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Marcelo Angelo Cirillo
Federal University of Lavras, Brazil, macufla@gmail.com

Lúcia Pereira Barroso
University of São Paulo, Brazil

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Robust Regression Estimates in the Prediction of Latent Variables in Structural Equation Models

Marcelo Angelo Cirillo
Federal University of Lavras,
Lavras, Brazil

Lúcia Pereira Barroso
University of São Paulo,
São Paulo, Brazil

The incorporation of the robust regression methods Least Median Square (LMS) and Least Trimmed Squares (LTS) is proposed in structural equation modeling. Results show that, in situations of high deviations of symmetry, the evaluated methods would be recommended for applications including smaller sample sizes.

Key words: Accuracy, Monte Carlo simulation, normal asymmetry, precision.

Introduction

Structural equation modeling seeks to reproduce a population covariance matrix through the sample covariance associated with parameter constraints determined by a researcher. The most commonly used estimation methods are based on maximum likelihood, ordinary least squares and partial least squares. These methods, however, show a sensitivity to various factors, for example: violation of the normality assumption (Lei & Lomax, 2005), presence of outliers (Yuan & Bentler, 2001) and samples that show the effects of asymmetry and excess kurtosis compared to the multivariate normal distribution (Gao, et al., 2008).

It is reasonable to assume that the presence of these effects can be explained by observing outliers in the sample and it is also relevant to consider sample size because large data sets are subject to a large number of these observations. Moreover, the violation of the assumption of normality can also be caused by these observations; therefore, a practical (but not always feasible) alternative is to apply a transformation to the data. Gao, et al. (2008) state that – depending on the transformation to be used – the relationships between variables may be nonlinear. However, applying this statement to structural equation models, where the nature of the relationships between the variables are linear, the application of a transformation may complicate the interpretation of results as well as affect the quality of model fit.

Pilati and Laros (2007) state that, if the assumption of normality is not met, the researcher may choose to use other estimation methods such as the method of partial least squares (PLS) estimators. According to O'Loughlin & Coenders (2004) the disadvantage of this method is that they provide biased estimates. Cassel, et al. (1999) note that the properties of robustness must be further studied due to the fact that the estimates are sensitive to deviations from symmetry in observable variables, multicollinearity among observable variables and incorrect specification of measurement models.

As noted, not all variable distributions are normal and – in the presence of outliers – not all are appropriate to use for robust regression estimators that would be incorporated in structural equation models (SEMs). However, robust regression estimators that have a high breaking point are constructed based on the size of the subsets of data, thus, Hawkins, et al. (1984) state that this size should be minimal,

Marcelo Cirillo is Adjunct Lecturer IV in the Exact Sciences Department. E-mail him at: macufla@dex.ufla.br. Lúcia Barroso is Associate Professor in the in the Institute of Mathematics and Statistics. E-mail her at: lbarroso@ime.usp.br.

that is, the total number of observations belonging to each set should be equal to the number of model parameters. This recommendation is justified due to the fact that the proportion of subsets that will contain at least one observational discrepancy will grow rapidly as a function of the size of the subset. For this reason, by keeping the subsets as small as possible the number of estimates that are not influenced by outliers is maximized.

As a base for this theory, the method of Least Median of Squares (LMS) is cited (Gervini & Yohai, 2002). This method involves minimizing the sum of squares of the ordinary residuals obtained by the method in such a way that the subset formed from the residuals resulting in the lowest median value are considered for the minimization processes. Other robust regression estimators, adequate for incorporation in structural equation modeling (SEM) are defined by the Least Trimmed Squares (LTS) method (Agulló, 2001). This method proposes to minimize the sum of the first *h* residuals sorted and squared, and *h* the size of the subset defined by the number of model parameters and sample size (Maronna, et al., 2006).

This article incorporates robust estimators LMS and LTS in structural SEM. For this purpose, the performance of the incorporation of these estimators was evaluated by Monte Carlo simulation considering different degrees of asymmetry in the distributions of the observable variables and errors of the structural and measurement models.

Methodology

Error Distribution of the Structural Equation Model using Monte Carlo Simulation

Following the definition of a structural equation model (SEM), structural error and measurement models were generated following the asymmetric standard normal distribution with probability functions defined respectively as

$$f_{\zeta} = 2\phi(\zeta)\Phi(a\zeta),\ \zeta \in \Re,\ a \in \Re; \quad (1)$$

$$f_{\delta} = 2\phi(\delta)\Phi(a\delta),\ \delta \in \Re,\ a \in \Re; \quad (2)$$

and

$$f_{\varepsilon} = 2\phi(\varepsilon)\Phi(a\varepsilon),\ \varepsilon \in \Re,\ a \in \Re, \quad (3)$$

where $\phi(.)$ and $\Phi(.)$ represent the probability density function and standard normal cumulative distribution N(0,1) and *a* is the asymmetry parameter. Thus, the notation $\zeta \sim N(0, 1, a)$, $\delta \sim N(0,1,a)$ and $\varepsilon \sim N(0,1,a)$ was used. Arbitrarily, the values were set at $a = 20$, 0 and -20 implying the errors presented positive asymmetry, symmetry, and negative asymmetry, respectively; these situations are illustrated in Figures 1-3.

