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Median Based Modified Ratio Estimators with Known Quartiles of an Auxiliary Variable

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New median based modified ratio estimators for estimating a finite population mean using quartiles and functions of an auxiliary variable are proposed. The bias and mean squared error of the proposed estimators are obtained and the mean squared error of the proposed estimators are compared with the usual simple random sampling without replacement (SRSWOR) sample mean, ratio estimator, a few existing modified ratio estimators, the linear regression estimator and median based ratio estimator for certain natural populations. A numerical study shows that the proposed estimators perform better than existing estimators; in addition, it is shown that the proposed median based modified ratio estimators outperform the ratio and modified ratio estimators as well as the linear regression estimator.

Keywords: Bias, inter-quartile range, linear regression estimator, mean squared error, natural population, simple random sampling

Introduction

Consider a finite population $U = \{U_1, U_2, \ldots, U_N\}$ of N distinct and identifiable units. Let Y be a study variable with value Y_i measured on $U_i, i = 1, 2, 3, \ldots, N$ giving a vector $Y = \{Y_1, Y_2, \ldots, Y_N\}$. The goal is to estimate the population mean, $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$, with some desirable properties on the basis of a random sample of size *n* selected from the population *U*. The simplest estimator of population mean is the sample mean, obtained by using simple random sampling without

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replacement (SRSWOR), when there is no information on the auxiliary variable available. Let X be an auxiliary variable that is positively correlated with the study variable Y: Sometimes the information on auxiliary variable X, positively correlated with Y, may be utilized to obtain a more efficient estimator of the population mean (for further details on ratio estimators see Cochran, 1977 and Murthy, 1967.) When the population parameters of an auxiliary variable X, such as, population mean, coefficient of variation, coefficient of kurtosis, coefficient of skewness and median are known, ratio, product and linear regression estimators (and their modifications) have been proposed in the literature – many of which perform better than the SRSWOR sample mean for estimating the population mean of a study variable.

Subramani (2013a) proposed a median based ratio estimator by using the median of a study variable as auxiliary information, and it has been shown that this median based ratio estimator outperforms the usual SRSWOR sample mean, ratio estimator, modified ratio estimator and linear regression estimator. Based on Subramani's (2013a) median based ratio estimator, some new median based modified ratio estimators with known quartiles of the auxiliary variable are proposed.

The first quartile, also called lower quartile, is denoted by Q_1; the third quartile, also called the upper quartile, is denoted by Q_3. The lower quartile is a point where 25% of the observations are less than Q_1 and 75% are above Q_1. The upper quartile is a point where 75% observations are less than Q_3 and 25% are above Q_3. Quartiles are unaffected by extreme values unlike the population mean, variance, correlation coefficient, etc.

The inter-quartile range used as a measure of spread in a data set. The inter-quartile range of a distribution is the difference between the upper and lower quartiles. The formula for computing the inter-quartile range is

$$Q_r = Q_3 - Q_1 . \tag{1}$$

The semi-quartile range of a distribution is half the difference between the upper and lower quartiles, or half the inter-quartile range. The formula for computing the semi-quartile range is

$$Q_d = \frac{Q_3 - Q_1}{2} \tag{2}$$

Another measure, the quartile average, noted by Q_a, was suggested by Subramani and Kumarapandiyan (2012a) and is defined as

$$Q_a = \frac{Q_3 + Q_1}{2} \tag{3}$$

The notations and formulae used in this article are:

N:	Population size
n:	Sample size
Y:	Study variable
M:	Median of the study variable
X:	Auxiliary variable
Q_i :	i^{th} Quartile of auxiliary variable, i=1,3
ρ :	Correlation coefficient between X and Y
$\bar{X}, \bar{Y}$:	Population means
$\bar{x}, \bar{y}$:	Sample means
$\bar{M}$:	Average of sample medians of Y
m:	Sample median of Y
β :	Regression coefficient of Y on X
$B(.)$:	Bias of the estimator
$V(.)$:	Variance of the estimator
$MSE(.)$:	Mean squared error of the estimator
$PRE(e,p) = \frac{MSE(e)}{MSE(p)} * 100$:	Percent relative efficiency of the proposed estimator p with respect to the existing estimator e

The formulae for computing various measures including the variance and the covariance of the SRSWOR sample mean and sample median are:

$$V(\bar{y}) = \frac{1}{N_{C_n}} \sum_{i=1}^{N_{C_n}} \left(\bar{y}_i - \bar{Y}\right)^2 = \frac{1-f}{n} S_y^2, \; V(\bar{x}) = \frac{1}{N_{C_n}} \sum_{i=1}^{N_{C_n}} \left(\bar{x}_i - \bar{X}\right)^2 = \frac{1-f}{n} S_x^2,$$

$$MSE(m) = V(m) = \frac{1}{N_{C_n}} \sum_{i=1}^{N_{C_n}} \left(m_i - M\right)^2,$$

$$Cov(\bar{y}, \bar{x}) = \frac{1}{N_{C_n}} \sum_{i=1}^{N_{C_n}} \left(\bar{x}_i - \bar{X}\right)\left(\bar{y}_i - \bar{Y}\right) = \frac{1-f}{n} \frac{1}{N-1} \sum_{i=1}^{N} \left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right),$$

$$Cov(\bar{y}, m) = \frac{1}{N_{C_n}} \sum_{i=1}^{N_{C_n}} \left(m_i - M\right)\left(\bar{y}_i - \bar{Y}\right),$$

$$C_{xx}^{'} = \frac{V(\bar{x})}{\bar{X}^2}, \; C_{mm}^{'} = \frac{V(m)}{M^2}, \; C_{ym}^{'} = \frac{Cov(\bar{y}, m)}{M\bar{Y}}, \; C_{yx}^{'} = \frac{Cov(\bar{y}, \bar{x})}{\bar{X}\bar{Y}},$$

where $f = \frac{n}{N}$; $S_y^2 = \frac{1}{N-1} \sum_{i=1}^{N} \left(Y_i - \bar{Y}\right)^2$, $S_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} \left(X_i - \bar{X}\right)^2$.

