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Interaction-Driven Phenomena And Wigner Transition In Two-Dimensional Systems

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INTERACTION-DRIVEN PHENOMENA AND WIGNER TRANSITION IN TWO-DIMENSIONAL SYSTEMS

by

TALBOT KNIGHTON

DISSERTATION

Submitted to the Graduate School
of Wayne State University,
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Approved By:

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Advisor

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DEDICATION

To my family and friends.
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CHAPTER 1 INTRODUCTION

1.1 Introduction and Motivation

In 1936, Wigner predicted that electrons can form an ordered crystal structure in order to minimize the electron-electron Coulomb repulsion [27]. Since that time, electron-electron interaction has become a fascinating field for the many exotic and useful phases that it encompasses, such as both low-temperature and high temperature superconductive phases [28, 29], topological states [30–38], non-abelian particles with fractional statistics useful for quantum computing [39–41], etc. However, proof of the crowning jewel of a Wigner crystal has remained elusive.

The case of a two dimensional crystal is of particular interest for several reasons. Two-dimensional semi-conductor systems provide perhaps the only method for achieving a quantum Wigner crystal (WC). Furthermore, the question of two-dimensional phase transitions has a long-standing history. Mermin and Wagner have provided a rigorous proof that long-range order cannot be achieved [42, 43]. This appeared to be at odds with liquid crystal experiments and simulations that indicated transitions. This was later explained in terms of the unbinding of topological defect pairs by Kosterlitz and Thouless [33, 34, 44] (for which the most recent Nobel Prize in Physics was awarded in 2016). Nelson, Halperin, and Yong later expanded on these ideas, indicating that multiple phase transitions can occur as different types of defects become energetically favorable (dislocaitons and disclinations).

This dissertation demonstrates a pinning conduction threshold in the reentrant insulating phase (RIP) at filling factor $\nu = 1/3$ in the fractional quantum Hall regime. This develops rapidly when the holes are cooled below the melting temperature for a classical WC $E_C/137 = 330$ mK (where $E_C$ is the Coulomb energy). The data are consistent with a two-
stage transition through an intermediate phase from liquid to solid, and provide perhaps the
strongest transport evidence to date for WC at the RIP centered on $\nu = 0.37$.

There have been long standing theoretical and experimental challenges for studying this
strongly correlated state. Theoretically, it is difficult to calculate the effects of dominant
Coulomb interaction and manybody physics at large $r_s$ (where the interaction parameter
$r_s = E_C/E_F \propto 1/\sqrt{p}$ is the ratio of coulomb to kinetic energy). Experimentally, competing
effects such as thermal fluctuations and disorder make it difficult to achieve quantum systems
with strong interaction $r_s \gg 1$. However, recent progress in GaAs semiconductors allows us
to take a closer look.

In 1958, Phillip Anderson’s Nobel prize winning theory of localization dealt with electron-
disorder interaction [45]. This type of localization is ubiquitous due to the prevalence of dis-
order in most systems [46–50, 50–55]. Following works, based on scaling theory, showed that
all two-dimensional (2D) charge systems should be localized with wavefunctions decaying
exponentially over a characteristic length $\xi$ that depends on the amount of disorder [2, 56].
However, the Anderson picture ignores interaction effects. Subsequent theoretical studies
disagree upon whether localization is enhanced or destroyed for weakly interacting charges
[50, 57, 58]. The behavior of a weakly disordered system with large Coulomb energy is also
not well understood [59, 60].

To add to the complexity, experimentalists have discovered that a low temperature metal-
insulator transition (MIT) occurs at a critical density in 2D systems [2–4, 12, 34, 57, 58, 60–
71]. The critical density often depends on the amount of disorder in a given sample. On the
insulating side, the resistance diverges exponentially as $T \rightarrow 0$ indicating that the transport
mechanism is single-particle hopping consistent with the Anderson localization picture [12,
However, the existence of a metallic state and implication of a zero-temperature quantum phase transition in 2D directly contradicts the localization theories and is not yet understood. Furthermore, it is not known whether the metallic behavior will actually continue if the samples could be cooled to lower temperatures [58, 72].

From the above discussion, we see that lowering the charge density increases the interaction parameter $r_s$. However, ineffective screening at low densities leaves the charges susceptible to disorder-driven localization [48]. Another way to quench kinetic energy is to confine the charges to small cyclotron orbits through the application of a large perpendicular magnetic field. In two-dimensional systems, this produces the integer and fractional quantum Hall effects (IQHE and FQHE) each of which was awarded a Nobel Prize in physics (in 1980 and 1982 respectively) [30–32, 73–75]. The behavior of IQHE states can be described using a single particle picture. Insight from these studies helped fuel the development of topological band theory [38, 46, 76–80]. For the fractional states, Coulomb interaction plays a critical role, and the physics of such systems, especially the non-abelian even-denominator states at 5/2, etc., and is not yet well understood [40, 81, 82]. Furthermore, the topological classification for Coulomb dominated interacting systems is also not known [83–87].

In addition to these three branches (localization, MIT, quantum Hall effect), there are a host of other contributing factors. For example, the importance of spin can be seen in the Giant magneto resistance (GMR) [88, 89], Kondo effect [90–92], spin quantum Hall effect (SQHE) [93, 94, 94–98], etc. Electron-electron interaction mediated through phonons is responsible for the conventional superconductor materials [99]. The high critical temperature (HTC) superconductors are also believed to stem from electron-electron interactions [28, 29]. For these scenarios, theoretical predictions are mostly restricted to the weakly interacting
regime $r_s \lesssim 1$ where approximations make calculations easier. Thus, the field of strongly interacting systems is a great opportunity for experimentalists to be at the forefront of an exciting area of physics.

Experimental challenges have long hindered direct studies of strongly Correlated states driven by Coulomb interaction. One of the main causes is the ubiquitous presence of disorder leading to Anderson localization at low densities [52, 54]. At present, the highest purity systems are two-dimensional (e.g., charges trapped in a two-dimensional quantum well grown one layer of atoms at a time through molecular beam epitaxy [100], or suspended above the surface of liquid helium [1, 101]). Recent advances in GaAs growth allow extremely high purity doped and undoped systems with charge densities as low as $2 \times 10^{-8} \text{ cm}^{-2}$. These systems can achieve the predicted $r_s$ value for quantum WC, even in zero or low magnetic field, so that we can hope to achieve a crystal state [102].

A strong candidate for WC are the high field insulating phase (HFIP) and RIP found between Laughlin Liquids. These appear in the lowest Landau level (LL) for $n$-type GaAs, but are found at larger filling factors and smaller fields in high purity $p$-type GaAs (presumably due to the larger effective hole mass which increases $r_s$) or silicon MOSFET’s [11, 12, 63, 103–110]. Previous studies of the RIP have focused on microwave resonance, spontaneous noise generation, and also some results for nonlinear transport characteristics [10–13, 60, 63, 109–115].

In this work, we perform the first sensitive transport measurements of high purity $p$-type systems at the RIP near $\nu = 1/3$. At base temperature ($T = 11 \text{ mK}$), a sharp threshold in the current-voltage (IV) characteristics indicates pinning (equivalent to $\rho_{xx} \sim 250 \text{ M}\Omega/\square$) that is several orders of magnitude larger than previous observations. The $T$ dependence
shows a clear melting transition between 40 and 330 mK where the sample regains a linear (Ohmic) characteristic, and the resistance drops to $h/e^2 \approx 25.8$ kΩ by 300 mK. Both the IV threshold and non-activated melting distinguish the low-temperature state from an Anderson insulator. Furthermore, the threshold field and energy are consistent with a manybody tunneling process [116] of crystal domains having $10^3$-$10^4$ charges rather than a single particle mode. These results complement previous transport measurements taken in $n$-type systems at higher field [13, 109] and ultra-low density holes at zero field [114, 117], providing a strong footing for the observation of a WC.

The nonlinear transport described above indicates a phase transition, for which the giant pinning strength and low-temperature saturation both strongly suggest WC. However, other states such as a disordered Wigner glass (WG), hexatic, and micro-emulsion phases are also possible. Therefore, it would be highly advantageous to probe the periodicity of the triangular lattice using a scanning measurement. Any such scanning technique in GaAs must use a capacitively coupled probe to observe fluctuating charge density inside the quantum well, which lies beneath an AlGaAs dielectric. Furthermore, to resolve individual charges, we must have a spatial resolution on the order of 20 nm, which produces an extremely small capacitance. Thus, we investigate several different methods for preamplification in order to measure such a small capacitance. To test these methods, we perform quantum capacitance measurements on a Vanadium dioxide system across the metal insulator transition.

In the near future, we would like to do a more detailed study of the sub-threshold transport in the RIP to determine the de-pinning mechanism of the insulating RIP. Additionally, we would like to perform quantum capacitance measurement on this strongly correlated insulator.
We also do transport measurements on an anti-hall bar (modified Corbino disk) geometry in order to determine the edge-bulk correspondence and topological robustness of a charge system with relatively large coulomb interaction.

Finally, we discuss various projects related to 2D charges and strongly interacting systems, such as the fabrication and testing of dilute 2D holes in HIGFET devices and fabrication of graphene devices for optical detection.
CHAPTER 2 BACKGROUND

2.1 Prediction of a Quantum Wigner Crystal

Carl Wigner first predicted, in 1934 [27], the formation of a quantum electron solid [now known as a Wigner crystal (WC)] in highly degenerate \(k_B T\) much less than the Fermi Energy \(E_F\) systems at low charge densities. The crystal is a response to the coulomb repulsion between electrons, which is minimized if the charges fall into a triangular lattice. The importance of Coulomb energy can be roughly characterized by the interaction parameter

\[
r_s = \frac{E_C}{E_F} \tag{2.1}
\]

where the Coulomb and Fermi energies are

\[
E_C = \frac{e^2}{4\pi\epsilon} \sqrt{\pi p}, \text{ SI units} \tag{2.2}
\]

and

\[
E_F = \frac{2\pi\hbar^2 p}{g_s m^*} \tag{2.3}
\]

respectively so that \(r_s \propto 1/\sqrt{p}\). Here, \(p\) is the areal charge density, \(\epsilon = \epsilon_r\epsilon_0\) is the dielectric constant, \(g_s = 2\) is the spin degeneracy, and \(m^*\) is the effective hole mass. Taking \(\epsilon_r = 12.9\) for GaAs and \(m^* \in (0.2, 0.45)m_e\) (see footnote 2), we plot these two energies and estimate \(r_s\) as a function of density (Fig. 2.1). \(^1\) \(^2\)

\(^1\) There is another convenient way to represent the interaction parameter that is used by many theorists: \(r_s = a/a_B^*\), where \(a = 1/\sqrt{\pi p}\) is the radius for a circle having the average area-per-electron and \(a_B^* = 4\pi\hbar^2/m^*e^2\) in SI units. That is \(a_B^* = a_B \frac{\epsilon_r e}{m^* m_e}\), where \(a_B \sim 52.9177\) pm.

\(^2\) \(m^*\) is somewhat dependent on the density and applied magnetic field. For the densities in this study \(p \sim 4 \times 10^{10}\) cm\(^{-2}\), we take \(m^*\) to be about 0.35\(m_e\) in zero field [118].
Figure 2.1: Interaction parameter $r_s$ as a function of charge density density $p$ and $m^*$. We have taken $m^*$ to be a slowly decreasing function of $p$ to give a picture that is in qualitative agreement with the measured value. The critical value $r_s = 38$ for Wigner crystallization occurs at $p \sim 3 \times 10^9$ cm$^{-2}$. 

$E_C, E_F$ (meV) vs $p$ (cm$^{-2}$) 

$m^*/m_0$ vs $p$ (cm$^{-2}$) 

$r_s = E_C/E_F$ vs $p$ (cm$^{-2}$) 

$E_C$, $E_F$, $r_s$ 

$E_C$, $E_F$, $r_s$ 

$E_C$, $E_F$, $r_s$ 

$E_C$, $E_F$, $r_s$
2.2 Prediction and Discovery of a Classical Wigner Crystal

Before continuing with the discussion of a quantum WC, we discuss the prediction and subsequent observation of a classical WC. We stated above that for a degenerate system, the kinetic energy is dominated by $E_F \gg k_B T$. $E_F$ scales with $p$ so that lower densities increase the interaction parameter.

For a classical system, the kinetic energy is dominated by $k_B T \gg E_F$. A better measure of the Coulomb interaction is thus

$$\Gamma = \frac{E_C}{k_B T}. \quad (2.4)$$

In this case, $\Gamma \propto \sqrt{n}$ (electron charge density), so that higher densities are required to maximize $\Gamma$ and produce a classical WC [1, 119].

Charges suspended above a liquid Helium surface proved to be an ideal system for achieving a 2D classical WC. This interface is free from disorder so that Anderson localization (AL) does not effect the system, and very low charge densities (down to $10^5$ cm$^{-2}$) are available [1]. However, the limiting factor for larger $\Gamma$ is the surface energy of the liquid and how much charge can be applied before the system breaks down. Grimes and Adams find experimentally that the charges are trapped by a binding of 0.7 meV which allows densities of $10^5$ to $10^{10}$ cm$^{-2}$ (corresponding to $2 \leq \Gamma \leq 200$). They drive the system using an oscillating $\vec{E}$-field perpendicular to the surface. Vibrational modes appear below a certain ratio of $n/T$ corresponding to $\Gamma = 137 \pm 15$ (Fig. 2.2) indicating a phase transition to the

\footnote{Under these conditions, the particle interaction is point-like (classical) because the spatial extent perpendicular to the Helium surface is only 1 $\mu$m whereas the inter-particle spacing $a$ is much larger ($\approx 50 \mu$m).}
solid state [1]. At the time of their experiment, several theoretical papers had predicted that plasmon modes (perpendicular to the $^3$He surface) would be detected below a liquid-solid phase transition around $\Gamma \sim 100$. Additionally, resonance due to coupled ripplon modes (transverse) also play a role in the experiment [1, 120].

Figure 2.2: (a) Disappearing resonance modes of a classical electron Wigner crystal above the surface of liquid helium. (b) Portion of the electron liquid-solid phase boundary at $\Gamma = 135 \pm 15$. Figure adapted from Ref. [1].

2.3 Strongly Correlated Fermi Liquids

A low-temperature 2D charge system can be classified into 3 distinct regions based on $r_s$. Tanatar and Ceperly estimate that the 2D Fermi liquid to quantum WC transition occurs when $r_s^{(c)} \approx 38$, so that the ground state is a solid at zero temperature and zero $\vec{B}$-field when $r_s \geq 38$ [121]. On the opposite end of the spectrum, for small $r_s < 1$ and resistivity $\rho < h/e^2$, the system is a “good” metal and well understood [59]. In between, for $1 < r_s < r_s^{(c)}$ the system is a strongly correlated Fermi liquid that is still a mystery in many ways.

For $r_s$ in this intermediate range, there are four regions in the temperature dependence delineated by three energies (as tabulated in Tab. 2.1 below [59]): the Fermi energy $E_F$, the plasmon frequency $\hbar \Omega_p = \sqrt{E_F E_C} = \sqrt{r_s} E_F$, and the Coulomb energy $E_C = r_s E_F$. Notice that these energies all collapse to $E_F$ when $r_s = 1$ but have a wider spread at large $r_s$.

We now see that there are at least two conditions for forming a 2D quantum WC in zero
<table>
<thead>
<tr>
<th>Temperature Region</th>
<th>State Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T &lt; E_f$</td>
<td>Fermi-liquid</td>
</tr>
<tr>
<td>$E_F &lt; T &lt; \sqrt{r_s}E_F$</td>
<td>Highly correlated non-degenerate quantum liquid</td>
</tr>
<tr>
<td>$\sqrt{r_s}E_F &lt; T &lt; r_sE_F$</td>
<td>Highly correlated classical liquid</td>
</tr>
<tr>
<td>$r_sE_F &lt; T$</td>
<td>Classical electron gas</td>
</tr>
</tbody>
</table>

Table 2.1: Regions of Temperature dependence for a strongly correlated Fermi liquid.

magnetic field: $r_s \geq 38$ so that the ground state is a WC and $T \ll E_F/k_B$ so that the system is not thermally excited out of the ground state.

To satisfy $r_s > 38$ for a quantum system, the charge density must be lowered. This greatly decreases the magnitude of both $E_C$ and $E_F$ so that other energies can affect the ground state. Most notably, the dilute charges are unable to screen disorder effectively so that the system becomes increasingly susceptible to Anderson localization. It is well known that (contrary to the scaling theory of localization discussed in Sec. 2.6 below) 2D systems undergo a metal-insulator transition (MIT) upon lowering the density below a critical value $p_c [4]$ (see Sec. 2.7). However, the tendency towards disorder-induced localization in dilute systems makes it difficult to identify a strongly-correlated manybody insulator unambiguously.

2.4 Solid-Melting Transition in Two Dimensions

In 1966 and 1968, Mermin and Wagner showed that there can be no long-range crystallographic order in two dimensions due to pair-wise potentials of the form

$$
\phi_\lambda(\vec{r}) = \phi(\vec{r}) - \lambda r^2 |\nabla^2 \phi(\vec{r})|.
$$

(2.6)

In particular, the Fourier coefficients of the density go to zero logarithmically with increasing sample size

$$
\rho_k \sim 1/\ln N
$$

(2.7)
This appeared to be at odds with experiments in two-dimensional liquid crystals, which showed phase transitions. Kosterlitz and Thouless later discussed these transitions as the unbinding of topological defect pairs [33, 34, 44]. Nelson, Halperin, and Young later expanded on this work, showing that different types of defects (dislocations, and disclinations) can cause various phase transitions, so that the liquid-solid phase transition may occur through an intermediate hexatic phase (for which the translational order dies away exponentially, but the bond-orientational order has only a power law decay) [122–124].

For the case of a quantum solid driven by Coulomb interaction, several theoretical works have studied the melting transition of a WC ground state upon raising temperature. The melting temperature is expected to be close to \( E_C/137k_B \) by analogy with the classical case (Sec. 2.2 and Ref. [1]). Some modification due to quantum fluctuations (\( 4800/\sqrt{T} \) for GaAs [125]) is inevitable, but these are expected to be only on the order of a few percent. Early results assumed a first order transition [59, 126–132]. Many subsequent papers have demonstrated that the transition is likely second order and that various intermediate micro-emulsion (bubble, stripe, etc.) phases are possible in addition to the hexatic phase (see Refs. in [59]). However, the range of temperatures over which the transition occurs is not known and may be small.

