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The univariate time series models, in the case of unit root hypothesis, are more biased towards the acceptance of the Unit Root Hypothesis especially in a short time span. However, the panel data time series model is more appropriate in such situation. The Bayesian analysis of unit root testing for a panel data time series model is considered. An autoregressive panel data AR(1) model with linear time trend and augmentation term has been considered and derived the posterior odds ratio for testing the presence of unit root hypothesis under appropriate prior assumptions. A simulation study and real data analysis are carried out for the derived theorem.

Keywords: Panel data, stationarity, autoregressive time series, unit root, prior and posterior, posterior odds ratio

Introduction

Analysis of chronologically recorded data became more popular because of its usability, which combines cross-section information like a series collected from multiple locations, a group of people who are surveyed periodically over a given period of time, etc. This is called a panel data time series, which is also equally popular in the social sciences, having been used in economics to study the behavior of firms and wages of people over a time, as well as in marketing to study market share changes across different market structures (Yaffee, 2003; Hsiao, 2007). Panel data are more appropriate in comparison to the univariate model because of its

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applicability in controlling individual's heterogeneity, which is not controlled alone through univariate time series model. There are lots of works appearing in literature dealing with Bayesian unit root tests for univariate time series (DeJong & Whiteman, 1991; Schotman & van Dijk, 1991; Uhlig, 1994; Lubrano, 1995). The panel data time series model is able to study the dynamics of adjustment, which is capable of identifying and measures the effects that are simply not detectable in pure cross-section or pure time-series data. See also Harris and Tzavalis (1999), Maddala and Wu (1999), Kruiniger and Tzavalis (2002), Moon and Perron (2004), De Wachter, Harris, and Tzavalis (2007), Moon and Perron (2008), Madsen (2010), and De Blander and Dhaene (2012).

In an analysis of financial time series, testing the stationarity of series is very important. A series may be non-stationary due to time trend or unit root. The order of time trend may be reduced by one polynomial degree in the case of unit root. If there is a linear time trend then, under the unit root hypothesis, the model becomes difference stationary (Dickey & Fuller, 1979, 1981). The traditional theory of unit root associated with univariate approach has low power if it is close to unity, particularly in short term time span; see e.g. Shiller and Perron (1985). This makes the test more favorable to the acceptance of unit root hypothesis in various economic time series.

The literature on the unit root test has been extended to include the situations where panel data are available. Levin and Lin (1992, 1993) proposed to apply the unit root test on a pooled cross-sectional data set instead of an equation unit root test. The main inspiration behind the panel data unit root tests, as conferred by Maddala and Kim (1998), was to increase the power of the test by increasing the sample size through the panel data approach. These panel data unit root tests take advantage of cross-sectional information and escort to boost in power of the test. Asymptotic normality of the Dickey-Fuller test statistic for panel data with arbitrarily large cross-sectional dimensions and small fixed time series dimensions was obtained by Breitung and Meyer (1994), and Papell (1997) extended the panel data unit root test in case of purchase power parity to test the stationarity. Im, Pesaran, and Shin (1997) observed that the t-bar statistic has higher power than the Levin-Lin test by allowing a greater heterogeneity across individuals. Under the alternative hypothesis, they assumed the same long-run multiplier across countries and proposed a new test based on the mean group approach which was applied by Wu (2000) in panel data unit root tests to obtain support for the mean-reverting property of the current account series. Levin, Lin, and Chu (2002) analyzed the asymptotic and finite sample properties of the panel data unit root test when intercept and trend are allowed to vary across individuals.

Westerlund (2014) proposed a panel data unit root test where errors may be not only serial and/or cross-correlated, but also unconditionally heteroscedastic. Westerlund also shows evidence through Monte Carlo and suggested that the new test performs well in small samples in the case of panel data. The classical testing is predominantly based on the assumption that parameters are fixed and the population is finite; however, Bayesian analysis is free from such assumptions. The Bayesian approach for testing the unit root hypothesis was proposed by Sims (1988) and Sims and Uhlig (1991).

Kuma, Chaturvedi, and Afifa (2016) tested the stationarity of NAV series of NPS (new pension scheme) using unit root hypothesis and found that series are trend stationary. Karavias and Tzavalis (2016) discussed asymptotic local power properties in reference to the panel data unit root test for various fixed T and serially correlated error. They also studied the case considering the instrumental variable and found that variables are dominant in the case of the test based on the within-groups estimator. Schotman and van Dijk (1991) considered a panel data time series model with linear time trend incorporating augmentation term.

Here, a posterior odds ratio is derived for testing the unit root hypothesis considering prior assumption. A simulation study is carried out to explore the panel data unit root test. Considered here are three series in AR(1) panel data time series. First, take fixed values of intercept terms and generate the series for all possible combinations of fixed coefficients of time trend. Similarly, the simulation is repeated for taking a constant coefficient of time trend and generating the panel data for all possible combinations of fixed intercept terms. For numerical simplification, we have taken the prior odds ratio to be one and then we have obtained posterior odds ratios by applying the results of the theorem. The simulation results demonstrate that, for all combinations of selected values of the parameters, the derived posterior odds ratio correctly identifies the true hypothesis.

Model and Hypothesis

Consider the panel data time series model with linear time trend

$$y_{it} = \mu_i + \delta_i t + u_{it} \tag{1}$$

with $\{y_{it} : i = 1, 2, \ldots, n; t = 1, 2, \ldots, T\}$. This is a time series of observations on each of n cross sections, and its error is from an AR(1) process

$$u_{it} = \rho u_{it-1} + \varepsilon_{it} \tag{2}$$

Here, ρ is an autoregressive time series model and the ε_{it} are iid random variables, each following normal distribution with mean zero and variance τ^{-1}.

