Journal of Modern Applied Statistical Methods

Volume 14 | Issue 2 Article 17

11-1-2015

Approaches for Detection of Unstable Processes: A Comparative Study

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Cover Page Footnote

This work is supported by JSSMVP Mysore. I, sincerely thank to my Guide Dr.Swamy D.R, Professor and Head of the Department, Industrial Engineering &Management, JSSATE Bangalore and Co-Guide Dr P.Nagesh, Professor, Department of Management studies, SJCE, Mysore.

Journal of Modern Applied Statistical Methods
November 2015, Vol. 14 No. 2, 219-235.

ISSN 1538 − 9472

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A process is stable only when parameters of the distribution of a process or product characteristic remain same over time. Only a stable process has the ability to perform in a predictable manner over time. Statistical analysis of process data usually assume that data are obtained from stable process. In the absence of control charts, the hypothesis of process stability is usually assessed by visual examination of the pattern in the run chart. In this paper appropriate statistical approaches have been adopted to detect instability in the process and compared their performance with the run chart of considerably shorter length for assessing its patterns and ensuring the process stability.

Keywords: Process stability, run chart patterns, run test, unstable process

Introduction

The run chart is a most effective and widely used tool for monitoring the stability of a process by displaying the data to make process performance visible. As long as the series of points in time exhibit a random pattern, the process is assumed to have constant mean and standard deviation and no autocorrelation (i.e. stable). While run charts focus more on time pattern, a control chart focuses on acceptable limits of the process data. However, in many industrial situations, it becomes necessary to estimate process parameter whose stability cannot be monitored using control charts due to lack of data and time for establishing control limits. In the absence of properly established control charts, process stability can be evaluated with the help of run chart trend and its pattern, which can be detected by applying run rules and to conclude the assignable causes present in the process.

In run chart, each observation of a sample have a time variable representing the time of each data point is measured when data have time related behavior, run charts are familiar tools to visualize the process behavior. Also Deming (1986) pointed that when processes ought to behave randomly overtime, run charts can

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help to identify nonrandom behavior, which can unearth potential for improvement. Run charts can be used as one of the important tools for diagnosing and solving various industrial problems, nonrandom patterns are indicative of process instability. Depending on the causes of process instability the non-random patterns can be of different types. The SQC Handbook of Western Electric illustrated various types of unnatural or nonrandom patterns that may occur in the run chart (Western Electric, 1956). Among these, six types of non-random patterns of individual observations are upward shift, downward shift, increasing trend, decreasing trend, cyclic and systematic patterns.

Various statistical tools, such as Regression analysis, ANOVA method, SR test, INSR test, and Levene's test have been used to assess the process location and variation to detect statistical stability of the forging process. These tools have also been compared with run chart of considerably shorter length to assess the efficiency of the above statistical methods, and indicate the process stability.

Methodology

The methodology involves the following steps:

1. Understanding the basic concepts and tools to detect process stability of a manufacturing process.
2. Process data collection.
3. Approaches used for assessing the statistical stability of the process are
    a. Regression Analysis,
    b. SR method,
    c. INSR method,
    d. Run test
    e. ANOVA method
    f. Levene's test
4. Construction of Run chart using statistical software MINITAB
5. Compare the performance of the above approaches with Run chart.
6. Conclusion about the performance of the above methods.

Data collection and analysis

The data set pertaining to the critical quality characteristic i.e. inner diameter of piston rings for an automotive engine produced by forging process. The details of the operation and product specification are presented in Table 1. The required quality characteristic of 32 consecutive units are measured and presented in Table 2. The basic sample statistics are calculated and presented in Table 3.

Table 1. Product description

Part Name	Material	Operation	Specifications	Measuring Device
Piston ring	Cast steel	Forging	74.00 ± 0.05	Dial Gauge

*All dimensions are in mm.

Table 2. Measurements of Piston ring hole diameter in mm.

Sl. no.	Hole dia	Sl. no.	Hole dia	Sl. no.	Hole dia	Sl. no.	Hole dia
1	74.030	9	74.011	17	73.996	25	74.014
2	74.002	10	74.004	18	73.993	26	74.009
3	74.019	11	73.988	19	74.015	27	73.994
4	73.992	12	74.024	20	74.009	28	73.997
5	74.008	13	74.021	21	73.992	29	73.985
6	73.995	14	74.005	22	74.007	30	73.993
7	73.992	15	74.002	23	74.015	31	73.998
8	74.001	16	74.002	24	73.989	32	73.990

Table 3. Summary Statistics of the case study data.

