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An almost unbiased estimator using known value of some population parameter(s) is proposed. A class of estimators is defined which includes Singh and Solanki (2012) and Sahai and Ray (1980), Sisodiya and Dwivedi (1981), Singh, Cauhan, Sawan, and Smarandache (2007), Upadhyaya and Singh (1984), Singh and Tailor (2003) estimators. Under simple random sampling without replacement (SRSWOR) scheme the expressions for bias and mean square error (MSE) are derived. Numerical illustrations are given.

Keywords: Auxiliary information, bias, mean square error, unbiased estimator

Introduction

The precision of the estimates of the population mean or total of the study variable y can be considering improved by the use of known information on an auxiliary variable x which is highly correlated with the study variable y. Consider a finite population $U = U_1, U_2, \ldots, U_N$ of N units. Let y and x stand for the variable under study and auxiliary variable respectively. Let (y_i, x_i), $i = 1, 2, \ldots, n$ denote the values of the units included in a sample s_n of size n drawn by simple random sampling without replacement (SRSWOR). The auxiliary information has been used in improving the precision of the estimate of a parameter (see Sukhatme, Sukhatme, Sukhatme, & Ashok (1984) and the references cited therein). Among many methods, the ratio and product methods of estimation are good illustrations in this context.

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In order to have a survey estimate of the population mean $\bar{Y}$ of the study character y, assuming the knowledge of the population mean $\bar{X}$ of the auxiliary character x, the well-known ratio estimator is

$$t_R = \bar{y}\frac{\bar{X}}{\bar{x}} \tag{1}$$

Bahl and Tuteja (1991) suggested an exponential ratio type estimator as

$$t_{\exp} = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right] \tag{2}$$

Several authors have used prior value of certain population parameter(s) to find more precise estimates. Sisodiya and Dwivedi (1981), Sen (1978) and Upadhyaya and Singh (1984) used the known coefficient of variation (CV) of the auxiliary character for estimating population mean of a study character in ratio method of estimation. The use of prior value of coefficient of kurtosis in estimating the population variance of study character y was first made by Singh et al. (1973) Later used by Singh and Kakran (1993) in the estimation of population mean of study character. Singh and Tailor (2003) proposed a modified ratio estimator by using the known value of correlation coefficient. Kadilar and Cingi (2006) and Singh, Pandey, and Hirano (2008) have suggested modified ratio estimators by using different pairs of known value of population parameter(s).

Under SRSWOR, an almost unbiased estimator for estimating $\bar{Y}$ is proposed. To obtain the bias and MSE,

$$\bar{y} = \bar{Y}\left(1+e_0\right),\ \bar{x} = \bar{X}\left(1+e_1\right),$$

such that $E\,(e_0) = E\,(e_1) = 0$.

$$E\left(e_0^2\right) = f_1C_y^2,\ E\left(e_1^2\right) = f_1C_x^2,\ E\left(e_0e_1\right) = f_1\rho C_yC_x$$

where

$$f_1 = \left(\frac{1}{n} - \frac{1}{N}\right), S_y^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2, S_x^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2$$

$$C_y = \frac{S_y}{Y}, C_x = \frac{S_x}{X}, K_x = \rho_{yx}\left(\frac{C_y}{C_x}\right), \rho_{yx} = \frac{S_{yx}}{\left(S_y S_X\right)}, S_{yx} = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_i - \bar{X}\right)$$

The Proposed Estimator

Consider the following estimator

$$t_1 = \bar{y}\left(\frac{K_1\bar{X} + K_2K_3}{K_1\bar{x} + K_2K_3}\right)^{\alpha} \tag{3}$$

The bias and MSE expressions of the estimator t_1 up to the first order of approximation are, respectively, given by

$$B(t_1) = \bar{Y}f_1C_x^2\left[\frac{\alpha(\alpha+1)V_1^2}{2} - \alpha V_1 K_x\right] \tag{4}$$

$$\text{MSE}(t_1) = \bar{Y}^2 f_1\left[C_y^2 + C_x^2\left(\alpha^2V_1^2 + 2V_1\alpha K_x\right)\right] \tag{5}$$

