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Effect of Moment Gradient and Load Height With Respect to Centroid on the Reliability of Wide Flange Steel Beams Subject to Elastic Lateral Torsional Buckling

Christopher D. Eamon¹, Alexander W. Lamb², and Kapil Patki³

Abstract

The reliability of doubly-symmetric wide flange steel beams designed to the AISC Specification for Structural Steel Buildings subjected to elastic lateral torsional buckling was evaluated when considering variation in moment gradient and load height. The analysis considers continuous loads on spans subjected to various end moments with supports that are torsionally fixed and laterally supported, without additional intermediate restraints. Dead load, occupancy live load, and beam resistance random variables were considered. Beam lateral torsional buckling resistance was evaluated from numerical solution of a fundamental differential equation that accounts for the effect of moment gradient and load height. In some cases, it was found that use of the AISC design procedure results in significant inaccuracies for estimation of elastic lateral torsional buckling resistance, where underestimations occur in regions of reverse curvature bending and when loads are placed below the beam shear center, while large overestimations can occur when loads are placed above the beam shear center. These discrepancies result in significant variation in beam reliability. However, the use of accurate equivalent uniform moment factors can restore uniformity in notional reliability level.

KEYWORDS: Reliability, Steel structures, Stability

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1. Introduction

A wide range of literature describing the elastic lateral torsional buckling (LTB) behavior of structural steel beams based on analytical, numerical and experimental data is currently available [1-14]. Moment gradient between the supports, effect of load height with respect to shear center, buckling interaction, and out-of-plane restraints at member ends are some of the common issues considered while studying the lateral torsional stability of beams. Two of these considerations, moment gradient and placement of load height with respect to shear center, are of particular concern in this study and are further discussed below.

For flexural members loaded with non-uniform moment, an equivalent uniform moment factor approach is often considered. This represents the ratio of the critical moment for a member with a particular moment gradient to the critical moment for the member with a uniform moment [15], where the critical moment refers to that which causes an instability failure. The work of various researchers has developed this concept. For example, Nethercot and Rockey [16] used numerical data in an effort to describe a general procedure to determine the elastic critical moment of beams. More recently, Suryoatmono and Ho [9] and Lamb and Eamon [13] developed a generalized parametric solution procedure that can be used to solve the governing differential equation for elastic stiffness for a wide range of moment gradients that includes the load height effect. The expression proposed by [13] was since revised by Trahair [14].

Various international design standards address the effect of moment gradient, typically with simplified empirical expressions that can account for any arbitrary moment function, as well as the use of more precise formula for specific cases. Some of these many standards include: Eurocode 3: Design of Steel Structures [17]; the Australian Standard for Steel Structures, AS

4100 [18]; Canada's Design of Steel Structures (CSA-S16) [19] as well as the American Institute of Steel Construction's Specification for Structural Steel Buildings, AISC 360 [20], the focus of this paper. Some standards also adjust for the effect of load height on the beam; Eurocode 3, CSA-16, and AS 4100 are such examples. Despite the research conducted on this issue, however, some prominent design standards such as AISC 360 have not included the effect of load height in the development of equivalent moment factors (it should be mentioned that the code commentary of AISC 360 suggests that if the designer desires a more accurate solution considering load height, several alternative sources in the literature can be referenced for guidance; no specific provision is codified nor required, however). In particular, the expressions provided for the equivalent moment factor in these specifications implicitly consider loads to be acting at the shear center, neglecting the effect of load height throughout the depth of the cross-section. Moreover, the method to calculate equivalent moment factor in these specifications uses a general closed form expression which, although easy to use, for some load scenarios, produces results significantly different from the theoretical solution.

Such simplifications may have significant effect on the reliability of steel beams with regard to elastic LTB. In particular, based on deterministic analysis results, it is expected that lowering and raising vertical load placement from the shear center (i.e. at the centroid, for the symmetric sections considered here) of the beam, referred to as 'load height' in this paper (see Figure 1), will increase and decrease beam reliability, respectively. It is also expected that large deviations in reliability may occur when both positive and negative moments appear on the span [13]. However, the potential impacts that these effects may have on beam reliability have not been quantified. In fact, few studies have investigated the failure probability of structural steel members with regard to LTB in general. Ellingwood et al. [21] and Galambos and Ravindra [22]

