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Erratum

A previous version of this article incorrectly listed the first author's institutional affiliation as the University of Northern Colorado. This has been corrected to the University of Northern California.

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A Double EWMA Control Chart for the Individuals Based on a Linear Prediction

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Industrial process use single and double Exponential Weighted Moving Average control charts to detect small shifts in it. Occasionally there is a need to detect small trends instead of shifts, but the effectiveness to detect small trends. A new control chart is proposed to detect a small drift.

Keywords: EWMA, DEWMA, control charts, linear drift, forecast, average run length

Introduction

One of the most useful tools to assure the quality of a product or process in manufacture industry are quality control charts. Shewhart (1926) developed the control charts tool to identify when a process was producing a good or a defective product. Today, many control charts have developed to ensure quality through the control of certain characteristics of interest. The main idea of control charts is to detect as soon as possible when this characteristic has changed. In this sense, control charts are designed to detect a shift quickly. Several control charts have been designed to detect small shifts, while others were designed to detect big shifts. In practice, however, we occasionally wish to detect small trends, instead of shifts, in the process; this gradual changes may be due to tool wear or similar causes. Examples of this phenomenon are commonly observed in several manufacturing processes and administrative activities. The effectiveness of these methods to determine small trends in a process has not been thoroughly researched in the current literature. Knoth (2012) reviewed this literature and invited the statistical process community to extend research in this area in order to enhance the knowledge about drift detection.

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A double exponentially weighted moving average (DEWMA) control chart was initially developed by Shamma and Shamma (1992). Zhang and Chen (2005) presented an extension of the exponentially weighted moving average (EWMA) technique to a DEWMA technique. These two DEWMA techniques are the same, and so are their conclusions. Research regarding DEWMA was developed, taking the DEWMA control chart as a reference. For example, Mahmoud and Woodall (2010) conducted a study to compare some characteristics between the EWMA and the DEWMA. Alkahtani (2013) assessed the robustness of DEWMA and EWMA control charts for abnormal processes. Extensions for a multivariate DEWMA control chart case exist. For example, Alkahtani and Schaffer (2012) developed a multivariate DEWMA control chart for detecting shifts in the mean vector of a multivariate normal quality characteristic distribution.

The DEWMA control chart is constructed on the assumption of a data stream of X_i random values following a normal distribution, initially $X_i \sim N(\mu_0, \sigma)$; the DEWMA $S_i' = \lambda S_i + (1-\lambda) S_{i-1}'$ is then calculated, where $S_i = \lambda X_i + (1 - \lambda)S_{i-1}$ with its corresponding initial values. The control chart is built by plotting the value S_i' with its limits, using k times the variance of S_i'. The DEWMA value is plotted with the upper and lower limits versus i. The DEWMA and EWMA control charts work efficiently to detect small shifts when the mean of the process has changed slightly, and the classical Shewhart control chart works well to detect big shifts (more than twice the standard deviation of the process). What happens, however, if after a period of stability, the process has a permanent small change (a linear trend or drift in the stream of X_i)?

Brown (1962) proposed using the smoothing technique to forecast the demand of goods. These forecast methods are the basis of other more complex forecast methods. Smoothing techniques to produce forecasting are well known in business, where it is essential to predict the demand for goods and services (Hanke & Wichern, 2009). According to the fundamental theorem of exponential smoothing explained by Brown and Meyer (1961), a linear prediction can be forecast by a DEWMA with a linear relationship by the follow equation:

$$F_{i+t} = a_i + b_i t \tag{1}$$

where F_{i+t} is the forecast in the t period,

$$a_i = 2S_i - S_i' \tag{2}$$

and

$$b_i = \frac{\lambda}{1-\lambda}\left(S_i - S_i'\right) \tag{3}$$

Details of the development of these equations can be seen in Brown (1962) and Yates (1968). Also, a similar linear prediction equation like the one shown in equation (1) can be built using Holt forecast equations or moving average equations, or other forecast techniques as explained in Hanke and Wichern (2009).

The main idea is to build a three individual control charts under the null hypothesis of the in-control process, assuming the $X_i \sim N(\mu_0, \sigma^2)$. The first control chart is the a_i control chart, where the center line is $E(a_i)$ and the upper and lower limits are given by

$$E\left(a_i\right) \pm k\sqrt{\mathrm{Var}\left(a_i\right)}$$

This control chart tests the null hypothesis that the forecast level is equal to μ_0, (i.e. $E(a_i) = \mu_0$) at time t.

