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Specifying Asymmetric STAR models with Linear and Nonlinear GARCH Innovations: Monte Carlo Approach

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Economic and finance time series are typically asymmetric and are expected to be modeled using asymmetrical nonlinear time series models. Smooth Transition Autoregressive (STAR) models: Logistic (LSTAR) and Exponential (ESTAR) are known to be asymmetric and symmetric respectively. Under non-normal and heteroscedastic innovations, the residuals of these models are estimated using Generalized Autoregressive Conditionally Heteroscedastic (GARCH) models with variants which include linear and nonlinear forms. The small sample properties of STAR-GARCH variants are yet to be established but these properties are investigated using Monte Carlo (MC) simulation. An MC investigation was conducted to investigate the performance of selections of STAR-GARCH models by classical nonlinear selection approaches. The ARCH(1) and GARCH(1,1) models were the linear GARCH specifications while the Logistic Smooth Transition-ARCH (LST-ARCH(1,1)), Logistic Smooth Transition-GARCH (LST-GARCH(1,1)) and Asymmetric Nonlinear Smooth Transition-GARCH (ANST-GARCH(1,1)) models were the nonlinear GARCH specifications. The nonlinearity parameter in the variance equations and Autoregressive (AR) parameters were varied along with different sample sizes. With the assumption of normality, the results showed that the selection of LSTAR models were actually affected by the structure of the innovations and this improved as sample size increased. Misspecification tests showed that these models cannot be misrepresented in the real sense.

Keywords: Asymmetry, Monte Carlo simulations, nonlinear GARCH, Smooth transition autoregression, specification

Introduction

Smooth Transition Autoregressive (STAR) and Generalized Conditionally Heteroscedastic (GARCH) models are gaining their popularities in economics and

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finance. STAR models of Granger and Teräsvirta (1993) classify market into two phases of contraction and expansion, whereas GARCH model of Bollerslev (1986) is often used to study the behavior of asset returns or innovations of the ‘parent’ model. In that case, such a ‘parent’ model is the mean equation and the (GARCH) model is the variance equation. The innovations of the STAR model are expected follow normal distribution (homoscedasticity) but in case this is not true, the innovations are said to possess heteroscedasticity, which can be of various forms (Pavlidis, Paya and Peel, 2010). The mean and variance equations are then compounded as STAR-GARCH model.

Maximum Likelihood Estimation (MLE) of STAR-GARCH model was examined in Chan and McAleer (2002). The structural and statistical properties of the model were also established in the paper, even though the asymptotic normality and finite sample properties are still examined using Monte Carlo simulation approach. Chan and McAleer (2002) also considered the effects of misspecifying the transition functions (logistic or exponential) in the STAR model and the results obtained showed that greater bias will be induced in the GARCH estimates for the STAR-GARCH model whenever STAR mode is misspecified. Their results further showed that Logistic STAR model can easily be substituted for Exponential STAR model.

In the study of financial returns, negative returns tend to be followed by periods of higher volatility than positive returns of the same magnitude, that is negative and positive shocks exert different values for the leverage of a firm which on the long run realize different volatilities (Black, 1976). This property has therefore led to the development of GARCH variants that are robust to asymmetry. These variants are nonlinear in their structures due to the fact that the conditional variance is no longer specified as a linear function of lagged squared error and lagged variance. These common asymmetric variants are the Exponential GARCH (EGARCH) (Nelson, 1991), Asymmetric Power ARCH (APARCH) (Ding, *et al.*, 1993) and Glosten Jaganathan and Runkle (GJR-GARCH) (Glosten, *et al.*, 1993) models, but in this work we investigate GARCH variants which display regime switching dynamics.

This study is motivated by the work of Chan and McAleer (2002). We applied the linear GARCH and Smooth Transition specification of ARCH/GARCH models in a Monte Carlo simulation approach. Nonlinearities were first introduced in the ARCH functional form in Engle and Bollerslev (1986). They proposed in their model the dynamics of conditional variance, σ_t^2 as it changes with the squared residuals and the transition between different conditional variance determined by normal cumulative distribution function. A

few years later, Higgins and Bera (1992) developed a Nonlinear ARCH (NARCH) model which accommodated different functional forms to predict the conditional variance. Apart from the classical ARCH and GARCH models of Engle (1982) and Bollerslev (1986), Smooth Transition ARCH (STAR), Smooth Transition GARCH (ST-GARCH) and Asymmetric Nonlinear Smooth Transition GARCH (ANST-GARCH) models of Hagerud (1996; 1997), González-Rivera (1998) and Anderson *et al.* (1999) respectively are also considered. The ST-ARCH model was applied on the Nordic and Stockholm stock returns and found the model better than the linear GARCH model. González-Rivera (1998) used MC simulation experiment to study the model and applied the models on stock returns and exchange-rate data.