Figure 1: ζ, δ, ε~ N(0, 1, 20) Error Histogram

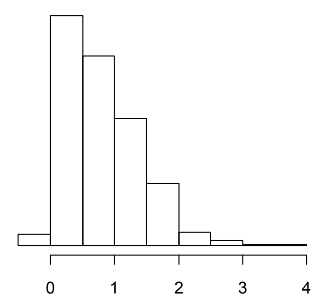


Figure 2: ζ, δ, ε~ N(0, 1) Error Histogram

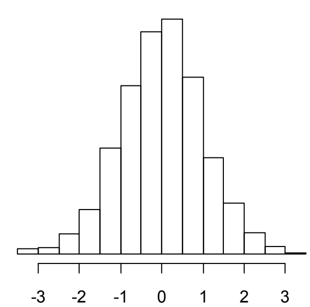
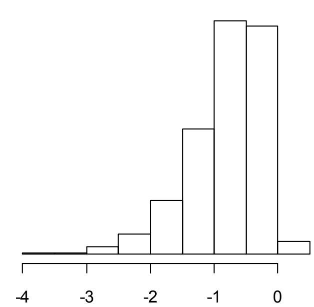


Figure 3: ζ, δ, ε~ N(0, 1, −20) Error Histogram



Structural Equation Models Used in the Monte Carlo Simulation

Keeping the specifications of the error distributions consistent with the proposed objective in the Monte Carlo simulation process, the structural model

$$\eta = \gamma_1 \xi_1 + \gamma_2 \xi_2 + \gamma_3 \xi_3 + \zeta, \tag{4}$$

where γ_i for i = 1, 2, 3 represents the regression coefficients and ζ is the structural error with distribution is N(0,1,*a*) was considered. The observable variables were defined by x_k (k = 1, ..., 9), such that each equation of the measurement model at x was composed by

$$\xi_1 = \pi_1 x_1 + \pi_2 x_2 + \pi_3 x_3 + \delta_1, \tag{5}$$

$$\xi_2 = \pi_4 x_4 + \pi_5 x_5 + \pi_6 x_6 + \delta_2, \tag{6}$$

$$\xi_3 = \pi_7 x_7 + \pi_8 x_8 + \pi_9 x_9 + \delta_3. \tag{7}$$

and where δ_i (i = 1, 2, 3) was generated by a normal distribution.

In the case of the measurement model for η, the definition of the equations was given as:

$$y_1 = \lambda_1 \eta + \varepsilon_1, \tag{8}$$

$$y_2 = \lambda_2 \eta + \varepsilon_2, \tag{9}$$

$$y_3 = \lambda_3 \eta + \varepsilon_3, \tag{10}$$

and

$$y_4 = \lambda_4 \eta + \varepsilon_4. \tag{11}$$

Based on these equations, along with assumptions of the structural model in which the expected values of the error vectors and latent variables are equal to zero, ζ and ξ_i, (i = 1, 2, 3) are uncorrelated, ε_j (j = 1, 2, 3, 4) are uncorrelated with η, ξ_i and δ_i, being δ_i uncorrelated with ξ_i, η and ε_j. With these specifications, the regression coefficient estimators were obtained by robust LMS and LTS methods and the sample generation was characterized by different degrees of asymmetry for x observable variables distributed as a Beta (α, β) distribution, whose parameters were fixed in accordance with Table 1.

Table 1: Beta (α, β) Distribution Used in the Simulation of the Observable Variables with Different Degrees of Asymmetry

Case	Distribution	Degree of Asymmetry
0	Beta (6, 6)	Perfectly Symmetrical
1	Beta (9, 4)	Moderate Asymmetry to the Right
2	Beta (9, 1)	High Asymmetry to the Right

The illustration of each case is shown in Figures 4-6 and the parametric values used in the simulation of the structural model are shown in Figure 7.

Figure 4: Beta (6, 6)

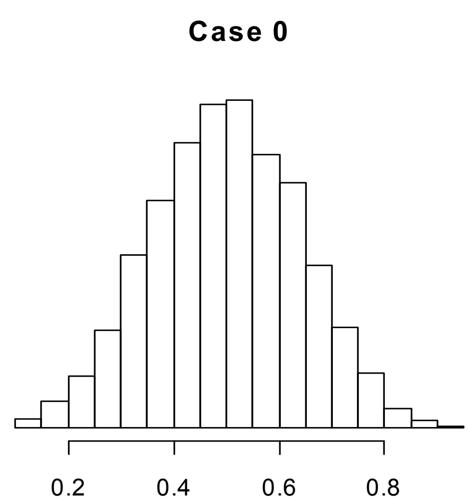


Figure 5: Beta (9, 4)