The second control chart is for b_i, a control chart for the forecast slope, where the center line of the control chart is $E(b_i) = 0$ and the upper and lower limits can be built as

$$E\left(b_i\right) \pm k\sqrt{\mathrm{Var}\left(b_i\right)}$$

This control chart tests the null hypothesis $b_i = 0$ at time t (i.e. the forecast linear trend is equal to zero) versus the alternative hypothesis $b_i \neq 0$ (i.e. the slope differs from zero). The main idea is to detect a change when the slope differs from zero.

The third control chart is the sum $F_{i+t} = a_i + b_i t$, a control chart for the forecast assuming a linear prediction, where the center line is $E(F_{i+t}) = E(a_i + b_i t) = E(a_i) + tE(b_i)$ and the upper and lower limits can be built as

$$E\left(F_t\right) \pm k\sqrt{\mathrm{Var}\left(F_t\right)}$$

This control chart tests the null hypothesis $F_t = \mu_0$ at time t (i.e. the forecast linear trend is equal to μ_0, the target value) versus the alternative hypothesis $F_t \neq \mu_0$ (i.e.

the mean level differs from the target). The principal idea is to try and detect a linear drift as soon as it occurs.

The Control Charts to Detect Small Shifts

The EWMA control chart was introduced by Roberts (1959). According to Lucas and Saccucci (1990), the EWMA control chart was a good alternative to the Shewhart control chart when the interest is in detecting small shifts. The EWMA is generally used with individual observations; therefore, this control chart will be discussed when $n = 1$.

The Exponentially Weighted Moving Average Control Chart

For a $X_i \sim \mathrm{N}(\mu_0, \sigma)$, $i = 1, 2,\ldots, n$, the EWMA control statistic S_i is explained by Montgomery (2007) as:

$$S_i = \lambda X_i + (1-\lambda) S_{i-1} \tag{4}$$

where $0 < \lambda < 1$ and $S_0 = \mu_0$. It can be shown

$$\mathrm{E}(S_i) = \mu_0 \tag{5}$$

and

$$\mathrm{Var}(S_i) = \frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\sigma^2 \tag{6}$$

For large values of i, the asymptotic variance becomes

$$\mathrm{Var}_{\text{asym}}(S_i) = \left(\frac{\lambda}{2-\lambda}\right)\sigma^2 \tag{7}$$

Therefore, the control limits, and center line become

$$\mathrm{UCL} = \mu_0 + k\sigma\sqrt{\frac{\lambda}{2-\lambda}}$$
$$\mathrm{CL} = \mu_0$$
$$\mathrm{LCL} = \mu_0 - k\sigma\sqrt{\frac{\lambda}{2-\lambda}}$$

The Double Exponentially Weighted Moving Average Control Chart

For a $X_i \sim N(\mu_0, \sigma)$, $i = 1, 2, \ldots, n$, the DEWMA control statistic S_i' was first developed by Shamma and Shamma (1992). It is defined as

$$S_i' = \lambda S_i + (1-\lambda) S_{i-1}' \tag{8}$$

$$S_i = \lambda X_i + (1-\lambda) S_{i-1} \tag{9}$$

where $0 < \lambda < 1$ and $S_0 = S_0' = \mu_0$. It can be shown that

$$\mathrm{E}(S_i') = \mu_0 \tag{10}$$

and

$$\begin{aligned}&\mathrm{Var}(S_i')\\ &= \lambda^4 \frac{1+(1-\lambda)^2 - (1-\lambda)^{2i}\left((i+1)^2 - (2i^2+2i-1)(1-\lambda)^2 + i^2(1-\lambda)^4\right)}{\left(1-(1-\lambda)^2\right)^3}\sigma^2\end{aligned} \tag{11}$$

For large values of i, the asymptotic variance becomes

$$\mathrm{Var}_{\mathrm{asym}}(S_i') = \frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}\sigma^2 \tag{12}$$

Then for large values of i the control limits become

$$\mathrm{UCL} = \mu_0 + k\sigma\sqrt{\frac{\lambda\left(2-2\lambda+\lambda^2\right)}{\left(2-\lambda\right)^3}}$$
$$\mathrm{CL} = \mu_0$$
$$\mathrm{LCL} = \mu_0 - k\sigma\sqrt{\frac{\lambda\left(2-2\lambda+\lambda^2\right)}{\left(2-\lambda\right)^3}}$$

Mahmoud and Woodall (2010) show how these variances can be obtained.