Write the model (1), incorporating an augmentation term and error equation (2) as:

$$y_{it} = \rho y_{it-1} + \left[(1-\rho)\mu_i + \rho\delta_i\right] + (1-\rho)\delta_i t + \sum_{j=1}^{k_i}\theta_{ij}\Delta y_{it-j} + \varepsilon_{it} \tag{3}$$

Considering $\alpha_i = [(1-\rho)\mu_i + \rho\delta_i]$ and $\beta_i = (1-\rho)\delta_i$,, and $\Delta y_{it} = y_{it} - y_{it-1}$. Rewrite the above model (3) as

$$y_{it} = \rho y_{it-1} + \alpha_i + \beta_i t + \sum_{j=1}^{k_i}\theta_{ij}\Delta y_{it-j} + \varepsilon_{it} \tag{4}$$

The interest is in testing the unit root hypothesis H_0: $\rho = 1$ against the alternative H_1: $\rho \in S$ with $S = \{a < \rho < 1; a > -1\}$. Under the null hypothesis of unit root, the model reduces to

$$\Delta y_{it} = \delta_i + \sum_{j=1}^{k_i}\theta_{ij}\Delta y_{it-j} + \varepsilon_{it} \tag{5}$$

Let $\mathbf{l}_t$ be a $T \times 1$ vector with all elements 1 and let $\boldsymbol{\xi}_T = (1, 2,\ldots, T)'$. Further, define

$$\begin{aligned} &\mathbf{\Delta y}_i = \left(\Delta y_{i1}, \Delta y_{i2}, \ldots, \Delta y_{it}\right)', \quad \mathbf{\Delta y} = \left(\mathbf{\Delta y}_1', \mathbf{\Delta y}_2', \ldots, \mathbf{\Delta y}_n'\right)', \\ &\mathbf{y} = \left(\mathbf{y}_1', \mathbf{y}_2', \ldots, \mathbf{y}_n'\right)', \quad \mathbf{y}_i = \left(y_{i1}, y_{i2}, \ldots, y_{iT}\right)', \quad \boldsymbol{\varepsilon} = \left(\boldsymbol{\varepsilon}_1', \boldsymbol{\varepsilon}_2', \ldots, \boldsymbol{\varepsilon}_n'\right)' \\ &\mathbf{Z} = \left[\left(\mathbf{I}_n \otimes \mathbf{l}_t\right)\left(\mathbf{I}_n \otimes \boldsymbol{\xi}_T\right)\right], \quad \boldsymbol{\alpha} = \left(\alpha_1, \alpha_2, \ldots, \alpha_n\right)', \quad \boldsymbol{\beta} = \left(\beta_1, \beta_2, \ldots, \beta_n\right)', \\ &\boldsymbol{\gamma} = \left(\boldsymbol{\alpha}, \boldsymbol{\beta}\right)', \quad \boldsymbol{\delta} = \left(\delta_1, \delta_2, \ldots, \delta_n\right)' \end{aligned} \tag{6}$$

Notice the i^{th} equation includes an augmentation term of order k_i. For writing the models (5) and (6) in matrix notations, let us write

$$k = \sum_{i=1}^{n} k_i, \quad \boldsymbol{\theta}_i = \begin{pmatrix} \theta_{i1} \\ \theta_{i2} \\ \vdots \\ \theta_{ik_i} \end{pmatrix}, \quad \mathbf{X}_i = \begin{pmatrix} \Delta y_{i\,0} & \Delta y_{i\,-1} & \cdots & \Delta y_{i\,1-k_i} \\ \Delta y_{i\,-1} & \Delta y_{i\,0} & \cdots & \Delta y_{i\,2-k_i} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta y_{i\,T-1} & \Delta y_{i\,T-2} & \cdots & \Delta y_{i\,T-k_i} \end{pmatrix},$$
$$\boldsymbol{\theta} = \begin{pmatrix} \boldsymbol{\theta}_1 \\ \boldsymbol{\theta}_2 \\ \vdots \\ \boldsymbol{\theta}_n \end{pmatrix}, \quad \mathbf{X} = \begin{pmatrix} \mathbf{X}_1' & 0 & \cdots & 0 \\ 0 & \mathbf{X}_2' & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{X}_n' \end{pmatrix} \tag{7}$$

Utilizing the above notations, along with the notations defined in (6), write the model under the null and alternative hypothesis as

$$\text{Under } \mathrm{H}_0 : \boldsymbol{\Delta}\mathbf{y} = \left(\mathbf{I}_n \otimes \mathbf{l}_T\right)\boldsymbol{\delta} + \mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon} \tag{8}$$

$$\text{Under } \mathrm{H}_1 : \mathbf{y} = \rho\mathbf{y}_{-1} + \mathbf{Z}\boldsymbol{\gamma} + \mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon} \tag{9}$$

where $\mathbf{y}_{-1} = \left(\mathbf{y}'_{-1,1}, \mathbf{y}'_{-1,2}, \ldots, \mathbf{y}'_{-1,n}\right)$ and $\mathbf{y}_{-1,i} = \left(y_{i0}, y_{i1}, \ldots, y_{iT-1}\right)'$.

Posterior Odds Ratio

The posterior odds ratio is now derived for the unit root hypothesis. Assume prior distributions as in Schotman and van Dijk (1991) for the following parameters of the model:

$$\begin{aligned} &\boldsymbol{\delta} \sim \mathrm{N}\left(0, \frac{1}{\vartheta\tau}\mathbf{I}_n\right), \quad \boldsymbol{\alpha} \sim \mathrm{N}\left((1-\rho)\,y_0, \frac{(1+\rho)}{(1-\rho)\tau}\mathbf{I}_n\right), \\ &\boldsymbol{\beta} \sim \mathrm{N}\left(0, \frac{(1-\rho)^2}{\vartheta\tau}\mathbf{I}_n\right), \\ &\mathrm{p}(\tau) \propto \frac{1}{\tau}, \quad 0 < \tau < \infty, \\ &\mathrm{p}(\rho) = \frac{1}{1-a}, \quad a < \rho < 1; a > -1 \\ &\mathrm{p}(\theta) \propto 1 \end{aligned} \tag{10}$$

where ϑ is considered to be a hyper parameter. Then prior distribution for $\boldsymbol{\gamma}$ is given by

$$\boldsymbol{\gamma} \sim \mathrm{N}\left((1-\rho)\boldsymbol{\phi}_0, \frac{1}{\tau}\mathrm{V}(\rho)^{-1} \right) \tag{11}$$

where, if $\mathbf{y}_0$ is the vector of initial observations,

$$\boldsymbol{\phi}_0 = \begin{pmatrix} \mathbf{y}_0 \\ 0 \end{pmatrix}, \quad \mathrm{V}(\rho) = \begin{pmatrix} \frac{(1+\rho)}{(1-\rho)}\mathbf{I}_n & 0 \\ 0 & \frac{\vartheta}{(1-\rho)^2}\mathbf{I}_n \end{pmatrix} \tag{12}$$

Assume that the prior probability in favor of H_0 is $P(\rho = 1) = p_0$ and prior probability in favor of H_1 is $P(\rho \in S) = 1 - p_0$.

