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Reliability Analysis of Reinforced Concrete Beams Exposed to Fire

Christopher D. Eamon[1] and Elin Jensen[2]

Abstract

A procedure for conducting reliability analysis of reinforced concrete beams subjected to a fire load is presented. This involves identifying relevant load combinations, specifying critical load and resistance random variables, and establishing a high-temperature performance model for beam capacity. Based on the procedure, an initial reliability analysis is conducted using currently available data. Significant load random variables are taken to be dead load, sustained live load, and fire temperature. Resistance is in terms of moment capacity, with random variables taken as steel yield strength, concrete compressive strength, placement of reinforcement, beam width, and thermal diffusivity. A semi-empirical model is used to estimate beam moment capacity as a function of fire exposure time, which is calibrated to experimental data available in the literature. The effect of various beam parameters were considered, including cover, beam width, aggregate type, compressive strength, dead to live load ratio, reinforcement ratio, support conditions, mean fire temperature, and other parameters. Using the suggested procedure, reliability was estimated from zero to four hours of fire exposure using Monte Carlo simulation. It was found that reliability decreased nonlinearly as a function of time, while the most significant parameters were concrete cover; span/depth ratio when axial restraints are present, mean fire temperature; and support conditions.

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Introduction

Every year building fires cause significant loss of human life and tremendous damage to property. In 2005 alone, fires caused 3,762 deaths, 17,925 civilian injuries, and $10.7 billion in property damage in the United States (NFPA 2011). In addition to fire prevention techniques, various means of fire damage mitigation are used. Some of these include providing the proper architectural planning of exits and escape routes; the use of active fire protection techniques such as sprinklers to reduce the number of severe fires; and, providing structural fire protection to achieve a minimum fire resistance rating, with the intent to allow structural members to maintain their integrity throughout the escape and firefighting phases. A fire rating is frequently expressed in terms of time; i.e. the time which a member is expected to maintain its structural integrity when subjected to a standard test fire.

Traditionally, a structural member's fire resistance rating is determined by either conducting a fire endurance test such as specified in ASTM E119 (2005), or by calculation, which can be used for limited cases when previous fire endurance test results exist for similar structures (ACI 1989, 2007; ASCE 2006). A fire rating, however, provides no quantitative measure of safety in terms of failure probability, and the reliability of reinforced concrete structures exposed to fire loads is largely unknown. This is not consistent with prevalent Load and Resistance Factor Design (LRFD) philosophy, where load and resistance factors in various load combinations were specifically developed using probabilistic principles to ensure a consistent and adequate level of safety for structural members of the same importance level. In the case of fire resistance, there is no guarantee that members have a consistent level of safety, and in fact it is well-known that significant performance variation results in traditional prescriptive fire load design methods (Meacham 1997; Kruppa 2000; Kodur and Dwaikat 2011).

Furthermore, significant differences in treatment of other loads when structural members are exposed to fire exist among otherwise usually consistent structural load standards. For example, in ASCE 7, Minimum Design Loads for Buildings and Other Structures (ASCE 2010), the recommended loads that a structural member should carry during a fire for its given fire rating are taken as 120% of the service dead load and 50% of the service live load. However, ACI 216.1-07, Code Requirements for Determining Fire Resistance of Concrete and Masonry Construction Assemblies (ACI 2007), recommends application of 100% of dead load and 100% of live (service) load. These inconsistencies illustrate the lack of a systematic, probabilistically calibrated approach for consideration of fire across the prevailing standards used for reinforced-concrete member design.

Recognizing these problems, the fire engineering community has become interested in adopting a more systematic and rational way to assess and achieve a consistent level of fire safety (SFPE 2002). The general framework that allows the achievement of this goal is performance based design, which in regard to fire engineering, is a robust method allowing probabilistic assessment that is founded on the principles of fire science, heat transfer, and structural analysis. A 2008 position paper by the International FORUM of Fire Research Directors on the application of performance-based design for fire code application identified five research priorities needed to be fulfilled to achieve inclusive PBD (Croce 2008). One of these priorities is of interest to the topic of this study: the estimation of uncertainty and means to incorporate it into (structural) risk analyses when considering fires.

Over the last several decades, there has been limited research on the probabilistic analysis of structures exposed to fire, though diverse types of analyses have been considered. These include Beck (1985), who studied the reliability of structural steel members exposed to fires;

Shetty et al. (1998) who applied reliability analysis to assess the fire safety of offshore structures; Teixeira and Soares (2006) who estimated the reliability of load bearing steel plates subjected to localized heat loads; and Vaidogas and Juocevicius (2008), who considered the reliability analysis of a timber structure exposed to fire. Based on an analysis of load frequency, Ellingwood (2005) summarized practical design load combinations that need to be considered for fire design.

Only a few studies were identified in the technical literature that considered the failure probabilities of reinforced concrete structural elements exposed to fire. The earliest among these is Ellingwood and Shaver (1977), who considered the reliability of a T-beam assuming that loads were deterministic and that resistance was given by a Weibull distribution. Later, Courge et al. (2004) studied the reliability of a concrete tunnel subjected to fire, while Sidibe et al. (2000) and Wang et al. (2008) considered the fire reliability of reinforced concrete columns. Recently, Wang et al. (2010, 2011) provided calculations for the 1-hour fire reliability of a reinforced concrete beam exposed to random dead and live loads, but treated beam resistance as deterministic. Jensen at al. (2010) presented a preliminary estimation of the reliability of a fire-exposed reinforced concrete beam, with both load and resistance random variables, a precursor to the work presented here. Currently, however, there exists no systematic assessment of the reliability of RC beams exposed to fire that have been designed to current (ACI 318) standards considering both load and resistance uncertainties, nor an examination of the changes in reliability as various important beam parameters change. Moreover, there is no general fire-based reliability model currently available for conducting this analysis. As a step toward performance based design, this study presents a procedure that can be used to estimate the reliability of reinforced concrete beams exposed to fire. Using the suggested procedure, currently available high-temperature performance models and random variable data are incorporated to estimate safety levels of reinforced concrete beams designed according to ACI 318 Code (2011) exposed to a standard fire. The potential effect of changing various design parameters on beam reliability when exposed to fire is also investigated.