Case 1

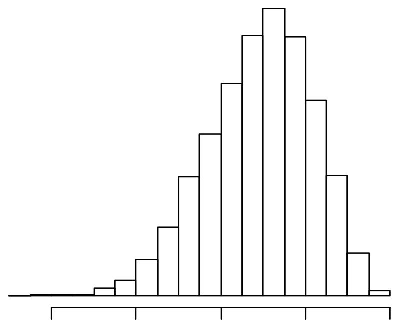
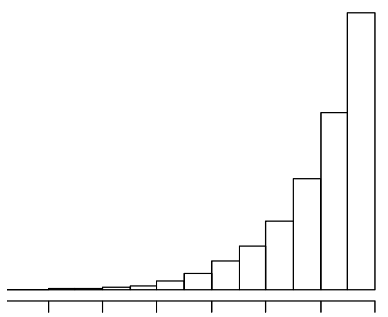


Figure 6: Beta (9, 1)

Case 2



Robust Regression Estimators Incorporated into the Structural Equation Model

The LMS estimators (Rousseeuw & Leroy, 2003) applied in SEM were incorporated considering structural model (4) and the measurement models related to the independent (models (5)-(7)) and the dependent (models (8)-(11)) variables. For sample sizes fixed at n = 50, 200 and 1,000, estimates were obtained by minimizing equations (12)-(14). Thus, with the exception of the structural model, each of the measurement model equations was specified by the index s = 1, ..., Q, where Q is the total number of equations and the parameter estimates of each equation were obtained by the LMS method.

$$\min\left\{\text{median}\left(\zeta_1^2,\ldots,\zeta_n^2\right)\right\}, \tag{12}$$

$$\min\left\{\text{median}\left(\varepsilon_{s1}^2,\ldots,\varepsilon_{sn}^2\right)\right\},\ s=1,\ldots,(Q=4), \tag{13}$$

and

$$\min\left\{\text{median}\left(\delta_{s1}^2,\ldots,\delta_{sn}^2\right)\right\} s=1,\ldots,(Q=3). \tag{14}$$

In the case of the LTS method (Rousseeuw & Leroy, 2003) the parameter estimates were obtained from expressions:

$$\min\left\{\sum_{r=1}^{h}\zeta_r^2,\ldots,\zeta_h^2\right\}; \tag{15}$$

$$\min\left\{\sum_{r=1}^{h}\varepsilon_{sr}^2,\ldots,\varepsilon_{sh}^2\right\};\ s=1,\ldots,(Q=4); \tag{16}$$

and

$$\min\left\{\sum_{r=1}^{h}\left(\delta_{sr}^2,\ldots,\delta_{sh}^2\right)\right\};\ s=1,\ldots,(Q=3). \tag{17}$$

In all situations, the values of *h* were recommended by Rousseeuw and Hubert (1994), according to the expression:

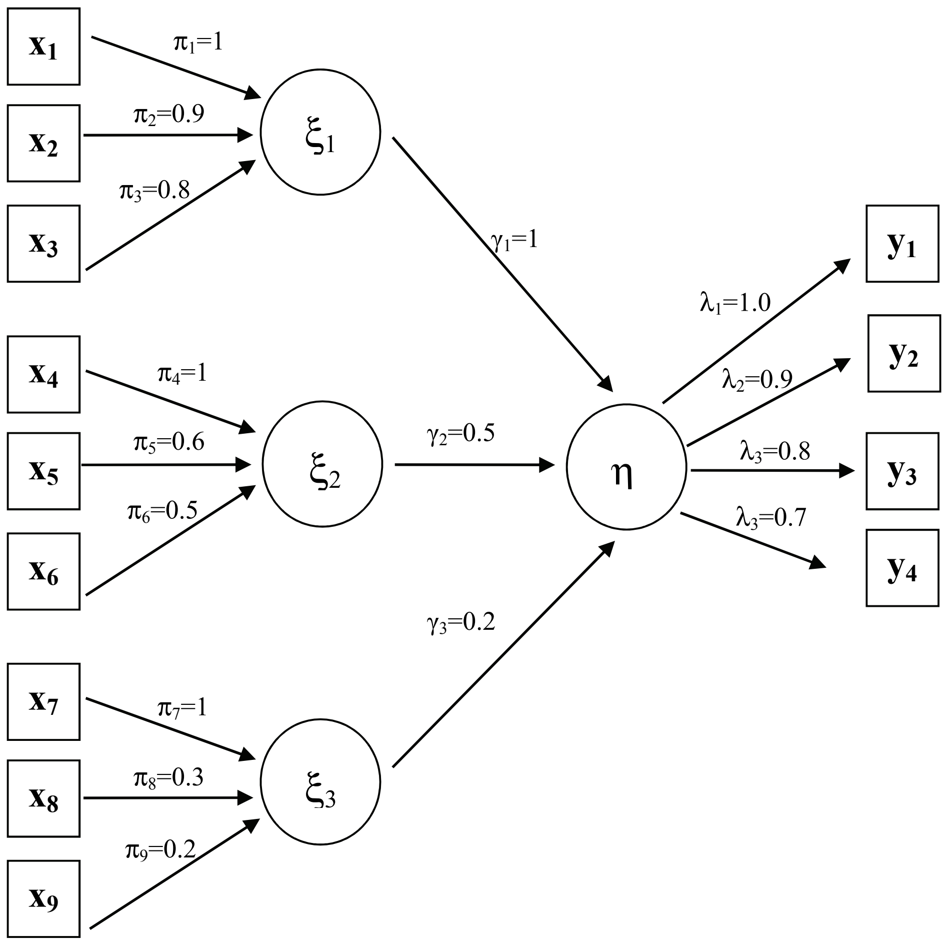
$$h=\left[\frac{n+p+1}{2}\right], \tag{18}$$

where *p* is the total number of parameters relating to each equation.