In the case of SRSWOR, the sample mean, $\bar{y}$, is used to estimate the population mean, $\bar{Y}$. That is, the estimator of $\bar{Y} = \hat{\bar{Y}}_r = \bar{y}$ with variance

$$V\left(\hat{\bar{Y}}_r\right) = \frac{1-f}{n} S_y^2. \tag{4}$$

The classical ratio estimator for estimating the population mean $\bar{Y}$ of a study variable Y is defined as $\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}} \bar{X} = \hat{R}\bar{X}$. The bias and mean squared error of $\hat{\bar{Y}}_R$ are:

$$B\left(\hat{\bar{Y}}_R\right) = \bar{Y}\left\{C_{xx}^{'} - C_{yx}^{'}\right\} \tag{5}$$

and

$$MSE\left(\hat{\bar{Y}}_R\right) = V(\bar{y}) + R^2 V(\bar{x}) - 2R\,Cov(\bar{y}, \bar{x}). \tag{6}$$

The other commonly used estimator using the auxiliary variable X is the linear regression estimator. The linear regression estimator and its variance with known regression coefficient are:

$$\hat{\bar{Y}}_{lr} = \bar{y} + \beta\left(\bar{X} - \bar{x}\right) \tag{7}$$

$$V\left(\hat{\bar{Y}}_{lr}\right) = V\left(\bar{y}\right)\left(1-\rho^2\right) \text{ where } \rho = \frac{Cov\left(\bar{y},\bar{x}\right)}{\sqrt{V\left(\bar{x}\right) * V\left(\bar{y}\right)}} \tag{8}$$

Subramani & Kumarapandiyan (2012a) suggested some modified ratio estimators using known quartiles and their functions of an auxiliary variable, these are:

$$\hat{\bar{Y}}_{RM1} = \bar{y}\left(\frac{\bar{X}+Q_1}{\bar{x}+Q_1}\right) \tag{9}$$

$$\hat{\bar{Y}}_{RM2} = \bar{y}\left(\frac{\bar{X}+Q_3}{\bar{x}+Q_3}\right) \tag{10}$$

$$\hat{\bar{Y}}_{RM3} = \bar{y}\left(\frac{\bar{X}+Q_r}{\bar{x}+Q_r}\right) \tag{11}$$

$$\hat{\bar{Y}}_{RM4} = \bar{y}\left(\frac{\bar{X}+Q_d}{\bar{x}+Q_d}\right) \tag{12}$$

$$\hat{\bar{Y}}_{RM5} = \bar{y}\left(\frac{\bar{X}+Q_a}{\bar{x}+Q_a}\right) \tag{13}$$

The bias and the mean squared error of the modified ratio estimators in (9) to (13) are:

$$B\left(\hat{\bar{Y}}_{RMi}\right) = \bar{Y}\left\{\theta_i^2 C'_{xx} - \theta_i C'_{yx}\right\} \tag{14}$$

$$MSE\left(\hat{\bar{Y}}_{RMi}\right)=V\left(\bar{y}\right)+R^2\theta_i^2V\left(\bar{x}\right)-2R\theta_iCov\left(\bar{y},\bar{x}\right) \tag{15}$$

where $R=\dfrac{\bar{Y}}{\bar{X}}$ and

$$\theta_1=\frac{\bar{X}}{\bar{X}+Q_1},\theta_2=\frac{\bar{X}}{\bar{X}+Q_3},\ \theta_3=\frac{\bar{X}}{\bar{X}+Q_r},\theta_4=\frac{\bar{X}}{\bar{X}+Q_d},\ \theta_5=\frac{\bar{X}}{\bar{X}+Q_a};i=1,2,3,4,5$$

Recently Subramani (2013a) suggested a median based ratio estimator for estimating $\bar{Y}$ when the median of the study variable Y is known. The estimator with its bias and mean squared error are:

$$\hat{\bar{Y}}_M=\frac{\bar{y}}{m}M \tag{16}$$

$$B\left(\hat{\bar{Y}}_M\right)=\bar{Y}\left\{C'_{mm}-C'_{ym}-\frac{Bias\left(m\right)}{M}\right\} \tag{17}$$

$$MSE\left(\hat{\bar{Y}}_M\right)=V\left(\bar{y}\right)+R'^2V\left(m\right)-2R'Cov\left(\bar{y},m\right)\text{ where }R'=\frac{\bar{Y}}{M}. \tag{18}$$

For further details on modified ratio estimators with known population parameters of an auxiliary variable, such as coefficient of variation, skewness, kurtosis, correlation coefficient, quartiles and their linear combinations, readers are referred to Kadilar and Cingi (2004, 2006a, b, 2009) Koyuncu and Kadilar (2009), Singh and Kakran (1993), Singh and Tailor (2003, 2005), Singh (2003), Sisodia and Dwivedi (1981), Subramani (2013a, b), Subramani and Kumarapandiyan (2012a, b, c, 2013), Tailor and Sharma (2009), Tin (1965), and Yan and Tian (2010).