Load Models

For design as well as reliability analysis, the various loads that a structure may be subjected to over its design lifetime must be considered, such as dead load, occupancy and roof live loads, snow load, wind, and earthquake, among others. When reliability analysis involving an extreme load such as fire is conducted, care must be taken to establish the governing load combination(s). Here, not only load magnitude, but frequency of occurrence, and in particular, probability of simultaneous occurrence with other loads, becomes important. Based on an analysis of the frequency of occurrence of these loads relative to that of a structurally significant fire, it can be determined that some load combinations with fire can be practically neglected for calculation of reliability indices β that are at or below typical code targets (approximately when $\beta \leq 3.5$ - 4). By examining the coincidence rates of various extreme loads (in the United States) with a structurally significant fire, Ellingwood (2005) excluded the need to consider fires coincident with extreme loads involving snow (coincidence rate 2×10^{-8}/yr – 1.7×10^{-7}/yr); earthquakes (coincidence rate 4.6×10^{-11}/yr – 9.2×10^{-12}/yr); winds (coincidence rate 3.7×10^{-9}/yr); roof live loads (coincidence rate 1.7×10^{-8}/yr), and transient occupancy live loads (coincidence rate 3.2×10^{-9}/yr). This leaves the sustained loads for consideration in combination with fire: dead load and sustained (occupancy) live load.

Statistical parameters for dead load, the permanent gravity loads on the structure, are well known and available in the literature. When different mean values are considered for a random variable used in a series of reliability analyses (for example, analyses considering different mean concrete strengths), it is often convenient to describe statistical parameters in terms of normalized values. Thus, bias factor λ is frequently reported in the literature, which represents the ratio of mean value to the nominal value. Similarly, coefficient of variation (COV) is often used, the ratio of standard deviation to mean value. Dead load is typically assigned a bias factor of 1.05 and COV of 0.10. It is normally distributed (Nowak and Szerszen 2003).

Occupancy live load has two components: transient live load and sustained live load. Transient live load represents extreme loads for rare, special events such as emergencies, crowding, or remodeling. This load component becomes important in typical reliability analysis of structural members used for code calibration, such as that conducted by Szerszen and Nowak (2003) for the 2002 ACI 318 Code. As noted above, this live load component is generally not important when considering fire due to its low coincidence probability, and is not considered further here. Sustained, or 'arbitrary-point-in-time' load, L_s, represents the typical load on the structure at any particular time, primarily representing movable items such as furniture, partitions, and other contents. Bias factor for sustained live load has been reported to range from approximately 0.24 – 0.50, depending on tributary area and occupancy type, with COV from 60-0.65. L_s is typically modeled with a gamma distribution (Ellingwood 2005; Nowak and Szerszen 2003). In this study, sustained live load is taken with bias factor of 0.24 and COV of 0.65, as assumed by Nowak and Szerszen (2003).

Depending on fuel load, ventilation, convective and radiative properties of the compartment, as well as other factors, fires will produce various temperature-time profiles. It is this resulting temperature profile which causes a temperature rise in the structural member and causes a loss of capacity as a function of time. To conduct consistent reliability analysis, it is useful to consider a standard fire profile. Therefore, throughout most of the analysis, the mean value of fire temperature $\overline{T}$ is taken to be that given by the standard fire temperature *(T)*-time *(t)* profile used for fire rating in ASTM E119 (2005), which can be approximated with eq. (1):

$$\overline{T}\,(°C) = 750\left(1 - e^{-3.79553\sqrt{t}}\right) + 170.41\sqrt{t} + T_0 \tag{1}$$

where t is time (hours) and T_0 is the ambient temperature, taken as 20 ºC. Methods are available that can relate the effect of any fire to that of a standard fire, if desired (Kodur et al. 2010). As fire temperature is considered a random variable (RV) in this study, statistical information regarding its variability (i.e. COV) is also needed. The variation in temperature experienced by a structural element in a fire depends on various parameters including fuel load, ventilation, room geometry, and other compartment characteristics. In this study, fire temperature COV was estimated for a typical range of compartment characteristics by determining how a variation in fuel load in the compartment affects component heat load. Based on a series of compartment burn tests, this relationship was developed by Hamarthy and Mehaffey (1984). As fuel load itself is essentially a function of the sustained live load (and a small portion of the dead load) in the building, representing combustible building components such as furniture, partitions, books and papers, etc., a set of sustained live load samples was first generated with Monte Carlo simulation (MCS), then, the heat load for each sample was calculated using Hamarthy and Mehaffey's relationship for a typical compartment, and the COV of the resulting heat loads was calculated. Depending on the compartment characteristics considered, COV of the resulting heat load was found to range from approximately 0.44 - 0.51, with the most representative case having a COV of 0.45. Note that there is a strong relationship between fuel load and fire heat load, and thus the live load L_s and temperature T random variables are not independent. In this study, both the independent as well as fully-dependent cases were considered. However, little difference was found in the reliability results between the two cases, and thus the relationship between these RVs was taken as fully dependent.

Resistance Model

Various failure modes may be considered for members subjected to fires, including stability-related criteria such as strength and deflection; integrity criteria to prevent fire and gasses from penetrating through the member, and insulation criteria that limit the temperature on the cold side of the member (NISTIR 2009; Kodur and Dwaikat 2011). Integrity and insulation criteria are generally more useful for partitions and walls, while limits on serviceability become difficult to quantify and are not typically used for reliability analysis. Thus in this study, resistance is based on strength. For tension-controlled, rectangular beams in which all steel reinforcement yields at ultimate capacity, nominal moment capacity as a function of temperature, $M_n(T)$, can be computed as (NISTIR 2009):

$$M_n(T) = (A_s' \cdot f_y(T_r)) \cdot (d - d') + (A_s - A_s') \cdot f_y(T_r) \cdot (d - \frac{a(T_c)}{2}) + M_r(T) \qquad (2)$$

where A_s and A_s' are the areas of tension and compression steel, while d and d' are the depths of the tension and compression steel centroids, respectively, which remain temperature-independent. Other properties, such as steel yield stress $f_y(T_r)$ and concrete compressive strength $f'_c(T_c)$ are a function of temperatures of each reinforcement bar (T_r), as well as temperature of the concrete (T_c), while $a(T_c)$ is the depth of the compressive stress block, given by

$a(T_c) = \dfrac{(A_s - A_s') \cdot f_y(T_r)}{0.85 f_c'(T_r) \cdot b}$, and $M_r(T)$ is additional resistance caused by axial and/or rotational thermally-induced restraints on the section.