2.5 Mott Insulator

How do impurity states interact with one-another and how do they effect electronic properties? We can treat the impurity states as forming a regular array, but very spread out (sub-lattice size \( b \)) relative to the lattice size \( a \) of the crystal in which they are embedded. Using a single electron picture reveals that the impurities form a half filled energy band (each impurity donates only one atom, and this does not fill the spin degeneracy). We
might naively expect this to have a metallic conduction. However, this ignores the coulomb repulsion between impurity sites, which is preventative of conduction in the narrow band case. To see this, consider the Coulomb energy cost $E_C = e^2/b$ for moving two electrons onto the same impurity site. If this is larger than the bandwidth of the impurity band $V_b$, then conduction cannot happen. This is known as a Mott insulator, leading to a metal-insulator transition that can be driven by externally applied pressure or additional doping.

As a simple picture, first consider that the impurity sub-lattice spacing is infinitely large so that the impurities levels have no splitting. Each impurity has two possible electron states: a single electron occupation (energy $E+0$), or a double electron occupation (add an electron of opposite spin to give the impurity an energy $E_0 + E_C$). The material transitions from an insulator to a metal as the lattice spacing decreases until the splitting bands from these two states overlap (at this point, the band gap due to Coulomb repulsion is overrun).

The model is quantitatively studied in the greatly simplified Hubbard model which only considers nearest neighbor interactions (See Ref. [54], page 29). In the case of Mott insulator, Coulomb repulsion provides an extra energy cost that prevents electrons from occupying states that are normally allowed by the Pauli exclusion principle.

### 2.6 Disorder and Localization

#### 2.6.1 Anderson Localization

To further study the nature of the impurity bands, we again ignore the Coulomb interaction, making the single electron approximation. We further complicated the scenario by allowing random variation of the energy level for each impurity within the impurity sub-lattice while still leaving their perfect periodic spacing in tact (Fig. 2.4). The energy at each impurity site $i$ is given a random value $\epsilon_i$ chosen from a uniform probability distribution
Figure 2.3: Illustration of a Mott transition in the Hubbard model. $E_0$ is an available energy state in a monovalent impurity lattice, and $E_C$ is the Coulomb energy cost for placing two electrons in the same unit cell. The energy bands for $E_0$ and $E_0 + E_C$ widen until they cross at the Fermi energy when the lattice constant shrinks to a critical value $b = b_c$ causing a metal-insulator transition.
$P = 1/W$ having a bandwidth of $W$. Note that the “narrow band” width $V_b$ in the Mott case is an energy splitting due to the finite distance between perfect impurity states (all having the same energy). In the Anderson case, the bandwidth is due to the random variation of impurity site energies. The Hamiltonian for this system (spin indices suppressed) is given by

$$H = \sum_j \left[ \epsilon_j a_j^\dagger a_j + \sum_{m \neq 0} I(m)a_j^\dagger a_{j+m} \right]$$

(2.8)

(where $I(m)$ is the overlap integral). It turns out that to mathematically solve this system is extremely challenging, and still not fully understood (especially if particle interactions are taken into account). However, the qualitative picture for a single electron is quite reliable and strongly supported by computational simulations.

In a semi-conductor system, the intrinsic material possesses a band gap which is prohibitive for charge transport. Thus, we must look to impurities and the electronic wave functions that they generate in order to produce conduction. To determine whether the disordered medium will conduct, we ask whether the impurity state wavefunctions are extended (metallic) or localized (insulating). The answer to this question depends critically on the shape of the tails of the wavefunctions. If the tails die slowly, the material may possess a metallic band of coherent states extending over a large or macroscopic area, whose description as a linear superposition of site states requires significant contribution from distant states. However, if the tails die exponentially, the states are localized with vanishingly small contribution from distant site functions.

It is actually very difficult to give an exact mathematical description of the localization condition. One method is to look at the amplitude of a wave function at site $i$ as $t \to$
∞. If the particle is localized, the wavefunction will remain non-zero over this site with exponentially vanishing tails at distant sites for all time. If it is not, the wavepacket will spread (the electron delocalizes), so that the amplitude at site $i$ vanishes over a time scale $\hbar/V_b$, where $V_b$ is the width of the energy band in which the state exists. This was the test originally used by Anderson in his seminal 1958 paper.

Anderson’s final result is that whether or not a state is localized depends on the dimensionless parameter $W/I$ where $W$ is the bandwidth of the uniform density impurity band and $I$ is the overlap integral. When $W/I$ drops below a critical value, delocalized states begin to form in the center of the impurity band. Thouless explained this in terms of a percolation of resonant states (states that are close in energy) as the amount of disorder decreases. That is, as the disorder bandwidth narrows, it becomes more and more likely for neighboring states to be close in energy, until at a critical value of $W/I$, a single cluster of resonant states can span the system. However, the validity of these results depends on several factors, such as dimensionality and electron-electron interaction (to avoid this difficulty, and in light of the experiments he was observing at the time, Anderson originally discussed the suppression of spin diffusion [see Anderson’s Nobel Lecture]), as we will see below in the discussion of the scaling theory of localization and two-dimensional metal-insulator transition.

We can illustrate the above ideas to get a taste of the flavor (without any mathematical rigor) by considering a disordered lattice in 1D as pictured in Fig. 2.4(a). Notice that states have been color-coded by the energy of their particular lattice site. For energies near the middle of the band (blue), it is relatively probable to find a nearby state of similar energy. At the edges of the band (yellow, purple) similar energy states are farther apart. For energies where the density of states is large, we are more likely to have a percolation of
resonant clusters that spans the sample. This type of thinking gives rise to a very important concept in semi-conductor physics: the concept of a mobility edge that separates regions of localized and extended states in energy space, based on the change in DOS as a function of energy [Fig. 2.4(b)]. The MIT across this transition through doping is known as the Mott-Anderson transition (or sometimes just Anderson transition), indicating the importance of both disorder and electron-electron interaction (see Ref. [54] and references therein).

2.6.2 Minimum Metallic Conductivity

The exact distinction between a conductor and an insulator is not yet well understood. This becomes especially important at zero temperature where insulating states are expected to have zero conductivity, but metallic states should exhibit low or zero resistance. The question is: is there a smooth transition between these two types of state? For many years a minimum metallic conductivity was postulated. Simply put, the mean free path of an electron cannot be infinitely small; it must be greater than or equal to one lattice spacing (or impurity sub-lattice spacing for a Mott insulator). So, the zero temperature conductivity for a metallic state must be non-zero (Ref. [54], pages 36-38). This implies that the conductivity may have to jump discontinuously from \( \sigma = \sigma_{\text{min}} \) to \( \sigma = 0 \) between metallic and insulating states at

![Figure 2.4](image-url)
zero temperature. However, this has been a highly criticized idea. An important paper by Abrahams, Licciardello, Anderson, and Ramakrishnan introduced the scaling theory of localization [2] (Sec. 2.6.3 below). This theory says that there are no metallic states in 1D or 2D systems, and thus no minimum metallic conductivity. At zero temperature, all states are localized by any disorder in the system (though the localization length may be larger than the sample under study if there are very few impurities). For 3D systems, the effects of disorder are not so dominant, and metallic states can exist below an impurity threshold.

Experimentally, however, there appears to be distinction between metallic and insulating states. The Metal insulator transition observed as a function of density in 2D by Kravchenko appears to be universal to a variety of 2D systems. It is impossible to know whether the states which appear to be metallic (resistance goes to zero as temperature is lowered) will turn out to have zero conductivities at temperatures below the available experimental range for semiconductor systems, though some results have been presented [72]. The metal insulator transition is a crucial result, highlighting the importance of electron-electron interaction which is neglected in Anderson’s model.

2.6.3 Scaling Theory

We now summarize the main results from the scaling theory of localization [2]. For the moment, we ignore electron electron interactions. In units of quantum resistance, the normalized conductivity is

$$G \equiv \frac{e^2}{\hbar \rho}$$

$$= G_0 + \delta G.$$
Here $G_0 \approx k_F l$ where $l$ is the mean free path, and the differential conductivity

$$\delta G = -\ln L_\phi / l$$

depends on the phase coherence length normalized by the mean free path. If the sample size $L$ is finite, $L_\phi$ should be replaced by the sample size $L$. The main result of the scaling theory is that

$$\frac{d \ln G}{d \ln L} = \beta(G)$$

(2.12)

where $\beta$ is some function of $G$ alone. Equation 2.11 implies that $\beta \sim -1/G \to 0$ as $G \to \infty$. The results of their work is shown in Fig. 2.5 where $\beta$ is plotted against the normalized local conductivity $\ln g$. For $d > 2$, there exists a critical conductivity $g_c$ (minimum metallic conductivity) such that systems with $g > g_c$ are metallic (having extended states regardless of the system size). For $d \leq 2$, the conductivity for all systems approaches zero as the system size increases. Thus, all two dimensional states should be localized, even for a minimally disordered system. This appears to be in direct conflict with the experimentally-observed metal-insulator transition, for which metallic states exists down to the lowest available temperature in most 2D systems [4].

2.6.4 Hopping Conduction

There are several types of hopping mechanism, each of which is expected to be valid for a different temperatures range. They are all marked by exponential temperature dependence of the resistivity

$$\sigma \propto \exp \left( \frac{T^*}{T} \right)^\nu$$

(2.13)
Figure 2.5: Plot of $\beta$ versus the normalized local conductance $g$ for dimensions $d = 1, 2,$ and $3$. Figure adapted from Ref. [2].
where $T^*$ is some characteristic temperature about which $\nu$ indicates the dominant mechanism. We describe three types of hopping below.

Arrhenius hopping is a classical process of charges jumping onto neighboring lattice sites due to thermal excitation of the charge carriers and phonons. It is a statistical process with conductivity proportional to the hopping probability. This gives rise to a Boltzmann factor $\sigma \propto \exp \left( \frac{T^*}{T} \right)$ so that $\nu = 1$ in Eqn. 2.13. Note that the Drude model is valid in this regime when the mean free path $l$ is larger than the Fermi wavelength so that $k_F l < 2\pi$ and the resistance $\rho$ is less than the quantum resistance $h/e^2$. $k_F l = 2\pi$ marks the upper limit for possible applicability of Boltzmann transport theory and corresponds to $\rho = h/e^2$ in the Drude model [59].

The Mott variable range hopping (VRH) mechanism assumes a constant density of states (DOS) $g(\epsilon) = g_0$ within a range $\epsilon_M$ of the Fermi energy and does not consider Coulomb interaction. Mott VRH focuses on the overlap of the exponentially decaying tails of the quantum wavefunctions. The overlap integral allows tunneling onto non-nearest-neighbor lattice sites that are a bit farther away [133–135]. This can be understood in terms of first-order perturbation theory, for which the wavefunctions are modified by a term proportional to $I/\Delta E$ where $I$ is the overlap interval (rapidly decreasing with distance) and $\Delta E$ is the energy differential between two unperturbed wavefunctions. Mott VRH occurs when the thermal energy is limited, so that the charges must “scan” a greater area to find charges with $\Delta E$ within the acceptable range. Not surprisingly, the available states within a given scanning radius depends on the number of spatial dimensions $d$, so that the final result $\nu = 1/(d + 1)$ depends on $d$.

Efros and Shklovskii later demonstrated that Coulomb repulsion opens an energy gap
$\epsilon_{\text{ES}}$ at $E_F$ (it does not maintain the constant value $g_0$ assumed in Mott VRH), assuming that overlap of the wavefucitons is negligible. This gives an exponent of $\nu = 1/2$ in all dimensions [52].

Conduction involves all charges within an energy range $k_B T$ of the Fermi energy $E_F$. As $T$ lowers, the thermal resolution of the density of states around the Fermi energy increases. The assumptions of Mott VRH and ES VRH are valid only when the distribution function $f(E)$ is sharp the energy scales $\epsilon_M$ or $\epsilon_{\text{ES}}$. Thus, the dominant transport mechanism depends on the temperature of the sample. For some systems, $\epsilon_{\text{ES}} \ll \epsilon_M \ll \Delta E_{\text{nearest neighbor}}$ so that Arrhenius, Mott VRH, and ES VRH hopping conduction can be observe in that order as the temperature is lowered. However, for very lightly doped semiconductors, the Coulomb energy is already large compared to $E_F$, thus there may be no range for which the Mott VRH picture is valid, and we can expect to see a transition from $\nu = 1$ to $\nu = 1/2$ directly. Furthermore, for very pure systems with little disorder and strong electron interaction, these hopping pictures likely do not provide an adequate picture for the conductance.

How do electron-electron interactions effect this picture? Corrections due to weak electron have been studied by several authors [136–142]. Efros and Shklovskii even predict that a WC in a disordered system will have residual hopping at finite temperature due to dislocations in the crystal (or a “soft” coulomb gap), and that activated temperature dependence with $\nu = 1/2$ can be expected [54]. But, it is important to note that the ES VRH is derived in the single particle picture. That is, it is assumed that the minimum energy excitation to allow conduction is the displacement of a single particle. For a strongly correlated system, this may not be the case [116]. Many results [59] (in particular, the pinned IV characteristics presented in this paper) are incompatible with the single particle picture, and highly correlated systems
with dominant Coulomb energy below are not yet well understood [59].

2.7 Metal Insulator Transition

The metal insulator transition (MIT) at zero $\vec{B}$-field is one of the foremost unsolved problems in 2D physics far-reaching consequences to our understanding of both disorder and electron-electron interactions [4, 59, 61]. Though the effect has been observed in many different systems by multiple groups, the theoretical interpretation is still highly debated [59].

As discussed in the previous section, disorder is expected to localize the wavefunctions, causing them to decay exponentially on a length scale $\xi$. The result is an activated temperature dependence $\rho \propto \exp (T^*/T)^\nu$ where $\nu = 1$ for Arrhenius hopping (nearest neighbor hopping through Boltzmann statistics at high temperature), 1/3 for Mott Variable Range hopping (at lower thermal energy quantum tunneling to states that are closer in energy but farther in distance becomes important), and 1/2 for Efros Shklovskii hopping (when temperature falls well below the Coulomb energy $k_B T \ll E_C$ so that the distribution function $f(E)$ is sharp enough to resolve a gap in the DOS at $E_F$ caused by Coulomb repulsion). Thus, it is expected that

$$\frac{\partial \rho}{\partial T} < 0 \quad \text{(2.14)}$$

at all densities so that $\rho$ diverges as $T \rightarrow 0$.

However, in most 2D systems it is observed that $d\rho/dT \rightarrow 0$ down to the lowest experimentally available temperatures for charge densities $n$ greater than some critical value $n_c$ [Fig. 2.6(a)] [4, 59, 61]. We call this metallic behavior because it is similar to the trend in normal metals to approach zero resistivity (in the absence of disorder) at low temperatures due to the freezing out of electron-phonon scattering. A simple scaling adjustment along the
horizontal axis \(T \rightarrow T/T_0\) where \(T_0\) is a function of \(|n-n_c|\), the density relative to \(n_c\) causes the metallic and insulating curves to overlap with like-states [Fig. 2.6(b)]. \(T_0\) is chosen so that the most insulating curve follows \(E_S\) hopping. These results suggest a \(T = 0\) quantum critical point with a phase transition (in the absence of thermal fluctuations) between metallic and insulating states that is not yet well understood.

![Figure 2.6: (a) Metal insulator transition for a silicon MOSFET. Resistance at the critical density \(n_c\) (not labeled) extrapolates to \(\rho = h/3e^2\) at \(T = 0\). (b) Data from (a) are scaled in the temperature direction by the substitution \(T \rightarrow T/T_0\) where \(T_0\) is a function of density \(|n-n_c|\). Figure adapted from Refs. [3, 4].](image)

The existence of a zero temperature metallic state directly conflicts with the scaling theory of localization. However, we cannot know if the \(\frac{d\rho}{dT} < 0\) trend will continue if lower experimental temperatures can be achieved.

On the insulating side, the theoretical interpretation is also complicated. Nearly all studies of the metal insulator transition in semi-conductor systems report activated dependence at densities lower than \(n_c\). Though coulomb interactions play a role, the ES hopping conduction is a disorder-driven single particle transport picture. Thus, it is not yet known whether electron-electron interactions can produce ground state with properties that are much dif-
ferent from an Anderson insulator (for example, manybody tunneling transport, such as the quantum phase slip envisioned by Bardeen for charge density waves in 1979 [116]). Aside from the WC, various other phases have been suggested (such as Wigner glass [143, 144], hexatic [124, 145], and emulsion phases [127–129]) that may result from the interplay between electron-electron interactions and disorder. There is no clear consensus in the literature on how to distinguish such states experimentally.

These uncertainties highlight the need for ultra-high purity systems to be tested at lower temperatures. In particular, $p$-type GaAs systems have a relatively large effective mass and a substantial dielectric constant $\epsilon_r = 12.9$ which enhances the Coulomb interaction. The record lowest densities (in semiconductor systems) to date are measured in are undoped GaAs/AlGaAs (heterojunction gated field effect transistors or HIGFET’s) with charges capacitively induced by a metallic gate. Nearly perfect lattice matching between GaAs and AlGaAs produces very little strain at the interface, and the absence of doping impurities minimizes disorder.

For such systems, it is found that the MIT (indicated by a change of sign in $d\rho/dT$) occurs at a critical hole density $p_c$ such that the interaction parameter $r_s \approx 37$ is at the expected value for WC [9, 121] and $\rho$ extrapolates to the quantum resistance $h/e^2$ at zero $T$ [146]. For lower densities, $\rho$ exhibits a power law dependence indicating that disorder plays a negligible role [102, 147]. For the lowest temperatures, IV relations show a strong many-body pinning/depinning process in the insulating regime with a sharp orders-of-magnitude drop in the differential resistance for currents in the pA range [5] (Fig. 2.7). The onset of insulating behavior is also accompanied by a sign change of the low-field magnetoresistance

---

4 This was also confirmed by the introduction of long-range disorder which immediately produces activated temperature dependence in the same system [48].
Furthermore, $\rho$ exhibits a sharp upturn even in the “metallic” state for temperatures lower than 30 mK, which proves the efficacy of a novel cooling method used for these studies (see Sec. 3.2.6), and sheds new light on the MIT [72]. Taken together, these results constitute a strong case for the observation of a zero-field WC.

2.8 Two-Dimensional Charges in a Magnetic Field

The HIGFET samples required to study zero-field WC are extremely challenging to fabricate. Fortunately, there is an easier way to shrink the kinetic energy (effectively increasing $r_s$) in more traditional high mobility samples without needing to achieve such low densities: application of a magnetic field.

An in-plane magnetic field polarizes the electron spins and drastically modifies the resistivity of a device. This effect (Fig. 2.8) was awarded the nobel prize in physics (2007) and lead to the development of magnetoresistive random-access memory (MRAM) memory storage devices.

Application of a perpendicular magnetic field also quenches the kinetic energy in a 2D system by confining charges to cyclotron orbits and curtailing their linear momentum. Also, at certain field strengths (depending on the charge density of the system), the system exhibits exotic behaviors known as the integer and fractional quantum Hall states (IQH and FQH respectively). Most of the Hall states (excluding the even-denominator fractional states) can be described in terms of a single-particle Fermi liquid picture (although, this requires the concept of a fractionally charged quasi-particle for the fractional states).

