follows:

$$\ln\left(\frac{\mathrm{P}\left(Y = j \mid x_1, x_2, \ldots, x_p\right)}{\mathrm{P}\left(Y < j \mid x_1, x_2, \ldots, x_p\right)}\right) = \alpha_j + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p\,, \tag{4}$$

where α_j are the intercepts or cut points and $\beta_1, \beta_2, \ldots, \beta_p$ are the logit coefficients. The CR model in Fagerland (2014) and Hosmer et al. (2013) followed this form although it was not called the backward CR model. Since the odds of being in a certain category relative to being below that category (i.e., P ($Y = j$) / P($Y < j$)) are the inversed odds of being below that category versus being in that category (i.e., P(Y < j) / P($Y = j$)), the equation (4) can be easily transformed to estimate the inversed odds as follows:

Table 1. Forward and backward CR models and the corresponding odds

Models	Sub-models	Odds

Forward CR models	Model A	$\frac{P(Y = j)}{P(Y > j)}$
	Model B (inversed odds)	$\frac{P(Y > j)}{P(Y = j)}$
Backward CR models	Model A	$\frac{P(Y = j)}{P(Y < j)}$
	Model B (inversed odds)	$\frac{P(Y < j)}{P(Y = j)}$

$$\ln\left(\frac{\mathrm{P}\left(Y < j \mid x_1, x_2, \ldots, x_p\right)}{\mathrm{P}\left(Y = j \mid x_1, x_2, \ldots, x_p\right)}\right) = -\alpha_j + \left(-\beta_1 X_1 - \beta_2 X_2 - \ldots - \beta_p X_p\right) \qquad (5)$$

The left side of the question expresses the logit or log odds of being below a category relative to being in that category. The signs before both the intercepts and logit coefficients on the right side of equation (5) are reversed from those in equation (4). Table 1 presents a comparison of the odds in the forward and backward CR models and their sub-models.

Methodology

Sample

The data were from the Educational Longitudinal Study of 2002 (ELS:2002). The ELS:2002 study, conducted by the National Center for Educational Statistics (NCES), investigated the changes over time of multiple variables of the high school students, from their sophomores to postsecondary school education and to their future careers. In the 2002 base-year of the study, nearly 16,000 sophomores from 752 high schools across the nation participated in the study by taking mathematics and reading tests and responding to surveys. The specific sample size for the current demonstration is 15,976. The ordinal outcome variable in this study was students' mathematics proficiency and the predictors were purposes of computer use (BYS45A, BYS45B, and BYS45C) and hours per day for computer use (BYS46A and BYS46B) in the dataset.

The outcome variable of interest, students' mathematics proficiency levels in high schools, was an ordinal categorical variable with five levels (1 = students can do simple arithmetical operations on whole numbers and the highest level

5 = students can solve complex multiple-step word problems and/or understand advanced mathematical material) (Ingels et al., 2004, 2005). Students needed to pass through the lower levels of proficiency before achieving the highest fifth level. In addition, those students who failed to reach level 1 were assigned to level 0. Table 2 provides the frequency of six mathematics proficiency levels (i.e., levels 0-5).

Data Analysis

To compare the use of the different models to the empirical data, we demonstrate four types of the CR models. First, the forward CR model was fitted with the Stata user-written `ocratio` command (Wolfe, 1998). The `eform` option was used to estimate the odds ratios and corresponding standard errors and the confidence intervals. Second, the backward CR model was fitted using the Stata user-written `ccrlogit` command (Fagerland, 2014). The `or` option was used to estimate the odds ratios in the backward CR model. Both `ocratio` and `ccrlogit` need to be installed first since they are user-written programs for different CR models. Third, the inversed odds ratios of the forward and backward CR models were computed. The results of the fitted models were interpreted and compared. Finally, the PO models were fitted using the Stata `ologit` command and the results were compared with those of the CR models.