Little information is available on other fire-induced failure modes such as shear and torsion, and are beyond the scope of this study. RVs important for reliability analysis for RC beams in flexure are steel yield strength f_y, depth of placement in the section, d, concrete compressive strength f_c', beam width b, and professional factor P. The professional factor is used to account for uncertainties in the typically conservative analysis models used to establish member strength; for example, assumptions that concrete crushes at a strain of 0.003, that steel is elasto-plastic, etc. (Melchers 1999). The statistical parameters for these RVs are taken from Nowak and Szerszen (2003), where distributions are reported as normal. There is insufficient statistical data to accurately determine the variation of steel yield and concrete compressive strength as a function of temperature (Beck 1985; Ellingwood and Shaver 1977). Therefore, the COVs of f_y and f_c' at elevated temperatures is taken as that at ambient temperature. For high temperature analysis, thermal diffusivity α, is also considered as an RV, with COV taken from Shin et al. (2002). Thermal diffusivity is given as:

$$\alpha = \frac{k}{\rho \cdot c_p} \tag{3}$$

where k is thermal conductivity, ρ is density, and c_p the specific heat capacity. Thermal diffusivity can be thought of as a measure of how quickly heat flows through a material, and can be calculated by eq. 3 for a particular specimen by experimentally measuring k, ρ, and c_p. Mean value for α is highly variable and dependent on the type of section and material properties considered. As mean α significantly impacts results, in this study it is determined with a special calibration procedure detailed below. A summary of the statistical parameters taken for RVs is given in Table 1.

In this study, fire acts on the bottom and sides of the beam section, while the top is assumed be in a different compartment or protected by a floor slab. When exposed to fire, loss of capacity occurs because of reduced strength of the steel and the concrete, although the former is much more significant when beams are tension-controlled as required by ACI-318. To describe the loss of strength of a reinforced concrete beam, a fire-based resistance model must account for two major effects: the change of temperature in the material at various points of importance, such as the steel reinforcement and in the compressive zone of the concrete; and how the change in temperature affects strength. The latter effect is generally modeled by fitting curves to experimental results, though there is much scatter in the data.

Various temperature-yield stress curves have been proposed for steel (for example, Luecke 2005; Eurocode 2002; BSI 1987). For this study, steel yield strength reduction factor r is taken as (BSI 1987):

$$r = \frac{720 - (T_r + 20)}{470}\,; 0 \le r \le 1 \tag{4}$$

For concrete, researchers have found differing compressive strength-temperature relationships (Harmathy 1993; Phan 2002; Abrams 1971; Castillo 1990; Diederich 1993; Harada et al. 1972; Lie 1992). However, in tension-controlled beams, concrete properties have minimal impact on moment capacity, which is governed by the tension steel, and the choice of concrete model used was found to have little influence on the final results. In this study, the reduced section (500 °C isotherm) method proposed by Anderberg (1978) is used, where $f_c'(T_c)$ is held constant in the analysis but the size of the effective compression block is reduced. Here, the initial compressive block size is based on the traditional Whitney model, where at section ultimate capacity, the compressive block contains a uniform stress of $0.85f_c'$ to a depth of $\beta_1 c$, where β_1 varies with concrete strength and c is the neutral axis depth, with a block width equal to the beam width b (ACI 2011). The 500°C isotherm represents the boundary in the beam section where temperature is exactly equal to 500°C, which varies as a function of time depending on external temperature, section shape, and thermal diffusivity. In the Anderberg method, the concrete compressive block size is reduced by eliminating the compressive capacity of the material at locations where internal temperatures are greater than 500 ºC ($f_c' = 0$), and concrete is given full compressive strength ($f_c'(T) = f_c'$) at locations in the section where temperature is less than 500 ºC. Although more refined models are available (e.g. Hertz 1981), a investigation revealed that minimal difference in $M_n(T)$ resulted when tension-controlled beams are considered.

A more difficult effect to model is the change in temperature throughout the section as external temperature and time changes. This is a function of section geometry, material density, specific heat, and other factors. If conduction is the only heat transfer mechanism and if thermal conductivity is constant, two-dimensional heat transfer and resulting temperature T with respect to time t and coordinate directions x and y within a section is governed by the following relationship:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \tag{5}$$

However, the above expression often becomes difficult to solve analytically. Thus, various models have been proposed to approximate this behavior, including finite element approaches (Bratina et al. 2005; Dwaikat and Kodur 2008; Wang et al. 2011) as well as semi-empirical approaches (Wickstrom 1986; Hertz 1981).

For the reliability analysis used in this paper, a large number of simulations is needed. This practically precludes use of involved FEA approaches, as the required computational effort becomes too great. However, FEA is generally only needed for complex, non-standard cases, while the empirical approaches available can often provide good results for regularly-shaped sections subjected to standard fires, which are of interest to this study. To determine how internal temperature changes in the section as a function of time (*t*; hours) and external temperature *(T)*, a specially calibrated version of Wickstrom's model (Wickstrom 1986) is used. The Wickstrom model was developed by conducting a series of finite element analyses of reinforced concrete sections exposed to fire, and determining the resulting concrete and reinforcement bar temperatures as a function of time (Wickstrom 1985). The analyses included a reinforced concrete material model that considered varying thermal conductivitv, the influence of water evaporation, and non-linear thermal boundary conditions. From the results of the analysis, curves were constructed to fit to the temperature data as a function of the fire time-temperature curve, individual rebar placement within the section, and thermal diffusivity. Similar approaches have been developed for different materials and design scenarios in the form of tables and charts by ACI (1989), PCI (Gustaferro, A.H. and Martin, L.D., 1989), ASCE (2005), and Eurocode (2002), among others. For the Wickstrom model, excellent agreement to the FEA results were reported for regular section shapes (Wickstrom 1986). For further verification, as part of this study, a selection of beam configurations described below were modeled with the thermal FEA code SAFIR (2011) and compared to results from Wickstrom's model. Here very good agreement was found (i.e. differences generally within a few percent). It should be noted that rectangular sections with typical configurations and material properties were considered for development of the model, as the beams used in this study, and for more complex scenarios, advanced techniques such as finite element or finite difference analysis should be used. In the Wickstrom model, the temperature of the steel reinforcement T_r is given by:

$$T_r = \left(n_w\left(n_x + n_y - 2n_x n_y\right) + \left(n_x n_y\right)\right)T \qquad (6)$$

where n_w represents the ratio of the beam surface temperature rise and that of the fire temperature, and n_x, and n_y are used to determine the ratio between the temperature rise on the surface to that of an interior point in the section, as a function of fire temperature, position and time. These values are defined as:

