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# Reliability Analysis of Reinforced Concrete Columns Exposed to Fire

Christopher D. Eamon<sup>1</sup> and Elin Jensen<sup>2</sup>

## Abstract

A reliability analysis is conducted on reinforced concrete columns subjected to fire load. From an evaluation of load frequency of occurrence, load random variables are taken to be dead load, sustained live load, and fire temperature. Resistance is developed for axial capacity, with random variables taken as steel yield strength, concrete compressive strength, placement of reinforcement, and section width and height. A rational interaction model based on the Rankine approach is used to estimate column capacity as a function of fire exposure time. Various factors were considered in the analysis such as fire type, load ratio, reinforcement ratio, cover, concrete strength, load eccentricity, and other parameters. Reliability was computed from zero to four hours of fire exposure using Monte Carlo simulation. It was found that reliability decreased nonlinearly as a function of time, while the most significant parameters were fire type, load ratio, eccentricity, and reinforcement ratio.

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## **1. Introduction**

The damage caused by building fires represents a substantial impact to the civil infrastructure. In the United States, fires resulted in 3,010 deaths, 17,050 civilian injuries, and \$12.5 billion in property damage in 2009 [1].

Fire mitigation involves a multi-faceted approach, including fire prevention efforts as well as protective measures for human life and property if a fire starts. Among these are the requirement of buildings to meet minimum standards for escape routes; the use of sprinklers and other active fire protection devices to limit fire severity; and providing fire resistance to structural components, such that the building remains stable to allow for escape and fire suppression. Fire resistance is often measured in terms of a fire rating, which is generally given as the time throughout which a structural component is expected to sustain a minimum specified load when subjected to a standard test fire.

In practice, fire resistance for a reinforced concrete member is typically provided by the design engineer by specifying a prescribed reinforcement cover for a given member size and aggregate type as specified in a code standard such as ASCE 29 [2]. A more precise fire rating may be determined for a specific member design with a fire endurance test such as given by ASTM E119 [3], calculation methods [2,4,5], or more sophisticated analyses such as finite element or finite difference methods. Regardless of the approach taken, the resulting fire rating gives no insight to actual safety level in terms of failure probability, and the reliability of reinforced concrete members under fire loads is largely unknown. This is problematic in the Load and Resistance Factor Design (LRFD) approach, in which probabilistic methods are used to set appropriate load and resistance factors for consistent minimum safety levels for members of the same level of importance. As LRFD codes have not been calibrated for fire loads, members

that have the same deterministic fire resistance rating may have significantly different levels of reliability. Moreover, even using existing prescriptive methods and measuring safety deterministically, a significant difference in member performance under fire may result [6-10].

Another area of concern is that codes which are meant to follow the same framework for structural loads and load factors have significantly different provisions when fire is considered. For example, ACI 216.1-07, Code Requirements for Determining Fire Resistance of Concrete and Masonry Construction Assemblies [4], recommends application of 100% of service dead load and 100% of service live load when evaluating member performance under fire, while ASCE 7, Minimum Design Loads for Buildings and Other Structures [11], specifies 120% of service dead load and 50% of service live load. Such inconsistencies are indicative of a lack of a systematic reliability-based approach for fire loads throughout the applicable standards for reinforced concrete structural design.

These issues spurred interest in developing a more consistent method to assess and maintain fire safety, within the general framework of performance based design (PBD) [12]. With regard to fire engineering, PBD is described as a robust method allowing probabilistic assessment that is founded on the principles of fire science, heat transfer, and structural analysis. In 2008, the International FORUM of Fire Research Directors identified the estimation of uncertainty and the means to incorporate it into risk analyses when considering fire a research priority [13]. Despite this, limited research exists on the reliability assessment of structures exposed to fire. Some of this research includes Beck [14] and Teixeira and Soares [15], who modeled the reliability of steel members and plates subjected to fire; Vaidogas and Juocevicius [16] as well as Hietaniemi [17], who conducted probabilistic analyses of timber components exposed to fire, and Shetty et al. [18] who assessed the reliability of offshore structures under fire

load. Other recent contributions include Ellingwood [19], who developed relevant design load combinations for fire design based on an analysis of load frequency, and Au et al. [20], who used subset simulation to assess fire risk analysis in a compartment.

A smaller number of studies exist that specifically consider reliability analysis of concrete components exposed to fire. Courge et al. [21] estimated the reliability of a fire-exposed concrete tunnel, while Ellingwood and Shaver [22], Wang et al. [23, 24], and Eamon and Jensen [25, 26] estimated the reliability of reinforced concrete beams exposed to fire. Sidibe et al. [27] estimated the reliability of concrete columns subjected to fire, but information regarding the method of column design as well as the random parameters were not specified. Moreover, loads were taken as deterministic and an approximate reliability approach was used, limiting the applicability of the results.

Currently, there exists no systematic assessment of the reliability of RC columns exposed to fire that have been designed to current ACI 318 [28] Code standards considering both load and resistance uncertainties, nor an examination of the changes in reliability as various important column parameters change. As a step towards performance based design, this study estimates the reliability of a selection of reinforced concrete columns designed according to ACI 318 Code exposed to fire. The intent is to estimate a baseline of current safety levels, as well as to examine how various parameters affect column reliability when exposed to fire. At present, the results may be used to assess the reliability of typical RC columns exposed to fire, as well as to determine the column characteristics needed to achieve a desired level of reliability for a given fire duration, within a reliability framework consistent with that used to calibrate the ACI 318 code.

## 2. Load Models

In general, various loads must be considered for design as well as reliability analysis, including dead load, occupancy and roof live loads, snow load, wind, and earthquake, among others. However, Ellingwood examined the coincidence rates of the extremes of these loads in the United States with a structurally significant fire [19], and determined that many of these load combinations can be practically neglected when considering reliability indices  $\beta$  that are close to or below typical code target levels (i.e. approximately for  $\beta \leq 3.5 - 4.0$ ). Of these combinations, it was found that fires coincident with extreme loads involving snow, earthquakes, winds, roof live loads, and transient occupancy live loads would not govern reliability and can thus be excluded from the analysis. The remaining loads that must be considered with a structurally significant fire are dead load and sustained (occupancy) live load.