developed initial resistance statistics for steel that can be used to evaluate LTB, while more recently, a statistical evaluation of LTB resistance properties of steel I-beams for Eurocode is presented by Silva et al. [23] and Robelo et al [24], wherein a new partial safety factor was proposed. Szalai and Papp [25] presented a new probabilistic evaluation of standard resistance models for the stability of columns and beams, while Badari [26] validated their method by examining a simply supported steel beam subjected to LTB. Most recently, Kala [27] studied the effects of random imperfections on steel beam LTB reliability. Currently, however, there exists no systematic probabilistic assessment of steel beams subjected to elastic LTB designed according to current AISC 360 standards that accounts for general moment gradient and load height effects. To address this issue, this study aims to estimate the reliability of typical wide-flange beams subjected to elastic LTB as designed according to the AISC 360 provisions, considering the effect of continuous moment gradients and load height.

2. Load Models

During its design lifetime, a structure is subjected to various loads such as dead load, occupancy and roof live loads, wind, snow, and earthquake loads, as well as others. Many interior beams in common braced frame steel construction are not subjected to significant lateral and environmental loads, and hence the load combination that frequently dominates is that of dead load and live load only, which is considered in this study.

Dead load (*DL*) statistical parameters used for code calibration are given consistently by various researchers [21, 28-30] where *DL* is described as normally distributed with bias factor (ratio of mean value to nominal, or code-specified value) of $\lambda=1.05$ and coefficient of variation (COV) of 0.10.

Occupancy live load represents the weight of people, furniture, partitions and other movable contents, and may be categorized into sustained (arbitrary-point-in-time) and transient (extreme event) components. Transient live load considers unusual occurrences of high load concentration such as a large number of people crowding together in a small room. It governs over the sustained effect with the load combination considered in this study, where 50 year maximum load statistics vary somewhat from one researcher to the next. However, when used for steel code calibration, statistics are generally taken as $\lambda=1.0$ and $COV=0.25$ [20, 27, 29], and it was typically assumed to follow a Gumbel distribution [21, 28], although Galambos [30] assumed it to be lognormal for ease of calculation. In this study, occupancy live load is taken as a Gumbel distribution with the above statistical parameters. However, it should be noted that the results were found to be relatively insensitive to type of live load distribution used.

3. Resistance Model

For reliability analysis, uncertainty in component resistance is traditionally developed from three sources: basic material properties (M); geometry during fabrication (F); and inaccuracies in the modeling method used to evaluate capacity, the professional (P) factor. The final bias factor for resistance, λ_R , is then taken as the product of the individual biases: $\lambda_R = \lambda_M \lambda_F \lambda_P$. Similarly, the COV of resistance, V_R , can be approximated as a function of its component COVs (V) as: $V_R = (V_M^2 + V_F^2 + V_P^2)^{1/2}$.

Ravindra and Galambos [31] report $\lambda_M=1.05$ and $V_M=0.10$ for rolled W shapes, while Galambos [30] later recommends $\lambda_M=1.06$ and $V_M=0.06$ from consideration of more recent material tests. However, these values were based on an analysis of variation in yield stress, which is relevant for flexural resistance based on yielding, but not for the elastic LTB failure

mode of interest in this study. Rather, Ellingwood et al. [21] and Nowak and Collins [32] provide statistical parameters for M specifically for elastic LTB as $\lambda_M = 1.0$ and $V_M = 0.06$. These values are identical with those for elastic modulus recommended by Galambos and Ravindra [22], the material parameter most relevant to elastic LTB. Therefore, in this study, statistical parameters for M are taken as $\lambda_M = 1.0$ and $V_M = 0.06$. Values for F , which are independent of failure mode, are consistently taken as $\lambda_F = 1.0$ and $V_F = 0.05$ by various researchers [21, 30, 32], and are similarly used in this study.