$n_w = 1 - 0.0616t^{-0.88}$

$n_{s\ (s=x,y)} = 0.18ln(\alpha_r t/s^2) - 0.81$

s is the distance of the center of the reinforcement bar considered to the outer edge of the concrete section, measured in the x or y coordinate direction, as appropriate (m), with a limit imposed of: $s \geq 2h - 3.6(0.0015t)^{0.5}$

α_r is the ratio of thermal diffusivity considered to a reference value of $0.417\text{x}10^{-6}$ m^2/s.

To account for reductions of concrete compressive strength, the position of the 500 ºC isotherm in the section is needed. For compressive blocks exposed to fire from the sides of the section, using the Wickstrom model, it is given by the following, measured from the outer edge of the beam (Purkiss 2007):

$$x_{500} = \sqrt{\frac{\alpha_r t}{\exp\left(4.5 + \frac{480}{0.18 n_w T}\right)}} \qquad (7)$$

Once this is located, the effective width of the compression block as a function of concrete temperature (T_c) becomes: $b(T_c) = b - 2x_{500}$. Here the '2' assumes that the fire is encroaching on both sides of the beam. As noted above, this reduced effective section width takes the place of reducing f_c' at higher temperatures.

An important factor in Wickstrom's model is the thermal diffusivity ratio α_r. It is known that α_r changes based on density, aggregate type, temperature, and other factors (Morabito 1989; Shin et al. 2002; Bentz et al. 2011; Purkiss 2007, Van Green et al. 1997). Increasing α_r results in faster heat transfer through the section, and a decrease in time to failure. A good estimation of α_r is important because capacity results are sensitive to this value, as shown in Figure 1. There is insufficient experimental data to reliably establish mean α_r for each of the beam cases considered. Thus, in this study, mean value for α_r is determined by calibration. In this procedure, mean α_r is determined such that Wickstrom's model, used in conjunction with eq. (2), provides a fire rating, or predicted time of failure, consistent with experimental results or more advanced numerical models. This resulting α_r value may not only represent the effects of different material thermal properties, but also how other factors not directly included in the Wickstrom model would effectively modify heat flow as well.

The α_r calibration is made to the fire rating method presented by Kodur and Dwaikat (2011), who recently developed an approach to determine the fire rating of typical reinforced concrete beams, based on a database of experimental tests and finite element simulations under standard fires. In their model, a base fire rating is obtained from the procedure given in AS 3600 (2001), which is a function of beam width and cover, and can be obtained to a high degree of fidelity, to the nearest minute rather than in half-hour or hour increments as with the ACI (1989) or ASCE (2006) approaches. This initial rating is then modified to account for various factors such as load ratio (in terms of applied load / nominal section capacity); reinforcement ratio; proportion of corner bars; beam area to beam heated perimeter; aggregate type; span to depth ratio; as well as support conditions including axial and rotational restraints. They reported a good fit to the experimental data, with significant improvements over existing code-based methods.

In this study, the Kodur and Dwaikat approach is first used to determine a nominal fire rating, or time to beam failure, to the nearest minute, for the specific beam being considered for analysis. Then, an effective α_r is determined in Wickstrom's model that would result in a matching prediction of time–to-failure, anchoring point 'B" on the horizontal line in Figure 1, for the particular beam considered. Depending on aggregate type and other section characteristics, typical values of effective α_r ranged from approximately 0.75-1.5, which appear reasonable, and are within spread of actual α values reported for different concrete materials (Purkiss 2007; Van Green et. al. 1997). Once the Wickstom model is calibrated to the specific beam case considered, it can then be used to estimate beam capacity as a function of time, between the fixed points A and B in Figure 1.

Because the Kodur and Dwaikat (K-D) approach was intended for design use, it was conservatively developed such that the lower bound of fire rating for most of the study beams was predicted. However, for the reliability analysis in this study, the K-D method is used for analysis rather than design, and the best estimate of fire rating is desired, rather than the lower bound. Based on the fire rating data of the study beams given by Kodur and Dwaikat (2011), the bias factor of their procedure can be determined by computing the mean ratio of the actual failure time of the beam samples to that predicted by the model. This was found to be λ=1.38 for simply-supported beams and λ=1.30 for rotationally-constrained beams. For axially-constrained beams, bias factor was found to vary with beam span/depth ratio, and is given by

$\lambda = 7.8\left(\frac{L}{d}\right)^{-0.64}$. Thus, when used in this study, the failure time predicted by the K-D approach was multiplied by the bias factors above to get best estimate of actual failure time.

In summary, beam capacity at any point in time considered (as shown in Figure 1), is generated from eq 2. The concrete and steel strengths used in eq 2 are found by use of eqs. 4 and 7, while the reinforcement temperatures needed for eq. 4 are found from eq 6. In eqs. 6 and 7, thermal diffusivity (expressed in terms of α_r) is an important input parameter, which varies for different beams, and for which no specific data are available. Thus, for each case, α_r is determined (by an iterative process) that would result in a beam failure (fire rating) occurring at the time given by Kodur (2011). Establishing this value for α_r sets a lower point on the time-capacity curve generated by the model (point B on Fig. 1). Thus, with points A (initial cold capacity) and B established, the model is considered calibrated, and can then used to determine values of capacity between points A and B prior to failure, for which the reliability analysis will be conducted.

Beams considered

By studying the load and resistance models used, it can be seen that the following parameters may effect capacity, and therefore reliability, of beams exposed to fire if designed to satisfy ACI 318: cover, beam width, aggregate type, f_c', $D/(D+L)$ ratio, reinforcement ratio, proportion of corner bars to total bars, support conditions, fire temperature, and heated perimeter to area ratio. Therefore, the reliability of various rectangular beams were studied, from $t = 0$ to $t = 4$ hours of fire exposure, by varying these parameters.