The STAR-GARCH and STAR-STGARCH Models

This article presents compounded regime switching and volatility models, with the regime switching model as the mean equation and volatility models as the variance equation. For a time series $y_t, t=1, \dots, N$ with $y_t \sim N(\mu, \sigma^2)$ in the structural model,

$$\hat{y}_t = f(\cdot) + \varepsilon_t \quad (1)$$

where $f(\cdot)$ is the function of y_t and ε_t is the innovation process, expected to be independently and identically distributed with mean 0 and variance 1 that is homoscedasticity case. In the case where this assumption of normal distribution fails, the innovations are estimated with volatility models.

The Mean Equation: STAR model

The Smooth Transition Autoregressive (STAR) model is introduced in Granger and Teräsvirta (1993) and the specification, estimation and evaluation of the model are itemized following standard procedures in Teräsvirta (1994). Since then, the model has been applied to study nonlinearity in business cycle (Teräsvirta and Anderson, 1992); Skalin and Teräsvirta 1996; 1998) and real exchange rates (Baum *et al.*, 1998; Liew *et al.*, 2002). The connection between business cycle-regimes and nonlinearity in the UK labour market is studied in Acemoglu and Scotts (1994). Öcal (2000) applied STAR model on the nonlinearities in growth rates of some selected UK macroeconomic time series

and suggest either two-regime or three-regime model for UK economy. Mourelle, Cuestas and Gil-Alana (2011), Shittu and Yaya (2011) and Yaya (2013) considered STAR model for Nigerian inflation series.

Apart from real life time series data that have been considered for the STAR model, Escribano and Jordá (2001) and Yaya and Shittu (2011) investigated the selection of STAR model by varying some of the parameters and conditions in the models and obtained results that serve as guide for nonlinear time series modelers; then, there is need to study, and if possible develop the structural and small sample properties of the STAR model.

The STAR model of order p is given as,

$$y_t = \phi_{10} + \sum_{i=1}^p \phi_{1i} y_{t-i} + \left(\phi_{20} + \sum_{i=1}^p \phi_{2i} y_{t-i} \right) F(y_{t-d}; \gamma, c) + \varepsilon_t \quad (2)$$

where ϕ_{10}, ϕ_{20} are the constants and ϕ_{1i}, ϕ_{2i} ($i = 1, \dots, p$) are the autoregressive parameters of order p . The transition function, $F(y_{t-d}; \gamma, c)$ causes the nonlinear dynamics in the model, and this are of logistic and exponential forms as given as,

$$F(y_{t-d}; \gamma, c) = \frac{1}{1 + \exp[-\gamma(y_{t-d} - c)]} \quad (3)$$

and

$$F(y_{t-d}; \gamma, c) = 1 - \exp[-\gamma(y_{t-d} - c)^2] \quad (4)$$

respectively, with $\gamma > 0$ in both cases. The logistic type is known to be asymmetric whereas the exponential type is symmetric. Economic and finance series often exhibit forms of asymmetries, and therefore Logistic STAR (LSTAR) model is often applied to model nonlinear dynamics in the series. In the transition functions, the transition variable is y_{t-d} with d assuming values 1, 2, ..., p . the value of d is varied in order to improve nonlinearity in the system when it is not known prior to model estimation. The slope, γ and intercept, c are parts of the nonlinearity parameters in the transition function. As γ assumes values from 1 to say 100, the nonlinearity becomes sharper, and the dynamics shift from lower linear region to upper linear region at faster rate, after being in the nonlinear state

for some period. At $\gamma = 1$, depending on the variance of y_t and size of c , discrimination between the nonlinear and linear series may not be significant. (Yaya and Shittu, 2011). The transition functions in (3) and (4) are bounded between 0 and 1, and this makes the STAR modelling of interesting application. When the transition function is at zero state, the entire system in (2) becomes linear, and at unity state, it is also linear. Most of the time, the transition function is such that $0 < F(y_{t-d}; \gamma, c) < 1$, which is a nonlinear state.

Specification between the asymmetric and symmetric transition function is often carried out using the approach outlined in Teräsvirta (1994). Though there is a newer specification approach proposed in Escribano and Jordá (2001), the approach of Teräsvirta (1994) is not dominated by that of Escribano and Jordá (2001). Further readings on the specification of STAR models are referred to the two articles as well as Luukkonen, Saikkonen and Teräsvirta (1988).

The Variance Equation: GARCH and ST-GARCH models

Apart from the issue of nonlinearity of the time series y_t , the innovations of the estimated model (mean equation) is often heteroscedastic for economic and finance series to be specific. Engle (1982) proposed the Autoregressive Conditionally Heteroscedastic (ARCH) model of order q for UK inflation.

$$\sigma_t^2 = w + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 \quad (5)$$

where σ_t^2 is the conditional variance, w is the constant and α_i ($i = 1, \dots, q$) are the parameters in the ARCH model. The ε_{t-i} are the residuals from the mean equation which are assumed to be heteroscedastic.

