Journal of Modern Applied Statistical Methods

Volume 16 | Issue 1 Article 13

2017

Factor Analysis by Limited Scales: Which Factors to Analyze?

Stan Lipovetsky
GfK North America, stan.lipovetsky@gfk.com

Recommended Citation

Lipovetsky, S. (2017). Factor analysis by limited scales: which factors to analyze? Journal of Modern Applied Statistical Methods, 16(1), 233-245. doi: 10.22237/jmasm/1493597520

Journal of Modern Applied Statistical Methods
May 2017, Vol. 16, No. 1, 233-245.
doi: 10.22237/jmasm/1493597520

ISSN 1538 − 9472

Factor Analysis by Limited Scales: Which Factors to Analyze?

Stan Lipovetsky
GfK North America
Minneapolis, MN

Factor Analysis (FA) and Principal Component Analysis (PCA) are well-known main tools of the multivariate statistics for data analysis, reduction, and visualization. Commonly, the analysis and interpretation of their solutions is performed for each of several main eigenvectors with variances explaining a big part of the total variability in data. The recommendation is to determine if all the main vectors are really needed in the analysis, or some of them should be skipped if they correspond to the absence of the analyzing features. A simple criterion for identifying redundant vectors of loadings is their negative correlation with the vector of mean values of the original variables. Limited Likert scales of measurements are considered, and it is shown variables correlations and variances are connected to the mean values. FA and PCA structures defined by subsets of highly related variables can correspond to the lower levels of Likert scales meaning the absence of the measured features, so these loading vectors could be senseless for interpretation. Numerical examples are discussed on marketing research data.

Keywords: FA, PCA, loadings, eigenvectors, interpretation

Introduction

Factor Analysis (FA), Principal Component Analysis (PCA), and also Singular Value Decomposition (SVD) are well-known main tools of the multivariate statistics for data analysis, reduction, and visualization, widely used already for many dozen years (for instance, Lawley & Maxwell, 1971; Timm, 1975; Harman, 1976; Dillon & Goldstein, 1984) and continuing to be described and developed in numerous works (Bartholomew & Knott, 1999; Skrondal & Rabe-Hesketh, 2004; Lipovetsky & Conklin 2005; Elden, 2007; Härdle & Hlávka, 2007; Motoda & Liu, 2008; Izenman, 2008; Härdle & Simar, 2012; Lipovetsky, 2009, 2012, 2015). The analysis and interpretation of their solutions is usually performed for several first

Stan Lipovetsky, Ph.D., is a Senior Research Director in the Marketing Sciences department. Email them at: stan.lipovetsky@gfk.com.

retained eigenvectors with bigger variances explaining a main part of the total variability in data.

Variables defined in Likert scales often applied in marketing research and other social measurements are considered. It is a limited scale of, for instance, four, five, seven, or ten levels for measuring characteristics of interest. The paper shows that the variables' mean values can influence their variances, correlations, and the loadings of FA or PCA. In some cases the FA and PCA loading structures defined by subsets of highly related variables can correspond to the levels of Likert scales which actually indicate the absence of the measured features, so such loading vectors could be redundant for analysis and interpretation. The paper suggests checking correlations of the main eigenvectors with the vector of means, and when some of these correlations are negative the related factors may be skipped from consideration if they correspond not to presence but to absence of the analyzing features.

Relation of Means, Standard Deviations, and Correlations for Limited Scales

Consider data from a real marketing research project on features and qualities of protein snacks and shakes, where 1034 respondents evaluated thirty-five attributes by four-point Likert scales with levels

$$\begin{cases} 4 \text{ - definitely applies to me} \\ 3 \text{ - applies to me somewhat} \\ 2 \text{ - does not really apply to me} \\ 1 \text{ - does not apply to me at all} \end{cases} \tag{1}$$

Table 1 presents descriptive statistics on these attributes: means and standard deviations (std).

The graph of std versus mean values is presented in Figure 1 and shows that standard deviations are smaller if mean values are closer to the margins 1 and 4 of this Likert scale. Note that there are less observations on the lower levels of the scale because respondents in marketing research mostly answer at the "better" side of scales. It is intuitively clear that it should be so, because there is simply no space for volatility when most of observations gravitate to one or another margin of a limited scale. Quadratic regression of standard deviation by mean values yields the model:

$$\text{std} = -0.52 + 1.40\,\text{mean} - 0.30\,\text{mean}^2 \qquad (2)$$

where the coefficient of multiple determination R^2 =0.88 and the *F*-statistic of 4264 are big, so the model is of a very high quality.