A Proposed Control Chart to Detect Small Change in the Trends

Double Exponentially Weighted Moving Average Based on a Linear Prediction

A new control charts is now proposed to detect linear trends. The double exponentially weighted moving average based on a linear prediction (DEWMABLP) is constructed assuming a stream of variables $X_i \sim \mathrm{N}(\mu_0, \sigma)$, then the DEWMA is $S_i' = \lambda S_i + \left(1-\lambda\right)S_{i-1}'$, where $S_i = \lambda X_i + \left(1-\lambda\right)S_{i-1}$ and the smooth linear forecast is

$$F_{i+t} = a_i + b_i t \tag{13}$$

where F_{i+t} is the forecast in the t period ahead,

$$a_i = 2S_i - S_i'$$

and

$$b_i = \frac{\lambda}{1-\lambda}\left(S_i - S_i'\right)$$

F_t is called the statistic of the DEWMABLP. It is possible to create three control charts: first, a control chart for the intercept a_i that will be similar than the EWMA control chart; second, a control chart for the slope b_i that is used to test if there is a linear drift or trend; and third, a control chart for a linear prediction t

periods ahead of i, F_t, that can be used to test if the statistic one period forecast ahead is or not in statistical control.

An Intercept Prediction DEWMA Control Chart (a_t)

The center line for an a_i control chart is the expected value of a_i. It is

$$\begin{aligned} \mathrm{E}(a_i) &= \mathrm{E}(2S_i - S_i') \\ &= \mathrm{E}(2S_i) - \mathrm{E}(S_i') \\ &= 2\,\mathrm{E}(S_i) - \mathrm{E}(S_i') \\ &= 2\mu_0 - \mu_0 \\ &= \mu_0 \end{aligned} \tag{14}$$

This can be verified using equations (5) and (10). Using equations (6) and (11), the variance of a_i can be obtained as

$$\mathrm{Var}_{\text{asym}}(a_i) = \mathrm{Var}(2S_i - S_i') \tag{15}$$

Brown (1962) showed the asymptotic variance for a predict value of a_i is

$$\mathrm{Var}_{\text{asym}}(a_i) = \frac{\lambda\left(1 + 4(1-\lambda) + 5(1-\lambda)^2\right)}{\left(1 + (1-\lambda)\right)^2}\sigma^2 \tag{16}$$

For large values of i the control limits for the a_i control chart become

$$\begin{aligned} \mathrm{UCL} &= \mu_0 + k\sigma\sqrt{\frac{\lambda\left(1 + 4(1-\lambda) + 5(1-\lambda)^2\right)}{\left(1 + (1-\lambda)\right)^2}\sigma^2} \\ \mathrm{CL} &= \mu_0 \\ \mathrm{LCL} &= \mu_0 - k\sigma\sqrt{\frac{\lambda\left(1 + 4(1-\lambda) + 5(1-\lambda)^2\right)}{\left(1 + (1-\lambda)\right)^2}\sigma^2} \end{aligned}$$

A Slope Prediction DEWMA Control Chart (b_t)

In a similar manner, the center line for b_i is the expected value of b_i. Using the equations (5) and (10) it can be shown that

$$\begin{aligned} \mathrm{E}(b_i) &= \mathrm{E}\left(\frac{\lambda}{1-\lambda}(S_i - S_i')\right) \\ &= \frac{\lambda}{1-\lambda}\left(\mathrm{E}(S_i) - \mathrm{E}(S_i')\right) \\ &= \frac{\lambda}{1-\lambda}(\mu_0 - \mu_0) \\ &= 0 \end{aligned} \tag{17}$$

The variance of b_i is defined as:

$$\begin{aligned} \mathrm{Var}(b_i) &= \mathrm{Var}\left(\frac{\lambda}{1-\lambda}(S_i - S_i')\right) \\ &= \left(\frac{\lambda}{1-\lambda}\right)^2 \mathrm{Var}(S_i - S_i') \end{aligned} \tag{18}$$