At very large fields and at intermediate fields between the fractional and (recently) integer Hall states, the system transitions to insulating phases with $\rho_{xx} \gg h/e^2$. The transition to an insulating phase is quite easily identified experimentally, both by the large magne-
Figure 2.7: (a) DC IV measurements in zero field [5]. (b) Contour plot of the differential resistance (on a logarithmic scale) as a function of $V$ and $T$ [we ignore regions of negative differential resistance, and use the color-coded dashed line estimates instead]. $E$-field values result from assuming voltage drops linearly inside the sample.
Figure 2.8: Giant Magnetoresistance. Figure adapted from Ref. [6].
toresistance and the characteristic temperature dependence $\frac{d\rho}{dT} \ll 0$. Furthermore, the strength of these insulating phases appears to increase (become more resistive) in higher purity systems and to form at lower $\vec{B}$-fields in systems of higher $r_s$. Thus, there is a long-standing suspicion that these insulating phases, formed by quenching the kinetic energy in relatively pure systems, are a form of WC. However, the wavefunctions have been heavily modified by the large magnetic field so that the relationship between these states and the zero-field insulating phase is not yet understood.

2.8.1 The Quantum Hall Effect

The integer and fractional quantum Hall effects (IQHE and FQHE) were discovered in 1980 and 1982 by K. von Klitzing and D. C. Tsui respectively for which they were separately awarded Nobel prizes in physics. It is well known that application of a perpendicular magnetic field causes charges to follow cyclotron orbits. For a 2D system, the Hamiltonian is analogous to that of a simple harmonic oscillator, so that the DOS (which is constant in zero $\vec{B}$-field) splits into a series of sharply defined energy levels separated by $\hbar \omega_c$. At certain integer and fractional factors $\nu = n/n_B$ [where $n$ is the charge density and $n_B = eB/\hbar$ is the degeneracy of a landau level (LL)], both the longitudinal resistivity $\rho_{xx}$ and longitudinal conductivity $\sigma_{xx}$ of the bulk drop to zero while the hall resistivity $\rho_{xy}$ takes on perfectly quantized values of $\hbar/\nu e^2$ (see Fig. 2.9).

However, the edges of the sample exhibit perfect (dissipationless) conduction due to the absence of backscattering states on a given edge. Skipping cyclotron orbits is the simplest way to picture this (Fig. 2.10). Due to the broken time reversal symmetry, electrons can only travel in a clock-wise (or counter-clockwise depending on the sign of $B$) direction. Figure 2.10 illustrates the skipping of cyclotron orbits along the edges of a sample and resulting
edge conduction.

![Visualization of a Hall bar and skipping orbitals.](image)

Figure 2.9: (a) Integer quantum Hall effect (adapted from Ref. [7]). (b) Fractional quantum Hall effect (adapted from Ref. [8]).

To gain a deeper understanding of the integer effect, we consider the Landau Levels a bit more closely. The Landau level (LL) energy is given by

\[
E = \hbar \omega_c (n + \frac{1}{2})
\]  

(2.15)

where \( \omega_c = eB/m^* \) (and \( m^* \) is the effective mass).\(^5\) For low integer filling factors \( \nu = \)

\(^5\) Many sources state that the Zeeman splitting in GaAs is much smaller than the LL spacing and can be
$\gamma_\mu^*B$ is comparable to $\hbar\omega^*$ so that the LL have split into fully spin-polarized states. However, the energy gap for the even states can still be larger than that for odd $\nu$. Thus, odd fillings will tend to disappear before even fillings as the sample is heated.

When the charges are in a quantum Hall state, the Fermi level lies in an energy gap throughout the sample bulk (so that the bulk is an excellent insulator). But, close to the edges of the sample, the energy levels must bend upwards sharply due to the confinement potential of the vacuum, which causes them to cross the Fermi energy $E_F$ [Fig. 2.11(a)]. These Fermi level crossings give rise to 1D conduction channels with either positive or negative group velocity (the chirality or helicity of the channel depends on the spatial derivative of the energy surface and the direction of the magnetic field) which reproduces the dissipationless transport picture of the cyclotron orbits. Other factors (such as electron-electron interaction and spin-orbit coupling) can cause each of these LL crossings to split into multiple channels that can be either forward or backward moving. However, the net number of conduction channels with positive group velocity along a given edge is strictly governed by the bulk-boundary correspondence: it must be equal to the difference in Chern number across the boundary. 6

Having established the dissipationless uni-directional transport along an edge state, it is straightforward [77–79] to calculate that the Hall voltage should be quantized in frac-

\[E = \hbar\omega^*(n+\frac{1}{2}) \pm g\mu_B^*B\]

where $\mu_B^* = e\hbar/2m^*$ and $g \approx 0.44$ for GaAs (though it can be much larger for strongly interacting charges [148, 149]).

6 The Chern number characterizes the Berry curvature of Eigenenergy surfaces for solutions of a given Hamiltonian. Two gapped Hamiltonians $H_1$ and $H_2$ are said to be of different topological classes if $H_1$ cannot be adiabatically (slowly enough that $\hbar\omega$ is less than the energy gap of either Hamiltonian) transformed into $H_2$ without closing the energy gap at $E_F$ [Fig. 2.11(b)]. Otherwise, they are of the same class. The edge state reconstruction of a Hall state may be quite complicated [19, 20, 46, 80, 85–87, 93, 96, 97, 150–165], but the net number of forward moving edges is determined by the difference between the Chern number of the Bulk and that of the vacuum [38, 87].
Figure 2.11: (a) Visualization of the Landau level energy surfaces inside the sample. (b) Visualization of Fermi level crossing at the boundary of two topologies.

The calculation is done simply by considering the transmission and reflection coefficients at the contacts.

Two dimensional systems provide unique opportunities to test the edge-bulk correspondence. The first uses a Corbino disk geometry to force charges to propagate through the sample bulk between inner and outer contacts. Section 6.1 discusses our results using a variation of this method. The second is to use electrostatic gating to prevent certain channels from crossing a region of high energy. In this way, contacts can be coupled to specific edge states (regardless of the bulk behavior) allowing detailed knowledge of the inter-LL scattering [165, 166] among other things.

The application of a $B$-field is not necessary to produce a quantized Hall effect or to produce a topological insulator. Many other topical phases are presently the subject of intense research, especially for strongly interacting systems where the physics is less understood.

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7 The simple Landau level picture cannot explain the fractional states, which arise due to Coulomb interaction between charges. However, it was found that a gauge transformation of the interacting Hamiltonian allows the odd-denominator fractional states to be discussed in terms of fractionally charged non-interacting quasi-particles that are composed of charges coupled to quanta of magnetic flux.

8 For example, the quantum spin Hall effect occurs in the absence of a $B$-field.
2.8.2 Insulating Phases in a Magnetic Field

It is well established that large enough magnetic field renders a 2D system into a high field insulating phase (HFIP) near or below filling factor $\nu = 1/7$ in the fractional quantum Hall regime [9, 11, 12, 59, 63, 107, 109, 167] (Fig. 2.12). At more intermediate field strengths, several energies ($E_C$, $E_F$, $\hbar \omega_c$, $g\mu_B B$, etc.) compete for the ground state.

![Schematic phase diagram of a disorder-less 2D electron/hole system at $T = 0$. The $x$ axis is filling factor and the $y$ axis is the effective $r_s$. Several possible reentrant phase transitions around the principle FQHE states are illustrated. We have assumed that at $\nu = 1/7$, the ground state is the Wigner crystal. Figure and caption from Ref. [9].](image)

High purity samples exhibit transitions to reentrant insulating phases (RIP) between dominant fractional [11, 15, 109] and integer [12, 63, 115] filling factors. These are suspected to be a form of WC rather than disorder-driven insulator since they appear more strongly at larger filling factors in higher purity systems with increased $r_s$. However, the activated temperature dependence for AC measurement of these states (Fig. 2.13) appears to follow the Arrhenius hopping of an Anderson insulator [10–12, 63, 109, 111, 112]. The connection between various RIP’s is not yet firmly understood [15], and in light of the large modification
of the electron wavefunctions by magnetic field, the relationship between RIP and zero field WC is also an open question.

2.13 [10, 112]

![Figure 2.13](image)

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Figure 2.13: (a) Magnetotransport showing the reentrant insulating phase (RIP) before $\nu = 1/5$ in an $n$-type system. (b) Activated temperature dependence at the RIP shown by closed circles. Figure adapted from Ref. [10].

Several different experimental methods have been used to probe for WC and manybody physics in these RIP phases. The earliest is AC transport measurement of the High field insulating phase (HFIP) [107]. Subsequent increases in sample purity allowed the same method to discover reentrant insulating phases at 1/5 in $n$-type GaAs (Fig. 2.13) [10, 112], 1/3 in $p$-type GaAs [11] (where larger effective hole mass relative to $n$ type increases $r_s$) (Fig. 2.14), and in the integer regime for both silicon [110, 115] and GaAs systems [12, 63] (Fig. 2.15 below). In addition to the AC measurement by lock-in amplifier, these phases have been subjected to swept DC offset $V_{DC}$ [109, 111]. A small kink was observed in the differential resistance for the RIP near $\nu = 1/5$ when $V_{DC}$ reaches a critical value $V_C$ 2.14. This was
interpreted as a pinning threshold, beyond which, the state is a sliding WC. Spontaneous noise generation for $V_{DC} > V_C$ (much like the vibration of a violin string when the bow is drawn across) are offered as supporting evidence. However, the differential resistance only drops by a few percent at $V_C$. Similar results are also observed in the $\nu = 1/3$ state (Fig. 2.14) [11].

![Figure 2.14](image)

Figure 2.14: (a) Low field Hall resistance. (b) Low field Magnetoresistance. (c) High field insulating phase and discovery of RIP near $\nu = 1/3$ in a $p$-type sample. (d),(e) Differential resistance for the RIP. Changes are less than 1 order of magnitude. Figure adapted from Ref. [11].

Soon after that, Refs. [13, 110] (see Fig. 2.16) demonstrated much stronger pinning thresholds in zero field (silicon MOSFET system) and the RIP near $\nu = 1/5$ ($n$-type GaAs). In particular, Ref. [13] observes a drop of 3 orders of magnitude when the in-plane electric field reaches $E \sim 100$ mV/cm (depending on the filling factor $\nu$) in the HFIP and the RIP near 1/5. Using a classical estimate of the sheer modulus for WC, they estimate the a solid state with domain size of 250 particles.

To understand the de-pinning mechanism (especially for insulators in a $B$-field), it may
Figure 2.15: (a) Zero-field metal-insulator transition (MIT). (b) Magnetoresistance with RIP near $\nu = 1$. (c) Activated temperature dependence of RIP. (d) Energy gap extrapolated from slope of the temperature dependence. Figure adapted from Ref. [12].

be important to study the transverse response [143]. Several studies have investigated the Hall voltage in response to a DC IV measurement. Ref. [168] shows that the quantized Hall resistance appears to retain its classical value $B/ne$. This agrees with other studies using magnetic field reversal to eliminate mixing between $\rho_{xx}$ and $\rho_{xy}$ from the AC measurements [169].

In all these non-linearity studies, the sample generates current dependent noise when pushed beyond the pinning threshold. This is often interpreted as evidence for vibrational modes of a sliding WC.

In addition to the transport and noise measurements, the absorption of microwave noise is a key feature of these insulating phases. The resonant peaks (Fig. 2.17) disappear at a temperature $T$ that depends on the filling factor $\nu$ rather than $B$ or $n$ individually [14]. However, the shape of the resonances do not provide conclusive evidence for WC.
Figure 2.16: (a) Magnetoresistance for Hall measurement. (b) \( B \)-field dependence of threshold IV characteristics at fields \( B = 15, 17.25, 18.2, 19.25, \) and 22 Tesla. (c) Temperature dependence of Threshold IV at \( T = 40, 104, 132, \) and 192 mK . (d) Absorption of high frequency signals indicates resonant modes. Figure adapted from Ref. [13].

Finally, recent results [15] show a puzzling disconnect between the RIP and HFIP (Fig. 2.18) indicating that the phase diagram is more complicated than previously thought [9]. Further study is required to understand the nature of these insulating states and to know their pinning/depinning and transport mechanisms.

2.9 Introduction to Quantum Capacitance

The energy cost for capacitively charging a system can be represented as an effective capacitance \( c_{eff} = Q/V \). For a good metal, the DOS is practically infinite, so that the charging cost is determined purely geometrically, and so that the effective capacitance \( c_{eff} = c_{geo} \). For other systems, many factors can produce significant modifications to the charging energy in non-metallic systems. As first discussed by Luryi in 1988 [170], this effect can be represented by a parallel term (called the “quantum” capacitance \( c_q \)) so that \( c_{eff}^{-1} = c_{geo}^{-1} + c_q^{-1} \).

If \( c_{geo} \) is known and \( c_{eff} \) is measured, one can calculate \( c_q \) (to an accuracy that improves with increasing \( c_{geo}/c_q \)). This directly yields the density of states \( g(E_F) = (\text{const.}) \times c_q \). Furthermore, if the charge carrier density \( n \) is known, one can calculate the electronic
Figure 2.17: Resonant absorption of microwaves by 2D holes from a coplanar waveguide. Figure adapted from Ref. [14].

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Figure 2.18: Experimental phase diagram for the high field insulating phase (HFIP) and RIP near $\nu = 1/3$. The figure shows a curious disappearance of the RIP at large $r_s$ leading to a disconnect between the RIP and HFIP. [15].
compressibility \( \kappa = c_q/n^2 \). We allow \( c_q \) to be either positive or negative, and note that \( c_q > 0 \) reduces and \( c_q < 0 \) increases the device capacitance relative to \( c_{geo} \). Both correspond to observable physical scenarios [17, 18, 24, 170–173]. For example, capacitively charging systems with a small density of states dopes the system (moves the Fermi-level relative to the band structure), and this floating of \( E_F \) reduces \( c_{eff} \) (Fig. 2.19) and corresponds to \( c_q > 0 \) [24]. On the opposite end of the spectrum, Coulomb interaction can produce a negative electronic compressibility due to over-screening of an applied electric field [16, 174]. This corresponds to a negative \( c_q \) (Fig. 2.20 below) so that \( c_{eff} > c_{geo} \).

![Figure 2.19: (a) Parallel plate capacitor formed by a metal plate [having large density of states (DOS)] and semiconductor 2D sample (small DOS) at zero bias. (b),(c) Applied bias essentially dopes the system by shifting the chemical potential in such a way that the total amount of charge stored on the device is reduced (relative to the applied bias). This lowers the effective device capacitance.](image)

Note that \( c_{eff} \) is dominated by the smaller of the two capacitances \( c_{geo} \) and \( c_q \). Thus, if \( c_{geo} \ll c_q \), \( c_q \) has little effect on the measured capacitance \( c_{eff} \). On the other hand, for \( c_{geo} \geq c_q \), the non-geometric factors can significantly effect the measured \( c_{eff} \). Note that we have intentionally used lowercase \( c_{geo} \) and \( c_q \) to represent *intrinsic* quantities (normalized per unit area). The extrinsic properties \( C_{geo} \) and \( C_q \) both scale proportionally to the (effective)
device area $A_{eff}$ so that the ratio $C_q/C_{geo} = c_q/c_{geo}$ does not improve with larger sample size.

Scaling the area does, however, provide one important advantage: it increases the measured sample capacitance $C_{eff}$ (an extrinsic property). It can be difficult to measure small capacitors with high accuracy (especially at low frequencies where the impedance $Z \sim 1/\omega C$ is large). Devices of larger area can be easier to measure.

However, if one wishes to probe a 1D sample or to spatially resolve electronic structure on the size of a single crystal domain, it may be impossible to increase the sample area. For example, little can be done to increase the surface area of a carbon nanotube \cite{18}. Instead, we require thinner dielectrics to increase $c_{geo}$.

Additionally, we note that when the DOS drops, the sample resistance may increase by orders of magnitude. This will affect the charging time of the circuit \cite{175}. In order to decouple the resistance changes from the capacitance changes, it is necessary to measure at low frequencies (low currents) where the in-plane voltage drop is negligible relative to the probe-to-sample (or gate-to-sample) voltage \cite{24}.

From an experimental standpoint, the above considerations lead us to the following challenges for capacitive probing of the non-geometric contributions to the charging energy of an electronic device:

1. In order to measure changes in the sample capacitance close to a metal-insulator transition, we must use low frequencies.

2. Typically, we want to use small-area probes for good spatial resolution. This means that the sample impedance will be very large and often quite difficult to measure. The
device size must be large enough that $c_{eff}A$ is a measurable quantity.

3. We require that $c_{geo} \geq c_q$. This is essentially a requirement for high quality thin dielectrics (characterized by $\epsilon/d$ where $\epsilon = \epsilon_r\epsilon_0$ is the dielectric constant and $d$ is the thickness of the insulating layer), which is a classically challenging problem.

In the following paragraphs, we look at results from several groups that have managed to overcome such difficulties in certain situations. We also look at several similar techniques used to perform spatially resolved scanning measurements of the electronic structure. Our own progress towards a spatially resolved quantum capacitance setup is presented in Chap. 4.

2.10 Quantum Capacitance and Negative Compressibility of Strongly Interacting Systems

After Luryi’s discussion in 1988 [170], Jim Eisenstein used several ingenious experimental methods to demonstrate the effects of electron-electron interaction producing a negative electronic compressibility [Fig. 2.20(a)] [16, 174].

Figure 2.20: A 2D system can show either positive or negative electronic compressibility (the vertical scale is in arbitrary units). Figure adapted from Refs. [16, 17].
Ashoori’s group later used very thin dielectrics to demonstrate that the electron-electron interaction can produce up to a 40% increase in the device capacitance [Fig. 2.20(b)] [17]. The McCueen group also performed a fascinating measurement demonstrating the DOS for a 1D system using a gated carbon nanotube (Fig. 2.21) [18]. Various other measurements on 2D systems were also performed by other groups [24, 171, 173, 176].

Figure 2.21: Quantum capacitance measurement of a carbon nanotube. Figure adapted from Ref. [18].

These measurements provide excellent insight into the electronic structure of certain
systems. They do not, however, give us a spatially resolved image or any way to asses the uniformity of these systems. For that, we turn to scanning measurements in the following section.

2.11 Scanning Techniques Using Capacitive and Other Probes

There are many methods available for scanning of electronic states across a sample. Here, we highlight several that are particularly relevant to our measurement:

Scanning tunneling microscopy (STM) is a well known and ubiquitous technique. This can achieve spacial resolution down to the atomic scale, and also give a look at the density of states (Fig. 2.22) [19]. Though the method has many advantages, there are also several drawbacks. First, the tunneling is only suitable for probing surface states, and cannot access the charges in a 2D quantum well embeded in a semiconductor wafer. Second, the method is best suited for metallic samples. Third, the tunneling depends non-linearly on properties of the probe tip, and it can be difficult to accurately de-convolute the tip behavior from the final results.