Finding Pearson's pair correlations between all the attributes and stacking them into one matrix together with the corresponding mean values we can consider how correlations depend on mean values. To make such a consideration more clear, we can find 5% quantiles of the means and correlations and present them on one graph – see Figure 2. It shows that there evidently are two areas of higher correlations related to bigger and to smaller mean values.

Fourth-degree polynomial regression corresponding to the plot in Figure 2 yields the model:

$$\text{cor} = -73.30 + 114.60\,\text{mean} - 66.35\,\text{mean}^2 + 16.97\,\text{mean}^3 - 1.625\,\text{mean}^4 \qquad (3)$$

with coefficient of multiple determination R^2 = 0.58 and *F*-statistic 5.1, so the model is of a good quality as well. The smaller std at the margins of the limited scale presented in Figure 1 are translated onto the bigger correlation (as covariance divided by standard deviations of the correlated variables) in Figure 2.

Table 1. Means and std for 35 attributes measure by 4-point Likert scale

attribute	mean	std	attribute	mean	std
1	2.77	1.06	19	2.94	1.03
2	2.86	1.06	20	2.94	1.00
3	2.54	1.08	21	2.06	1.06
4	2.85	1.03	22	2.84	1.03
5	2.81	1.05	23	2.47	1.12
6	2.92	0.99	24	3.08	0.91
7	2.81	1.04	25	2.99	0.98
8	2.91	1.04	26	2.77	1.03
9	2.39	1.08	27	2.27	1.11
10	2.34	1.07	28	3.19	0.90
11	3.07	0.90	29	2.96	0.98
12	3.00	0.98	30	2.99	0.94
13	3.07	0.92	31	2.84	0.97
14	2.97	0.98	32	3.14	0.91
15	2.60	1.08	33	3.03	0.91
16	2.68	1.09	34	2.45	1.11
17	2.78	1.03	35	2.46	1.11
18	1.91	1.08			

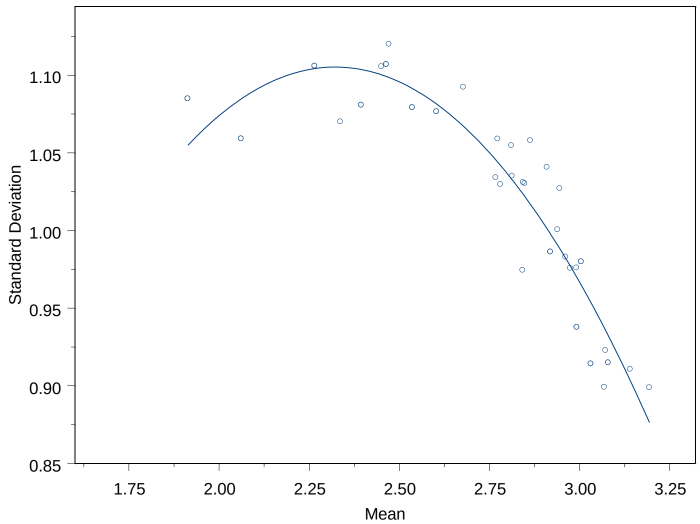

Figure 1. Standard deviation versus mean for attributes measured by Likert 4-point scale