When described as a random variable, dead load  $D$  is frequently characterized with a bias factor  $\lambda$  (ratio of mean value to nominal value) of 1.05 and coefficient of variation (COV) of 0.10. It is normally distributed [29].

Statistics for live load are generally developed for two situations; transient and sustained loads. The transient case represents an extreme load for infrequent, atypical events such as emergencies, crowding, or remodeling. Transient live load is used in the reliability analysis of structural members not simultaneously subjected to other extreme loads [29]; as discussed above, it generally does not govern reliability in combination with other extreme loads such as fire, due to the low coincidence probability. The other live load component, the sustained, or arbitrary-point-in-time load,  $L_s$ , represents the typical live load on the structure, generated by items such as furniture, partitions, and other contents. Sustained live load bias factors range from about 0.24 – 0.50, depending on the tributary area of the structural component evaluated as well as building

occupancy type, with COV from 0.60-0.65. It is frequently assumed to follow a gamma distribution [19, 29]. For the analysis presented in this study,  $L_s$  is modeled with a gamma distribution with bias factor of 0.24 and COV of 0.65, as assumed by Nowak and Szerszen in the calibration of the ACI 318 Code [29].

Depending on fuel load and type, fire bed location, ventilation, convective and radiative properties of the compartment, as well as other factors, a structural member will be subjected to various temperature-time profiles. Although fire occurrence data exists, statistics that describe the details of variation in the time-temperature profile of actual building fires are unavailable. Therefore, to conduct the reliability analysis, idealized fires must be considered. Here, various fire curves are available, including the well-known standard fire given in ASTM E119 [3], the similar ISO 834 fire [30], as well as parametric fires such as those given in the Eurocode [31], which are thought to more accurately represent realistic, post-flashover fires with heating as well as cooling phases. Therefore, in this study, fire temperature  $T$  is considered a random variable, with mean value at any exposure time  $t$  considered, taken as the temperature developed from the fire temperature ( $T$ )-time ( $t$ ) profile from a selection of the idealized fires above. In particular, Eurocode parametric fires developed from fuel loads from 400-3000 MJ/m<sup>2</sup> and ventilation factors  $F_v$  of 0.02 and 0.04, as well as the standard fire (ASTM E119), will be considered for analysis, as shown in Figure 1.

The equivalent fuel load used is based on that expected to be associated with a maximum 50-year fire, a typical time period that is used for reliability assessment of structural components in buildings [19, 29]. Although the maximum expected 50 year fire temperature profile is currently unknown and not directly obtainable from available survey data, it is estimated in this study by noting that fuel load is largely dependent on live load, where maximums often occur

during remodeling or construction, when items and furniture are temporarily moved into a small area. As noted above, mean (sustained) live load has a bias factor of approximately 0.24, while 50-year maximum live load has a bias factor of approximately 1.0 [26]. Although the exact relationship between fuel load and live load is unavailable, given that they are closely related, it is assumed that the ratio of 50-year maximum-to-sustained fuel load is similar to the ratio of 50-year maximum-to-sustained live load; i.e. a ratio of about 1:0.24. Given that the range of mean sustained fuel loads for room in office buildings is from about 420 – 1100 MJ/m<sup>2</sup> [32], dividing by 0.24 produces a range of expected 50-year mean maximum fuel loads from about 1750-4580 MJ/m<sup>2</sup>.

The burning phase temperature of a Eurocode parametric fire is given by:

$$T = 1325(1 - 0.324\exp(-0.2t) - 0.204\exp(-1.7t) - 0.472\exp(-19t)) \quad (1)$$

This expression is based on a thermal inertia of concrete of 1900 W s<sup>0.5</sup>/m<sup>2</sup>K, and  $F_v$  of 0.04. To account for other  $F_v$ , time  $t$  in eq. 1 is multiplied by the factor  $(F_v/0.04)^2$ . The burning period  $t_d$  (hours) of the fire is given by  $t_d = 0.00013e_t/F_v$ , where  $e_t$  is fuel load multiplied by the ratio of the floor area to the total surface area of the chamber. Here, a floor area to total surface area ratio of 0.2 was considered, which is taken to represent typical office room proportions.

Interestingly, when the upper range of the 50-year maximum fuel load noted above of about 4500 MJ/m<sup>2</sup> is used with eq. (1), a time-temperature profile nearly identical to that of the E119 standard is produced up to and beyond 3 hours of burn time. Thus, the E119 fire appears to reasonably represent a potential 50-year mean maximum fire.

To use fire temperature as a random variable for reliability analysis, an estimate of its variability, measured here in terms of COV, is also needed. Fuel load, room geometry and ventilation, as well as various other fire and compartment characteristics will have an impact on

fire temperature variability. Unfortunately, there exists no statistical data for most of these parameters in the technical literature. However, some useful relationships are available. In particular, according to Hamarthy and Mehaffey [33], the normalized heat load  $H'$  ( $s^{0.5}K$ ) experienced by compartment surfaces can be used as a measure of the temperature  $T$  that a structural element within the compartment will experience in a fire. Based on a series of compartment burn tests, Hamarthy and Mehaffey determined that normalized heat load can be estimated with:

$$H' = (1 \times 10^6) \frac{11\delta + 1.6}{A_t \sqrt{\kappa \rho c_p} + 935 \sqrt{\Phi_v L_f A_f}} (L_f A_f) \quad (2)$$

In eq. (2),  $\delta$  is the fraction of fuel energy released in the room, given by:  $\delta = 0.79 \sqrt{H_c^3 / \Phi_v} \leq 1.0$ ;  $A_t$  is the inside surface area of room boundary ( $m^2$ );  $\sqrt{\kappa \rho c_p}$  is the surface averaged thermal inertia inside the compartment boundary ( $J/m^2 s^{0.5} K$ );  $\Phi_v$  is the ventilation factor, given by:  $\Phi_v = \rho_a A_v \sqrt{g H_v}$  ( $kg/s$ );  $H_c$  is room height ( $m$ );  $g$  is the gravitational constant ( $9.8 m/s^2$ );  $H_v$  is the compartment vent height ( $m$ );  $A_v$  is the area of the compartment vent openings ( $m^2$ );  $\rho_a$  is the density of air ( $1.23 kg/m^3$ );  $A_f$  is the compartment floor area ( $m^2$ ); and  $L_f$  is the fuel load in the compartment ( $kg/m^2$ ). Most of these parameters may vary widely as they depend on the geometry of a specific chamber, and are not useful for classification as random variables for general reliability analysis. However, general statistical data does exist for fuel load ( $L_f$ ), which has a significant influence on maximum fire temperature. Thus, COV of fire temperature is estimated in this study by determining the statistical relationship between fuel load and heat load using eq. (2), for a range typical compartment characteristics.