Following Singh and Solanki (2012), consider the following estimator

$$t_2 = \bar{y}\left\{2 - \left(\frac{\bar{x}}{\bar{X}}\right)^{\beta}\exp\left[\lambda\left(\frac{\left(K_4\bar{X} + K_5\right) - \left(K_4\bar{x} + K_5\right)}{\left(K_4\bar{X} + K_5\right) + \left(K_4\bar{x} + K_5\right)}\right)\right]\right\} \tag{6}$$

The bias and MSE expressions of the estimator t_2 up to the first order of approximation are, respectively, given by

$$B(t_2) = \bar{Y}f_1C_x^2\left[\frac{\lambda V_2\beta}{2} - \frac{\beta(\beta-1)}{2} - \frac{\lambda(\lambda+2)V_2^2}{8} - \beta K_x + \frac{\lambda V_2 K_x}{2}\right] \tag{7}$$

$$\mathrm{MSE}(t_2) = \bar{Y}^2 f_1\left[C_y^2 + C_x^2\left(\beta^2 + \frac{\lambda^2 V_2^2}{4} - \beta\lambda V_2\right) - 2K_x C_x^2\left(\beta - \frac{\lambda V_2}{2}\right)\right] \tag{8}$$

α, λ, and β are suitable chosen constants. Also K_1, K_3, K_4, K_5 are either real numbers or function of known parameters of the auxiliary variable x such as C_x, $\beta_2(x)$, ρ_{yx} and K_x.. K_2 is an integer which takes values +1 and -1 for designing the estimators and

$$\left.\begin{aligned} V_1 &= \frac{K_1\bar{X}}{K_1\bar{X} + K_2K_3} \\ V_2 &= \frac{K_4\bar{X}}{K_4\bar{X} + K_5} \end{aligned}\right\}$$

The estimators t_1 and t_2 are biased estimators. In some applications bias is disadvantageous. Following these estimators we have proposed almost unbiased estimator of $\bar{Y}$.

Almost Unbiased Estimator

Suppose

$$t_0 = \bar{y} \quad t_1 = \bar{y}\left(\frac{K_1\bar{X} + K_2K_3}{K_1\bar{x} + K_2K_3}\right)^{\alpha} \quad t_2 = \left\{2 - \left(\frac{\bar{x}}{\bar{X}}\right)^{\beta} \exp\left[\lambda\left(\frac{(K_4\bar{X} + K_5) - (K_4\bar{x} + K_5)}{(K_4\bar{X} + K_5) + (K_4\bar{x} + K_5)}\right)\right]\right\}$$

such that t_0, t_1, $t_2 \in W$, where W denotes the set of all possible estimators for estimating the population mean $\bar{Y}$. By definition, the set W is a linear variety if

$$t_p = \sum_{i=0}^{3} w_i t_i \in W \tag{9}$$

such that,

$$\sum_{i=0}^{3} w_i = 1 \text{ and } w_i \in R \tag{10}$$

where w_i (i = 0 , 1 , 2 , 3) denotes the constants used for reducing the bias in the class of estimators, H denotes the set of those estimators that can be constructed from t_i (i = 0 , 1 , 2 , 3) and R denotes the set of real numbers.

Expressing t_p in terms of e's,

$$t_p = \bar{Y}\left[\begin{array}{l} 1+e_0+w_1\left(\dfrac{\alpha(\alpha+1)V_1^2e_1^2}{2}-\alpha V_1e_1-\alpha V_1e_0e_1\right) \\ +w_2\left(-\beta e_1-\dfrac{\beta(\beta-1)e_1^2}{2}+\dfrac{\lambda V_2e_1}{2}+\dfrac{\lambda V_2\beta e_1^2}{2}-\dfrac{\lambda(\lambda+2)V_2^2e_1^2}{8}-\beta e_0e_1+\dfrac{\lambda V_2e_0e_1}{2}\right)\end{array}\right] \tag{11}$$