Figure 2.22: (a) Scanning tunneling microscopy (STM) image of graphene with atomic resolution (inset). (b) Density of states obtained from tunneling measurement with swept tip-sample DC bias. Figure adapted from Ref. [19].
Microwave reflection is another method that has been recently applied for imaging the edge states of a topological insulator [20]. To achieve spatial resolution, a waveguide and sensor for reflected light are mounted on a scanning probe head [Fig. 2.23(a)]. The resistance of the sample beneath the probe is calculated from the magnitude and phase shift of the reflected signal [Fig. 2.23(b) shows the calibration curves]. The scanning measurements are performed at various filling factors in the integer quantum Hall regime [Fig. 2.23(c)]. The resulting map of the resistivity is given in [Fig. 2.23(d)]. The high measurement frequency of this measurement allows excellent spatial resolution. However, it cannot be used on top-gated samples, such as the HIGFET devices described above in Sec. 2.7, where it would be interesting to try to observe the formation of a WC lattice.

Figure 2.23: (a) Schematic of the microwave reflectivity measurement setup. (b) Calibration for converting reflectivity to longitudinal resistivity. (c) Hall measurement of a graphene sample. (d) Scanned image of the resistivity showing the conductive edge states. Figure adapted from Ref. [20].
A point-like gate can be used to modify the local charge density in a 2D sample [Fig. 2.24(a)] [21]. This is useful for scanning the current distribution of charges flowing through a narrow path such as a quantum point contact [Fig. 2.24(b)]. Changes in the conductivity are measured as the tip is scanned over the region. This gives a detailed map of the electrical current showing quantum interference patterns. If the charges are embedded in a 2D well distant $d$ beneath the surface, resolution is limited to the length scale of $d$. Also, this technique will not work for top-gated samples, because the metallic layer would shield the 2D layer from the electrostatic field. Presumably, a similar technique utilizing a magnetized tip could be used to measure the spin current in the quantum spin Hall effect (QHSE) or in spintronics devices.

Figure 2.24: (a) Scanning point-like gate probe. (b) Image of the current density. Figure adapted from Ref. [21].

The final method we want to highlight is an AC coupled capacitive probe using a scanning atomic force microscope (AFM) tip. This is a method used by several groups (the Tessmer group and Goldhaber-Gordon groups) for achieving spatial resolution of the electronic density of states or resistivity in 2D wells beneath the surface. The technique can also be used to study the effects of local dopants on the density of states [22].

Chapter 4 reports our progress towards achieving a scanning capacitance probe similar
to the one described by the Tessmer group. It turns out that there are many technical challenges for this project.

2.12 2D Crystal and Melting

Mermin Wagner theory may not appli
CHAPTER 3 EXPERIMENTAL METHODS

This chapter summarizes the experimental techniques used in this work.

3.1 Sample Fabrication

This dissertation deals with four types of samples: narrow (20 nm) AlGaAs/GaAs/AlGaAs quantum square wells, GaAs/AlGaAs heterojunctions, exfoliated graphene, and VO$_2$ thin-films. We fabricate these samples in a class 100 cleanroom using photolithography as described in the GaAs subsection below.

3.1.1 GaAs-AlGaAs Molecular Beam Epitaxy

Most of this work is performed in two-dimensional systems formed by GaAs-AlGaAs square wells grown by Lauren Pfeiffer and Ken West at Princeton University using molecular beam epitaxy (MBE). GaAs has a zinc blende structure with a lattice constant of 5.65 Å [177, 178] and a direct band-gap of 1.52eV at room temperature 3.1 [23]. Substitution doping of Ga by Al has almost no effect on the lattice constant [177, 178], but widens the band gap (see Fig. 3.1) [23]. Thus, atomically smooth layered structures with minimal strain or disorder can be grown using molecular beam epitaxy. These materials are ideal for building quantum wells and super-lattice structures with minimal disorder that confine charges in the direction of crystal growth (the z-direction).

In particular, our system consists of a 20 nm wide quantum square well. Two layers of carbon dopants (delta-doping at a distance of 120 nm symmetrically above and below the well) serve as acceptors. These trap excess charges at low temperatures that create an electric field and associated band bending to move the Fermi energy into the valence band of the narrow GaAs layer. The well is then sparsely populated by holes [Fig. 3.2(a)].

Mobile charges in a GaAs 2D well can move anywhere perpendicular to the wafer growth
Figure 3.1: Band gap of $\text{Al}_x\text{Ga}_{1-x}\text{As}$ as a function of the molar fraction $x$ of Aluminum at room temperature (adapted from Ref. [23]).
Figure 3.2: (a) Current flow through middle of long hall bar is uniform and linear. (b) Band diagram for our \( p \)-type \( \text{AlGaAs/GaAs/AlGaAs} \) quantum well.
direction. To be useful for experiment, these mobile charges must be further restricted by etching patterns into the wafer to a depth beneath the 2D surface. In fact, even if one etches close to the well, a depletion region forms (presumably due to the electric field of trapped surface charges and strain left by the etching process [177, 178]) that prevents charges from moving outside of the desired geometry. Patterns are chosen to maximize the amount of control we have over the electrons, allowing us to stress them in various ways and observe the responses as accurately as possible. For example, a long Hall bar with current leads at the ends and voltage leads near the center [Fig. 3.2(b)] produces uniform current distribution for accurate Hall measurement and allows charges to transport along the edges of the sample [74, 177, 178]. On the other hand, a Corbino disk forces charges to travel through the bulk of the sample.

In addition to the etching process (a one-time procedure), metal gates can be used to dynamically modify the sample geometry through an applied electric field. For example, quantum point contacts use gates to pinch or squeeze the current path through a Hall bar so that the edge states from opposite sides come close enough to interact.

After layout of the sample has been chosen, the sample is fabricated using photolithographic techniques in a class 100 cleanroom (Figs. 3.3 and 3.4). The procedure for creating a Hall bar is as follows:

1. Spin photoresist (PR) onto surface of sample (∼4000 rpm),

2. Soft bake (95 °C for 95 sec),

3. Mask align and UV exposure (∼12s),

4. Develop pattern (1min),


Figure 3.3: (a) Karl-Suss MJB3 mask aligner for exposing the photoresist to patterned UV light. (b) Chemical hoods for solvents (left) and acids (right). (c) Thermal annealing chamber for making Ohmic contact. The white stripe is a silica coated heating element upon which the sample is placed. (d) Kurt J. Lesker Nano36 thermal deposition chamber. (e) Nilfisk vacuum with hepa filter.
Figure 3.4: (a) Compressed air system for providing clean dry air (CDA) required by the MJB3 mask aligner and the Nano36. (b) Process gass (dry nitrogen and forming gas) used for nitrogen guns, MJB3, and thermal annealer. Also shown are the sealed feedthroughs where the gas enters the cleanroom. (c) Water filtration system for providing 1MΩ, 17MΩ, and 18MΩ deionized (DI) water inside the acid hood. (d) Conditions monitoring system showing room temperature, humidity, and pressure relative to the building. (e) Water filtration and boiler for introducing moisture to the cleanroom air supply on dry winter days. Air enters the cleanroom through hepa filters in the ceiling of the process chamber.
5. Remove oxide using NaOH solution (20s),

6. Etch Hallbar shape using SO$_4$ solution (~1 min),

7. Repeat steps 1-5 to open windows for metal contacts,

8. Thermal deposition of metal contacts in vacuum chamber at $10^{-7}$ torr, and

9. Anneal metal contacts (460 °C for 1 min).

More details about the fabrication process can be found in my lab partner’s dissertation (Zhe Wu, 2017). There are also a variety of helpful and comprehensive books about modern processing methods for GaAs [177, 178].

It is important to emphasize the value of the final step and the importance of having Ohmic contacts. If the metal contacts have a non-linear transfer characteristic, it can completely obscure the transport measurement of charges inside the 2D well. Annealing allows the deposited material to diffuse into the surrounding GaAs. This eliminates the Schottkey barrier formed at a metal-semi-conductor junction by providing sufficient impurity states or local doping (in the vicinity of the contacts) to allow charges to pass between the metal charge reservoirs and the quantum well with little resistance.

To measure the quantum Hall effect in a pristine system requires that the annealed contacts access edge states on the outer edge of the sample. This is due to the fascinating property of these topological insulating states: they can only conduct on the edges of the sample. Contacts interior to the Hall bar do not have access to the edge states. Furthermore, as discussed above in the section about quantum Hall measurement 2.8.1, various gating methods can be used to restrict the edge states that each contact can access. Contact
placement and gating provide another level of control over the experiment that must be carefully considered during the fabrication.

Similar considerations apply for performing lithography on graphene. Vanadium dioxide reacts with many chemicals used in this process and must be handled carefully. We found that the AZ series photoresist reacts with VO$_2$ film, but that the Shipley series does not. The photoresist developer is often enough to etch the VO$_2$ film. However, there are better methods for doing this. The lithography of VO$_2$ films would be better discussed by with Zhe Wue or with students from MSU where the film was grown [24, 179].

3.1.2 Exfoliated Graphene

We also study the Hall effect in exfoliated graphene. This material is promising for many applications due to its high room-temperature mobility, flexibility, transparency, etc. and interesting for research purposes due to the mass-less dispersion relation.

By now, probably even elementary school children know that graphene can be made using the “Scotch tape method.” Layers of graphite are peeled by repeatedly sticking/unsticking pieces of tape until single-layer graphene is abundant. The tape is pressed onto some substrate (we use heavily doped silicon with 285 nm oxide on the surface) where some of the pieces will stick. Contacts are then made to the graphene using photolithography as outlined in the following section. Figure 3.5 shows an exfoliated graphene sample with gold contacts. The sample is approximately 40x100 µm.

3.1.3 Vanadium Dioxide

Finally, we also work with vanadium dioxide thin films and nano-beams. One of the challenges for making VO$_2$ is that many other vanadium oxides are also stable, and the growth parameters must be strictly controlled to get the right product. We experimented
Figure 3.5: Exfoliated graphene sample with gold contacts.
with growing films directly into a Hallbar pattern or other useful shape (skipping the photolithography steps), but found that the surface temperature of the SiO$_2$ substrate was too difficult to control using our method. Thus, we used photolithography (described in the GaAs section above) to define the shapes of our samples. We photolithographic processes similar to the above described method with one notable difference: we found that the AZ developer used for GaAs reacts with VO$_2$, so we used the Shipley developer instead.

![Figure 3.6](image_url)

Figure 3.6: (a) Cross-section of a vanadium dioxide (VO$_2$) thin film sample sample. (b) Optical image of VO$_2$ thin film with two gold wires attached using silver paint. Figures taken from Ref. [24].

The films are grown on silicon-dioxide/p-doped silicon substrate [Fig. 3.6(a)] using pulsed laser deposition [24]. A metallic vanadium target is ablated using a KrF excimer laser (LambdaPhysik LPX 200, $\lambda = 248$ nm) at 10 Hz repetition rate and 350 mJ pulse energy, while the temperature at the substrate is held constant at 595 °C. For conditions similar to those of Ref. [179], the deposition time is 10 min. To make Ohmic contacts to the VO2, it is sufficient to glue gold wire to the film using silver paint [Fig. 3.6(b)]. A layer of SiO$_2$ dielectric (thickness $d = 285$ nm) separates the film from a heavily boron doped silicon substrate (with a low resistivity 15 mΩ cm) that serves as a back gate. Indium contacts are soldered to the gate at 450 °F, yielding a contact resistance of approximately 10 Ω. The area of the VO$_2$ film is roughly 6 mm$^2$, so we expect to measure a geometric capacitance $C_{geo} = \epsilon A/d \approx 700$ pF in the metallic regime (the SiO$_2$ dielectric constant is $\epsilon = 3.7\epsilon_0$). Both
transport and capacitance measurements are performed at high vacuum (≈ 10 − 5 mbar) in a chamber isolated from vibration. Resistive heaters are used to drive T variation of the thermal reservoir at rates of 0.10.3 K/min with resolution 0.01 K. Excellent thermal contact between the substrate and thermal reservoir eliminates T lag at such low ramp rates.

3.2 Cryogenics

Our GaAs samples have a Fermi temperature (TF ∼ 3.1 K) much lower than room temperature and must be cooled to observe the quantum effects. Generally, we perform a quick test in a physical properties measurement system (PPMS), a helium dewar (using a dipping stick), or a flow cryostat to determine whether the contacts remain Ohmic at low temperature. If everything checks out, we can use a closed cycle dilution refrigerator from Leiden cryogenics to achieve a base temperature of 10mK. To actually cool the 2D electrons down to such a low temperature is highly challenging because the phonon-electron coupling freezes out below about 100mK. Thus, to effectively cool the sample requires excellent heat-sinking of the leads. We discuss these methods in the following subsections.

3.2.1 Sample Mounting

The samples are mounted by hand using 1mil gold wire and silver paint (Fig. 3.7). The wires can be cut to length using a razor blade or pair of scissors. The silver paint must be vigorously shaken to ensure that the nanoparticles are uniformly distributed in the solution. If the paint has been sitting for a while, it may be necessary to apply a bit of paint thinner and to use an ultrasonic bath to re-suspend the colloid. A drop of silver paint is placed on a glass slide. The cut wires are picked up by precision tweezers, dipped in the silver paint, and applied to the sample. It is often helpful to place one large drop and one small drop on the slide next to each other. The small drop dries more quickly, and will get stickier.
By dipping a gold wire back and forth between the two drops, it is possible to adjust the liquidity of the accumulated silver paint at the tip of the wire. Also, if the glass slide is too far from the sample, the paint will dry as the wire is carried from the slide to the vacuum chuck that holds the sample in place while the wires are applied. It is sometimes necessary to hold the silver paint very close to the sample, so that minimal motion is required between dipping the wire and placing it on the sample.

After the long gold leads are attached to the contacts, the vacuum chuck is released, and the sample is transferred to a holder for cooling. The dangling ends of each gold wire are attached to a gold pin using either indium or silver paint. The pin is wired to a feed-through to the outside of the cryostat.

After the sample is successfully wired, it is important to check the dry silver paint left on the glass slide for conductivity. If the colloid was properly suspended, the silver paint will dry to a hard conductive substance. If not, it will form a gummy insulating substance. It is also important to minimize air time of the contacts to avoid pollution, so we immediately place the sample under vacuum after wiring it up.

3.2.2 Helium Dewar Dipping Stick

Figure 3.8 shows a homemade dipping stick that provides a quick method for preliminary testing of samples in a helium dewar. A long stainless steel dipping tube has a piece of g10 fiberglass board inside the tip for sample mounting. Samples are attached to the board using vacuum grease, and wired to several pins that have been epoxied to the board. Thin copper wires attach the pins to a feedthrough at the top of the probe. The wires have enough extra length that the g-10 board can be removed from the end of the tube for mounting the sample, without unsoldering them. After wiring, the board is re-inserted, and held in place
Figure 3.7: Mounting station for thermally anchoring the sample to a holder and electrically attaching the leads. On the table are several precision tweezers, glass slides, gold wire, and silver paint used for the mounting process.

be friction. The tube also has a pressure release valve for safety, and a flushing port. A carbon resistor has been sanded flat and attached to the back side of the g-10 board using GE varnish. The resistor serves as a thermometer. It is approximately calibrated using an exponential curve and three temperature points (Room temperature, 77K, and 4.2K). By slowly adjusting the height of the dipping stick as it is inserted, we can cool the sample at a controlled rate.

3.2.3 Flow Cryostat

We utilize a Cryo Industries optical flow cryostat (Fig. 3.9) and a Lakeshore temperature controller that together provide a range of operation from 3K to 380K. Cryogenic liquids (helium or nitrogen) can be flushed through the cold finger for cooling, and a small heater with PID control loop provides heating. The cryostat has 16 uniaxial copper wires with formvar coating. It also has four RG178 coaxial lines with Lemo ffa.00.250 feedthrough
Figure 3.8: (a) Wire feedthrough and safety valve. (b) Sliding shield and probe cap. The valve and orange plastic fitting is for venting the stick. A cap keeps the sample in a low moisture environment. (c) G10 board for holding the sample. (d) thermometer epoxied on to the g10 board.
connectors. The coaxial lines can be used for capacitance measurement.

### 3.2.4 Dilution Refrigerator

To reach the lowest temperatures, we use a Dilution Refrigerator from Leiden Cryogenics. Figure 3.10 shows a diagram of the system, and pictures are given in Fig. 3.11. The refrigerator has several cooling stages which all use different principles of operation. First the stages are pre-cooled by a pulse tube down to 3K. Then, gaseous helium-3 and helium-4 are liquefied inside the fridge. Pumping on the circulation lines continues to lower the temperature as the evaporating helium draws heat from inside the fridge. Eventually a phase separation occurs between helium-3 and helium-4 so that two solutions appear, one with high He-3 concentration and the other with low He-3 concentration (Fig. 3.12). Cooling power at the lowest temperatures is the result of the endothermic process that occurs as the lighter He-3 rich solution is pumped through the heavier He-4 rich solution that has a lower vapor pressure [25].

### 3.2.5 Heat Sinking

Figures 3.13(a) and (b) show sample holders used for cooling in our dilution refrigerator. Panel 3.13(c) compares the cooling results of the two holders.

The holder in Fig. 3.13(a) is about an inch in diameter and two inches tall. It is screwed on to the cold finger of a top-loading probe for cold insertion to the fridge (top-right of the photograph). Two samples sit in the center, and are wired to two rows of gold pins. The gold pins are gold plated single in-line package (SIP) header pins with beryllium copper springs for contact. Beneath the holder, two rows of SIP pin headers are plugged into the sample holder. Strands of very fine copper wire insulated by a thin layer of formvar are inserted into the female side of the SIP header and soldered into place. These are wrapped around the
Figure 3.9: Cryoindustries flow cryostat. Optical ports at the bottom have been sealed with conductive tape for better electrical shielding.
Figure 3.10: Diagram of closed cycle dilution refrigerator from Leiden Cryogenics.

cold finger and secured using vacuum grease and Teflon tape for cooling. Remaining lengths of loose wire are tied down using dental floss to prevent them from catching on the fridge and breaking (the wires will be brittle at low temperature) while the probe is inserted. A red LED provides light to the sample. At low temperature, the electrons are frozen into a configuration that may be disordered. Photons from the LED can provide a bit of energy that allows the frozen system to change its configuration. It is hoped that light will allow the system to relax from a disordered state, and that the electron mobility inside the 2D well will increase. However, if the lighting is non-uniform or is too intense, it can actually increase the level of disorder (especially for ultrapure systems).