The effect of elastic LTB modeling accuracy, represented with factor P , is of particular interest in this study. In previous elastic LTB reliability analysis efforts, statistics for a single value of P have been used. For example, Ellingwood et al. [21] and Nowak and Collins [32] provide $\lambda_P = 1.03$ and $V_P = 0.09$. Similarly, although developed for flexural yielding rather than elastic LTB, Galambos [30] expands this slightly to differentiate between beams subjected to uniform moments ($\lambda_P = 0.99$ and $V_P = 0.06$) and those subjected to a moment gradient ($\lambda_P = 1.16$ and $V_P = 0.12$). Ravindra and Galambos [30], Rosowsky et al. [28], and Galambos [30] took the distribution of inelastic flexural resistance to be lognormally distributed. However, the most appropriate distribution for elastic modulus, and therefore elastic LTB resistance, is not apparent [22, 33]. Correspondingly, a normal distribution is assumed in this study, though it was found that changing distribution type has a relatively small effect on results.

As discussed in detail below, the use of a single value for λ_P is not particularly accurate in many circumstances, as a wide variation exists between the nominal and "actual" elastic LTB capacities, depending on moment gradient and the vertical load position on the section with respect to the shear center. To determine elastic LTB resistance more precisely, beam behavior is fundamentally described using Euler-Bernoulli flexure theory. The fundamental end

conditions can be described as “fork” supports, where translations are fixed; torsion is fixed ($\phi = 0$), where ϕ refers to rotation about the longitudinal axis (z direction) of the beam; and all other rotations (and higher order deformations), including those about the vertical and lateral axes are taken as free, as well as warping ($\frac{d^2\phi}{dz^2} = 0$). Variable end moments are included in the solution process, as discussed below. This allows consideration of the effect of rigid as well as partially-restrained conditions that can be achieved with welded or bolted connections (see, for example, Brunesi et al. [34, 35]). The differential equation describing the resulting lateral torsional buckling behavior of the beam under these constraint conditions that also accounts for load height is then given as [14]:

$$\frac{d^4\phi}{dz^4} - \frac{GJ}{EC_w} \frac{d^2\phi}{dz^2} + \frac{y_v w_y(z)}{EC_w} \phi - \frac{m_x^2(z)}{E^2 I_y C_w} \phi = 0 \quad (1)$$

In eq. 1., E is Young's modulus; I_y is moment of inertia about the weak (lateral) axis; G is shear modulus; J is the torsion constant; and C_w is the warping constant. Load height is represented by y_v , and the applied moments and loads are represented as functions $m_x(z)$ and $w_y(z)$, respectively. In this study, these applied moments and loads correspond to the three generalized types of load distributions considered in Figure 1.

In the figure, the Type 1 load corresponds to an end moment applied on the right beam end; the Type 2 corresponds to a moment applied on the left; and the Type 3 represents end moments applied at both ends of the span. Moreover, w is the applied load magnitude on the span itself; z is the variable component in the length direction of the beam (also appearing in eq. 1); n is the linearity factor, where $n=0$ corresponds to a uniform load and $n=1$ a linear (triangular) load; m the value of the applied end moment; and β is the end moment factor which adjusts

moment magnitude. Here, β values of 0 and 1.0 represent end moments corresponding to simple and fully fixed support conditions, respectively, while intermediate values of β can be used to represent partial moment fixity. Increasing β beyond 1.0 represents the application of additional end moment imposed on the member, while reversing the sign of β will reverse the end moment direction. By changing these factors, various common load types may be recovered such as a uniformly distributed load with possible end moments and a linearly increasing load distribution with variable end moments. Although only these common load types are considered in this study, more complex load distributions of any order can be considered by increasing the linearity factor n . Note that the reason end moments m appear somewhat complex algebraically, as nonlinear functions of n , is to allow convenient parametric solution of eq. 1. However, by independently adjusting parameter β as desired, any end moment values can be paired with the desired interior load function on the span [13]. Thus, in summary, the analysis considers continuous loads on spans subjected to various end moments with supports that are torsionally fixed and laterally supported, without additional intermediate restraints.

Once a desired load distribution is chosen for consideration, the corresponding moment functions are developed for a beam of length L and inserted into eq. 1. For example, the moment function corresponding to the Type 1 distribution shown in Figure 1 can be shown to be:

$$m_x(z) = -\frac{wz^{n+2}}{(n+1)(n+2)L^n} + \left[\beta \frac{wL}{(n+2)(n+4)} + \frac{wL}{(n+1)(n+2)} \right] z \quad (2)$$

The smallest load value (w) to cause the stiffness of the beam to approach zero is the critical elastic lateral torsional buckling load. For ease of use, this load is converted into a critical moment, M_{cr} which represents the maximum moment on the actual moment distribution

considered, then normalized using an equivalent uniform moment factor (EUMF) approach, as given by eq. 3.