Subtracting $\bar{Y}$ from both sides of equation (11) and then taking expectation of both sides, the bias of the estimator t_p is obtained up to the first order of approximation, as

$$\begin{aligned} B(t_p) &= \bar{Y}f_1w_1C_x^2\left(\frac{\alpha(\alpha+1)V_1^2}{2}-\alpha V_1K_x\right) \\ &+\bar{Y}f_1w_2C_x^2\left(\frac{\lambda V_2\beta}{2}-\frac{\beta(\beta-1)}{2}-\frac{\lambda(\lambda+2)V_2^2}{8}-\beta K_x+\frac{\lambda V_2K_x}{2}\right)\end{aligned} \tag{12}$$

From (11), we have

$$\left(t_p-\bar{Y}\right)=\bar{Y}\left[e_0-w_1\alpha V_1e_1-w_2\left(\beta e_1+\frac{\lambda V_2e_1}{2}\right)\right] \tag{13}$$

Squaring both sides of (13) and then taking expectation, the MSE of the estimator t_p up to the first order of approximation is obtained, as

$$\text{MSE}(t_p)=\bar{Y}^2f_1\left[C_y^2+C_x^2\left(Q^2-2QK_x\right)\right] \tag{14}$$

which is a minimum when

$$Q=K_x \tag{15}$$

where

$$Q = w_1 \alpha V_1 + w_2 \left(\beta - \frac{\lambda V_2}{2} \right) \tag{16}$$

Putting the value of $Q = K_x$ in (14), the optimum value of estimator as t_p (optimum) is obtained. Thus, the minimum MSE of t_p is given by

$$\min.\text{MSE}\left(t_p\right) = \bar{Y}^2 f_1 C_y^2 \left(1 - \rho_{yx}^2\right) \tag{17}$$

which is same as that of traditional linear regression estimator.

From (10) and (16), there are two equations and three unknowns. It is not possible to find the unique values for w_i's, 1 = 0,1,2. In order to get unique values of w_i's, impose the linear restriction

$$\sum_{i=0}^{2} w_i B\left(t_i\right) = 0 \tag{18}$$

where $B(t_i)$ denotes the bias in the i^{th} estimator.

Equations (10), (16) and (18) can be written in the matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & \alpha V_1 & \beta - \dfrac{\lambda V_2}{2} \\ 0 & B\left(t_1\right) & B\left(t_2\right) \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 \\ k_x \\ 0 \end{bmatrix} \tag{19}$$

Using (19), the unique values of w_i's, 1 = 0,1,2 are

$$\left.\begin{aligned} & \frac{\alpha V_1\left[\alpha V_1 A_2 - A_1 X_1\right] - X_1 K_x A_1 - X_2 \alpha V_1\left[\alpha V_1 A_2 - A_1 X_1\right] - \alpha V_1 K_x A_1}{\alpha V_1\left[\alpha V_1 A_2 - A_1 X_1\right]} \\ w_0 = w_1 &= \frac{X_1 K_x A_1}{\alpha V_1\left[\alpha V_1 A_2 - A_1 X_1\right]} + X_2 \\ w_2 &= \frac{K_x A_1}{\left[\alpha V_1 A_2 - A_1 X_1\right]} \end{aligned}\right\}$$

where,

$$\left.\begin{aligned}
A_1 &= \frac{\alpha(\alpha+1)V_1^2}{2} - \alpha V_1 K_x \\
A_2 &= \frac{\lambda V_2 \beta}{2} - \frac{\beta(\beta-1)}{2} - \frac{\lambda(\lambda+2)V_2^2}{8} - \beta K_x + \frac{\lambda V_2 K_x}{2} \\
X_1 &= A_1\left[\beta - \frac{\lambda V_2}{2}\right] \\
X_2 &= \frac{K_x}{\alpha V_1}
\end{aligned}\right\}$$

Use of these w_i's, 1 = 0, 1, 2 remove the bias up to terms of order $o\,(n^{-1})$ at (9).

Empirical Study

For the empirical study, consider the data sets by Kadilar and Cingi (2006) (population 1) and Khoshnevisan, Singh, Chauhan, Sawan, and Smarandache (2007) (population 2).

Data Statistics

Population	N	n	$\bar{Y}$	$\bar{X}$	C_y	C_x	ρ_{yx}	$\beta_2(x)$
Population 1	106	20	2212.59	27421.7	5.22	2.1	0.86	34.57
Population 2	20	8	19.55	18.8	0.355	0.394	-0.92	3.06

Table 1. Values of w_i

w_i	Population 1	Population 2
w_0	2.104965	3.692323
w_1	-6.48599	1.379436
w_2	5.381022	-4.07176

The percent relative efficiencies (PRE) of various estimators of $\bar{Y}$ are computed and presented in Table 2 below.

