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Inverted Exponential Distribution Under a Bayesian Viewpoint

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The objective of this study was to examine the properties of Bayes estimators of the parameter, reliability function and hazard rate under the symmetric and asymmetric loss functions for the inverted exponential model. The Bayes predictive interval and the Bayes estimate of shift point are also determined. A simulation study was carried out to study the properties of the Bayes estimators.

Key words: Bayes estimators, LINEX loss function, squared error loss function, prediction limits.

Introduction

The exponential distribution is frequently used in lifetime data analysis, but its suitability is restricted to constant hazard (failure) rates. For situations where a failure rate is monotonically increasing or decreasing, the two-parameter Weibull and the Gamma distributions are popular for analyzing lifetime data. Both distributions have increasing and decreasing hazard rates depending on the shape parameter. However, one of the major disadvantages of the Gamma distribution is that its distribution and survival functions cannot be expressed in a closed form if the shape parameter is not an integer. Moreover, there are terms involving the incomplete Gamma function, thus, it is necessary to obtain distribution, survival or hazard functions by numerical integration. This makes the Gamma distribution less popular compared to the Weibull distribution, which has a closed form for the hazard and survival functions, but the Weibull distribution also has disadvantages. Bain & Engelhardt (1991) demonstrated that the maximum likelihood estimators of the Weibull distribution might not behave properly for all parametric ranges.

Recently two new distributions have been introduced: the generalized Exponential (two - parameter) and the inverted Exponential (one - parameter) distributions. The generalized exponential distribution can be used effectively in situations where a skewed distribution is needed. Gupta & Kundu (1999, 2002) and Raqab & Ahsanullah (2001) investigated several properties of the two parameter generalized exponential distribution.

It is remarkable that most of the Bayesian inference procedures have been developed with the usual squared error loss function (SELF), which is symmetrical and associates equal importance to losses due to overestimation and underestimation of equal magnitude. However, such a restriction may be unrealistic in the most situations of practical importance. For example, in estimating reliability and hazard rate functions, an overestimation is usually much more serious than an underestimation. The use of a symmetrical loss function in Bayesian framework might be inappropriate (Parsian & Kirmani, 2002).

A useful asymmetric loss function known as the LINEX loss function (LLF) was introduced by Varian (1975) and has been used in several studies. The LLF for any parameter θ is given by

$$L\left(\bar{\Delta}\right) = e^{a\bar{\Delta}} - a\,\bar{\Delta} - 1;\ \bar{\Delta} = \hat{\theta} - \theta \qquad (1.1)$$

where $a(\neq 0)$ is the shape parameter and $\hat{\theta}$ is any estimate of the parameter θ.

The sign and magnitude of $'a'$ represents the direction and degree of

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asymmetry respectively. The positive (negative) value of $'a'$ is used when overestimation is more (less) serious than underestimation. The LLF (1.1) is approximately squared error and almost symmetric if $|a|$ is near zero. Many authors have discussed estimation procedures under a LLF criterion, however a few recently presented studies using Bayesian and/or LLF criterions, for example see Xu & Shi (2004), Ahmadi, et al. (2005), Son & Oh (2006), Singh, et al. (2007) and Prakash (2011).

Present article examine the properties of Bayes estimators for the r^{th} power of the parameter θ, reliability function, hazard rate and the shift point. Both the symmetric (SELF) and asymmetric (LLF) loss functions were considered and the behavior of the future observations is predicted in terms of the predictive interval.

The Model and the Prior Distributions

The model considered is the inverted Exponential distribution with a distribution function

$$F(x;\theta)=e^{-1/x\theta};\ x>0,\theta>0. \tag{2.1}$$

This distribution has no finite moments. The reliability function and hazard rate for a specific mission time $t(>0)$ are obtained as

$$\psi(t)=1-e^{-1/t\theta}$$

and

$$\rho(t)=\frac{1}{t^2\theta}\left(e^{1/t\theta}-1\right)^{-1}.$$

If $x_1, x_2, ..., x_n$ are n independent random samples from model (2.1), then the likelihood function is obtained as

$$L\left(x_1,x_2,...,x_n\,|\,\theta\right)=\frac{1}{\theta^2}\prod_{i=1}^{n}x_i^{-2}\exp\left(-\frac{T_n}{\theta}\right),$$

$$T_n=\left\{\sum_{i=1}^{n}x_i^{-1}\right\}. \tag{2.2}$$

The maximum likelihood estimate (MLE) of the parameter θ is $\hat{\theta}=\frac{1}{n}T_n$. Further, $x_i^{-1};\ i=1,2,\quad ...,n$ are iid Exponential with parameter θ, and the distribution of T_n is a Gamma distribution with a probability density function (pdf)

$$f\left(T_n\right)=\frac{T_n^{n-1}}{\Gamma n}e^{-\frac{T_n}{\theta}}\theta^{-n};\ T_n>0. \quad 2.3)$$