Bollerslev (1986) proposed the generalized version of Engle's model which is named the Generalized Autoregressive Conditionally Heteroscedastic (GARCH) model of order (p, q) given as,

$$\sigma_t^2 = w + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^r \beta_j \sigma_{t-j}^2 \quad (6)$$

where β_j ($j = 1, \dots, r$) are the parameters in the GARCH term. In the ARCH(q) and GARCH(q, r) models in (5) and (6), $w > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$ and the

existence of covariance-stationarity is $\sum_{i=1}^q \alpha_i < 1$ for ARCH(q) and $\sum_{i=1}^q \alpha_i + \sum_{j=1}^r \beta_j < 1$ for GARCH(q, r) model.

Hagerud (1996; 1997) and González-Rivera (1998) considered introducing regime switching functional forms in the ARCH/GARCH systems. Their propositions are further developed in Lundbergh and Teräsvirta (1999). Hagerud (1996) proposed Smooth Transition-ARCH (q) (STARARCH) model,

$$\sigma_t^2 = w + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 [1 - F(\varepsilon_{t-i})] + \sum_{i=1}^q \delta_i \varepsilon_{t-i}^2 F(\varepsilon_{t-i}) \quad (7)$$

where w and α_i are as defined in ARCH model. The additional parameter, $\delta_i (i=1, \dots, q)$ defines the model in two-regimes. The transition function, with the transition variable ε_{t-i} is of logistic and exponential as well. These are given as,

$$F(\varepsilon_{t-i}) = \frac{1}{1 + \exp(-\theta \varepsilon_{t-i})} \quad (8)$$

and

$$F(\varepsilon_{t-i}) = 1 - \exp(-\theta \varepsilon_{t-i}^2) \quad (9)$$

for the two forms respectively with $\theta > 0$ in both cases. The two transition functions in (8) and (9) will generate different data dynamics for the conditional variance. The logistic form in (8) will produce a return process where the dynamics of the conditional variance differ depending on the signs of the innovations (Hagerud, 1997). As $\varepsilon_{t-j} \rightarrow -\infty$, the logistic function equals to $-1/2$ and as $\varepsilon_{t-j} \rightarrow +\infty$, the function equals to $1/2$. The exponential function in (9) is symmetric with respect to the sign of the error term, hence it generates data for which the dynamics of the conditional variance depends only on the magnitude of the innovations. As $|\varepsilon_{t-j}| \rightarrow \infty$, the impact of ε_{t-1}^2 on σ_t^2 changes smoothly from α_i to δ_i in both logistic ST-ARCH(q) and ST-GARCH(q, r) when the function equals 1, and as $\varepsilon_{t-j} = 0$, the logistic function equals 0. Also, as the parameter θ

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becomes larger, both the logistic ST-ARCH and ST-GARCH functions approach step functions which equal 0 for negative ε_{t-1} and 1 for positive ε_{t-1} , therefore, for logistic function, $-1/2 \leq F(\cdot) \leq 1/2$ and for exponential function, $0 \leq F(\cdot) \leq 1$.

For positive conditional variance in the logistic ST-ARCH model, the condition $\alpha_i \geq \frac{1}{2}|\delta_i|$ and for stationarity of the innovations ε_t , $\sum_{i=1}^q \left[\alpha_i - \frac{1}{2}|\delta_i| + \max(\delta_i, 0) \right] < 1$. For the positive conditional mean in the exponential ST-ARCH, $\alpha_i + \delta_i \geq 0$ and for stationarity of the innovations ε_t , $\sum_{i=1}^q [\alpha_i + \max(\delta_i, 0)] < 1$ (Hagerud, 1997).

The generalized form of the model called Smooth Transition-GARCH (q, r) (ST-GARCH) is proposed in Hagerud (1997) and González-Rivera (1998) as,

$$\sigma_t^2 = w + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 [1 - F(\varepsilon_{t-i})] + \sum_{i=1}^q \delta_i \varepsilon_{t-i}^2 F(\varepsilon_{t-i}) + \sum_{j=1}^r \beta_j \sigma_{t-j}^2 \quad (10)$$

with the transition functions in (8) and (9) for the logistic and exponential cases respectively. The ST-GARCH model only included the GARCH term, σ_{t-j}^2 .

For positive conditional variance in the logistic ST-GARCH model, all the covariance stationarity condition of GARCH(p, q) model hold here in ST-GARCH, and apart from these, $\alpha_i \geq \frac{1}{2}|\delta_i|$ for the logistic case and for the stationarity of the innovations ε_t , $\sum_{i=1}^q \left[\alpha_i - \frac{1}{2}|\delta_i| + \max(\delta_i, 0) \right] + \sum_{i=1}^r \beta_j < 1$. For the positive conditional mean in the exponential ST-GARCH, $\alpha_i + \delta_i \geq 0$ and for stationarity of the innovations ε_t , $\sum_{i=1}^q [\alpha_i + \max(\delta_i, 0)] < 1$ (Hagerud, 1997).