Table 2. PRE of different estimators of $\bar{Y}$ with respect to $\bar{y}$

Choice of scalars													
w_0	w_1	w_2	K_1	K_2	K_3	K_4	K_5	α	β	λ	**Estimator**	**PRE (POPI)**	**PRE (POPII)**
1	0	0									$\bar{y}$	100	100
0	1	0	1	1	0			1			t_R	212.8	24.69
			1	1	0			-1			t_{exp}	53.94	583.07
0	0	1							1	0	$t_{1(1,0)}$	212.8	23.39
									-1	0	$t_{1(-1,0)}$	53.94	527.29
						1	0		1	1	$t_{2(1,1)}$	143.99	42.93
						1	0		1	-1	$t_{2(1,-1)}$	306.54	14.63
						1	0		0	1	$t_{2(0,1)}$	72.12	348.58
						1	0		0	-1	$t_{2(0,-1)}$	143.97	42.93
			1	1	1	1	1	1	1	1	t_P **Optimum**	**384.02**	**651.04**

Proposed Estimators in Two Phase Sampling

When $\bar{X}$ is unknown, it is sometimes estimated from a preliminary large sample of size *n*' on which only the characteristic *x* is measured (for details see Singh et al., 2007). Then, a second phase sample of size *n* ($n < n'$) is drawn on which both *y* and *x* characteristics are measured. Let $\bar{x} = \frac{1}{n'}\sum_{i=1}^{n'} x_i$ denote the sample mean of *x* based on first phase sample of size *n*', $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, be the sample means of *y* and *x* respectively based on second phase of size *n*.

In two-phase sampling the estimator t_p will take the following form

$$t_{pd} = \sum_{i=0}^{3} h_i t_{id} \in H \tag{20}$$

Such that,

$$\sum_{i=0}^{3} h_i = 1 \text{ and } h_i \in R \tag{21}$$

where,

$$t_{0d} = \bar{y}, t_{1d} = \bar{y}\left(\frac{K_1\bar{x}' + K_1K_2}{K_1\bar{x} + K_2K_3}\right)^m, t_{2d} = \bar{y}\left\{2 - \left(\frac{\bar{x}}{\bar{x}'}\right)^q \exp\left[\gamma\left(\frac{(K_4\bar{x}' + K_5) - (K_4\bar{x} + K_5)}{(K_4\bar{x}' + K_5) + (K_4\bar{x} + K_5)}\right)\right]\right\}$$

The bias and MSE expressions of the estimator t_{1d} and t_{2d} up to the first order of approximation are, respectively, given by

$$B(t_{1d}) = \bar{Y}\left[\frac{m(m-1)R_1^2 f_2C_x^2}{2} + \frac{m(m+1)R_1^2 f_1C_x^2}{2} - m^2R_1^2 f_2C_x^2 + mR_1 f_3K_xC_x^2\right] \quad (22)$$

$$\text{MSE}(t_{1d}) = \bar{Y}^2\left[f_1C_y^2 + m^2R_1^2 f_3C_x^2 - 2mR_1K_x f_3C_x^2\right] \quad (23)$$

$$B(t_{2d}) = \bar{Y}\left[-\frac{q(q-1) f_1C_x^2}{2} + \frac{q(q+1) f_2C_x^2}{2} + qf_2K_xC_x^2 + q^2 f_2C_x^2 + f_3\gamma R_2K_xC_x^2 + f_3\gamma R_2qC_x^2\right] \quad (24)$$

$$\text{MSE}(t_{2d}) = \bar{Y}^2\left[f_1C_y^2 + L_1^2 f_3C_x^2\right] \quad (25)$$

where

$$\left.\begin{aligned} R_1 &= \frac{K_1\bar{X}}{K_1\bar{X} + K_2K_3} \\ R_2 &= \frac{K_4\bar{X}}{2\left[K_4\bar{X} + K_5\right]} \\ L_1 &= q - \gamma A_2 \end{aligned}\right\} \quad (26)$$