It is assumed that, from a Bayesian viewpoint, there is clearly no way in which it can be stated that one prior is better than another (Arnold & Press, 1983). More frequently the case is that attention to a given flexible family of priors is restricted and a prior is chosen from that family. Thus, in present case the conjugate prior of the parameter θ is considered as inverted Gamma distribution and is given as:

$$g_1(\theta)=\frac{\beta^{\alpha}}{\Gamma\alpha}\theta^{-(\alpha+1)}e^{-\frac{\beta}{\theta}};\ \alpha>0,\ \beta>0. \tag{2.4}$$

Further, in a situation where a researcher has no or very little prior information about the parameter θ, a family of priors defined as

$$g_2(\theta)=\theta^{-\delta};\ \delta>0, \tag{2.5}$$

If $\delta=0$ a diffuse prior is obtained, and if $\delta=1$ a non–informative prior is obtained. The posterior density of θ under the prior $g_1(\theta)$ is given by

$$Z_1(\theta)=\frac{\left(T_n+\beta\right)^{\alpha+n}}{\Gamma(\alpha+n)}e^{-\frac{(T_n+\beta)}{\theta}}\theta^{-(\alpha+n+1)}. \tag{2.6}$$

This is an inverted Gamma distribution with parameters $(\alpha+n)$ and $\left(T_n+\beta\right)$. Similarly, the posterior density of θ corresponding to $g_2(\theta)$ is

$$Z_2(\theta) = \frac{(T_n)^{\delta+n-1}}{\Gamma(\delta+n-1)} e^{-\frac{T_n}{\theta}} \theta^{-(\delta+n)}. \quad (2.7)$$

Bayes Prediction Limits

Predicting the nature of the future behavior of an observation when sufficient information regarding the past and present behavior of an event or an observation are known or given is an important problem in lifetime models. Statistical prediction limits have many applications in quality control and reliability problems and the determination of these limits has been extensively investigated. It may be desirable to obtain confidence limits not only for any parameter of a distribution, but also for a future observation drawn from the same model. Such limits are called prediction limits.

If a $100\varepsilon\%$ prediction limit for an additional observation is desired, for example Y, given a random sample $\underline{X} = (x_1, x_2, ..., x_n)$ from model (2.1), the problem is equivalent to determining the region $R(\underline{X})$ such that $R(\underline{X})$ covers the average proportion ε of the distribution of Y.

A wealth of literature is available regarding predictive inference for future failure distributions; examples of studies involving predictive inference for future observations include: Aitchison & Dunsmore (1975), Bain (1978), Sinha (1990), Raqab (1997), Cramer & Kamps (1998), Raqab & Madi (2002), Ahmed et al. (2007) and Prakash & Prasad (2010).

In the context of prediction, it may be stated that (l, u) is a $100(1-\varepsilon)\%$ prediction interval for a future observation Y if

$$\Pr(l \le Y \le u) = 1-\varepsilon; \quad (3.1)$$

where l and u are the lower and upper prediction limits for the random variable Y, and $1-\varepsilon$ is termed the confidence prediction coefficient.

The predicative distribution of a future observation Y may be obtained from model (2.1) by simplifying

$$h(y|\underline{X}) = \int_\theta f(y;\theta)\cdot Z_1(\theta)\, d\theta$$

$$\Rightarrow h(y|\underline{X}) = (n+\alpha)y^{-2} \frac{(T_n+\beta)^{n+\alpha}}{(T_n+\beta+y^{-1})^{n+\alpha+1}}, \quad (3.2)$$

and $100(1-\varepsilon)\%$ equal tail prediction interval is obtained by solving

$$\int_0^l h(y|\underline{X})\, dy = \int_u^\infty h(y|\underline{X})\, dy = \frac{\varepsilon}{2}. \quad (3.3)$$

