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A New Exponential Approach for Reducing the Mean Squared Errors of the Estimators of Population Mean Using Conventional and Non-Conventional Location Parameters

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A New Exponential Approach for Reducing the Mean Squared Errors of the Estimators of Population Mean Using Conventional and Non-Conventional Location Parameteres

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Classes of ratio-type estimators *t* (say) and ratio-type exponential estimators t_e (say) of the population mean are proposed, and their biases and mean squared errors under large sample approximation are presented. It is the class of ratio-type exponential estimators t_e provides estimators more efficient than the ratio-type estimators.

Keywords: study variable, auxiliary variable, bias, mean squared error.

Introduction

The use of auxiliary information at the estimation stage of a survey improves the precision of the estimate(s) of the parameter(s) under investigation. The problem of estimating the population mean or total using population mean of an auxiliary variable has been extensively discussed. Out of many ratio, product and regression methods of estimation are good examples in this context. The ratio method of estimation is most effective for estimating population mean of the study variable when there is a linear relationship between study variable and auxiliary variable and they have the positive (high) correlation. However, if the correlation between study variable and auxiliary variable is negative (high) the product method of estimation can be employed.

Let $U = (U_1, U_2, \ldots, U_N)$ be the finite population of size N and the variables under study and auxiliary be denoted by y and x respectively. Let $\left(\hat{\bar{Y}}, \hat{\bar{X}}\right)$ be the population means of (y, x) respectively. It is desired to estimate the population mean $\bar{Y}$

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using information on population parameters such as mean $(\bar{X})$, coefficient of variation (C_x), coefficient of skewness $(\beta_1(x))$, kurtosis $(\beta_2(x))$, deciles, quartiles, median, midrange (MR), Walsh average (i.e. Hodges-Lehman estimator) (HL) (and tri mean (TM) etc, associated with auxiliary variable x and the correlation coefficient ρ between y and x. In this context, the reader is referred to Searls (1964), Das and Tripathi (1980), Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh and Tailor (2003), Singh et al (2004), Kadilar and Cingi (2004, 2006) , Yan and Tian (2010), Subramani and Kumarapandian (2012a,2012b,2012c), Jeelani et al (2013), Ekpenyoung and Enang (2015), Subramani et al (2015) and Abid et al (2016a,b,c).

Define:

N : Population size.

n : Sample size.

$f=\frac{n}{N}$: Sampling fraction.

$\theta=\frac{(1-f)}{n}$.

$S_y^2=(N-1)^{-1}\sum_{i=1}^{N}(y_i-\bar{Y})^2$: Population Variance of the study variable y.

$S_x^2=(N-1)^{-1}\sum_{i=1}^{N}(x_i-\bar{X})^2$: Population variance of the auxiliary variable x.

$C_y=S_y/\bar{Y}$: Coefficient of variation of the study variable y.

$C_x=S_x/\bar{X}$: Coefficient of variation of the auxiliary variable x.

$S_{xy}=(N-1)^{-1}\sum_{i=1}^{N}(x_i-\bar{X})(y_i-\bar{Y})$: Covariance between y and x.

$\rho=S_{xy}/(S_xS_y)$: Correlation coefficient between x and y.

$C=\rho\ C_y/C_x$,

M_d: Population median of x.

Q_i : i^{th} population quartile (i=1,2,3).

T_m : $\frac{(Q_1+Q_2+Q_3)}{2}$: Tri mean.

$H_l = \text{median}\left(\left(X_j + X_k \right) / 2, 1 \le j \le k \le N \right)$ Hodges-Lehmann estimator.

$X_{(1)}$: Lowest order statistic in a population of size N,

$X_{(N)}$: Highest order statistic in a population of size N,

$M_r = \dfrac{X_{(1)} + X_{(N)}}{2}$: Mid range,

$Q_r = \left(Q_3 - Q_1 \right)$: Inter-quartile range,

$Q_d = \dfrac{Q_3 - Q_1}{2}$: Semi-quartile range,

$\beta_1(x) = \dfrac{Ni\sum_{i=1}^{N} \left(x_i - \overline{X} \right)^3}{(N-1)(N-2) S_x^3}$: Coefficient of skewness of the auxiliary variable x,

$\beta_2(x) = \left[\dfrac{N(N+1)\sum_{i=1}^{N} \left(x_i - \overline{X} \right)^4}{(N-1)(N-2)(N-3) S_x^4} - \dfrac{3(N-1)^2}{(N-2)(N-3)} \right]$: Coefficient of kurtosis of the auxiliary variable x,

$\Delta = \left[\beta_2(x) - \beta_1^2(x) + \dfrac{2(N-1)^2}{(N-2)(N-3)} \right]$,

$Q_a = \dfrac{Q_3 + Q_1}{2}$: Quartile average,

$R = \dfrac{\overline{Y}}{\overline{X}}$: Population ratio of means,

$G = \dfrac{4}{N-1} \sum_{i=1}^{N} \dfrac{(2i - N - 1)}{2N} X_{(i)}$: Gini's Mean Difference,

$D = \dfrac{2\sqrt{\pi}}{N(N-1)} \sum_{i=1}^{N} \left(i - \dfrac{N+1}{2} \right) X_{(i)}$: Downton's method,

$S_{pw} = \dfrac{\sqrt{\pi}}{N^2} \sum_{i=1}^{N} (2i - N - 1) X_{(i)}$: Probability Weighted Moments,

$\mu_r(x) = (1/N) \sum_{i=1}^{N} \left(x_i - \overline{X} \right)^r$: r being non-negative integer.

We are interested in estimating the population mean $\bar{Y}$ of the study variable *y* (taking value y_i for i=1, 2,…, *N*) from a simple random sample size n drawn without replacement from the population *U*. We use the notation $\bar{y}$ and $\bar{x}$ for the sample means, which are unbiased estimators of the population mean $\bar{Y}$ and $\bar{X}$, respectively. We also denote:

$s_{xy} = (n-1)^{-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$: Sample covariance between *y* and *x*.

$s_x^2 = (n-1)^{-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$: Sample variance of *x*.

$\hat{\beta} = \frac{s_{xy}}{s_x^2}$: Sample regression coefficient estimate of the population regression coefficient $\beta = \frac{S_{xy}}{S_x^2}$ of y on x.

$\hat{R} = \frac{\bar{y}}{\bar{x}}$: Ratio of sample means.

$\hat{\bar{Y}} = \left[\bar{y} + \hat{\beta}\left(\bar{X} - \bar{x}\right)\right]$: Regression estimator of the population mean $\bar{Y}$.

Existing Modified Ratio Estimators and The Suggested Class of Ratio Estimators

The usual unbiased estimator for population mean $\bar{Y}$ is defined by

$$t_0 = \bar{y} . \tag{1}$$

whose MSE is given by

$$MSE(\bar{y}) = \frac{(1-f)}{n} S_y^2 . \tag{2}$$

The classical ratio estimator for the population mean $\bar{Y}$ in presence of known population mean $\bar{X}$ of the auxiliary variable x is defined by

$$\bar{y}_R = \bar{y} \frac{\bar{X}}{\bar{x}} \quad \bar{x} \neq 0 . \tag{3}$$

To the first degree of approximation, the bias and MSE of the ratio estimator $\bar{y}_R$ are respectively given by

$$B(\bar{y}_R) = \frac{(1-f)}{n} . \frac{1}{\bar{X}} \left(RS_x^2 - \rho\, S_x\, S_y\right) \tag{4}$$

and

$$MSE(\bar{y}_R) = \frac{(1-f)}{n}\left(S_y^2 + R^2 S_x^2 - 2\, R\, \rho\, S_y\, S_x\right). \tag{5}$$

In Table 1, the modified versions of the ratio estimator reported by Kadilar and Cingi (2004) are given, Kadilar and Cingi (2006) –type estimator, Yan and Tian (2010), Subramani and Kumarapandiyan (2012a, 2012b, 2012c, 2012d), Jeelani et al. (2013) and Abid et al. (2016) along with their biases and mean squared errors (MSEs) to the first degree of approximation, as reported in Abid et al. (2016).

Note the estimators $t_j\,(j = 1 \; to \; 65)$ members of the following class of estimators of the population mean $\bar{Y}$ defined by

$$t = \hat{\bar{Y}}\left(\frac{a\bar{X} + b}{a\bar{x} + b}\right) \qquad = \left[\bar{y} + \hat{\beta}\left(\bar{X} - \bar{x}\right)\right]\left(\frac{a\bar{X} + b}{a\bar{x} + b}\right), \tag{6}$$

where $\hat{\beta}$ is the sample estimate of the population regression coefficient β of y on x, $a(\neq 0)$ and b are real numbers (constants) or the functions of population parameters such as population total $X\left(= N\bar{X}\right)$, population standard deviation S_x, variance S_x^2, coefficient of variation C_x,

Coefficient of skewness $\beta_1(x)$ and kurtosis $\beta_2(x)$, correlation coefficient ρ, Δ, quartiles, deciles, median, mode, midrange, Trimean and Hodgs-Lehmann (HL) estimator etc.

Some unknown members of the suggested class of ratio-type estimators t are given in Table 2

To obtain the bias and mean squared error of the proposed class of estimators ‘t’ we write

$$\bar{y} = \bar{Y}(1+e_0),\; \bar{x} = \bar{X}(1+e_1),\; s_{xy} = S_{xy}(1+e_2),\; s_x^2 = S_x^2(1+e_3)$$

such that

$$E(e_i) = 0 \text{ for all } i = 1,2,3;$$

and

$$E\left(e_0^2\right) = \frac{(1-f)}{n}C_y^2,\; E\left(e_1^2\right) = \frac{(1-f)}{n}C_x^2,\; E(e_0 e_1) = \frac{(1-f)}{n}CC_x^2,$$

$$E(e_1 e_2) = \frac{N(N-n)}{(N-1)(N-2)}\frac{\mu_{21}}{n}\frac{1}{\bar{X}S_{xy}} = \frac{(N-n)}{n(N-2)}\frac{\mu_{21}}{\bar{X}\mu_{11}},$$

Table 1. Known modified ratio estimators for the population mean $\bar{Y}$.

S.No.	Estimator	MSE of (.)	Population Ratio
1.	$t_1 = \frac{\hat{\bar{Y}}}{\bar{x}}.\bar{X}$ Kadilar and Cingi (2004)	$\theta\left[R_1^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right]$	$R_1 = \frac{\bar{Y}}{\bar{X}} = R$
2.	$t_2 = \frac{\hat{\bar{Y}}}{\left(\bar{x}+C_x\right)}\left(\bar{X}+C_x\right)$ Kadilar and Cingi (2004)	$\theta\left[R_2^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right]$	$R_2 = \frac{\bar{Y}}{\left(\bar{X}+C_x\right)}$
3.	$t_3 = \frac{\hat{\bar{Y}}}{\left(\bar{x}+\beta_2(x)\right)}\left(\bar{X}+\beta_2(x)\right)$ Kadilar and Cingi (2004)	$\theta\left[R_3^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right]$	$R_3 = \frac{\bar{Y}}{\bar{X}+\beta_2(x)}$
4.	$t_4 = \frac{\hat{\bar{Y}}}{\left(\bar{x}\beta_2(x)+C_x\right)}\left(\bar{X}\beta_2(x)+C_x\right)$ Kadilar and Cingi (2004)	$\theta\left[R_4^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right]$	$R_4 = \frac{\bar{Y}\beta_2(x)}{\left(\bar{X}\beta_2(x)+C_x\right)}$
5.	$t_5 = \frac{\hat{\bar{Y}}}{\left(\bar{x}C_x+\beta_2(x)\right)}\left(\bar{X}C_x+\beta_2(x)\right)$ Kadilar and Cingi (2004)	$\theta\left[R_5^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right]$	$R_5 = \frac{\bar{Y}C_x}{\left(\bar{X}C_x+\beta_2(x)\right)}$
6.	$t_6 = \frac{\hat{\bar{Y}}}{\left(\bar{x}+\rho\right)}\left(\bar{X}+\rho\right)$ Kadilar and Cingi (2006) –type	$\theta\left[R_6^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right]$	$R_6 = \frac{\bar{Y}}{\bar{X}+\rho}$
7.	$t_7 = \frac{\hat{\bar{Y}}}{\left(\bar{x}C_x+\rho\right)}\left(\bar{X}C_x+\rho\right)$ Kadilar and Cingi (2006) –type	$\theta\left[R_7^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right]$	$R_7 = \frac{\bar{Y}C_x}{\left(\bar{X}C_x+\rho\right)}$
8.	$t_8 = \frac{\hat{\bar{Y}}}{\left(\bar{x}\rho+C_x\right)}\left(\bar{X}\rho+C_x\right)$ Kadilar and Cingi (2006) –type	$\theta\left[R_8^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right]$	$R_8 = \frac{\bar{Y}\rho}{\left(\bar{X}\rho+C_x\right)}$
9.	$t_9 = \frac{\hat{\bar{Y}}}{\left(\bar{x}\beta_2(x)+\rho\right)}\left(\bar{X}\beta_2(x)+\rho\right)$ Kadilar and Cingi (2006) –type	$\theta\left[R_9^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right]$	$R_9 = \frac{\bar{Y}\beta_2(x)}{\left(\bar{X}\beta_2(x)+\rho\right)}$
10.	$t_{10} = \frac{\hat{\bar{Y}}}{\left(\bar{x}\rho+\beta_2(x)\right)}\left(\bar{X}\rho+\beta_2(x)\right)$ Kadilar and Cingi (2006) –type	$\theta\left[R_{10}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right]$	$R_{10} = \frac{\bar{Y}\rho}{\left(\bar{X}\rho+\beta_2(x)\right)}$
11.	$t_{11} = \frac{\hat{\bar{Y}}}{\left(\bar{x}+\beta_1(x)\right)}\left(\bar{X}+\beta_1(x)\right)$ Yan and Tian (2010)	$\theta\left[R_{11}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right]$	$R_{11} = \frac{\bar{Y}}{\left(\bar{X}+\beta_1(x)\right)}$