Expressing (20) in terms of e's,

$$t_{pd} = \bar{Y}\left[\begin{array}{l} 1+e_0+w_1\left(\dfrac{m(m+1)R_1^2e_1^2}{2}-mR_1e_1-mR_1e_0e_1+mR_1e_0e_1+\dfrac{m(m-1)R_1^2e_1'^2}{2}+mR_1e_0e_1'\right) \\ +w_2\left(-qe_1-\dfrac{q(q-1)e_1^2}{2}+qe_1'+q^2e_1e_1'-\dfrac{q(q+1)e_1'^2}{2}-\gamma R_2\left(e_1'-e_1\right)+\gamma R_2\left(e_0e_1-e_0e_1'\right)-qe_0e_1\right) \end{array}\right]$$

Subtracting $\bar{Y}$ from both sides of equation (22) and then taking expectation of both sides, the bias of the estimator t_{pd} is obtained up to the first order of approximation, as

$$B\left(t_{pd}\right)=\bar{Y}\left[B\left(t_{1d}\right)+B\left(t_{2d}\right)\right] \tag{27}$$

also,

$$\left(t_{pd}-\bar{Y}\right)=\bar{Y}\left[e_0+w_1\left[mR_1e_1'-mR_1e_1\right]+w_2\left(-qe_1+qe_1'-\gamma R_2e_1'+\gamma R_2e_1\right)\right] \tag{28}$$

Squaring both sides of (28) and then taking expectation, the MSE of the estimator t_{pd} is obtained up to the first order of approximation, as

$$\text{MSE}\left(t_{pd}\right)=\bar{Y}^2 f_1C_y^2+L_2^2 f_3C_x^2-2L_2 f_3K_xC_x^2 \tag{29}$$

It is a minimum when

$$L_2=K_x \tag{30}$$

where

$$L_2=h_1mR_1+h_2\left(q-\gamma R_2\right) \tag{31}$$

Putting the value of $L_2 = K_x$ in (28), the optimum value of estimator as t_{pd} (optimum) is obtained. Thus, the minimum MSE of t_{pd} is given by

$$\min.\text{MSE}\left(t_{pd}\right)=\bar{Y}^2C_y^2\left(f_1-f_3\rho_{yx}^2\right), \tag{32}$$

which is same as that of traditional linear regression estimator.

From (21) and (31), there are two equations in three unknowns. It is not possible to find the unique values for h_i's, 1 = 0,1,2. In order to get unique values of h_i's, impose the linear restriction

$$\sum_{i=0}^{2} h_i B(t_i) = 0 \tag{33}$$

where $B(t_i)$ denotes the bias in the i^{th} estimator.

Equations (21), (31) and (33) can be written in the matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & mR_1 & q-\gamma R_2 \\ 0 & B(t_{1d}) & B(t_{2d}) \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} 1 \\ K_x \\ 0 \end{bmatrix} \tag{34}$$

Using (34), we get the unique values of h_i's, i = 0,1,2 as

$$\left.\begin{aligned} h_0 &= 1 - h_1 - h_2 \\ h_1 &= \frac{k_x}{mR_1} - \frac{N_1 K_x (q-\gamma R_2)}{N_1 q - mR_1 N_2 - N_1 \gamma R_2} \\ h_2 &= \frac{K_x N_1}{\left[N_1 q - mR_1 N_2 - N_1 \gamma R_2\right]} \end{aligned}\right\}$$

where,

$$N_1 = \frac{m(m-1)R_1^2 f_2 C_x^2}{2} + \frac{m(m+1)\mathrm{R}_1^2 \mathrm{f}_1 \mathrm{C}_\mathrm{x}^2}{2} - m^2 \mathrm{R}_1^2 \mathrm{f}_2 \mathrm{C}_\mathrm{x}^2 + mR_1 f_3 K_x C_x^2$$

$$N_2 = \left[-\frac{q(q-1) f_1 C_x^2}{2} + \frac{q(q+1) f_2 C_x^2}{2} + qf_2 K_x C_x^2 + q^2 f_2 C_x^2 + f_3 \gamma R_2 K_x C_x^2 + f_3 \gamma R_2 q C_x^2\right]$$

Use of these h_i's, i = 0, 1, 2 remove the bias up to terms of order $o(n^{-1})$ at (20).