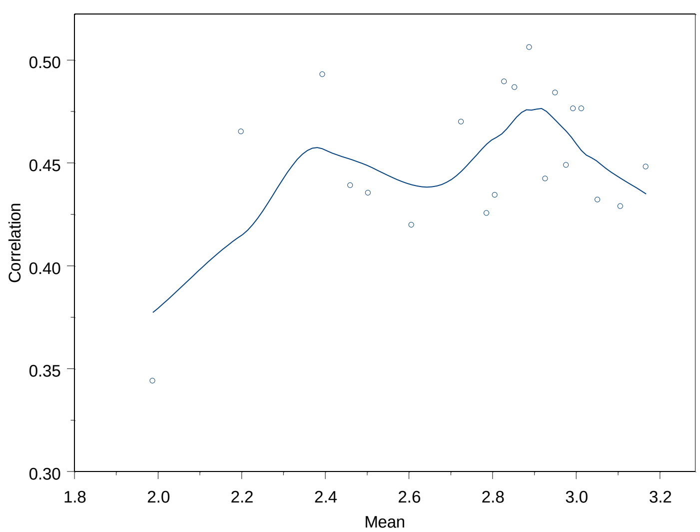

Figure 2. Correlations vs. means for attributes by Likert 4-point scale

Factor Loadings and Their Correlations with Mean Values

Big and approximately equal correlations correspond to the block-diagonal structure of the entire correlation matrix of all variables, where the inter-block correlations are bigger than the outer-block correlations (by absolute value). All pair correlations of the items in this example are positive and varying in the range from 0.35 to 0.55. If some big correlations would be negative, it is always possible to change the variables to the opposite direction by flipping the scale, so all correlations become positive. Let us first briefly describe some results from positive matrix theory.

Due to the Perron-Frobenius theory for a positive matrix's eigenvectors (Salton, 1988; Lipovetsky, 2009; Horn & Johnson, 2013), the first eigenvector of a positive correlation matrix has positive elements and the larger ones identify the variables more related among themselves than with others identified by smaller loadings. Absence of zero elements shows that the matrix is irreducible, or by permutation of variables the matrix cannot be presented in a block-diagonal form when each diagonal block consists of highly correlated subsets of the variables, and the non-diagonal blocks contain zeros. However, higher loadings define a subset of closely-related variables, and the rest of variables with lower loadings could belong to another subset of closely-related variables. In practice, a matrix of correlation can only be approximately presented in a block-diagonal form with higher correlations within the diagonal blocks and with lower correlations in the non-diagonal blocks. If the first eigenvector identifies by the highest elements one of the diagonal blocks, the second eigenvector should correspond to another diagonal block and, due to the Perron-Frobenius theory, it can have positive elements of the variables belonging to this block. The next main eigenvectors can relate to other diagonal blocks and, again, each of them can be flipped by sign.

Let us consider how the results of factor analysis can correspond to different ranges of the mean values shown in Figure 2. FA loadings for 3, 4, and 5-factor solutions obtained in a maximum likelihood approach with additional varimax rotation are presented in Table 2.

The main loadings in Table 2 are colored by dark green. Table 2 also shows the item means, and correlations between them and FA loadings. We see that in each FA solution there is a strong negative correlation of the loadings with mean values of attributes. It can be interpreted as follows.