As noted earlier, fuel load is generally a function of the live load (and a small portion of dead load), representing combustible items such as partitions, furniture, books and papers, wall hangings, etc. To understand the statistical relationship between live load  $L$  (i.e. fuel load) and normalized heat load  $H'$ , and thus resulting fire temperature  $T$ , a set of reasonable values for room geometry were chosen for consideration in eq. (2), where a compartment of 4 m x 4 m x 2.5 m, with  $A_v = 1 \text{ m}^2$  and  $H_v = 1.5 \text{ m}$ , was used. Next, a set of live load samples were generated with Monte Carlo Simulation (MCS), using the statistics presented above for live load. Then, the heat load for each live load sample was calculated using eq. (2), and the COV of the resulting heat loads was calculated. For the assumed geometric compartment parameters given above, COV of  $H'$  was found to be 0.45. To explore the effect of different compartment geometries, a large range of room parameter values was considered (spanning approximately an order of magnitude). However, it was found that the resulting COV was not particularly sensitive to changes in these parameters; COV ranged from approximately 0.44 - 0.51. Therefore, for the typical compartment of consideration, COV for temperature  $T$  was taken to be 0.45.

Note that, as per eq. (2), there is a strong relationship between  $H'$  and  $L_f$ . Increasing fuel load increases  $H'$  (and therefore temperature), and thus temperature  $T$  is dependent on live load  $L$ . However, little difference was found in the actual reliability results whether a fully-dependent or independent relationship between  $L$  and  $T$  was considered. Thus, reliability results are calculated assuming the fully-dependent case.

### **3. Resistance Model**

When components are subjected to fire, several different failure modes may be considered such as stability-related criteria based on the capacity of the structure; integrity

criteria to prevent fire and smoke from penetrating through the member, and insulation criteria based on a temperature limit on the cold side of the member [3, 10, 33]. Integrity and insulation criteria are generally more useful for partitions and walls, while limits on deflection become difficult to quantify and are not typically used for reliability analysis. Thus in this study, resistance is based on strength.

When exposed to fire, loss of capacity occurs because of reduced strength of the steel and the concrete, although the former becomes more significant as load eccentricity increases. Although various semi-empirical [2, 4, 10, 34-38] as well as finite element [39-41, 24] models exist for beams, little information is available for columns. Of the US guidelines, ASCE 29 [2] provides reinforced concrete column fire ratings to 1-hour increments as a prescriptive function of cover, section dimensions, and aggregate type. ACI 216-R [4] provides a summary of column fire test data, but gives no specific guidance as to the use of the data. Later, Dotreppe et al. [42], Tan and Tang [43], and Kodur and Raut [44] proposed semi-empirical models for regular columns exposed to fire, developing high temperature capacity reduction factors from finite element and experimental results.

For the reliability analysis procedure used in this study, a large number of simulations is required. This renders complex FEA approaches infeasible, as the required computational effort becomes impractically large. However, FEA and other advanced techniques are generally needed only for evaluation of complex, non-standard scenarios. For the simpler, typical cases of interest in this study, where regularly-shaped columns are subjected to temperature curves similar to standard fires, the available analytical approaches can often provide good results. Therefore, for this study, the model proposed by Tan and Tang [43] is used for strength evaluation. This model is based on the Rankine method [45] which develops an interaction

relationship between the plastic ultimate capacity of the column and its elastic buckling load under high temperatures.

This method expresses axial capacity  $P_n$  as the product of three factors: cold-strength plastic collapse load  $P_p$ ; a plastic load reduction factor  $u_{pr}$ ; and a modified buckling coefficient  $N_r$ :

$$P_n = P_p u_{pr} N_r \quad (3)$$

Here,  $u_{pr}$  is a function of the plastic collapse load, the axial load at balanced failure (i.e. simultaneous crushing of concrete and yielding of steel in tension), load eccentricity, and eccentricity at balanced failure; while  $N_r$  is a function of the plastic collapse load and elastic buckling load, as modified by a degradation of steel and concrete properties caused by fire exposure up to time  $t$ .

In this model, structural temperatures are not directly calculated, but the capacity reduction factor  $N_r$  is a linear function of factors  $\beta_c$  and  $\beta_{yr}$ , which are taken from Dotreppe et al [42], and modified by Tan and Tang.  $\beta_c$  is given as:

$$\beta_c(t_e) = \frac{\gamma(t_e)}{\sqrt{1 + (0.3A_c^{-0.5} t_e)^{A_c^{-0.25}}}} \quad (4)$$

where  $A_c$  is the cross-sectional area of the concrete section ( $\text{mm}^2$ );

$\gamma(t_e) = 1 - 0.3t_e \geq 0.85$ , and effective exposure time  $t_e = \alpha_{agg} \alpha_{fire} t$  (hours).  $\alpha_{agg}$  is a factor that adjusts for aggregate type and  $\alpha_{fire}$  adjusts for fires that deviate from ISO 834.  $\beta_{yr}$  is given as:

$$\beta_{yr}(t_e) = \gamma(t_e) \left( 1 - \frac{0.9t_e}{0.046c + 0.11} \right) \geq 0 \quad (5)$$

where  $c$  is concrete cover (mm). The original eqs. presented by Dotreppe et al. [42] are reported to have been curve-fit to finite element results which accounted for the energy required

to evaporate water and the non-uniform distribution of temperature throughout the column section. The equations were then refined and calibrated to a set of 78 experimental tests of columns, collected from Hass [46], Lie and Woollerton [47], and Dotreppe et al. [48], to account for additional factors including spalling. The final model, which assumes fire acts on all sides of the column, was found to have good agreement to the tests. Complete details of the model and validation results are given elsewhere [43].