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Appendix A.

Some members (ratio-type) of the class t_1 when $w_0 = 0$, $w_1 = 1$, $w_2 = 0$, $\alpha = 1$

K_1	K_3	PRE's $K_2 = 1$	PRE's $K_2 = -1$
1	C_x	212.80	212.82
1	$\beta_2(x)$	212.60	213.02
$\beta_2(x)$	C_x	212.81	212.81
C_x	$\beta_2(x)$	212.71	212.91
1	ρ_{yx}	212.81	212.82
$N\bar{X}$	S_x	212.81	212.81
$N\bar{X}$	f	212.80	212.82
$\beta_2(x)$	K_x	212.60	213.02
N	K_x	212.81	212.81
N	1	212.71	212.91
N	C_x	212.81	212.82
N	ρ_{yx}	212.81	212.81
N	S_x	212.80	212.82
N	f	212.60	213.02
N	$g = 1 - f$	212.81	212.81
N	K_x	212.71	212.91
n	ρ_{yx}	212.81	212.82
n	S_x	212.81	212.81
n	f	212.80	212.82
n	$g = 1 - f$	212.60	213.02
n	K_x	212.81	212.81
$\beta_2(x)$	$\bar{X}$	212.71	212.91
$N\bar{X}$	$\bar{X}$	212.81	212.82
N	$\bar{X}$	212.81	212.81
n	$\bar{X}$	212.81	212.81

Appendix B.

Some members (product-type) of the class t_1 when w_0 = 0, w_1 = 1, w_2 = 0, α = - 1

K_1	K_3	PRE's $K_2 = 1$	PRE's $K_2 = -1$
1	C_x	550.91	501.92
1	$\beta_2(x)$	646.03	314.33
$\beta_2(x)$	C_x	535.22	519.18
C_x	$\beta_2(x)$	582.35	91.18
1	ρ_{yx}	466.00	579.15
$N\bar{X}$	S_x	528.52	526.06
$N\bar{X}$	f	527.30	527.28
$\beta_2(x)$	K_x	510.01	543.74
N	K_x	527.15	527.43
N	1	530.39	524.16
N	C_x	528.52	526.03
N	ρ_{yx}	524.41	530.15
N	S_x	549.55	503.49
N	f	527.53	527.06
N	$g = 1 - f$	530.16	524.40
N	K_x	524.70	529.87
n	ρ_{yx}	520.05	534.38
n	S_x	579.44	465.58
n	f	527.88	526.71
n	$g = 1 - f$	534.42	520.01
n	K_x	520.77	533.69
$\beta_2(x)$	$\bar{X}$	622.76	146.09
$N\bar{X}$	$\bar{X}$	530.39	524.16
N	$\bar{X}$	580.14	464.59
n	$\bar{X}$	632.80	363.64

Appendix C.

Some members (product-type) of the class t_2 when w_0 = 0, w_1 = 0, w_2 = 1

K_4	K_5	PRE'S
K_1	K_3	$(\beta=-1, \lambda=-1)$
1	C_x	358.00
1	$\beta_2(x)$	423.38
$\beta_2(x)$	C_x	351.94
C_x	$\beta_2(x)$	357.48
1	ρ_{yx}	324.10
$N\bar{X}$	S_x	349.09
$N\bar{X}$	f	348.58
$\beta_2(x)$	K_x	341.45
N	K_x	348.52
N	1	349.89
N	C_x	349.09
N	ρ_{yx}	347.37
N	S_x	358.21
N	f	348.68
N	$g=1-f$	349.79
N	K_x	347.49
n	ρ_{yx}	345.56
n	S_x	372.38
n	f	348.82
n	$g=1-f$	351.60
n	K_x	345.86
$\beta_2(x)$	$\bar{X}$	345.86
$N\bar{X}$	$\bar{X}$	349.89

In addition to above estimators a large number of estimators can also be generated from the proposed estimators just by putting different values of constants w_i's, h_i's K_1, K_2, K_3, K_4, K_5, α, β and λ.