Table 1. *continued*

S.No.	Estimator	MSE of (.)	Population Ratio
12.	$t_{12} = \frac{\hat{\bar{Y}}}{\left(\bar{x}\beta_1(x)+\beta_2(x)\right)}\left(\bar{X}\beta_1(x)+\beta_2(x)\right)$ Yan and Tian (2010)	$\theta\left[R_{12}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{12} = \frac{\bar{Y}\beta_1(x)}{\left(\bar{X}\beta_1(x)+\beta_2(x)\right)}$
13.	$t_{13} = \frac{\hat{\bar{Y}}}{\left(\bar{x}+M_d\right)}\left(\bar{X}+M_d\right)$ Subramani and Kumarapandian (2012a)	$\theta\left[R_{13}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{13} = \frac{\bar{Y}}{\left(\bar{X}+M_d\right)}$
14.	$t_{14} = \frac{\hat{\bar{Y}}}{\left(C_x\bar{x}+M_d\right)}\left(\bar{X}C_x+M_d\right)$ Subramani and Kumarapandiyan (2012a)	$\theta\left[R_{14}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{14} = \frac{\bar{Y}C_x}{\left(\bar{X}C_x+M_d\right)}$
15.	$t_{15} = \frac{\hat{\bar{Y}}}{\left(\beta_1(x)\bar{x}+M_d\right)}\left(\bar{X}\beta_1(x)+M_d\right)$ Subramani and Kumarapandiyan (2012b)	$\theta\left[R_{15}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{15} = \frac{\beta_1(x)\bar{Y}}{\left(\bar{X}\beta_1(x)+M_d\right)}$
16.	$t_{16} = \frac{\hat{\bar{Y}}}{\left(\beta_2(x)\bar{x}+M_d\right)}\left(\bar{X}\beta_2(x)+M_d\right)$ Subramani and Kumarapandiyan (2012c)	$\theta\left[R_{16}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{16} = \frac{\beta_2(x)\bar{Y}}{\left(\bar{X}\beta_2(x)+M_d\right)}$
17.	$t_{17} = \frac{\hat{\bar{Y}}}{\left(\bar{x}+D_1\right)}\left(\bar{X}+D_1\right)$ Subramani and Kumarapandiyan (2012d)	$\theta\left[R_{17}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{17} = \frac{\bar{Y}}{\left(\bar{X}+D_1\right)}$
18.	$t_{18} = \frac{\hat{\bar{Y}}}{\left(\bar{x}+D_2\right)}\left(\bar{X}+D_2\right)$ Subramani and Kumarapandiyan (2012d)	$\theta\left[R_{18}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{18} = \frac{\bar{Y}}{\left(\bar{X}+D_2\right)}$
19.	$t_{19} = \frac{\hat{\bar{Y}}}{\left(\bar{x}+D_3\right)}\left(\bar{X}+D_3\right)$ Subramani and Kumarapandiyan (2012d)	$\theta\left[R_{19}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{19} = \frac{\bar{Y}}{\left(\bar{X}+D_3\right)}$
20.	$t_{20} = \frac{\hat{\bar{Y}}}{\left(\bar{x}+D_4\right)}\left(\bar{X}+D_4\right)$ Subramani and Kumarapandiyan (2012d)	$\theta\left[R_{20}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{20} = \frac{\bar{Y}}{\left(\bar{X}+D_4\right)}$
21.	$t_{21} = \frac{\hat{\bar{Y}}}{\left(\bar{x}+D_5\right)}\left(\bar{X}+D_5\right)$ Subramani and Kumarapandiyan(2012d)	$\theta\left[R_{21}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{21} = \frac{\bar{Y}}{\left(\bar{X}+D_5\right)}$
22.	$t_{22} = \frac{\hat{\bar{Y}}}{\left(\bar{x}+D_6\right)}\left(\bar{X}+D_6\right)$ Subramani and Kumarapandiyan (2012d)	$\theta\left[R_{22}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{22} = \frac{\bar{Y}}{\left(\bar{X}+D_6\right)}$

continued

Table 1. *continued*

S.No.	Estimator	MSE of (.)	Population Ratio
23.	$t_{23}=\frac{\hat{\bar{Y}}}{(\bar{x}+D7)}(\bar{X}+D_7)$ Subramani and Kumarapandiyan (2012d)	$\theta\left[R_{23}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{23}=\frac{\bar{Y}}{(\bar{X}+D_7)}$
24.	$t_{24}=\frac{\hat{\bar{Y}}}{(\bar{x}+D_8)}(\bar{X}+D_8)$ Subramani and Kumarapandiyan (2012d)	$\theta\left[R_{24}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{24}=\frac{\bar{Y}}{(\bar{X}+D_8)}$
25.	$t_{25}=\frac{\hat{\bar{Y}}}{(\bar{x}+D_9)}(\bar{X}+D_9)$ Subramani and Kumarapandiyan (2012d)	$\theta\left[R_{25}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{25}=\frac{\bar{Y}}{(\bar{X}+D_9)}$
26.	$t_{26}=\frac{\hat{\bar{Y}}}{(\bar{x}+D_{10})}(\bar{X}+D_{10})$ Subramani and Kumarapandiyan (2012d)	$\theta\left[R_{26}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{26}=\frac{\bar{Y}}{(\bar{X}+D_{10})}$
27.	$t_{27}=\frac{\hat{\bar{Y}}}{(\bar{x}\beta_1(x)+Q_d)}(\bar{X}\beta_1(x)+Q_d)$ Jeelani et al (2013)	$\theta\left[R_{27}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{27}=\frac{\bar{Y}\beta_1(x)}{(\bar{X}\beta_1(x)+Q_d)}$
28.	$t_{28}=\hat{\bar{Y}}\frac{(\rho\bar{X}+M_d)}{(\rho\bar{x}+M_d)}$ Subramani and Kumarapandiyan (2014)	$\theta\left[R_{28}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{28}=\frac{\rho\bar{Y}}{(\rho\bar{X}+M_d)}$
29.	$t_{29}=\frac{\hat{\bar{Y}}}{(\beta_2(x)\bar{x}+Q_1)}(\bar{X}\beta_2(x)+Q_1)$ Subramani et al (2014)	$\theta\left[R_{29}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{29}=\frac{\beta_2(x)\bar{Y}}{(\beta_2(x)\bar{X}+Q_1)}$
30.	$t_{30}=\frac{\hat{\bar{Y}}}{(\bar{x}\beta_2(x)+Q_3)}(\bar{X}\beta_2(x)+Q_3)$ Subramani et al (2014)	$\theta\left[R_{30}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{30}=\frac{\beta_2(x)\bar{Y}}{(\beta_2(x)\bar{X}+Q_3)}$
31.	$t_{31}=\frac{\hat{\bar{Y}}}{(\bar{x}\beta_2(x)+Q_r)}(\bar{X}\beta_2(x)+Q_r)$ Subramani et al (2014)	$\theta\left[R_{31}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{31}=\frac{\beta_2(x)\bar{Y}}{(\beta_2(x)\bar{X}+Q_r)}$
32.	$t_{32}=\frac{\hat{\bar{Y}}}{(\bar{x}\beta_2(x)+Q_d)}(\bar{X}\beta_2(x)+Q_d)$ Subramani et al (2014)	$\theta\left[R_{32}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{32}=\frac{\beta_2(x)\bar{Y}}{(\beta_2(x)\bar{X}+Q_d)}$
33.	$t_{33}=\frac{\hat{\bar{Y}}}{(\bar{x}\beta_2(x)+Q_a)}(\bar{X}\beta_2(x)+Q_a)$ Subramani et al (2014)	$\theta\left[R_{33}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{33}=\frac{\beta_2(x)\bar{Y}}{(\beta_2(x)\bar{X}+Q_a)}$

Table 1. *continued*

S.No.	Estimator	MSE of (.)	Population Ratio
34.	$t_{34}=\frac{\hat{\bar{Y}}}{\left(\bar{x}\beta_1(x)+Q_1\right)}\left(\bar{X}\beta_1(x)+Q_1\right)$ Subramani et al (2014)	$\theta\left[R_{34}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{34}=\frac{\beta_1(x)\bar{Y}}{\left(\beta_1(x)\bar{X}+Q_1\right)}$
35.	$t_{35}=\frac{\hat{\bar{Y}}}{\left(\bar{x}\beta_1(x)+Q_3\right)}\left(\bar{X}\beta_1(x)+Q_3\right)$ Subramani et al (2014)	$\theta\left[R_{35}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{35}=\frac{\beta_1(x)\bar{Y}}{\left(\beta_1(x)\bar{X}+Q_3\right)}$
36.	$t_{36}=\frac{\hat{\bar{Y}}}{\left(\bar{x}\beta_1(x)+Q_r\right)}\left(\bar{X}\beta_1(x)+Q_r\right)$ Subramani et al (2014)	$\theta\left[R_{36}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{36}=\frac{\beta_1(x)\bar{Y}}{\left(\beta_1(x)\bar{X}+Q_r\right)}$
37.	$t_{37}=\frac{\hat{\bar{Y}}}{\left(\bar{x}\beta_1(x)+Q_a\right)}\left(\bar{X}\beta_1(x)+Q_a\right)$ Subramani et al (2014)	$\theta\left[R_{37}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{37}=\frac{\beta_1(x)\bar{Y}}{\left(\beta_1(x)\bar{X}+Q_a\right)}$
38.	$t_{38}=\frac{\hat{\bar{Y}}}{\left(\rho\bar{x}+Q_1\right)}\left(\rho\bar{X}+Q_1\right)$ Subramani et al (2014)	$\theta\left[R_{38}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{38}=\frac{\rho\bar{Y}}{\left(\rho\bar{X}+Q_1\right)}$
39.	$t_{39}=\frac{\hat{\bar{Y}}}{\left(\rho\bar{x}+Q_3\right)}\left(\rho\bar{X}+Q_3\right)$ Subramani et al (2014)	$\theta\left[R_{39}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{39}=\frac{\rho\bar{Y}}{\left(\rho\bar{X}+Q_3\right)}$
40.	$t_{40}=\frac{\hat{\bar{Y}}}{\left(\rho\bar{x}+Q_r\right)}\left(\rho\bar{X}+Q_r\right)$ Subramani et al (2014)	$\theta\left[R_{40}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{40}=\frac{\rho\bar{Y}}{\left(\rho\bar{X}+Q_r\right)}$
41.	$t_{41}=\frac{\hat{\bar{Y}}}{\left(\rho\bar{x}+Q_d\right)}\left(\rho\bar{X}+Q_d\right)$ Subramani et al (2014)	$\theta\left[R_{41}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{41}=\frac{\rho\bar{Y}}{\left(\rho\bar{X}+Q_d\right)}$
42.	$t_{42}=\frac{\hat{\bar{Y}}}{\left(\rho\bar{x}+Q_a\right)}\left(\rho\bar{X}+Q_a\right)$ Subramani et al (2014)	$\theta\left[R_{42}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{42}=\frac{\rho\bar{Y}}{\left(\rho\bar{X}+Q_a\right)}$
43.	$t_{43}=\frac{\hat{\bar{Y}}}{\left(\bar{x}+T_m\right)}\left(\bar{X}+T_m\right)$ Abid et al (2016a)	$\theta\left[R_{43}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{43}=\frac{\bar{Y}}{\left(\bar{X}+T_m\right)}$
44.	$t_{44}=\frac{\hat{\bar{Y}}}{\left(\bar{x}C_x+T_m\right)}\left(\bar{X}C_x+T_m\right)$ Abid et al (2016a)	$\theta\left[R_{44}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{44}=\frac{\bar{Y}C_x}{\left(\bar{X}C_x+T_m\right)}$

continued

Table 1. *continued*

S.No.	Estimator	MSE of (.)	Population Ratio
45.	$t_{45}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+T_m)}(\bar{X}\rho+T_m)$ Abid et al (2016a)	$\theta\left[R_{45}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{45}=\frac{\bar{Y}\rho}{(\bar{X}\rho+T_m)}$
46.	$t_{46}=\frac{\hat{\bar{Y}}}{(\bar{x}+M_r)}(\bar{X}+M_r)$ Abid et al (2016a)	$\theta\left[R_{46}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{46}=\frac{\bar{Y}}{(\bar{X}+M_r)}$
47.	$t_{47}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+M_r)}(\bar{X}C_x+M_r)$ Abid et al (2016a)	$\theta\left[R_{47}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{47}=\frac{\bar{Y}C_x}{(\bar{X}+M_r)}$
48.	$t_{48}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+M_r)}(\bar{X}\rho+M_r)$ Abid et al (2016a)	$\theta\left[R_{48}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{48}=\frac{\bar{Y}\rho}{(\bar{X}\rho+M_r)}$
49.	$t_{49}=\frac{\hat{\bar{Y}}}{(\bar{x}+H_l)}(\bar{X}+H_l)$ Abid et al (2016a)	$\theta\left[R_{49}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{49}=\frac{\bar{Y}}{(\bar{X}+H_l)}$
50.	$t_{50}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+H_l)}(\bar{X}C_x+H_l)$ Abid et al (2016a)	$\theta\left[R_{50}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{50}=\frac{\bar{Y}C_x}{(\bar{X}C_x+H_l)}$
51.	$t_{51}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+H_l)}(\bar{X}\rho+H_l)$ Abid et al (2016a)	$\theta\left[R_{51}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{51}=\frac{\bar{Y}\rho}{(\bar{X}\rho+H_l)}$
52.	$t_{52}=\frac{\hat{\bar{Y}}}{(\bar{x}+G)}(\bar{X}+G)$ Abid et al (2016b)	$\theta\left[R_{52}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{52}=\frac{\bar{Y}}{(\bar{X}+G)}$
53.	$t_{53}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+G)}(\bar{X}\rho+G)$ Abid et al (2016b)	$\theta\left[R_{53}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{53}=\frac{\bar{Y}\rho}{(\bar{X}\rho+G)}$
54.	$t_{54}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+G)}(\bar{X}C_x+G)$ Abid et al (2016b)	$\theta\left[R_{54}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{54}=\frac{\bar{Y}C_x}{(\bar{X}C_x+G)}$
55.	$t_{55}=\frac{\hat{\bar{Y}}}{(\bar{x}+D)}(\bar{X}+D)$ Abid et al (2016b)	$\theta\left[R_{55}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{55}=\frac{\bar{Y}}{(\bar{X}+D)}$