The base beam for consideration is taken as a rectangular section with b = 305 mm (12 in), h = 610 mm (24 in), f_c'= 28 MPa (4 ksi) with siliceous aggregate, 4 - #9 tension steel bars and 38 mm (1.5 in) cover to a #3 stirrup on the sides and bottom (total cover to tension bar 48 mm (1.875 in)). The base beam simply spans 4.5 m (15 ft) and is uniformly loaded with a $D/(D+L)$ ratio of 0.50. Variations of this beam are reported in the results section. All beams are minimally designed according to ACI 318 in terms of moment capacity ($\phi M_n = M_u$), with the design load combination relevant to this study, as discussed above: $1.2D + 1.6L$. All beams are tension-controlled, with $\phi = 0.90$. Note that a T-beam configuration does not alter the reliability calculations, and thus results would also apply to T-beams with the same parameters considered.

Reliability Analysis

In this study, direct Monte Carlo simulation is used to calculate reliability. For a given beam design and time after fire initiation for which reliability is to be computed, for simulation *i*, the sampling process becomes:

1. Load and resistance RVs are sampled based on the statistical parameters given in Table 1, and basic beam parameters (M_n, cover, width, etc.) are calculated.

2. The time to failure (fire rating) of the sampled beam is determined with the K-D approach, and adjusted with the appropriate bias factor for the support conditions considered.

3. Wickstrom's model is calibrated so it can be used to determine beam capacities before failure. As the relationship between α_r and $M_n(T)$ is nonlinear, this requires a process of iteration, whereby a Newton Raphson procedure is used to determine α_r that results in Wickstrom's model matching the fire rating predicted from the K-D result for the beam.

4. Using the calibrated Wickstrom model from step 3, moment capacity $M_n(T)$ of the sampled beam at time t at which reliability is to be computed is determined. Time t ranges from 0 to 4 hours.

5. Evaluate the limit state function $g = M_n(T) - D_M - L_{sM}$, where $M_n(T)$ is determined from step 4, and dead load D_M and live load L_{sM} moments are computed based on the sampled load RV values from step 1. For simulation i, it is recorded if $g < > 0$.

6. Repeat for n simulations.

7. Using the Monte Carlo process, failure probability p_f is then determined by: *(number of samples $g < 0$)/n*. Generalized reliability index is then reported in the results as $\beta = -\Phi^{-1}(p_f)$, where Φ is the standard normal cumulative distribution function.

The number of simulations n varies in the analysis to maintain sufficient accuracy and precision, depending on the expected failure probability. The number of simulations ranged from $1\text{x}10^6$-$1\text{x}10^{10}$, depending on the time and beam considered.

Results

In this study, reliability index β is used to measure safety level. Most components designed by LRFD have calculated reliability indices between 3.5 and 4.5, with 3.5 and 4.0 being code target levels for beams and columns in ACI 318, respectively. However, it should be emphasized that, due to modeling simplifications and limited statistical data to characterize RVs, the p_f usually obtained from reliability analysis are generally not used to represent failure probabilities of actual structures, which are typically significantly higher than the theoretically calculated values. Rather, β are more practically used as a tool allowing consistent comparison of safety level rather than direct p_f assessment.

Reliability indices (β) as a function of time are given in Figures 2-7. For all beams, the base cold strength reliability index (t = 0) is approximately 5.4. Note that this is much higher than the cold-strength values reported by Nowak and Szerszen (2003) in the ACI 318 Code calibration, which ranged from approximately 3.5-4.4 for the $D + L$ load combination (for designs with $\phi = 0.90$). The reason for the discrepancy is the live load model used. Recall, based on an analysis of load coincidence probability, arbitrary-point-in-time loads (i.e. dead load and sustained live load) combined with fire will govern reliability when β-values are less than about 3.5, whereas for the ACI Code calibration (neglecting fire load), transient live load (i.e. 50 year maximum) is considered, which is accompanied by a significantly higher bias factor. Therefore, it should be kept in mind that values on the graphs represent reliabilities of beams exposed to fire (T) in combination with arbitrary-point-in-time dead and live load values: $D + L_s + T$. For reliability indices beyond about 3.5, results will be governed by load combinations other than fire, with values shown in Nowak and Szerszen (2003).

Figure 2 shows the effect of $D/(D+L)$ ratio. As can be seen, increasing this load ratio generally decreases reliability across all times. A similar effect was observed by Nowak and Szerszen (2003) for cold strength beam reliability, and occurs because live load is accompanied by the higher load factor used for design, and thus the beam is designed (slightly) less conservatively as dead load proportion increases. The general trend is an initial concave down shape, following the M_n capacity change as a function of time similar to Figure 1. However, near the 1 hour mark, the curve changes concavity and becomes asymptotic to a minimum value. This lower bound (as well as the upper bound of approximately 5.4) occurs primarily because of the high COV of live load (0.65). In probabilistic analysis, as COV of the RVs increase, the maximum possible value of β (either positive or negative) decreases. Although the curves appear close together, significant differences are present near mean failure time, which is at the point on the graph where $\beta = 0$ (i.e. $p_f = 0.50$; note a negative generalized β results in $p_f > 0.50$); consider the difference between the *D/(D+L)* case of 0.90 and 0.50, where mean failure times are approximately 2.8 hrs and 4.0 hrs, respectively. Similarly, reliability index varies by about 0.5 between the minimum and maximum load ratios considered at 3-4 hours. Thus *D/(D+L)* load ratio has a substantial influence on reliability.

Concrete cover is recognized as a critical measure of fire endurance, and this is borne out in the reliability indices presented in Figure 3, which shows large differences in mean failure times, close to 2 hours, when cover is changed from 38 mm (1.5 in) to 25 mm (1.0 in). Although a cover of 25 mm (1.0 in) will generally not satisfy ACI-318 design criteria for beams, slabs may be designed as such.

Figure 3 also shows the affect of beam width, which is clearly a significant factor. Changing from b = 305 mm (12 in) to b = 710 mm (28 in) results in an increase in mean failure time close to 1 hour for reliability levels approaching zero (i.e. close to failure time). This is a direct result of the resistance model, which predicts significant increases in fire resistance as beam width increases.