Brown (1962) gave the asymptotic variance of b_i for large values of i as

$$\mathrm{Var}_{\mathrm{asym}}(b_i) = \sigma^2 \frac{2\lambda^3}{\left(1+(1-\lambda)\right)^2} \tag{19}$$

Then, for large values of i the control limits for the b_i chart become

$$\begin{aligned} \mathrm{UCL} &= k\sigma\sqrt{\frac{2\lambda^3}{\left(1+(1-\lambda)\right)^3}} \\ \mathrm{CL} &= 0 \\ \mathrm{LCL} &= -k\sigma\sqrt{\frac{2\lambda^3}{\left(1+(1-\lambda)\right)^3}} \end{aligned}$$

A Linear Trend Prediction Double EWMA Control Chart (F_t)

Using equations (14) and (17), it can be shown that the expected value of F_t is

$$\begin{aligned} \mathrm{E}\left(F_{i+t}\right) &= \mathrm{E}\left(a_i + b_i t\right) \\ &= \mathrm{E}\left(a_i\right) + t\,\mathrm{E}\left(b_i\right) \\ &= \mu_0 + 0 \\ &= \mu_0 \end{aligned}$$

The variance of F_t is

$$\begin{aligned} \mathrm{Var}\left(F_{i+t}\right) &= \mathrm{Var}\left(a_i + b_i t\right) \\ &= \mathrm{Var}\left(a_i\right) + \mathrm{Var}\left(b_i t\right) + 2\,\mathrm{Cov}\left(a_i, b_i t\right) \end{aligned} \tag{20}$$

The covariance term 2Cov(a_i, $b_i t$) in the previous equation was investigated via simulation to verify the possible independence between a_i and b_i. Simulation for the covariance between a_i and b_i were performed for several values of the smooth parameter λ, considering a process under the in-control null hypothesis. The simulation yielded values very close to zero for the Cov(a_i, b_i). These results suggest that the covariance Cov(a_i, b_i) can be considered negligible. Nevertheless, For t = 1, Brown (1962) gives the asymptotic covariance of Cov(a_i, b_i):

$$\mathrm{Cov}\left(a_i, b_i\right) = \frac{\lambda^2\left(1 + 3\left(1-\lambda\right)\right)}{\left(1 + \left(1-\lambda\right)\right)^3}\sigma^2 \tag{21}$$

Substituting equations (16), (19), and (21) in equation (20) it is possible to obtain the asymptotic variance of the F_t as

$$\begin{aligned} &\mathrm{Var}_{\mathrm{asym}}\left(F_t\right) \\ &= \sigma^2\left(\frac{\lambda\left(1 + 4\left(1-\lambda\right) + 5\left(1-\lambda\right)^2\right)}{\left(1 + \left(1-\lambda\right)\right)^2} + \frac{2\lambda^3}{\left(1 + \left(1-\lambda\right)\right)^3} + \frac{\lambda^2\left(1 + 3\left(1-\lambda\right)\right)}{\left(1 + \left(1-\lambda\right)\right)^3}\right) \end{aligned} \tag{22}$$

Then, for large values of i the control limits and the center line for the F_t control chart become

$$
\begin{aligned}
&\text{UCL}\\
&= \mu_0 + k\sigma\sqrt{\frac{\lambda\left(1+4(1-\lambda)+5(1-\lambda)^2\right)}{\left(1+(1-\lambda)\right)^2}+\frac{2\lambda^3}{\left(1+(1-\lambda)\right)^3}+\frac{\lambda^2\left(1+3(1-\lambda)\right)}{\left(1+(1-\lambda)\right)^3}}\\
&\text{CL} = \mu_0 \qquad (23)\\
&\text{LCL}\\
&= \mu_0 - k\sigma\sqrt{\frac{\lambda\left(1+4(1-\lambda)+5(1-\lambda)^2\right)}{\left(1+(1-\lambda)\right)^2}+\frac{2\lambda^3}{\left(1+(1-\lambda)\right)^3}+\frac{\lambda^2\left(1+3(1-\lambda)\right)}{\left(1+(1-\lambda)\right)^3}}
\end{aligned}
$$

Design of a Double EWMA Base on Linear Prediction Control Chart F_t

The design parameters for this chart are constructed with k times the multiple of sigma, the standard deviation used in the control limits, and the value of λ, the smooth parameter. It is possible to choose these parameters to give a mean performance of average run length (ARL) under the null hypothesis (H_0), i.e. ARL_0, for a certain number. For example, an $ARL_0 = 370$ is the equivalent of an ARL of a Shewhart control chart under H_0 for 3σ as its control limits; the DEWMABLP can be designed with $k = 2.16$ and $\lambda = 0.10$ to obtain an $ARL_0 = 373 \approx 370$.