A similar ST-GARCH (p, q) is proposed in Anderson, et al. (1999) and applied recently in Nam, et al. (2002). This is given as,

$$\begin{aligned} \sigma_t^2 = & \left(w_{10} + \sum_{i=1}^q \alpha_{1i} \varepsilon_{t-i}^2 + \sum_{j=1}^r \beta_{1j} \sigma_{t-j}^2 \right) [1 - F(\varepsilon_{t-i})] \\ & + \left(w_{20} + \sum_{i=1}^q \alpha_{2i} \varepsilon_{t-i}^2 + \sum_{j=1}^r \beta_{2j} \sigma_{t-j}^2 \right) F(\varepsilon_{t-i}) \end{aligned} \quad (11)$$

This is a variant of GARCH model in regime switching functional form. The parameters and the conditions of existence of GARCH as defined for the GARCH specification in (6) holds for the ST-GARCH model. The model in (11) is defined only for the asymmetric function (8), and therefore, the ST-GARCH model is otherwise known as Asymmetric nonlinear Smooth Transition-GARCH (ANST-GARCH) model (Nam, *et al.*, 2002). Franses and van Dijk (2003) showed that there is similarity between the ST-GARCH (q, r) model of Hagerud (1997), even in the conditions of existence of conditional volatility and stationarity. Our selection of asymmetric variants of GARCH in this paper is based on similarity with STAR model and their abilities to realize smooth changing dynamics.

Structure of the Data Generating Process and Nonlinearity Tests

The structure of the Data Generating Process (DGP) model used in the simulation is first explained analytically using a particular STAR model used in Granger and Teräsvirta (1993), Teräsvirta, Lin and Granger (1993), Teräsvirta (1994), Escribano, Franses and van Dijk (1998), Escribano and Jordá (2001) and Lopes and Salazar (2006). The DGP is examined by varying the nonlinearity parameters in the models. From the results, nonlinearity tests are described. The DGP is,

$$y_t = 1.8y_{t-1} - 1.06y_{t-2} + (\phi_{20} - 0.9y_{t-1} + 0.795y_{t-2})F(y_{t-d}; \gamma, c) + \varepsilon_t \quad (12)$$

where $\varepsilon_t \sim N(0, 0.1\sigma_t)$ and $F(y_{t-d}; \gamma, c)$ is either the logistic or exponential transition function as given in (3) and (4) respectively. The ϕ_{20} is the intercept in the nonlinear part of the Autoregressive model.

Following Teräsvirta (1994), the LSTAR transition function in (3) is approximated by the third order Taylor's series expansion as,

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$$F(y_{t-d}; \gamma, c) \approx -\left(\frac{1}{4}c\gamma + \frac{1}{48}\gamma^3c^3\right) + \left(\frac{1}{4}\gamma + \frac{c^2\gamma^3}{16}\right)y_{t-d} - \frac{1}{16}c\gamma^3y_{t-d}^2 + \frac{1}{48}\gamma^3y_{t-d}^3 + R. \quad (13)$$

where R is the remainder series.

Substituting $\gamma = 100$ and $c = 0.2$, then (13) becomes

$$F(y_{t-d}; \gamma, c) \approx -20838.33 + 2525y_{t-d} - 12500y_{t-d}^2 + 20833.33y_{t-d}^3 \quad (14)$$

this is then substituted in (12) to obtain

$$\begin{aligned} y_t = & (-20838.3\phi_{20} + 18751.8y_{t-1} - 16567.5y_{t-2}) \\ & + (2525\phi_{20} - 2272.5y_{t-1} + 2007.4y_{t-2})y_{t-d} \\ & + (-12500\phi_{20} - 11250y_{t-1} + 9937.5y_{t-2})y_{t-d}^2 \\ & + (20833.3\phi_{20} - 18750y_{t-1} + 16562.5y_{t-2})y_{t-d}^3 + R^* \end{aligned} \quad (15)$$

The expansion in (15) can be generalized as,

$$y_t = \phi' y_t^{(2)} + \beta_1' y_t^{(2)} y_{t-d} + \beta_2' y_t^{(2)} y_{t-d}^2 + \beta_3' y_t^{(2)} y_{t-d}^3 + \mathcal{G}_t \quad (16)$$

where \mathcal{G}_t is some noise process and $y_t^{(2)}$ is the AR process of order 2 and β_1 , β_2 and β_3 are the parameters of the nonlinear regression model. From (16), the LSTAR model is specified if the parameter β_2 is not significant at α -level or if it is the least significant among the three betas. Otherwise, ESTAR is specified.