Define

$$\begin{aligned} &\boldsymbol{\Sigma} = \mathbf{I}_{nT} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}', \quad \mathbf{A} = (\mathbf{I}_n \otimes \mathbf{l}_T)'\boldsymbol{\Sigma}(\mathbf{I}_n \otimes \mathbf{l}_T) + \vartheta\mathbf{I}_n, \\ &\varsigma = \Delta\mathbf{y}'\left(\boldsymbol{\Sigma} - \boldsymbol{\Sigma}(\mathbf{I}_n \otimes \mathbf{l}_T)\mathbf{A}^{-1}(\mathbf{I}_n \otimes \mathbf{l}_T)'\boldsymbol{\Sigma}\right)\Delta\mathbf{y}, \quad \mathrm{B}(\rho) = \mathbf{Z}'\boldsymbol{\Sigma}\mathbf{Z} + \mathrm{V}(\rho), \\ &\varsigma(\rho) = (\mathbf{y} - \rho\mathbf{y}_{-1})'\boldsymbol{\Sigma}(\mathbf{y} - \rho\mathbf{y}_{-1}) \\ &\qquad -\left(\mathbf{Z}'(\mathbf{y} - \rho\mathbf{y}_{-1}) + (1-\rho^2)\boldsymbol{\phi}_0\right)'\mathrm{B}(\rho)\left(\mathbf{Z}'(\mathbf{y} - \rho\mathbf{y}_{-1}) + (1-\rho^2)\boldsymbol{\phi}_0\right) \end{aligned} \tag{13}$$

Theorem: An AR(1) panel data time series model with linear time trend and augmentation term is difference stationary or trend stationary equivalent to H_0: $\rho = 1$ against the alternative H_1: $\rho \in S$ with $S = \{a < \rho < 1; a > -1\}$ with prior odds ratio $p_0 / (1 - p_0)$ can be tested by the posterior odds ratio is given by

$$\beta_{01} = \frac{p_0}{1-p_0}\frac{1-a}{|\mathbf{A}|^{\frac{1}{2}}\varsigma^{\frac{nT-k}{2}}}\left[\int_a^1 \frac{(1+\rho)^{\frac{n}{2}}}{(1-\rho)^{\frac{3n}{2}}|\mathrm{B}(\rho)|^{\frac{1}{2}}(\varsigma(\rho))^{\frac{nT-k}{2}}}d\rho\right]^{-1} \tag{14}$$

Proof: The likelihood function under the unit root hypothesis is given by

$$\mathrm{p}\left(\mathbf{y}\,|\,\boldsymbol{\delta},\tau\right)=\frac{\tau^{\frac{nT}{2}}}{\left(2\pi\right)^{\frac{nT}{2}}}\exp\left[-\frac{\tau}{2}\left\{\left(\Delta\mathbf{y}-\left(\mathbf{I}_n\otimes\mathbf{l}_T\right)\boldsymbol{\delta}-\mathbf{X}\boldsymbol{\theta}\right)'\left(\Delta\mathbf{y}-\left(\mathbf{I}_n\otimes\mathbf{l}_T\right)\boldsymbol{\delta}-\mathbf{X}\boldsymbol{\theta}\right)\right\}\right] \tag{15}$$

Combining the likelihood function with the prior distributions (10) of parameters, we get

$$\mathrm{p}\left(\mathbf{y}\,|\,\mathrm{H}_0\right)=\int_0^\infty\int_{\mathbb{R}^n}\int_{\mathbb{R}^{nk}}\frac{\tau^{\frac{nT+n}{2}-1}\boldsymbol{\vartheta}^{\frac{n}{2}}}{\left(2\pi\right)^{\frac{nT+n}{2}}}\exp\left[-\frac{\tau}{2}\left\{\left(\Delta\mathbf{y}-\left(\mathbf{I}_n\otimes\mathbf{l}_T\right)\boldsymbol{\delta}-\mathbf{X}\boldsymbol{\theta}\right)'\left(\Delta\mathbf{y}-\left(\mathbf{I}_n\otimes\mathbf{l}_T\right)\boldsymbol{\delta}-\mathbf{X}\boldsymbol{\theta}\right)+\vartheta\boldsymbol{\delta}'\boldsymbol{\delta}\right\}\right]d\boldsymbol{\theta}\,d\boldsymbol{\delta}\,d\tau \tag{16}$$