Hence, the Bayes prediction limits and length of the Bayes predictive interval are obtained as

$$l = \left[(T_n+\beta)\left\{\left(\frac{\varepsilon}{2}\right)^{-1/(\alpha+n)} - 1\right\}\right]^{-1}, \quad (3.4)$$

$$u = \left[(T_n+\beta)\left\{\left(1-\frac{\varepsilon}{2}\right)^{-1/(\alpha+n)} - 1\right\}\right]^{-1} \quad (3.5)$$

and

$$I = u - l. \quad (3.6)$$

Bayes Estimators for Reliability Function and Hazard Rate

The Bayes estimates of $\psi(t)$ and $\rho(t)$ under the SELF corresponding to the posterior $Z_1(\theta)$ are obtained as

$$\psi_1 = E_P(\psi(t)) = 1 - \left(1 + \frac{1}{t(T_n+\beta)}\right)^{-(\alpha+n)} \quad (4.1)$$

and

$$\rho_1 = E_P(\rho(t)) = \frac{1}{t^2} I(0, \infty, \rho_{S1});$$

$$\rho_{S1} = z(e^{z/t} - 1)^{-1}, \quad (4.2)$$

where

$$I(z_1, z_2, f_z) = \frac{(T_n + \beta)^{\alpha+n}}{\Gamma(\alpha+n)} \int_{z_1}^{z_2} f_z \cdot e^{-(T_n+\beta)z} z^{\alpha+n-1} dz.$$

Here f_z is a function of z and suffix P indicates the expectation taken under posterior density.

Similarly, the Bayes estimators of the reliability function and hazard rate under the LLF-criterion corresponding to the posterior $Z_1(\theta)$ are obtained by solving

$$\psi_2 = -\frac{1}{a}\ln E_P\left(e^{-a\psi(t)}\right)$$
$$= -\frac{1}{a}\ln\left(e^{-a} I(0, \infty, \psi_{L1})\right),$$
$$\psi_{L1} = \exp\left(ae^{-z/t}\right)$$

and

$$\rho_2 = -\frac{1}{a}\ln E_P\left(e^{-a\rho(t)}\right)$$
$$= -\frac{1}{a}\ln I\ (0, \infty, \rho_{L1}),$$
$$\rho_{L1} = \exp\left(\frac{a}{t^2}\rho_{S1}\right).$$

The expressions of the risks for these estimators under the SELF and the LLF loss criterions are $R_{(S)}(\psi_i)$, $R_{(L)}(\psi_i)$, $R_{(S)}(\rho_i)$ and $R_{(L)}(\rho_i)$; $i = 1, 2$. Note that these do not exist in closed form. However, a numerical study has been carried out in later section.

The Bayes Estimator for Shift Point

In order to obtain information about their endurance, manufactured items such as mechanical or electronic components, are often put to life tests and life times are observed periodically. Physical systems manufacturing different items are often subject to random fluctuations and it may happen that, at some point, there is a change in the parameter. The objective of this study was to determine when and where this change starts occurring; this is called the shift point inference problem. Bayesian modeling may play an important role in the study of such shift point problems (Broemeling & Tsurumi, 1987; Jani & Pandya, 1999).

Consider first a sequence of independent random sample of size n such as x_1, $x_2, ..., x_m, x_{m+1}, ..., x_n$ from model (2.1) with a reliability function $\psi_1(t)$ at mission time $t(>0)$. If it is later found that there was a change in the system at some point in time m, this will be reflected in the sequence after x_m by a change in the reliability $\psi_2(t)$ at mission time t.

Thus, from model (2.1), the pdf of the random samples $x_1, x_2, ..., x_m$ of size m is given by

$$f(x_i; \theta_1) = \frac{1}{\theta_1 x_i^2}\exp\left(-\frac{1}{\theta_1 x_i}\right); \quad i = 1, 2, ..., m, \theta_1 > 0. \tag{5.1}$$

Similarly, the remaining $x_{m+1}, x_{m+2}, ..., x_n$ components of size $(n-m)$ follow model (2.1) with the pdf

$$f(x_i; \theta_2) = \frac{1}{\theta_2 x_i^2}\exp\left(-\frac{1}{\theta_2 x_i}\right); \quad i = m+1, m+2, ..., n, \theta_2 > 0. \tag{5.2}$$

If prior information regarding the parameter is considered as the conjugate prior, then prior $g_1(\theta)$ is redefined as

$$g_3(\theta_i) = \frac{\beta^\alpha}{\Gamma\alpha}\theta_i^{-(\alpha+1)} e^{-\frac{\beta}{\theta_i}}; \quad \alpha > 0,\ \beta > 0, i = 1, 2. \tag{5.3}$$