Table 2. Proficiency categories, descriptions, and frequencies for the ELS:2002 sample (N = 15,976)

Proficiency category	Description	Frequency
0	Did not reach level 1	842 (5.27%)
1	Do simple arithmetical operations on whole numbers	3882 (24.30%)
2	Do simple operations with decimals, fractions, powers, and root	3422 (21.42%)
3	Do simple problem solving	4521 (28.30%)
4	Understand intermediate-level mathematical concepts and/or find multi-step solutions to word problems	3196 (20.01%)
5	Solve complex multiple-step word problems and/or understand advanced mathematical material	113 (0.71%)

Results

Forward CR Models

To demonstrate the use of forward CR models, five predictors (fun, schwork, learn, hoursch, and houroth) were used in the models to estimate the odds of being in a particular category of students' mathematics proficiency levels relative to being above that category. Table 3 provides the result of the two forward CR models with the five predictor variables using the Stata `ocratio` command (Wolfe, 1998). The log likelihood ratio chi-square test, LR $\chi^2_{(5)} = 1{,}702.22$, $p < .001$, which indicated that the model provided a better fit than the null model in predicting mathematics proficiency.

The coefficients of the four predictor variables on mathematics proficiency (Model B) were significant. The estimated logit coefficient for using computers for fun (fun), $\beta = .304$, $z = 23.73$, $p < .001$; the logit coefficient for using computers for school work (schwork), $\beta = .297$, $z = 21.32$, $p < .001$; the coefficient for hours per day on using computers for school work (hoursch) $\beta = –.111$, $z = –9.59$, $p < .001$; and finally, the coefficient for hours per day of using computers on the others (houroth), $\beta = –.127$, $z = –15.13$, $p < .001$. However, the coefficient for using computers to learn on their own (learn) was not significant, $\beta = –.021$, $z = –1.89$, $p > .05$.

Table 3. Results of the forward CR models with five predictor variables using Stata ocratio: Forward CR Model A (*Y* = cat. *j* vs. *Y* > cat. *j*) and Model B (*Y* > cat. *j* vs. *Y* = cat. *j*)

	Model A		Model B	
Variables	**–b (se(–b))**	**OR**	**b (se(b))**	**OR**
α_1	–1.415		1.415	
α_2	0.586		–0.586	
α_3	0.920		–0.920	
α_4	2.178		–2.178	
α_5	5.339		–5.339	
fun	–0.304 (0.013)**	0.738	0. 304 (0.013)**	1.355
schwork	–0.297 (0.014)**	0.723	0. 297 (0.014)**	1.346
learn	0.021 (0.011)	1.021	–0.021 (0.011)	0.979
hoursch	0.111 (0.012)**	1.117	–0.111 (0.012)**	0.895
houroth	0.127 (0.008)**	1.136	–0.127 (0.008)**	0.880
LR R^2	0.04		0.04	
Model fit[a]	$\chi^2_{(5)} = 1{,}702.22$**		$\chi^2_{(5)} = 1{,}702.22$**	

Note: [a] Likelihood ratio test
** Significant at $p < .01$

Interpreting the Odds Ratios of Stopping in a Particular Category (Forward CR Model A)

The forward CR Model A estimates the odds of being in a particular category relative to being above that category, which are the exponentiated negative logit coefficient exp($-\beta$) for a one-unit change in a predictor, so the odds ratios in Model A are the inversed odds ratios in Model B.

Two predictors were associated with the odds of being in a proficiency level rather than being above that level. The odds of stopping in a proficiency level rather than being in higher proficiency levels decreased by a factor of .738 with a one-unit increase in using computers for fun and decreased by a factor of .723 with a one-unit increase in using computers for schoolwork. In other words, students spent more time in using computers for fun and schoolwork had larger odds of being in higher proficiency levels. However, two other predictors were positively associated with the logits of stopping in a proficiency level. Students who spent excessive hours a day on computer for schoolwork and others were associated with the conditional odds of stopping in a mathematics proficiency level (ORs = 1.117 and 1.136 for hoursch and houroth, respectively). Finally, students who spent more time on using computers to learn on their own (learn) did not influence the odds of stopping in a particular mathematics proficiency level (OR = 1.021) since they were not significant.