In the model, a standard fire exposure is accounted for. In the present paper, fire temperature is taken as a random variable. This implies deviation from a standard fire, and must be accounted for in the resistance model used. An adjustment can be made in the Tan and Tang model for different fire exposures by expressing the fire exposure time  $t$  with an effective time  $t_e$  [43], where fires with a higher temperature than the standard fire up to a particular time  $t$  would have  $t_e$  greater than  $t$ . There are various ways to determine  $t_e$  for nonstandard fires, including empirical approaches as described in Eurocode 1 [31], as well as the equal area, maximum temperature, and minimum load capacity methods, as summarized in Kodur et al. [34]. However, Kodur et al. [34] found these various methods to be unreliable, and presented an approach for fire equivalency based on the concept of equivalent energy. The method involves computing the heat flux of the study fire and comparing it to the standard fire. In this approach, the energy  $E$  transferred to a structural member by a particular fire can be expressed as:

$$E = \alpha A \int (4\sigma\epsilon T^4 + h_c T) dt \quad (6)$$

where  $\alpha$  and  $A$  are constants;  $\sigma$  is the Stefan-Boltzmann constant ( $5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$ );  $\epsilon$  is emissivity (taken as 0.50);  $h_c$  is the convective heat transfer coefficient (taken as  $25 \text{ W/m}^2\text{K}$ ); and  $T$  is the fire temperature (K). Equivalent time can then be determined by setting  $E$  for the study fire at the desired exposure time  $t$  equal to  $E$  for the standard fire, and determining  $t_e$  for

the standard fire necessary for the equality to hold. In the current paper, the Kodur et al. [34] method is used to determine the time equivalency  $t_e$  for use in the Tan and Tang [43] resistance model for the random fire, up to the time  $t$  considered. Eq. (6) is evaluated by numerical integration of the standard and random fire time-temperature curves. The random fire time-temperature curve is determined by applying a factor to the standard fire temperature curve necessary to generate a heat flux equal to the realization of random variable  $T$ .

An axial capacity reduction curve for a typical column exposed to a standard fire as predicted by the Tan and Tang [43] approach is given in Figure 2. Note that the column capacity decreases rapidly then becomes asymptotic to a minimum level. This implies that the column has residual capacity after fire exposure. As compared to the test results, this model was found to be slightly conservative, with a mean under-prediction of axial capacity by a factor of 0.92 [43]. This results in a bias factor of approximately 1.09, which is used as a high-temperature professional factor in the reliability analysis.

Random variables important for reliability analysis for RC columns are steel yield strength  $f_y$ , depth of placement in the section,  $d$ , concrete compressive strength  $f_c'$ , section height  $h$  and width  $b$ , and professional factor  $P$ , the latter of which accounts for uncertainties in the analysis model used for design. The statistical parameters for these random variables are taken from Nowak and Szerszen [29] for cast-in-place columns, where distributions are reported as normal. There is insufficient statistical data to accurately determine the variation of steel yield and concrete compressive strength as a function of temperature [14, 22]. Therefore, the COVs of  $f_y$  and  $f_c'$  at elevated temperatures are taken as those at ambient temperature. A summary of the statistical parameters taken for resistance random variables are given in Table 1.

#### 4. Columns considered

An examination of the load and resistance models used can reveal that the following input parameters in the analysis may effect capacity, and therefore reliability, of columns exposed to fire if designed to ACI 318: cover, section dimensions and bar placement, aggregate type,  $f_c'$ ,  $D/(D+L)$  ratio, reinforcement ratio, slenderness ratio, and load eccentricity. Therefore, the reliability of various rectangular tied columns were studied, from  $t=0$  to  $t=4$  hours of fire exposure, by varying these parameters.

The base column for consideration (Figure 2) is taken as a rectangular section with width  $b=305$  mm (12 in), height  $h=305$  mm (12 in),  $f_c'=31$  MPa (4.5 ksi) with siliceous aggregate, two layers of 3-#8 steel bars with 38 mm (1.5 in) cover to a #3 tie (total cover to tension bar approximately 50 mm (2 in)). According to ASCE 29 [2] this column has a 2 hour fire rating. The base column has height of 3 m (10 ft) and is concentrically loaded with a  $D/(D+L)$  ratio of 0.50. Variations of this column are reported in the results section. All column variations are minimally designed according to ACI 318 [28] ( $\phi P_n = P_u$ ) for the combined effects of axial force and moment for eccentric loads, with the design load combination relevant to this study, as discussed above:  $1.2D + 1.6L$ . Note that this represents a design load combination valid for ambient or raised temperature; ACI 318 provides no basic load combination specifically for fires.

#### 5. Reliability Analysis

For reliability analysis, direct Monte Carlo simulation is used. The relevant limit state  $g$  is:

$$g = P_n(T) - D - Ls \quad (7)$$

where  $P_n(T)$  is determined using the Tan and Tang [43] resistance model described above, as a function of the random variables given in Table 1. Once a column design is selected for evaluation and its design strength determined according to ACI 318, its corresponding nominal (service) axial loads can be determined, depending on the  $D/(D+L)$  ratio considered. The corresponding mean values for the dead load random variable  $D$  and sustained live load random variable  $L_S$  used in the reliability analysis are then determined by multiplying the nominal loads by the appropriate bias factors. Thus, all cases considered are minimally designed; i.e. no column considered is under or over-designed for the loads applied according to ACI 318, so reliability indices for the different variations can be consistently compared with reference to the ACI 318 code standard.