Table 1. *continued*

S.No.	Estimator	MSE of (.)	Population Ratio
56.	$t_{56}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+D)}(\bar{X}\rho+D)$ Abid et al (2016b)	$\theta\left[R_{56}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{56}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D)}$
57.	$t_{57}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+D)}(\bar{X}C_x+D)$ Abid et al (2016b)	$\theta\left[R_{57}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{57}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D)}$
58.	$t_{58}=\frac{\hat{\bar{Y}}}{(\bar{x}+S_{pw})}(\bar{X}+S_{pw})$ Abid et al (2016b)	$\theta\left[R_{58}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{58}=\frac{\bar{Y}\rho}{(\bar{X}+S_{pw})}$
59.	$t_{59}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+S_{pw})}(\bar{X}\rho+S_{pw})$ Abid et al (2016b)	$\theta\left[R_{59}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{59}=\frac{\bar{Y}\rho}{(\bar{X}\rho+S_{pw})}$
60.	$t_{60}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+S_{pw})}(\bar{X}C_x+S_{pw})$ Abid et al (2016b)	$\theta\left[R_{60}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{60}=\frac{\bar{Y}C_x}{(\bar{X}C_x+S_{pw})}$
61.	$t_{61}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+D_1)}(\bar{X}\rho+D_1)$ Abid et al (2016c)	$\theta\left[R_{61}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{61}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_1)}$
62.	$t_{62}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+D_2)}(\bar{X}\rho+D_2)$ Abid et al (2016c)	$\theta\left[R_{62}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{62}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_2)}$
63.	$t_{63}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+D_3)}(\bar{X}\rho+D_3)$ Abid et al (2016c)	$\theta\left[R_{63}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{63}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_3)}$
64.	$t_{64}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+D_4)}(\bar{X}\rho+D_4)$ Abid et al (2016c)	$\theta\left[R_{64}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{64}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_4)}$
65.	$t_{65}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+D_5)}(\bar{X}\rho+D_5)$ Abid et al (2016c)	$\theta\left[R_{65}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{65}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_5)}$
66.	$t_{66}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+D_6)}(\bar{X}\rho+D_6)$ Abid et al (2016c)	$\theta\left[R_{66}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{66}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_6)}$

continued

Table 1. *continued*

S.No.	Estimator	MSE of (.)	Population Ratio
67.	$t_{67}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+D_7)}(\bar{X}\rho+D_7)$ Abid et al (2016c)	$\theta\left[R_{67}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{67}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_7)}$
68.	$t_{68}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+D_8)}(\bar{X}\rho+D_8)$ Abid et al (2016c)	$\theta\left[R_{68}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{68}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_8)}$
69.	$t_{69}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+D_9)}(\bar{X}\rho+D_9)$ Abid et al (2016c)	$\theta\left[R_{69}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{69}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_9)}$
70.	$t_{70}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+D_{10})}(\bar{X}\rho+D_{10})$ Abid et al (2016c)	$\theta\left[R_{70}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{70}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_{10})}$
71.	$t_{71}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+D_1)}(\bar{X}C_x+D_1)$ Abid et al (2016c)	$\theta\left[R_{71}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{71}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_1)}$
72.	$t_{72}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+D_2)}(\bar{X}C_x+D_2)$ Abid et al (2016c)	$\theta\left[R_{72}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{72}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_2)}$
73.	$t_{73}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+D_3)}(\bar{X}C_x+D_3)$ Abid et al (2016c)	$\theta\left[R_{73}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{73}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_3)}$
74.	$t_{74}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+D_4)}(\bar{X}C_x+D_4)$ Abid et al (2016c)	$\theta\left[R_{74}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{74}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_4)}$
75.	$t_{75}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+D_5)}(\bar{X}C_x+D_5)$ Abid et al (2016c)	$\theta\left[R_{75}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{76}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_5)}$
76.	$t_{76}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+D_6)}(\bar{X}C_x+D_6)$ Abid et al (2016c)	$\theta\left[R_{76}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{76}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_6)}$
77.	$t_{77}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+D_7)}(\bar{X}C_x+D_7)$ Abid et al (2016c)	$\theta\left[R_{77}^2S_x^2+S_y^2(1-\rho^2)\right]$	$R_{77}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_7)}$

Table 1. *continued*

S.No.	Estimator	MSE of (.)	Population Ratio
78.	$t_{78}=\frac{\hat{\bar{Y}}}{\left(\bar{x}C_x+D_8\right)}\left(\bar{X}C_x+D_8\right)$ Abid et al (2016)	$\theta\left[R_{78}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{78}=\frac{\bar{Y}C_x}{\left(\bar{X}C_x+D_8\right)}$
79.	$t_{79}=\frac{\hat{\bar{Y}}}{\left(\bar{x}C_x+D_9\right)}\left(\bar{X}C_x+D_9\right)$ Abid et al (2016c)	$\theta\left[R_{79}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{79}=\frac{\bar{Y}C_x}{\left(\bar{X}C_x+D_9\right)}$
80.	$t_{80}=\frac{\hat{\bar{Y}}}{\left(\bar{x}C_x+D_{10}\right)}\left(\bar{X}C_x+D_{10}\right)$ Abid et al (2016c)	$\theta\left[R_{80}^2S_x^2+S_y^2\left(1-\rho^2\right)\right]$	$R_{80}=\frac{\bar{Y}C_x}{\left(\bar{X}C_x+D_{10}\right)}$

$$E\left(e_1e_3\right)=\frac{N\left(N-n\right)}{\left(N-1\right)\left(N-2\right)}\frac{\mu_{30}}{n}\frac{1}{\bar{X}S_x^2}=\frac{\left(N-n\right)}{n\left(N-2\right)}\frac{\mu_{30}}{\bar{X}\mu_{20}},$$

where $\mu_{rs}=E\left[\left(x_i-\bar{X}\right)^r\left(y_i-\bar{Y}\right)^s\right]$, $C=\rho_{yx}\frac{C_y}{C_x}$, $C_y=\frac{S_y}{\bar{Y}}$, $C_x=\frac{S_x}{\bar{X}}$ and

$\rho_{yx}=\frac{S_{xy}}{\left(S_xS_y\right)}$, (r,s) being non-negative integers.

Expressing ‘t’ defined by (6) in terms of e’s

$$t=\bar{Y}\left[1+e_0-\left(\frac{\beta\bar{X}}{\bar{Y}}\right)e_1\left(1+e_2\right)\left(1+e_3\right)^{-1}\right]\left(1+\tau e_1\right)^{-1}.$$

where $$\tau=\frac{\left(a\bar{X}\right)}{\left(a\bar{X}+b\right)}.$$

Assume $\left|e_1\right|<1$ and $\left|e_3\right|<1$ so that we $\left(1+e_3\right)^{-1}$ and $\left(1+\tau e_1\right)^{-1}$ are expandable. Expanding the right hand side of (7), multiplying and neglecting terms of *e's* having power greater than two we have

$$t=\bar{Y}\left[1+e_0-\tau e_1+\tau^2e_1^2-\tau e_0e_1-c\left(e_1+e_1e_2-e_1e_3-\tau e_1^2\right)\right]$$

Table 2. Some unknown members of the class of ratio type estimators *t*.

S.No.	Estimator	Values of constants a	b
1.	$t_1^* = \hat{\bar{Y}}\frac{(\beta_1(x)\bar{X}+\rho)}{(\beta_1(x)\bar{x}+\rho)}$	$\beta_1(x)$	ρ
2.	$t_2^* = \hat{\bar{Y}}\frac{(\beta_1(x)\bar{X}+C_x)}{(\beta_1(x)\bar{x}+C_x)}$	$\beta_1(x)$	C_x
3.	$t_3^* = \hat{\bar{Y}}\frac{(\bar{X}C_x+\beta_1(x))}{\bar{x}C_x+\beta_1(x)}$	C_x	$\beta_1(x)$
4.	$t_4^* = \hat{\bar{Y}}\left(\frac{\bar{X}\beta_2(x)+\beta_1(x)}{\bar{x}\beta_2(x)+\beta_1(x)}\right)$	$\beta_2(x)$	$\beta_1(x)$
5.	$t_5^* = \hat{\bar{Y}}\left(\frac{\bar{X}\rho+\beta_1(x)}{\bar{x}\rho+\beta_1(x)}\right)$	ρ	$\beta_1(x)$
7.	$t_7^* = \hat{\bar{Y}}\left(\frac{M_d\bar{X}+C_x}{\bar{x}M_d+C_x}\right)$	M_d	C_x
8.	$t_8^* = \hat{\bar{Y}}\left(\frac{M_d\bar{X}+\beta_2(x)}{\bar{x}M_d+\beta_2(x)}\right)$	M_d	$\beta_2(x)$
9.	$t_9^* = \hat{\bar{Y}}\left(\frac{\bar{X}M_d+\beta_1(x)}{\bar{x}M_d+\beta_1(x)}\right)$	M_d	$\beta_1(x)$
10.	$t_{10}^* = \hat{\bar{Y}}\left(\frac{M_d\bar{X}+\rho}{\bar{x}M_d+\rho}\right)$	M_d	ρ
11.	$t_{11}^* = \hat{\bar{Y}}\left(\frac{\bar{X}+Q_d}{\bar{x}+Q_d}\right)$	1	Q_d
12.	$t_{12}^* = \hat{\bar{Y}}\left(\frac{\bar{X}C_x+Q_d}{\bar{x}C_x+Q_d}\right)$	C_x	Q_d
14.	$t_{14}^* = \hat{\bar{Y}}\left(\frac{\bar{X}M_d+Q_d}{\bar{x}M_d+Q_d}\right)$	M_d	Q_d
15.	$t_{15}^* = \hat{\bar{Y}}\left(\frac{Q_d\bar{X}+C_x}{Q_d\bar{x}+C_x}\right)$	Q_d	C_x
16.	$t_{16}^* = \hat{\bar{Y}}\left(\frac{Q_d\bar{X}+\beta_2(x)}{Q_d\bar{x}+\beta_2(x)}\right)$	Q_d	$\beta_2(x)$

continued

Table 2. *continued*

S.No.	Estimator	Values of constants	
		a	b
17.	$t_{17}^{*}=\hat{\bar{Y}}\left(\frac{Q_d\bar{X}+\beta_1(x)}{Q_d\bar{x}+\beta_1(x)}\right)$	Q_d	$\beta_1(x)$
18.	$t_{18}^{*}=\hat{\bar{Y}}\left(\frac{Q_d\bar{X}+\rho}{Q_d\bar{x}+\rho}\right)$	Q_d	ρ
19.	$t_{19}^{*}=\hat{\bar{Y}}\left(\frac{Q_d\bar{X}+M_d}{Q_d\bar{x}+M_d}\right)$	Q_d	M_d
20.	$t_{20}^{*}=\hat{\bar{Y}}\left(\frac{\beta_1(x)\bar{X}+Q_d}{\beta_1(x)\bar{x}+Q_d}\right)$ Kumarapandiyan and Subramani (2016)-type	$\beta_1(x)$	Q_d
21.	$t_{21}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_2(x)+T_m}{\bar{x}\beta_2(x)+T_m}\right)$	$\beta_2(x)$	T_m
22.	$t_{22}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_1(x)+T_m}{\bar{x}\beta_1(x)+T_m}\right)$	$\beta_1(x)$	T_m
23.	$t_{23}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_d+T_m}{\bar{x}M_d+T_m}\right)$	M_d	T_m
24.	$t_{24}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_d+T_m}{\bar{x}Q_d+T_m}\right)$	Q_d	T_m
25.	$t_{25}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}T_m+C_x}{\bar{x}T_m+C_x}\right)$	T_m	C_x
26.	$t_{26}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}T_m+\beta_2(x)}{\bar{x}T_m+\beta_2(x)}\right)$	T_m	$\beta_2(x)$
27.	$t_{27}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}T_m+\rho}{\bar{x}T_m+\rho}\right)$	T_m	ρ
28.	$t_{28}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}T_m+\beta_1(x)}{\bar{x}T_m+\beta_1(x)}\right)$	T_m	$\beta_1(x)$
29.	$t_{29}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}T_m+M_d}{\bar{x}T_m+M_d}\right)$	T_m	M_d