Table 2. Attribute means, FA loadings, and correlations

		FA-3			FA-4				FA-5				
item	mean	F1	F2	F3	F1	F2	F3	F4	F1	F2	F3	F4	F5
1	2.77	0.12	0.59	0.38	0.60	0.07	0.33	0.22	0.52	0.09	0.33	0.19	0.51
2	2.86	0.13	0.56	0.51	0.55	0.12	0.49	0.18	0.46	0.14	0.51	0.14	0.40
3	2.54	0.20	0.64	0.41	0.64	0.18	0.38	0.19	0.58	0.18	0.43	0.18	0.16
4	2.85	0.32	0.41	0.62	0.41	0.27	0.56	0.31	0.34	0.28	0.59	0.29	0.16
5	2.81	0.24	0.50	0.64	0.49	0.22	0.62	0.24	0.44	0.22	0.65	0.24	0.10
6	2.92	0.73	0.16	0.10	0.14	0.79	0.15	0.04	0.16	0.78	0.15	0.04	-0.03
7	2.81	0.66	0.20	0.01	0.19	0.64	0.00	0.15	0.20	0.64	0.01	0.13	0.05
8	2.91	0.24	0.42	0.68	0.40	0.23	0.69	0.20	0.36	0.22	0.71	0.20	0.06
9	2.39	0.32	0.67	0.34	0.66	0.29	0.32	0.19	0.67	0.26	0.37	0.22	-0.06
10	2.34	0.30	0.67	0.29	0.67	0.26	0.25	0.22	0.65	0.24	0.30	0.23	0.04
11	3.07	0.56	0.21	0.42	0.22	0.44	0.28	0.50	0.18	0.46	0.28	0.48	0.15
12	3.00	0.52	0.27	0.45	0.29	0.35	0.24	0.64	0.27	0.35	0.26	0.64	0.09
13	3.07	0.70	0.11	0.23	0.10	0.65	0.19	0.27	0.10	0.66	0.19	0.25	0.02
14	2.97	0.72	0.14	0.16	0.12	0.77	0.20	0.08	0.12	0.77	0.20	0.06	0.02
15	2.60	0.44	0.52	0.34	0.54	0.34	0.24	0.39	0.53	0.33	0.27	0.40	0.03
16	2.68	0.57	0.18	0.09	0.18	0.56	0.08	0.16	0.20	0.54	0.09	0.17	-0.08
17	2.78	0.46	0.34	0.29	0.35	0.39	0.20	0.34	0.33	0.38	0.22	0.33	0.07
18	1.91	0.09	0.62	0.06	0.63	0.06	0.03	0.10	0.62	0.05	0.07	0.11	0.12
19	2.94	0.24	0.38	0.69	0.35	0.24	0.72	0.18	0.31	0.23	0.74	0.18	0.05
20	2.94	0.26	0.40	0.63	0.39	0.24	0.61	0.23	0.31	0.26	0.63	0.21	0.20
21	2.06	0.35	0.52	0.01	0.52	0.34	0.02	0.06	0.53	0.32	0.06	0.08	-0.02
22	2.84	0.34	0.45	0.60	0.44	0.29	0.56	0.29	0.41	0.28	0.59	0.30	0.00
23	2.47	0.06	0.68	0.27	0.69	0.03	0.23	0.15	0.63	0.04	0.24	0.11	0.45
24	3.08	0.53	0.20	0.48	0.22	0.36	0.28	0.66	0.20	0.36	0.29	0.65	0.06
25	2.99	0.48	0.28	0.52	0.29	0.37	0.40	0.47	0.27	0.37	0.42	0.47	0.00
26	2.77	0.46	0.42	0.46	0.43	0.37	0.36	0.41	0.43	0.35	0.40	0.43	-0.07
27	2.27	0.24	0.69	0.23	0.68	0.25	0.26	0.04	0.69	0.22	0.32	0.06	-0.06
28	3.19	0.40	0.14	0.63	0.14	0.33	0.55	0.39	0.11	0.33	0.55	0.39	0.00
29	2.96	0.67	0.21	0.33	0.20	0.65	0.32	0.23	0.18	0.66	0.32	0.22	0.07
30	2.99	0.63	0.20	0.40	0.20	0.54	0.32	0.39	0.17	0.56	0.32	0.37	0.12
31	2.84	0.61	0.25	0.29	0.26	0.55	0.22	0.31	0.23	0.56	0.23	0.29	0.13
32	3.14	0.59	0.12	0.28	0.12	0.54	0.24	0.27	0.10	0.56	0.23	0.25	0.12
33	3.03	0.55	0.20	0.46	0.22	0.44	0.33	0.48	0.17	0.46	0.33	0.45	0.19
34	2.45	0.21	0.65	0.38	0.64	0.22	0.40	0.09	0.61	0.20	0.45	0.10	0.06
35	2.46	0.25	0.63	0.25	0.63	0.23	0.23	0.17	0.59	0.22	0.27	0.16	0.12
cor		0.56	-0.79	0.49	-0.79	0.48	0.38	0.56	-0.85	0.52	0.31	0.51	0.05

As is well-known, the vectors of loadings in FA, PCA, and SVD, as eigenvectors of eigenproblems for covariance, correlation, or non-centered second-moment matrices, are defined up to an arbitrary normalizing constant – particularly, up to sign change of all their elements that flips the vectors to opposite direction. It is so for maximum likelihood and other methods of estimation, with orthogonal, oblique, and rotated solutions as well. Negative correlations of some vectors of loading with mean values of attributes can be observed practically in any FA or PCA solution, but it does not eliminate such factors from analysis and interpretation on the basis of this correlation sign only. However, for Likert scales it could indicate that negative correlation of a factor's loadings with the vector of the variables' means occurs because this factor is constituted by the variables with the values mostly on the "lower", or "non-relevant" levels. For instance, such a factor can consist of the attributes getting mostly the lower 1 and 2 levels in the scale of "does not apply to me" meaning in (1).