Both holders cool the sample through the contact with the back of the device while the sample is under vacuum. If the sample is being tested at 4K (a quick test to see if the leads annealed properly), it is sufficient to press the sample onto the holder with a bit of
Figure 3.11: Dilution Refrigerator from Leiden Cryogenics at Wayne State University.
Figure 3.12: Phase diagram for $^3$He-$^4$He mixture taken from Ref. [25].
Dow Corning silicone vacuum grease or Apiezon low temperature vacuum grease. For a real measurement at base temperature, the sample is glued to the holder using silver paint. However, this cooling method suffers from an inherent challenge: the freezing out of electron-phonon coupling below 100 mK. This is a well known feature of GaAs systems which limits the cooling power at low temperature.

Dotted lines in Fig. 3.13(c) show 3 resistance traces at different densities for a single sample in a single cooling cycle using the holder in panel (a). The low-temperature saturation effect is an artifact of the inefficient cooling method. Though the fridge thermometer (which is thermally anchored to the mixing chamber plate) reaches a base temperature of ≈ 20 mK, the sample stops cooling at ≥ 100 mK.

Figure 3.13(b) provides a much improved cooling method. Note that the wires have been buried in a long groove in the sample holder and buried in high quality silver paint. The paint provides a larger surface area for thermal contact between the wires and the sample holder so that electrons injected into the quantum well are pre-cooled. We can also see the result of this method from the solid lines in Fig. 3.13(c). Notice that the re-measured red and blue traces are continue to exhibit changes down the low temperature. Some slight flattening of the curves appears when the fridge temperature is about 30mK, and a naive interpretation would indicate that 30mK is our new base temperature.

However, we know that the temperature of the 2D charges cannot be so low. In fact, it is well known that the MIT turnover (the maximum in the blue trace) should appear at 100-200mK. In our graph, it does not appear until the fridge temperature reaches 45-50mK. Thus, we estimate that the actual sample temperature only reaches about 100mK with this improved cooling method (and that it was much higher when cooled using the previous
method). In the following section, we will discuss a way to remove the insulating formvar barrier and to greatly increase the thermal contact area for each wire.

Figure 3.13: (a,b) Sample holder for dilution refrigerator cooling through contact with the back of the sample. The wires on holder (b) are embedded in silver paint for an improved thermal link to the cold finger. (c) Comparison of the cooling capability of holders (a) and (b).

### 3.2.6 Helium-3 Cell

Effective cooling of the sample depends critically on the thermalisation of the 2D holes with the mixing chamber (MC) plate. To achieve this, we utilize a vacuum-tight, liquid helium-3 sample-immersion cell mounted at the lower end of a cold finger (made of pure silver) with its top end fastened to the MC plate (Fig. 3.14). The cell has a volume of 2.5 cm$^3$ and is fabricated with polyethylenimine (PEI) material. The roof is a sintered silver cylindrical block extension of the cold finger made by compressed pure silver microparticles that are perfectly thermalized with the cold finger. During operation, helium-3 gas is continuously fed through a capillary into the cell where it condenses until the volume is completely filled and the sintered silver block is saturated. The sintered silver roof provides more than 30 m$^2$ contact area for cooling the helium-3 bath which provides a thermal link.
to the immersed sample with minimal Kapitza resistance [note that the Kaptiza resistance with liquid helium-4/helium-3 mixtures is higher, and it is usually difficult to cool the sample below 30 mK even though the bath $T$ is lower]. Direct cooling of the 2D holes is achieved via efficient heat-sinking of the metal contacts through sintered silver pillars (wrapped around each silver wire inside the cell) providing roughly 2 m$^2$ surface area per lead. The large surface area of these pillars also provides excellent filtering of microwave noise. A thermometer inside the cell verifies that the temperature differential between the bath and the MC is less than 0.1mK at all times, and effective cooling of the 2D holes below 30mK has been demonstrated [72].

We have also designed and constructed a separate $^3$He gas handling system for supplying gas to the cell. The tubing and valves are all HiP high pressure components. We use very narrow tubes so that the $^3$He must fill only minimal volume. A cold trap catches impurities. Ports for flushing gas and a turbo pump allow us to clean out the system. A scroll pump allows us to recover the gas from the cell and put it back into the tank. The diagram for this system is shown in Fig. 3.15, and pictures of the system are displayed in Fig. 3.16.

3.3 Electronics

To determine the state of electrons inside the quantum well, we probe the electron response to various external stimuli: temperature reduction, electric fields, magnetic fields, etc. Since the motion of electrons is an electrical current, we do this by performing sensitive electrical measurements. Here I describe the various techniques we use to study the electrons’b quantum transport properties.
Figure 3.14: Helium-3 Cell used for optimum sample cooling. An optical image of the sintered silver pillars is shown. The capillary used for transferring helium to/from the cell is not drawn.
Figure 3.15: Diagram of the $^3$He gas handling system.

Figure 3.16: Front and back of the $^3$He gas handling system.
3.3.1 AC Measurement by Lock-in Amplifier

Noise is present in every electrical measurement. It comes from a variety of sources: the 60Hz noise of our electrical grid is present everywhere, and Wifi, Bluetooth, and cellular devices produce noise at much higher frequencies. Additionally, many electrical components have an intrinsic noise level that cannot be avoided (such as Johnson noise in a resistor and Shot noise from the random motion of electrons inside a material). Some types of noise can be useful. For example, shot noise is proportional to the magnitude of the charge on an individual charge carrier, and noise measurements have been used to identify quasi-particles that effectively carry a fractional unit of charge (relative to that of a single electron). Such particles may be useful for quantum computing. Also, under certain conditions, applying a constant (DC) driving force to an electron system produces AC noise at various harmonic frequencies. This is analogous to the vibration of a violin string when a bow is drawn across. Such noise is important for identifying electronic phase transitions, because the noise indicates the presence of pinning and restorative forces which compete with the driving electric field to excite vibrational modes. Such modes exist in an electron solid (and, perhaps, in a strongly-correlated electron liquid) but not in an electron gas.

An experimental setup (test fixture and electrical instruments) must be chosen with an appropriate bandwidth for the type of measurement to be performed. For the measurements in this thesis, we are generally interested in the low frequency response. Thus, we can eliminate high frequency noise through the use of low-pass filters. Furthermore, we often want to drive the sample with an AC signal and to measure the response only at that frequency. This is precisely the intended role of a lock-in amplifier.

A lock-in amplifier produces a high quality sine (or square) wave voltage output that
is used to apply a driving force to the charges (electrons or holes) in a sample (output impedance for most instruments is typically about 600 Ω). At low frequencies, this voltage output can also be converted to an effective current source by placing a large ballast resistor in series with the sample (this does not work at higher frequencies because the large impedance lengthens the response time of the circuit and limits the bandwidth). The amplifier input senses the sample response at the driving frequency using either single or differential voltage modes or using current sensing mode and virtual ground (the difference between these two modes has to do with the type of feedback of the amplifier at the sensing terminal). The sensing terminal can be either DC or AC coupled. As described below, any DC offset will be subtracted from the final output (reading) of the instrument, but AC coupling is useful for eliminating large DC offsets without overloading the sensing circuitry, and the SR830 can withstand up to 50V offset in AC coupled mode.

The lock-in amplifier senses the sample response only at the driving frequency by using a multiplexer and a long-time averaging. To get a rough mathematical idea of how this will work, consider the orthogonality relationship used to project out a single frequency component in Fourier analysis:

$$\frac{1}{T} \int_0^T \exp\left(\frac{2\pi}{T} (n) t\right) \exp\left(\frac{2\pi}{T} (m) t\right) dt = \delta_{nm}$$  \hspace{1cm} (3.1)$$

For a lock-in amplifier, we replace the factor $\exp\left(\frac{2\pi}{T} (m) t\right)$ by a square wave modulation signal $f_m(t)$ that switches between ±1 at the driving frequency of the lock-in, and we replace the integration interval $T$ by an averaging time $\tau$. The orthogonality relation is no longer exact in this case. However, for large averaging times, the non-zero contributions for many oscillation
periods will tend to cancel out for all frequencies except the modulation frequency $\omega_m$. To state this as an equation, we say that as $\tau \to \infty$

$$
\frac{1}{\tau} \int_0^T \exp \left( \frac{2\pi}{T} n t \right) \exp \left( \frac{2\pi}{T} m t \right) dt \to \delta_{nm}.
$$

To see why this is the case, let $T_m \ll \tau$ be the time period of the modulation signal, and $T_n$ be the time period for any Fourier component of the noise in the circuit. The integral from 0 to $\tau$ has many integrals over the time period $T_m$. On average, the sum of these integrals cannot be large because of the phase shifting of the two signals caused by the period difference $T_m - T_n$ (which changes the integral from positive to negative or vice versa in a repeating pattern).

A natural question to ask is, How low long should the averaging time be (in terms of the ratio $\tau/T_m$) to eliminate noise and produce a clean signal? It depends on the type of noise in the circuit. The cancellation at frequencies close to $\omega_m$ takes a long time because the phase shift consecutive time integrals over $T_m$ is small, and many consecutive integrals will add constructively, rather than destructively. Harmonics of the modulation frequency are difficult to cancel for the same reason (due to an effect similar to aliasing). In most experimental setups, 60Hz noise is the dominant factor. Thus, we generally measure at frequencies such as 7 Hz or 13 Hz that do not have 60 Hz as a harmonic. This allows better noise performance with shorter averaging times. A general rule of thumb is to keep $\tau/T_m \geq 7$. However, it takes about 6 time constants to settle on a new value, so $\tau$ must also be small enough to allow a reasonable measurement time.

Figure 3.17 shows a block diagram schematic of a lock-in amplifier. The measured signal
is multiplied by a reference signal of ±1 and then time-averaged using \(RC\) filters. The result is a phase-sensitive measurement of the \(rms\) value of the input signal at the frequency of the reference signal.

![Lock-in amplifier schematic](image)

Figure 3.17: Lock-in amplifier schematic. Figure adapted from *The Art of Electronics* by Hook and Hall.

The lock-in is, in some sense, a very narrow band-pass filter. The narrowness of the filter is limited by the phase resolution of the lock-in (0.01° for the SR830). Also, the phase resolution is important when trying to make a precise determination of, say, the dissipation factor for a capacitor when measuring the balance point of a capacitance bridge.

The lock-in amplifier is an excellent tool for AC measurement, allowing clean recovery of a small signal from potentially much larger noise. The maximum noise to signal ratio at the input, known as dynamic gain, from which the lock-in can recover a signal is up to 6 orders of magnitude.

One of the fundamental assumptions of the lock-in amplification technique is that the device under test (DUT) is approximately linear. In fact, some non-linearity can arise from physical effects, causing the sample to respond at various harmonics of the applied driving force. Many Lock-ins can be adjusted to measure at various harmonics of the driving force,
which can be beneficial in some circumstances. Another use for the lock-in amplifier is to perform AC+DC measurement to obtain the differential resistance (described below).

3.3.2 DC Measurement by Electrometer

DC measurement provides several advantages for measuring highly resistive and non-linear devices. At zero frequency, the effects of stray capacitance are minimized: it affects the response time (and possibly the stability) of the circuit, but it does not introduce any leakage currents. A DC bias sweep can be performed as slowly as required by the circuit response time, and allows the non-linear response of the circuit to be accurately determined.

Compared to AC techniques, DC measurements have the added complication of DC drift and DC offset. DC drift occurs for a variety of reasons, one of them being slight changes in temperature which can cause small voltages or change the output of the device. To combat DC drift, it is better to perform measurements quickly, but this is not always possible. DC offset occurs because the feedback loop of the instrument responds differently for various load resistors. To combat the offset, we perform a DC bias sweep and obtain the resistance from the slope of the curve, rather than an individual point. The pinned electronic crystal that we measure in this work exists within a narrow window of $\pm 4$ pA where the effective resistance is extremely high. To measure such a large resistance requires an electrometer, which is a DC source-meter designed for measuring extremely high impedances. The Keithley 6430 source-meter used in our measurements can resolve currents as low as 100aA and voltages of 1µV for the sensitivity ranges used in our experiments. It has an input impedance of $10^{16} \Omega$, so that leakage current through the instrument is minimal. The meter is capable of sourcing either current or voltage. For our experiment, the current source provides higher resolution because the voltage range on the instrument was minimum, but the current range was not
pushing the instrument limits. In our measurement, the results from both drive types agree.

One limitation of electrometers is that they are not very accurate for low impedance devices. In our results section, we will show that the differential resistance of our sample drops by nearly 4 orders of magnitude from 1 GΩ to about 200 kΩ. The log-log scale of Fig. 3.18 demonstrates that the Keithley 6430 is suitable for observing impedance changes over many decades. However, the measurement accuracy drops rapidly for resistors less than 100 kΩ as demonstrated in Fig. 3.19.

![Figure 3.18: (a) IV characteristics of resistors over several decades of impedance. (b) Logarithmic plot of the measured slope ($R_{\text{exp}}$) vs. the nominal resistance value $R_{\text{real}}$.](image)

### 3.3.3 AC+DC Measurement

Combining the above two techniques, DC bias sweep with lock-in measurement, provides another powerful tool for characterizing a device. The AC signal used for this result is chosen to be small enough that the response of the device is approximately linear. Then, a
Figure 3.19: Percent error between actual value and measured value for resistors over a range of decades.
DC signal is superposed to sweep this AC measurement across the spectrum of excitations. The result is that we can use the noise rejection capabilities of a lock-in amplifier to sense the differential resistance of the device at different points along the non-linear curve. Essentially, it is an analog method to take the derivative of the DC sweep. In some circumstances this provides the best resolution for small changes in the resistance.

### 3.3.4 Capacitance Measurement

Capacitance measurement using a homemade capacitance bridge will be discussed in the following chapter.

### 3.3.5 Filters, Shielding, and Ground Loops

It is important to filter out high frequency noise. Large noise can be filtered out of the measurement by lock-in amplifier, but the noise still affects the sample. It can cause Joule heating, or it can push the sample beyond a pinning threshold that we wish to study. To do this, we use a series of low-pass RC filters with cutoff frequency $f_c$ in the range $40$ kHz for low-impedance sourcing. For a high impedance source, the cutoff frequency is much lower. To see this, note that the filter capacitors can discharge through either the sample $R_{\text{sample}}$ or the source loop $R_{\text{source}} + R_{\text{filters}}$. The cutoff frequency

$$
 f_c = \frac{1}{(R_{\text{sample}} || (R_{\text{filters}} + R_{\text{source}})) C}. 
$$

(3.3)

is dominated by whichever resistance is smallest. Currents at frequencies above $f_c$ pass through the filters instead of the sample. It is important to remember that using a large impedance source (such as a large ballast resistor on a lock-in output) severely limits the bandwidth of the signal. This can cause measurement artifacts in the form of phase shifts and signal attenuation when the sample impedance grows large in an insulating phase.
Proper shielding reduces the environmental noise from reaching the sample and should always be used. This is especially important for measuring high impedance or small capacitance samples, which are greatly affected by the surrounding environment if they are not fully shielded.

To eliminate ground loops, all instruments should be grounded to a single point. This may require the use of ground breaking isolators on the power-supplies, and manually wiring the chassis ground of the instruments.
CHAPTER 4 CAPACITANCE MEASUREMENT BRIDGE

4.1 Preface

Both the DOS $g(E)$ and the electronic compressibility $\kappa$ can be measured by capacitive charging of a device [16, 18, 170, 174].

However, several challenges make this a difficult task. First, the systems of interest undergo an MIT that can lengthen the response time of a circuit. This causes the charging energy of a device to appear artificially high if the rising resistance causes the circuit time constant $1/f_{roll}$ (where $f_{roll}$ is the roll-off -3dB point of the $RC$ roll-off frequency) to change significantly compared to the period $1/f_{drive}$ of the driving signal. To combat this effect, we must choose $f_{drive}$ to be small, which makes it difficult to achieve high resolution measurement (due to a lower number of averaged measurement cycles and increased $1/f$ noise). Second, we must use a low excitation voltage to resolve changes in $g(E)$, which makes the measurement susceptible to noise. Third, the probe-to-sample capacitance is quite small, meaning that the impedance $Z = 1/\omega C$ is large. Thus, signal recovery requires some method of pre-amplification (either a high impedance amplifier that is physically close to the sample, or a low impedance amplifier elsewhere in the circuit). These demands fall outside the range of most LCR meters and require the use of a homemade capacitance bridge.

This chapter presents our progress towards integrating a capacitance bridge and pre-amplification technique with an AFM tip to achieve a spatially resolved measurement of the DOS. Section 4.2 gives an overview of the bridge circuit. Section 4.3 details a voltage pre-amplification technique using a high electron mobility transistor (HEMT) that can be operated at cryogenic temperatures. This section also discusses two other forms of pre-amplification based on current and charge measurements and compares the possible res-
olution by all three measurement types. Finally, Sec. 4.4 details a computer automated balancing algorithm for the bridge.

4.2 Bridge Overview

The basic bridge circuit using a voltage sensing instrument at the balance point is represented in Fig. 4.1.

![Figure 4.1: A basic bridge circuit. The two independent voltage sources can be supplied by a low-noise high-resolution dual-channel function generator, or they can be approximated by a combination of power and ratio transformers.](image)

This figure includes two independent voltage sources (which could be a supplied by a resolution voltage supply, or a precision ratio transformer), a sample with impedance $Z_{\text{sample}}$, reference impedance $Z_{\text{ref}}$, and a meter with finite input impedance $Z_{\text{shunt}}$. Summing the currents leaving the balance point, we see that

$$\frac{V_b - V_{\text{ref}}}{Z_{\text{ref}}} + \frac{V_b - V_{\text{ref}}}{Z_{\text{ref}}} + \frac{V_b - 0}{Z_{\text{shunt}}} = 0. \quad (4.1)$$

Balancing the bridge so that $V_b = 0$ eliminates the shunt impedance $Z_{\text{shunt}}$ from the equation. This allows us to solve for

$$Z_{\text{sample}} = \frac{V_{\text{ref}}}{V_{\text{sample}}}Z_{\text{ref}}. \quad (4.2)$$
After $Z_{\text{sample}}$ is known, we can obtain the meter/sensing cable shunting impedance to ground from

\[
\frac{1}{Z_{\text{shunt}}} = \frac{V_b - V_{\text{sample}}}{V_b} \frac{1}{Z_{\text{sample}}} + \frac{V_b - V_{\text{ref}}}{V_b} \frac{1}{Z_{\text{ref}}}. \tag{4.3}
\]

The shunting impedance is constant (assuming the sensing line cable does not flex or move during the experiment), so that the bridge can be operated “off-balance if $Z_{\text{shunt}}$ is known. In this case, the sample impedance is given by

\[
\frac{1}{Z_{\text{sample}}} = -\frac{V_b - V_{\text{ref}}}{V_b} \frac{1}{Z_{\text{ref}}} - \frac{V_b}{V_b - V_{\text{sample}}} \frac{1}{Z_{\text{shunt}}}. \tag{4.4}
\]

To determine the bridge sensitivity, we must take into account the signal attenuation due to the voltage divider effects of $Z_{\text{shunt}}$. Suppose that $Z_{\text{shunt}}$ represents a 100pF cable capacitance, and $Z_{\text{ref}}, Z_{\text{sample}} \approx 1pF$. We use the superposition principle to consider the separate contributions of $V_{\text{sample}}$ and $V_{\text{ref}}$ to the voltage at the balance point. A signal $V_{\text{sample}}$ encounters a $1/100$ voltage divider between $Z_{\text{sample}}$ and $Z_{\text{shunt}}||Z_{\text{ref}} \approx Z_{\text{shunt}}$ (since $Z_{\text{shunt}} \ll Z_{\text{ref}}$). Thus, a signal $V_{\text{sample}} = 1 \text{ mV}$ produces only $10 \mu\text{V}$ at the balance point. The contribution from $V_{\text{ref}}$ is similarly attenuated. Thus, the “full-scale” signal of the bridge (when $V_{\text{sample/\text{ref}}}$ are maximized) is reduced by two orders of magnitude.