At the end of the analysis, failure probability  $p_f$  is then determined by dividing the number of samples where  $g < 0$  by  $n$ , where  $n$  is the number of simulations conducted. Generalized reliability index is then reported in the results as  $\beta = -\Phi^{-1}(p_f)$ , where  $\Phi$  is the standard normal cumulative distribution function. The number of simulations  $n$  varies in the analysis to maintain sufficient accuracy and precision, depending on the expected failure probability. The number of simulations ranged from  $1 \times 10^6$  -  $1 \times 10^{10}$  (with associated range of  $p_f$  evaluated from about  $1.3 \times 10^{-3}$  -  $1.9 \times 10^{-8}$  for  $\beta$  from about 3 - 5.5) depending on the fire exposure time and column considered.

## 6. Results

Reliability indices ( $\beta$ ) as a function of fire exposure time are given in Figures 3-10. For compression-controlled columns (low load eccentricities), the base cold strength reliability index ( $t=0$ ) is approximately 5.8, whereas for tension-controlled, beam-like columns (high load

eccentricities), the base cold strength reliability index is approximately 5.4. These values represent upper limits of cold-strength safety for most columns reasonably designed and subjected to sustained live load  $L_s$ , as maximum reliability is limited by the large COV associated with  $L_s$ , as well as concrete compressive strength to a lesser degree. It should be noted that these reliability indices are greater than the cold-strength values given by Nowak and Szerszen [29] for cast-in-place columns as used for the ACI 318 Code calibration, which ranged from approximately 4.7-5.3 for the  $D+L$  load combination. The discrepancy is a result of the live load model used. As discussed above, for consideration with a 50-year fire, sustained live load is used (eq 7); this combination governs reliability over that of a 50-year fire and 50-year (transient) live load combination, once adjusting for the very low load coincidence probability of these two extreme events. However, for the ACI Code calibration, where fire was not considered, the load combination that governs is transient live load with dead load. As noted above, transient live load has a significantly higher bias factor than sustained live load (by a ratio of 1:0.24). Therefore, it should be emphasized that the reliability indices on the graphs represent those of columns exposed to an extreme fire ( $T$ ) in combination with arbitrary-point-in-time live (and dead) loads. Thus, for reliability indices beyond about 4.7-5.3 for most columns (other than those that are highly eccentric), results will be governed by load combinations other than fire (i.e. dead load and extreme, or transient live load), with values given by Nowak and Szerszen [29].

The effect of the different fire types considered on the base column is shown in Figure 3. As expected, the lower-temperature Eurocode parametric fire ( $F_v=0.02$ ) results in greatest reliability throughout the exposure time  $t$ , whereas the  $F_v=0.04$  parametric fires and the standard fire show nearly identical reliability results (the curves overlap on Figure 3 such the differences are not visible) up to the cooling phases of the parametric fires. This result is expected, given

the close temperature profile of the fires until the parametric fires begin their cooling phases (at approximately  $t=0.3$ , 0.6, and 0.9 hours for the fuel loads above). At this point, column reliability for the parametric fires begins to increase as the steel reinforcement temperatures decrease (not shown on graph). These points, and the accompanying minimum expected reliability from the burning phases of these fires, are indicated on the graph. Since the overall results in Figure 3 are somewhat similar, results of the remaining parametric investigations in Figures 4-10 are shown for the standard fire curve only.

Figure 4 shows the effect of  $D/(D+L)$  ratio. The general trend follows the  $P_n$  capacity change as a function of time similar to Figure 1. As can be seen, increasing this load ratio generally decreases reliability across all times. Nowak and Szerszen [29] also found this trend for cold strength column reliability. It occurs because live load is given a higher load factor for design, and this results in column designs that are slightly less conservative for larger proportions of dead load. Beyond about 1 hour of fire exposure, reliability index drops by nearly 2 as the  $D/(D+L)$  load ratio changes from 0.30 to 0.90, a substantial influence on safety.

Figure 5 illustrates the effect of reinforcement ratio. Here, reinforcement ratio was varied from 0.013 to 0.053, with higher reliabilities accompanied with lower reinforcement ratios. Similar in significance to changing load ratio, at exposure times beyond about 2 hours, reliability index drops by nearly 2 as reinforcement ratio increases from 0.013 to 0.053. The benefit of a low reinforcement ratio is due to the more detrimental result that fire has on steel strength than concrete, and thus columns with a higher proportion of steel have a correspondingly greater strength loss as temperature rises. Note here that increasing reinforcement ratio provides the column with greater nominal capacity, and thus as all cases considered are minimally designed to ACI 318, appropriately greater loads were applied to the high-reinforcement ratio column for

analysis. As a higher proportion of the total load is carried by the steel than concrete as compared to a low reinforcement ratio column, a greater proportion of total load capacity loss was seen from the high reinforcement ratio columns.

The effect of load eccentricity  $e$  is given in Figure 6, where  $e$  is varied from 0 to 1015 mm (40 in). Note the balanced eccentricity value for this column, where the steel and concrete fail simultaneously, is approximately 230 mm (9 in). Thus, compression controlled, transition, and tension-controlled sections are considered in the Figure according to ACI-318, where the first two cases ( $e=0$  and  $e=215$  mm (8.5 in)) have a strength reduction factor  $\phi = 0.65$ ; the case where  $e=370$  (14.5 in) has  $\phi = 0.80$ , and where  $e \geq 510$  mm (20 in),  $\phi = 0.90$ . As seen in the figure, a large difference in reliability results across the eccentricity values, from 2-3, with higher eccentricities producing lower reliabilities, though the effect becomes less sensitive at higher eccentricities, with no increases in reliability once the section becomes tension-controlled. This occurs for two reasons. First, higher eccentricities place a higher reliance on the steel to carry the resulting moment, which clearly cannot be resisted by the concrete alone when steel capacity is lost as temperature increases. Second, as noted above, the strength reduction factor  $\phi$  decreases as eccentricity decreases, resulting in more conservative designs. Moreover, ACI 318 limits nominal capacity to 0.80 of capacity under axial load only (i.e.  $e=0$ ) for tied columns, increasing the safety of the low eccentricity columns.