Table 2. *continued*

S.No.	Estimator	Values of constants	
		a	b
30.	$t_{30}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}T_m+Q_d}{\bar{x}T_m+Q_d}\right)$	T_m	Q_d
31.	$t_{31}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_2(x)+M_r}{\bar{x}\beta_2(x)+M_r}\right)$	$\beta_2(x)$	M_r
32.	$t_{32}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_1(x)+M_r}{\bar{x}\beta_1(x)+M_r}\right)$	$\beta_1(x)$	M_r
33.	$t_{33}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_d+M_r}{\bar{x}M_d+M_r}\right)$	M_d	M_r
34.	$t_{34}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_d+M_r}{\bar{x}Q_d+M_r}\right)$	Q_d	M_r
35.	$t_{35}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_r+C_x}{\bar{x}M_r+C_x}\right)$	M_r	C_x
36.	$t_{36}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_r+\beta_2(x)}{\bar{x}M_r+\beta_2(x)}\right)$	M_r	$\beta_2(x)$
37.	$t_{37}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_r+\rho}{\bar{x}M_r+\rho}\right)$	M_r	ρ
38.	$t_{38}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_r+\beta_1(x)}{\bar{x}M_r+\beta_1(x)}\right)$	M_r	$\beta_1(x)$
39.	$t_{39}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_r+M_d}{\bar{x}M_r+M_d}\right)$	M_r	M_d
40.	$t_{40}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_r+Q_d}{\bar{x}M_r+Q_d}\right)$	M_r	Q_d
41.	$t_{41}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}T_m+M_r}{\bar{x}T_m+M_r}\right)$	T_m	M_r
42.	$t_{42}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_r+T_m}{\bar{x}M_r+T_m}\right)$	M_r	T_m
43.	$t_{43}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_2(x)+H_l}{\bar{x}\beta_2(x)+H_l}\right)$	$\beta_2(x)$	H_l

continued

Table 2. *continued*

S.No.	Estimator	Values of constants	
		a	b
44.	$t_{44}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_1(x)+H_l}{\bar{x}\beta_1(x)+H_l}\right)$	$\beta_1(x)$	H_l
45.	$t_{45}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_d+H_l}{\bar{x}M_d+H_l}\right)$	M_d	H_l
46.	$t_{46}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_d+H_l}{\bar{x}Q_d+H_l}\right)$	Q_d	H_l
47.	$t_{47}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}H_l+C_x}{\bar{x}H_l+C_x}\right)$	H_l	C_x
48.	$t_{48}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}H_l+\beta_2(x)}{\bar{x}H_l+\beta_2(x)}\right)$	H_l	$\beta_2(x)$
49.	$t_{49}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}H_l+\rho}{\bar{x}H_l+\rho}\right)$	H_l	ρ
50.	$t_{50}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}H_l+\beta_1(x)}{\bar{x}H_l+\beta_1(x)}\right)$	H_l	$\beta_1(x)$
51.	$t_{51}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}H_l+M_d}{\bar{x}H_l+M_d}\right)$	H_l	M_d
52.	$t_{52}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}H_l+Q_d}{\bar{x}H_l+Q_d}\right)$	H_l	Q_d
53.	$t_{53}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}H_l+M_r}{\bar{x}H_l+M_r}\right)$	H_l	M_r
54.	$t_{54}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}T_m+H_l}{\bar{x}T_m+H_l}\right)$	T_m	H_l
55.	$t_{55}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}H_l+T_m}{\bar{x}H_l+T_m}\right)$	H_l	T_m
56.	$t_{56}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_r+H_l}{\bar{x}M_r+H_l}\right)$	M_r	H_l
57.	$t_{57}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}+Q_a}{\bar{x}+Q_a}\right)$	1	Q_a

Table 2. *continued*

S.No.	Estimator	Values of constants	
		a	b
58.	$t_{58}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}C_x+Q_a}{\bar{x}C_x+Q_a}\right)$	C_x	Q_a
62.	$t_{62}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_d+Q_a}{\bar{x}M_d+Q_a}\right)$	M_d	Q_a
63.	$t_{63}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_d+Q_a}{\bar{x}Q_d+Q_a}\right)$	Q_d	Q_a
64.	$t_{64}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_d+C_x}{\bar{x}Q_d+C_x}\right)$	Q_d	C_x
65.	$t_{65}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_a+\beta_2(x)}{\bar{x}Q_a+\beta_2(x)}\right)$	Q_a	$\beta_2(x)$
66.	$t_{66}^{*}=\hat{\bar{Y}}\left(\frac{Q_a\bar{X}+\rho}{\bar{x}Q_a+\rho}\right)$	Q_a	ρ
67.	$t_{67}^{*}=\hat{\bar{Y}}\left(\frac{Q_a\bar{X}+\beta_1(x)}{\bar{x}Q_a+\beta_1(x)}\right)$	Q_a	$\beta_1(x)$
68.	$t_{68}^{*}=\hat{\bar{Y}}\left(\frac{Q_a\bar{X}+M_d}{\bar{x}Q_a+M_d}\right)$	Q_a	M_d
69.	$t_{69}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_a+Q_d}{\bar{x}Q_a+Q_d}\right)$	Q_a	Q_d
70.	$t_{70}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}T_m+Q_a}{\bar{x}T_m+Q_a}\right)$	T_m	Q_a
71.	$t_{71}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_a+T_m}{\bar{x}Q_a+T_m}\right)$	Q_a	T_m
72.	$t_{72}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_a+M_r}{\bar{x}Q_a+M_r}\right)$	Q_a	M_r
73.	$t_{73}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_r+Q_a}{\bar{x}M_r+Q_a}\right)$	M_r	Q_a
74.	$t_{74}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}H_l+Q_a}{\bar{x}H_l+Q_a}\right)$	H_l	Q_a

continued

Table 2. *continued*

S.No.	Estimator	Values of constants	
		a	b
75.	$t_{75}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_a+H_l}{\bar{x}Q_a+H_l}\right)$	Q_a	H_l
76.	$t_{76}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_r+Q_a}{\bar{x}Q_r+Q_a}\right)$	Q_r	Q_a
77.	$t_{77}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_a+Q_r}{\bar{x}Q_a+Q_r}\right)$	Q_a	Q_r
78.	$t_{78}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}+S_x}{\bar{x}+S_x}\right)$ Singh (2003)-type	1	S_x
79.	$t_{79}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}C_x+S_x}{\bar{x}C_x+S_x}\right)$	C_x	S_x
80.	$t_{80}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_2(x)+S_x}{\bar{x}\beta_2(x)+S_x}\right)$ Singh (2003)-type	$\beta_2(x)$	S_x
81.	$t_{81}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_1(x)+S_x}{\bar{x}\beta_1(x)+S_x}\right)$ Singh (2003) –type	$\beta_1(x)$	S_x
82.	$t_{82}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\rho+S_x}{\bar{x}\rho+S_x}\right)$	ρ	S_x
83.	$t_{83}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_d+S_x}{\bar{x}M_d+S_x}\right)$	M_d	S_x
84.	$t_{84}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_d+Q_a}{\bar{x}Q_d+Q_a}\right)$	Q_d	Q_a
85.	$t_{85}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+C_x}{\bar{x}S_x+C_x}\right)$	S_x	C_x
86.	$t_{86}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+\beta_2(x)}{\bar{x}S_x+\beta_2(x)}\right)$	S_x	$\beta_2(x)$
87.	$t_{87}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+\rho}{\bar{x}S_x+\rho}\right)$	S_x	ρ

Table 2. *continued*

S.No.	Estimator	Values of constants	
		a	b
88.	$t_{88}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+\beta_1(x)}{\bar{x}S_x+\beta_1(x)}\right)$	S_x	$\beta_1(x)$
89.	$t_{89}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+M_d}{\bar{x}S_x+M_d}\right)$	S_x	M_d
90.	$t_{90}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+Q_d}{\bar{x}S_x+Q_d}\right)$	S_x	Q_d
91.	$t_{91}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}T_m+S_x}{\bar{x}T_m+S_x}\right)$	T_m	S_x
92.	$t_{92}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+T_m}{\bar{x}S_x+T_m}\right)$	S_x	T_m
93.	$t_{93}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+M_r}{\bar{x}S_x+M_r}\right)$	S_x	M_r
94.	$t_{94}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_r+S_x}{\bar{x}M_r+S_x}\right)$	M_r	S_x
95.	$t_{95}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}H_l+S_x}{\bar{x}H_l+S_x}\right)$	H_l	S_x
96.	$t_{96}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+H_l}{\bar{x}S_x+H_l}\right)$	S_x	H_l
97.	$t_{97}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_r+S_x}{\bar{x}Q_r+S_x}\right)$	Q_r	S_x
98.	$t_{98}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+Q_r}{\bar{x}S_x+Q_r}\right)$	S_x	Q_r
99.	$t_{99}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_a+S_x}{\bar{x}Q_a+S_x}\right)$	Q_a	S_x
100.	$t_{100}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+Q_a}{\bar{x}S_x+Q_a}\right)$	S_x	Q_a
101.	$t_{101}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}+X}{\bar{x}+X}\right)$	1	$X\left(=N\bar{X}\right)$

continued

Table 2. *continued*

S.No.	Estimator	Values of constants	
		a	b
102.	$t_{102}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}C_x+X}{\bar{x}C_x+X}\right)$	C_x	$X\left(=N\bar{X}\right)$
103.	$t_{103}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_2(x)+X}{\bar{x}\beta_2(x)+X}\right)$	$\beta_2(x)$	$X\left(=N\bar{X}\right)$
104.	$t_{104}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_1(x)+X}{\bar{x}\beta_1(x)+X}\right)$	$\beta_1(x)$	$X\left(=N\bar{X}\right)$
105.	$t_{105}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\rho+X}{\bar{x}\rho+X}\right)$	ρ	$X\left(=N\bar{X}\right)$
102.	$t_{102}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}C_x+X}{\bar{x}C_x+X}\right)$	C_x	$X\left(=N\bar{X}\right)$
103.	$t_{103}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_2(x)+X}{\bar{x}\beta_2(x)+X}\right)$	$\beta_2(x)$	$X\left(=N\bar{X}\right)$
104.	$t_{104}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_1(x)+X}{\bar{x}\beta_1(x)+X}\right)$	$\beta_1(x)$	$X\left(=N\bar{X}\right)$
105.	$t_{105}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\rho+X}{\bar{x}\rho+X}\right)$	ρ	$X\left(=N\bar{X}\right)$
106.	$t_{106}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_d+X}{\bar{x}M_d+X}\right)$	M_d	$X\left(=N\bar{X}\right)$
107.	$t_{107}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_d+X}{\bar{x}Q_d+X}\right)$	Q_d	$X\left(=N\bar{X}\right)$
108.	$t_{108}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}X+C_x}{\bar{x}X+C_x}\right)$	$X\left(=N\bar{X}\right)$	C_x
109.	$t_{109}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}X+\beta_2(x)}{\bar{x}X+\beta_2(x)}\right)$	$X\left(=N\bar{X}\right)$	$\beta_2(x)$
110.	$t_{110}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}X+\rho}{\bar{x}X+\rho}\right)$	$X\left(=N\bar{X}\right)$	ρ
111.	$t_{111}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}X+\beta_1(x)}{\bar{x}X+\beta_1(x)}\right)$	$X\left(=N\bar{X}\right)$	$\beta_1(x)$