The median based ratio estimator proposed by Subramani (2013a) is extended and, as a result, some new median based modified ratio estimators $\hat{\bar{Y}}_{SP1}$, $\hat{\bar{Y}}_{SP2}$, $\hat{\bar{Y}}_{SP3}$, $\hat{\bar{Y}}_{SP4}$ and $\hat{\bar{Y}}_{SP5}$ with known quartiles and their functions of auxiliary variables are proposed.

Proposed Median Based Modified Ratio Estimators

The proposed median based modified ratio estimators for estimating a population mean $\bar{Y}$ based on Subramani's (2013a) ratio estimator are:

$$\hat{\bar{Y}}_{SP1} = \bar{y}\left(\frac{M+Q_1}{m+Q_1}\right) \tag{19}$$

$$\hat{\bar{Y}}_{SP2} = \bar{y}\left(\frac{M+Q_3}{m+Q_3}\right) \tag{20}$$

$$\hat{\bar{Y}}_{SP3} = \bar{y}\left(\frac{M+Q_r}{m+Q_r}\right) \tag{21}$$

$$\hat{\bar{Y}}_{SP4} = \bar{y}\left(\frac{M+Q_d}{m+Q_d}\right) \tag{22}$$

and

$$\hat{\bar{Y}}_{SP5} = \bar{y}\left(\frac{M+Q_a}{m+Q_a}\right). \tag{23}$$

To the first degree of approximation, the bias and mean squared error of $\hat{\bar{Y}}_{SPj}$ are derived as:

$$B\left(\hat{\bar{Y}}_{SPj}\right) = \bar{Y}\left\{\theta_j^{'2}C_{mm}^{'} - \theta_j^{'}C_{ym}^{'} - \theta_j^{'}\frac{Bias(m)}{M}\right\},\ j=1,2,3,4,5, \tag{24}$$

$$MSE\left(\hat{\bar{Y}}_{SPj}\right) = V(\bar{y}) + R'^2\theta_j^{'2}V(m) - 2R'\theta_j^{'}Cov(\bar{y},m),\ j=1,2,3,4,5 \tag{25}$$

where

$$R^{'} = \frac{\bar{Y}}{M},\ \theta_1^{'} = \frac{M}{M+Q_1},\ \theta_2^{'} = \frac{M}{M+Q_3},\ \theta_3^{'} = \frac{M}{M+Q_r},\ \theta_4^{'} = \frac{M}{M+Q_d},\ \theta_5^{'} = \frac{M}{M+Q_a}.$$

See Appendix A for detailed derivation of the bias and the mean squared error of $\hat{\bar{Y}}_{SPj}$.

Efficiency Comparisons

Comparison with SRSWOR Sample Mean

The conditions (see Appendix B) for which the proposed estimators $\hat{\bar{Y}}_{SPj}$, $j=1,2,3,4,5$ are more efficient than the SRSWOR sample mean $\hat{\bar{Y}}_r$ were derived from expressions (25) and (4) and are:

$$MSE\left(\hat{\bar{Y}}_{SPj}\right) \le V\left(\hat{\bar{Y}}_r\right) \text{ if } 2C'_{ym} \ge \theta'_j C'_{mm};\ j=1,2,3,4,5. \quad (26)$$

Comparison with Ratio Estimators

The conditions (see Appendix B) for which the proposed estimators $\hat{\bar{Y}}_{SPj}$, $j=1,2,3,4,5$ are more efficient than the usual ratio estimator $\hat{\bar{Y}}_R$ were derived from expressions (25) and (6) and are:

$$MSE\left(\hat{\bar{Y}}_{SPj}\right) \le MSE\left(\hat{\bar{Y}}_R\right) \text{ if } \theta_j^{'2} C'_{mm} - \theta_i^2 C'_{xx} \le 2\left(\theta'_j C'_{ym} - \theta_i C'_{yx}\right); i,\ j=1,2,3,4,5. \quad (27)$$

Comparison with Modified Ratio Estimators

From expressions (25) and (15), the conditions (see Appendix B) for which the proposed estimators $\hat{\bar{Y}}_{SPj}$, $j=1,2,3,4,5$ are more efficient than the existing modified ratio estimator $\hat{\bar{Y}}_{RMi}$, $i=1,2,3,4,5$ were derived and are:

$$MSE\left(\hat{\bar{Y}}_{SPj}\right) \le MSE\left(\hat{\bar{Y}}_{RMi}\right)$$

if

$$MSE\left(\hat{\bar{Y}}_{SPj}\right) \le MSE\left(\hat{\bar{Y}}_{RMi}\right) \text{ if } \theta_j^{'2} C'_{mm} - \theta_i^2 C'_{xx} \le 2\left(\theta'_j C'_{ym} - \theta_i C'_{yx}\right);\ i,\ j=1,2,3,4,5 \quad (28)$$

Comparison with Linear Regression Estimator

From expressions (25) and (8), the conditions (see Appendix B) for which the proposed estimators $\hat{\hat{Y}}_{SPj}$, $j = 1, 2, 3, 4, 5$ are more efficient than the usual linear regression estimator $\hat{\hat{Y}}_{lr}$ were derived and are:

$$MSE\left(\hat{\hat{Y}}_{SPj}\right) \leq V\left(\hat{\hat{Y}}_{lr}\right) \text{ if } 2\theta_j^{'} C_{ym}^{'} - \theta_j^{'2} C_{mm}^{'} \geq \frac{\left[C_{yx}^{'}\right]^2}{C_{xx}^{'}};\ j = 1,\ 2. \tag{29}$$