Sample size	Mean	Median	Minimum	Maximum	Range	Std. Deviation
32	74.003	74.002	73.985	74.03	0.045	0.0115

Statistical Approaches to Detect Instability

Regression analysis

One way to quantify the change in location is to fit a straight line to the data using an index variable as the independent in the regression. In this case, the observed values are in the sequential run order and they are collected at equally spaced time intervals. In this study, index variable are $X = 1, 2, 3, \ldots N$ where N is the number of observations. If there is no significant drift in the location over time, the slope parameter would be zero. The scatter diagram of the data reveals a negative linear association. Therefore, it can be proceeded to find the equation of the regression line using MINITAB statistical software.

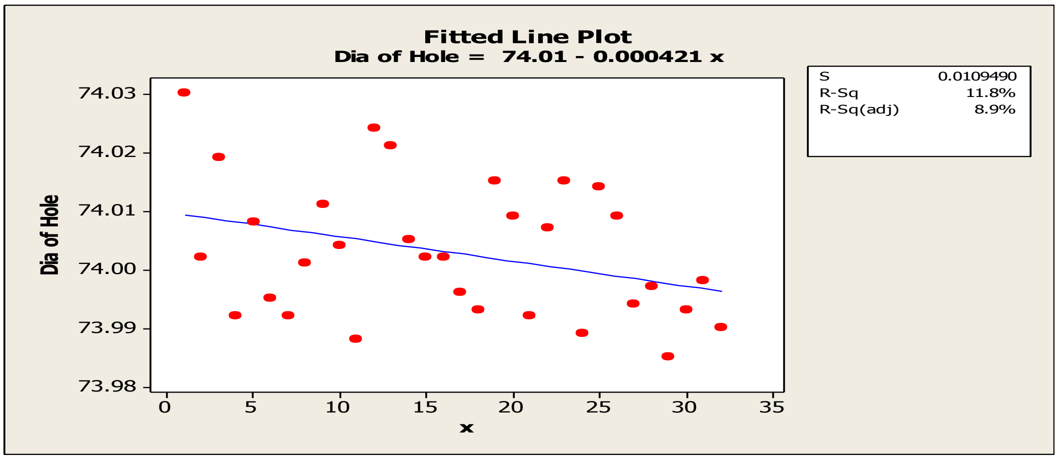

Figure 1. Output of regression analysis table for case study data.

The regression equation is Dia. of Hole = 74.0 - 0.000421 × (X)

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	0.0004831	0.0004831	4.03	0.054
Residual Error	30	0.0035964	0.0001199		
Total	31	0.0040795			

In the output of the regression analysis table for the case study data, the F-statistic is 4.03. The table value is 4.17 for F (0.05, 1, 30). Since $F_{calculated}$ is less than F_{table} value, and the p-value is greater than 0.05. It may be concluded that there is evidence that slope is almost equal to zero and ensure the process is stable over time.

SR method (standard deviation ratio method)

The SR test is derived from the square of the ratio of the standard deviation estimated using all the observations and the standard deviation estimated using sub group ranges/standard deviations/individual moving ranges. The basis of the SR test is that if the process is stable, all the approaches would yield similar estimates for the process standard deviation. In this case statistic, SR is computed as the ratio of the estimate of the long term variance and the estimate of the short term variance. The estimated sample variance based on the N observations will indicate the long term variance and the estimated variance based on the moving range (MR) method will reveal the short term variance.
Thus,

$$SR = \frac{\frac{1}{N-1}\sum_{i=1}^{N}\left(y-\bar{y}\right)^2}{\left(\overline{MR}/1.128\right)} \tag{1}$$

$$\bar{y} = \sum_{i=1}^{N} Y_i / N \tag{2}$$

$$\overline{MR} = \sum_{i=1}^{N-1}\left|y_{i+1}-y_i\right| / \left(N-1\right) \tag{3}$$

Ramirez and Runger (2006) assumed that an approximate F-distribution for SR, where the effective degree of freedom associated with the numerator and denominator are considered as (N-1) and $0.62 \times (N\text{-}1)$ respectively and accordingly, it is recommended as an approximate F-test for SR.