To check it, let us reshape Table 2 by sorting FA loadings due to the descending order of the items mean values – the results are presented in Table 3. Indeed, it is easy to see by Table 3 that in any FA solution the factors negatively correlated with mean values have the main loadings on the attributes with minimum mean values, in the range below about the mean point 2.5 in the scale (1). But those values correspond to meaningless attributes in this study because they are related to the "non-applied to respondent" levels.

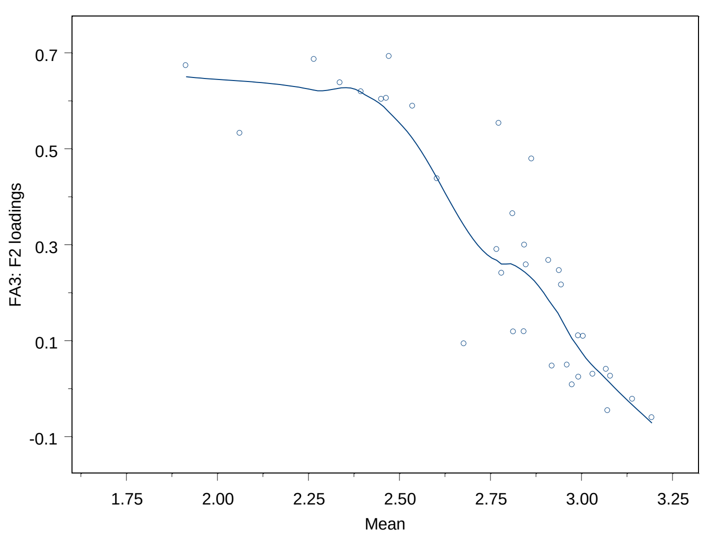

Figure 3. FA-3 solution for 35 attributes with the second factor loadings vs. means