Assessing the Performance of DEWMABLP

In order to assess the performance of this new chart, its performance was compared with the performance of the EWMA, DEWMA, and Shewhart control charts. This comparison was made using the average run length under out-of-control (ARL_1).

Design Parameters for EWMA, DEWMA, and DEWMABLP Control Charts

A Monte Carlo simulation with 10,000 replications with an ARL_0 in-control was fixed approximately to 370 for all control charts under study: EWMA, DEWMA, DEWMABLP, and Shewhart control charts. In order to be fair, all charts were set to an $ARL_0 \approx 370$. Table 1 shows the parameters for λ and k for EWMA, DEWMA, and DEWMABLP control charts to give an $ARL_0 \approx 370$. Also, the standard deviation of ARL_0 is displayed.

Table 1. Average run length under H_0 for several λ and k

Control Chart	λ	k	ALR_0	$sd(ARL_0)$
EWMA	0.01	1.980	372.5	450.1
EWMA	0.05	2.511	371.8	372.1
EWMA	0.10	2.710	369.6	367.2
EWMA	0.15	2.800	371.1	366.0
EWMA	0.20	2.862	370.0	367.4
DEWMA	0.01	1.300	374.6	533.8
DEWMA	0.05	1.920	373.3	382.6
DEWMA	0.10	2.220	368.4	370.5
DEWMA	0.15	2.408	373.6	369.5
DEWMA	0.20	2.530	374.9	380.7
$F_t = a_t + b_t t$	0.01	1.725	375.2	557.0
$F_t = a_t + b_t t$	0.05	2.035	377.5	399.4
$F_t = a_t + b_t t$	0.10	2.160	373.3	386.0
Ft = at + bt	0.15	2.240	379.1	385.4
Ft = at + bt	0.20	2.287	374.5	373.7

Simulations were conducted to compare the performance of the EWMA, DEWMA, DEWMALP, and Shewhart control charts. The ARL under linear drift (ARL_1) for several slopes, that is the out-of-control, were compared between all this control charts. The control chart with lowest ARL_1 is considered the best chart.

The simulation was written in R. A stream of X_t for $t = 1$, 2,.., 100 observations were created such that X_t are independent and identically distributed as $X_t \sim N(\mu_0 = 10, \sigma = 1)$, and then another stream X_t was created such that of X_t for $t = 101,\ldots, 200$ observations where $X_t \sim N(\mu_0 = 10 + \beta t\sigma, \sigma = 1)$. This procedure was repeated several times using values of 0, 0.025, 0.05, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, 0.75, and 1 for the slope β.

Results

A summary of the simulation is presented in Tables 2 to 5. Table 2 shows the ARL_1 for these several slopes and $\lambda = 0.20$. In the same manner, Tables 3, 4, and 5 show the ARL_1 for these several slopes and $\lambda = 0.10$, 0.05, and 0.01, respectively.

When $\lambda = 0.2$, it can be observed in Table 2 that the ARL_1 of the F_t control chart is less than the ARL_1 of the other control charts only when the slope $\beta > 0.20$. For $\lambda = 0.10$ in Table 3, the ARL_1 of the F_t control chart is less than the ARL_1 of the other control charts when the slope $\beta \geq 0.10$. In similar way, for $\lambda = 0.05$ in Table 4, the ARL_1 of the F_t control chart is less than the ARL_1 of the other control charts for slope values $\beta \geq 0.05$. Finally, for $\lambda = 0.01$ in Table 5, the ARL_1 of the

F_t control chart is less than the ARL_1 of the other control charts for slope values between $0.025 \leq \beta < 0.200$. For values $\beta \geq 0.200$, the ARL_1 of Shewhart chart has the best performance.