Table 4. Calculation of Moving Range for the case study data.

Sl. no.	Hole dia (y_i)	$MR\left|\left(y_{i+1}-y_i\right)\right|$	Sl. no.	Hole dia (y_i)	$MR\left|\left(y_{i+1}-y_i\right)\right|$
1	74.030	-	17	73.996	0.006
2	74.002	0.028	18	73.993	0.003
3	74.019	0.017	19	74.015	0.022
4	73.992	0.027	20	74.009	0.006
5	74.008	0.016	21	73.992	0.017
6	73.995	0.013	22	74.007	0.015
7	73.992	0.003	23	74.015	0.008
8	74.001	0.009	24	73.989	0.016
9	74.011	0.010	25	74.014	0.025
10	74.004	0.007	26	74.009	0.005
11	73.988	0.016	27	73.994	0.015
12	74.024	0.036	28	73.997	0.003
13	74.021	0.003	29	73.985	0.007
14	74.005	0.016	30	73.993	0.008
15	74.002	0.003	31	73.998	0.005
16	74.002	0.000	32	73.990	0.008

$$\begin{aligned} \bar{y} &= \frac{2368.09}{32} = 74.0029, \\ \overline{MR} &= \frac{0.373}{31} = 0.0120, \\ \sigma' &= \frac{\overline{MR}}{d_2} \end{aligned} \tag{4}$$

d_2 = 1.128, Statistical constant for n = 2 (Montgomery, 2009, p.702)

$$\sigma' = \frac{0.012}{1.128} = 0.0106.$$

$$\sum\left|\left(y_{i+1}-y_i\right)\right| = 0.373$$

$$F\left(0.05,31,19.22\right) = 1.93F\left(tab\right)$$

Because $SR = 0.012$, i.e., (F *calculated*), F (*calculated*) < F (*table*). Hence, it is concluded that the process is said to be stable.

Instability ratio test (INSR)

The instability ratio is defined as the ratio of the number of data points that have one or more violation of the Western Electric (1956) rules to the total number of data points plotted in the process behavior chart for the time period under assessment. The motivation for the INSR test is that if the process is stable, then it operates with common cause variation only and over time the observations move randomly about the central line and typically remain within the upper and lower control limits. The pattern exhibited in the run chart is called a random pattern.

Appearance of a nonrandom pattern, which can be detected by applying run rules, is indicative that there is either an assignable cause present in the process or the process output's variation has increased. Ramirez and Runger (2006) considered that the four most popular Western Electric (1956) rules for application of INSR method. Rules are as follows:

- 1 point out side of 3σ limits,
- 8 points in a row on one side of the central line,
- 2 of 3 points 2σ and beyond on the same side of the central line,
- 4 of 5 points 1σ and beyond on the same side of the central line.

Then the test statistic, INSR, is noted as follows

$$INSR = \frac{\text{Total number of violations with respect to the four rules in the chart}}{\text{Total number of observations plotted in the chart}} \times 100 \quad (5)$$

Table 5. Calculation of Moving Range for the case study data.

Sl. no.	(y_i)	$MR\left|(y_{i+1}-y_i)\right|$	Sl. no.	(y_i)	$MR\left|(y_{i+1}-y_i)\right|$
1	74.030	-	17	0.006	0.006
2	74.002	0.028	18	0.003	0.003
3	74.019	0.017	19	0.022	0.022
4	73.992	0.027	20	0.006	0.006
5	74.008	0.016	21	0.017	0.017
6	73.995	0.013	22	0.015	0.015
7	73.992	0.003	23	0.008	0.008
8	74.001	0.009	24	0.016	0.016
9	74.011	0.010	25	0.025	0.025
10	74.004	0.007	26	0.005	0.005
11	73.988	0.016	27	0.015	0.015
12	74.024	0.036	28	0.003	0.003
13	74.021	0.003	29	0.007	0.007
14	74.005	0.016	30	0.008	0.008
15	74.002	0.003	31	0.005	0.005
16	74.002	0.000	32	0.008	0.008