Table 3. Factor loadings sorted by mean values

		FA-3			FA-4				FA-5				
item	**mean**	**F1**	**F2**	**F3**	**F1**	**F2**	**F3**	**F4**	**F1**	**F2**	**F3**	**F4**	**F5**
28	3.19	0.40	0.14	0.63	0.14	0.33	0.55	0.39	0.11	0.33	0.55	0.39	0.00
32	3.14	0.59	0.12	0.28	0.12	0.54	0.24	0.27	0.10	0.56	0.23	0.25	0.12
24	3.08	0.53	0.20	0.48	0.22	0.36	0.28	0.66	0.20	0.36	0.29	0.65	0.06
11	3.07	0.56	0.21	0.42	0.22	0.44	0.28	0.50	0.18	0.46	0.28	0.48	0.15
13	3.07	0.70	0.11	0.23	0.10	0.65	0.19	0.27	0.10	0.66	0.19	0.25	0.02
33	3.03	0.55	0.20	0.46	0.22	0.44	0.33	0.48	0.17	0.46	0.33	0.45	0.19
12	3.00	0.52	0.27	0.45	0.29	0.35	0.24	0.64	0.27	0.35	0.26	0.64	0.09
25	2.99	0.48	0.28	0.52	0.29	0.37	0.40	0.47	0.27	0.37	0.42	0.47	0.00
30	2.99	0.63	0.20	0.40	0.20	0.54	0.32	0.39	0.17	0.56	0.32	0.37	0.12
14	2.97	0.72	0.14	0.16	0.12	0.77	0.20	0.08	0.12	0.77	0.20	0.06	0.02
29	2.96	0.67	0.21	0.33	0.20	0.65	0.32	0.23	0.18	0.66	0.32	0.22	0.07
19	2.94	0.24	0.38	0.69	0.35	0.24	0.72	0.18	0.31	0.23	0.74	0.18	0.05
20	2.94	0.26	0.40	0.63	0.39	0.24	0.61	0.23	0.31	0.26	0.63	0.21	0.20
6	2.92	0.73	0.16	0.10	0.14	0.79	0.15	0.04	0.16	0.78	0.15	0.04	-0.03
8	2.91	0.24	0.42	0.68	0.40	0.23	0.69	0.20	0.36	0.22	0.71	0.20	0.06
2	2.86	0.13	0.56	0.51	0.55	0.12	0.49	0.18	0.46	0.14	0.51	0.14	0.40
4	2.85	0.32	0.41	0.62	0.41	0.27	0.56	0.31	0.34	0.28	0.59	0.29	0.16
22	2.84	0.34	0.45	0.60	0.44	0.29	0.56	0.29	0.41	0.28	0.59	0.30	0.00
31	2.84	0.61	0.25	0.29	0.26	0.55	0.22	0.31	0.23	0.56	0.23	0.29	0.13
5	2.81	0.24	0.50	0.64	0.49	0.22	0.62	0.24	0.44	0.22	0.65	0.24	0.10
7	2.81	0.66	0.20	0.01	0.19	0.64	0.00	0.15	0.20	0.64	0.01	0.13	0.05
17	2.78	0.46	0.34	0.29	0.35	0.39	0.20	0.34	0.33	0.38	0.22	0.33	0.07
1	2.77	0.12	0.59	0.38	0.60	0.07	0.33	0.22	0.52	0.09	0.33	0.19	0.51
26	2.77	0.46	0.42	0.46	0.43	0.37	0.36	0.41	0.43	0.35	0.40	0.43	-0.07
16	2.68	0.57	0.18	0.09	0.18	0.56	0.08	0.16	0.20	0.54	0.09	0.17	-0.08
15	2.60	0.44	0.52	0.34	0.54	0.34	0.24	0.39	0.53	0.33	0.27	0.40	0.03
3	2.54	0.20	0.64	0.41	0.64	0.18	0.38	0.19	0.58	0.18	0.43	0.18	0.16
23	2.47	0.06	0.68	0.27	0.69	0.03	0.23	0.15	0.63	0.04	0.24	0.11	0.45
35	2.46	0.25	0.63	0.25	0.63	0.23	0.23	0.17	0.59	0.22	0.27	0.16	0.12
34	2.45	0.21	0.65	0.38	0.64	0.22	0.40	0.09	0.61	0.20	0.45	0.10	0.06
9	2.39	0.32	0.67	0.34	0.66	0.29	0.32	0.19	0.67	0.26	0.37	0.22	-0.06
10	2.34	0.30	0.67	0.29	0.67	0.26	0.25	0.22	0.65	0.24	0.30	0.23	0.04
27	2.27	0.24	0.69	0.23	0.68	0.25	0.26	0.04	0.69	0.22	0.32	0.06	-0.06
21	2.06	0.35	0.52	0.01	0.52	0.34	0.02	0.06	0.53	0.32	0.06	0.08	-0.02
18	1.91	0.09	0.62	0.06	0.63	0.06	0.03	0.10	0.62	0.05	0.07	0.11	0.12
cor		0.56	-0.79	0.49	-0.79	0.48	0.38	0.56	-0.85	0.52	0.31	0.51	0.05

So we can see by negative correlations of FA loadings and means that it is possible to identify the variables gravitating to the levels of "does not really apply to me" and "does not apply to me at all". Such attributes do not supply useful information elicited from the respondents. Thus, the factors negatively correlated with means can be skipped from the analysis and interpretation. For illustration, the loadings of the second factor in the solution with three factors (FA-3 solution, the factor F2 in Table 3) are shown in Figure 3, which clearly describes a negative pattern of correlation.

Cleaning data from inadequate variables always helps to a meaningful statistical analysis, so FA can be re-run without the redundant variables of mostly the irrelevant levels on the limited scale. It is also interesting to note that the PCA loadings even without rotation produce similar to FA correlations with means. For instance, correlations of three first PCA vectors with the vector of means are 0.72, -0.62, and 0.31, so very close to three factor solution's correlations given at the last row in Table 3.

Table 4. Correlations of means and FA loadings for several factor solutions with 45 attributes measure by 7-point Likert scale

	F1	F2	F3	F4	F5	F6
FA-3	0.89	0.08	-0.89			
FA-4	0.87	0.03	-0.87	0.19		
FA-5	0.82	0.14	-0.94	0.21	0.17	
FA-6	0.83	0.15	-0.95	0.21	0.14	0.07