Similar nonlinearity test to the above is developed in Escribano and Jordá (2001). Here there is suggestion to apply second order Taylor's series expansion of the ESTAR function in (4) to approximate the transition function. The approximation is given as,

$$\begin{aligned} F(y_{t-d}; \gamma, c) \approx & \left(\gamma c^2 - \frac{1}{2}\gamma^2 c^4\right) + (2c^3\gamma^2 - 2c\gamma)y_{t-d} \\ & + \left(\gamma - 3c^2\gamma^2\right)y_{t-d}^2 + 2c\gamma^2y_{t-d}^3 - \frac{1}{2}\gamma^2y_{t-d}^4 + R. \end{aligned} \quad (17)$$

Substituting $\gamma = 100$ and $c = 0.2$, then (17) becomes

$$F(y_{t-d}; \gamma, c) \approx -12 + 120y_{t-d} - 1100y_{t-d}^2 + 4000y_{t-d}^3 + 5000y_{t-d}^4 \quad (18)$$

this is then substituted in (12) to obtain

$$\begin{aligned} y_t = & (-12\phi_{20} + 12.4y_{t-1} - 11.6y_{t-2}) \\ & + (120\phi_{20} - 108y_{t-1} + 95.4y_{t-2})y_{t-d} \\ & + (-1100\phi_{20} - 990y_{t-1} + 874.5y_{t-2})y_{t-d}^2 \\ & + (4000\phi_{20} - 3600y_{t-1} + 3180y_{t-2})y_{t-d}^3 \\ & + (5000\phi_{20} - 4500y_{t-1} + 3975y_{t-2})y_{t-d}^4 + R^* \end{aligned} \quad (19)$$

The expansion in (19) can be generalized as,

$$y_t = \phi' y_t^{(2)} + \beta_1' y_t^{(2)} y_{t-d} + \beta_2' y_t^{(2)} y_{t-d}^2 + \beta_3' y_t^{(2)} y_{t-d}^3 + \beta_4' y_t^{(2)} y_{t-d}^4 + \mathcal{G}_t \quad (20)$$

Here, the parameters are of order 2 and LSTAR is specified once the odd parameters β_1 and β_3 are most significant. Otherwise, ESTAR is specified if the parameters β_2 and β_4 are most significant.

Monte Carlo Simulation Experiment

The Data Generating Process (DGP) defined as,

$$y_t = 1.8y_{t-1} - 1.06y_{t-2} + (\phi_{20} - 0.9y_{t-1} + 0.795y_{t-2})F(y_{t-d}; \gamma, c) + \varepsilon_t \quad (21)$$

with the nonlinear transition functions $F(y_{t-d}; \gamma, c) = \frac{1}{1 + \exp[-100(y_{t-1} - 0.2)]}$

and $F(y_{t-d}; \gamma, c) = 1 - \exp[-100(y_{t-1} - 0.2)^2]$ for Logistic STAR and Exponential STAR respectively. In the DGP, the autoregressive parameter ϕ_{20} is varied as $\phi_{20} = \{0, 0.2, 0.5\}$ and the innovations are assumed to have non-constant variance, that is $\varepsilon_t \sim N(0, 0.1\sigma_t)$. The values of ϕ_{20} are chosen such that the DGP will

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realized stationary series. At $F(s_i; \gamma, c) = 0$, the resulting linear model has complex roots that are less than unity in absolute term, hence the process becomes nonstationary and there is possibility of explosion. At $F(s_i; \gamma, c) = 1$, the behavior of the process is influenced by the values of ϕ_{20} . For example, when $\phi_{20} = \{0, 0.2\}$, the resulting characteristic equation has complex roots that are less than unity in absolute terms, hence the system reverts back to stationary region. At $\phi_{20} = 0.5$, the roots of the characteristic equations are real and the system realize nonstationary series.

The variance equations used in the simulations are the ARCH (1), GARCH (1,1), STARCH (1), ST-GARCH (1,1) and ANST-GARCH (1,1) are:

$$\sigma_t^2 = 0.02 + 0.3\varepsilon_{t-1}^2 \quad (22)$$

$$\sigma_t^2 = 0.02 + 0.3\varepsilon_{t-1}^2 + 0.6\sigma_{t-1}^2 \quad (23)$$

$$\sigma_t^2 = 0.02 + 0.3\varepsilon_{t-1}^2 [1 - F(\varepsilon_{t-1})] + 0.5\varepsilon_{t-1}^2 \quad (24)$$

$$\sigma_t^2 = 0.02 + 0.3\varepsilon_{t-1}^2 [1 - F(\varepsilon_{t-1})] + 0.5F(\varepsilon_{t-1}) + 0.6\sigma_{t-1}^2 \quad (25)$$

$$\begin{aligned} \sigma_t^2 = & 0.05 + 0.5\varepsilon_{t-1}^2 + 0.3\sigma_{t-1}^2 [1 - F(\varepsilon_{t-1})] \\ & + (0.02 + 0.3\varepsilon_{t-1}^2 + 0.6\sigma_{t-1}^2)F(\varepsilon_{t-1}) \end{aligned} \quad (26)$$

The logistic and exponential functions for the innovations ε_t are $F(\varepsilon_{t-1}) = \frac{1}{1 + \exp(-\theta\varepsilon_{t-1})}$ and $F(\varepsilon_{t-1}) = 1 - \exp(-\theta\varepsilon_{t-1}^2)$ respectively. In each case, the nonlinear parameter in the variance equations in (14) to (18) is varied as $\theta = \{1, 5, 10\}$. The experiment is carried out over 1,000 replications with sample sizes $N = \{50, 100, 200, 500, 1000\}$. Initialization problem is catered for by discarding the first 100 observations in each replication.