Comparison with Median Based Ratio Estimator

From expressions (25) and (18), the conditions (see Appendix B) for which the proposed estimators $\hat{\hat{Y}}_{SPj}$, $j = 1, 2, 3, 4, 5$ are more efficient than the existing modified ratio type estimator $\hat{\hat{Y}}_{M}$ were derived and are:

$$MSE\left(\hat{\hat{Y}}_{SPj}\right) \leq MSE\left(\hat{\hat{Y}}_{M}\right) \text{ if } 2C_{ym}^{'} \leq \left(\theta_j^{'} + 1\right) C_{mm}^{'};\ j = 1, 2, 3, 4, 5. \tag{30}$$

Numerical Comparison

The conditions for which the proposed median based modified ratio estimators performed better than the other usual estimators considered in this study have been obtained. In order to show that the proposed estimators perform better than the other estimators, numerical comparisons were made to determine the efficiencies of the proposed estimators. Two populations were used to assess the efficiencies of the proposed median based modified ratio estimators with that of the existing estimators. Populations 1 and 2 are from Singh and Chaudhary (1986, p. 177). The parameter values and constants computed for the populations are given in Table 1, the bias for the proposed and existing estimators computed for the two populations are given in Table 2 and the mean squared errors are given in Table 3.

Table 1. Parameter values and constants for 2 different populations

Parameter	n = 3		n = 5	
	Pop 1	Pop 2	Pop 1	Pop 2
N	34.0000	34.0000	34.0000	34.0000
n	3.0000	3.0000	5.0000	5.0000
N_{C_n}	5,984.0000	5,984.0000	278,256.0000	278,256.0000
$\bar{Y}$	856.4118	856.4118	856.4118	856.4118
$\bar{M}$	747.7223	747.7223	736.9811	736.9811
M	767.5000	767.5000	767.5000	767.5000
$\bar{X}$	208.8824	199.4412	208.8824	199.4412
Q_1	94.2500	99.2500	94.2500	99.2500
Q_3	254.7500	278.0000	254.7500	278.0000
Q_r	160.5000	178.7500	160.5000	178.7500
Q_d	80.2500	89.3750	80.2500	89.3750
Q_a	174.5000	188.6250	174.5000	188.6250
R	4.0999	4.2941	4.0999	4.2941
R'	1.1158	1.1158	1.1158	1.1158
θ_1	0.6891	0.6677	0.6891	0.6677
θ_2	0.4505	0.4177	0.4505	0.4177
θ_3	0.5655	0.5274	0.5655	0.5274
θ_4	0.7224	0.6905	0.7224	0.6905
θ_5	0.5448	0.5139	0.5448	0.5139
θ_1'	0.8906	0.8855	0.8906	0.8855
θ_2'	0.7508	0.7341	0.7508	0.7341
θ_3'	0.8270	0.8111	0.8270	0.8111
θ_4'	0.9053	0.8957	0.9053	0.8957
θ_5'	0.8148	0.8027	0.8148	0.8027
$\mathrm{var}(\bar{y})$	163,356.4086	163,356.4086	91,690.3713	91,690.3713
$\mathrm{var}(\bar{x})$	6,884.4455	6,857.8555	3,864.1726	3,849.2480
$\mathrm{var}(m)$	101,518.7738	101,518.7738	59,396.2836	59,396.2836
$\mathrm{cov}(\bar{y}, m)$	90,236.2939	90,236.2939	48,074.9542	48,074.9542
$\mathrm{cov}(\bar{y}, \bar{x})$	15,061.4011	14,905.0488	8,453.8187	8,366.0597
ρ	0.4491	0.4453	0.4491	0.4453

Table 2. Bias of existing and proposed estimators

Estimators		*n* = 3		*n* = 5	
		Pop 1	Pop 2	Pop 1	Pop 2
Existing	$\hat{\bar{Y}}_R$	63.0241	72.9186	35.3748	40.9285
	$\hat{\bar{Y}}_{RM1}$	14.4774	15.9291	8.1261	8.9409
	$\hat{\bar{Y}}_{RM2}$	-5.0570	-5.4535	-2.8385	-3.0610
	$\hat{\bar{Y}}_{RM3}$	2.4369	1.6513	1.3678	0.9269
	$\hat{\bar{Y}}_{RM4}$	18.4357	18.8016	10.3478	10.5531
	$\hat{\bar{Y}}_{RM5}$	0.8276	0.5910	0.4645	0.3317
	$\hat{\bar{Y}}_M$	52.0924	52.0924	57.7705	57.7705
Proposed	$\hat{\bar{Y}}_{SP1}$	32.0179	31.1618	43.0405	42.3993
	$\hat{\bar{Y}}_{SP2}$	11.4953	9.4306	27.2167	25.5531
	$\hat{\bar{Y}}_{SP3}$	21.9708	19.6375	35.4268	33.6263
	$\hat{\bar{Y}}_{SP4}$	34.5121	32.8700	44.9012	43.6773
	$\hat{\bar{Y}}_{SP5}$	20.1662	18.4422	34.0355	32.6983