The five cut points in Model A were –1.415, .586, .920, 2.178, and 5.339. They were the estimated intercepts in the underlying binary models due to different comparisons between categories. The forward CR model (Model A) compares category 0 with categories 1 and above, category 1 with categories 2 and above, category 2 with categories 3 and above, category 3 with categories 4 and 5, and category 4 with category 5. The cut points in Model B were the same in magnitude as those in Model A but are opposite in sign since Model B estimated the odds of being beyond a category versus being in that category. Thus, the category comparisons in these two models had opposite directions.

Interpreting the Odds Ratios of Being Above a Particular Category Versus Being In that Category (Forward CR Model B)

Two predictors were positively associated with the logits or log odds of being beyond a proficiency level. In terms of odds ratios (OR), the odds of being beyond a proficiency level increased by a factor of 1.355 with a one-unit increase in using computers for fun and increased by a factor of 1.346 with a one-unit increase in using computers for schoolwork. However, two other predictors were negatively associated with the logits of being above a proficiency level. Students who spent excessive hours a day on computer for schoolwork and others were associated with the conditional odds of stopping in a mathematics proficiency level (ORs = .895 and .880 for hoursch and houroth, respectively). In addition, students who spent more time on using computers to learn on their own did not influence the conditional odds of stopping in a particular mathematics proficiency level versus being above that proficiency level (OR = .979) since they were not significant.

Backward CR Models

The backward CR model also has two forms. One estimates the odds of being in a certain category *j* relative to being below that category and the other estimates the inversed odds of comparing lower categories and a particular category. The results of the backward CR models using `ccrlogit` are presented in Table 4.

The log likelihood ratio chi-square test of the backward CR model, LR $\chi^2_{(5)} = 1{,}580.62$, $p < .001$, which indicated that the model provided a better fit than the null model in predicting mathematics proficiency.

The coefficients of all the five predictor variables in the backward CR model (Model A in Table 4) were significant. The estimated logit coefficient for using computers for fun (fun), $\beta = .299$, $z = 24.03$, $p < .001$; the logit coefficient for using computers for schoolwork (schwork), $\beta = .254$, $z = 19.30$, $p < .001$; the coefficient

Table 4. Results of the backward CR models with five predictor variables using Stata ccrlogit: Model A (Y = cat. j vs. Y < cat. j) and Model B (Y < cat. j vs. Y = cat. j)

	Model A		Model B	
Variables	**b (se(b))**	**OR**	**–b (se(–b))**	**OR**
α_1	0.348		–0.348	
α_2	–1.756		1.756	
α_3	–2.106		2.106	
α_4	–2.952		2.952	
α_5	–6.503		6.503	
fun	0.299 (0.012)**	1.348	–0.299 (0.012)**	0.742
schwork	0.254 (0.013)**	1.289	–0.254 (0.013)**	0.776
learn	–0.025 (0.010)*	0.976	0.025 (0.010)*	1.025
hoursch	–0.108 (0.012)**	0.897	0.108 (0.012)**	1.115
houroth	–0.121 (0.008)**	0.886	0.121 (0.008)**	1.129
LR R^2	0.037		0.037	
Model fit[a]	$\chi^2_{(5)} = 1{,}580.62$**		$\chi^2_{(5)} = 1{,}580.62$**	

Note: [a] Likelihood ratio test
Significant at: ** $p < .01$; * $p < .05$

for using computers to learn on their own (learn), $\beta = -.025$, $z = -2.26$, $p < .05$; the coefficient for hours per day on using computers for schoolwork (hoursch) $\beta = -.108$, $z = -9.07$, $p < .001$; and the coefficient for hours per day of using computers on the others (houroth), $\beta = -.121$, $z = -14.29$, $p < .001$.

Interpreting the Odds Ratios of Being In a Particular Category Versus Being Below that Category (Backward CR Model A)

To understand the odds ratios of being in a particular versus being below that category, our demonstration data analysis revealed that two predictors were positively associated with the log odds of reaching a particular category versus being below that category. In terms of odds ratios (OR), the odds of being in a proficiency level increased by a factor of 1.348 with a one-unit increase in using computers for fun and increased by a factor of 1.289 with a one-unit increase in using computers for schoolwork.