Let $\widehat{\boldsymbol{\theta}}=\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\left(\Delta\mathbf{y}-\left(\mathbf{I}_n\otimes\mathbf{l}_T\right)\boldsymbol{\delta}\right)$. Then, write (16) as

$$\begin{aligned}\mathrm{p}\left(\mathbf{y}\,|\,\mathrm{H}_0\right)=&\int_0^\infty\int_{\mathbb{R}^n}\int_{\mathbb{R}^{k}}\frac{\tau^{\frac{nT+n}{2}-1}\boldsymbol{\vartheta}^{\frac{n}{2}}}{\left(2\pi\right)^{\frac{nT+n}{2}}}\exp\left[-\frac{\tau}{2}\left\{\left(\Delta\mathbf{y}-\left(\mathbf{I}_n\otimes\mathbf{l}_T\right)\boldsymbol{\delta}\right)'\left(\mathbf{I}_{nk}-\mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\right)\left(\Delta\mathbf{y}-\left(\mathbf{I}_n\otimes\mathbf{l}_T\right)\boldsymbol{\delta}\right)\right.\right.\\&\left.\left.+\vartheta\boldsymbol{\delta}'\boldsymbol{\delta}+\left(\boldsymbol{\theta}-\widehat{\boldsymbol{\theta}}\right)'\mathbf{X}'\mathbf{X}\left(\boldsymbol{\theta}-\widehat{\boldsymbol{\theta}}\right)\right\}\right]d\boldsymbol{\theta}\,d\boldsymbol{\delta}\,d\tau \qquad (17)\\&=\frac{\boldsymbol{\vartheta}^{\frac{n}{2}}\left|\mathbf{XX}\right|^{-\frac{1}{2}}\Gamma\left(\frac{nT-k}{2}\right)}{\pi^{\frac{nT-k}{2}}\left|\mathbf{A}\right|^{\frac{1}{2}}\boldsymbol{\varsigma}^{\frac{nT-k}{2}}}\end{aligned}$$

Further, under H_1, the likelihood function is given by

$$\mathrm{p}\left(\mathbf{y} \mid \boldsymbol{\gamma}, \rho, \tau\right) = \frac{\tau^{\frac{nT}{2}}}{\left(2\pi\right)^{\frac{nT}{2}}} \exp\left[-\frac{\tau}{2}\left\{\left(\mathbf{y} - \rho\mathbf{y}_{-1} - \mathbf{Z}\boldsymbol{\gamma} - \mathbf{X}\boldsymbol{\theta}\right)' \left(\mathbf{y} - \rho\mathbf{y}_{-1} - \mathbf{Z}\boldsymbol{\gamma} - \mathbf{X}\boldsymbol{\theta}\right)\right\}\right] \tag{18}$$

Combining the likelihood function with the prior distribution, (10), (11), and (12) leads to

$$\mathrm{p}\left(\mathbf{y} \mid \mathrm{H}_1\right) = \int_a^1 \int_0^\infty \int_{\mathbb{R}^{2n}} \int_{\mathbb{R}^k} \frac{\tau^{\frac{nT}{2}+n-1} \boldsymbol{\vartheta}^{\frac{n}{2}} \left|\mathrm{V}\left(\rho\right)\right|^{\frac{1}{2}}}{\left(2\pi\right)^{\frac{nT}{2}+n} \left(1-a\right)} \exp\left[-\frac{\tau}{2}\left\{\left(\mathbf{y} - \rho\mathbf{y}_{-1} - \mathbf{Z}\boldsymbol{\gamma} - \mathbf{X}\boldsymbol{\theta}\right)' \left(\mathbf{y} - \rho\mathbf{y}_{-1} - \mathbf{Z}\boldsymbol{\gamma} - \mathbf{X}\boldsymbol{\theta}\right) + \left(\boldsymbol{\gamma} - \left(1-\rho\right)\boldsymbol{\phi}_0\right)' \mathrm{V}\left(\rho\right)\left(\boldsymbol{\gamma} - \left(1-\rho\right)\boldsymbol{\phi}_0\right)\right\}\right] d\boldsymbol{\theta}\, d\boldsymbol{\gamma}\, d\tau\, d\rho$$

writing

$$\tilde{\boldsymbol{\theta}} = \left(\mathbf{X}'\mathbf{X}\right)^{-1} \mathbf{X}'\left(\mathbf{y} - \rho\mathbf{y}_{-1} - \mathbf{Z}\boldsymbol{\gamma} - \mathbf{X}\boldsymbol{\theta}\right)$$
$$\tilde{\boldsymbol{\gamma}} = \mathrm{B}\left(\rho\right)^{-1}\left[\mathbf{Z}'\boldsymbol{\Sigma}\left(\mathbf{y} - \rho\mathbf{y}_{-1}\right) + \left(1-\rho^2\right)\boldsymbol{\phi}_0\right]$$