Table 3. Variance/mean squared error of existing and proposed estimators

Estimators		*n* = 3		*n* = 5	
		Pop 1	Pop 2	Pop 1	Pop 2
Existing	$\hat{\bar{Y}}_r$	163356.4086	163356.4086	91690.3713	91690.3713
	$\hat{\bar{Y}}_R$	155577.8155	161802.8878	87324.3215	90818.3961
	$\hat{\bar{Y}}_{RM1}$	133203.7861	134261.9210	74765.9957	75359.9173
	$\hat{\bar{Y}}_{RM2}$	131205.2291	131950.5079	73644.2252	74062.5432
	$\hat{\bar{Y}}_{RM3}$	130523.6191	131018.5135	73261.6440	73539.4239
	$\hat{\bar{Y}}_{RM4}$	134530.4901	135259.2456	75510.6618	75919.7060
	$\hat{\bar{Y}}_{RM5}$	130420.4186	130968.9816	73203.7186	73511.6221
	$\hat{\bar{Y}}_{lr}$	130408.9222	130964.1249	73197.2660	73508.8959
	$\hat{\bar{Y}}_M$	88379.0666	88379.0666	58356.9234	58356.9234
Proposed	$\hat{\bar{Y}}_{SP1}$	84266.7092	84147.8927	54798.7634	54675.1252
	$\hat{\bar{Y}}_{SP2}$	83413.6960	83642.1970	52826.6580	52784.4688
	$\hat{\bar{Y}}_{SP3}$	83266.0122	83175.3231	53543.5010	53322.4123
	$\hat{\bar{Y}}_{SP4}$	84643.7479	84390.4264	55174.2962	54924.5239
	$\hat{\bar{Y}}_{SP5}$	83190.4430	83153.4557	53369.8070	53221.3731

The percentage relative efficiencies of the proposed estimators with respect to the existing estimators were also obtained and are shown in Tables 4-5.

Table 4. Percentage Relative Efficiency of $\hat{\hat{Y}}_{SPj}$ for Population 1

Existing Estimators	For sample size *n*=3 Proposed Estimators					For sample size *n*=5 Proposed Estimators				
	$\hat{\hat{Y}}_{SP1}$	$\hat{\hat{Y}}_{SP2}$	$\hat{\hat{Y}}_{SP3}$	$\hat{\hat{Y}}_{SP4}$	$\hat{\hat{Y}}_{SP5}$	$\hat{\hat{Y}}_{SP1}$	$\hat{\hat{Y}}_{SP2}$	$\hat{\hat{Y}}_{SP3}$	$\hat{\hat{Y}}_{SP4}$	$\hat{\hat{Y}}_{SP5}$
$\hat{Y}_r$	193.86	195.84	196.19	192.99	196.36	167.32	173.57	171.24	166.18	171.80
$\hat{Y}_R$	184.63	186.51	186.84	183.80	187.01	159.35	165.30	163.09	158.27	163.62
$\hat{Y}_{RM1}$	158.07	159.69	159.97	157.37	160.12	136.44	141.53	139.64	135.51	140.09
$\hat{Y}_{RM2}$	155.70	157.29	157.57	155.01	157.72	134.39	139.41	137.54	133.48	137.99
$\hat{Y}_{RM3}$	154.89	156.48	156.75	154.20	156.90	133.69	138.68	136.83	132.78	137.27
$\hat{Y}_{RM4}$	159.65	161.28	161.57	158.94	161.71	137.80	142.94	141.03	136.86	141.49
$\hat{Y}_{RM5}$	154.77	156.35	156.63	154.08	156.77	133.59	138.57	136.72	132.68	137.16
$\hat{Y}_{lr}$	154.76	156.34	156.62	154.07	156.76	133.57	138.56	136.71	132.67	137.15
$\hat{Y}_M$	104.88	105.95	106.14	104.41	106.24	106.49	110.47	108.99	105.77	109.34

Table 5: Percentage Relative Efficiency of $\hat{\hat{Y}}_{SPj}$ for Population 2

Existing Estimators	For sample size n=3 Proposed Estimators					For sample size n=5 Proposed Estimators				
	$\hat{\hat{Y}}_{SP1}$	$\hat{\hat{Y}}_{SP2}$	$\hat{\hat{Y}}_{SP3}$	$\hat{\hat{Y}}_{SP4}$	$\hat{\hat{Y}}_{SP5}$	$\hat{\hat{Y}}_{SP1}$	$\hat{\hat{Y}}_{SP2}$	$\hat{\hat{Y}}_{SP3}$	$\hat{\hat{Y}}_{SP4}$	$\hat{\hat{Y}}_{SP5}$
$\hat{Y}_r$	194.13	195.30	196.40	193.57	196.45	167.70	173.71	171.95	166.94	172.28
$\hat{Y}_R$	192.28	193.45	194.53	191.73	194.58	166.11	172.06	170.32	165.35	170.64
$\hat{Y}_{RM1}$	159.55	160.52	161.42	159.10	161.46	137.83	142.77	141.33	137.21	141.60
$\hat{Y}_{RM2}$	156.81	157.76	158.64	156.36	158.68	135.46	140.31	138.90	134.84	139.16
$\hat{Y}_{RM3}$	155.70	156.64	157.52	155.25	157.56	134.50	139.32	137.91	133.89	138.18
$\hat{Y}_{RM4}$	160.74	161.71	162.62	160.28	162.66	138.86	143.83	142.38	138.23	142.65
$\hat{Y}_{RM5}$	155.64	156.58	157.46	155.19	157.50	134.45	139.27	137.86	133.84	138.12
$\hat{Y}_{lr}$	155.64	156.58	157.46	155.19	157.50	134.45	139.26	137.86	133.84	138.12
$\hat{Y}_M$	105.03	105.66	106.26	104.73	106.28	106.73	110.56	109.44	106.25	109.65