Table 2. *continued*

S.No.	Estimator	Values of constants	
		a	b
112.	$t_{112}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}X+M_d}{\bar{x}X+M_d}\right)$	$X\left(=N\bar{X}\right)$	M_d
113.	$t_{113}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}X+Q_d}{\bar{x}X+Q_d}\right)$	$X\left(=N\bar{X}\right)$	Q_d
114.	$t_{114}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}T_m+X}{\bar{x}T_m+X}\right)$	T_m	$X\left(=N\bar{X}\right)$
115.	$t_{115}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}X+T_m}{\bar{x}X+T_m}\right)$	$X\left(=N\bar{X}\right)$	T_m
116.	$t_{116}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}X+M_r}{\bar{x}X+M_r}\right)$	$X\left(=N\bar{X}\right)$	M_r
117.	$t_{117}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_r+X}{\bar{x}M_r+X}\right)$	M_r	$X\left(=N\bar{X}\right)$
118.	$t_{118}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}H_l+X}{\bar{x}H_l+X}\right)$	H_l	$X\left(=N\bar{X}\right)$
119.	$t_{119}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}X+H_l}{\bar{x}X+H_l}\right)$	$X\left(=N\bar{X}\right)$	H_l
120.	$t_{120}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_r+X}{\bar{x}Q_r+X}\right)$	Q_r	$X\left(=N\bar{X}\right)$
121.	$t_{121}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}X+Q_r}{\bar{x}X+Q_r}\right)$	$X\left(=N\bar{X}\right)$	Q_r
122.	$t_{122}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}X+S_x}{\bar{x}X+S_x}\right)$	$X\left(=N\bar{X}\right)$	S_x
123.	$t_{123}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+X}{\bar{x}S_x+X}\right)$	S_x	$X\left(=N\bar{X}\right)$
124.	$t_{124}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_a+X}{\bar{x}Q_a+X}\right)$	Q_a	$X\left(=N\bar{X}\right)$
125.	$t_{125}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}X+Q_a}{\bar{x}X+Q_a}\right)$	$X\left(=N\bar{X}\right)$	Q_a

continued

Table 2. *continued*

S.No.	Estimator	Values of constants	
		a	b
126.	$t_{126}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}+\Delta}{\bar{x}+\Delta}\right)$	1	Δ
127.	$t_{127}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}C_x+\Delta}{\bar{x}C_x+\Delta}\right)$	C_x	Δ
128.	$t_{128}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_2(x)+\Delta}{\bar{x}\beta_2(x)+\Delta}\right)$	$\beta_2(x)$	Δ
129.	$t_{129}^{*}=\hat{\bar{Y}}\setminus\left(\frac{\bar{X}\beta_1(x)+\Delta}{\bar{x}\beta_1(x)+\Delta}\right)$	$\beta_1(x)$	Δ
130.	$t_{130}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\rho+\Delta}{\bar{x}\rho+\Delta}\right)$	ρ	Δ
131.	$t_{131}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}M_d+\Delta}{\bar{x}M_d+\Delta}\right)$	M_d	Δ
132.	$t_{132}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_d+\Delta}{\bar{x}Q_d+\Delta}\right)$	Q_d	Δ
133.	$t_{133}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\Delta+C_x}{\bar{x}\Delta+C_x}\right)$	Δ	C_x
134.	$t_{134}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\Delta+\beta_2(x)}{\bar{x}\Delta+\beta_2(x)}\right)$	Δ	$\beta_2(x)$
135.	$t_{135}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\Delta+\rho}{\bar{x}\Delta+\rho}\right)$	Δ	ρ
136.	$t_{136}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}Q_a+\Delta}{\bar{x}Q_a+\Delta}\right)$	Q_a	Δ
137.	$t_{137}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\Delta+Q_a}{\bar{x}\Delta+Q_a}\right)$	Δ	Q_a
138.	$t_{138}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\Delta+S_x}{\bar{x}\Delta+S_x}\right)$	Δ	S_x
139.	$t_{139}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}S_x+\Delta}{\bar{x}S_x+\Delta}\right)$	S_x	Δ

Table 2. *continued*

S.No.	Estimator	Values of constants	
		a	b
140.	$t_{140}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}\Delta + \beta_1(x)}{\bar{x}\Delta + \beta_1(x)}\right)$	Δ	$\beta_1(x)$
141.	$t_{141}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}\Delta + M_d}{\bar{x}\Delta + M_d}\right)$	Δ	M_d
142.	$t_{142}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}\Delta + Q_d}{\bar{x}\Delta + Q_d}\right)$	Δ	Q_d
143.	$t_{143}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}T_m + \Delta}{\bar{x}T_m + \Delta}\right)$	T_m	Δ
144.	$t_{144}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}\Delta + T_m}{\bar{x}\Delta + T_m}\right)$	Δ	T_m
145.	$t_{145}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}\Delta + M_r}{\bar{x}\Delta + M_r}\right)$	Δ	M_r
146.	$t_{146}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}M_r + \Delta}{\bar{x}M_r + \Delta}\right)$	M_r	Δ
147.	$t_{147}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}H_l + \Delta}{\bar{x}H_l + \Delta}\right)$	H_l	Δ
148.	$t_{148}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}\Delta + H_l}{\bar{x}\Delta + H_l}\right)$	Δ	H_l
149.	$t_{149}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}Q_r + \Delta}{\bar{x}Q_r + \Delta}\right)$	Q_r	Δ
150.	$t_{150}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}\Delta + Q_r}{\bar{x}\Delta + Q_r}\right)$	Δ	Q_r
151.	$t_{151}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}\Delta + X}{\bar{x}\Delta + X}\right)$	Δ	$X\left(=N\bar{X}\right)$
152.	$t_{152}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X}X + \Delta}{\bar{x}X + \Delta}\right)$	$X\left(=N\bar{X}\right)$	Δ
153.	$t_{153}^{*} = \hat{\bar{Y}}\left(\frac{\bar{X} + Q_1}{\bar{x} + Q_1}\right)$ Al-Omar et al (2009)	1	Q_1

continued

Table 2. *continued*

S.No.	Estimator	Values of constants	
		a	b
154.	$t_{154}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}+Q_3}{\bar{x}+Q_3}\right)$ Al-Omar et al (2009)	1	Q_3
156.	$t_{156}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_1(x)+Q_2}{\bar{x}\beta_1+Q_2}\right)$ Kumarapandiyan and Subramani (2016)-type	$\beta_1(x)$	Q_2
159.	$t_{159}^{*}=\hat{\bar{Y}}\left(\frac{\bar{X}\beta_1(x)+Q_d}{\bar{x}\beta_1+Q_d}\right)$ Kumarapandiyan and Subramani (2016)-type	$\beta_1(x)$	Q_d

Or

$$\left(t-\bar{Y}\right)=\bar{Y}\left[e_0-\tau e_1+\tau^2e_1^2-\tau e_0e_1-c\left(e_1+e_1e_2-e_1e_3-\tau e_1^2\right)\right]. \tag{8}$$

Taking expectation of both sides of (2) we get the bias of 't' to the the first degree of approximation as

$$B(t)=\frac{(1-f)}{n}\left[R_J^2\frac{S_x^2}{\bar{Y}}-\frac{N}{(N-2)}\beta\left(\frac{\mu_{21}}{\mu_{11}}-\frac{\mu_{30}}{\mu_{20}}\right)\right]$$

$$=\frac{(1-f)}{n}(A-B) \tag{9}$$

where $R_J=\frac{a\bar{Y}}{\left(a\bar{X}+b\right)}$, $A=R_J^2\left(\frac{S_x^2}{\bar{Y}}\right)$ and $B=\frac{N}{(N-2)}\beta\left(\frac{\mu_{21}}{\mu_{11}}-\frac{\mu_{30}}{\mu_{20}}\right)$.

The correct biases of the estimators listed in Table 1 and 2 can be obtained from (9) just by putting the suitable values of (a, b). The biases of the estimators belonging to the class of estimators 't' is negligible if the sample size n is sufficiently large $(i.e.\quad n\rightarrow N)$. It should be noted that the biases of the estimators t_1 to t_{45} listed in Table 1 reported in Subramani and Kumarapandian (2012a,b,c,d) and Abid et al (2016a,b,c) are not correct.

Squaring both sides of (8) and neglecting terms of e's having power greater than two

$$\left(t-\bar{Y}\right)=\bar{Y}^{2}\left[e_0^2-\tau^2 e_1^2+C^2 e_1^2-2\tau e_0 e_1-2Ce_0 e+2\tau Ce_1^2\right] \tag{10}$$

Taking expectation of both sides of (10), obtain the MSE of '*t*' to the first degree of approximation as

$$MSE\left(t\right)=\frac{(1-f)}{n}\left[R_J^2 S_x^2+S_y^2\left(1-\rho^2\right)\right] \tag{11}$$

The MSE of the estimators belonging to class of estimators '*t*' can be obtained from (11) just by putting the suitable values of (*a, b*).

The proposed class of estimators '*t*' is more efficient than the usual unbiased estimator $\bar{y}$ if

$$\text{MSE }(t)<\text{MSE }\left(\bar{y}\right)$$

i.e. if

$$R_J^2 S_x^2<\beta^2 \quad . \tag{12}$$

The members of the proposed class of estimators '*t*' is better than the usual unbiased estimator $\bar{y}$ as along as the condition (12) is satisfied. Further from (5) and (11)

$$\text{MSE}\left(t\right)<\text{MSE }\left(\bar{y}_r\right)$$

i.e. if

$$R_J^2<\left(R-\beta\right)^2 . \tag{13}$$

The members of the proposed class of estimators '*t*' is more efficient than the usual ratio estimator $\bar{y}_R$ as long as the condition (13) is satisfied.

Suggested Class of Ratio-Type Exponential Estimators

Define a class of ratio-type exponential estimators for the population mean $\bar{Y}$ as

$$t_e=\hat{\bar{Y}}\exp\left\{\frac{a\left(\bar{X}-\bar{x}\right)}{a\left(\bar{X}+\bar{x}\right)+2b}\right\}$$

$$=\left\{\bar{y}+\hat{\beta}\left(\bar{X}-\bar{x}\right)\right\}\exp\left\{\frac{a\left(\bar{X}-\bar{x}\right)}{a\left(\bar{X}+\bar{x}\right)+2b}\right\}, \tag{14}$$

where (*a, b*) are same as defined for the class of estimators '*t*' at (1). A large number of estimators can be identified from the proposed class of estimators t_e for suitable values of (*a, b*). Some members of the proposed class of estimators t_e corresponding to the members of the class of estimators t are listed in Table 3.
Expressing t_e in terms of e's we have

$$t_e=\bar{Y}\left[1+e_0-Ce_1\left(1+e_2\right)\left(1+e_3\right)^{-1}\right]\exp\left\{-\frac{\tau e_1}{2}\left(1+\frac{\tau}{2}e_1\right)^{-1}\right\} \tag{15}$$

where,

$$C=\left(\frac{\beta\bar{X}}{\bar{Y}}\right)=\rho\frac{C_y}{C_x}.$$

Expanding the right hand side of (15), multiplying out and neglecting terms of e's having power greater than two we have

$$t_e=\bar{Y}\left[1+e_0-\frac{\tau e_1}{2}-C\left(e_1+e_1e_2-e_1e_3\right)-\frac{\tau e_0e_1}{2}+\frac{\tau}{8}(3\tau+4C)e_1^2\right]$$

or

$$\left(t_e-\bar{Y}\right)=\bar{Y}\left[e_0-\frac{\tau e_1}{2}-Ce_1-C\left(e_1e_2-e_1e_3\right)-\frac{\tau e_0e_1}{2}+\frac{\tau}{8}(3\tau+4C)e_1^2\right]. \tag{16}$$

Taking expectation of both sides of (16) we get the bias of 't_e'to the first degree of approximation, we have

$$B\left(t_e\right)=\frac{(1-f)}{n}\left[\frac{3}{8}R_J^2\left(\frac{S_x^2}{\bar{Y}}\right)-\frac{N}{(N-2)}\beta\left(\frac{\mu_{21}}{\mu_{11}}-\frac{\mu_{30}}{\mu_{20}}\right)\right],$$

$$=\frac{(1-f)}{n}\left(\frac{3}{8}A-B\right), \tag{17}$$

where

$$R_J=\frac{a\bar{Y}}{\left(a\bar{X}+b\right)}$$, A and B are same as defined earlier.

Table 3. Some members of the class of estimators t_e corresponding to the estimators listed in Table 1.

S.No.	Estimators	MSE	Values of Constants a	b	Population Ratio
1.	$t_{1e}=\hat{\bar{Y}}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	$\theta\left[R_1^2\frac{S_x^2}{4}+S_y^2\left(1-\rho^2\right)\right]$	1	0	$R_1=\frac{\bar{Y}}{\bar{X}}=R$
2.	$t_{2e}=\hat{\bar{Y}}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}+2C_x}\right)$	$\theta\left[R_2^2\frac{S_x^2}{4}+S_y^2\left(1-\rho^2\right)\right]$	1	C_x	$R_2=\frac{\bar{Y}}{\left(\bar{X}+C_x\right)}$
3.	$t_{3e}=\hat{\bar{Y}}\exp\left(\frac{\left(\bar{X}-\bar{x}\right)}{\bar{X}+\bar{x}+2\beta_2(x)}\right)$	$\theta\left[R_3^2\frac{S_x^2}{4}+S_y^2\left(1-\rho^2\right)\right]$	1	$\beta_2(x)$	$R_3=\frac{\bar{Y}}{\bar{X}+\beta_2(x)}$
4.	$t_{4e}=\hat{\bar{Y}}\exp\left(\frac{\left(\bar{X}-\bar{x}\right)\beta_2(x)}{\beta_2(x)\left(\bar{X}+\bar{x}\right)+2C_x}\right)$	$\theta\left[R_4^2\frac{S_x^2}{4}+S_y^2\left(1-\rho^2\right)\right]$	$\beta_2(x)$	C_x	$R_4=\frac{\bar{Y}\beta_2(x)}{\left(\bar{X}\beta_2(x)+C_x\right)}$
5.	$t_{5e}=\hat{\bar{Y}}\exp\left(\frac{C_x\left(\bar{X}-\bar{x}\right)}{C_x\left(\bar{X}+\bar{x}\right)+2\beta_2(x)}\right)$	$\theta\left[R_5^2\frac{S_x^2}{4}+S_y^2\left(1-\rho^2\right)\right]$	C_x	$\beta_2(x)$	$R_5=\frac{\bar{Y}C_x}{\left(\bar{X}C_x+\beta_2(x)\right)}$
6.	$t_{6e}=\hat{\bar{Y}}\exp\left(\frac{\left(\bar{X}-\bar{x}\right)}{\bar{X}+\bar{x}+2\rho}\right)$	$\theta\left[R_6^2\frac{S_x^2}{4}+S_y^2\left(1-\rho^2\right)\right]$	1	ρ	$R_6=\frac{\bar{Y}}{\bar{X}+\rho}$
7.	$t_{7e}=\hat{\bar{Y}}\exp\left(\frac{C_x\left(\bar{X}-\bar{x}\right)}{C_x\left(\bar{X}+\bar{x}\right)+2\rho}\right)$	$\theta\left[R_7^2\frac{S_x^2}{4}+S_y^2\left(1-\rho^2\right)\right]$	C_x	ρ	$R_7=\frac{\bar{Y}C_x}{\left(\bar{X}C_x+\rho\right)}$
8.	$t_{8e}=\hat{\bar{Y}}\exp\left(\frac{\rho\left(\bar{X}-\bar{x}\right)}{\rho\left(\bar{X}+\bar{x}\right)+2C_x}\right)$	$\theta\left[R_8^2\frac{S_x^2}{4}+S_y^2\left(1-\rho^2\right)\right]$	ρ	C_x	$R_8=\frac{\bar{Y}\rho}{\left(\bar{X}\rho+C_x\right)}$
9.	$t_{9e}=\hat{\bar{Y}}\exp\left(\frac{\beta_2(x)\left(\bar{X}-\bar{x}\right)}{\beta_2(x)\left(\bar{X}+\bar{x}\right)+2\rho}\right)$	$\theta\left[R_9^2\frac{S_x^2}{4}+S_y^2\left(1-\rho^2\right)\right]$	$\beta_2(x)$	ρ	$R_9=\frac{\bar{Y}\beta_2(x)}{\left(\bar{X}\beta_2(x)+\rho\right)}$
10.	$t_{10e}=\hat{\bar{Y}}\exp\left(\frac{\rho\left(\bar{X}-\bar{x}\right)}{\rho\left(\bar{X}+\bar{x}\right)+2\beta_2(x)}\right)$	$\theta\left[R_{10}^2\frac{S_x^2}{4}+S_y^2\left(1-\rho^2\right)\right]$	ρ	$\beta_2(x)$	$R_{10}=\frac{\bar{Y}\rho}{\left(\bar{X}\rho+\beta_2(x)\right)}$
11.	$t_{11e}=\hat{\bar{Y}}\exp\left(\frac{\left(\bar{X}-\bar{x}\right)}{\left(\bar{X}+\bar{x}\right)+2\beta_1(x)}\right)$	$\theta\left[R_{11}^2\frac{S_x^2}{4}+S_y^2\left(1-\rho^2\right)\right]$	1	$\beta_2(x)$	$R_{11}=\frac{\bar{Y}}{\left(\bar{X}+\beta_1(x)\right)}$