Further, the prior distribution for shift point m is considered to be discrete uniform over the set $\{1, 2, ..., n-1\}$. Hence, the joint posterior density for the parameters θ_1, θ_2 and m is

$$Z_3(\theta_1, \theta_2, m) = k^{-1}\exp\left(-\frac{\omega_1}{\theta_1} - \frac{\omega_2}{\theta_2}\right)\theta_1^{-m-\alpha-1}\theta_2^{-n+m-\alpha-1}; \tag{5.4}$$

where

$$k^{-1} = \sum_{m=1}^{n-1}\Delta,$$

$$\Delta = \left(\frac{\Gamma(m+\alpha)\,\Gamma(n-m+\alpha)}{\omega_1^{m+\alpha}\ \omega_2^{n-m+\alpha}}\right),$$

$$\omega_1 = \beta + \sum_{i=1}^{m} x_i^{-1}$$

and

$$\omega_2 = \beta + \sum_{i=m+1}^{n} x_i^{-1}.$$

This case may be verified without considering shift point situations with $\theta_1 = \theta_2$.

The marginal posterior density for shift point m is

$$Z_4(m) = k^{-1}\Delta. \tag{5.5}$$

Therefore, the Bayes estimator for shift point m under the SELF and LLF are obtained respectively as (suffixes S and L indicates the loss criterion selected as the SELF and LLF respectively)

$$\hat{m}_S = k^{-1}\sum_{m=1}^{n-1}(m\Delta)$$

and

$$\hat{m}_L = -\frac{1}{a}\ln\left\{k^{-1}\sum_{m=1}^{n-1}\left(e^{-am}\Delta\right)\right\}. \tag{5.6}$$

If no further information regarding $\theta_i;\ i = 1, 2$ is available and they are assumed as *a priori* independent random variables, then the non-informative prior is considered from (2.7) with $(\delta = 1)$ such that

$$g_4(\theta_i) = \frac{1}{\theta_i};\ i=1, 2,$$

The Bayes estimators for shift point m under SELF and LLF are obtained from (5.6) by replacing $\beta = 0 = \alpha$ as:

$$\hat{m}'_S = k_1^{-1}\sum_{m=1}^{n-1}(m\Delta')$$

and

$$\hat{m}'_L = -\frac{1}{a}\ln\left\{k_1^{-1}\sum_{m=1}^{n-1}\left(e^{-am}\Delta'\right)\right\}. \tag{5.7}$$

where

$$k_1^{-1} = \sum_{m=1}^{n-1}\Delta',$$

$$\Delta' = \frac{\Gamma m\Gamma(n-m)}{\omega_3^{m}\ \omega_4^{n-m}},$$

$$\omega_3 = \sum_{i=1}^{m} x_i^{-1}$$

and

$$\omega_4 = \sum_{i=m+1}^{n} x_i^{-1}.$$

The Bayes Estimator for Parameter θ

The Bayes estimator for θ^r (r being any integer) obtained corresponding to the posterior $Z_1(\theta)$ under the SELF is

$$\hat{\theta}_S^r = \frac{\Gamma(n+\alpha-r)}{\Gamma(n+\alpha)}\left(T_n+\beta\right)^r. \tag{6.1}$$

In particular, the Bayes estimators for the parameters $\theta (r=1)$ and $\frac{1}{\theta}(r=-1)$ are given respectively as

$$\hat{\theta}_S = \frac{T_n+\beta}{n+\alpha-1}$$

and

$$\hat{\theta}_S^{-1} = \frac{n+\alpha}{T_n + \beta} \tag{6.2}$$

Similarly, the Bayes estimator for θ^r under the LLF is obtained with respect to the posterior $Z_1(\theta)$ by solving

$$\hat{\theta}_L^r = -\frac{1}{a} \ln I\left(0, \infty, e^{-a\theta^r}\right). \tag{6.3}$$

The Bayes estimator for parameter θ does not exist in a closed form. However, the Bayes estimator for $\frac{1}{\theta}$ is given as

$$\hat{\theta}_L^{-1} = \left(\frac{n+\alpha}{a}\right) \ln\left(1 + \frac{a}{T_n + \beta}\right). \tag{6.4}$$

Note that, all results discussed thus hold for the posterior distribution $Z_2(\theta)$ if $\alpha(=\delta - 1)$ and $\beta\ (= 0)$ are substituted.