Tables 4 and 5 show that the percent relative efficiencies of the proposed estimators, with respect to existing estimators, range in general from 104.41 to 196.45. In particular, the PRE ranges from 166.18 to 196.45 for comparing with the SRSWOR sample mean; ranging from 158.27 to 194.58 for comparing with the ratio estimator; ranging from 132.68 to 162.66 for comparing with the modified ratio estimators; ranging from 132.67 to 157.50 for comparing with the linear regression estimator and ranging from104.41 to 110.56 for comparing with the median based ratio estimator. This demonstrates that the proposed estimators perform better than the existing SRSWOR sample mean, ratio, modified ratio and linear regression estimators for the two populations considered. Further it is observed from the numerical comparisons that the following inequalities hold:

$$MSE\left(\hat{\bar{Y}}_{SPj}\right) \leq MSE\left(\hat{\bar{Y}}_{M}\right) \leq V\left(\hat{\bar{Y}}_{lr}\right) \leq MSE\left(\hat{\bar{Y}}_{RMi}\right) \leq MSE\left(\hat{\bar{Y}}_{R}\right) \leq V\left(\hat{\bar{Y}}_{r}\right)$$

Conclusion

This article proposed some new median based modified ratio estimators using known quartiles and their functions of the auxiliary variable. The conditions for which the proposed estimators are more efficient than the existing estimators were derived. Further the percentage relative efficiencies of the proposed estimators with respect to existing estimators were shown to range in general from 104.41 to 196.45 for certain natural populations available in the literature. It is usually believed that the linear regression estimator is the optimum estimator for estimating the population mean whenever an auxiliary variable exists that is positively correlated with that of a study variable. However, it was shown that the proposed median based modified ratio estimators outperform not only the ratio and modified ratio estimators but also the linear regression estimator. Based on results of this study, the proposed median based modified ratio estimators are recommended for estimating finite population means.

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Appendix A

The derivation of the bias and the mean squared error of $\bar{Y}_{SP1}$ are given below:
Consider

$$\hat{\bar{Y}}_{SP1} = \bar{y}\left(\frac{M+Q_1}{m+Q_1}\right) \tag{A1}$$

Let $e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$ and $e_1 = \frac{m-M}{M}$

$$\Rightarrow E(e_0)=0;\ E(e_1)=\frac{\bar{M}-M}{M}=\frac{Bias(m)}{M} \tag{A2}$$

$$\Rightarrow E\left(e_0^2\right)=\frac{V(\bar{y})}{\bar{Y}^2};\ E\left(e_1^2\right)=\frac{V(m)}{M^2};\ E\left(e_0e_1\right)=\frac{Cov(\bar{y},m)}{\bar{Y}M} \tag{A3}$$

The estimator $\bar{Y}_{SP1}$ can be written in terms of e_0 and e_1 as

$$\hat{\bar{Y}}_{SP1} = \bar{Y}\left(1+e_0\right)\left(\frac{M+Q_1}{M\left(1+e_1\right)+Q_1}\right)$$

$$\Rightarrow \hat{\bar{Y}}_{SP1} = \bar{Y}\left(1+e_0\right)\left(\frac{M+Q_1}{\left(M+Q_1\right)+Me_1}\right)$$

$$\Rightarrow \hat{\bar{Y}}_{SP1} = \bar{Y}\left(1+e_0\right)\left(\frac{1}{1+\left(\frac{M}{M+Q_1}\right)e_1}\right)$$

$$\Rightarrow \hat{\bar{Y}}_{SP1} = \bar{Y}\left(1+e_0\right)\left(\frac{1}{1+\theta_1'e_1}\right);\ \text{where } \theta_1' = \frac{M}{M+Q_1}$$

$$\Rightarrow \hat{\bar{Y}}_{SP1} = \bar{Y}\left(1+e_0\right)\left(1+\theta_1' e_1\right)^{-1}$$

Neglecting the terms of higher order, we have

$$\hat{\bar{Y}}_{SP1} = \bar{Y}\left(1+e_0\right)\left(1-\theta_1' e_1 + \theta_1'^2 e_1^2\right)$$

$$\Rightarrow \hat{\bar{Y}}_{SP1} = \bar{Y} + \bar{Y}e_0 - \bar{Y}\theta_1' e_1 - \bar{Y}\theta_1' e_0 e_1 + \bar{Y}\theta_1'^2 e_1^2$$

$$\Rightarrow \hat{\bar{Y}}_{SP1} - \bar{Y} = \bar{Y}e_0 - \bar{Y}\theta_1' e_1 - \bar{Y}\theta_1' e_0 e_1 + \bar{Y}\theta_1'^2 e_1^2 \quad \text{(A4)}$$

Taking expectations on both sides of (A4) we have,

$$E\left(\hat{\bar{Y}}_{SP1} - \bar{Y}\right) = \bar{Y}E\left(e_0\right) - \bar{Y}\theta_1' E\left(e_1\right) - \bar{Y}\theta_1' E\left(e_0 e_1\right) + \bar{Y}\theta_1'^2 E\left(e_1^2\right)$$

$$\Rightarrow E\left(\hat{\bar{Y}}_{SP1} - \bar{Y}\right) = \bar{Y}\left\{\theta_1'^2 C'_{mm} - \theta_1' C'_{ym} - \theta_1' \frac{Bias(m)}{M}\right\} \text{ from (A2) and (A3)}$$

$$\Rightarrow Bias\left(\hat{\bar{Y}}_{SP1}\right) = \bar{Y}\left\{\theta_1'^2 C'_{mm} - \theta_1' C'_{ym} - \theta_1' \frac{Bias(m)}{M}\right\} \quad \text{(A5)}$$