Table 3. *continued*

S.No.	Estimators	MSE	Values of Constants a	b	Population Ratio
12.	$t_{12e}=\hat{\bar{Y}}\exp\left(\frac{\beta_1(x)(\bar{X}-\bar{x})}{\beta_1(x)(\bar{X}+\bar{x})+2\beta_2(x)}\right)$	$\theta\left[R_{12}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_1(x)$	$\beta_2(x)$	$R_{12}=\frac{\bar{Y}\beta_1(x)}{(\bar{X}\beta_1(x)+\beta_2(x))}$
13.	$t_{13e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2M_d}\right)$	$\theta\left[R_{13}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	M_d	$R_{13}=\frac{\bar{Y}}{(\bar{X}+M_d)}$
14.	$t_{14e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2M_d}\right)$	$\theta\left[R_{14}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	M_d	$R_{14}=\frac{\bar{Y}C_x}{(\bar{X}C_x+M_d)}$
15.	$t_{15e}=\hat{\bar{Y}}\exp\left(\frac{\beta_1(x)(\bar{X}-\bar{x})}{\beta_1(x)(\bar{X}+\bar{x})+2M_d}\right)$	$\theta\left[R_{15}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_1(x)$	M_d	$R_{15}=\frac{\beta_1(x)\bar{Y}}{(\bar{X}\beta_1(x)+M_d)}$
16.	$t_{16e}=\hat{\bar{Y}}\exp\left(\frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2M_d}\right)$	$\theta\left[R_{16}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_2(x)$	M_d	$R_{16}=\frac{\beta_2(x)\bar{Y}}{(\bar{X}\beta_2(x)+M_d)}$
17.	$t_{17e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_1}\right)$	$\theta\left[R_{17}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	D_1	$R_{17}=\frac{\bar{Y}}{(\bar{X}+D_1)}$
18.	$t_{18e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_2}\right)$	$\theta\left[R_{18}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	D_2	$R_{18}=\frac{\bar{Y}}{(\bar{X}+D_2)}$
19.	$t_{19e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_3}\right)$	$\theta\left[R_{19}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	D_3	$R_{19}=\frac{\bar{Y}}{(\bar{X}+D_3)}$
20.	$t_{20e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_4}\right)$	$\theta\left[R_{20}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	D_4	$R_{20}=\frac{\bar{Y}}{(\bar{X}+D_4)}$
21.	$t_{21e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_5}\right)$	$\theta\left[R_{21}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	D_5	$R_{21}=\frac{\bar{Y}}{(\bar{X}+D_5)}$
22.	$t_{22e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_6}\right)$	$\theta\left[R_{22}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	D_6	$R_{22}=\frac{\bar{Y}}{(\bar{X}+D_6)}$
23.	$t_{23e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_7}\right)$	$\theta\left[R_{23}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	D_7	$R_{23}=\frac{\bar{Y}}{(\bar{X}+D_7)}$

Table 3. *continued*

S.No.	Estimators	MSE	Values of Constants a	b	Population Ratio
24.	$t_{24e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_8}\right)$	$\theta\left[R_{24}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	D_8	$R_{24}=\frac{\bar{Y}}{(\bar{X}+D_8)}$
25.	$t_{25e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_9}\right)$	$\theta\left[R_{25}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	D_9	$R_{25}=\frac{\bar{Y}}{(\bar{X}+D_9)}$
26.	$t_{26e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_{10}}\right)$	$\theta\left[R_{26}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	D_{10}	$R_{26}=\frac{\bar{Y}}{(\bar{X}+D_{10})}$
27.	$t_{27e}=\hat{\bar{Y}}\exp\left(\frac{\beta_1(x)(\bar{X}-\bar{x})}{\beta_1(x)(\bar{X}+\bar{x})+2Q_d}\right)$	$\theta\left[R_{27}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_1(x)$	Q_d	$R_{27}=\frac{\bar{Y}\beta_1(x)}{(\bar{X}\beta_1(x)+Q_d)}$
28.	$t_{28e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2M_d}\right)$	$\theta\left[R_{28}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	M_d	$R_{28}=\frac{\rho\bar{Y}}{(\rho\bar{X}+M_d)}$
29.	$t_{29e}=\hat{\bar{Y}}\exp\left(\frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2Q_1}\right)$	$\theta\left[R_{29}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_2(x)$	Q_1	$R_{29}=\frac{\bar{Y}\beta_2(x)}{(\bar{X}\beta_2(x)+Q_1)}$
30.	$t_{30e}=\hat{\bar{Y}}\exp\left(\frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2Q_3}\right)$	$\theta\left[R_{30}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_2(x)$	Q_3	$R_{30}=\frac{\bar{Y}\beta_2(x)}{(\bar{X}\beta_2(x)+Q_3)}$
31.	$t_{31e}=\hat{\bar{Y}}\exp\left(\frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2Q_r}\right)$	$\theta\left[R_{31}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_2(x)$	Q_r	$R_{31}=\frac{\bar{Y}\beta_2(x)}{(\bar{X}\beta_2(x)+Q_r)}$
32.	$t_{32e}=\hat{\bar{Y}}\exp\left(\frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2Q_d}\right)$	$\theta\left[R_{32}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_2(x)$	Q_d	$R_{32}=\frac{\bar{Y}\beta_2(x)}{(\bar{X}\beta_2(x)+Q_d)}$
33.	$t_{33e}=\hat{\bar{Y}}\exp\left(\frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2Q_a}\right)$	$\theta\left[R_{33}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_2(x)$	Q_a	$R_{33}=\frac{\bar{Y}\beta_2(x)}{(\bar{X}\beta_2(x)+Q_a)}$
34.	$t_{34e}=\hat{\bar{Y}}\exp\left(\frac{\beta_1(x)(\bar{X}-\bar{x})}{\beta_1(x)(\bar{X}+\bar{x})+2Q_1}\right)$	$\theta\left[R_{34}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_1(x)$	Q_1	$R_{34}=\frac{\bar{Y}\beta_1(x)}{(\bar{X}\beta_1(x)+Q_1)}$
35.	$t_{35e}=\hat{\bar{Y}}\exp\left(\frac{\beta_1(x)(\bar{X}-\bar{x})}{\beta_1(x)(\bar{X}+\bar{x})+2Q_3}\right)$	$\theta\left[R_{35}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_1(x)$	Q_3	$R_{35}=\frac{\bar{Y}\beta_1(x)}{(\bar{X}\beta_1(x)+Q_3)}$

continued

Table 3. *continued*

S.No.	Estimators	MSE	Values of Constants a	b	Population Ratio
36.	$t_{36e}=\hat{\bar{Y}}\exp\left(\frac{\beta_1(x)(\bar{X}-\bar{x})}{\beta_1(x)(\bar{X}+\bar{x})+2Q_r}\right)$	$\theta\left[R_{36}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_1(x)$	Q_r	$R_{36}=\frac{\bar{Y}\beta_1(x)}{(\bar{X}\beta_1(x)+Q_r)}$
37.	$t_{37e}=\hat{\bar{Y}}\exp\left(\frac{\beta_1(x)(\bar{X}-\bar{x})}{\beta_1(x)(\bar{X}+\bar{x})+2Q_a}\right)$	$\theta\left[R_{37}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	$\beta_1(x)$	Q_a	$R_{37}=\frac{\bar{Y}\beta_1(x)}{(\bar{X}\beta_1(x)+Q_a)}$
38.	$t_{38e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2Q_1}\right)$	$\theta\left[R_{38}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	Q_1	$R_{38}=\frac{\rho\bar{Y}}{(\rho\bar{X}+Q_1)}$
39.	$t_{39e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2Q_3}\right)$	$\theta\left[R_{39}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	Q_3	$R_{39}=\frac{\rho\bar{Y}}{(\rho\bar{X}+Q_3)}$
40.	$t_{40e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2Q_r}\right)$	$\theta\left[R_{40}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	Q_r	$R_{40}=\frac{\rho\bar{Y}}{(\rho\bar{X}+Q_r)}$
41.	$t_{41e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2Q_d}\right)$	$\theta\left[R_{41}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	Q_d	$R_{41}=\frac{\rho\bar{Y}}{(\rho\bar{X}+Q_d)}$
42.	$t_{42e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2Q_a}\right)$	$\theta\left[R_{42}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	Q_a	$R_{42}=\frac{\rho\bar{Y}}{(\rho\bar{X}+Q_a)}$
43.	$t_{43e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2T_m}\right)$	$\theta\left[R_{43}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	T_m	$R_{43}=\frac{\bar{Y}}{(\bar{X}+T_m)}$
44.	$t_{44e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2T_m}\right)$	$\theta\left[R_{44}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	T_m	$R_{44}=\frac{\bar{Y}C_x}{(\bar{X}C_x+T_m)}$
45.	$t_{45e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2T_m}\right)$	$\theta\left[R_{45}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	T_m	$R_{45}=\frac{\bar{Y}\rho}{(\bar{X}\rho+T_m)}$
46.	$t_{46e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2M_r}\right)$	$\theta\left[R_{46}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	M_r	$R_{46}=\frac{\bar{Y}}{(\bar{X}+M_r)}$
47.	$t_{47e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2M_r}\right)$	$\theta\left[R_{47}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	M_r	$R_{47}=\frac{\bar{Y}C_x}{(\bar{X}+M_r)}$

Table 3. *continued*

S.No.	Estimators	MSE	Values of Constants		Population Ratio
			a	b	
48.	$t_{48e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2M_r}\right)$	$\theta\left[R_{48}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	M_r	$R_{48}=\frac{\bar{Y}\rho}{(\bar{X}\rho+M_r)}$
49.	$t_{49e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2H_1}\right)$	$\theta\left[R_{49}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	H_1	$R_{49}=\frac{\bar{Y}}{(\bar{X}+H_l)}$
50.	$t_{50e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2H_1}\right)$	$\theta\left[R_{50}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	H_1	$R_{50}=\frac{\bar{Y}C_x}{(\bar{X}C_x+H_l)}$
51.	$t_{51e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2H_1}\right)$	$\theta\left[R_{51}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	H_1	$R_{51}=\frac{\bar{Y}\rho}{(\bar{X}\rho+H_l)}$
52.	$t_{52e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2G}\right)$	$\theta\left[R_{52}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	G	$R_{52}=\frac{\bar{Y}}{(\bar{X}+G)}$
53.	$t_{53e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2G}\right)$	$\theta\left[R_{53}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	G	$R_{53}=\frac{\bar{Y}\rho}{(\bar{X}\rho+G)}$
54.	$t_{54e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2G}\right)$	$\theta\left[R_{54}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	G	$R_{54}=\frac{\bar{Y}C_x}{(\bar{X}C_x+G)}$
55.	$t_{55e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D}\right)$	$\theta\left[R_{55}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	D	$R_{55}=\frac{\bar{Y}}{(\bar{X}+D)}$
56.	$t_{56e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2D}\right)$	$\theta\left[R_{56}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	D	$R_{56}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D)}$
57.	$t_{57e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2D}\right)$	$\theta\left[R_{57}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	D	$R_{57}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D)}$
58.	$t_{58e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2S_{pw}}\right)$	$\theta\left[R_{58}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	1	S_{pw}	$R_{58}=\frac{\bar{Y}\rho}{(\bar{X}+S_{pw})}$
59.	$t_{59e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2S_{pw}}\right)$	$\theta\left[R_{59}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	S_{pw}	$R_{59}=\frac{\bar{Y}\rho}{(\bar{X}\rho+S_{pw})}$

continued

Table 3. *continued*

S.No.	Estimators	MSE	Values of Constants a	b	Population Ratio
60.	$t_{60e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2S_{pw}}\right)$	$\theta\left[R_{60}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	S_{pw}	$R_{60}=\frac{\bar{Y}C_x}{(\bar{X}C_x+S_{pw})}$
61.	$t_{61e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2D_1}\right)$	$\theta\left[R_{61}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	D_1	$R_{61}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_1)}$
62.	$t_{62e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2D_2}\right)$	$\theta\left[R_{62}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	D_2	$R_{62}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_2)}$
63.	$t_{63e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2D_3}\right)$	$\theta\left[R_{63}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	D_3	$R_{63}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_3)}$
64.	$t_{64e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2D_4}\right)$	$\theta\left[R_{64}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	D_4	$R_{64}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_4)}$
65.	$t_{65e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2D_5}\right)$	$\theta\left[R_{65}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	D_5	$R_{65}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_5)}$
66.	$t_{66e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2D_6}\right)$	$\theta\left[R_{66}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	D_6	$R_{66}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_6}$
67.	$t_{67e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2D_7}\right)$	$\theta\left[R_{67}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	D_7	$R_{67}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_7)}$
68.	$t_{68e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2D_8}\right)$	$\theta\left[R_{68}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	D_8	$R_{68}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_8)}$
69.	$t_{69e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2D_9}\right)$	$\theta\left[R_{69}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	D_9	$R_{69}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_9)}$
70.	$t_{70e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2D_{10}}\right)$	$\theta\left[R_{70}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	ρ	D_{10}	$R_{70}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_{10})}$