Statistical Measurements to Compare the Results

Based on the procedure described, the robust estimators were incorporated into the structural equation model and a program was constructed in software R version 2.11.1 (R Development Core Team, 2010), in which 1,000 Monte Carlo simulations were performed for each sample size for each degree of asymmetry in the errors distributions (see Figures 1-3) and

Figure 7: Graphical Representation of the Structural Equation Model
with the Parametric Values Used in the Monte Carlo Simulation Process



in the observable variables distributions (see Figures 4-6). To evaluate the accuracy for each parameter of structural equation (4), the relative deviations were computed as:

$$B = \frac{\sum_{b=1}^{1000}\left(\frac{\gamma_{ib} - \hat{\gamma}_{ib}}{\gamma_{ib}}\right)}{1000}, \; i = 1, 2, 3. \tag{19}$$

In terms of the precision of the study, the range given in (20) was used considering the difference between the highest and lowest standard deviation value $S\gamma_i$ obtained by means of the empirical distribution of estimates of each parameter γ_i (i = 1, 2, 3) generated in the simulation process:

$$A_{sd} = \max\left(S_{\gamma_i}\right) - \min\left(S_{\gamma_i}\right), \quad i = 1, 2, 3 \tag{20}$$

Results

Considering the observable variables generated with perfect symmetry (Case 0; see Table 2), it was observed that, in the situation where errors were symmetrical (Figure 2), the results were similar; the three estimators being considered accurate for all methods evaluated in terms of sample sizes.

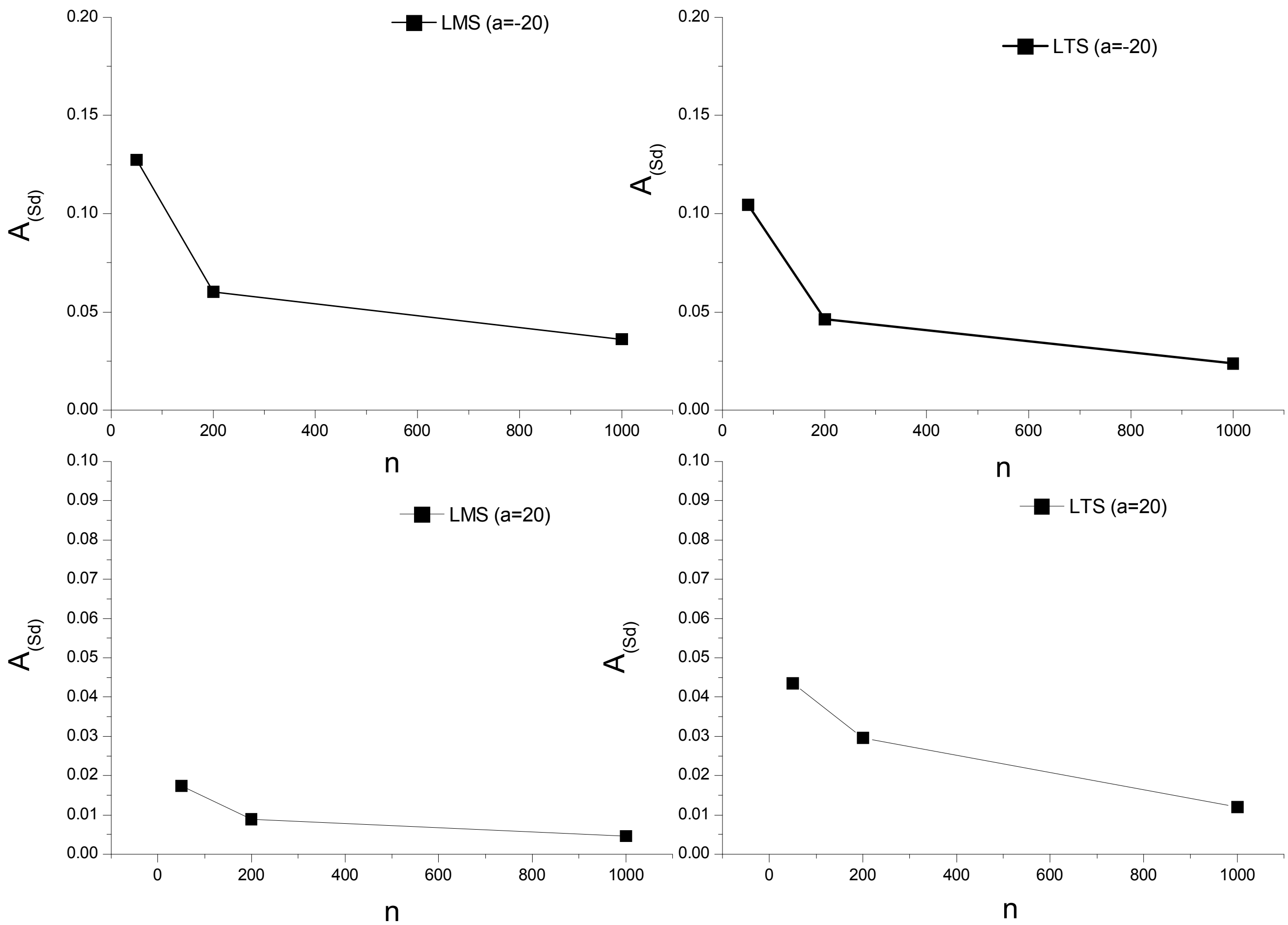
Table 2: Estimates of the Relative Bias (B) Considering Different Degrees of Asymmetry of the Distribution of Errors with the Observable Variables Generated by Beta (6, 6)