However, the other three predictors were negatively associated with the log odds of being in a proficiency level rather than being in lower proficiency levels. Students who spent more time on using computers to learn on their own were associated with the odds of being in lower proficiency levels (OR = .976). Further, the odds of being in a proficiency level versus being below that level decreased by a factor of .897 with a one-unit increase in spending excessive hours a day on computer for schoolwork, and decreased by a factor of .886 with a one-unit increase in spending more hours a day on computers for other things.

Interpreting the Odds of Being Below a Particular Category Versus Being In that Category (Backward CR Model B)

Compared to Model A in the backward CR model, Model B estimates the inversed odds, the odds of being below a particular category relative to reaching that category. By exponentiating the negative logit coefficient $\exp(-\beta)$ in Model B, we obtain the odds ratio of being below a particular category, which can be interpreted as the change in the odds for a one-unit change in a predictor.

Specifically, in the demonstration data, two predictors were negatively associated with the log odds of being below a particular category relative to being in that category. In terms of odds ratio (OR), the odds of being below a proficiency level decreased by a factor of .742 with a one-unit increase in using computers for fun and decreased by a factor of .776 with a one-unit increase in using computers for schoolwork. However, the other three predictors were positively associated with the log odds of being below a proficiency level rather than reaching that proficiency level. Students who spent more time on using computers to learn on their own were associated with the odds of being in lower proficiency levels (OR = 1.025). The odds of being below a proficiency level versus reaching that level increased by a factor of 1.115 with a one-unit increase in spending excessive hours a day on computer for schoolwork, and increased by a factor of 1.129 with a one-unit increase in spending more hours a day on computers for other things.

A Comparison of the Results between the Forward and Backward CR Models

Presented in Tables 3 and 4 are the results of the forward and backward CR models, respectively. The estimated logit coefficients in the forward CR Model A were different from those in the backward CR Model A in both magnitude and sign. The forward CR Model A compares a particular category with higher categories, whereas the backward CR Model A compares a particular category with lower categories. Therefore, the signs before the logit coefficients were opposite between these two models.

The estimated coefficients in the forward CR Model A looked similar to those in the backward CR Model B, but they were different in nature, because the former model estimated the odds of being in a particular category versus being above that category, whereas the latter estimated the odds of being in lower categories relative to being in that category.

The logit coefficient for using computers to learn on their own (learn) was not significant in the forward CR models. However, it was significant in the backward CR models.

The results of the two sub-models of the forward CR model were the same in magnitude but were the opposite in sign since these two models estimated the inversed odds, as were the results of the two sub-models of the backward CR model. Therefore, the model equations should be matched with the corresponding odds.

A Comparison of the Results between the CR Models and PO models

Presented in Table 5 are the results of the two PO models with the five predictor variables using the Stata `ologit` command. The PO Model A estimated the cumulative odds of being at or below a particular category versus being above that category, whereas the PO Model B estimated the inversed odds, the odds of being in higher categories rather than being at or below that category. The results of the PO models were different from those of the forward and backward CR models since the PO models and CR models estimated different types of the odds. These results

Table 5. Results of the proportional odds (PO) models with five predictor variables using Stata ologit: Model A ($Y \leq$ cat. j vs. $Y >$ cat. j) and Model B ($Y >$ cat. j vs. $Y \leq$ cat. j)

	Model A		Model B	
Variables	**b (se(b))**	**OR**	**–b (se(–b))**	**OR**
α_1	–1.078		1.078	
α_2	1.138		–1.138	
α_3	2.133		–2.133	
α_4	3.530		–3.530	
α_5	7.155		–7.155	
fun	–0.390 (0.016)**	0.678	0.390 (0.016)**	1.476
schwork	–0.359 (0.017)**	0.698	0.359 (0.017)**	1.432
learn	0.030 (0.014)*	1.031	–0.030 (0.014)*	0.970
hoursch	0.145 (0.015)**	1.156	–0.145 (0.015)**	0.865
houroth	0.162 (0.011)**	1.176	–0.162 (0.011)**	0.850
LR R^2	0.041		0.041	
Model fit[a]	$\chi^2_{(5)} = 1{,}747.58^{**}$		$\chi^2_{(5)} = 1{,}747.58^{**}$	