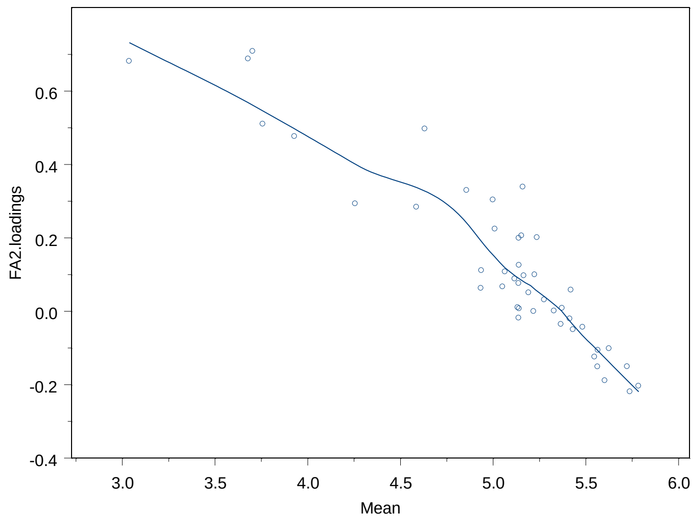

Figure 4. FA-3 solution for 45 attributes with the third factor loadings versus means

Table 5. FA-3 loadings sorted by means for 45 attributes measured by 7-point Likert scale

item	mean	F1	F2	F3	item	mean	F1	F2	F3
21	5.79	0.62	0.19	-0.02	13	5.14	0.48	0.31	0.30
22	5.74	0.75	0.15	-0.01	15	5.14	0.45	0.55	0.28
20	5.72	0.70	0.11	0.04	28	5.14	0.28	0.70	0.20
41	5.63	0.57	0.15	0.07	31	5.14	0.41	0.66	0.20
18	5.60	0.65	0.41	0.04	37	5.14	0.52	0.16	0.35
26	5.57	0.70	0.37	0.13	29	5.13	0.32	0.68	0.21
43	5.56	0.69	0.42	0.09	12	5.12	0.46	0.34	0.26
10	5.55	0.63	0.40	0.10	40	5.07	0.51	0.17	0.26
14	5.48	0.65	0.36	0.18	17	5.05	0.24	0.70	0.25
32	5.43	0.59	0.48	0.18	36	5.01	0.53	0.17	0.38
5	5.42	0.73	0.24	0.28	38	5.00	0.57	0.17	0.47
3	5.41	0.60	0.37	0.19	25	4.94	0.41	0.47	0.29
42	5.37	0.59	0.19	0.19	35	4.93	0.34	0.66	0.26
45	5.37	0.64	0.28	0.17	44	4.86	0.45	0.36	0.50
7	5.33	0.54	0.45	0.21	2	4.63	0.37	0.20	0.62
24	5.28	0.56	0.46	0.25	39	4.59	0.27	0.43	0.42
16	5.24	0.56	0.15	0.36	30	4.26	0.01	0.60	0.40
33	5.23	0.57	0.16	0.27	23	3.93	0.18	0.32	0.57
19	5.22	0.43	0.64	0.22	8	3.76	0.08	0.41	0.60
27	5.19	0.59	0.23	0.23	9	3.70	0.11	0.18	0.76
34	5.17	0.52	0.47	0.31	6	3.68	0.11	0.22	0.74
1	5.16	0.56	0.20	0.51	11	3.04	0.03	0.10	0.70
4	5.15	0.63	0.23	0.40	cor		0.89	0.08	-0.89

In another data set from the same marketing research project, forty five attributes had been measured by a 7-point Likert scale, from 7 meaning "extremely important" to 1 meaning "not at all important." A general structure of the relations between mean values and FA loadings is very similar to that described above for the smaller set of attributes. Table 4 presents the correlations between mean values and factors loadings from three factor solution (FA-3 in the first row) to six factor solution (FA-6 in the last row).

It is useful to note that PCA loading correlated with mean values also yield negative values. PCA constructed by correlation matrix gives three first correlations 0.93, 0.66, and -0.21, and PCA by covariance matrix produces correlations 0.97, -0.29, and -0.11. By Table 4 we see that adding more factors does not change the correlations of the first three factors (F1, F2, and F3 in the first columns) with the mean values of attributes. So, for illustration on the FA loading sorted by means of the attributes, it is sufficient to use the FA-3 solution which is presented in Table 5. This solution demonstrates that the negative correlation of the loadings with means is observed for the third factor mostly defined by the attributes with mean values below the mid-point of the scale. So there is no need to consider and interpret this 3rd factor defined mostly by the "non-important" attributes. The last factor's loadings for 45 attributes solution from Table 5 is presented in Figure 4 with decreasing loadings profiled by the mean values.