which obtains

$$
\begin{aligned}
\mathrm{p}(\mathbf{y}\,|\,\mathrm{H}_1) &= \int_a^1\int_0^\infty\int_{\mathbb{R}^{2n}}\int_{\mathbb{R}^k} \frac{\tau^{\frac{nT}{2}+n-1}\vartheta^{\frac{n}{2}}\left|\mathrm{V}(\rho)\right|^{\frac{1}{2}}}{(2\pi)^{\frac{nT}{2}+n}(1-a)} \exp\Big[-\frac{\tau}{2}\Big\{(\mathbf{y}-\rho\mathbf{y}_{-1}-\mathbf{Z}\boldsymbol{\gamma})'\boldsymbol{\Sigma}(\mathbf{y}-\rho\mathbf{y}_{-1}-\mathbf{Z}\boldsymbol{\gamma}) \\
&\qquad +(\boldsymbol{\gamma}-(1-\rho)\boldsymbol{\phi}_0)'\mathrm{V}(\rho)(\boldsymbol{\gamma}-(1-\rho)\boldsymbol{\phi}_0)\Big\}+(\boldsymbol{\theta}-\tilde{\boldsymbol{\theta}})'\mathbf{X}'\mathbf{X}(\boldsymbol{\theta}-\tilde{\boldsymbol{\theta}})\Big]d\boldsymbol{\theta}\,d\boldsymbol{\gamma}\,d\tau\,d\rho \\
&= \int_a^1\int_0^\infty\int_{\mathbb{R}^{2n}} \frac{\tau^{\frac{nT-k}{2}+n-1}\vartheta^{\frac{n}{2}}\left|\mathrm{V}(\rho)\right|^{\frac{1}{2}}\left|\mathbf{X}'\mathbf{X}\right|^{-\frac{1}{2}}}{(2\pi)^{\frac{nT}{2}+n}(1-a)} \exp\Big[-\frac{\tau}{2}\Big\{(\mathbf{y}-\rho\mathbf{y}_{-1}-\mathbf{Z}\boldsymbol{\gamma})'\boldsymbol{\Sigma}(\mathbf{y}-\rho\mathbf{y}_{-1}-\mathbf{Z}\boldsymbol{\gamma}) \\
&\qquad +(\boldsymbol{\gamma}-(1-\rho)\boldsymbol{\phi}_0)'\mathrm{V}(\rho)(\boldsymbol{\gamma}-(1-\rho)\boldsymbol{\phi}_0)\Big\}\Big]d\boldsymbol{\gamma}\,d\tau\,d\rho \\
&= \int_a^1\int_0^\infty\int_{\mathbb{R}^{2n}} \frac{\tau^{\frac{nT-k}{2}+n-1}\vartheta^{\frac{n}{2}}\left|\mathrm{V}(\rho)\right|^{\frac{1}{2}}\left|\mathbf{X}'\mathbf{X}\right|^{-\frac{1}{2}}}{(2\pi)^{\frac{nT}{2}+n}(1-a)} \exp\Big[-\frac{\tau}{2}\Big\{\varsigma(\rho)+(\boldsymbol{\gamma}-\tilde{\boldsymbol{\gamma}})'\mathrm{B}(\rho)(\boldsymbol{\gamma}-\tilde{\boldsymbol{\gamma}})\Big\}\Big]d\boldsymbol{\gamma}\,d\tau\,d\rho \\
&= \int_a^1\int_0^\infty \frac{\tau^{\frac{nT-k}{2}-1}\vartheta^{\frac{n}{2}}(1+\rho)^{\frac{n}{2}}\left|\mathbf{X}'\mathbf{X}\right|^{-\frac{1}{2}}}{(2\pi)^{\frac{nT-k}{2}}(1-\rho)^{\frac{3n}{2}}\left|\mathrm{B}(\rho)\right|^{\frac{1}{2}}(1-a)} \exp\Big[-\frac{\tau}{2}\varsigma(\rho)\Big]d\tau\,d\rho \\
&= \frac{\vartheta^{\frac{n}{2}}\left|\mathbf{X}'\mathbf{X}\right|^{-\frac{1}{2}}\Gamma\left(\frac{nT-k}{2}\right)}{\pi^{\frac{nT-k}{2}}(1-a)} \int_a^1 \frac{(1+\rho)^{\frac{n}{2}}}{(1-\rho)^{\frac{3n}{2}}\left|\mathrm{B}(\rho)\right|^{\frac{1}{2}}\varsigma(\rho)^{\frac{nT-k}{2}}}\,d\rho
\end{aligned}
\tag{19}
$$

Using (17) and (19), obtain the expression (14) required of the theorem.

Table 1. Posterior odds ratio and autoregressive coefficient value

β_{01}	$\hat{\rho}$	SE$(\hat{\rho})$	$\hat{\sigma}^2$
1.25E-214	0.674	1.23E+03	7.83E+06

Numerical Illustration

To understand the need and worthiness of proposed study, empirical as well as simulation studies are frequently used methodology, therefore an empirical analysis for the model under study is explored and then a simulation study is also applied for the same.

Empirical Analysis

The objective is to develop the testing procedure in order to test the unit root hypothesis for the panel data time series model with linear time trend incorporating an augmentation term of order one. Consider the Bayesian procedure for testing the unit root hypothesis and obtain the posterior odds ratio and empirical analysis to justify the Bayesian testing procedure for proposed model. To fulfill the objective, data were taken from *Statistics at a Glance 2014* (Ministry of Agriculture, India, 2015). The import series of fertilizers were analyzed, namely Nitrogen (N), Phosphate (P), and Potash (K), in a panel dataset covering the period from 1980-81 to 2013-14.

Consider the series of fertilizers as a time series in three panels where panels are taken in respect to different fertilizers. Test the hypothesis whether the observed fertilizer series are difference stationary or trend stationary or, equivalently, the unit root hypothesis $H_0: \rho = 1$ against the alternative $H_1: \rho \in S$ with $S = \{a < \rho < 1; a > -1\}$. Let $\{y_{it} : i = 1, 2, \ldots, n; t = 1, 2, \ldots, T\}$ be the recorded import series of fertilizers are assumed by the model (5) for analysis purpose.

The posterior odds ratio, estimated value of $\hat{\rho}$ with SE$(\hat{\rho})$, and error variance $(\hat{\sigma})$ are recorded on Table 1, and the maximum likelihood estimates of autoregressive regression coefficients: intercept, trend, and $\hat{\rho}$ are given in Table 2 with variance covariance matrix of regression coefficients.

As the posterior odds ratio works on the comparison of probabilities, it is clear from Table 1 that observed posterior odds ratio under study is less than one which indicates that the unit root hypothesis $H_0: \rho = 1$ is rejected and we may accept the alternative hypothesis $H_1: \rho \in S$ with $S = \{a < \rho < 1; a > -1\}$.Therefore, series of fertilizers under study are trend stationary. For justification of real data analysis, a simulation study has also conducted.

Simulation Study

Data were generated from the panel data time series model:

$$y_{it} = \rho y_{it-1} + \left[(1-\rho)\mu_i + \rho\delta_i\right] + (1-\rho)\delta_i t + \sum_{j=1}^{k_i}\theta_{ij}\Delta y_{it-j} + \varepsilon_{it}$$

with $i = 1, 2,\dots, n$; $t = 1, 2,\dots, T$. For generating a panel data time series of size 25, consider three panels with $\varepsilon_{it} \sim N(0, 1)$, where the initial observations are $y_{10} = 1000$, $y_{20} = 1500$, and $y_{30} = 2000$, the coefficient of augmentation term with order 1; $\theta_{11} = \theta_{21} = \theta_{31} = 1$.We have generated the data for two situations. In the first situation, take the fixed values of the intercept term, which are $\{\mu_1 = 750, \mu_2 = 1000, \mu_3 = 1250\}$ and generate the series for all the possible 27 combinations of values $\{\delta_1, \delta_2, \delta_3\} = \{1, 1.25, 1.5\}$. Similarly, in the second situation, take the fixed values of the coefficient of time trend $\{\delta_1 = 1, \delta_2 = 1.25, \delta_3 = 1.5\}$ and generate the series for all the 27 possible combinations of $\{\mu_1, \mu_2, \mu_3\} = \{750, 1000, 1250\}$.