The effect of siliceous or carbonate aggregate type is shown in Figure 4. The use of siliceous aggregates generally increases thermal diffusivity over carbonate aggregates, resulting in a faster rate of temperature rise in concrete and reinforcement. In Figure 4, two *D/(D+L)* ratios are considered (0.3, 0.9) with the use of both aggregate types (*'Sil'* and *'Carb'* in the graphs). It can be seen that aggregate type has a measurable effect on reliability, but not to the extent that the extreme differences in *D/(D+L)* load ratios have.

The number of corner bars to total tension bars *("c/t")* is also shown in Figure 4, which shows results for the beam with 2, 4, and 8 total bars. Following the trend near the lower portion of the curve, mean time to failure appears to differ by approximately 0.5 hrs, though resulting differences in reliability index at these times is small. The effect of compression steel area was found to be is insignificant, as were changes in the ratio of the heated beam perimeter to beam cross-sectional area.

As expected for tension-controlled beams, concrete compressive strength makes little difference in reliability, as shown in Figure 5, with a small decrease for higher strength concretes. This change is due to bias factor, which as shown in Table 1, decreases for high strength concretes. Even so, here the resulting increase in mean concrete strength itself is not directly important. Rather, the higher bias factors in lower strength concretes result in a smaller stress block depth relative to the nominal value, which slightly increases the moment arm *'jd'*, and thus increases section moment capacity to a greater degree over the nominal value.

Figure 5 also shows the results of altering reinforcement ratios. Three ratios are considered, approximately bounding the ACI Code-allowed minimum and maximum (such that ϕ remains ~ 0.90) values for the beam. In general, differences in reliability are small, with slight increases accompanied by higher reinforcement ratios. As noted earlier, all beams are designed to ACI minimum requirements ($\phi M_n = M_u$), such that beams with higher reinforcement ratios are accompanied by appropriately higher design loads for consistent reliability analysis. Increasing reinforcement ratio provides no significant reliability gain for cold strength analysis; however, slight advantages are realized for fire resistance based on the results of Kodur and Dwaikat (2011).

Figure 6 demonstrates the effects of changing the axial restraint force and span/depth ratio of the section. If an axial restraint is imposed below the neutral axis of the beam, a thermally-induced thrust is generated at higher temperatures that results in a negative moment at midspan of a simply-supported beam, effectively increasing moment capacity. The values in Figure 6 are based on an axial restraint imposed at the beam centroid (305 mm (12 in) from the top; the neutral axis at ultimate capacity is approximately 180 mm (7 in) from the top). For a fixed span/depth ratio (L/d = 13 in the base beam), the effect of axial restraints appears small. Here, three axial restraint ratios a_x (stiffness of the axial restraint / axial stiffness of the section) are considered, 0.1, 0.2, and 0.3. However, as also shown in the figure, adjusting span/depth ratio *(L/d)* for a fixed axial restraint ratio, taken as 0.10, has a large effect, changing reliability index by about 2 or more throughout the range of times considered. When L/d ratios increase, beam deflections also increase, decreasing the distance between the (fixed) axial constraint force position at the ends of the beam and the (deflected) neutral axis of the beam, and lowering the resulting negative moment gained from the thermal thrust. For large L/d ratios, such as the L/d = 18 in Figure 6, deflections are large enough to cause the beam neutral axis depth to pass below the position of the axial restraint when ultimate loads are applied. Thus in the large L/d case, the axial restraint decreases moment capacity as temperature increases; as described above, the reverse occurs when L/d is small. Full rotational restraints (i.e. fixed moment reactions) provide a similarly large effect on reliability, as shown in Figure 6 ("*ROT=0*").

Although most fire load analysis in the US is based on the standard (ASTM E119) fire time-temperature profile, it may also be of interest to study the effect of different fires on beam reliability. For this study, three additional mean fire temperatures were considered, as given in the insert of Figure 7. These fires are based on the Eurocode (2002) parametric fires, which can more accurately represent realistic, post-flashover fires with heating as well as cooling (i.e. decreasing temperature) phases. These fire curves were developed from a fuel load of 3000 MJ/m^2 and varying ventilation factor F_v from 0.02, 0.04, and 0.08 $m^{1/2}$. For reference, note that, as shown in the Figure 7 insert, the standard fire is closely replicated by the heating phase of the F_v = 0.04 $m^{1/2}$ fire. As the K-D model used to calibrate α_r for previous results in this study is based on a standard fire with continuously increasing mean temperature, accuracy may be lost if the calibration process is applied to nonstandard fires with cooling phases. Therefore, for consideration of these non-standard fires, the model calibration was made to rebar temperatures determined from finite element analysis using the commercial thermal code SAFIR (2011) rather than the K-D model. For consideration of the cooling phase, it is assumed that concrete strength remains degraded to that reached at the peak fire temperature, but steel yield stress recovers as rebar temperature decreases. Results are given in Figure 7, where it can be seen that the F_v = 0.04 $m^{1/2}$ fire matches the standard fire results until about t = 2 hours, when the cooling phase begins and reliability increases slightly. From about t = 1 hour onward, the F_v = 0.02 $m^{1/2}$ fire provides a constant increase in reliability index by about 0.5 over the standard fire, as expected as it provides a similarly constant reduction in temperature as shown in Figure 7. The F_v = 0.08 $m^{1/2}$ fire gives initially lower reliability up until about t = 1.8 hours, then increases as steel temperatures decrease, until reliability reaches a level slightly less than the initial cold strength value (β = 5.4 as compared to β = 5.0) close to t= 3.8 hours. Beyond about 4 hours, reliability remains constant at approximately β = 5.1, a decrease caused by permanent damage sustained to the concrete.