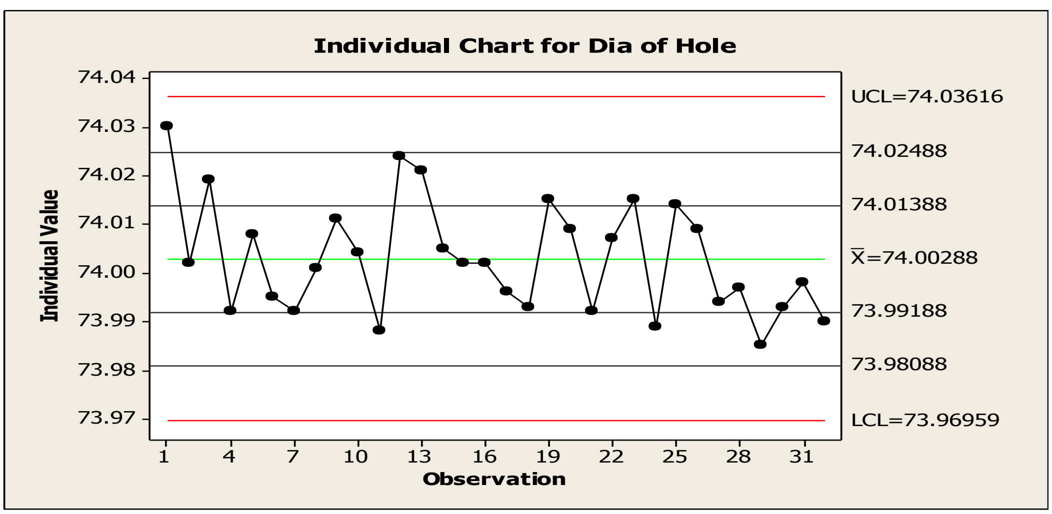

Figure 2. Run chart with 1σ, 2σ and 3σ control limits.

Process mean (μ) that represents the central line and the standard deviation (σ) that determines the distances of the control limits from the central line are usually unknown, and so these may be estimated from the N observations. The process means (μ) and standard deviation (σ) are estimated using arithmetic mean and moving ranges respectively.

Interpretation

a) 1 point out side of 3σ limits, (in Figure 2 no points violate this rule).
b) 8 points in a row on one side of the central line, (in Figure 2 no points violate this rule).
c) 2 of 3 points 2σ and beyond on the same side of the central line, (in Figure 2 no points violate this rule).
d) 4 of 5 points 1σ and beyond on the same side of the central line, (in Figure 2 no points violate this rule).
e) As no points violating the above 4 rules, INSR = 0.00, cutoff value for Run chart length ($N = 32$) is 3.125% [8], so the process is said to be stable.

Variation

To detect a change in variation in the process, Levene's test has been used it is based on the median rather than the mean. It assesses the assumptions that variance of the population from which different samples are drawn are equal. It tests the null hypothesis that the population variances are equal. If the resulting *p*-value of Levene's test is less than critical value (0.05), the obtained differences in the sample variances are unlikely to have occurred based on random sampling from a population with equal variances thus the null hypothesis of equal variances is rejected and it is concluded that there is a difference between the variances in the population. It also tests whether two sub samples in a given population have equal or different variances based on *p*-values.

Hypothesis Testing: Null hypothesis H_0 ; $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4$ (There is no change in variance)

Alternate hypothesis, H_0 ; $\sigma_1 \neq \sigma_2 \neq \sigma_3 \neq \sigma_4$ (There is change in variance)

Levine's Test has been carried out using the MINITAB software. Since the *p*-value is greater than 0.05, the null hypothesis is accepted and hence that there is no change in variance among the 4 sets in the sample data of 32 consecutive units.

ANOVA

This approach is to compare within subgroup variation to between subgroup variation to detect a difference in subgroup means and aimed at detecting changes in the process mean only. In this case study, N=32 individual observations are collected and the ANOVA method is applied by forming subgroups of size 2 using consecutive observations, i.e. there will be N/2 subgroups. Then the test statistic F is computed as the ratio of the mean sum of squares of subgroups (MS subgroup) and the mean sum of squares of errors (MS error).