Using the derived theorem for the model (3), the presence of unit root in autoregressive panel data is tested with linear time trend and augmentation term for the values of $\rho = \{0.90, 0.92, 0.94, 0.96, 0.98\}$. The posterior odds ratio and estimated value of $\widehat{\rho}$ with $\mathrm{SE}(\widehat{\rho})$ for testing unit root hypothesis, i.e. series is difference stationary (equivalent to: H_0: $\rho = 1$) against the alternative, i.e. the series is trend stationary (equivalent to H_1: $\rho \in S$ with $S = \{a < \rho < 1; a > -1\}$) are calculated. Because the posterior odds ratio is decreasing with the increasing value of ρ, therefore we have reported the results only for $\rho = \{0.90, 0.94, 0.98\}$ in Tables 3 to 5, considering equal prior probability for the null and alternative hypothesis.

AR(1) panel data time series model is generated considering the trend stationary model and we got all posterior odds ratios less than one. In a Bayesian testing procedure using the posterior odds ratio, the concept of accepting the hypothesis is directly decided with respect to the hypothesis which has more chance in comparison to other. Here, if the probability of the alternate is more than the probability of null that means the results are confirming that all the generated series under the setup of trend stationary are concluded trend stationary. Therefore, it may be concluded the results support the derived theorem for identifying the unit root hypothesis correctly.

Conclusion

The posterior odds ratio was derived for testing the unit root hypothesis in panel data time series models with a linear trend and augmentation term. The simulation and empirical study correctly tested the hypothesis. This may be extended for the cases of non-linear time trends, a model with non-normal errors, as well as other multivariate time series models.

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Appendix A: Tables 2-5

Table 2. Variance and covariance matrix for fertilizers panel data

Coeff						**Variance Covariance Matrix** $(\hat{\Sigma})$					
$\hat{\rho}$	6.75E-01	1.50E+06	5.90E+07	4.10E+08	-8.80E+08	-2.60E+08	-2.20E+08	-1.90E+08	-8.40E+05	-6.50E+05	-1.10E+06
$\hat{\mu}_1$	-1.74E+01	5.90E+07	1.30E+13	1.60E+10	-3.50E+10	-9.00E+11	-8.60E+09	-7.40E+09	2.70E+08	-2.60E+07	-4.20E+07
$\hat{\mu}_2$	2.20E+01	4.10E+08	1.60E+10	1.30E+13	-2.40E+11	-6.90E+10	-9.50E+11	-5.10E+10	-2.20E+08	3.40E+08	-2.90E+08
$\hat{\mu}_3$	4.49E+02	-8.80E+08	-3.50E+10	-2.40E+11	1.40E+13	1.50E+11	1.30E+11	-8.00E+11	4.90E+08	3.80E+08	-9.20E+08
$\hat{\beta}_1$	6.36E+01	-2.60E+08	-9.00E+11	-6.90E+10	1.50E+11	1.30E+11	3.80E+10	3.30E+10	-3.60E+07	1.10E+08	1.90E+08
$\hat{\beta}_2$	4.08E+01	-2.20E+08	-8.60E+09	-9.50E+11	1.30E+11	3.80E+10	1.10E+11	2.70E+10	1.20E+08	-2.20E+06	1.60E+08
$\hat{\beta}_3$	1.82E+01	-1.90E+08	-7.40E+09	-5.10E+10	-8.00E+11	3.30E+10	2.70E+10	1.10E+11	1.10E+08	8.20E+07	2.80E+08
$\hat{\theta}_{11}$	4.04E-01	-8.40E+05	2.70E+08	-2.20E+08	4.90E+08	-3.60E+07	1.20E+08	1.10E+08	9.00E+06	3.60E+05	6.00E+05
$\hat{\theta}_{12}$	1.52E-01	-6.50E+05	-2.60E+07	3.40E+08	3.80E+08	1.10E+08	-2.20E+06	8.20E+07	3.60E+05	6.70E+06	4.70E+05
$\hat{\theta}_{13}$	1.38E-01	-1.10E+06	-4.20E+07	-2.90E+08	-9.20E+08	1.90E+08	1.60E+08	2.80E+08	6.00E+05	4.70E+05	7.90E+06

Table 3. For ρ = 0.90, posterior odds ratio and estimated autoregressive coefficient for different combinations of intercept (μ) and coefficient of time trend value (δ)