Table 3. *continued*

S.No.	Estimators	MSE	Values of Constants a	b	Population Ratio
71.	$t_{71e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2D_1}\right)$	$\theta\left[R_{71}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	D_1	$R_{71}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_1)}$
72.	$t_{72e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2D_2}\right)$	$\theta\left[R_{72}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	D_2	$R_{72}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_2)}$
73.	$t_{73e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2D_3}\right)$	$\theta\left[R_{73}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	D_3	$R_{73}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_3)}$
74.	$t_{74e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2D_4}\right)$	$\theta\left[R_{74}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	D_4	$R_{74}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_4)}$
75.	$t_{75e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2D_5}\right)$	$\theta\left[R_{75}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	D_5	$R_{75}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_5)}$
76.	$t_{76e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2D_6}\right)$	$\theta\left[R_{76}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	D_6	$R_{76}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_6)}$
77.	$t_{77e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2D_7}\right)$	$\theta\left[R_{77}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	D_7	$R_{77}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_7)}$
78.	$t_{78e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2D_8}\right)$	$\theta\left[R_{78}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	D_8	$R_{78}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_8)}$
79.	$t_{79}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2D_9}\right)$	$\theta\left[R_{79}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	D_9	$R_{79}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_{19})}$
80.	$t_{80e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2D_{10}}\right)$	$\theta\left[R_{80}^2\frac{S_x^2}{4}+S_y^2(1-\rho^2)\right]$	C_x	D_{10}	$R_{80}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_{10})}$

The bias of t_e at (17) is negligible if the sample size n is sufficiently large. The bias of the members of the proposed class of estimators can be obtained easily from (17) just by putting suitable values of the scalars (*a, b*). Squaring both sides of (16) and neglecting terms of e's having power greater than two

$$\left(t_e-\overline{Y}\right)^2=\overline{Y}^2\left[e_0^2+\frac{\tau^2e_1^2}{4}+C^2e_1^2-\tau e_0e_1-2Ce_0e+\tau Ce_1^2\right] \tag{18}$$

Taking expectation of both sides of (18) we get the MSE of t_e to the first degree of approximation as

$$MSE\left(t_e\right)=\frac{(1-f)}{n}\left[R_J^2\left(\frac{S_x^2}{4}\right)+S_y^2\left(1-\rho^2\right)\right]. \tag{19}$$

The MSE of the members of the proposed class of estimators t_e can be easily obtained from (19) just by putting the suitable values of (*a, b*).

Remark 1 Motivated by Swain (2014), define a class of ratio-type estimators for population mean $\overline{Y}$ as

$$t_s=\hat{\overline{Y}}\left(\frac{a\overline{X}+b}{a\overline{x}+b}\right)^{1/2}. \tag{20}$$

Thus the form of the estimators *t* and t_S taking into consideration, we define a class of ratio-cum-product-type estimators for population mean $\overline{Y}$ as

$$t_g=\hat{\overline{Y}}\left(\frac{a\overline{X}+b}{a\overline{x}+b}\right)^{g}, \tag{21}$$

where *g* is a scalar taking real values. Note for *g(>0)* the class of estimators t_g generates the ratio-type estimators while for g*(<0)* it generates product-type estimators.

To the first degree of approximation the bias and MSE of t_e are respectively given by

$$\begin{aligned}B\left(t_g\right)&=\frac{(1-f)}{n}\left[R_J^2\frac{S_x^2}{\overline{Y}}\frac{g(g+1)}{2}-\frac{\beta N}{(N-2)}\left(\frac{\mu_{21}}{\mu_{11}}-\frac{\mu_{30}}{\mu_{20}}\right)\right]\\&=\frac{(1-f)}{n}\left[\frac{g(g+1)}{2}A-B\right],\end{aligned} \tag{22}$$

$$MSE\left(t_g\right)=\frac{(1-f)}{n}\left[g^2R_J^2S_x^2+S_y^2\left(1-\rho^2\right)\right], \tag{23}$$

where A and B are same as defined earlier. Putting $g=\frac{1}{2}$ in (22) and (23) we get the bias and MSE of the Swain's (2014)-type estimator t_S at (20) respectively as

$$B\left(t_s\right)=\frac{(1-f)}{n}\left[\frac{3}{8}A-B\right] \tag{24}$$

and

$$MSE\left(t_s\right)=\frac{(1-f)}{n}\left[\frac{R_J^2S_x^2}{4}+S_y^2\left(1-\rho^2\right)\right]. \tag{25}$$

From (17), (19), (24) and (25), note the bias and MSE of the Swain's (2014) type estimators t_S and ratio-type exponential estimator t_e defined in (14) are same up to first order of approximation.
From (11) and (23)

$$MSE(t)-MSE\left(t_g\right)=\frac{(1-f)}{n}R_J^2S_x^2+S_x^2\left(1-g^2\right)$$

which is positive if

$$1-g^2>0$$

i.e if

$$g^2<1$$

i.e if

$$-1<g<1. \tag{26}$$

The members of the class of ratio-cum-product type estimators t_g is more efficient than the corresponding members of the proposed class of ratio-type estimators t as long as the condition (26) is satisfied. From (19) and (23)

$$MSE(t)-MSE\left(t_g\right)=\frac{(1-f)}{n}R_J^2S_x^2+\left(\frac{1}{4}-g^2\right)$$

which is positive if

i.e. if

$$\left(\frac{1}{4}-g^2\right)>0\ g^2>\frac{1}{4}$$

i.e. if $$-\frac{1}{2}<g<\frac{1}{2}. \tag{27}$$

The proposed class of ratio-cum-product type estimators t_g would always be more efficient than the corresponding members of the ratio-type exponential estimator t_e as long as the condition (27) is satisfied.

Remark 2 It follows from (23) that either the estimator is ratio-type (*i.e.* t defined by (6)) or product-type defined by

$$\begin{aligned} t^{**} &= \hat{\bar{Y}}\left(\frac{a\bar{x}+b}{a\bar{X}+b}\right), \\ &= \left[\bar{y}+\hat{\beta}\left(\bar{X}-\bar{x}\right)\right]\left(\frac{a\bar{x}+b}{a\bar{X}+b}\right), \end{aligned} \tag{28}$$

the mean squared errors of ratio-type (t) and product-type (t^{**}) to the first degree of approximation are turn out to be the same *i.e.*

$$MSE\left(t\right)=MSE\left(t^{**}\right)=\frac{\left(1-f\right)}{n}\left[R_J^2S_x^2+S_y^2\left(1-\rho^2\right)\right]. \tag{29}$$

The proposed class of estimators is always better than the ratio-type (t) and product-type (t^{**}) estimators as long as the condition:

$$-1<g<1 \tag{30}$$

is satisfied.

Remark 3 Define a generalized version of the ratio-cum-product-type exponential estimator t_e for the population mean $\bar{Y}$ as

$$\begin{aligned} t_{ge} &= \hat{\bar{Y}}\exp\left\{\frac{g\left(\bar{X}^{*}-\bar{x}*\right)}{\left(\bar{X}^{*}+\bar{x}*\right)}\right\} \\ &= \left[\bar{y}+\hat{\beta}\left(\bar{X}-\bar{x}\right)\right]\exp\left\{\frac{ga\left(\bar{X}-\bar{x}\right)}{\left(\bar{X}+\bar{x}\right)+2b}\right\}, \end{aligned} \tag{31}$$

where $\bar{x}^* = (a\bar{x} + b)$ such that $E(\bar{x}^*) = \bar{X}^* = a\bar{X} + b$; and g is a scalar taking real values. For $g(>0)$ t_{ge} generates ratio-type exponential estimator, and for $g(>0)$ it generates product-type exponential estimator.

To the first degree of approximation, the bias and mean squared error of the proposed class of ratio-cum-product-type exponential estimators t_{ge} are respectively given by

$$B(t_{ge}) = \frac{(1-f)}{n}\left[\frac{g(g+2)}{8} R_J^2 \frac{S_x^2}{\bar{Y}} - \frac{N}{(N-2)}\beta\left(\frac{\mu_{21}}{\mu_{11}} - \frac{\mu_{30}}{\mu_{20}}\right)\right]$$
$$= \frac{(1-f)}{n}\left[\frac{g(g+2)}{8} A - B\right] \tag{32}$$

and

$$MSE(t_{ge}) = \frac{(1-f)}{n}\left[\left(\frac{g^2}{4}\right) R_J^2 S_x^2 + S_y^2(1-\rho^2)\right]. \tag{33}$$

From (23) and (33)

$$MSE(t_g) - MSE(t_{ge}) = \frac{3}{4}\frac{(1-f)}{n} g^2 R_J^2 S_x^2 \tag{34}$$

which is always positive.

It follows from (34) that the members of the proposed class of ratio-cum-product-type exponential estimators t_{ge} is always better than the corresponding members of the suggested class of ratio-cum-product type estimators t_e.
From (19) and (33) we have

$$MSE(t_e) - MSE(t_{ge}) = \frac{(1-f)}{n} R_J^2 \frac{S_x^2}{4}(1-g^2)$$

which is positive if

$$(1-g^2) > 0$$

i.e. if $$-1 < g < 1$$

i.e. if $$|g| < 1. \tag{35}$$

The members of the proposed class of ratio-cum product-type exponential estimators is better than the corresponding members of the suggested ratio-type exponential estimator t_e as long as the condition (35) is satisfied. It can be also proved that the members proposed of the proposed class of ratio-cum-product-type estimators t_{ge} is also better than the corresponding members of the product-type exponential estimator defined by

$$t_e^{**} = \hat{\bar{Y}} \exp\left\{\frac{a\left(\bar{x}-\bar{X}\right)}{a\left(\bar{X}+\bar{x}\right)+2b}\right\}$$

$$= \left\{\bar{y}+\hat{\beta}\left(\bar{X}-\bar{x}\right)\right\}\exp\left\{\frac{a\left(\bar{x}-\bar{X}\right)}{a\left(\bar{X}+\bar{x}\right)+2b}\right\} \tag{36}$$

as long as the condition: $|g|<1$ in (35) is satisfied.

Efficiency Comparison

From (2) and (19)

$$MSE\left(\bar{y}\right)-MSE\left(t_e\right)=\frac{(1-f)}{n}S_x^2\left(\beta^2-\frac{R_J^2}{4}\right)$$

which is positive if

$$\beta^2-\frac{R_J^2}{4}>0$$

i.e. if

$$\frac{R_J^2}{4}<\beta^2. \tag{37}$$

From (5) and (19) e

$$MSE\left(\bar{y}_R\right)-MSE\left(t_e\right)=\frac{(1-f)}{n}\left[\left(\beta-R\right)^2-\frac{R_J^2}{4}\right]$$

which is non-negative if

$$\left[\left(\beta-R\right)^2-\frac{R_j^2}{4}\right]>0$$

i.e. if
$$\frac{R_j^2}{4} < (\beta - R)^2 \tag{38}$$

Further from (11) and (19) we have

$$MSE(t) - MSE(t_e) = \frac{3}{4}\frac{(1-f)}{n} R_j^2 S_x^2 \tag{39}$$

which is always positive.

Thus from (37) – (39) it follows that the members of the proposed class of estimators t_e is :

(i) more efficient than the usual unbiased estimator $\overline{y}$ as long as the condition (37) is satisfied.

(ii) more efficient than the usual ratio estimator $\overline{y}_R$ as long as the condition (38) is satisfied.

(iii) is always better than the corresponding members of the *t*-family of estimators.