LS Method									
n	$\zeta; \varepsilon_j; \delta_i \sim N(0, 1)$			$\zeta; \varepsilon_j; \delta_i \sim N(0, 1, 20)$			$\zeta; \varepsilon_j; \delta_i \sim N(0, 1, -20)$		
	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3
50	0.0023	-0.0007	-0.0426	0.1561	0.2666	0.6163	-0.5592	-0.5830	0.1774
200	-0.0017	-0.0015	0.0076	0.1557	0.2684	0.6019	-0.5597	-0.5406	0.2067
1000	-0.0001	-0.0004	0.0081	0.1551	0.2755	0.5831	-0.5625	-0.5335	0.2511
LMS Method									
n	$\zeta; \varepsilon_j; \delta_i \sim N(0, 1)$			$\zeta; \varepsilon_j; \delta_i \sim N(0, 1, 20)$			$\zeta; \varepsilon_j; \delta_i \sim N(0, 1, -20)$		
	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3
50	0.0081	-0.0481	-0.0202	0.0994	0.2009	0.4488	-0.4828	-0.5342	0.0770
200	0.0091	-0.0346	0.0044	0.0927	0.1684	0.3605	-0.4619	-0.3981	0.0890
1000	0.0006	0.0027	0.0087	0.0852	0.1642	0.3227	0.0782	0.1142	0.1043
LTS Method									
n	$\zeta; \varepsilon_j; \delta_i \sim N(0, 1)$			$\zeta; \varepsilon_j; \delta_i \sim N(0, 1, 20)$			$\zeta; \varepsilon_j; \delta_i \sim N(0, 1, -20)$		
	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3
50	0.0245	-0.0596	-0.0452	0.0999	0.1863	0.4770	-0.4754	-0.5223	0.0238
200	-0.0007	-0.0156	-0.0039	0.0834	0.1557	0.3758	-0.4334	-0.3834	0.0548
1000	-0.0018	-0.0033	0.0164	0.0712	0.1479	0.3087	-0.3976	-0.3352	0.1547

This result is consistent with findings by Cassel, et al. (1999) that the parameter estimates resulting from the application of the Partial Least Squares method showed similar results, that is, estimates were not affected by increasing sample size. However, in considering the errors distributed as the normal asymmetric with the parameters of asymmetry $a = 20$ (see Figure 1) and $a = -20$ (see Figure 3), it was observed that the values of the biases in situations where the LMS and LTS methods were used were smaller those when considering the method of Least Squares (LS) estimation. However, it should be emphasized that the robust regression methods in the case of negative asymmetry of the errors retained results that characterize an inaccuracy and show evidence of a tendency to provide inflated or deflated estimates related to the parameters of the structural equation. This deficiency was corrected by increasing the sample size to n = 1,000 with emphasis on the LMS method which provided more accurate results compared to the parametric values imposed on the structural model.

Because results were more accurate for robust methods, in situations where the errors were generated by asymmetric normal distributions precision was evaluated using the statistic given by (20). Thus, the results illustrated in Figure 8 show that the LMS and

Figure 8: Precision of the Robust Estimates Considering Positive and Negative Asymmetric Errors with Observable Variables Generated by Beta (6, 6)



LTS methods presented similar precision when the errors have a negative asymmetry effect, and the LMS method showed better performance in the case of positive asymmetry.

Considering the distribution of observable variables with a moderate degree of asymmetry (Case 1), the results in Table 3 show that, in the situation where structural errors and measurement were simulated assuming normality without deviations of asymmetry (Figure 2), the LMS and LTS methods continued to provide good accuracy. However, when considering the errors generated by the normal asymmetric distribution, it was observed that the robust estimation methods were less accurate than those outlined in Table 2, but showed better performance than the least squares estimator. Maintaining the focus on the study of the precision afforded by the robust LMS and LTS methods (see Figure 9) the performance of the two estimators is similar.