We can estimate the bridge resolution using the superposition principle as follows: Let the attenuation factor of the reference signal discussed in the preceding paragraph be denoted as

\[
A_{\text{ref}} = \frac{Z_{\text{shunt}}||Z_{\text{sample}}}{Z_{\text{ref}} + Z_{\text{shunt}}||Z_{\text{sample}}}, \tag{4.5}
\]
and similarly for $A_{\text{ref} \leftrightarrow \text{sample}}$. Then, the signal at the balance point is

$$V_b = A_{\text{ref}} V_{\text{ref}} + A_{\text{sample}} V_{\text{sample}}.$$  \hfill (4.6)

Let $V_{b, \text{noise}}$ be the noise level at the balance point and $\delta V_{\text{ref,sample}}$ be the resolution (minimum voltage change) of the voltage source. Then the bridge resolution for percent change in sample impedance can be estimated as

$$\frac{\max(V_{b, \text{noise}}, A_{\text{ref}} \delta V_{\text{ref}}, A_{\text{sample}} \delta V_{\text{sample}})}{\min(A_{\text{ref}} V_{\text{ref}}, A_{\text{sample}} V_{\text{sample}})}.$$  \hfill (4.7)

For best resolution, we want a small ratio. First, we consider the limiting factor for achieving a large denominator. Note that for $Z_{\text{sample}} \neq Z_{\text{ref}}$ either $V_{\text{ref}}$ or $V_{\text{sample}}$ must be small. Also, if $Z_{\text{shunt}}$ is small compared to $Z_{\text{ref}}$ or $Z_{\text{sample}}$, then either $A_{\text{ref}}$ or $A_{\text{sample}}$ must be small. Therefore, the best case (and simplest) scenario when $Z_{\text{sample}} \approx Z_{\text{ref}} \ll Z_{\text{shunt}}$ and the bridge is nearly balanced so that $V_{\text{ref,sample}}$ are both near full scale. Knowing that small $A_{\text{ref,sample}}$ is not advantageous, we see that the smallness of the numerator is limited by the resolution of the voltage supplies and by the noise at the bridge balance point. These factors can only be improved by low noise electronics and a well shielded circuit. Signal filters can be used to reduce $V_{b, \text{noise}}$, but these can cause phase shifts and attenuation that must be accounted for.

### 4.3 Amplification Techniques

At low frequencies, small capacitors represent a very large impedance. To recover a meaningful signal requires that we use a pre-amplifier. The following sections discuss the use of voltage, current, and charge sensitive amplifiers in this bridge circuit. Each has its
own benefits and limitations. Before going into all of the details, we present the final results here. Figure 4.2 gives the predicted resolution limit for the voltage and charge amplifiers based on parameters that I tested in the lab (the exact experimental setup will be detailed in the following subsections) in terms of Farads as a function of volt excitation at a frequencies $f \geq 1 \text{ kHz}$. The noise floor for the SR830 lock-in amplifier measuring at $f \approx 1000 \text{ Hz}$ is 300 nV$_{\text{rms}}$ when connected to the output of the HEMT voltage pre-amplifier having a signal gain of 2 and stray capacitance of 500 fF at the pre-amplifier input. The noise floor can be greatly reduced (by 1 order of magnitude at least) at higher frequencies or by using longer averaging times. The base level noise of the Femto HQA-15M-10T charge pre-amplifier is 1 $\mu$V when measured by the lock-in. This can be improved by inserting a band-pass filter between the pre-amp and the lock-in (most of the 15 MHz bandwidth is just adding noise to our single-frequency measurement).

The results in Fig. 4.2 are independent of frequency for $f$ greater than some minimum cutoff value as discussed below (about 250 Hz for the charge amplifier and probe dependent for the voltage pre-amp). This is because, the ratio of $q/V$ is fixed by a capacitor. However, the rate of charge flow varies with frequency. Thus, the resolution of a current sensitive measurement will depend on $f$. For current amplification, there is a tradeoff: Larger $f$ means larger signal, but the bandwidth of a current a current pre-amplifier decreases with higher gain. By comparing the noise specifications, gains, and bandwidths of various Femto trans-impedance pre-amplifier products, we expect that the maximum performance for a current-based amplification scheme will be similar to the other two methods for frequencies $f = 1$-10 kHz and gains of $10^8$-$10^9$ V/A.

From this summary, we see that the results from all methods are fairly similar in terms of
Figure 4.2: Smallest capacitance changes resolvable using the voltage (dashed red line) and charge (for various input capacitances) pre-amplifiers. We assume the voltage source of the bridge circuit to have 5 digits of accuracy. The flat part of the yellow line reflects this limitation.
signal resolution. One drawback of the voltage pre-amplification method is that the reference capacitor and HEMT pre-amplifier must be placed as close to the sample as possible (within 1-2 mm). Otherwise, stray capacitance from the balance point to ground (in particular, the capacitance between the center pin and shield of a coaxial cable connected to $V_b$ is usually about 100 pF) adds to $Z_{shunt}$ and attenuates the signal. Thus, the final HEMT circuit must be characterized at cryogenic temperatures.

It is quite an undertaking to produce a robust voltage pre-amplifier that can operate reliably at low temperatures, but both the current and charge amplification methods avoid this difficulty by driving the $V_b$ to be a virtual ground. The virtually grounded balance point does not experience effects of $Z_{shunt}$, which drops from the bridge equation when $V_b = 0$ (Eqn. 4.1). In this case, the output of the pre-amp is proportional to the excess charge or current that it must supply in order to force the bridge to balance. This gives the same information as a direct measurement of the imbalance voltage with one major advantage: cable capacitance is no longer an issue (at moderate frequencies) and the pre-amplifier can be placed outside the cryostat. Thus, we can choose from a range of ready-made options without having to develop our own. In fact, the Femto HQA-15M-10T pre-amplifier is already used by some low-temperature AFM companies to measure tuning fork resonance in order to control the scanning distance of an AFM tip. The quickest path to a spatially resolved capacitance measurement might be simply to modify this setup to scan the tip at fixed height above the sample (eliminating the need for tuning fork feedback). The charge pre-amp can then be incorporated into the bridge circuit for capacitance measurement. However, this brings up several obvious questions that need to be answered:

First, is it possible to scan across the sample with no feedback in the $z$-direction? This
is a huge challenge. To have any hope of measuring quantum effects, we must have the tip very close to the sample so that the geometric capacitance is large (for reasons described in Sec. 2.9). Any surface roughness, dust, or misalignment between the xy-planes of the sample and AFM head will cause the AFM tip to crash. To avoid this, we can first scan the sample using the z-direction feedback at room temperature to make sure it is aligned with the head. We can then scan the sample (either at cryogenic or room temperature) to find a dust-free flat portion that suits our needs, though it is anticipated that more challenges may arise.

Second, how can we decouple the resonant vibrations of the tuning fork and eliminate surface topology effects from our capacitance measurement? Since the electronic states we want to probe depend on $B$ field, it may be possible to decouple these effects. The surface topology can be scanned in zero $B$ field or when the sample is in a metallic state. Then, the sample can be re-scanned over the same portion under the appropriate magnetic field. The topology signal can be subtracted from the measurement. This is the approach taken by Tessemer’s group.

4.3.1 Voltage Preamplifier

Voltage amplifiers are always placed in parallel to the device under test. For this reason, the ideal input impedance is infinite (or at least many orders of magnitude larger than that of the sample) so that the amplifier does not add an additional conduction path to the circuit. In reality, both the cable capacitance and input resistance of a preamplifier yield finite impedance. Typical lock-ins (SR830 and EGG or Princeton Applied Research 5210(09)) have 10-100 MΩ resistors at the input which is in parallel to a minimum stray capacitance of about 25 pF that is inherent to the sensing FETs. Additionally, cable capacitance of
the experimental setup (usually several hundred pF) also affects the circuit by adding in parallel to the 25 pF. This can easily reduce the input impedance of the lock-ins to be below that of the device under test (DUT) which attenuates and distorts the signal. Figure 4.3 demonstrates the effect of stray capacitance on the balance point of the bridge circuit using color-coded nodes. We can see that the stray capacitance from the balance point (purple) to the grounded shield of the coaxial cable and test fixture (green) appear in the circuit as a stray capacitance (orange). If this stray capacitance is larger than the sample capacitance, it acts as a voltage divider which attenuates the voltage at the balance point. This can make small signals impossible to measure.

![Figure 4.3: (a) “Realistic” view of bridge circuit. (b) Simplified bridge diagram.](image)

Figure 4.3: (a) “Realistic” view of bridge circuit. (b) Simplified bridge diagram.

Figure 4.4 demonstrates the use of a voltage preamplifier to remedy the stray capacitance problem. In the figure, the DUT is the capacitance between an AFM tip and a 2D sample. Note the use use of a voltage preamplifier to buffer the signal from the balance point and eliminate cable and meter input capacitance from \( Z_{shunt} \) in the bridge equation. As highlighted by orange arrows in the figure, the remaining stray capacitance still plays an important role in our ability to measure tiny value. The following discussion gives more details for the amplifier circuit which relies on the low input capacitance (250-500 fF) of a high electron mobility transistor (HEMT) capable of low-temperature operation. The use
of a transistor to drive the voltage on the center pin of the sensing cable prevents the cable capacitance from attenuating the signal at the balance point.

Figure 4.4: Demonstrates that placing a voltage preamplifier close to the the sample and reference capacitors eliminates the cable capacitance. Also, note that the AFM tip is used for the bridge balance point while the sample (assumed to be a gated 2D device) is driven by $V_{\text{sample}}$. If, instead, we had connected the voltage source to the tip and the sample to the balance point, our signal would be shorted out by the large stray capacitance to the gate.

We now discuss the preamplifier circuit and the actual components used to realize it. We use a single HEMT transistor, which has a small input capacitance (the unpackaged Fujitsu fhx35x has 250 pF, and the LG package has 500 pF). Figure 4.5 demonstrates the setup used for DC characterization of the transistor. Up to half of the transistors we purchased have an unacceptably large leakage current and fail to pinch off at large gate voltage (it is impossible to deplete the conduction channel). These defective units were unsuitable for use as an amplifier. Other transistors performed excellently, with typical characteristics displayed in Fig. 4.6. To produce this plot, the IV characteristic curves were obtained at various gate voltages with $R_G = 100 \ \text{M}\Omega$ and $R_D = 0 \ \Omega$. Then, a python program was used to plot the load lines for various $R_D = 0 \ \Omega$. The transistor operates along these load lines (the intersection of the IV curves for various gate voltages with the load line of the resistor give
the operating point of the transistor within a given circuit).

Figure 4.5: Experimental setup for DC characterization of the high electron mobility transistor (HEMT).

The transistor must be properly biased to allow ac voltage amplification. Figure 4.7 shows the voltage amplification characteristics for both DC and AC input signals. Figure 4.7(a) shows the DC voltage across the transistor $V_{out}$ as a function of $V_{gate}$. Thick solid lines show the load line intercept for a range of values of $R_D$. To determine the AC response [Fig. 4.7(b)], we look at the differential changes or derivative of these curves (the AC signal
is small compared to the DC bias). We see that larger $R_D$ produces a sharper upturn in the DC output near the pinch-off region where the carriers are nearly fully depleted. The corresponding AC gain is large at this DC bias point. A DC bias voltage must be superposed onto the balance point AC signal relative to the transistor source in order to bring the circuit to this operating range. Also, note that there is a limit to the maximum $R_D$ that is useful. If the AC gain of the circuit is narrow compared to the size of the AC signal, the resulting output will be distorted (i.e. the peaks and troughs of a sine wave will be attenuated relative to the central values). For the signals in our measurement ($100\mu V$ to $2 \text{ mV}$) this will not be a problem.

We can gain a bit more understanding into the circuit by examining the frequency dependence of the amplifier. The transistor can function into the GHz range, so we expect
Figure 4.7: HEMT voltage output for both DC and AC signals. That the gain should be frequency-independent (above some minimum cutoff frequency to be explained below). Figure 4.8 shows the measured response for a small AC coupled signal when the transistor is biased for maximum AC gain. The blue trace was taken using the bench top DC power supplies using the setup outlined in blue. The resulting signal is a poorly reproduced sine wave and with frequency amplitude gain. To obtain the green trace, we replace the benchtop DC instruments by a battery voltage source. The gain becomes very close to the expected value with no frequency dependence (above a lower cutoff limit explained below) and the sine wave looks pristine. We conclude that the benchtop DC supplies are not designed to meet the transient power demands for supplying the transistors high frequency AC output. However, the battery supply produces a clean signal suitable for sensitive measurement.
Figure 4.8: Comparison of the gain, signal quality, and frequency response of the HEMT amplifier when powered by bench-top DC supplies (blue) and battery supplies (green) under identical conditions. The circuit diagram outlined in blue shows the experimental setup for the blue trace. For the green trace, we substitute the battery supply described in the text for the bench-top instruments. Oscilloscope signal images are taken at 40 kHz.
We now take a closer look at the circuit. Figure 4.9 shows the circuit schematic for both the battery power supply and the HEMT amplifier. The battery supply provides a relatively stable voltage even as the battery ages. The design relies on a fixed voltage drop across a red light emitting diode (LED) to bias a 2N3904 transistor which produces a constant current through resistors $R_1$ and $R_2$. These are used to provide the DC bias for the transistor. $R_d$ is adjusted to fix the load-line and operation point of the transistor. $R_G$ is used to provide a DC bias to the transistor gate without shorting the AC coupled signal to ground. Obviously, the size of $R_G$ is a limiting factor for the input impedance of this amplifier. If the sample impedance is greater than $R_G$, the signal will be attenuated. The transistor gate-source resistance for the HEMT is effectively about 500 MOhm (determined by looking at the leakage current), so there is no point to choose $R_G$ much larger than 200MΩ because, it will no longer be the limiting factor.

Figure 4.9: A full schematic of the HEMT amplifier circuit. The left half shows the battery supply which remains outside the test fixture. The right shows the all components that must be placed close to the sample inside the test fixture.

Figures 4.10 gives a design for incorporating the HEMT preamplifier into an existing AFM head with tuning height sensing. The design requires four coaxial cables inside of the
test fixture. The components should be placed as closely as possible together to minimize stray capacitance. Note that the AFM tip should be electrically isolated from the tuning fork and all other electrical components in order to minimize stray capacitance at the balance point.

Figure 4.10: (a) Diagram for a pre-amplifier placed close to an AFM scanning head. Cables correspond to the schematic in Fig. 4.9. (b) Position of components on AFM head to minimize stray capacitance. (c) Picture of existing AFM head and tuning fork scanning tip to be incorporated with the HEMT preamplifier.

Figure 4.11 gives the frequency dependence of the amplifier for various different AC coupled inputs. This will allow us to determine whether or not we have met our goal of minimizing stray capacitance. At low frequency, the gain rolls off due to the rising impedance of the AC coupled input relative to $R_G$. We can use the -3dB point (at which the gain reduces to half of its high frequency value) in order to determine the capacitance of the AC coupling (or DUT). At the -3dB point, the magnitude of the sample (or capacitor) impedance $1/\Omega C$ is equal to $R_G$ so that the signal is attenuated by a factor of 0.5. Using this relation, we calculate the input capacitance and compare the observed values to the nominal values used for our tests. We see fairly good agreement down to 1pF. Below this point, it is difficult to get a good absolute value without further tweaking and minimization of stray capacitance. But, the purpose of this test is not to determine the absolute accuracy of our amplifier.
Instead, we just want to determine what resolution we can hope to achieve.

Figure 4.11: Frequency dependence of the battery supplied HEMT amplifier for an AC input signal of 1 mV and various AC couplings. The transistor is operated in a range where the AC gain is about 2, and this matches with the 250 pF AC coupled signal.

Figure 4.11 also demonstrates that the high frequency gain depends on the input capacitance. We find that 80% of the gain variation can be accounted for by considering the voltage divider effect of the residual stray capacitance (0.5 pF) from the gate to the source of the fmx35LG transistor and packaging. Thus, the circuit is behaving more-or-less as expected.

We have already given the final result in terms of achievable resolution in Sec. 4.3 and Fig. 4.2 above. Here we want to discuss a few ways in which the result can be improved:
First, the actual output of the HEMT has very low noise but only a small gain (-1). Thus, it may be possible to improve on the values in this graph by inserting a low noise commercial pre-amp on the outside of the test fixture. The cable capacitance and input impedance of the pre-amp are no longer an issue after the signal has been buffered by the HEMT. This will allow us boost the signal level above the noise floor of the lock-in inputs if the commercial preamp has less noise than the lock-in (otherwise, we will not gain anything).

Second, a band-pass filter can be used to limit the bandwidth of the measurement and reduce the noise. However, it is important to remember that filtration can affect the phase of the signal, which is important both for balancing the bridge and for accurately characterizing the sample impedance.

Third, the DC power supply for this transistor must be improved. Using a 9V battery, the DC bias point drifts significantly on the time scale of tens of minutes. A real measurement designed to capture small changes in the capacitance must have steady gain for long periods of time. To meet this requirement, a precision DC supply capable of sourcing power and maintaining a steady voltage at the frequencies in our measurement (using capacitors or low-pass filters) must be incorporated. This will not improve the resolution of the bridge, but it is necessary for to achieve steady gain.