$$EUMF = \frac{M_{cr}}{M_{0_cr}} \quad (3)$$

The EUMF is the ratio of the maximum applied moment needed to cause LTB instability (i.e. the critical load) based on the considered load distribution and boundary conditions, to the basic strength. The basic strength, M_{0_cr} is the LTB resistance of a simply supported member subject to a constant moment distribution. The basic strength is given as [36, 37]:

$$M_{0_cr} = \frac{\pi}{L} \sqrt{EI_y GJ \left(1 + \frac{EC_w}{GJ} \frac{\pi^2}{L^2} \right)} \quad (4)$$

The EUMF represents a convenient way that LTB solutions can be expressed for beams exposed to moment gradients, as if known, the EUMF can be simply multiplied by the basic strength of the specific member under consideration to determine its elastic LTB resistance. Equivalent uniform moment factor approaches are considered by various design codes. For example, AISC 360 presents the following expression, which is similar to that proposed by Kirby and Nethercot [8], as a function of the maximum moment (M_{max}) and moments at the quarter points of the span (M_a , M_b , and M_c),

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_a + 4M_b + 3M_c} \leq 3.0 \quad (5)$$

ISC refers to C_b as the moment gradient factor, which is equivalent in concept to the EUMF, and allows approximate consideration of the effects of arbitrary moment distributions.

In the standard code procedure, to determine the nominal resistance for elastic LTB, C_b is multiplied by the basic strength [20]:

$$M_n = C_b M_{0_cr} \quad (6)$$

Other standards use a similar approach. For example, in place of C_b , CSA-S16 specifies a factor ω_2 , which for non-linear moment gradients is:

$$\omega_2 = \frac{4M_{max}}{\sqrt{M_{max}^2 + 4M_a^2 + 7M_b^2 + 4M_c^2}} \leq 2.5 \quad (7)$$

Similarly, AS 4100 provides an adjustment factor α_m for an arbitrary nonlinear moment gradient:

$$\alpha_m = \frac{1.7M_{max}}{\sqrt{M_a^2 + M_b^2 + M_c^2}} \leq 2.5 \quad (8)$$

Fewer standards account for load height, but those that do generally apply additional adjustment factors to reduce capacity. CSA-S16, for example, accounts for loads placed on the top flange of the beam (as opposed to be beam centroid) by setting $\omega_2 = 1.0$ and increasing the effective length of the beam by a factor of 1.2 for simple supports and by 1.4 for all other end conditions, when calculating the critical lateral torsional buckling load. In comparison, AS 4100 specifies an additional adjustment factor α_s (to be used in conjunction with α_m), as a function of the yield moment capacity (M_s) and the critical elastic lateral torsional buckling moment (M_{oa}), calculated with an effective beam length factor of 1.4:

$$\alpha_s = 0.6 \left[\sqrt{\left[\left(\frac{M_s}{M_{oa}} \right)^2 + 3 \right]} - \left(\frac{M_s}{M_{oa}} \right) \right] \quad (9)$$

The results of eq. 1. are compared to those of CSA-S16, AS 4100, and AISC 360 and shown in Figures 2 and 3. To evaluate eq. 1., a finite difference analysis, implemented in Excel,

is used to determine the minimum (critical) load w for each specific load type and load height considered. In particular, a central difference approximation with the first non-zero terms of a Taylor series polynomial expansion to describe the differential operators is used. Solution convergence occurs when, after increasing the number of discretized beam segments used in the analysis, the critical moment remains practically unchanged (within 0.5%). This typically required 40 segments. To verify solution validity, a selection of cases with known solutions (such as that of a simply supported beam with a uniformly distributed load, as well as that with equal and unequal end moments applied) were considered and compared to the finite difference solutions. In all cases, the finite difference procedure recovered the known solutions [36, 37, 38].