Supplementing these findings, in a comparative study of maximum likelihood estimators (MLE) and robust estimation methods applied in the setting of a confirmatory factor analysis, Zhong & Yuan (2011) concluded that, in the case of non-normal distributions, the MLE estimates are biased and inefficient and the use of robust methods provide an improvement on these properties; however, they stressed that the

Table 3: Estimates of Relative Bias (B) Considering Different Degrees of Asymmetry of the Distribution of Errors with the Observable Variables Generated by Beta (9, 4)

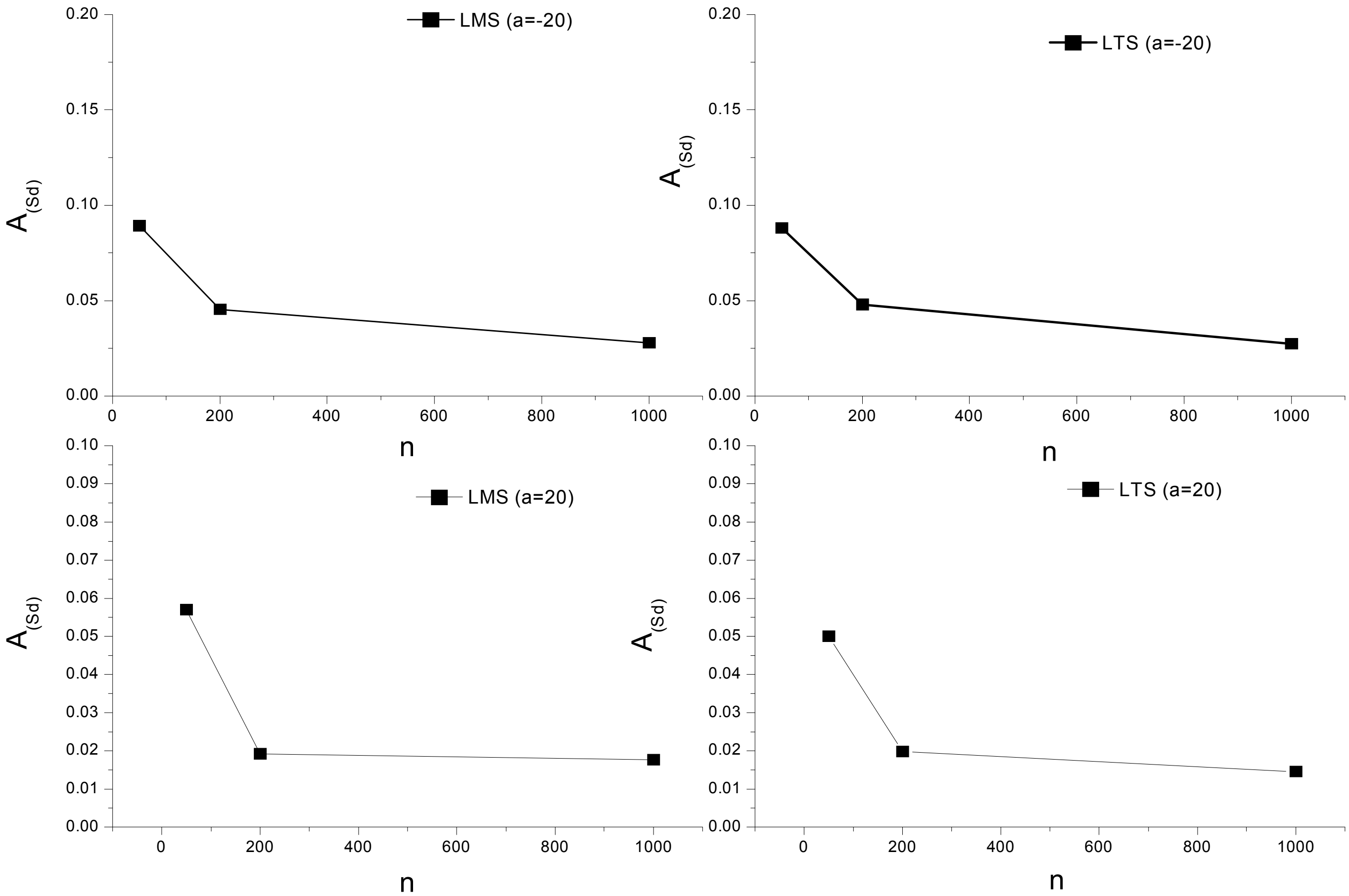
LS Method									
n	$\zeta;\ \varepsilon_j;\ \delta_i \sim N(0, 1)$			$\zeta;\ \varepsilon_j;\ \delta_i \sim N(0, 1, 20)$			$\zeta;\ \varepsilon_j;\ \delta_i \sim N(0, 1, -20)$		
	γ_1	γ_2	γ_3		γ_1	γ_2	γ_3		γ_1
50	0.0000	0.0024	-0.0078	50	0.0000	0.0024	-0.0078	50	0.0000
200	-0.0018	0.0047	0.0043	200	-0.0018	0.0047	0.0043	200	-0.0018
1000	-0.0007	0.0051	-0.0064	1000	-0.0007	0.0051	-0.0064	1000	-0.0007
LMS Method									
n	$\zeta;\ \varepsilon_j;\ \delta_i \sim N(0, 1)$			$\zeta;\ \varepsilon_j;\ \delta_i \sim N(0, 1, 20)$			$\zeta;\ \varepsilon_j;\ \delta_i \sim N(0, 1, -20)$		
	γ_1	γ_2	γ_3		γ_1	γ_2	γ_3		γ_1
50	0.0035	-0.0042	-0.0505	50	0.0035	-0.0042	-0.0505	50	0.0035
200	-0.0054	0.0187	0.0039	200	-0.0054	0.0187	0.0039	200	-0.0054
1000	-0.0008	-0.0023	0.0137	1000	-0.0008	-0.0023	0.0137	1000	-0.0008
LTS Method									
n	$\zeta;\ \varepsilon_j;\ \delta_i \sim N(0, 1)$			$\zeta;\ \varepsilon_j;\ \delta_i \sim N(0, 1, 20)$			$\zeta;\ \varepsilon_j;\ \delta_i \sim N(0, 1, -20)$		
	γ_1	γ_2	γ_3		γ_1	γ_2	γ_3		γ_1
50	-0.0134	0.0130	-0.0012	50	-0.0134	0.0130	-0.0012	50	-0.0134
200	-0.0045	0.0082	0.0160	200	-0.0045	0.0082	0.0160	200	-0.0045
1000	-0.0037	-0.0026	0.0385	1000	-0.0037	-0.0026	0.0385	1000	-0.0037