Comparison to Existing Codes and Standards

ASCE 29 rates most of the beams studied at 3 hours; ACI 216 rates most of the same beams from 3.5 - 4+ hours, and the Australian Code (AS 3600), in contrast, rates most of the beams at 2.24 hours (with some exceptions for each code). Specifically consider Figure 3, for which the above ratings apply. At a 2.24 hour fire rating, for a middle *D/(D+L)* ratio of 0.5, reliability index is approximately 0.75. Here, there is a small, but measureable safety margin (p_f = 0.23). However, at about 3 hours, β = 0.32 (p_f = 0.37), while at 4 hours, β = 0 (p_f = 0.50). This implies that the ACI and ASCE methods, which are based on ASTM E119, have practically no safety margin. That is, if a beam is exposed to a structurally significant fire, the fire ratings obtained from these methods will often result in a substantial number of beam failures before this time is achieved (the expected failure proportion at a time of interest can be determined by reading β from the figures and converting to p_f). This is not unexpected, as ASTM E119, upon which the ACI and ASCE methods are based, does not specify a safety factor in the rating process.

However, this does not imply that reinforced concrete beams are necessarily unsafe with regard to fire load. For this kind of evaluation, fire rate of occurrence must also be considered. As with fire ratings, the reliability results above are calculated under the certain presence of a fire load. That is, the load combination $D + L_s + T$ was computed with the assumption that the beam will be exposed to a structurally significant fire in its design lifetime. Depending on occupancy and other fire mitigation techniques such as sprinklers and fire-fighting, however, from a design point of view, it may be overly-conservative to assume that the probability of a structurally significant fire = 1.0. The development of a rigorous fire frequency-of-occurrence model accounting for various occupancies, compartment characteristics, and activation of fire mitigation techniques is a complex task and beyond the scope of this paper. However, the effect of frequency of occurrence on reliability can be reasonably approximated. To account for fire frequency of occurrence, reliability can be estimated with: $\beta = -\Phi^{-1}\left(\left(p_f\, p_{fire}\right)\right)$, where p_{fire} is the probability of occurrence of a structurally significant fire and p_f is the probably of failure of the beam (as previously calculated in Figures 2-7), given that a structurally significant fire has occurred (with an upper bound of the arbitrary-point-in-time cold strength reliability index).

Conclusions

A procedure was suggested for reliability analysis of reinforced concrete beams exposed to fire. This involves identifying relevant load combinations, specifying critical load and resistance random variables, and establishing a high-temperature performance model for beam capacity. Using the procedure, an initial analysis was conducted for various reinforced concrete beams designed according to ACI 318 that are exposed to fire. Based on the load and resistance models used, it was found that most beams had a cold-strength reliability index of approximately 5.4 while exposed to dead load and sustained live load. Reliability rapidly decreased as a function of time for the first 1-2 hours after fire exposure, and continued to decrease at a slower rate thereafter, to become asymptotic to a minimum reliability index that ranges between 0 and -1 for most cases. The most significant parameters on reliability were concrete cover; span/depth ratio when axial restraints are present, rotational restraints, and mean fire temperature. Moderately important were *D/(D+L)* ratio, beam width, and aggregate type, while concrete compressive strength, reinforcement ratio, proportion of corner bars, compression steel area, heated perimeter/section area ratio, and axial restraint level generally had minor effects on reliability.

Since the mean failure time was found to be close to that predicted by the ACI 216 and ASCE 29 methods for most beams, it may be unconservative to rely upon fire ratings obtained from these methods, as depending on reasonable levels of uncertainty in loads and resistance, a significant proportion of beam failures would be expected to occur before the predicted fire rating is met. This perhaps suggests further consideration of the need for a fire load combination in design, similar to that presented by Ellingwood (2005) or Section C2.0 of ASCE 7.

To improve the fidelity of future results, it is suggested that further research is needed to better characterize critical uncertainties at high temperatures, including variation in expected fire temperature, thermal diffusivity, and material strengths. Furthermore, before fire-related temperature effects could be feasibly incorporated into design load combinations, the development of a rigorous fire frequency of occurrence model is needed.

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Table 1. Random Variables

RV*	bias factor	mean value for base beam	COV
D	1.05	65.6 kN/m (4.47 kip/ft)	0.05
L_s	0.24	15.0 kN/m (1.02 kip/ft)	0.65
T	1.0	per eq. (1)	0.45
f_y	1.145	474 MPa (68.7 ksi)	0.05
d	0.99	562 mm (22.1 in)	0.04
f_c'	1.10-1.23**	34 MPa (4.92 ksi)	0.145
b	1.01	308 mm (12.1 in)	0.04
α	1.0	0.417×10^{-6} m^2/s (4.54×10^{-6} ft^2/s)	0.06
P	1.02	1.02	0.06

*All distributions are normal, except L_s, which is gamma.
**given as a function of f_c' (ksi): for $f_c' \leq 55$ MPa (8 ksi),
$\lambda = -0.0081fc'^3 + 0.1509f_c'^2 - 0.9338f_c' + 3.0649$, which
results in $\lambda = 1.23$ for $f_c' = 28$ MPa (4 ksi); for $f'_c > 55$ MPa, $\lambda = 1.10$.

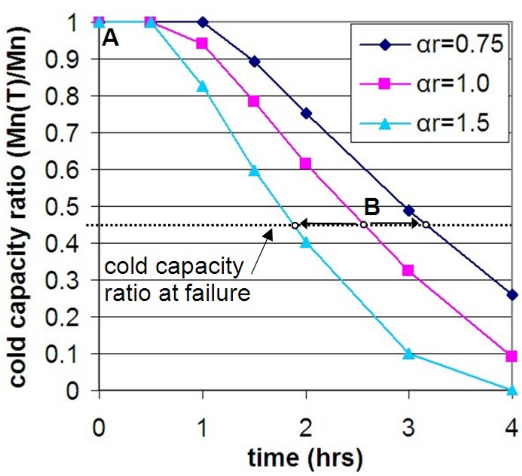

Fig. 1. shows how α_r changes time to failure for a typical beam, which is given by the time corresponding to the intersection of the horizontal dashed line and the capacity curves.