The solution was conducted for a typical beam (W14 x 132) subjected to elastic LTB loaded at the centroid (Figure 2) and on the top flange (Figure 3), where the ratio of M_{cr} (adjusted for moment ratio and load height) and M_{0_cr} are presented for a Type 1 load. As shown in Figures 2 and 3, when the beam is loaded at the centroid, each of the simplified code approaches considered produce nearly identical and accurate results for end moment factors β greater than about -1. For $\beta < -1$, various degrees of conservatism exist, which appear to be quite high in some cases. It should be noted, however, that many beams may fail in yield before reaching the large theoretical LTB capacities given in Figure 2. When the beam is loaded on the top flange (Figure 3), differences between theoretical and code predictions of capacity become more inconsistent, where CSA-S16 and AS 4100 are generally conservative, considerably so in many cases, for all values of β considered. In contrast, AISC 360 is generally unconservative in most cases, as expected, as it does not directly account for top flange loading.

As shown in the figures, although very useful, the drawback of these expressions is that they inaccurately describe LTB resistance under certain load situations. Focusing on the AISC procedure, a more detailed consideration of its results is given in Figures 4-6. In these figures, values are presented for $n=0$ and $n=1$ load types (see Figure 1), as well as load application at the shear center (“0”), and on the top (“+”) or bottom (“-“) flange of beams with load heights of 3.5 in (90 mm), and 7 in (180 mm). Although not exact, the use of linear interpolation for other load placements was found to be reasonable. Considering the case where load is placed at the shear center (lines $n=0$, 0 and $n=1$, 0 in Figures 4-6), which the AISC procedure implicitly assumes, it can be seen that the code method provides a good match to the theoretical capacity for end moment factors close to $\beta > -1$ for Type 1 loads (Figure 4) and $\beta < 1$ for Type 2 and 3 loads (Figures 5 and 6). However, beyond these limits, the code method becomes significantly conservative, under-predicting elastic LTB capacity by over 50%. These regions correspond to load distributions that cause reverse curvature bending (i.e. when both positive and negative moments appear on the span).

When load height is changed, as expected, loads placed above the beam centroid decrease LTB capacity by causing an additional destabilizing moment (twist). As shown, some of the negative (below centroid) load placements theoretically provide very large increases in LTB capacity, often more than twice that if the load is placed at the centroid. Such increases are due to the stabilizing effect of load placement, which acts to resist LTB. However, such large capacity increases may not be necessarily realized, since the elastic LTB capacity may eventually exceed the yield moment, indicating that the beam may fail in yielding before elastic LTB. Note that the results are based on consideration of a W14x132. However, the resulting capacity ratios

are similar for most beam sizes and spans for which elastic LTB is expected to occur, which are of interest to this study.

Based on the procedure above, for this study, the constant value for professional factor bias λ_P assumed in previous reliability analyses of elastic LTB is replaced with that determined for each specific load case considered. This is essentially the ratio of the calculated EUMF to the AISC EUMF as shown in Figures 4-6. Mean resistance is thus taken as $\overline{M_{cr}} = \lambda_R M_n$, where the development of λ_R is discussed above. COV for P is taken as that suggested by Ellingwood et al. [21] and Nowak and Collins [32], where $V_P=0.09$. Accounting for the additional material and fabrication factor variances discussed above, the resulting COV for resistance is $V_R=0.12$.

4. Reliability Analysis and Results

Dead load (DL), live load (LL), and critical elastic LTB moment capacity (M_{cr}) are the random variables considered in the analysis, with statistical parameters summarized in Table 1. Expressing DL and LL in terms of corresponding moment effects M_{DL} and M_{LL} , the resulting limit state function is:

$$g = M_{cr} - M_{DL} - M_{LL} \quad (10)$$

For the reliability analysis, the nominal moment capacity is calculated according to AISC 360, with strength reduction factor taken as $\phi_r=0.9$, using the governing load combination of $1.2D + 1.6L$ or $1.4D$ specified in ASCE 7 [39]. The iterative First Order Reliability Method (FORM) [40] was used for reliability index evaluation. For verification, 1×10^6 Monte Carlo Simulations (MCS) were used to compute failure probability p_f of a selection of cases, then results transformed to reliability index β with the standard normal transformation $\beta = -\Phi^{-1}(p_f)$ for

comparison. As expected, it was found that the MCS results well-matched the FORM reliability index estimate.