The experiment was carried out in two scenarios:

1. When the LSTAR DGP was used to realize LSTAR series with the specifications of the variance equations (Tables 1-3). When ESTAR model was misspecified for LSTAR model (Tables 4-6).
2. The relative frequencies of selecting an asymmetric STAR model with a particular variance equation are computed on every 1,000 replications at 5% nominal significant level.

The relative frequencies of selecting an asymmetric STAR model with a particular variance equation are computed on every 1,000 replications at 5% nominal significant level.

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When the LSTAR DGP is used to realise LSTAR series.

Table 1. Selection Frequencies of models at different ϕ_{20} with fixed $\theta = 1$

$\phi_{20} = 0, \theta = 1$											
<i>N</i>	<i>LSTAR-ARCH</i>		<i>LSTAR-GARCH</i>		<i>LSTAR-LSTAR</i>		<i>LSTAR-LSTGARCH</i>		<i>LSTAR-ANLSTGARCH</i>		
	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	
50	0.522	0.934	0.692	0.879	0.551	0.918	0.683	0.875	0.697	0.875	
100	0.627	0.988	0.772	0.961	0.643	0.980	0.777	0.963	0.803	0.956	
200	0.707	0.999	0.903	0.996	0.741	0.998	0.902	0.996	0.917	0.994	
500	0.869	1.000	0.990	1.000	0.872	1.000	0.992	1.000	0.994	1.000	
1000	0.949	1.000	1.000	1.000	0.966	1.000	1.000	1.000	1.000	1.000	

$\phi_{20} = 0.2, \theta = 1$											
<i>N</i>	<i>LSTAR-ARCH</i>		<i>LSTAR-GARCH</i>		<i>LSTAR-LSTAR</i>		<i>LSTAR-LSTGARCH</i>		<i>LSTAR-ANLSTGARCH</i>		
	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	
50	-	-	-	-	-	-	-	-	-	-	
100	-	-	-	-	-	-	-	-	-	-	
200	-	-	-	-	-	-	-	-	-	-	
500	-	-	0.598	0.500	-	-	0.623	0.500	0.590	0.494	
1000	-	-	0.672	0.554	-	-	0.630	0.507	0.670	0.557	

$\phi_{20} = 0.5, \theta = 1$											
<i>N</i>	<i>LSTAR-ARCH</i>		<i>LSTAR-GARCH</i>		<i>LSTAR-LSTAR</i>		<i>LSTAR-LSTGARCH</i>		<i>LSTAR-ANLSTGARCH</i>		
	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	
50	-	-	-	-	-	-	-	-	-	-	
100	-	-	-	-	-	-	-	-	-	-	
200	-	-	-	-	-	-	-	-	-	-	
500	-	-	-	-	-	-	-	-	-	-	
1000	-	-	-	-	-	-	-	-	-	-	

Note: Table 1 presents the results of the selections of LSTAR models with forms of heteroscedastic innovation processes. EJP performed better than TP in selecting the LSTAR models at zero intercept, $\phi_{20} = 0$ of the DGP. Both LSTAR-GARCH and LSTAR-ANLSTGARCH models were detected at frequencies higher than that of other model variants. As ϕ_{20} increased beyond 0, there was failure in model specifications as a result of matrix inversion problems encountered by the simulator. The results were worse when computed at the nonstationary region ($\phi_{20} = \{0.2, 0.5\}$) of the DGP.

Table 2. Selection Frequencies of models at different ϕ_{20} with fixed $\theta = 5$

$\phi_{20} = 0, \theta = 5$											
N	LSTAR-ARCH		LSTAR-GARCH		LSTAR-LSTARCH		LSTAR-LSTGARCH		LSTAR-ANLSTGARCH		
	TP	EJP	TP	EJP	TP	EJP	TP	EJP	TP	EJP	
50	0.522	0.934	0.692	0.879	0.549	0.921	0.682	0.868	0.697	0.878	
100	0.627	0.988	0.772	0.961	0.647	0.979	0.789	0.963	0.811	0.954	
200	0.707	0.999	0.903	0.996	0.751	0.997	0.907	0.997	0.918	0.995	
500	0.869	1.000	0.990	1.000	0.878	1.000	0.993	1.000	0.992	1.000	
1000	0.949	1.000	1.000	1.000	0.962	1.000	1.000	1.000	1.000	1.000	