Figure 7 presents results considering different numbers of reinforcing bars (2, 3, and 5 bars per each side, symmetrically distributed), while keeping reinforcement ratio constant. As expected, with no eccentricity, steel distribution has no effect on reliability. However, with a mild eccentricity present ( $e=200$  mm (8")), such that the column is still compression-controlled, reliability index drops slightly, by about 0.5, from the 5-bar to 2-bar per side cases.

The effect of concrete strength is given in Figure 8, where compressive strengths from 28 MPa (4 ksi) to 55 MPa (8 ksi) are considered. As shown in the figure, increasing concrete strength generally increases reliability, an effect that occurs despite a small increase in bias factor for lower concrete strengths. Keeping reinforcing ratio constant, this result is due to the increased proportion of load that the higher-strength concrete carries as compared to the steel, an effect practically similar to lowering reinforcement ratio (see Figure 5).

Concrete cover is an important parameter for fire endurance in beams, but this does not have nearly the same significance on column reliability, as shown in Figures 9 and 10. In columns, significant load carrying capability can often be maintained with loss of steel, unlike most beam members. The results demonstrate minor differences, where changes in reliability from cover changes from 50 mm (2 in) to 100 mm (4 in) are small, with higher reliabilities associated with larger cover, as expected. In these cases, cover was changed by adding additional concrete on the shell of the column, keeping the column core size the same. In Figure 9, two sets of values are given, one for code-designed columns with the tied-column reduction factor  $k=0.80$  applied, and another set of identical columns for which the reduction factor  $k$  is not applied, resulting in an artificially large design load. The second set of values are more useful for comparison to Figure 10, which show the results of cover when eccentric loads are considered. For the columns in Figure 10,  $k=0$  if designed according to ACI 318. Therefore, comparing the second set of values in Figure 8 to those in Figure 10 may provide more insight to the effect of eccentricity and cover together, as the influence of the  $k$  factor is removed. Comparing these values, it appears that the eccentric columns have lower reliability, as expected, an effect noted in the results of Figure 6. Also notice that in Figures 9 and 10, the differences in reliability index tend to increase as fire exposure time increases, similar to results shown in

Figures 3-8, but only up to about 3 hours of fire time exposure. Beyond this time, reliability indices for the different cases of cover tend to converge. This occurs because, at approximately 3 hours, the insulating effect of the larger cover cases is overcome, resulting in similar steel temperatures to that for the smaller cover cases.

Several other effects were also considered in the reliability analysis, including the effect of siliceous or carbonate aggregate type; column section size (note that ASCE 29 uses column section size as one of the three factors used to determine fire rating, it does not distinguish among sizes greater than 305-356 mm (12-14 in)); column slenderness ratio ( $kl/r$ ); and bending about the strong or weak reinforcement axis of the base column pictured in Figure 2. It was found that these parameters had minor to no significant effect on reliability.

## **7. Comparison to Existing Codes and Standards**

Although ACI 216R [4] as well as the PCI Design Handbook [49] provide detailed calculation procedures to determine the fire resistance of reinforced concrete flexural members, no specific criteria are given for reinforced concrete columns. However, ASCE 29 [2] provides gross ratings for columns based on the criteria of cover, section size, and aggregate type. Using ASCE 29 criteria, all columns considered in this study have a 2 hour fire rating, with the exception of those with larger cover shown in Figures 9 and 10, which increase fire rating up to 3 hours for 75 mm (3 in) cover and 4 hours for 100 mm (4 in) cover. Throughout the figures, the lowest reliability index associated with a 2-hour (per ASCE 29) column is approximately 2 (Figure 4 for the case of  $D/(D+L) = 0.90$ ). Note the extreme eccentricities of Figure 6 are excluded here, where the columns behave more like beams and different models for fire resistance may be more appropriate. The highest fire rating for a 2-hour column appears in

Figure 5 for the lowest reinforcement ratio case ( $\rho=0.013$ ), and gives a reliability index of approximately 5. This range of 2-5 among the columns considered represents a very large difference in reliability index, and many of the columns considered in this study fall below the code target for cold-strength columns of 4.0 [29]. Of course, even a larger range of reliabilities is possible if different combinations of fire types or column characteristics are considered than those in this study.

Although many of the column reliabilities fell below cold-strength code targets when exposed to fires for a duration that they were rated for, the members are not necessarily inadequate in terms of safety level. This is because the reliabilities reported in this study were calculated for a certain fire load event in the assumed design lifetime (i.e. 50-year) of the column. However, due to the building occupancy type as well as the use of fire mitigation techniques such as sprinklers and fire-fighting, the probability of the column experiencing a structurally significant fire may be much less than certainty.

A statistically accurate fire occurrence frequency model may account for different occupancy types, compartment characteristics, and presence of fire mitigation systems. This is a complex task requiring development of building survey data and analysis which is beyond the scope of this study. However, the affect of fire occurrence frequency on reliability can be estimated from the expression:  $\beta = -\Phi^{-1}(\Phi(-\beta_f)p_{fire})$ , where  $\beta_f$  is the reliability of the column due to a certain structurally significant fire, as presented in Figures 3-10, and  $p_{fire}$  is the probability that the structurally significant fire will occur. Here the upper bound of  $\beta$  is limited to the cold strength reliability index given in [29].

## 8. Conclusions

A reliability analysis was conducted for various reinforced concrete columns designed according to ACI 318 that are exposed to fire. Based on the load and resistance models used, it was found that most columns with low load eccentricity had a cold-strength reliability index of approximately 5.8 while exposed to dead load and sustained live load. Reliability rapidly decreased as a function of time for the first 1-2 hours after fire exposure, and continued to decrease at a slower rate thereafter, to become asymptotic to a minimum reliability index that ranges between 2 and 4 for most cases, though some exceptions exist. The most significant parameters on reliability were  $D/(D+L)$  ratio, reinforcement ratio, and load eccentricity. Moderately effective in affecting reliability were number of reinforcing bars, concrete compressive strength and cover, while aggregate type, slenderness ratio, section size, and axis of bending had minor effects on reliability.