For comparison, Figure 7 first presents the reliability results for beams subjected to uniform moment and moment gradients, when capacity is governed by yielding. Results are presented for various $D/(D+L)$ ratios because it is known that load proportion affects reliability, despite the attempt to minimize this variation with the use of different load factors and combinations in ASCE 7 for Load and Resistance Factor Design (LRFD). The gradual drop, then sudden rise in reliability that occurs at a $D/(D+L)$ ratio of about 0.9 is due to the change in beam capacity being governed by the load combination $1.4D$ rather than $1.2D + 1.6L$. The sudden change in slope at this load ratio as well as the general shape of the reliability graph shown in Figure 5 is thus not specific to steel members, but affects any structure designed using the ASCE 7 load combinations and can be seen in reinforced concrete members as well (see Szerszen and Nowak [41]). This primarily occurs due to the significantly different uncertainties (i.e. differences in COV) present in live and dead load, from which total variance in the limit state function changes as the D/L proportion changes, but constant load factors are used to establish design load.

For the results shown in Figure 7, reliability indices were calculated using the updated resistance statistics given by Galambos [30] noted above, which result in $\lambda_R=1.05$, $V_R=0.10$ for uniform moment and $\lambda_R=1.23$, $V_R=0.14$ for beams subjected to moment gradients, with a lognormal distribution. As expected, the results are essentially the same as those presented by Galambos [30], where a typical notional reliability index of approximately 2.6 for beams subjected to uniform moment and closer to 3 for beams subjected to moment gradients is expected of AISC 360 [28, 30]. Figure 7 also provides results for LTB. In this case, the more

precise analytical method to determine elastic LTB capacity is not applied. Rather, as in previous reliability investigations, a constant bias factor for P is taken directly from Ellingwood et al. [21] and Nowak and Collins [32] as $\lambda_P=1.03$. Used in conjunction with $\lambda_M=1.0$ and $\lambda_F=1.0$ as discussed above, this results in $\lambda_R=1.03$ (with $V_R=0.12$). Note that the lower reliabilities associated with LTB occur because for this failure mode, as discussed above, resistance is governed by elastic modulus rather than yield strength, resulting in a lower bias factor and, to a lesser extent, a normally distributed probability density rather than lognormal, both of which act to reduce reliability index. Thus, the values presented in Figure 7 correspond to the intended level of notional reliability in the AISC code.

When elastic LTB is evaluated more precisely (per eq. 1), reliability results are presented in Figures 8-16 for different load height positions (at beam centroid (0), and above (+) and below (-) the centroid at load heights of 3.5 in (90 mm), and 7 in (180 mm)) and for linearity factors $n=0$ and $n=1$, across a range of end moment factors from -2 to 2. Results are presented for three $D/(D+L)$ ratios: 0.40 (Figures 8-10), 0.55 (Figures 11-13), and 0.9 (Figures 14-16), to illustrate a range of typical, high, and low values for LTB reliability, respectively, as suggested in Figure 7. Note that practically, beam reliability cannot exceed that corresponding to beam flexural yielding, limiting some of the large capacity increases shown in Figures 4-6. This reliability limit is shown on Figures 8-16 as the horizontal line at a reliability index of approximately 3, and is taken from the yield capacity limits (for non-uniform moment gradient) shown in Figure 7 at the corresponding $D/(D+L)$ ratio considered. Also shown on Figures 8-16 is the LTB reliability level assumed by code, as taken from Figure 7. As shown in the figures, the nominal reliability index for elastic LTB assumed by code (i.e. if resistance is computed using the approximation provided by eq. 7, and which does not adjust for load height), is fairly consistent,

as expected for LRFD. It ranges from about 2.0-2.4, and is only a function only of load ratio $D/(D+L)$.

When the more accurate analytical procedure is considered, applying load at the beam centroid (lines designated $n=0, 0$ and $n=1, 0$ on the figures) produces very consistent results that match the code-assumed reliability level for a range of end moment factors $\beta > -0.8$ for load Type 1 and $\beta < 0.8$ for Types 2 and 3. However, for end moment factors that exceed this range, reliability dips slightly, then quickly increases until the reliability limit based on section yielding is reached at around an index of 3, significantly greater than the code-assumed value. Such results mirror the actual to code capacity ratio changes as shown in Figures 4-6.