$\phi_{20} = 0.2, \theta = 5$											
N	LSTAR-ARCH		LSTAR-GARCH		LSTAR-LSTARCH		LSTAR-LSTGARCH		LSTAR-ANLSTGARCH		
	TP	EJP	TP	EJP	TP	EJP	TP	EJP	TP	EJP	
50	-	-	-	-	-	-	-	-	-	-	
100	-	-	-	-	-	-	-	-	-	-	
200	-	-	-	-	-	-	-	-	-	-	
500	-	-	0.598	0.500	-	-	0.620	0.496	0.605	0.488	
1000	-	-	0.672	0.554	-	-	0.631	0.483	0.687	0.556	

$\phi_{20} = 0.5, \theta = 5$											
N	LSTAR-ARCH		LSTAR-GARCH		LSTAR-LSTARCH		LSTAR-LSTGARCH		LSTAR-ANLSTGARCH		
	TP	EJP	TP	EJP	TP	EJP	TP	EJP	TP	EJP	
50	-	-	-	-	-	-	-	-	-	-	
100	-	-	-	-	-	-	-	-	-	-	
200	-	-	-	-	-	-	-	-	-	-	
500	-	-	-	-	-	-	-	-	-	-	
1000	-	-	-	-	-	-	-	-	-	-	

Note: Increasing θ as 2 in Table 2, similar results to that of Table 2 were obtained. This implies that little increase in the nonlinearity of the residuals may not have significant effect on the specification of STAR models with Smooth Transition GARCH. The results were also worse at $\theta = 5$

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Table 3. Selection Frequencies of models at different ϕ_{20} with fixed 10

$\phi_{20} = 0, \theta = 10$

<i>N</i>	<i>LSTAR-ARCH</i>		<i>LSTAR-GARCH</i>		<i>LSTAR-LSTARCH</i>		<i>LSTAR-LSTGARCH</i>		<i>LSTAR-ANLSTGARCH</i>	
	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>
50	0.522	0.934	0.692	0.879	0.551	0.917	0.694	0.871	0.712	0.881
100	0.627	0.988	0.772	0.961	0.641	0.780	0.790	0.962	0.823	0.958
200	0.707	0.999	0.903	0.996	0.753	0.998	0.912	0.996	0.923	0.999
500	0.869	1.000	0.990	1.000	0.888	1.000	0.991	1.000	0.988	1.000
1000	0.949	1.000	1.000	1.000	0.961	1.000	0.999	1.000	1.000	1.000

$\phi_{20} = 0.2, \theta = 10$

<i>N</i>	<i>LSTAR-ARCH</i>		<i>LSTAR-GARCH</i>		<i>LSTAR-LSTARCH</i>		<i>LSTAR-LSTGARCH</i>		<i>LSTAR-ANLSTGARCH</i>	
	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>
50	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-
200	-	-	-	-	-	-	-	-	-	-
500	-	-	0.598	0.500	-	-	0.616	0.496	0.605	0.477
1000	-	-	0.672	0.554	-	-	0.644	0.450	0.690	0.530

$\phi_{20} = 0.5, \theta = 10$

<i>N</i>	<i>LSTAR-ARCH</i>		<i>LSTAR-GARCH</i>		<i>LSTAR-LSTARCH</i>		<i>LSTAR-LSTGARCH</i>		<i>LSTAR-ANLSTGARCH</i>	
	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>
50	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-
200	-	-	-	-	-	-	-	-	-	-
500	-	-	-	-	-	-	-	-	-	-
1000	-	-	-	-	-	-	-	-	-	-

Note: Table 3 gives similar results to Tables 1 and 2. As we see in the previous results that correct model specifications were carried out at intercept $\phi_{20} = 0$ and at this point, the process realized stationary time series.

When ESTAR model is misspecified for LSTAR model

Table 4. Selection Frequencies of models at different ϕ_{20} with fixed $\theta = 1$

$\phi_{20} = 0, \theta = 1$										
<i>N</i>	LSTAR-ARCH		LSTAR-GARCH		LSTAR-LSTARCH		LSTAR-LSTGARCH		LSTAR-ANLSTGARCH	
	TP	EJP	TP	EJP	TP	EJP	TP	EJP	TP	EJP
50	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-
200	-	-	-	-	-	-	-	-	-	-
500	-	-	0.750	0.569	-	-	0.752	0.570	0.767	0.607
1000	-	-	0.784	0.638	-	-	0.785	0.630	0.790	0.631

$\phi_{20} = 0.2, \theta = 1$										
<i>N</i>	LSTAR-ARCH		LSTAR-GARCH		LSTAR-LSTARCH		LSTAR-LSTGARCH		LSTAR-ANLSTGARCH	
	TP	EJP	TP	EJP	TP	EJP	TP	EJP	TP	EJP
50	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-
200	-	-	-	-	-	-	-	-	-	-
500	-	-	0.614	0.470	-	-	0.614	0.470	0.656	0.522
1000	-	-	0.673	0.554	-	-	0.667	0.543	0.718	0.563