Numerical Analysis

To assess and study the properties of the proposed Bayes estimators and prediction interval, the random samples are generated as follows:

1. For the given values of prior parameters α and β, generate θ using the prior density $g_1(\theta)$. The values of α and β are chosen to maintain the prior variance at 1.00 and the considered values are $(\beta, \alpha) =$ (02, 03), (10, 06) and (30, 11).

2. Using θ obtained in (1), generate 10,000 random samples size n = 5, 10, 15 from the considered model (2.1).

Bayes Prediction Interval

The Bayes prediction intervals were obtained with the level of significance $\varepsilon = 99\%, 95\%, 90\%$ and results are presented in Table 1. The intervals tend to be wider as the sample size n increases when other parametric values are fixed. The opposite trend was observed when a combination of the prior parameters increases. It is also noted that when the confidence level decreases the intervals also decrease.

Bayes Estimators for Reliability Function

Results for $a = 0.50, 1.00, 1.50$ and $t = 2.50$ (hours) are presented in Table 2. As Table 2 shows, the risk of Bayes estimators ψ_1 and ψ_2 decrease as sample size n increases under both loss criteria, SELF and LLF. In addition, the risk of ψ_1 increases as 'a' increases under a LLF loss criterion. A similar trend was observed for ψ_2 when 'a' increases under both loss criteria. A decreasing trend in risk was observed when a set of prior parameters increased only for ψ_1 under both loss criterions with other fixed parametric values. The magnitude of the risk is nominal for both estimators under the LLF.

Bayes Estimators for Hazard Rate

The numerical findings are presented in Table 3 for a similar set of values of 'a' and t. The performances of Bayes estimators ρ_1 and ρ_2 are similar to the Bayes estimators of the reliability functions ψ_1 and ψ_2 when sample size n or 'a' increase respectively. The magnitude of the risk is nominal for both estimators under the LLF loss criterion.

Bayes Estimators for Shift Point

For a similar set of values considered earlier with $a = 0.25, 0.50, 1.00, 1.50,$ samples were generated for $n = 10, 15, 20$ and results are presented in Tables 4 and 5. It was observed that, when sample size n increases, the magnitude of the Bayes estimator (under SELF) increases but the increment in magnitude is nominal (robust). Further, an opposite trend was observed when values of the set of prior parameters increase. Similar properties have been noted for the Bayes estimate of the shift point under LLF, and a decreasing trend in the magnitude of the estimate has also been observed when 'a' increases.

Bayes Estimators for the r^{th} Power of the Parameter

The numerical findings presented in Tables 6 and 7 are for $a = 0.50, 1.00, 1.50$ and $r = \pm 1, \pm 2$. Based on results show in the tables it may be concluded that the magnitude of the risk increases (decreases) when $a(n)$ increases when other parametric values are fixed. The increasing trend in the magnitude has also been observed when prior parameters increase (only for $r = -1, -2$). Further, the magnitude of the risk is smaller for these estimators under the LLF.

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Table 1: Bayes Prediction Limits

n	β, α	$\varepsilon = 99\%$		$\varepsilon = 95\%$		$\varepsilon = 90\%$	
		l	u	l	u	l	u
05	02,03	1.2272	1.2648	1.2662	1.4722	1.2825	1.7349
	10,06	0.8442	0.8697	0.9292	1.0784	0.8212	1.1070
	30,11	0.5941	0.6118	0.5458	0.6326	0.5265	0.7077
10	02,03	2.2409	2.3083	2.0216	2.3447	1.3851	1.8645
	10,06	1.3329	1.3727	1.2600	1.4604	1.0571	1.4209
	30,11	0.8032	0.8271	0.7453	0.8631	0.6920	0.9287
15	02,03	2.0570	2.1284	2.7406	3.1751	1.7060	2.2914
	10,06	1.4661	1.5097	1.4084	1.6311	1.3485	1.8098
	30,11	0.8807	0.9068	0.9548	1.1052	0.7790	1.0445