Table 2. Average run length under different slopes, $\lambda = 0.20$

k	**3.000**	**2.862**	**2.530**	**2.287**
Slope	**Shewhart**	**EWMA**	**DEWMA**	$\boldsymbol{F_t}$
0.000	370.00	370.00	374.85	374.50
0.025	50.12	30.80	**29.41**	36.40
0.050	30.64	19.64	**19.36**	21.97
0.100	18.41	**12.59**	13.14	13.42
0.150	13.59	**9.79**	10.62	10.16
0.200	11.00	**8.27**	9.25	8.41
0.300	8.11	6.49	7.60	**6.45**
0.400	6.51	5.46	6.63	**5.37**
0.500	5.52	4.84	6.02	**4.71**
0.750	4.05	3.84	5.04	**3.67**
1.000	3.27	3.27	4.43	**3.11**

Table 3. Average run length under different slopes, $\lambda = 0.10$

k	**3.000**	**2.710**	**2.220**	**2.160**
Slope	**Shewhart**	**EWMA**	**DEWMA**	$\boldsymbol{F_t}$
0.000	370.0	369.6	368.4	373.3
0.025	50.15	**28.99**	29.83	30.78
0.050	30.68	**19.12**	20.98	19.52
0.100	18.52	12.85	15.27	**12.70**
0.150	13.67	10.16	12.74	**9.91**
0.200	11.01	8.62	11.21	**8.33**
0.300	8.14	6.92	9.47	**6.62**
0.400	6.56	5.90	8.39	**5.62**
0.500	5.52	5.21	7.70	**4.94**
0.750	4.04	4.22	6.57	**3.96**
1.000	**3.29**	3.63	5.83	3.40

Table 4. Average run length under different slopes, λ = 0.05

k	**3.000**	**2.511**	**1.920**	**2.035**
Slope	**Shewhart**	**EWMA**	**DEWMA**	F_t
0.000	370.00	371.80	373.33	377.51
0.025	50.17	**29.19**	24.78	29.39
0.050	30.60	19.70	18.69	**19.36**
0.100	18.55	13.62	18.69	**13.18**
0.150	13.65	10.83	15.74	**10.44**
0.200	11.07	9.36	14.23	**8.93**
0.300	8.13	7.51	12.09	**7.14**
0.400	6.52	6.46	10.85	**6.11**
0.500	5.49	5.75	9.93	**5.43**
0.750	**4.06**	4.67	8.54	4.40
1.000	**3.27**	4.02	7.63	3.78

Table 5. Average run length under different slopes, λ = 0.01

k	**3.000**	**1.980**	**1.300**	**1.735**
Slope	**Shewhart**	**EWMA**	**DEWMA**	F_t
0.000	370.0	372.5	374.6	375.2
0.025	49.94	33.41	56.08	**33.14**
0.050	30.79	23.45	43.63	**23.09**
0.100	18.49	16.56	34.15	**16.20**
0.150	13.72	13.50	29.56	**13.18**
0.200	**11.09**	11.65	26.70	11.37
0.300	**8.12**	9.45	23.08	9.21
0.400	**6.54**	8.14	20.76	7.92
0.500	**5.51**	7.28	19.18	7.09
0.750	**4.07**	5.96	16.71	5.78
1.000	**3.29**	5.16	15.11	5.02

Conclusions

The new DEWMABLP control chart works better than the other control charts to detect linear trends in the cases where a small linear trend is present. It works better when slopes are between 0.05 and 0.75 times the standard deviation. The EWMA control chart also performed well, but with an ARL_1 slightly higher than DEWMABLP. The DEWMA works better for small shifts, but works poorly for linear drifts. Also, it is observed that the performance of the DEWMABLP overcomes the performance of the Shewhart, EWMA, and DEWMA control charts when a linear drift is present and the slope of this linear drift is greater than the parameter lambda of the DEWMABLP, EWMA, and DEWMA. It can be concluded that the new DEWMABLP control chart can be used as an alternative when it is suspected that a linear drift can occur in the process after a period of stability. Of course, the DEWMABLP is not designed to detect a shift in the process; therefore, if a shift and a drift are expected at the same time, it should be used in combination with other control charts. This is a similar practice as utilizing both the Shewhart and EWMA control charts with the intention to detect both small and big shifts.

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