estimates of standard errors must be studied. In this sense, the study of the precision of the estimators is justified, confirming the claims made by Kaplan (1969) referring to the fact that the refinement of precision does not necessarily lead to increased reliability. In addition, in some situations, there could occur a reduction in the reliability of the estimates by a reduction of the accuracy because in practical situations the researcher may encounter systematic errors, for example, errors in the definition of the measurement or recording of the observation and random errors associated with uncontrollable variations.

When a high degree of asymmetry is imposed on the observable variables through distribution Beta (9, 1) (Case 2; see Table 4), maintaining the structural error and measurement errors distributed according to a normal symmetry (Figure 2), the LMS and LTS methods are accurate and similar results are obtained using the LS method. However, considering the asymmetric error (see Figures 1 and 3), based on the bias, the robust methods were more suitable for use in replacement of the LS method.

In the same situations with different degrees of asymmetry (see Table 1) assuming the measurement errors of the equations ((5)-(7)) and ((8)-(11)) are uniformly distributed, Cassel, et al. (1999) evaluated the biases of the parameter estimates of the structural equation model

Figure 9: Precision of the Robust Estimates Considering the Positive and Negative Asymmetric Errors with Observable Variables Generated by Beta (9, 4)



resulting from the application of Partial Least Squares method and concluded that the increase in the degree of asymmetry reflected an increase in bias and this is more emphasized in a situation of a high degree of asymmetry (Case 2; see Figure 6). Results obtained in this work are consistent with the Cassel, et al. (1999) findings and there is statistical evidence to argue that robust regression methods are consistent and worthy of being adapted to be used in structural equation modeling.

Relevant to the discussion of the estimates accuracy provided by the LMS and LTS methods it is important to note that, in the violation of the normality assumption, there are numerous works relating other methods that can be used as alternatives. Engel, et al. (2003) stated that the maximum likelihood method can be applied when the normality deviation is relatively moderate. In this situation the authors recommend $n > 400$. Moreover, Muthén and Muthén (2002) stated that the maximum likelihood estimators can be consistent but not necessarily efficient.

For variables with non-normal distributions, the recommendations are divergent. The minimum sample size for the weighted least squares method to be reliable, should be at least $n = 1,000$ (Hoogland & Boomsma, 1998), and depending on the model and data being analyzed; in some cases even exceeding 4,000 or 5,000 (Boomsma & Hoogland, 2001; Hu, et al., 1992). In general, when considering small sample sizes, Bollen, et

Table 4: Estimates of Relative Biases (B) Considering Different Degrees of Asymmetry of the Distribution of Errors with the Observable Variables Generated by Beta (9, 1)

