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*BRIEF REPORT*

# Bayesian Analysis for Component Manufacturing Processes

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In manufacturing processes various machines are used to produce the same product. Based on the age, make, etc., of the machines the output may not always follow the same distribution. An attempt is made to introduce Bayesian techniques for a two machine problem. Two cases are presented in this article.

Key words: Stochastic models, Bayesian Analysis, MVUE, Posterior distribution.

## Introduction

Stochastic models can be better understood through the application of parametric, Bayesian and interval estimations. In this article, Bayesian Analysis of two machines producing the same component is attempted. If the first machine follows a distribution $D_1$ and the second machine follows distribution $D_2$, and $\lambda_1$ and $\lambda_2$ are the proportions of production for the two machines, then the total production equals $\lambda_1 + \lambda_2 = 1$.

In the final lot, a mixture of components from both the machines pooled together will have a distribution given by a linear combination of the two distributions as D = $\lambda_1 D_1 + \lambda_2 D_2$.

Case I

Assumptions:

1. The two machines produce components where the rate of production is not i.i.d.

2. The total lot collected has an observable distribution with an unknown parameter.

3. The number of components observed at sampled points in time is a discrete NB (N.p) distribution.

4. The log normal prior distribution of p is given by

$$\frac{1}{\beta p\sqrt{2\pi}}\, e^{-\frac{1}{2}((\frac{Logp-\alpha}{\beta})^2}$$

and is denoted by $\Lambda(\alpha,\beta^2)$.

If the number of components produced at sampled points of time ($t_1, t_2, .......t_n$) is ($c_1, c_2, .......c_n$) then D follows a negative binomial distribution given by

$$p_x = \binom{x+N-1}{N-1} p^N q^x$$

where

$$x = 0, 1, 2, 3\ldots,$$

and

$$p + q = 1. \qquad (1.1)$$

Based on (1.1) the likelihood function of the number of components is given by

$$L(p/c_1, c_2, .......c_n) = \prod_{i=1}^{n} \binom{x_i+N-1}{N-1} p^{nN} q^X,$$

where

$$X = \sum_{i=1}^{n} x_i \qquad (1.2)$$

for L to be the maximum likelihood estimator

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$$\frac{\partial Log L}{\partial p} = 0 .$$

Hence,

$$\hat{p} = \frac{N}{(N + \bar{X})} ,$$

where

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{n} x_i}{n} .$$

The sum of independent variables with a negative binomial distribution follows a negative binomial distribution (nN.p) with a probability mass function

$$f(x, p) = p(X = x)$$
$$= \binom{x + nN - 1}{nN - 1} p^{nN} q^X$$
$$x = 1, 2, 3, ... \tag{1.3}$$

where

$$E(X) = \frac{nN(1 - p)}{p} \tag{1.4a}$$

and

$$Var(X) = \frac{nN(1 - p)}{p^2} \tag{1.4b}$$

For large values of n, E( $\hat{p}$ ) = p variance tends to 0, hence, the MVUE of p is $\hat{p}$.

Posterior Distribution

If the prior density of p is a log normal distribution given by

$$\tau(p/\alpha, \beta) = \frac{1}{\beta p\sqrt{2\pi}} e^{-\frac{1}{2}((\frac{Logp-\alpha}{\beta})^2} , \tag{2.1}$$

where $\alpha$ is real, $\beta \succ 0$, and 0 < x < ∞ with mean $\alpha + \frac{\beta^2}{2}$ denoted by $\Lambda(\alpha, \beta^2)$, the marginal pdf of X is

$$f_X(x) = \int_0^1 f(x, p)\tau(p/\alpha, \beta)dp$$
$$= \int_0^1 f(x.p)\frac{1}{\beta p\sqrt{2\pi}} e^{-\frac{1}{2}((\frac{Logp-\alpha}{\beta})^2} dp \tag{2.2}$$

Therefore, the posterior distribution of p given by

$$\frac{\text{Likelihood function * Prior}}{\text{Normalising constant}}$$

is

$$q(p/c_1, c_2, ..., c_n) = \frac{f(x, p)\tau(p/\alpha, \beta)dp}{\int_0^1 f(x, p)\tau(p/\alpha, \beta)dp} \tag{2.3}$$

Case II

The numbers of components produced at discrete points of time in a given interval are observed if autoregressive processes of order n are considered. The initial observations preceding the sampled data must be determined first, which may not be possible in practical cases. If a first order AR model defined by $X_i = cX_{i-1} + g_i$ is considered where $c$ is the parameter to be estimated, i = 1, 2, 3…, and $g_i$ is the Gaussian noise, i.i.d. of normal variates with N(0, $\sigma^2$ ) and stationary for $c$ <1, then the backward shift operator defined by **B** $X_i = X_{i-1}$ results in $X_i = (1 - cB)^{-1} g_i$.The product of n observations has a multivariate normal distribution with mean zero and variance matrix.

$\sigma^2 =$

$$\begin{pmatrix} (1-c^2)^{-1} & c(1-c^2)^{-1} & \ldots\ldots\ldots\ldots & c^{n-1}(1-c^2)^{-1} \\ 0 & (1-c^2)^{-1} & c(1-c^2)^{-1}\ldots. & c^{n-2}(1-c^2)^{-1} \\ 0 & 0 & (1-c^2)^{-1}\ldots & c^{n-3}(1-c^2)^{-1} \\ 0 & 0 & 0 & (1-c^2)^{-1} \end{pmatrix}$$

Let

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -c & 1 & 0 & 0 & 0 \\ 0 & -c & 0 & 0 & 0 \\ & & & & \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

then

$P\sum P^i =$

$$\sigma^2 \begin{pmatrix} (1-c^2)^{-2} & 0 & 0 & 0\ldots. & 0 \\ 0 & 1 & 0 & 0\ldots. & 0 \\ 0 & 0 & 1 & 0\ldots.. & 0 \\ & & & & \\ 0 & 0\,,,,,,,,,,,,,,\ldots.,,1 & & & \end{pmatrix} = \sigma^2 D,$$

which is the covariance matrix of Y=PX which has a multivariate normal dist with zero mean. Because $X=\prod_{i=1}^{n} X_i$ the joint pdf of its components is

$$\frac{1}{(\sigma\sqrt{2\pi})^n} exp(\frac{1}{2\sigma^2}(1-c^2)X_1^{\,2} - \sum_{i=2}^{n}(X_i X_{i-1})^n.$$

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References

Alexander, Mood, & Boes. (1974). *Theory of Statistics*. New Delhi, India: Tata McGraw Hill.

Broemiling, L. D. (1993). *Bayesian analysis of linear models*. New York: Marcel Dekker Inc.

Radhakrishna, R. C. (2001). *Linear statistical inference and its applications* (*2nd Ed.*). New York: Wiley-Interscience.

Zellner, A. (1996). *An introduction to Bayesian inference to ecometrics*. New York: John Wiley.