Bias Comparison the Estimators *t* and t_e

It follows from (9) and (17)

$$|B(t_e)| < |B(t)|$$

if

$$\left|\frac{3}{8}A - B\right| < |A - B| \tag{40}$$

Since

$$\left|\frac{3}{8}A - B\right| < \frac{3}{8}|A| + |B| \tag{41}$$

and

$$|A - B| < \backslash |A| + |B|. \tag{42}$$

Therefore, from (40), (41) and (42)

$$\left|B\left(t_e\right)\right| < \left|B\left(t\right)\right|$$

if

$$\frac{3}{8}|A| + |B| < |A| + |B|$$

i.e. if $\frac{3}{8}|A| < |A|$

i.e. if $\frac{5}{8}|A| > 0$ (43)

which is always true. The members of the proposed t_e- family of estimators are less biased as well as more efficient than the corresponding members of the *t*- family. Hence, the members of the proposed class of estimators t_e is more efficient than the corresponding known members due to Kadilar and Cingi (2004), Kadilar and Cingi (2006)- type, Yan and Tian (2010), Subramani and Kumarapandian (2012a, 2012b, 2012c, 2012d), Jeelani et al (2013) and Abid et al (2016a, 2016b, 2016c) of the class of estimators *t*.

Empirical Study

In support of the theoretical results, MSEs of some known estimators listed in Table 1 were computed, and corresponding estimators listed in Table 3. Natural data sets were those considered by Kadilar and Cingi (2004) and Abid et al (2012b). The findings are shown in Table 4.

Population- Source: Kadilar and Cingi (2004) and Abid et al (2016b, p.361).

y: Apple production

x: Number of apple trees

$N = 106$,	$n = 40$,	$\bar{Y} = 2212.59$,	$\bar{X} = 27421.70$
$\rho = 0.860$,	$S_y = 11551.53$,	$C_y = 5.22$,	$S_x = 57460.61$,
$C_x = 2.10$	$\beta_2(x) = 34.572$,	$\beta_1(x) = 2.122$,	$M_d = 7297.50$
$Q_d = 12156.25$,	$G = 40201.69$	$S_{pw} = 35298.810$,	$D = 35634.990$

Table 4. Mean Squared Errors of some known members of the class of ratio-cum-product estimators t and the corresponding members of the class of ratio-cum-product-type estimators t_e

Known Estimators	MSE (t)	Rank	Corresponding members of t_e	MSE (t_e)	Rank
$t_1 = \frac{\hat{\bar{Y}}}{\bar{x}} . \bar{X}$ Kadilar and Cingi (2004)	875480.89	XXVI	$t_{1e} = \hat{\bar{Y}} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right)$	624527.57	XXVI
$t_2 = \frac{\hat{\bar{Y}}}{(\bar{x} + C_x)} (\bar{X} + C_x)$ Kadilar and Cingi (2004)	875429.64	XXI	$t_{2e} = \hat{\bar{Y}} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x} + 2C_x} \right)$	624514.75	XXI
$t_3 = \frac{\hat{\bar{Y}}}{(\bar{x} + \beta_2(x))} (\bar{X} + \beta_2(x))$ Kadilar and Cingi (2004)	874638.77	XVI	$t_{3e} = \hat{\bar{Y}} \exp\left(\frac{(\bar{X} - \bar{x})}{\bar{X} + \bar{x} + 2\beta_2(x)} \right)$	624317.04	XVI
$t_4 = \frac{\hat{\bar{Y}}}{(\bar{x}\beta_2(x) + C_x)} (\bar{X}\beta_2(x) + C_x)$ Kadilar and Cingi (2004)	875479.40	XXIV	$t_{4e} = \hat{\bar{Y}} \exp\left(\frac{(\bar{X} - \bar{x})\beta_2(x)}{\beta_2(x)(\bar{X} + \bar{x}) + 2C_x} \right)$	624527.20	XXIV
$t_5 = \frac{\hat{\bar{Y}}}{(\bar{x}C_x + \beta_2(x))} (\bar{X}C_x + \beta_2(x))$ Kadilar and Cingi (2004)	875079.48	XVII	$t_{5e} = \hat{\bar{Y}} \exp\left(\frac{C_x(\bar{X} - \bar{x})}{C_x(\bar{X} + \bar{x}) + 2\beta_2(x)} \right)$	624427.21	XVII
$t_6 = \frac{\hat{\bar{Y}}}{(\bar{x} + \rho)} (\bar{X} + \rho)$ Kadilar and Cingi (2006) -type	875459.90	XXII	$t_{6e} = \hat{\bar{Y}} \exp\left(\frac{(\bar{X} - \bar{x})}{\bar{X} + \bar{x} + 2\rho} \right)$	624522.32	XXII
$t_7 = \frac{\hat{\bar{Y}}}{(\bar{x}C_x + \rho)} (\bar{X}C_x + \rho)$ (2006 Kadilar and Cingi) -type	875470.89	XXIII	$t_{7e} = \hat{\bar{Y}} \exp\left(\frac{C_x(\bar{X} - \bar{x})}{C_x(\bar{X} + \bar{x}) + 2\rho} \right)$	624525.07	XXIII
$t_8 = \frac{\hat{\bar{Y}}}{(\bar{x}\rho + C_x)} (\bar{X}\rho + C_x)$ Kadilar and Cingi (2006) -type	875421.30	XIX	$t_{8e} = \hat{\bar{Y}} \exp\left(\frac{\rho(\bar{X} - \bar{x})}{\rho(\bar{X} + \bar{x}) + 2C_x} \right)$	624512.67	XIX
$t_9 = \frac{\hat{\bar{Y}}}{(\bar{x}\beta_2(x) + \rho)} (\bar{X}\beta_2(x) + \rho)$ Kadilar and Cingi (2006) –type	875480.28	XXV	$t_{9e} = \hat{\bar{Y}} \exp\left(\frac{\beta_2(x)(\bar{X} - \bar{x})}{\beta_2(x)(\bar{X} + \bar{x}) + 2\rho} \right)$	624527.41	XXV
$t_{10} = \frac{\hat{\bar{Y}}}{(\bar{x}\rho + \beta_2(x))} (\bar{X}\rho + \beta_2(x))$ Kadilar and Cingi (2006) -type	874501.98	XV	$t_{10e} = \hat{\bar{Y}} \exp\left(\frac{\rho(\bar{X} - \bar{x})}{\rho(\bar{X} + \bar{x}) + 2\beta_2(x)} \right)$	624282.84	XV
$t_{11} = \frac{\hat{\bar{Y}}}{(\bar{x} + \beta_1(x))} (\bar{X} + \beta_1(x))$ Yan and Tian (2010)	875429.11	XX	$t_{11e} = \hat{\bar{Y}} \exp\left(\frac{(\bar{X} - \bar{x})}{(\bar{X} + \bar{x}) + 2\beta_1(x)} \right)$	624514.62	XX

continued

Table 4. *continued*

Known Estimators	MSE (t)	Rank	Corresponding members of t_e	MSE (t_e)	Rank
$t_{12}=\frac{\hat{\hat{Y}}}{\left(\bar{x}\beta_1(x)+\beta_2(x)\right)}\left(\bar{X}\beta_1(x)+\beta_2(x)\right)$ Yan and Tian (2010)	875083.64	*XVIII*	$t_{12e}=\hat{\hat{Y}}\exp\left(\frac{\beta_1(x)\left(\bar{X}-\bar{x}\right)}{\beta_1(x)\left(\bar{X}+\bar{x}\right)+2\beta_2(x)}\right)$	624428.25	*XVIII*
$t_{13}=\frac{\hat{\hat{Y}}}{\left(\bar{x}+M_d\right)}\left(\bar{X}+M_d\right)$ Subramani and Kumarapandian (2012a)	749604.60	*X*	$t_{13e}=\hat{\hat{Y}}\exp\left(\frac{\left(\bar{X}-\bar{x}\right)}{\left(\bar{X}+\bar{x}\right)+2M_d}\right)$	593058.50	*X*
$t_{14}=\frac{\hat{\hat{Y}}}{\left(C_x\bar{x}+M_d\right)}\left(\bar{X}C_x+M_d\right)$ Subramani and Kumarapandian (2012a)	804446.62	*XII*	$t_{14e}=\hat{\hat{Y}}\exp\left(\frac{C_x\left(\bar{X}-\bar{x}\right)}{C_x\left(\bar{X}+\bar{x}\right)+2M_d}\right)$	606769.00	*XII*
$t_{15}=\frac{\hat{\hat{Y}}}{\left(\beta_1(x)\bar{x}+M_d\right)}\left(\bar{X}\beta_1(x)+M_d\right)$ Subramani and Kumarapandian (2012b)	805062.37	*XIII*	$t_{15e}=\hat{\hat{Y}}\exp\left(\frac{\beta_1(x)\left(\bar{X}-\bar{x}\right)}{\beta_1(x)\left(\bar{X}+\bar{x}\right)+2M_d}\right)$	606922.94	*XIII*
$t_{16}=\frac{\hat{\hat{Y}}}{\left(\beta_2(x)\bar{x}+M_d\right)}\left(\bar{X}\beta_2(x)+M_d\right)$ Subramani and Kumarapandian (2012c)	870388.46	*XV*	$t_{16e}=\hat{\hat{Y}}\exp\left(\frac{\beta_2(x)\left(\bar{X}-\bar{x}\right)}{\beta_2(x)\left(\bar{X}+\bar{x}\right)+2M_d}\right)$	623254.46	*XIV*
$t_{27}=\frac{\hat{\hat{Y}}}{\left(\bar{x}\beta_1(x)+Q_d\right)}\left(\bar{X}\beta_1(x)+Q_d\right)$ Jeelani et al (2013)	769827.97	*XI*	$t_{27e}=\hat{\hat{Y}}\exp\left(\frac{\beta_1(x)\left(\bar{X}-\bar{x}\right)}{\beta_1(x)\left(\bar{X}+\bar{x}\right)+2Q_d}\right)$	598114.34	*XI*
$t_{37}=\frac{\hat{\hat{Y}}}{\left(\bar{x}+G\right)}\left(\bar{X}+G\right)$ Abid et al (2016b)	595897.22	*IV*	$t_{37e}=\hat{\hat{Y}}\exp\left(\frac{\left(\bar{X}-\bar{x}\right)}{\left(\bar{X}+\bar{x}\right)+2G}\right)$	554631.65	*IV*
$t_{38}=\frac{\hat{\hat{Y}}}{\left(\bar{x}\rho+G\right)}\left(\bar{X}\rho+G\right)$ Abid et al (2016b)	586615.71	*I*	$t_{38e}=\hat{\hat{Y}}\exp\left(\frac{\rho\left(\bar{X}-\bar{x}\right)}{\rho\left(\bar{X}+\bar{x}\right)+2G}\right)$	552311.27	*I*
$t_{39}=\frac{\hat{\hat{Y}}}{\left(\bar{x}C_x+G\right)}\left(\bar{X}C_x+G\right)$ Abid et al (2016b)	656912.92	*VII*	$t_{39e}=\hat{\hat{Y}}\exp\left(\frac{C_x\left(\bar{X}-\bar{x}\right)}{C_x\left(\bar{X}+\bar{x}\right)+2G}\right)$	569885.57	*VII*
$t_{40}=\frac{\hat{\hat{Y}}}{\left(\bar{x}+D\right)}\left(\bar{X}+D\right)$ Abid et al (2016b)	604155.24	*V*	$t_{40e}=\hat{\hat{Y}}\exp\left(\frac{\left(\bar{X}-\bar{x}\right)}{\left(\bar{X}+\bar{x}\right)+2D}\right)$	556696.15	*V*
$t_{41}=\frac{\hat{\hat{Y}}}{\left(\bar{x}\rho+D\right)}\left(\bar{X}\rho+D\right)$ Abid et al (2016b)	593942.29	*II*	$t_{41e}=\hat{\hat{Y}}\exp\left(\frac{\rho\left(\bar{X}-\bar{x}\right)}{\rho\left(\bar{X}+\bar{x}\right)+2D}\right)$	554142.92	*II*

Table 4. *continued*

Known Estimators	MSE (t)	Rank	Corresponding members of t_e	MSE (t_e)	Rank
$t_{42}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+D)}(\bar{X}C_x+D)$ Abid et al (2016b)	668560.20	*VIII*	$t_{42e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2D}\right)$	572797.39	*VIII*
$t_{43}=\frac{\hat{\bar{Y}}}{(\bar{x}+S_{pw})}(\bar{X}+S_{pw})$ Abid et al (2016b)	604835.42	*VI*	$t_{43e}=\hat{\bar{Y}}\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2S_{pw}}\right)$	556866.20	*VI*
$t_{44}=\frac{\hat{\bar{Y}}}{(\bar{x}\rho+S_{pw})}(\bar{X}\rho+S_{pw})$ Abid et al (2016b)	594549.99	*III*	$t_{44e}=\hat{\bar{Y}}\exp\left(\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2S_{pw}}\right)$	554294.84	*III*
$t_{45}=\frac{\hat{\bar{Y}}}{(\bar{x}C_x+S_{pw})}(\bar{X}C_x+S_{pw})$ Abid et al (2016b)	669486.16	*IX*	$t_{45e}=\hat{\bar{Y}}\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2S_{pw}}\right)$	573028.89	*IX*

It is observed from Table 4 there is considerable reduction in MSEs of the proposed estimators (t_{1e} *to* t_{16e}, t_{27e}, t_{37e} *to* t_{45e}) as compared to the corresponding known estimators (t_1 *to* t_{13}, t_{27}, t_{37} *to* t_{45}). That is the members of the proposed class of ratio-cum-product-type exponential estimators t_e is more efficient than the corresponding members of the class of ratio-cum-product-type estimators t. The proposed estimators t_{38e} followed by the estimator t_{41e} have the smallest MSE among all the estimators considered in Table 4. Thus, the proposed class of ratio-cum-product-type exponential estimators t_e is justified.

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