Table 2: Risks for Bayes Estimate of Reliability Function

n	a	β, α	ψ_1	$R_S(\psi_1)$	$R_L(\psi_1)$	ψ_2	$R_S(\psi_2)$	$R_L(\psi_2)$
05	0.50	02,03	0.1125	14.7296	1.0523	0.0123	14.7012	1.0502
		10,06	0.0903	14.7612	1.0541	0.0780	14.8141	1.0607
		30,11	0.0584	14.8300	1.0578	0.0055	14.2740	1.0266
	1.00	02,03	0.1059	14.7296	2.8066	0.2646	14.7132	2.8041
		10,06	0.0818	14.7612	2.8106	0.1418	14.8245	2.8180
		30,11	0.0559	14.8300	2.8191	0.0567	14.5478	2.7834
	1.50	02,03	0.1190	14.7296	4.6586	0.2088	14.7194	4.6564
		10,06	0.0847	14.7612	4.6647	0.1302	14.8335	4.6724
		30,11	0.0561	14.8300	4.6777	0.0339	14.6418	4.6416
10	0.50	02,03	0.1827	5.9753	0.4270	0.8166	6.0077	0.4283
		10,06	0.1179	5.9827	0.4274	1.3328	5.8511	0.4193
		30,11	0.0768	6.0081	0.4288	0.5438	5.1131	0.3785
	1.00	02,03	0.1348	5.9753	1.1389	0.2641	6.0863	1.1399
		10,06	0.1057	5.9827	1.1398	0.6477	5.9076	1.1298
		30,11	0.0729	6.0081	1.1429	0.2365	5.5525	1.0852
	1.50	02,03	0.1668	5.9753	1.8905	0.0643	6.1807	1.8912
		10,06	0.1119	5.9827	1.8919	0.5157	5.9292	1.8813
		30,11	0.0734	6.0081	1.8967	0.0950	5.7037	1.8382
15	0.50	02,03	0.1932	0.4478	0.0321	1.5324	0.4472	0.0372
		10,06	0.1434	0.4489	0.0321	1.0564	0.5884	0.0393
		30,11	0.0856	0.4517	0.0323	0.8032	0.3951	0.0291
	1.00	02,03	0.2639	0.4478	0.0856	0.3801	0.4658	0.0913
		10,06	0.1468	0.4489	0.0857	0.3046	0.6159	0.0938
		30,11	0.0824	0.4517	0.0861	0.2365	0.4217	0.0822
	1.50	02,03	0.2005	0.4478	0.1421	0.7962	0.4796	0.1480
		10,06	0.1315	0.4489	0.1423	0.2168	0.6929	0.1505
		30,11	0.0870	0.4517	0.1429	0.0306	0.4310	0.1388

Table 3: Risks for Bayes Estimate of Hazard Rate

n	a	β, α	ρ_1	$R_S(\rho_1)$	$R_L(\rho_1)$	ρ_2	$R_S(\rho_2)$	$R_L(\rho_2)$
05	0.50	02,03	0.1181	14.1954	1.0230	0.3540	14.1651	1.0208
		10,06	0.1238	14.1827	1.0223	0.0195	14.2119	1.0294
		30,11	0.1398	14.2215	1.0244	0.1485	13.6402	0.9914
	1.00	02,03	0.1411	14.1954	2.7397	0.1463	14.1760	2.7368
		10,06	0.1216	14.1827	2.7381	0.3392	14.2227	2.7464
		30,11	0.1291	14.2215	2.7430	0.1184	13.9077	2.7026
	1.50	02,03	0.1157	14.1954	4.5564	0.0612	14.1813	4.5534
		10,06	0.1148	14.1827	4.5540	0.2712	14.2315	4.5631
		30,11	0.1317	14.2215	4.5615	0.1289	13.9996	4.5181
10	0.50	02,03	0.1944	5.7601	0.4152	0.0918	5.7696	0.4169
		10,06	0.0770	5.7694	0.4157	1.2395	5.6419	0.4077
		30,11	0.1676	5.8155	0.4182	0.3497	4.8807	0.3650
	1.00	02,03	0.1936	5.7601	1.1119	0.1672	5.7782	1.1138
		10,06	0.0883	5.7694	1.1130	0.6489	5.6963	1.1031
		30,11	0.1692	5.8155	1.1189	0.1737	5.3099	1.0537
	1.50	02,03	0.1841	5.7601	1.8493	0.1399	5.7825	1.8514
		10,06	0.0745	5.7694	1.8510	0.4315	5.7171	1.8404
		30,11	0.1730	5.8155	1.8599	0.0694	5.4577	1.7899
15	0.50	02,03	0.1314	0.4286	0.0310	1.1623	0.5327	0.0364
		10,06	0.1164	0.4258	0.0308	0.3103	0.4731	0.0386
		30,11	0.1303	0.4386	0.0316	0.5889	0.3813	0.0283
	1.00	02,03	0.2328	0.4286	0.0832	0.8862	0.5482	0.0897
		10,06	0.1980	0.4258	0.0828	0.0166	0.5017	0.0921
		30,11	0.1870	0.4386	0.0844	0.0024	0.4074	0.0804
	1.50	02,03	0.0677	0.4286	0.1384	0.5436	0.5566	0.1455
		10,06	0.1858	0.4258	0.1379	0.3606	0.5479	0.1479
		30,11	0.1560	0.4386	0.1403	0.0739	0.4165	0.1360