Table 6. Analysis of Variance

Sl. no.	x_1	x_2	$\bar{x}_i$	$\bar{x}$	$(\bar{x}_i - \bar{x})^2$	$(x_{ji} - \bar{x}_i)^2$	$(x_{ji} - \bar{x})^2$
1	74.030	74.002	74.016	73.996	0.0004	0.000392	0.001192
2	74.019	73.992	74.0055	73.996	9.03E-05	0.000365	0.000545
3	74.008	73.995	74.0015	73.996	3.02E-05	8.45E-05	0.000145
4	73.992	74.001	73.9965	73.996	3.00E-07	4.05E-05	0.000041
5	74.011	74.004	74.0075	73.996	0.000132	2.45E-05	0.000289
6	73.988	74.024	74.006	73.996	0.0001	0.000648	0.000848
7	74.021	74.005	74.013	73.996	0.000289	0.000128	0.000706
8	74.002	74.002	74.002	73.996	0.000036	0.000000	0.000072
9	73.996	73.993	73.9945	73.996	2.30E-06	4.50E-06	0.000009
10	74.015	74.009	74.012	73.996	0.000256	0.000018	0.00053
11	73.992	74.007	73.9995	73.996	1.23E-05	0.000113	0.000137
12	74.015	73.989	74.002	73.996	0.000036	0.000338	0.00041
13	74.014	74.009	74.0115	73.996	0.00024	1.25E-05	0.000493
14	73.994	73.997	73.9955	73.996	3.00E-07	4.50E-06	0.000005
15	73.985	73.993	73.989	73.996	0.000049	0.000032	0.00013
16	73.901	73.87	73.8855	73.996	0.01221	0.000481	0.024901

Table 7. Resulted values from the ANOVA Analysis.

SS_{Factor} = 0.0277684	MS_{Factor} = 0.001
SS_E = 0.0026845	MS_E = 0.002
SS_T = 0.03045	F_o = 0.98

From F_{table}, $F_{critical} = 2.39$ and $F_{calculated} = 0.98$. Since $F_{cal.} < F_{0.05,15,16}$, the process position in time relating to a hole diameter data is not subjected to significant changes.

Run test for randomness in the sequence.

It tests the runs up and down or the runs above and below the mean by comparing the actual values to expect values. The statistic for comparison is the chi-square test [6]. All observations in the sample larger than the median value are given a positive sign and those below the median are given negative sign. A succession of values with the same sign is called a run and the number of runs '*a*' in the sequence of data points is found and it from the test statistic. For $n > 30$, this test statistic can be compared with a normal distribution with mean and the variance, the test is two-tailed. Data: Sample size: 32 observations, Median: 74.002

Table 8. Values above and below the median.

74.030	74.002	74.019	73.992	74.008	73.995	73.992	74.001
-	+	-	+	-	-	+	+
74.011	74.004	73.988	74.024	74.021	74.005	74.002	74.002
-	+	+	-	-	-	-	-
73.996	73.993	74.015	74.009	73.992	74.007	74.015	73.989
-	+	-	-	+	+	-	+
74.014	74.009	73.994	73.997	73.985	73.993	73.998	73.990
-	-	+	-	+	+	+	-

H_0: The sequence is produced in a random manner.
H_1: The sequence is not produced in a random manner.
Number of observations, $N = 32$, Number of runs, $a = 18$

$$\mu_a = \frac{2N - 1}{3} \qquad (6)$$

$$\sigma_a^2 = \frac{16N - 29}{90} \qquad (7)$$

$$\mu_a = \frac{2(32)-1}{3} = 21$$

$$\sigma_a^2 = \frac{16(32)-29}{90} = 5.37$$

For $N > 20$, the distribution of 'a' (number of runs) is reasonably approximated by a normal distribution, $N\left(\mu_a, \sigma_a^2\right)$. This approximation can be used to test the independence of the observations. In this case the standardized normal test statistic is developed by subtracting the mean from the observed number of runs 'a' and dividing by the standard deviation.

The test statistic is as follows.

$$Z_0 = \frac{a - \mu_a}{\sigma_a} \tag{8}$$

$$Z_0 = \frac{18-21}{2.32} = -1.30$$

Test statistic: Z_0 = -1.30, Significance level: α = 0.05
Critical value: $Z_{1-\alpha/2}$ = 1.96, Reject H_0, if $|Z| > 1.96$.

In this case, the test statistic (-1.30) is inside the critical region, the null hypothesis cannot be rejected and hence it is concluded that the data is random. The critical value $Z_{0.025} = 1.96$. Because $|Z_0| < Z_{0.025}$, the independence (randomness) of the sequence of the observations cannot be rejected.