$\phi_{20} = 0.5, \theta = 1$										
<i>N</i>	LSTAR-ARCH		LSTAR-GARCH		LSTAR-LSTARCH		LSTAR-LSTGARCH		LSTAR-ANLSTGARCH	
	TP	EJP	TP	EJP	TP	EJP	TP	EJP	TP	EJP
50	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-
200	-	-	-	-	-	-	-	-	-	-
500	-	-	-	-	-	-	-	-	-	-
1000	-	-	-	-	-	-	-	-	-	-

Note: Tables 4-6 give the results of specifying ESTAR for LSTAR in the DGP in (12). At $\phi_{20} = 0$, the simulator could not specify LSTAR and it reported matrix inversion problems. Also, TP performed better than EJP in selecting LSTAR from ESTAR DGP

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Table 5. Selection Frequencies of models at different ϕ_{20} with fixed $\theta = 5$

$\phi_{20} = 0, \theta = 5$

<i>N</i>	<i>LSTAR-ARCH</i>		<i>LSTAR-GARCH</i>		<i>LSTAR-LSTARCH</i>		<i>LSTAR-LSTGARCH</i>		<i>LSTAR-ANLSTGARCH</i>	
	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>
50	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-
200	-	-	-	-	-	-	-	-	-	-
500	-	-	0.750	0.569	-	-	0.755	0.579	0.769	0.604
1000	-	-	0.784	0.638	-	-	0.786	0.641	0.790	0.629

$\phi_{20} = 0.2, \theta = 5$

<i>N</i>	<i>LSTAR-ARCH</i>		<i>LSTAR-GARCH</i>		<i>LSTAR-LSTARCH</i>		<i>LSTAR-LSTGARCH</i>		<i>LSTAR-ANLSTGARCH</i>	
	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>
50	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-
200	-	-	-	-	-	-	-	-	-	-
500	-	-	0.614	0.470	-	-	0.612	0.482	0.659	0.527
1000	-	-	0.673	0.554	-	-	0.657	0.546	0.718	0.563

$\phi_{20} = 0.5, \theta = 5$

<i>N</i>	<i>LSTAR-ARCH</i>		<i>LSTAR-GARCH</i>		<i>LSTAR-LSTARCH</i>		<i>LSTAR-LSTGARCH</i>		<i>LSTAR-ANLSTGARCH</i>	
	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>
50	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-
200	-	-	-	-	-	-	-	-	-	-
500	-	-	-	-	-	-	-	-	-	-
1000	-	-	-	-	-	-	-	-	-	-

Note: The results obtained here are similar to that of Table 4.

Table 6. Selection Frequencies of models at different ϕ_{20} with fixed $\theta = 10$

$\phi_{20} = 0, \theta = 10$

<i>N</i>	<i>LSTAR-ARCH</i>		<i>LSTAR-GARCH</i>		<i>LSTAR-LSTARCH</i>		<i>LSTAR-LSTGARCH</i>		<i>LSTAR-ANLSTGARCH</i>	
	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>
50	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-
200	-	-	-	-	-	-	-	-	-	-
500	-	-	0.750	0.569	-	-	0.760	0.578	0.768	0.609
1000	-	-	0.784	0.638	-	-	0.789	0.644	0.788	0.625

$\phi_{20} = 0.2, \theta = 10$

<i>N</i>	<i>LSTAR-ARCH</i>		<i>LSTAR-GARCH</i>		<i>LSTAR-LSTARCH</i>		<i>LSTAR-LSTGARCH</i>		<i>LSTAR-ANLSTGARCH</i>	
	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>
50	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-
200	-	-	-	-	-	-	-	-	-	-
500	-	-	0.614	0.470	-	-	0.612	0.482	0.656	0.527
1000	-	-	0.673	0.554	-	-	0.655	0.527	0.714	0.552

$\phi_{20} = 0.5, \theta = 10$

<i>N</i>	<i>LSTAR-ARCH</i>		<i>LSTAR-GARCH</i>		<i>LSTAR-LSTARCH</i>		<i>LSTAR-LSTGARCH</i>		<i>LSTAR-ANLSTGARCH</i>	
	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>	<i>TP</i>	<i>EJP</i>
50	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-
200	-	-	-	-	-	-	-	-	-	-
500	-	-	-	-	-	-	-	-	-	-
1000	-	-	-	-	-	-	-	-	-	-

Note: The results obtained here seem to improve insignificantly over that of Table 5.

Conclusion

This study considered the specification of asymmetric Smooth Transition Autoregressive (STAR) models with linear and nonlinear GARCH innovations. The GARCH error specifications are those proposed already in the literature.

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Specifications of the Logistic STAR-GARCH (LSTAR-GARCH) variants were carried out using the usual STAR specification procedures. The empirical results showed strong support for modelling STAR models with different GARCH error specifications. The results further showed that STAR model in STAR-GARCH model cannot be misrepresented in the real sense.

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