Figure 1. Effect of Thermal Diffusivity in the Wickstrom Model

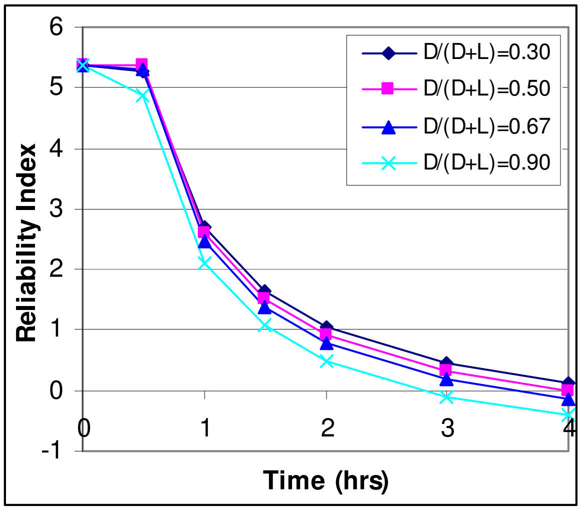

Figure 2. Effect of D/(D+L) Ratio

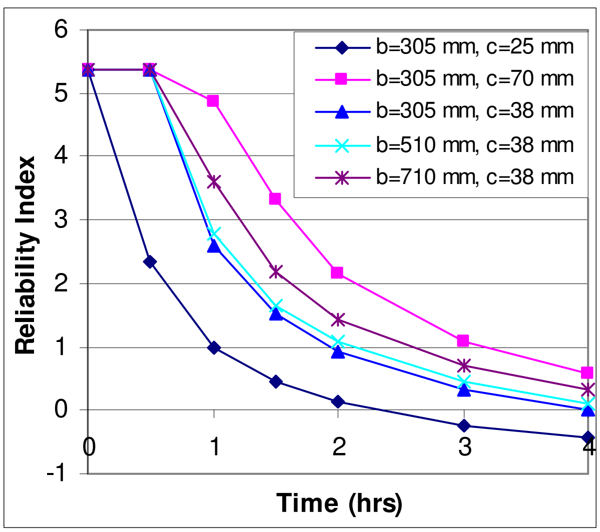

Figure 3. Effect of Concrete Cover and Beam Width

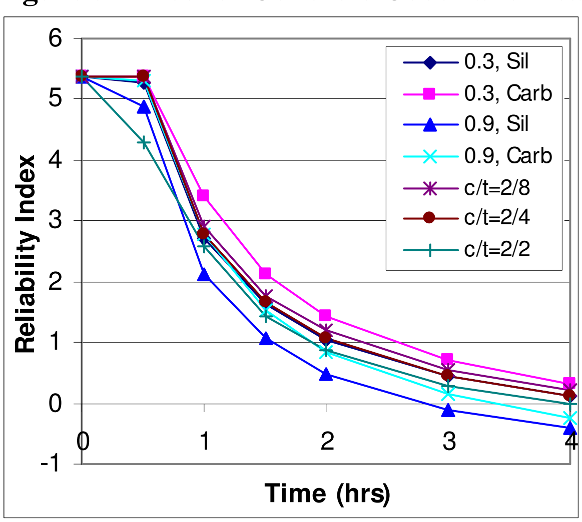

Figure 4. Effect of Aggregate Type and Proportion of Corner Bars

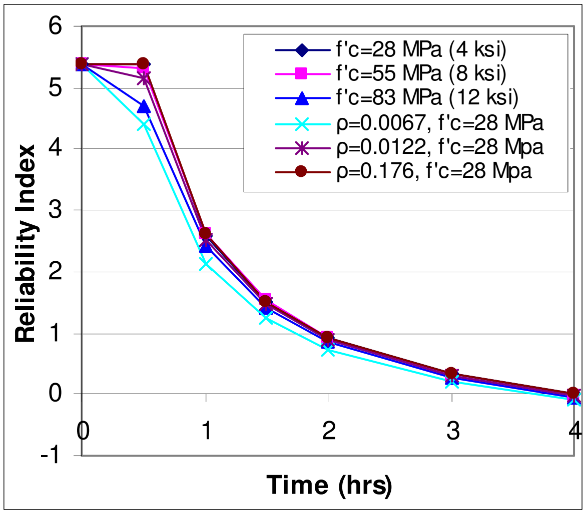

Figure 5. Effect of Concrete Compressive Strength and Reinforcement Ratio

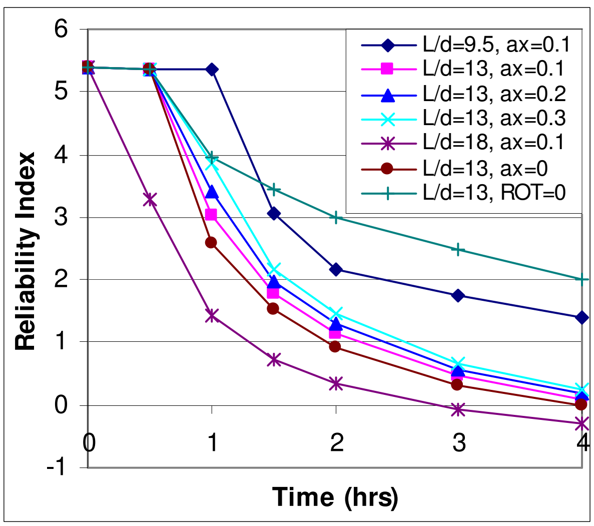

Figure 6. Effect of Span/Depth Ratio and Restraint Level

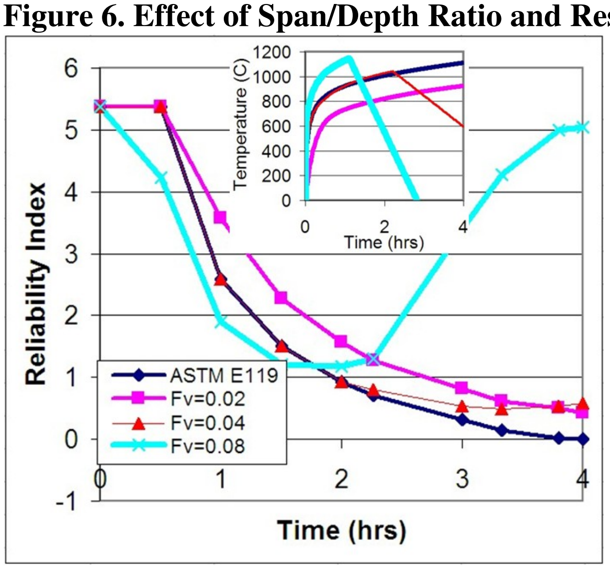

Figure 7. Effect of Fire Type