Run chart analysis

A run chart is a line graph of data plotted over time. By collecting and charting data over time, trends or patterns in the process can be revealed. As run charts do not use control limits, they cannot exhibit if a process is stable. However, they can show that how the process is running. The run chart can be a valuable tool at the beginning of a manufacturing process, as it reveals important information about a process before collecting the enough data to create reliable control limits. Figure 3 shows the Run chart for the case study data constructed using statistical software MINITAB to assess the stability of the process.

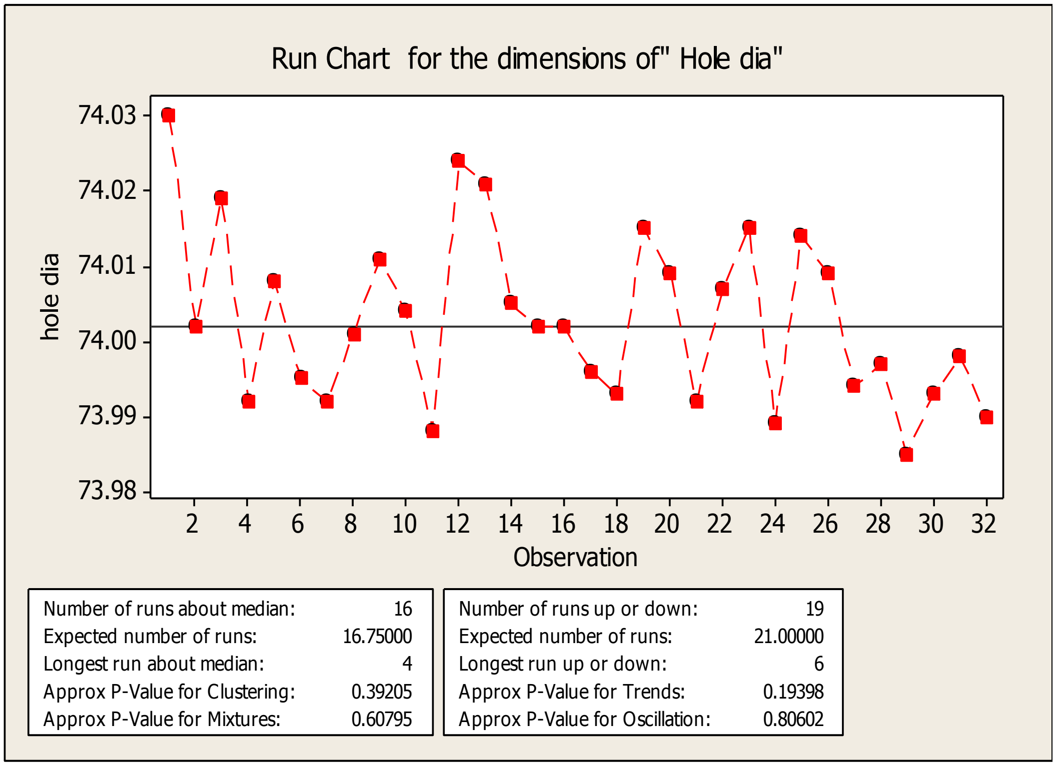

Figure 3. Construction of run chart using MINITAB-Statistical software.

The two tests (actual number of runs about median and number of runs up and down) have been conducted to check the randomness. In both the tests i.e., actual number of runs about median and number of runs up and down are close to the expected number of runs. It implies that the data come from random distribution. Clusters are groups of points in one area of the charts, cluster indicate variation due to special causes such as measurement problem. In this case, approximate *p*-value is 0.39205, it is greater than 0.05, hence it may be concluded that there is no clustering in the data. Process stability can be assured by observing the oscillation of data above and below the center line rapidly. In this case, Approximate *p*-value is 0.80602, it is greater than 0.05, so it may be conclude that there is no oscillating pattern in the data.

A mixture is characterized by an absence of points near the center line. It often indicates combined data from two populations or two processes operating at different levels. In this case, approximate *p*-value is 0.60795, it is greater than 0.05, hence it may be conclude that the data does not come from different process.