For fixed intercept term: μ_1 = 750; μ_2 = 1000; μ_3 = 1250						For fixed coefficient of time trend: δ_1 = 1.0; δ_2 = 1.25; δ_3 = 1.5					
δ_1	δ_2	δ_3	$\hat{\rho}$	SE$(\hat{\rho})$	β_{01}	μ_1	μ_2	μ_3	$\hat{\rho}$	SE$(\hat{\rho})$	β_{01}
1.00	1.00	1.00	0.9000	8.39E-04	3.13E-160	750	750	750	0.90002	8.34E-04	1.13E-140
1.00	1.00	1.25	0.9000	6.75E-04	2.86E-170	750	750	1000	0.90000	6.02E-04	3.06E-150
1.00	1.00	1.50	0.9000	8.43E-04	1.55E-170	750	750	1250	0.90002	8.39E-04	1.61E-160
1.00	1.25	1.00	0.9000	6.05E-04	2.80E-170	750	1000	750	0.90003	8.38E-04	1.24E-145
1.00	1.25	1.25	0.9000	9.45E-04	1.67E-170	750	1000	1000	0.90003	8.41E-04	3.02E-157
1.00	1.25	1.50	0.9000	8.43E-04	1.26E-170	750	1000	1250	0.90003	8.43E-04	1.26E-170
1.00	1.50	1.00	0.9000	8.43E-04	1.92E-170	750	1250	750	0.90000	6.01E-04	6.30E-149
1.00	1.50	1.25	0.9000	6.05E-04	1.66E-170	750	1250	1000	0.90003	8.43E-04	3.34E-162
1.00	1.50	1.50	0.9000	9.45E-04	9.94E-171	750	1250	1250	0.90003	8.16E-04	1.51E-140
1.25	1.00	1.00	0.8999	8.96E-04	3.59E-170	1000	750	750	0.90003	8.24E-04	1.49E-141
1.25	1.00	1.25	0.9000	8.43E-04	1.92E-170	1000	750	1000	0.90000	8.27E-04	1.81E-151
1.25	1.00	1.50	0.9001	5.83E-04	1.67E-170	1000	750	1250	0.90003	8.30E-04	3.80E-162
1.25	1.25	1.00	0.9000	8.43E-04	2.14E-170	1000	1000	750	0.90003	8.28E-04	1.17E-146
1.25	1.25	1.25	0.9000	6.05E-04	1.85E-170	1000	1000	1000	0.90000	6.04E-04	1.23E-158
1.25	1.25	1.50	0.9000	8.43E-04	1.14E-170	1000	1000	1250	0.90003	8.33E-04	7.28E-173
1.25	1.50	1.00	0.9000	8.43E-04	1.74E-170	1000	1250	750	0.90003	8.30E-04	4.00E-150
1.25	1.50	1.25	0.9000	8.43E-04	1.27E-170	1000	1250	1000	0.90000	6.05E-04	7.83E-164
1.25	1.50	1.50	0.9000	8.43E-04	9.27E-171	1000	1250	1250	0.90000	9.46E-04	2.28E-181
1.50	1.00	1.00	0.9000	6.05E-04	2.83E-170	1250	750	750	0.90003	8.45E-04	1.65E-178
1.50	1.00	1.25	0.9000	8.43E-04	1.74E-170	1250	750	1000	0.90003	8.19E-04	4.68E-150
1.50	1.00	1.50	0.9000	8.43E-04	1.27E-170	1250	750	1250	0.90003	8.21E-04	3.82E-160
1.50	1.25	1.00	0.9000	8.43E-04	1.94E-170	1250	1000	750	0.90000	6.02E-04	2.10E-145
1.50	1.25	1.25	0.9000	8.43E-04	1.42E-170	1250	1000	1000	0.90003	8.22E-04	6.34E-157
1.50	1.25	1.50	0.9000	8.43E-04	1.03E-170	1250	1000	1250	0.90000	6.06E-04	5.76E-170
1.50	1.50	1.00	0.9000	8.43E-04	1.58E-170	1250	1250	750	0.90000	6.03E-04	1.00E-148
1.50	1.50	1.25	0.8999	1.21E-03	1.81E-170	1250	1250	1000	0.90003	8.24E-04	8.17E-162
1.50	1.50	1.50	0.9000	8.43E-04	8.40E-171	1250	1250	1250	0.90003	8.27E-04	9.48E-178

Table 4. For ρ = 0.94, posterior odds ratio and estimated autoregressive coefficient for different combinations of intercept (μ) and coefficient of time trend value (δ)

For fixed intercept term: μ_1 = 750; μ_2 = 1000; μ_3 = 1250						For fixed coefficient of time trend: δ_1 = 1.0; δ_2 = 1.25; δ_3 = 1.5					
δ_1	δ_2	δ_3	$\hat{\rho}$	$SE(\hat{\rho})$	β_{01}	μ_1	μ_2	μ_3	$\hat{\rho}$	$SE(\hat{\rho})$	β_{01}
1.00	1.00	1.00	0.9400	8.10E-04	1.05E-181	750	750	750	0.94000	8.13E-04	4.33E-160
1.00	1.00	1.25	0.9400	9.88E-04	2.00E-192	750	750	1000	0.93998	6.06E-04	2.73E-170
1.00	1.00	1.50	0.9400	8.09E-04	1.64E-193	750	750	1250	0.94000	8.10E-04	2.49E-182
1.00	1.25	1.00	0.9400	6.03E-04	1.65E-192	750	1000	750	0.94000	8.12E-04	2.66E-165
1.00	1.25	1.25	0.9400	7.17E-04	9.46E-193	750	1000	1000	0.94000	8.11E-04	2.01E-178
1.00	1.25	1.50	0.9400	8.09E-04	1.06E-193	750	1000	1250	0.94000	8.09E-04	1.06E-193
1.00	1.50	1.00	0.9400	8.09E-04	2.69E-193	750	1250	750	0.93998	6.04E-04	4.84E-168
1.00	1.50	1.25	0.9400	6.03E-04	5.43E-193	750	1250	1000	0.94000	8.10E-04	3.06E-183
1.00	1.50	1.50	0.9400	7.17E-04	3.14E-193	750	1250	1250	0.94000	8.08E-04	5.31E-159
1.25	1.00	1.00	0.9400	5.95E-04	1.16E-191	1000	750	750	0.94000	8.11E-04	1.25E-160
1.25	1.00	1.25	0.9400	8.09E-04	2.66E-193	1000	750	1000	0.94000	8.10E-04	1.27E-171
1.25	1.00	1.50	0.9400	8.94E-04	9.88E-193	1000	750	1250	0.94000	8.08E-04	2.26E-183
1.25	1.25	1.00	0.9400	8.09E-04	3.40E-193	1000	1000	750	0.94000	8.10E-04	6.20E-166
1.25	1.25	1.25	0.9400	6.03E-04	6.93E-193	1000	1000	1000	0.93998	6.01E-04	8.86E-179
1.25	1.25	1.50	0.9400	8.09E-04	8.73E-194	1000	1000	1250	0.94000	8.07E-04	3.46E-195
1.25	1.50	1.00	0.9400	8.09E-04	2.22E-193	1000	1250	750	0.94000	8.09E-04	3.91E-169
1.25	1.50	1.25	0.9400	8.09E-04	1.12E-193	1000	1250	1000	0.93998	5.99E-04	7.29E-184
1.25	1.50	1.50	0.9400	8.09E-04	5.65E-194	1000	1250	1250	0.93996	7.13E-04	3.78E-203
1.50	1.00	1.00	0.9400	6.03E-04	1.76E-192	1250	750	750	0.94000	8.08E-04	8.02E-202
1.50	1.00	1.25	0.9400	8.09E-04	2.21E-193	1250	750	1000	0.94000	8.07E-04	2.50E-169
1.50	1.00	1.50	0.9400	8.09E-04	1.12E-193	1250	750	1250	0.94000	8.05E-04	4.35E-180
1.50	1.25	1.00	0.9400	8.09E-04	2.83E-193	1250	1000	750	0.94000	5.97E-04	1.51E-163
1.50	1.25	1.25	0.9400	8.09E-04	1.43E-193	1250	1000	1000	0.94000	8.06E-04	1.81E-176
1.50	1.25	1.50	0.9400	8.09E-04	7.23E-194	1250	1000	1250	0.93998	5.95E-04	8.22E-190
1.50	1.50	1.00	0.9400	8.09E-04	1.84E-193	1250	1250	750	0.91998	7.14E-04	6.71E-154
1.50	1.50	1.25	0.9400	9.83E-04	5.91E-194	1250	1250	1000	0.94000	8.05E-04	5.40E-181
1.50	1.50	1.50	0.9400	8.09E-04	4.67E-194	1250	1250	1250	0.94000	8.03E-04	7.19E-198