4.3.2 Current Preamplifier

As an alternative to the above approach, a current pre-amplifier can be used to recover the signal at the balance point. The current preamp uses resistive feedback to produce a virtual ground at the input of an operation amplifier. In this case, the bridge balance point is driven to ground potential, and the effects of cable capacitance are minimized. However, using the Femto LCM-200k-20M transimpedance amplifier, we find that the output signal
is quite noisy. We believe this is due to pick-up from the sample and cable capacitance. To remedy the situation, a large resistor, comparable to the sample impedance, can be placed in series with the current pre-amplifier. Unfortunately, this large input impedance basically produces a voltage pre-amp that is far away from the sample instead of bringing a virtual ground to the bridge balance point. This defeats the purpose of using a current pre-amp. However, even with the noisy signal, we were sometimes able to get a measurement with comparable resolution to that of the voltage pre-amp, and this option can be further explored.

4.3.3 Charge Preamplifier

The charge amplifier is essentially an integrating op-amp as shown in Fig. 4.12. The Miller effect ensures that the feedback capacitance far exceeds that of the cable which creates a virtual ground so that measurement is unaffected by long cables. Thus, the pre-amplifier can be placed relatively far from the sample, outside the test fixture. For the Femto HQA-15M-10T preamplifier, the gain is given in units of Volts per pico-Coulomb, which (in the case of an AC coupled signal through a capacitive sample) can be converted to a voltage gain per pF. Since the feedback component is capacitive (with very small resistance), the RC time constant is small, and the amplifier has a band-width of 250 Hz to 25 MHz. The signal from the amplifier is a bit noisy so a band-pass filter should be used to limit the bandwidth to a reasonable range about the desired measurement frequency.

Not shown in the figure, is a 1GΩ resistor that ties the amplifier input to ground. This is used to bleed off any DC offset that would saturate the pre-amplifier. Below 250Hz, the feedback circuit impedance grows larger than 10GΩ and the amplifier cannot operate. That is, rather than having a driven virtual ground at the input, the feedback signal is shorted
out by the 1GΩ resistor. This lower frequency cutoff point does not depend on the sample impedance. However, the sample impedance should be much higher than that of the feedback resistor for good operation (otherwise the preamp would saturate). The charge preamplifier produces a similar performance to the above two methods which we have discussed.

4.4 Balancing Algorithm

The balancing algorithm for the bridge relies on linearity of the bridge components (including the sample). The AC signals have magnitude and phase, and can be represented as phasors in a two-dimensional space. The circuit equation can thus be represented as pair of coupled linear equations. The procedure for finding the balance point is identical to the procedure middle school students use to solve a 1D linear equation. Experimental errors are eliminated through iteration. In Sec. 4.2, above, we saw that the balance-point is given by

\[ V_b = \frac{Z_{\text{equiv}}}{Z_{\text{sample}}} V_{\text{sample}} + \frac{Z_{\text{equiv}}}{Z_{\text{ref}}} V_{\text{ref}} \]  

(4.8)
where

$$\frac{1}{Z_{\text{equiv}}} = \left[ \frac{1}{Z_{\text{ref}}} + \frac{1}{Z_{\text{sample}}} + \frac{1}{Z_{\text{meter}}} \right].$$  \hspace{1cm} (4.9)$$

We want to represent voltages as 2D vectors, and to represent their coefficients as active/passive rotation matrices in the complex plane (which also rescale the vectors). To do so, we write

$$\vec{V}_b = \bar{M}_{\text{sample}} \vec{V}_{\text{sample}} + \bar{M}_{\text{ref}} \vec{V}_{\text{ref}} \hspace{1cm} (4.10)$$

where

$$\vec{V}_i = [\text{Re}(V_i), \text{Im}(V_i)], i \in \{\text{ref, sample}\} \hspace{1cm} (4.11)$$

and

$$M_i = [\text{Re}(\frac{Z_{\text{equiv}}}{Z_i}) - \text{Im}(\frac{Z_{\text{equiv}}}{Z_i}), \text{Im}(\frac{Z_{\text{equiv}}}{Z_i})\text{Re}(\frac{Z_{\text{equiv}}}{Z_i})], i \in \{\text{ref, sample}\}. \hspace{1cm} (4.12)$$

To simplify notation into a more familiar form, we make the change of notation $\bar{M} \equiv \bar{M}_{\text{sample}}$ and $\bar{b} \equiv \bar{M}_{\text{ref}} \vec{V}_{\text{ref}}$ (which is a constant), so that

$$\vec{V}_b = \bar{M} \vec{V}_{\text{sample}} + \bar{b}, \hspace{1cm} (4.13)$$

where $\bar{b}$ and $\bar{M}$ are approximately constant if the sample impedance does not change too drastically. Of course, these constants are only measured with finite accuracy so that it is impossible to calculate exactly what is the output (ideally $\vec{V}_{\text{sample}} = -\bar{M}^{-1}\bar{b}$) that will balance the bridge in a single step.
We take an iterative approach as follows: We now insert superscripts into Eqn. 4.13

\[ V_{b}^{(n)} = \bar{M} V_{sample}^{(n)} + \bar{b}. \]  

(4.14)

Note that

\[ \bar{b} = V_{b}^{(n)} - \bar{M} V_{sample}^{(n)}, \forall n. \]  

(4.15)

In particular, at iterative step \( k \) with non-zero balance point voltage \( V_{b}^{k} \), we want to achieve \( V_{b}^{k+1} = 0 \) on the next step if possible. Using the above equations, we calculate that the next output should be

\[ V_{sample}^{k+1} = -\bar{M}^{-1} \bar{b} \]  

(4.16)

\[ = -\bar{M}^{-1} \left( V_{b}^{k} - \bar{M} V_{sample}^{k} \right) \]  

(4.17)

So that finally, our iterative method relies on the equation

\[ V_{sample}^{k+1} = V_{sample}^{k} - \bar{M}^{-1} V_{b}^{k}. \]  

(4.18)

To make this method work, we must first obtain the circuits linear response matrix \( \bar{M} \).
We take the phase of the reference signal to be zero so that

\[ \vec{V}_{ref} = \begin{pmatrix} \Re V_{ref} \\ \Im V_{ref} \end{pmatrix} \]

\[ = |\vec{V}_{ref}| \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \] (4.19)

and

\[ \vec{V}_{sample} = \begin{pmatrix} \Re V_{sample} \\ \Im V_{sample} \end{pmatrix} \] (4.20)

Now, if we hold \( \vec{V}_{ref} \) to be constant, so that \( \vec{V}_b = \vec{V}_b(\vec{V}_{sample}) \), it is easy to see that \( \bar{M} \) is given by

\[
\bar{M} = \begin{bmatrix}
\vec{V}_b \left( \Delta V \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) - \vec{V}_b \left( \vec{0} \right) \\
\Delta V \\
\end{bmatrix},
\]

\[
\begin{bmatrix}
\vec{V}_b \left( \Delta V \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) - \vec{V}_b \left( \vec{0} \right)
\Delta V
\end{bmatrix}, \] (4.22)

where \( \Delta V \) is the rms voltage amplitude of \( V_{sample} \) (when it is non-zero). Thus, the linear
response of the circuit can an be obtained from three measurements

\[
\vec{V}_b \begin{bmatrix}
\Delta V \\
0 \\
0
\end{bmatrix},
\]

(4.23)

\[
\vec{V}_b \begin{bmatrix}
\Delta V \\
1 \\
0
\end{bmatrix}, \text{ and}
\]

(4.24)

\[
\vec{V}_b \begin{bmatrix}
\Delta V \\
0 \\
1
\end{bmatrix}.
\]

(4.25)

Larger \( \Delta V \) allows us to measure \( \bar{M} \) more accurately because noise level in each of these measurements is less important compared to the overall change.

Note that, as long as our guesses bring us somewhat closer to the balance point, \( \vec{V}_b \to 0 \) and \( \vec{V}_{sample}^{k+1} \to \vec{V}_{sample}^k \) as \( k \) increases. This iterative process makes it is possible to balance the bridge even when \( \bar{M} \) is somewhat inaccurate. Even if the sample impedance changes by 10% during a measurement, we likely do not need to re-perform the above measurements and calculate \( \bar{M} \). This saves a lot of time in the balancing algorithm, because we only need to re-obtain \( \bar{M} \) if the sample impedance changes are so large that the bridge drifts out of balance because the iteration no longer converges.
CHAPTER 5 PINNED INSULATING PHASE

5.1 Preface to RIP Results

The quantum Wigner Crystal (WC) [27] is a solid phase of electrons that emerges at temperatures $T \ll E_F/k_B T$ in systems for which the inter-particle Coulomb interaction energy $E_C = \frac{e^2}{\sqrt{\pi p}/4\pi \epsilon}$ dominates the zero-point energy $E_F = \frac{\pi \hbar^2 p/m^*}{2}$ (where $p$ is the charge density). Quantum Monte Carlo simulations anticipate solidification in zero magnetic $B$-field when the ratio $r_s = E_C/E_F$ reaches a critical value of 37 [9, 121]. For GaAs holes, this occurs at extremely low density $p_c \approx \frac{3 \times 10^9}{cm^2}$ when $E_C \approx 1$ meV (12K) and $E_F = 27\mu eV$ (or 300mK) are both small (taking the effective mass to be $m^* \approx 0.25 m_e$). This corresponds to a large average charge spacing, $2a = 2(\pi p)^{-1/2} \approx 200$ nm, so that reduced screening ability leaves the system susceptible to Anderson localization [2, 45]. Most systems cannot achieve a WC at large $r_s$, but tend towards Wigner glass (WG) instead [53, 144, 180]. Other intermediate phases (hexatic, micro-emulsion, etc.) are also likely [59, 122, 124, 129].

Application of a perpendicular magnetic field provides a way to effectively increase $r_s$ at larger $p$ by confining the charges to cyclotron orbitals, which limits their linear kinetic energy. This leads to various insulating phases, such as the high field insulating phase (HFIP) and reentrant insulating phases (RIP) between Laughlin liquids [10–12, 63, 110, 112, 115]. The arsenal of techniques applied to the study of RIP phases includes, microwave absorption from a co-planar waveguide, non-linear IV characteristics, differential resistance measurements, noise generation, and compressibility measurement (references in Sec. 2.8.2). However, two bench mark characteristics of a WC, pinning and melting, are not firmly established.  

Furthermore, utilizing a $B$-field to quench the kinetic energy greatly modifies the ele-

\textsuperscript{1} Ref. [13] is a notable exception, which demonstrates strong pinning in an $n$-type system for a RIP at $\nu = 2.1$ close to the 1/5 stage in a study that is complementary to our own.
tron wavefunctions, which (in the single-particle picture) are confined to length scales of $l_B = \sqrt{\frac{\hbar}{eB}}$ (note that $l_B$ shrinks below the inter-particle spacing $a$ for $\nu \leq 2$), so that the connection between a zero-field WC and RIP and HFIP phases is not clear. Recent experimental work has also revealed a disconnect between the RIP and HFIP phases [15], indicating that the phase diagram for $B \neq 0$ is more complicated than previously believed [9, 12, 63, 181].

Here we report rigorous pinning of charges in the $\nu = 1/3$ RIP phase. Large in-plane bias forces the state to conduct in a way that is consistent with a manybody transport mechanism. Melting of the state follows a piecewise (non-activated) temperature dependence, that is consistent with a two-stage transition through an intermediate phase. These results can be directly compared with Refs. [5, 13, 114] in the HFIP and zero-field insulating states, and are critical for understanding strongly strongly correlated systems.

5.2 Sample Description

The samples (Fig. 5.1) are $p$-type 20 nm GaAs/AlGaAs quantum square wells with $\delta$-doping both above and below. A hall bar having dimensions 0.5 x 2.5 mm is photolithographically and chemically wet etched into the well. Gold contacts are deposited using a high purity Kurt J. Lesker Nano36 thermal evaporator while the wafer is mounted to a water-cooled platen. Subsequent thermal annealing of the contacts at 460 °C for one minute in a reducing forming-gas environment (10% hydrogen, 90% nitrogen) produces consistent Ohmic contacts. Contact resistance is $\sim 400 \Omega$ per lead at the base temperature of 10 mK. These are cooled using the liquid helium-3 immersion cell [Fig. 5.2(a)] discussed in Sec. 3.2.6 above. Signal lines are attached to sintered silver pillars which provide 2 m$^2$ of surface area for cooling and excellent filtration of microwave noise. Tests are performed in a fully
shielded room, which eliminates noise and prevents Joule heating of the sample. Reference [72] demonstrates the effectiveness of these cooling methods. Hall measurements use standard 4-probe lock-in techniques with current excitations of 0.1-1 nA at frequencies $f \leq 13$ Hz. DC current-voltage (IV) characteristics are obtained using a Keithley 6430 DC electrometer with resolutions of 1 fA and 1 $\mu$V.

![Figure 5.1: (a) Etched Hall bar dimensions and Hall measurement electronics configuration. (b) Band structure for the quantum square well.](image)

**5.3 Results**

Figure 5.2(b) shows the quantum Hall states and Shubnikov de Haas oscillations (SDH) at a base temperature of 11 mK. Excellent sample quality can be seen in the SdH oscillations, which begin at fields as small as 0.05 T. Fourier analysis and temperature dependence of these ripples give a carrier density of $p = \nu e B / 4.0 \times 10^{10}$ cm$^{-2}$ (this varies slightly from sample to sample and between cooling cycles) and indicate that the heavy hole band is degenerate with an effective mass of approximately $m^* \approx 0.4$ (depending somewhat on the strength of the magnetic field) [118]. The mobility is $\mu = 1 / p e \rho = 3 \times 10^6$ cm$^2$/Vs.

A pronounced RIP phase appears at filling factor $\nu = 0.37$ close to the 1/3 state. At this
filling, $\rho_{xy}$ dips sharply beneath the classical value $B/ne$. The dip likely arises from mixing of the overwhelming magnetoresistance $\rho_{xx} \gg \rho_{xy}$ into the transverse signal [168, 169]. This effect may be important for understanding the transverse force required to de-pin a WC or WG in magnetic field [143, 168], and this can be re-visited for our system in a future study.

Here, we focus on sensitive current-voltage (IV) characteristics measurements of the magnetoresistance. For such a large impedance, the current-driven AC measurement (using a large ballast resistor on the lock-in output) shown in Fig. 5.1 not suitable for several reasons: First, the large sample resistance couples with stray capacitance in the signal lines to phase shift and attenuate the signal. Second, the lock-in amplifier output signal is already beyond the pinning threshold of the sample. In the following paragraphs, we describe DC IV results allowing sub-femtoamp resolution with magnetic field fixed at the center of the peak $B = 4.5$ T.

We use a 2-terminal setup, driving voltage between the current leads and sensing current, and assume that losses in the contacts are negligible (these are only 400 $\Omega$ at zero field). All voltages reported below (labeled as $V$ or $V/\square$) have been converted to a per-square value by dividing the Keithely 6430 output by the by the Hall bar aspect ratio, 5. We estimate the resulting in-plane electric field by assuming that voltage drops linearly across the resistive sample so that $E = V/L$ (where $L = 0.5$ mm is the side-length of one square).

Figure 5.3 shows the IV characteristics over a range of $T$. At base $T\ll 11$ mK, we see a remarkable threshold with a sharp drop in the differential resistance $d\rho_{xx}$ that is more than 3 orders of magnitude. No hysteresis or current switching effects are observed on the scale of the graph. The drop in $d\rho_{xx}$ occurs near a critical current of $I_c \sim \pm 4$ pA for voltage $V_c \approx 1$ mV allowing us to characterise the pinning window with an impedance $V_c/i_c = 240$ M$\Omega/\square$ that
Figure 5.2: (a) Helium-3 cell (see Sec. 3.2.6 for a detailed description). (b) Hall measurement showing the magnetoresistance $\rho_{xx}$ (red) and Hall resistance $\rho_{xy}$ (blue). Low-field Shubnikov de Haas (SdH) oscillations of $\rho_{xx}$ appear in the inset.

is 2-3 orders of magnitude larger than that of more disordered systems. At this field, the power dissipation is $\leq 2 \times 10^{-16}$ Watts, ruling out appreciable Joule-heating.

Figure 5.4 displays the differential conductance $d\sigma_{xx} = dI/dV$ (and differential resistance $d\rho_{xx} = 1/d\sigma_{xx}$) as a function of DC voltage $V$ (and the corresponding $E$-field). At $T = 11 \text{ mK}$, $d\sigma_{xx}$ increases exponentially above a critical voltage $V_c = 0.5$ mV but then levels off at around $e^2/h$ for larger $V \sim 2$ meV. Upon heating, both the de-pinning field and the threshold sharpness soften until the sample regains a linear IV characteristic at $T \approx 125$ mK. Further heating of the sample to 300 mK produces a flat (Ohmic) characteristic with $d\sigma_{xx} \approx e^2/h$, showing that the state has fully melted.

Figure 5.5 demonstrates the temperature dependence of $d\rho_{xx}$. Dashed black lines produce

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Footnotes:

2. **RIPNoise**

3. The use of a low impedance (voltage) source with current sensing is generally suitable for measuring a high impedance sample. However this puts our Keithley 6430 meter in the lowest voltage sourcing range and causes the drive signal to be less stable. This is the cause for noise in Fig. 5.4. Nevertheless, the magnitude of the sub-threshold impedance has been confirmed by both AC+DC and current drive methods. A more detailed study of the sub-threshold pinning characteristics will be pursued in a future study with a small adjustment to the measurement electronics setup.
Figure 5.3: Current-voltage characteristics at $B = 4.5$ T taken using a 2-terminal measurement between the current leads. Division by the Hall bar aspect ratio has been used to convert voltage to a per-square value. The derivative of this curve gives the differential longitudinal resistivity $d\rho_{xx}$ in units of $\Omega/\square$. 
Figure 5.4: Differential conductance $d\sigma = dI/dV$ and differential resistance $d\rho_{xx} = 1/d\sigma_{xx}$ at $B = 4.5$ T calculated from the data in Fig. 5.3. Voltage is given as a per-square value ($V/[] = V_{\text{applied}}w/L$ where $w/L$ is the hallbar aspect ratio). The electric field (top axis) is assumed to be $E = V_{\text{applied}}/L$. Note that the major horizontal grid-lines correspond to both $d\sigma_{xx}$ and $d\rho_{xx}$ but that the minor (dashed) lines only pertain to $d\sigma_{xx}$. A horizontal black line gives the quantum conductance (resistance). Noise at the bottom of the graph is due to the voltage drive noise of the Keithley 6430 in the lowest measurement setting. For future measurements, it would be more suitable to use an external battery supply or to use the current drive of the Keithley 6430 (which is more stable).
curves for fixed DC voltage, and vertical colored lines correspond to $T$ in Figs. 5.3 and 5.4.

The zero-bias (topmost) curve demonstrates several interesting features:

Figure 5.5: (a),(b) Temperature dependence of the differential resistance for fixed DC bias voltages at $B = 4.5$ T. (a) Shows the full temperature range demonstrating the low-temperature saturation. (b) focuses on temperatures $T > 25$ mK to emphasize the non-activated behavior.