When load height is decreased from the beam centroid, such as from hanging loads from the beam, elastic LTB reliability dramatically increases, but, as noted above, beam flexural reliability is limited to failure by yielding, practically limiting the theoretically possible large benefits in safety to a reliability index of approximately 3. When load height is increased, however, marked decreases in reliability appear. Consider the case where uniform load ($n=0$) is applied 3.5 in above the beam centroid. Here, the typical drop in reliability index is about 0.5, and for an end moment factor of about -0.9 for load Type 1 and +0.9 for load Types 2 and 3, reliability index drops even further to approximately 1-1.5 for load ratios from 0.40-0.55 and as low as 0.5 for a load ratio of 0.9. Of course, increasing load height further continues to decrease reliability. For a load applied on the top flange of a 14 in (380 mm) deep beam, for example, reliability index drops below zero in some cases. Such an event occurs when the mean load effect exceeds mean resistance, and the mean of the limit state function (eq. 10) becomes negative. The possibility of this occurring is easy to see from Figures 4-6, where very large decreases in capacity are realized (even greater than 50% for a Type 3 load, when load position

is at the top flange of a 14 in (360 mm) deep beam, for example) that the code design procedure does not account for. This is particularly problematic at a high $D/(D+L)$ ratio of 0.9, where the load combination $1.2D + 1.6L$ is governed by $1.4D$, and the low load factor of 1.4 controls the design load effect, resulting in an under-designed beam when considering the actual elastic LTB capacity.

As noted earlier, it was observed that results were not particularly sensitive to random variable distribution type. For example, consider a typical $D/(D+L)$ ratio of 0.55 with load height of 7". Changing live load distribution from normal to lognormal results in a mean difference in reliability index of 0.02, with a maximum difference of 0.20 considering all load Types 1-3 and end moment factors from -2 to 2 (where the maximum difference occurs for a Type 3 load with $n=1$ and an end moment factor of 1.13). Similarly, changing resistance distribution type from normal to lognormal results in a mean difference in reliability index of 0.03, with a maximum difference of 0.06 (for a Type 1 load with $n=0$ and an end moment factor of -1.75).

Here it should be emphasized that these reliability indices, very low in some cases, do not necessarily mean that the corresponding beam designs will readily fail in elastic LTB. As with nearly all probabilistic assessments of design code safety, the reliability indices calculated are meant to be taken as notional rather than actual indicators of reliability; due to a lack of precise statistical information as well as numerous idealizations in load and resistance modeling, converting the resulting reliability indices to failure probability is not expected to indicate actual probabilities of failure. For the case of LTB, there are various factors not accounted for in theoretical stability analysis that may increase reliability. For example, when beams support slabs, even if non-composite and the connection is taken as insufficient to rely upon for strength

analysis, significant restraint can be provided to the beam top flange that resists LTB. Correspondingly, if an unbraced girder supports cross-beams or joists which bear upon the top flange, although no direct lateral restraint is present, some tipping resistance is clearly formed between the contact surfaces [42, 43]. Moreover, intermediate braces or supports which bound the unsupported length of the beam typically provide some level of warping restraint which is generally ignored. Similarly, other uncertainties not accounted for in structural reliability analysis due to lack of data, such as design and construction errors [44], may decrease the assumed reliability level.

Therefore, results should not be considered as quantification of actual reliability, but rather used to examine the degree of consistency in notional reliability level, which is specifically meaningful for LRFD code calibration. Specifically, in Figures 8-16, it can be seen that a large variation in notional reliability level exists, with a range of reliability index from close to zero to approximately 3, a wide range that is undesirable in an LRFD-based code, for which a much more consistent range of about 2-2.5 for elastic LTB (Figure 7) is expected. This large variation in elastic LTB reliability is of concern, and suggests that further refinement of the design procedure would be beneficial.

Note that only dead and live loads were considered in this analysis. However, in different design scenarios, other load combinations such as those involving wind or seismic forces may govern. From a reliability standpoint, if a seismic (or wind) load effect is approximated as an equivalent static load, then no difference in shape of the curves shown in Figures 8-16 will result; the only effect would be to shift the resulting reliability indices down, assuming that the seismic load governed over the load combination presently considered. However, if potentially higher fidelity is desired when seismic load is considered, where a true

dynamic analysis is conducted, then a significantly more complex model than that provided by eq. 1 is required. In this case, one possible approach would be to first conduct a dynamic analysis, solving the beam equation of motion for a set of displacements as a function of time. Then, loads $w_y(z)$ and $m_x(z)$ within eq. 1 could be rewritten in terms of equivalent loads developed from the displacements. Another approach is to abandon eq. 1 entirely, and conduct a geometrically nonlinear, dynamic finite element analysis to assess critical moment. A true dynamic analysis would allow for a better understanding of how elastic LTB reliability changes under cyclic loads. However, the end moments, as well as gravity load to some extent, are not constant throughout the analysis. This may result in a different pattern of reliability index graphed as a function of (static) end moment factor than that presented, especially if a moment reversal occurs.