Table 5. For ρ = 0.98, posterior odds ratio and estimated autoregressive coefficient for different combinations of intercept (μ) and coefficient of time trend value (δ)

For fixed intercept term: $\mu_1 = 750; \mu_2 = 1000; \mu_3 = 1250$						For fixed coefficient of time trend: $\delta_1 = 1.0; \delta_2 = 1.25; \delta_3 = 1.5$					
δ_1	δ_2	δ_3	$\hat{\rho}$	$SE(\hat{\rho})$	β_{01}	μ_1	μ_2	μ_3	$\hat{\rho}$	$SE(\hat{\rho})$	β_{01}
1.00	1.00	1.00	0.9800	7.17E-04	4.57E-176	750	750	750	0.97999	7.29E-04	2.65E-154
1.00	1.00	1.25	0.9800	6.08E-04	1.47E-188	750	750	1000	0.98000	5.17E-04	3.63E-166
1.00	1.00	1.50	0.9800	7.11E-04	3.52E-189	750	750	1250	0.97999	7.16E-04	3.84E-178
1.00	1.25	1.00	0.9800	5.09E-04	5.04E-188	750	1000	750	0.97999	7.24E-04	3.12E-159
1.00	1.25	1.25	0.9800	6.03E-04	2.79E-187	750	1000	1000	0.97999	7.17E-04	5.46E-174
1.00	1.25	1.50	0.9800	7.11E-04	8.96E-190	750	1000	1250	0.97999	7.11E-04	8.96E-190
1.00	1.50	1.00	0.9800	7.11E-04	2.40E-188	750	1250	750	0.98000	5.15E-04	1.66E-160
1.00	1.50	1.25	0.9800	5.09E-04	1.46E-189	750	1250	1000	0.97999	7.12E-04	1.62E-176
1.00	1.50	1.50	0.9800	6.03E-04	4.16E-189	750	1250	1250	0.98000	7.26E-04	1.14E-149
1.25	1.00	1.00	0.9800	5.18E-04	3.22E-187	1000	750	750	0.97999	7.27E-04	2.41E-153
1.25	1.00	1.25	0.9800	7.11E-04	2.36E-188	1000	750	1000	0.98000	7.20E-04	4.02E-165
1.25	1.00	1.50	0.9799	4.65E-04	4.56E-190	1000	750	1250	0.97999	7.14E-04	2.23E-176
1.25	1.25	1.00	0.9800	7.11E-04	6.05E-188	1000	1000	750	0.97999	7.22E-04	3.90E-158
1.25	1.25	1.25	0.9800	5.09E-04	3.81E-189	1000	1000	1000	0.98000	5.16E-04	2.91E-172
1.25	1.25	1.50	0.9800	7.11E-04	6.46E-190	1000	1000	1250	0.97999	7.09E-04	6.89E-188
1.25	1.50	1.00	0.9800	7.11E-04	1.77E-188	1000	1250	750	0.97999	7.17E-04	1.16E-159
1.25	1.50	1.25	0.9800	7.11E-04	1.74E-189	1000	1250	1000	0.98000	5.13E-04	8.90E-175
1.25	1.50	1.50	0.9800	7.10E-04	1.71E-190	1000	1250	1250	0.98000	5.98E-04	4.82E-194
1.50	1.00	1.00	0.9800	5.10E-04	1.14E-187	1250	750	750	0.97999	7.06E-04	1.91E-195
1.50	1.00	1.25	0.9800	7.11E-04	1.89E-188	1250	750	1000	0.97999	7.20E-04	5.67E-160
1.50	1.00	1.50	0.9800	7.11E-04	2.03E-189	1250	750	1250	0.98000	7.13E-04	1.35E-169
1.50	1.25	1.00	0.9800	7.11E-04	4.87E-188	1250	1000	750	0.98000	5.24E-04	6.50E-154
1.50	1.25	1.25	0.9800	7.11E-04	4.96E-189	1250	1000	1000	0.97999	7.14E-04	3.69E-166
1.50	1.25	1.50	0.9800	7.11E-04	5.03E-190	1250	1000	1250	0.98000	5.15E-04	6.13E-179
1.50	1.50	1.00	0.9800	7.11E-04	1.41E-188	1250	1250	750	0.98000	5.21E-04	3.39E-155
1.50	1.50	1.25	0.9800	6.74E-04	4.30E-190	1250	1250	1000	0.97999	7.10E-04	1.07E-168
1.50	1.50	1.50	0.9800	7.10E-04	1.32E-190	1250	1250	1250	0.97999	7.04E-04	2.86E-184