Trends are sustained and systematic sources of variation characterized by a group of points that drifts either up or down. Trends may warn that a process is about to go out of control and may be due to worn tools. In this case, approximate *p*-values is 0.19398, it is greater than 0.05, hence it is concluded that there is no trend in the data. The tests for non-random pattern are significant at the 0.05 level. All *p*-values for all the tests are greater than 0.05 (α) which suggests that the data come from a random distribution and process is stable.

Discussion

The data set pertaining to the quality characteristic i.e. inner diameter of piston rings for an automotive engine produced by forging process. Measurements for inner diameter of 32 consecutive units are measured and recorded. The various approaches have been used on the data in order to assess the stability of the forging process. Tests with respect to location, variation, randomness and sequence of data has been done through Regression analysis, ANOVA test, Run test, Levene's test, SR test, INSR test. The scatter plot reveals a least magnitude of negative linear association (almost zero).

In Regression analysis, R^2 value is 11.8%; it is can be stated that 11.8% of the total variation in the hole diameter occurs because of the variation in the observations sequence and remaining 88.2% is due to randomness and other causes of variation and also reveals that the relationship between the variables i.e. hole diameter and time is not significant. Also the F-test indicates that there is no considerable slope in the line.

In Levene's test, P-valve is greater than 0.05, so the null hypothesis cannot be rejected that there is no change in variance among the 4 sets in the sample data of 32 consecutive units.

In case of Instability ratio test, Calculated Instability Ratio (INSR) = 0.00, cutoff value for Run chart length ($N = 32$) is 3.125% [8], as instability ratio value is less than cutoff value, the process is said to be stable. In SR method, the test statistic SR is computed and compared with the F (table) value. F-Test for SR, conclude that the process is stable as SR = 0.012 i.e. (F calculated) is less than F (0.05, 31, 19.22) = 1.93 i.e., (F table). In case of ANOVA method, $N = 32$ individual observations, it is applied by forming subgroups of size 2 using consecutive observations, i.e. there will be $N/2$ subgroups.

Then the test statistic F is computed as the ratio of the mean sum of squares of subgroups (MS subgroup) and the mean sum of squares of errors (MS error). From F_{table}, $F_{critical} = 2.39$ and $F_{calculated} = 0.98$. Since $F_{calculated} < F_{0.05,15,16}$, the process position in time relating to a hole diameter is not subjected to significant changes. Run Test for randomness of the sequence is concluded that the data is random. The Table 9 presents the summary of results of the various statistical methods.

Table 9. Summary results of the statistical method.

Sl. no.	Statistical method	Result	Stable/Unstable
1	Regression	*F*(calculated) < *F*(table), p > 0.05	Stable
2	SR-method	*F*(calculated) < *F*(table)	Stable
3	Instability Ratio method	Instability ratio < cutoff value,	Stable
4	Levene's Test	$p > 0.05$	Stable
5	ANOVA method	*F*(calculated) < *F*(table),	Stable
6	Run Test	Z_0(calculated) < $Z_{1-\alpha/2}$(table),	Stable
7	Run Chart	$p > 0.05$,All cases	Stable

Alternative approaches were presented to assess the stability of the process and compared with the run chart. Process stability has been detected using the approaches such as Regression analysis, SR method, INSR method, Levene's test, ANOVA method. Even though all the approaches yield the same result (i.e., process is stable), above mentioned approaches have their own advantages and limitations. As the exact distribution of SR is not known and assumed an approximate F-distribution for SR, it can be applied only when the number of observations is larger than or equal to 32. The advantage of ANOVA approach is that the F-test conducted using the 'between' and 'within' sums of squares is well defined and it is applicable even when the available number of observations is small but it requires practitioner's to have background in statistics. Run test indicated that the data points are independent and random, hence it is concluded that there is no shift in location. INSR Test is more effective test as it uses rules similar to run chart and it works well for large number of samples. For small number of samples like 32-100 subgroups it leads to a Type-I error (i.e. probability of declaring a stable process as unstable) as high as 0.35. Ramirez and Runger recommended taking the 95th percentile point of the distribution of INSR as the cutoff value. With aim to increase the effectiveness, it has been recommended using the ANOV and the INSR tests. All the statistical methods indicates the presence of statistical stability in the case study data but run chart using the statistical software MINITAB gives more effective and accurate result compared to the other methods for assessing stability of the process.

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