Another important issue is with regard to failure mode. Although the concern of this study is elastic LTB, a more common mode of failure may be inelastic LTB, where the section experiences partial yielding in combination with lateral instability. Although this analysis is beyond the scope of this study, some observations can be made. First, the expectation is that differences between code and theoretical solutions would decrease as yield becomes more prominent, as the simple code expressions used to compute yield capacity well-approximate exact solutions. Thus, peak ratios of $M_{cr} / M_{0_{cr}}$ shown in Figures 4-6 are expected to decrease, and in the extreme case of failure by pure yielding, ratios will approach 1.0 for all end moment factors. Second, with respect to reliability, as shown on Figures 8-16 and as noted earlier, maximum reliability index is limited to that which would be achieved from yield failure. Although this precise value varies with $D/(D+L)$ ratio, as shown in Figure 7, it is close to 3, as shown by the limiting upper horizontal line in Figures 8-16. Thus, as failure mode becomes

more yield-dominated, the reliability curves shown in Figures 8-16 would shift upward such that they become closer to the pure-yield reliability limit of approximately 3. Therefore, the results presented for elastic LTB can be thought to represent worst-case results, where discrepancies between code and exact results are most prominent.

5. Summary and Conclusion

In this study, a reliability analysis of doubly symmetric steel beams designed to AISC 360 standards subjected to elastic LTB failure was conducted. Random variables included dead load, occupancy live load, and beam resistance. LTB behavior was described with a differential equation that accounts for moment gradient and load height from the beam shear center, and a central difference procedure was used to solve the equation for critical moment and develop fundamental LTB resistance. It was found that the current simplified AISC design procedure that relies upon the moment gradient factor C_b provides significant over as well as under estimation of LTB capacity when non-uniform moment functions are considered, where most underestimations occur in regions of reverse curvature bending. Moreover, as AISC does not currently account for the effect of load height with its codified capacity equations, it was also found that even greater capacity discrepancies may occur when loads are placed above or below the beam shear center. These discrepancies similarly lead to a large variation in notional beam reliability if subjected to elastic LTB, an undesirable outcome for the LRFD-calibrated AISC 360 specification. Such discrepancies can be eliminated by refining the AISC design procedure that estimates LTB resistance.

Additionally, the LTB body of knowledge would benefit from a detailed investigation of several other important considerations as well. Some of these include an expansion of results to

include consideration of cantilever beams, concentrated loads and twist; different lateral and torsional constraint conditions; camber; and atypical beam geometries. Other significant considerations to address are initial beam curvature due to fabrication tolerances or other causes, as well as inclusion of inelastic response.

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Table 1. Random Variables

Random Variable		Bias Factor	COV	Distribution	Figure #
Fundamental Resistance Random Variables					
Material, yield	M	1.06	0.06	normal	7
Material, elastic LTB	M	1.0	0.06	normal	7-16
Fabrication	F	1.0	0.05	normal	7-16
Professional, yield, uniform moment	P	0.99	0.06	normal	7
Professional, yield, moment gradient	P	1.16	0.12	normal	7
Professional, elastic LTB	P	1.03	0.09	normal	7
Professional, exact, elastic LTB	P	*	0.09	normal	8-16
Derived Resistance Random Variables					
Resistance, uniform moment, yield	M_{cr}	1.05	0.10	lognormal	7
Resistance, moment gradient, yield	M_{cr}	1.23	0.14	lognormal	7
Resistance, intended, elastic LTB	M_{cr}	1.03	0.12	lognormal	7
Resistance, exact, elastic LTB	M_{cr}	*	0.12	normal	8-16
Load Random Variables					
Dead Load	DL	1.05	0.10	normal	7-16
Live Load	LL	1.0	0.25	Gumbel	7-16

* λ_P varies as shown in Figures 4-6. Resistance bias factor is then calculated as: $\lambda_R = \lambda_M \lambda_F \lambda_P$

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