As significant variations in reliability for members of the same importance level designed to the same code standard are generally undesirable, it may be worthwhile to further investigate the need for additional ACI code calibration with an appropriate fire load combination, similar to that presented by Ellingwood [19] or Section C2 of ASCE 7 [11]. Before this work can be done, however, additional research is needed to better characterize the uncertainties in natural fire temperatures on the structural member as well as member resistance and thermal properties under high temperatures. Moreover, a fire occurrence frequency model is needed. This would allow development of appropriate load factors for possible consideration of fire as an additional load effect for design.

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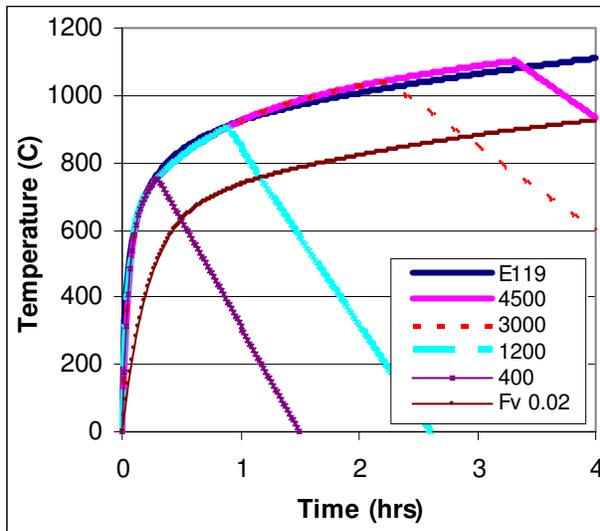
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**Table 1. Resistance Random Variable Parameters**

random variable	bias factor	COV
$f_y$	1.145	0.05
$d$	0.99	0.04
$f_c'$	1.10-1.23*	0.145
$b$	1.01	0.04
$h$	1.01	0.04
$P$	1.02, 1.09**	0.06

\*given as a function of  $f_c'$  (ksi): for  $f_c' \leq 55$  MPa (8 ksi),  $\lambda = -0.0081f_c'^3 + 0.1509f_c'^2 - 0.9338f_c' + 3.0649$ , which results in  $\lambda = 1.23$  for  $f_c' = 28$  MPa (4 ksi); for  $f_c' > 55$  MPa,  $\lambda = 1.10$ .  
 \*\*1.02 for cold strength, 1.09 when exposed to fire.