Note: [a] Likelihood ratio test
Significant at: ** $p < .01$; * $p < .05$

suggested that reversing the ordinal response variable in PO models only changed the signs of the coefficients. However, different category comparisons in CR models might change both the sign and the magnitude of the coefficients.

Conclusion

The forward and backward CR models compare different categories, so the results are different. The forward CR models focus on the comparisons between a particular category and higher categories, whereas the backward CR models compare a particular category with lower categories. Each model has sub-models when the inversed odds are estimated. The estimated coefficients in the two sub-models of the forward or backward CR model are the same in magnitude but are the opposite in sign. In practice, to select either sub-model will answer the same research question, but the estimated odds will be inversed. The CR models and the PO models estimate different types of odds since the former models estimate the conditional odds while the latter models estimate the cumulative odds.

These results extended the CR model (Agresti, 2010; Allison, 2012; Fienberg, 1980; Fullerton, 2009; Fullerton & Xu, 2016; Greenland, 1994; Liu et al., 2011; Liu, 2016; Long & Freese, 2006, 2014; O'Connell, 2006) in proposing and exploring different types of the CR models and making a clear distinction among them. There are different category comparisons when fitting the CR models and it is important to ensure these comparisons can be correctly matched with model equations.

In the educational research example, the CR models are useful when analyzing the educational attainment data such as different levels of diploma or ordinal proficiency data, where there are progressions toward higher degree or proficiency levels. The demonstration clarifies the confusion on different types of CR models for analysis of ordinal data.

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References

Agresti, A. (2007). *An introduction to categorical data analysis* (2nd edition). New York: John Wiley & Sons. doi: 10.1002/0470114754

Agresti, A. (2010). *Analysis of ordinal categorical data* (2nd edition). New York: John Wiley & Sons. doi: 10.1002/9780470594001

Agresti, A. (2013). *Categorical data analysis* (3rd edition). New York: John Wiley & Sons.

Allison, P. D. (2012). *Logistic regression using the SAS system: Theory and application* (2nd edition). Cary, NC: SAS Institute.

Ananth, C. V., & Kleinbaum, D. G. (1997). Regression models for ordinal responses: A review of methods and applications. *International Journal of Epidemiology, 26*(6), 1323-1333. doi: 10.1093/ije/26.6.1323

Armstrong, B. B., & Sloan, M. (1989). Ordinal regression models for epidemiological data. *American Journal of Epidemiology, 129*(1), 191-204. doi: 10.1093/oxfordjournals.aje.a115109

Bender, R., & Benner, A. (2000). Calculating ordinal regression models in SAS and S-Plus. *Biometrical Journal, 42*(6), 677-699. doi: 10.1002/1521-4036(200010)42:6<677::AID-BIMJ677>3.0.CO;2-O

Buis, M. L. (2013, October 25). seqlogit: Stata module to fit a sequential logit model. *Statistical Software Components*. Boston, MA: Boston College Department of Economics. Retrieved from https://econpapers.repec.org/RePEc:boc:bocode:s456843

Cole, S. R., & Ananth, C. V. (2001). Regression models for unconstrained, partially or fully constrained continuation odds ratios. *International Journal of Epidemiology, 30*(6), 1379-1382. doi: 10.1093/ije/30.6.1379

Fagerland, M. W. (2014). adjactlogit, ccrlogit, and ucrlogit: Fitting ordinal logistic regression models. *The Stata Journal, 14*(4), 947-964. doi: 10.1177/1536867X1401400414

Fienberg, S. E. (1980). *The analysis of cross-classified categorical data*. Cambridge, MA: MIT Press.