Table 4: Bayes Estimate of Shift Point Under SELF

Prior Density	(β, α) ↓ n →	10	15	20
Conjugate	02,03	4.9919	5.0069	5.0169
	10,06	4.9493	4.9641	4.9740
	30,11	4.4590	4.4724	4.4813
Non-Informative	00,00	4.8946	4.9112	4.9472

Table 5: Bayes Estimate of Shift Point Under LLF

Prior Density	n	(β, α) ↓ a →	0.25	0.50	1.00	1.50
Conjugate Prior	10	02,03	4.0051	3.2058	3.1096	3.0163
		10,06	3.7951	3.1965	3.1006	3.0076
		30,11	3.7196	3.1806	3.0852	2.9926
	15	02,03	4.2321	3.6035	3.4954	3.3905
		10,06	3.8554	3.2342	3.1372	3.0431
		30,11	3.7907	3.2191	3.1225	3.0288
	20	02,03	4.5051	3.7436	3.6313	3.5224
		10,06	3.8615	3.3673	3.2663	3.1683
		30,11	3.8356	3.3249	3.2252	3.1284
Non-Informative Prior	10	00,00	3.9477	3.2412	3.1440	3.0497
	15	00,00	4.0255	4.0184	3.9657	3.8467
	20	00,00	4.0926	4.0783	4.0424	3.9502

Table 6: Risks for the Bayes Estimate of θ^r

n	a	β, α	r = −2				r = 2			
			$R_{(S)}(\hat{\theta}_S^r)$	$R_{(L)}(\hat{\theta}_S^r)$	$R_{(S)}(\hat{\theta}_L^r)$	$R_{(L)}(\hat{\theta}_L^r)$	$R_{(S)}(\hat{\theta}_S^r)$	$R_{(L)}(\hat{\theta}_S^r)$	$R_{(S)}(\hat{\theta}_L^r)$	$R_{(L)}(\hat{\theta}_L^r)$
05	0.50	02,03	13.288	0.9700	13.343	0.9726	10.285	4.5467	1.6360	1.9821
		10,06	13.719	0.9961	13.804	1.0061	11.384	7.0496	0.7671	1.5262
		30,11	14.246	1.0258	13.706	0.9949	14.270	9.1108	2.0452	2.9854
	1.00	02,03	13.288	2.6178	13.440	2.6363	10.285	6.0364	2.3876	2.5377
		10,06	13.719	2.6776	13.816	2.6959	11.384	9.8003	1.6753	2.2181
		30,11	14.246	2.7460	13.981	2.7115	14.270	9.7628	5.7040	3.6989
	1.50	02,03	13.288	4.4315	13.518	4.4154	10.285	8.4251	7.7043	2.8013
		10,06	13.719	4.4615	13.859	4.4899	11.384	11.112	6.9306	2.4745
		30,11	14.246	4.5661	14.082	4.5336	14.270	10.550	5.9482	4.2190
10	0.50	02,03	5.4663	0.3986	5.4705	0.3995	3.9248	0.9999	0.1222	0.0180
		10,06	5.5413	0.4029	5.3734	0.3929	3.6741	0.9096	0.2512	0.0331
		30,11	5.7225	0.4131	4.8579	0.3637	9.1533	3.4073	0.6589	0.8112
	1.00	02,03	5.4663	1.0738	5.4805	1.0754	3.9248	1.2036	0.1302	0.0354
		10,06	5.5413	1.0838	5.4371	1.0701	3.6741	8.8628	0.2709	0.0474
		30,11	5.7225	1.1071	5.2914	1.0512	9.1533	4.6310	0.6659	0.8903
	1.50	02,03	5.4663	1.3599	5.4858	1.7949	3.9248	2.0292	0.1518	0.1044
		10,06	5.5413	1.8063	5.4661	1.7915	3.6741	9.0630	0.2838	0.0675
		30,11	5.7225	1.8419	5.4438	1.7871	9.1533	5.4232	0.7181	0.9334
15	0.50	02,03	0.3864	0.0286	0.4302	0.0342	0.0087	0.0009	0.0368	0.0038
		10,06	0.3975	0.0292	0.4434	0.0364	0.0045	0.0005	0.0443	0.0045
		30,11	0.4208	0.0306	0.3550	0.0267	0.1127	0.0203	0.0034	0.0004
	1.00	02,03	0.3864	0.0666	0.4416	0.0847	0.0087	0.0028	0.0462	0.0097
		10,06	0.3975	0.0791	0.4818	0.0873	0.0045	0.0017	0.0628	0.0104
		30,11	0.4208	0.0822	0.3807	0.0769	0.1127	0.1251	0.0045	0.0019
	1.50	02,03	0.3864	0.1225	0.4565	0.1379	0.0087	0.0054	0.0525	0.0182
		10,06	0.3975	0.1322	0.5402	0.1406	0.0045	0.0034	0.0737	0.0178
		30,11	0.4208	0.1369	0.3900	0.1307	0.1127	0.4628	0.0070	0.0062