**Figure 1. Idealized Fires**

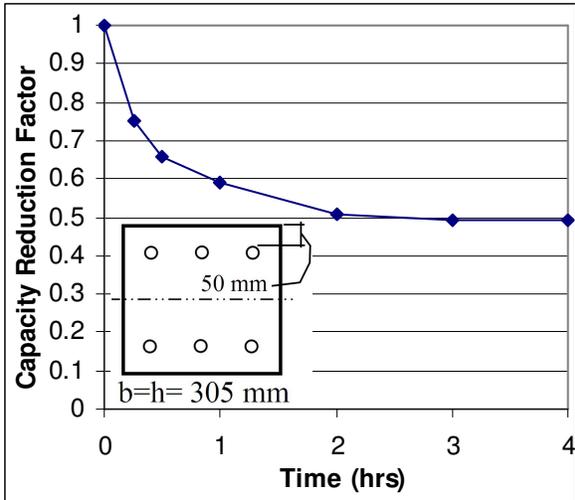


Figure 2. Typical Capacity Reduction of a Column Exposed to a Standard Fire

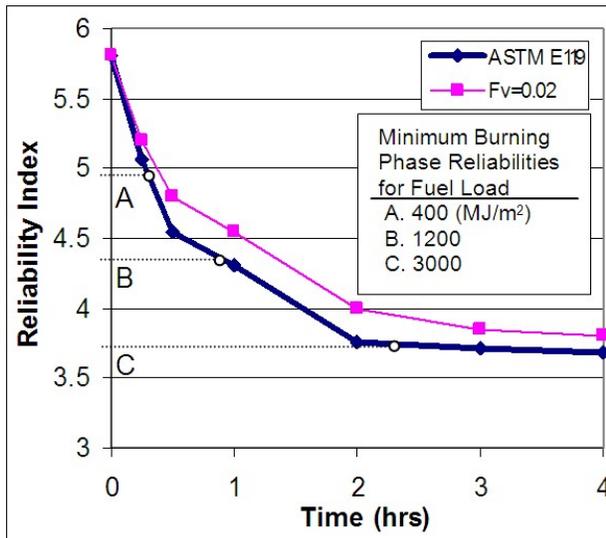


Figure 3. Effect of Fire Type

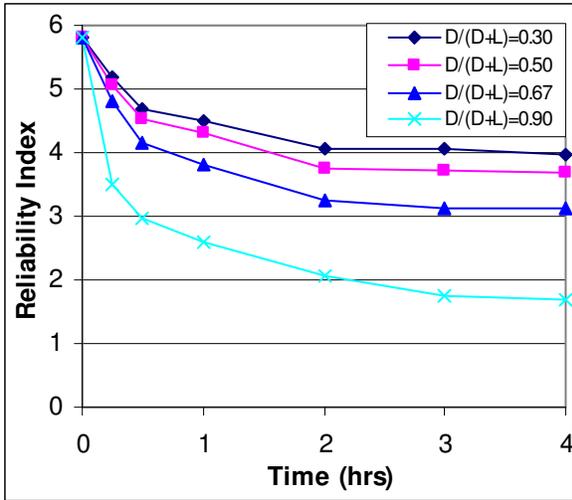


Figure 4. Effect of D/(D+L) Ratio

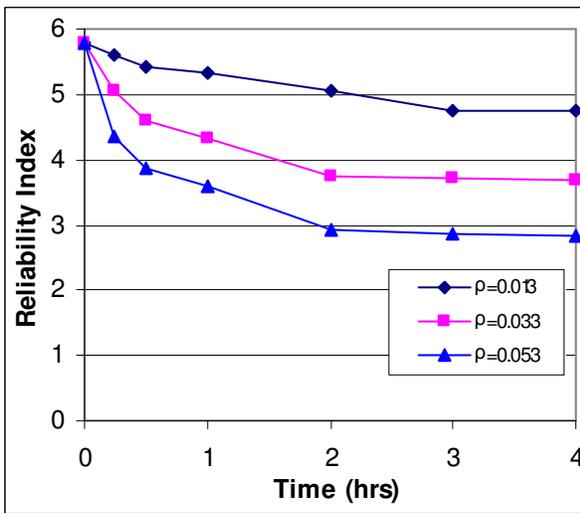


Figure 5. Effect of Reinforcement Ratio

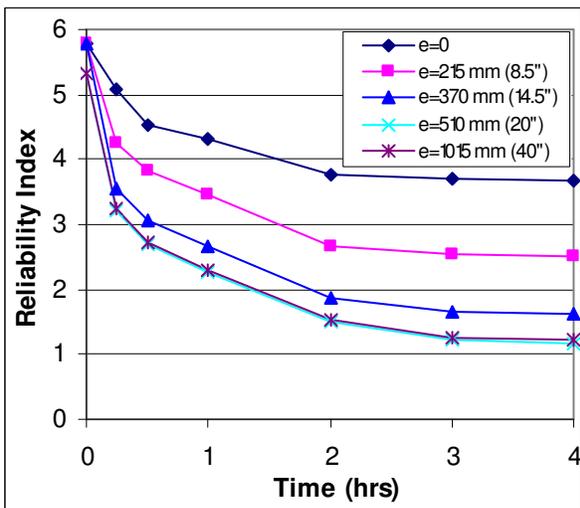


Figure 6. Effect of Load Eccentricity

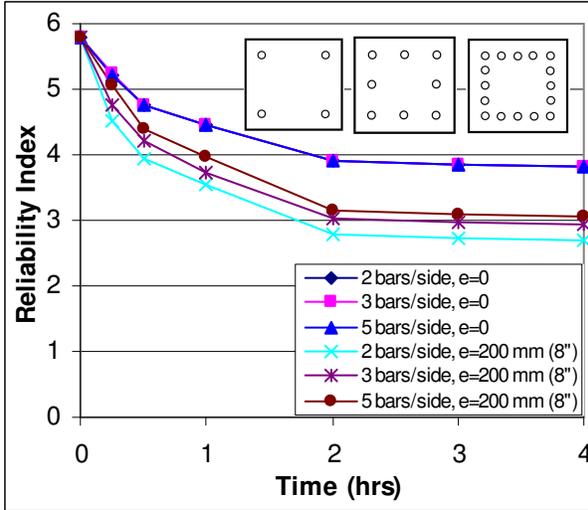


Figure 7. Effect of Number of Reinforcing Bars

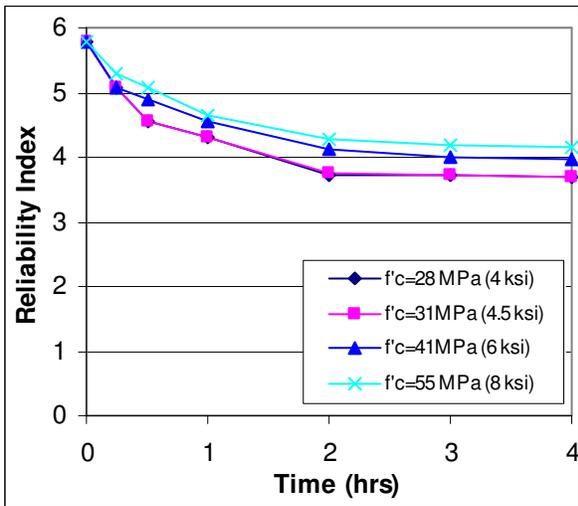


Figure 8. Effect of Concrete Compressive Strength

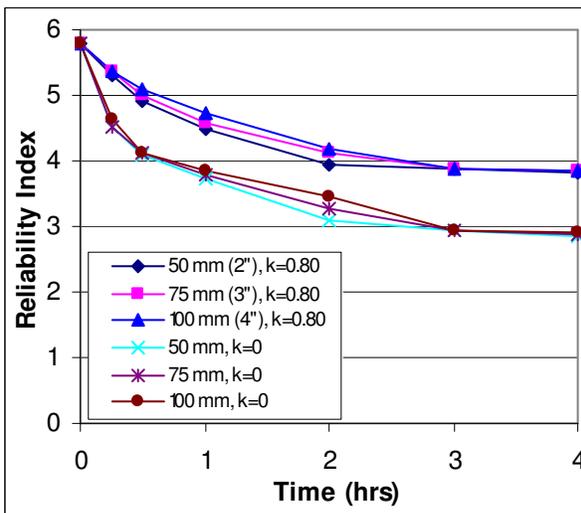


Figure 9. Effect of Cover, No Eccentricity

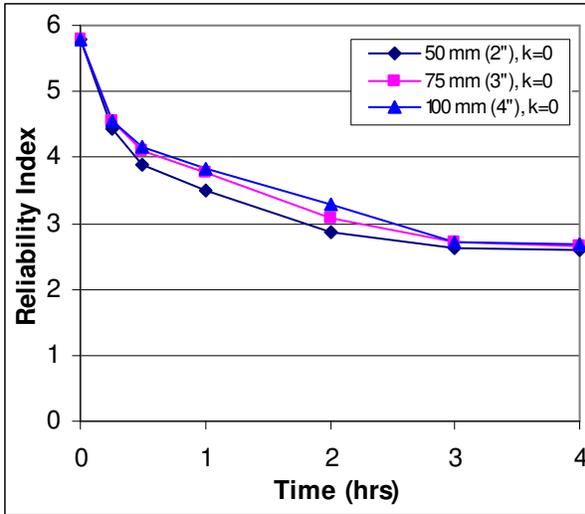


Figure 10. Effect of Cover with Eccentricity