First, we note the low $T$ saturation. That is $d\rho_{xx}$ changes very little when the temperature $T$ is more than doubled from 11 to 40 mK. Low-temperature saturation at $T$ only 1/3 of the classical melting temperature ($E_C/137$, see Ref. [1]) helps to distinguish this phase as a probable WC rather than intermediate phase.  

Melting of the threshold begins at $T_{m1} \approx 40$ mK. The IV characteristics are linear by 125 mK with a slope that drops to the quantum resistance $h/e^2$ at $T_{m2} \approx 300$ mK, indicating that the crystal has fully melted. In the range 40-300 mK the $T$ dependence is non-activated, indicating the minimal role of disorder in our system [Fig. 5.5(b)]. This suggests that our pinned state may be a WC instead of a disordered WG. However, even in the absence of disorder, the crystal is not expected to have rigorous long-range order due to the Mermin-Wagner theorem [42].

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4 It is important to note that this is the actual temperature of the holes in the well. The effectiveness of our cooling method has been uniquely verified in a previous paper [72].
Results of Figs. 5.4 and 5.5 are combined to produce the contour plot in Fig. 5.6. In the bottom left-hand corner is a WC, which can be pushed into sliding or melted states by application of $V$ and $T$.

![Contour plot of log$_{10}$[$d\rho_{xx}$ (Ω/□)].](image)

Figure 5.6: Contour plot of log$_{10}$[$d\rho_{xx}$ (Ω/□)].

In these results, we see several indications that a quantum mechanism, rather than classical depinning process, is necessary to describe our results:

First, Efros predicts that residual mobile carriers in a disordered WC will exhibit ES hopping. However, the temperature dependence of the RIP does not fit an activated ($\rho_{xx} \propto \exp(T^*/T)^\nu$) curve for the phonon assisted ($\nu = 1$), Mott (variable range hopping, $\nu = 1/3$), or Efros Shklovskii ($\nu = 1/2$) models.

Second, the classical sliding of a pinned domain structure should exhibit hysteresis [53]. However, hysteresis is not a feature of these curves.

Third, we can consider the energy differential $\Delta U_a = eEa$ between adjacent lattice sites
(distance $a$ apart) due to the applied $\vec{E}$-field. We estimate $E$ as the average electric field in the device, assuming that the voltage drops linearly through the conducting 2DH layer. Figure 5.7 re-scales the $x$-axis of Fig. 5.4 to reflect this parameter. The energy scale (nano eV) has also been converted to temperature $T = E/k_B$ on the top of the graph. Note that this energy is minuscule compared to the experimental temperature of the system. If single-particle hopping was a viable conduction mechanism, the thermally induced carriers would completely smear out the threshold behavior. However, let us consider the energy scale for collective motion of an entire domain. Ohmic transport is recovered at a relatively large melting temperature $T_m \approx 125$. This is comparable to $1000\Delta U_a$, suggesting an average domain size of about 1000 particles, corresponding to an area of roughly $(1.6 \, \mu m)^2$.

![Graph showing differential resistance as a function of the energy differential $\Delta U_a = eEa$ between adjacent lattice cites (distance $a$ apart) due to the applied $V$ and resulting $E$-field.](image)

Figure 5.7: Differential resistance as a function of the energy differential $\Delta U_a = eEa$ between adjacent lattice cites (distance $a$ apart) due to the applied $V$ and resulting $E$-field.

Finally, we can consider the energy for shearing the crystal by a distance of one
lattice spacing as discussed in Ref. [13]. The number of particles \( N_D \) in a single domain is

\[
N_D = \frac{Ka}{eE_T}
\]

(5.1)

where \( a = 1/\sqrt{\pi n} \) is the areal charge density, \( K \) is the sheer modulus, and \( E_T \) is the threshold field. Taking \( K \) to be the classical value (as an upper limit), Ref. [13] finds that \( N_D \leq 500 \) for \( E_T = 250 \text{ mV/cm} \) and density \( n = 9 \times 10^6 \). In our experiment, \( p = 4.0 \times 10^{10} \text{ cm}^{-2} \) and \( E_T = 12.4 \text{ mV/cm} \) correspond to \( N_D \leq 1.4 \times 10^4 \) charges in a single domain. This gives a domain area of roughly \((5 \mu\text{m})^2\).

The above results for both \( V \) and \( T \) dependence are summarized in the contour plot 5.6 representing \( \log_{10}[\rho_{xx}(\Omega/\square)] \). This serves as somewhat of a phase diagram for the system. The bottom left is a pinned WC. Yellow regions denote mixed or hexatic phases, and the blue region is a melted WC or strongly correlated liquid with sheet resistance \( \sim h/e^2 \).

In Fig. 5.6 it is impossible to distinguish the melted state from the low temperature state pushed over the threshold since these two have about the same resistance \( h/e^2 \). Does pressuring the WC cause melting? Or, does it produce a sliding state? Spontaneous noise generation has been taken as a sign of sliding charge density wave or sliding crystal in previous studies [13, 109, 114]. A next step for this research would be to test whether the sample generates noise when pushed beyond the conduction threshold in the pinned state.

Upon heating, the Kosterlitz-Thouless transition mechanism is expected to play a part in the melting [33, 34] or the system may pass through several intermediate phases [59, 124, 128, 129]). Our results are consistent with a transition from WC to intermediate phases around 40 mK and complete melting to the Fermi liquid regime at 300 mK where the differential
resistance drops to $h/e^2$.

The presence of a WC at the RIP near $\nu = 1/3$ is a long-sought and highly studied topic, making the findings in this chapter the most important contribution of this dissertation. The results complement previous studies of the zero-field MIT, RIP at other filling factors in both $n$-type and $p$-type samples, and works on the HFIP.

Our results are likely the strongest transport evidence to date for a pinned WC at filling factor $\nu = 0.37$. More detailed transport studies of the sub-pinning regime would be an excellent next step for this research. The state can also be further characterized using microwave resonance studies and other measurements that are critical for fully understanding the many-body insulator. This is an opportunity for experimentalists to uncover important results in an area that has been historically challenging but also highly interesting.
CHAPTER 6 RELATED PROJECTS

6.1 Bulk Conduction in the Integer Quantum Hall States

Two dimensional systems provide a unique opportunity to test the insulating bulk phase of a topological insulator by using a Corbino disk. For such a sample, the inner and outer edges are separated by the bulk phase. One can apply a potential between these edges to test transport through the bulk. Instead of a disk, we use a rectangular anti-Hall bar geometry (Fig. 6.1). This allows us to measure the magnetoresistance resistance at the inner and outer edges simultaneously. This allows us to observe the opening and closing of the energy gap of the insulating bulk phase as increasing $\vec{B}$-field sweeps the sample through a Hall state.

Figure 6.1: Experimental setup for measuring the bulk conduction properties using separated edges in a Corbino-disk-like geometry.
Figure 6.2(a) shows the differential resistance measurement using the setup in Fig. 6.1 at the filling factors indicated in Fig. 6.2(b) which displays the corresponding Hall resistances. We find that the applied DC voltage causes the conduction to jump in discrete steps at energies $\approx \hbar \omega_c$. Figure 6.3 zooms in on one measurement to show this more clearly. The tunneling mechanism appears to be a resonant feature of the edge state potential as discussed in more detail in Refs. [26].

Figure 6.2: (a)-(i) Differential resistance $R_d$ corresponding to various filling factors with locations indicated by color-coded vertical dashed lines in the panel on the right. Figure adapted from [26].
Figure 6.3: Zoom-in view of the differential conductance at filling factor $\nu = 1.00$.

6.2 Quantum Capacitance Measurement of Vanadium Dioxide

Vanadium dioxide is a fascinating material that is well known for undergoing a MIT near room temperature. After more than 50 years of study, there is still disagreement as to the cause of the transition (foremost theories are some combination of the structural Peierls and electronic Mott transitions). A number of factors (strain, electric field, crystal uniformity, temperature, etc.) are known to affect both the temperature and amount of hysteresis in the electronic transition. This makes VO$_2$ a promising material for many devices ranging from mechanical memory to fast switching in optical devices.

We are the first to perform quantum capacitance measurement in order to directly observe changes in the DOS across the VO$_2$ MIT 6.4. This approach promises to improve our understanding of the long-standing puzzle over the transition mechanism. Further discussion
Figure 6.4: (a) The measured capacitance $C_{\text{eff}}$ ($C_{\text{tot}}$) with slightly $T$-dependent geometric contribution extrapolated from the metallic regime to temperatures below the MIT as a dashed line. (b) Inverse quantum capacitance $C_q$ calculated from the measurement. Dashed lines represent the error margin in each graph. Figure adapted from [24].
and experimental details are available in Refs. [24, 175].

6.3 Undoped GaAs/AlGaAs Heterojunctions

Previous works by my advisor Jian Huang using undoped HIGFET devices have achieved the lowest measured densities in GaAs systems while retaining excellent mobility due to the ultrapure condition of the quantum well. Below are preliminary results for extending the study of a zero-field WC using these devices at Wayne State.

![HIGFET Turn On](image)

**Figure 6.5**: Turn on of HIGFET transistor. The black curve shows zero leakage current. The red curve gives the in-plane current. Carrier density is given by the gate-to-2D-layer capacitance $C_{gate}$ as $(V_{gate} - V_{TurnON})C_{gate}$ and agrees with the density measured using the Hall effect.

First, we demonstrate the ability to fabricate a working transistor with capacitively induced charges. Figure 6.5 gives the turn-on characteristics of a HIGFET device by displaying both in-plane and leakage currents. Note that the leakage current is at least 3 orders of magnitude smaller than the in-plane current of the device. The leakage current is actually too
small to detect using a picoammeter. This is excellent, because we want to investigate the IV characteristics at sub-picoamp in-plane currents. We have also performed both Hall and T-dependence measurement on these devices (Fig. 6.5). Unfortunately, at the time that these results were taken, we did not yet have a working $^3$He cell to achieve the necessary temperature for WC.

![Graph of Hall measurement of a Higfet device.](image)

Figure 6.6: Hall measurement of a Higfet device.

6.4 Graphene Devices

Graphene, the first stable 2D material ever discovered, is of interest for its linear dispersion relation and high mobility (among many other advantageous mechanical, electrical, and optical properties, etc.). There has been a major drive in recent years to push for composite devices, bringing systems of different dimensionality or quantum states together in close proximity to produce novel quantum effects at the interface. One proposal is to coat graphene with light-sensitive quantum dots. Upon illumination, Electron-hole pairs will form
such that the electron is trapped on a quantum dot but the hole is mobile in the graphene. This is essentially a transient doping (on the timescale of the electron-hole recombination time $\tau_{\text{recomb}}$) that can alter the resistivity of the device. Due to the large hole mobility in graphene, $\tau_{\text{recomb}}$ provides enough time for a mobile hole to cross the graphene sample $\approx 10^8$ times. Thus, the theoretical gain of the device is extremely high. Sensitivity to these transient doping effects is maximized if the Fermi energy is tuned to the Dirac point (via an applied DC bias through a metallic gate) so that the sheet has minimal background carriers and larger resistivity in a dark environment. An experimental setup for testing such a device is shown in Fig. 6.8.

Ability to minimize charge carriers depends on the quality and uniformity of the graphene sample, which translates to sharpness of the Dirac point. Layers grown by chemical vapor deposition always have a high amount of defects and relatively low mobility that is not suitable for such devices. Instead, we use an exfoliation method to fabricate large-sized
Figure 6.8: (a) Optical measurement setup for photosensitive device of graphene coated by quantum dots. (b) Changes in conductance measured at the frequency of the optical chopper. In practice a 4-terminal measurement would be used.
graphene sheets and photolithography to apply gold contacts (Fig. 6.9).

![Figure 6.9: 100 x 40 µm exfoliated graphene with thermally deposited gold contacts.](image)

Measuring the in-plane resistance while sweeping a gate voltage allows the density and the mobility to be calculated (Fig. 6.10). Away from the Dirac point, the mobility is $\mu \approx$ with little change from room-temperature down to 77mK. At a temperature of 4K, we perform Hall measurement in a fixed field of 7 Tesla using a physical properties measurement system (PPMS). The density agrees with the calculated results from the device capacitance to within one percent.

Note that the application of a large magnetic field greatly sharpens the Dirac point.
Figure 6.10: Hall Effect at 4.2K in exfoliated graphene. The density and mobility are $\rho = 1 \times 10^{12} \text{ cm}^{-2}$ and $\mu = 1.8 \times 10^4 \text{ cm}^2/\text{Vs}$ for gate voltage $V_g = 20 \text{ V}$ at room temperature.

Furthermore, the slope of the Hall voltage in $B$ field changes rapidly at with small variations in charge density. Thus, we suspect that a Hall voltage measurement in strong $B$ field near the Dirac point may provide a much more sensitive measurement of the illumination-induced transient doping. This can be investigated further in a future study.
CHAPTER 7 SUMMARY

The consequences of electron-electron interactions are far-reaching and universal to many systems. In particular, the the 2D metal-insulator transition is universal to many 2D systems, but not yet well understood [4, 59, 61, 182]. Present research seeks to understand the role of e-e interactions, and whether these can drive a strongly correlated ground state such as Wigner crystal [27, 182] or Wigner glass [143, 144, 180], which are expected to strongly pinned and insulating. For such systems, the Coulomb energy $E_C$ competes with many other factors, such as the kinetic energy $E_F$, disorder potential, polarizing magnetic field, etc. so that a variety of other phases have been proposed (hexatic [122], micro-emulsion [59, 128, 129], etc.). Thus, there is great disagreement over the proper theoretical interpretation for the experimentally rich observations in 2D systems at low temperature [59]. One of the major challenges to experimental progress in this area, is the difficulty in fabricating high purity systems with dilute charges. In this regime, the decreased screening ability tends to leave the system susceptible to disorder induced localization [48]. Furthermore, making Ohmic contact to dilute charge systems represents a significant challenge. This work utilizes high mobility $p$-type GaAs systems with density in the range $4.0 \times 10^{10}$. These demonstrate excellent mobility at low charge densities, while the large effective hole mass enhances the interaction parameter $r_s = E_C/E_F$. These systems allow us to identify pinning behavior in the reentrant insulating phase near filling factor $\nu = 1/3$ in the fractional quantum Hall regime and to observe several signatures of WC without suffering from localization effects (Chap. 5). Additionally, these samples provide a unique opportunity to probe the transport between two edges of a topological insulator that are perfectly separated by a bulk insulating phase in a Corbino-disk-like geometry for integer filling factors in the quantum Hall regime.
6.1 [26]. We also push for a spatially resolved capacitive measurement technique that can yield new insights into the domain size of a WC and electronic structure of topological states (Chap. 4). Preliminary work in ultradilute HIGFETs and in exfoliated graphene devices are also presented (Chap. 6). These studies are critical to understanding the physics of strongly correlated charges and their relation to topological phases, which is a fascinating area of intense current research [85–87, 183, 184].

Chapter 4 gives a summary of the experimental progress towards performing a scanning quantum capacitance measurement using a modified AFM head. We compare the merits of several pre-amplification techniques based on voltage, current, and charge sensitive amplifier. Further work is required to complete this project by incorporating the measurement components with the AFM scanning head.

Chapter 5 demonstrates the freezing of 2D holes ($p = 4.0 \times 10^{10}$ cm$^{-2}$ and $\mu = 3 \times 10^6$ cm$^2$/Vs inside a 20 nm quantum square well) into a pinned state at low temperature. Threshold IV characteristics show a sharp drop in the differential resistance that is orders of magnitude larger than previous studies of the RIP near 1/3 (which are more consistent with intermediate hexatic, microemulsion states, or even glassy states). The threshold develops in a non-activated fashion that is consistent with a second order liquid-solid phase transition for $T$ between 30 and 125 mK. Upon heating to 300 and when a large in-plane DC field is applied, the resistivity drops to $h/e^2$ and regains a linear character. These features are strong evidence for a second order phase transition to Wigner solid at low temperatures in the RIP phase near $\nu = 1/3$.

Finally, Chapter 6 describes various other projects related to this work that I was also heavily involved in. Measurement of the robustness of the insulating bulk in a Hall state
appears in Sec. 6.1 and further details are available in Ref. [26]. Section 6.2 discusses quantum capacitance measurements in Vanadium dioxide thin films (further details available in [175]). We also give preliminary results for HIGFET device measurement (Sec. 6.3) and exfoliated graphene (Sec. 6.4).

These works advance our understanding of strongly correlated systems occurring in both magnetic field and in zero field. Future work will focus on sub-threshold features in the RIP IV characteristics with improved measurement resolution. We would also like to finish the scanning capacitance measurement project to achieve spatial resolution. Our goal to test ultrapure HIGFET devices at low densities is attainable with the $^{3}$He cell to provide low temperatures. So far, the quantum dots on graphene have not responded to illumination at a fast enough rate to be useful. Further testing is needed to identify the root cause, whether the quantum dots we have used are not suitable, or if there is an interface problem between the dots and the graphene. Vanadium dioxide continues to be an interesting material for many applications, and warrants further study.
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The formation of a quantum Wigner Crystal (WC) is one of the most anticipated predictions of electron-electron interaction. This is expected to occur in zero magnetic field when the Coulomb energy $E_C$ dominates over the Fermi energy $E_F$ (at a ratio $r_s \equiv E_C/E_F \sim 37$) for temperatures $T \ll E_F/k_B$. The extremely low $T$ and ultra dilute carrier concentrations necessary to meet these requirements are difficult to achieve. Alternatively, a perpendicular magnetic $B$-field can be used to quench the kinetic energy. As $B$ increases, various energies compete to produce the ground state. High purity systems with large interaction $r_s > 1$ tend to exhibit reentrant insulating phases (RIP) between the integer and fractional Hall states. These are suspected to be a form of WC, but the evidence is not yet conclusive.

We use transport measurements to identify a conduction threshold in the RIP at filling factor $\nu = 0.37$ (close to the $1/3$ state) that is several orders of magnitude larger than the pinning observed in many other systems. We analyze the temperature and electric $E$-field dependence of this insulating phase and find them to be consistent with a second-order phase transition to WC. The measurements are performed on dilute holes $p = 4 \times 10^{10}$ cm$^{-2}$ of mobility $\mu = 1/pe\rho \sim 2.5 \times 10^6$ cm$^2$/Vs in 20 nm GaAs/AlGaAs quantum square wells.
We also discuss various other projects related to the study of topological states and strongly interacting charges: direct testing of the bulk conduction in a developing quantum Hall state using a corbino-disk-like geometry (or “anti-Hall bar”); preliminary results for ultra dilute charges in undoped heterojunction insulated gated field effect transistors; quantum capacitance measurement of the density of states across the vanadium dioxide metal insulator transition; progress towards a scanning capacitance measurement using the tip of an atomic force microscope; and graphene devices for optical detection.
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