The derivation of mean squared error of $\bar{Y}_{SP1}$ is given below:

$$MSE\left(\hat{\bar{Y}}_{SP1}\right) = E\left(\hat{\bar{Y}}_{SP1} - \bar{Y}\right)^2 = E\left(\bar{Y}e_0 - \bar{Y}\theta_1' e_1\right)^2$$

$$\Rightarrow MSE\left(\hat{\bar{Y}}_{SP1}\right) = \bar{Y}^2\left\{E\left(e_0^2\right) + \theta_1'^2 E\left(e_1^2\right) - 2\theta_1' E\left(e_0 e_1\right)\right\}$$

$$\Rightarrow MSE\left(\hat{\bar{Y}}_{SP1}\right) = \bar{Y}^2\left\{\frac{V\left(\bar{y}\right)}{\bar{Y}^2} + \theta_1'^2 \frac{V\left(m\right)}{M^2} - 2\theta_1' \frac{Cov\left(\bar{y}, m\right)}{\bar{Y}M}\right\}$$

$$\Rightarrow MSE\left(\hat{\bar{Y}}_{SP1}\right) = V\left(\bar{y}\right) + \frac{\bar{Y}^2}{M^2}\theta_1'^2 V\left(m\right) - 2\frac{\bar{Y}}{M}\theta_1' Cov\left(\bar{y}, m\right)$$

$$\Rightarrow MSE\left(\hat{\bar{Y}}_{SP1}\right) = V\left(\bar{y}\right) + R'^2 \theta_1'^2 V\left(m\right) - 2R'\theta_1' Cov\left(\bar{y}, m\right); R' = \frac{\bar{Y}}{M} \quad \text{(A6)}$$

In the Similar manner, the bias and mean squared error of $\hat{\bar{Y}}_{SP2}, \hat{\bar{Y}}_{SP3}, \hat{\bar{Y}}_{SP4}$ and $\hat{\bar{Y}}_{SP5}$ can be obtained.

Appendix B

The conditions for which the proposed estimators perform better than the existing estimators are derived here and are given below:

Comparison with that of SRSWOR sample mean

Consider $MSE\left(\hat{\bar{Y}}_{SPj}\right) \leq V\left(\hat{\bar{Y}}_r\right)$

$$\Rightarrow V\left(\bar{y}\right) + R'^2 \theta_j'^2 V\left(m\right) - 2R'\theta_j' Cov\left(\bar{y}, m\right) \leq V\left(\bar{y}\right)$$

$$\Rightarrow R'^2 \theta_j'^2 V\left(m\right) - 2R'\theta_j' Cov\left(\bar{y}, m\right) \leq 0$$

$$\Rightarrow R'^2 \theta_j'^2 V\left(m\right) \leq 2R'\theta_j' Cov\left(\bar{y}, m\right)$$

$$\Rightarrow Cov\left(\bar{y}, m\right) \geq \frac{R'\theta_j' V\left(m\right)}{2}, \; j = 1, 2, 3, 4, 5$$

$$\Rightarrow Cov\left(\bar{y}, m\right) \geq \frac{\bar{Y} M \theta_j' C_{mm}'}{2}$$

$$\Rightarrow 2C_{ym}' \geq \theta_j' C_{mm}', \; j = 1, 2, 3, 4, 5$$

Comparison with that of Ratio Estimator

Consider $MSE\left(\hat{\bar{Y}}_{SPj}\right) \le MSE\left(\hat{\bar{Y}}_{R}\right)$

$$\Rightarrow V(\bar{y}) + R'^2 \theta_j'^2 V(m) - 2R'\theta_j' Cov(\bar{y}, m) \le V(\bar{y}) + R^2 V(\bar{x}) - 2RCov(\bar{y}, \bar{x})$$

$$\Rightarrow R'^2 \theta_j'^2 V(m) - 2R'\theta_j' Cov(\bar{y}, m) \le R^2 V(\bar{x}) - 2RCov(\bar{y}, \bar{x})$$

$$\Rightarrow R'^2 \theta_j'^2 V(m) - R^2 V(\bar{x}) \le 2R'\theta_j' Cov(\bar{y}, m) - 2RCov(\bar{y}, \bar{x})$$

$$\Rightarrow \frac{\bar{Y}^2}{M^2} \theta_j'^2 V(m) - \frac{\bar{Y}^2}{\bar{X}^2} V(\bar{x}) \le 2\frac{\bar{Y}}{M} \theta_j' Cov(\bar{y}, m) - 2\frac{\bar{Y}}{\bar{X}} Cov(\bar{y}, \bar{x})$$

$$\Rightarrow \theta_j'^2 \frac{V(m)}{M^2} - \frac{V(\bar{x})}{\bar{X}^2} \le 2\left\{\theta_j' \frac{Cov(\bar{y}, m)}{\bar{Y}M} - \frac{Cov(\bar{y}, \bar{x})}{\bar{Y}\bar{X}}\right\}$$

$$\Rightarrow \theta_j'^2 C_{mm}' - C_{xx}' \le 2\left\{\theta_j' C_{ym}' - C_{yx}'\right\};\ j = 1, 2, 3, 4, 5$$