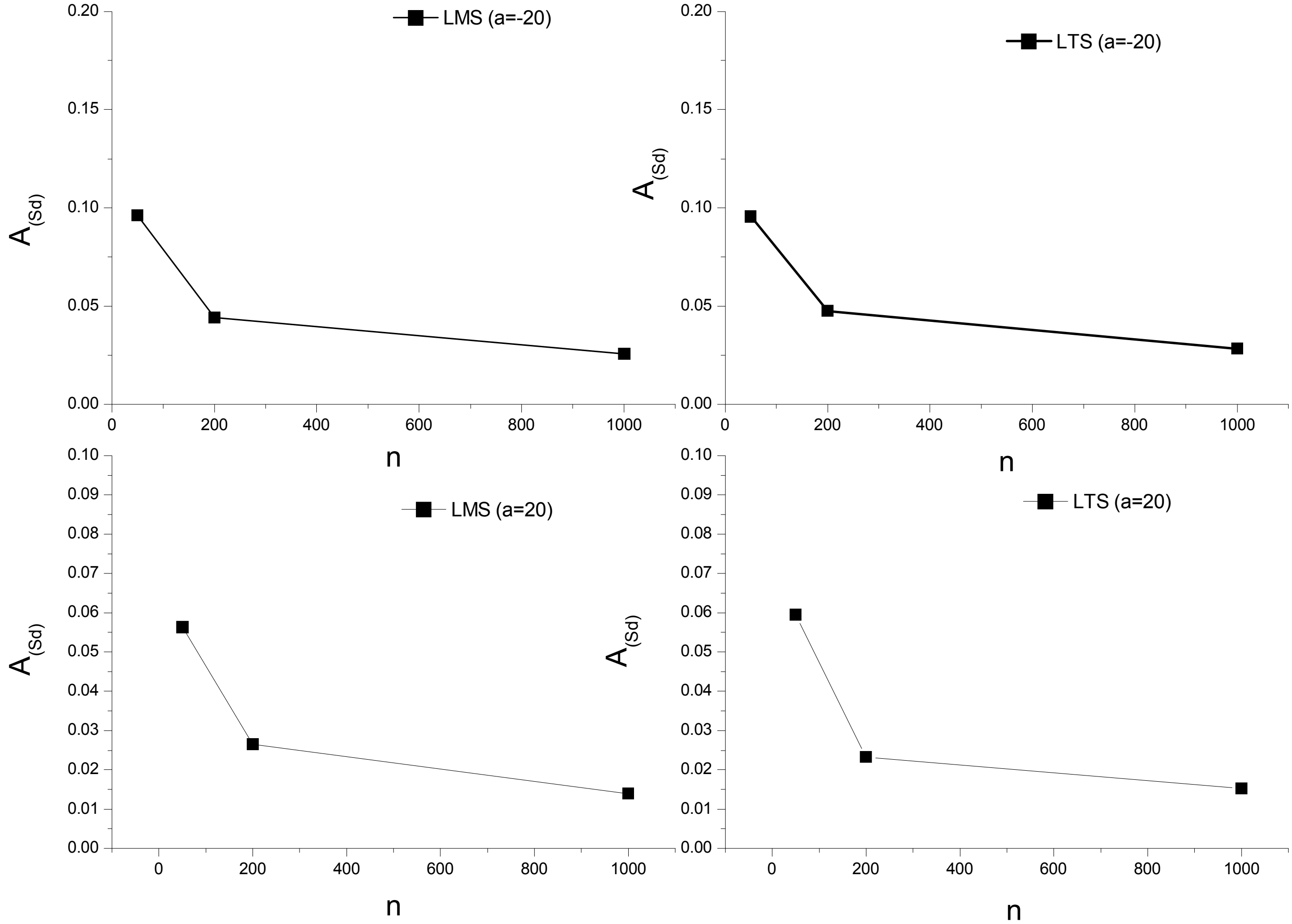
LS Method									
n	$\zeta; \varepsilon_j; \delta_i \sim N(0, 1)$			$\zeta; \varepsilon_j; \delta_i \sim N(0, 1, 20)$			$\zeta; \varepsilon_j; \delta_i \sim N(0, 1, -20)$		
	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3
50	-0.0030	0.0059	-0.0037	-0.0030	0.0059	-0.0037	-0.0030	0.0059	-0.0037
200	-0.0004	0.0035	-0.0044	-0.0004	0.0035	-0.0044	-0.0004	0.0035	-0.0044
1000	0.0001	0.0007	-0.0009	0.0001	0.0007	-0.0009	0.0001	0.0007	-0.0009
LMS Method									
n	$\zeta; \varepsilon_j; \delta_i \sim N(0, 1)$			$\zeta; \varepsilon_j; \delta_i \sim N(0, 1, 20)$			$\zeta; \varepsilon_j; \delta_i \sim N(0, 1, -20)$		
	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3
50	-0.0022	-0.0019	0.0717	-0.0022	-0.0019	0.0717	-0.0022	-0.0019	0.0717
200	-0.0039	0.0106	-0.0166	-0.0039	0.0106	-0.0166	-0.0039	0.0106	-0.0166
1000	0.0028	-0.0083	-0.0160	0.0028	-0.0083	-0.0160	0.0028	-0.0083	-0.0160
LTS Method									
n	$\zeta; \varepsilon_j; \delta_i \sim N(0, 1)$			$\zeta; \varepsilon_j; \delta_i \sim N(0, 1, 20)$			$\zeta; \varepsilon_j; \delta_i \sim N(0, 1, -20)$		
	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3
50	-0.0160	0.0236	0.0612	-0.0160	0.0236	0.0612	-0.0160	0.0236	0.0612
200	0.0006	0.0046	-0.0094	0.0006	0.0046	-0.0094	0.0006	0.0046	-0.0094
1000	-0.0002	0.0015	-0.0110	-0.0002	0.0015	-0.0110	-0.0002	0.0015	-0.0110

al. (2007) recommended that results should be interpreted with caution. Considering these arguments, it may be stated that the use of the robust estimation methods evaluated in this work are worthy of being considered in SEM, as the results show that the increase in the degree of asymmetry in the distribution of observed variables provided an improvement in the precision of the LMS and LTS estimates.

Conclusion

In situations where the observed variables and errors in a structural equation model show asymmetry deviations, the robust estimation methods LMS and LTS are suitable for application, especially with smaller sample sizes.

Figure 10: Precision of the Robust Estimates Considering Positive and Negative Asymmetric Errors with Observable Variables Generated by Beta (9, 1)



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