Fullerton, A. S. (2009). A conceptual framework for ordered logistic regression models. *Sociological Methods and Research, 38*(2), 306-347. doi: 10.1177/0049124109346162

Fullerton, A. S., & Xu, J. (2016). *Ordered regression models: Parallel, partial, and non-parallel alternatives*. Boca Raton, FL: Chapman & Hall/CRC.

Greenland, S. (1994). Alternative models for ordinal logistic regression. *Statistics in Medicine, 13*(16), 1665-1677. doi: 10.1002/sim.4780131607

Hilbe, J. M. (2009). *Logistic regression models*. Boca Raton, FL: Chapman & Hall/CRC.

Hosmer, D. W., Lemeshow, S., & Sturdivant, R. X. (2013). *Applied logistic regression* (3rd edition). New York: John Wiley & Sons. doi: 10.1002/9781118548387

Ingels, S. J., Pratt, D. J., Roger, J., Siegel, P. H., & Stutts, E. (2004, February). *ELS: 2002 base year data file user's manual* (NCES 2004-405). Washington, DC: National Center for Education Statistics. Retrieved from https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2004405

Ingels, S. J., Pratt, D. J., Roger, J., Siegel, P. H., & Stutts, E. (2005, November). *Education longitudinal study: 2002/04 public use base-year to first follow-up data files and electronic codebook system* (NCES 2006-346). Washington, DC: National Center for Education Statistics. Retrieved from https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2006346

Liao, T. F. (1994). *Interpreting probability models: Logit, probit, and other generalized linear models*. Thousand Oaks, CA: Sage.

Liu, X. (2009). Ordinal regression analysis: Fitting the proportional odds model using Stata, SAS and SPSS. *Journal of Modern Applied Statistical Methods, 8*(2), 632-645. doi: 10.22237/jmasm/1257035340

Liu, X. (2016). *Applied ordinal logistic regression using Stata: From single-level to multilevel modeling*. Thousand Oaks, CA: Sage.

Liu, X., O'Connell, A. A., & Koirala, H. (2011). Ordinal regression analysis: Predicting mathematics proficiency using the continuation ratio model. *Journal of Modern Applied Statistical Methods*, *10*(2), 513-527. doi: 10.22237/jmasm/1320120600

Long, J. S. (1997). *Regression models for categorical and limited dependent variables*. Thousand Oaks, CA: Sage.

Long, J. S., & Freese, J. (2006). *Regression models for categorical dependent variables using Stata* (2nd edition). College Station, TX: Stata Press.

Long, J. S., & Freese, J. (2014). *Regression models for categorical dependent variables using Stata* (3rd edition). College Station, TX: Stata Press.

McCullagh, P. (1980). Regression models for ordinal data. *Journal of the Royal Statistical Society: Series B (Methodological), 42*(2), 109-142. doi: 10.1111/j.2517-6161.1980.tb01109.x

McCullagh, P., & Nelder, J. A. (1989). *Generalized linear models* (2nd edition). London: Chapman and Hall.

O'Connell, A. A. (2000). Methods for modeling ordinal outcome variables. *Measurement and Evaluation in Counseling and Development, 33*(3), 170-193. doi: 10.1080/07481756.2000.12069008

O'Connell, A. A. (2006). *Logistic regression models for ordinal response variables*. Thousand Oaks, CA: Sage.

Powers, D. A., & Xie, Y. (2000). *Statistical models for categorical data analysis*. San Diego, CA: Academic Press.

Tutz, G. (1991). Sequential models in categorical regression. *Computational Statistics & Data Analysis, 11*(3), 275-295. doi: 10.1016/0167-9473(91)90086-H

Tutz, G. (2012). *Regression for categorical data*. New York: Cambridge University Press.

Wolfe, R. (1998). Continuation-ratio models for ordinal response data. *Stata Technical Bulletin, 8*(44), 18-21. Retrieved from: https://econpapers.repec.org/RePEc:tsj:stbull:y:1999:v:8:i:44:sg86