Comparison with that of Modified Ratio Estimators

Consider $MSE\left(\hat{\bar{Y}}_{SPj}\right) \le MSE\left(\hat{\bar{Y}}_{RMi}\right)$

$$\Rightarrow V(\bar{y}) + R'^2 \theta_j'^2 V(m) - 2R'\theta_j' Cov(\bar{y}, m) \le V(\bar{y}) + R^2 \theta_i^2 V(\bar{x}) - 2R\theta_i Cov(\bar{y}, \bar{x})$$

$$\Rightarrow R'^2 \theta_j'^2 V(m) - 2R'\theta_j' Cov(\bar{y}, m) \le R^2 \theta_i^2 V(\bar{x}) - 2R\theta_i Cov(\bar{y}, \bar{x})$$

$$\Rightarrow R'^2 \theta_j'^2 V(m) - R^2 \theta_i^2 V(\bar{x}) \le 2R'\theta_j' Cov(\bar{y}, m) - 2R\theta_i Cov(\bar{y}, \bar{x})$$

$$\Rightarrow \frac{\bar{Y}^2}{M^2} \theta_j'^2 V(m) - \frac{\bar{Y}^2}{\bar{X}^2} \theta_i^2 V(\bar{x}) \le 2\frac{\bar{Y}}{M} \theta_j' Cov(\bar{y}, m) - 2\frac{\bar{Y}}{\bar{X}} \theta_i Cov(\bar{y}, \bar{x})$$

$$\Rightarrow \theta_j^{'2} \frac{V(m)}{M^2} - \theta_i^2 \frac{V(\bar{x})}{\bar{X}^2} \le 2\left\{ \theta_j^{'} \frac{Cov(\bar{y}, m)}{\bar{Y}M} - \theta_i \frac{Cov(\bar{y}, \bar{x})}{\bar{Y}\bar{X}} \right\}$$

$$\Rightarrow \theta_j^{'2} C_{mm}^{'} - \theta_i^2 C_{xx}^{'} \le 2\left\{ \theta_j^{'} C_{ym}^{'} - \theta_i C_{yx}^{'} \right\}; i, j = 1, 2, 3, 4, 5$$

Comparison with that of Linear Regression Estimator

Consider $MSE\left(\hat{\bar{Y}}_{SPj}\right) \le V\left(\hat{\bar{Y}}_{lr}\right)$

$$\Rightarrow V(\bar{y}) + R^{'2} \theta_j^{'2} V(m) - 2R' \theta_j^{'} Cov(\bar{y}, m) \le V(\bar{y})\left(1 - \rho^2\right)$$

$$\Rightarrow R^{'2} \theta_j^{'2} V(m) - 2R' \theta_j^{'} Cov(\bar{y}, m) \le -V(\bar{y})\left(\frac{\left[Cov(\bar{y}, \bar{x})\right]^2}{V(\bar{x}) * V(\bar{y})} \right)$$

$$\Rightarrow 2R' \theta_j^{'} Cov(\bar{y}, m) - R^{'2} \theta_j^{'2} V(m) \ge \frac{\left[Cov(\bar{y}, \bar{x})\right]^2}{V(\bar{x})}; j = 1, 2, 3, 4, 5$$

$$\Rightarrow 2\frac{\bar{Y}}{M} \theta_j^{'} Cov(\bar{y}, m) - \frac{\bar{Y}^2}{M^2} \theta_j^{'2} V(m) \ge \frac{\left[Cov(\bar{y}, \bar{x})\right]^2}{V(\bar{x})}$$

$$\Rightarrow 2\bar{Y}^2 \theta_j^{'} C_{ym}^{'} - \bar{Y}^2 \theta_j^{'2} C_{mm}^{'} \ge \frac{\left[Cov(\bar{y}, \bar{x})\right]^2}{V(\bar{x})}$$

$$\Rightarrow 2\theta_j^{'} C_{ym}^{'} - \theta_j^{'2} C_{mm}^{'} \ge \frac{\left[C_{yx}^{'}\right]^2}{C_{xx}^{'}}, j = 1, 2, 3, 4, 5$$

Comparison with that of Median Based Ratio Estimator

Consider $\mathrm{MSE}\left(\hat{\bar{Y}}_{SPj}\right) \leq \mathrm{MSE}\left(\hat{\bar{Y}}_{M}\right)$

$$\Rightarrow V\left(\bar{y}\right)+R'^{2}\theta_j'^{2}V\left(m\right)-2R'\theta_j'Cov\left(\bar{y},m\right)\leq V\left(\bar{y}\right)+R'^{2}V\left(m\right)-2R'Cov\left(\bar{y},m\right)$$

$$\Rightarrow R'^{2}\theta_j'^{2}V\left(m\right)-2R'\theta_j'Cov\left(\bar{y},m\right)\leq R'^{2}V\left(m\right)-2R'Cov\left(\bar{y},m\right)$$

$$\Rightarrow R'^{2}\theta_j'^{2}V\left(m\right)-R'^{2}V\left(m\right)\leq 2R'\theta_j'Cov\left(\bar{y},m\right)-2R'Cov\left(\bar{y},m\right)$$

$$\Rightarrow R'V\left(m\right)\left(\theta_j'^{2}-1\right)\leq 2\left(\theta_j'-1\right)Cov\left(\bar{y},m\right)$$

$$\Rightarrow R'V\left(m\right)\left(\theta_j'-1\right)\left(\theta_j'+1\right)\leq 2\left(\theta_j'-1\right)Cov\left(\bar{y},m\right)$$

$$\Rightarrow Cov\left(\bar{y},m\right)\leq \frac{R'\left(\theta_j'+1\right)V\left(m\right)}{2}\ \ Since\ \ \theta_j'<1;\ j=1,2,3,4,5$$

$$\Rightarrow Cov\left(\bar{y},m\right)\leq \frac{\bar{Y}M\left(\theta_j'+1\right)C_{mm}'}{2}\ \ Since\ \ \theta_j'<1$$

$$\Rightarrow 2C_{ym}'\leq\left(\theta_j'+1\right)C_{mm}',\ j=1,2,3,4,5$$