Table 7: Risks for the Bayes Estimate of θ^r

n	a	β, α	r = −1				r = 1			
			$R_{(S)}(\hat{\theta}_S^r)$	$R_{(L)}(\hat{\theta}_S^r)$	$R_{(S)}(\hat{\theta}_L^r)$	$R_{(L)}(\hat{\theta}_L^r)$	$R_{(S)}(\hat{\theta}_S^r)$	$R_{(L)}(\hat{\theta}_S^r)$	$R_{(S)}(\hat{\theta}_L^r)$	$R_{(L)}(\hat{\theta}_L^r)$
05	0.50	02,03	12.002	0.8972	12.029	0.8983	2.0144	0.1875	2.3556	0.2156
		10,06	12.224	0.9110	12.332	0.9193	1.6934	0.1622	2.1786	0.2043
		30,11	12.723	0.9401	12.246	0.9116	0.7526	0.0784	0.7404	0.0763
	1.00	02,03	12.002	2.4474	12.082	2.4583	2.0144	0.5928	2.7107	0.7636
		10,06	12.224	2.4806	12.333	2.4953	1.6934	0.5180	2.4282	0.7162
		30,11	12.723	2.5485	12.510	2.5191	0.7526	0.2664	1.0455	0.3536
	1.50	02,03	12.002	4.1081	12.129	4.1356	2.0144	1.1044	3.0264	1.5302
		10,06	12.224	4.1595	12.337	4.1834	1.6934	0.9617	2.6799	1.4354
		30,11	12.723	4.2639	12.610	4.2399	0.7526	0.5180	1.3107	0.8213
10	0.50	02,03	4.8368	0.3626	4.8418	0.3629	0.9737	0.0933	1.2338	0.1153
		10,06	4.8905	0.3657	4.7321	0.3559	0.9049	0.0880	1.1338	0.1057
		30,11	5.0740	0.3764	4.2866	0.3297	0.5090	0.0525	0.2913	0.0298
	1.00	02,03	4.8368	0.9901	4.8420	0.9903	0.9737	0.2977	1.3538	0.3923
		10,06	4.8905	0.9976	4.7943	0.9843	0.9049	0.2833	1.2602	0.3689
		30,11	5.0740	1.0225	4.6961	0.9708	0.5090	0.1769	0.4723	0.1598
	1.50	02,03	4.8368	1.6626	4.8461	1.6646	0.9737	0.5517	1.4573	0.7545
		10,06	4.8905	1.6742	4.8244	1.6604	0.9049	0.5279	1.3707	0.7208
		30,11	5.0740	1.7125	4.8414	1.6639	0.5090	0.3412	0.6191	0.3880
15	0.50	02,03	0.3405	0.0259	0.3434	0.0312	0.1351	0.0121	0.1536	0.0164
		10,06	0.3488	0.0264	0.4113	0.0333	0.1200	0.0110	0.2155	0.0179
		30,11	0.3695	0.0276	0.3085	0.0239	0.0756	0.0073	0.0843	0.0080
	1.00	02,03	0.3405	0.0713	0.3881	0.0778	0.1351	0.0369	0.1644	0.0427
		10,06	0.3488	0.0725	0.4367	0.0803	0.1200	0.0338	0.2174	0.0445
		30,11	0.3695	0.0754	0.3326	0.0702	0.0756	0.0236	0.1025	0.0298
	1.50	02,03	0.3405	0.1203	0.3939	0.1273	0.1351	0.0659	0.1861	0.0728
		10,06	0.3488	0.1221	0.4865	0.1300	0.1200	0.0608	0.2617	0.0746
		30,11	0.3695	0.1265	0.3414	0.1205	0.0756	0.0438	0.1117	0.0578

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