Another interesting example of factor analysis performed on eighty adjectives measured by a 5-point Likert scale for characterizing the beauty of a mathematical proof can be found in Inglis and Aberdein (2014), with the second factor excluded from interpretation because of correspondence to lower levels of description accuracy.

Summary

The work considers the possibility to identify factors which can be skipped from interpretation and further application. The analysis is based on correlations of factor loadings with means of variables constituting the factors. Although the factor and principal component loadings are defined up to their sign, the correlations of factor loadings with variables' means permit the identification of factors consisting mostly of variables measured in Likert scales related to non-relevant values. The variables' means can influence the variances and correlations, which in turn define the factor loadings. In some factors the loading structure defined by subsets of highly-related variables can correspond to the "non-important" levels by Likert scale. Factor loadings after rotation to a simpler structure contain mostly the positive elements, so their negative correlations with the attribute means is a convenient indicator of the redundant factors which can be skipped from further analysis. Thus, depending on the content of a scale levels, there are studies with all main factors making sense, so they can be interpreted and used. Negative correlation of the loadings with mean values of variables in such a case simply shows that lower-level observations define this factor. But on the other hand, there could be studies where factors negatively correlated with mean values can be excluded from consideration because they rather correspond to the absence of analyzing features.

References

Bartholomew, D. J., & Knott, M. (1999). *Latent variable models and factor analysis* (2nd ed.). London, UK: Arnold.

Dillon, W. R., & Goldstein, M. (1984). *Multivariate analysis: Methods and applications*. New York, NY: Wiley.

Elden, L. (2007). *Matrix methods in data mining and pattern recognition*. Philadelphia, PA: SIAM. doi: 10.1137/1.9780898718867

Härdle, W., & Hlávka, Z. (2007). *Multivariate statistics: Exercises and solutions*. New York, NY: Springer. doi: 10.1007/978-3-642-36005-3

Härdle, W. K., & Simar, L. (2012). *Applied multivariate statistical analysis*. New York, NY: Springer. doi: 10.1007/978-3-642-17229-8

Harman, H. H. (1976). *Modern factor analysis*. Chicago, IL: University of Chicago Press.

Horn, R. A., & Johnson, C. R. (2013). *Matrix analysis* (2nd ed,). New York, NY: Cambridge University Press.

Inglis, M., & Aberdein, A. (2014). Beauty is not simplicity: An analysis of mathematicians' proof appraisals. *Philosophia Mathematica, 23*(1), 87-109. doi: 10.1093/philmat/nku014

Izenman, A. J. (2008). *Modern multivariate statistical techniques*. New York, NY: Springer. doi: 10.1007/978-0-387-78189-1

Lawley, D. N., & Maxwell, A. E. (1971). *Factor analysis as a statistical method*. New York, NY: American Elsevier.

Lipovetsky, S. (2009). PCA and SVD with nonnegative loadings. *Pattern Recognition*, *42*(1), 68-76. doi: 10.1016/j.patcog.2008.06.025

Lipovetsky, S. (2012). Dual PLS analysis. *International Journal of Information Technology & Decision Making, 11*(05), 879-891. doi: 10.1142/s0219622012500241

Lipovetsky, S. (2015). MANOVA, LDA, and FA criteria in clusters parameter estimation. *Cogent Mathematics*, *2*. doi: 10.1080/23311835.2015.1071013

Lipovetsky, S. & Conklin, M. (2005). Singular value decomposition in additive, multiplicative, and logistic forms. *Pattern Recognition, 38*(7), 1099-1110. doi: 10.1016/j.patcog.2005.01.010

Motoda, H., & Liu, H. (Eds.). (2008). *Computational methods of feature selection*. Boca Raton, FL: Chapman & Hall/CRC. doi: 10.1201/9781584888796

Salton, G. (1988). *Automatic text processing: The transformation, analysis, and retrieval of information by computer*. Reading, MA: Addison-Wesley.

Skrondal, A., & Rabe-Hesketh, S. (2004). *Generalized latent variable modeling*. Boca Raton, FL: Chapman & Hall/CRC.

Timm, N. H. (1975). *Multivariate analysis with applications in education and psychology*. Monterey, CA: Brooks/Cole.