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Resistance Factors for Ductile FRP-reinforced Concrete Flexural Members

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ABSTRACT

To prevent damage caused by corroding reinforcement, fiber reinforced polymer (FRP) reinforcing bars have been used in place of steel in a relatively small but increasing number of structures in the civil infrastructure. A concern with the use of traditional FRP bars, however, is the resulting lack of ductility. This problem has been overcome with the development of a new generation of composite reinforcement, ductile hybrid FRP (DHFRP) bars. However, standards that address the design of DHFRP bars are unavailable, and appropriate resistance factors for the use of DHFRP reinforcement are unknown. In this study, a reliability analysis is conducted on tension-controlled concrete flexural members reinforced with DHFRP, with the intent to estimate potential strength reduction factors. Flexural members considered include a selection of representative bridge decks and building beams designed to meet AASHTO LRFD and ACI-318 strength requirements and target reliability levels. Nominal moment capacity is calculated from standard analytical models and is taken as first DHFRP material failure. Statistical parameters for load and resistance random variables in the reliability model are consistent with previous code calibration efforts. The resulting resistance factors ranged from 0.61 to 0.64 for tension-controlled sections, which indicates a potential increase in allowed strength over flexural members using non-ductile bars.

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Introduction

The damage caused by corroding reinforcement is a prevailing problem in the civil infrastructure, where approximately 30% of bridges in the United States have significantly deteriorated due to reinforcement corrosion, requiring over $8 billion to repair (Won et al. 2007; FHWA 2001). Different approaches have been employed to limit corrosion and the resulting concrete damage, including increasing cover, utilizing epoxy-covered bars, changing mix porosities and adding admixtures to prevent chloride penetration, and implementing active cathodic protection systems. Although these methods have been shown to reduce corrosion in some cases, the problem remains prevalent, particularly in colder climates where chloride-containing deicing materials are used, as well as on concrete structures exposed to seawater (FHWA 2001; Smith and Virmani 1996).

The use of fiber-reinforced polymer (FRP) composites represents an alternative approach, where rather than attempt to protect the steel, it is partially or completely replaced with a non-corrosive, FRP alternative. In recent years, various bridges in North America have successfully used FRP reinforcement, and these structures can be found throughout the US from Michigan to Texas (Eamon et al. 2012). Although FRP reinforcement has been commercially available for more than a decade, FRP-reinforced structures represent a small portion of new and renovated concrete bridges. The small use of FRP relative to steel can be explained by various factors, including higher initial costs and designer unfamiliarity with the material and its design process, although design guidelines for FRP-reinforced structural members can be found in the *AASHTO LFRD Bridge Design Guide Specifications for GFRP-Reinforced Concrete Bridge Decks and Traffic Railings* (AASHTO 2009) and the *ACI Guide for the Design and Construction of Structural Concrete Reinforced with FRP Bars, ACI-440.1R* (ACI 2006). Other reasons for lack of use include performance concerns such as low elastic modulus, potential degradation in alkaline environments, and lack of reinforcement ductility. With appropriate selection of FRP materials, however, many of these drawbacks can be reduced or eliminated (Cheung and Tsang 2010).

The high initial cost of FRP remains a concern, although long term benefits can be significant. A recent life-cycle cost analysis of FRP-reinforced bridges revealed that, while FRP bars may be on the order of 6-8 times more expensive than steel, and the resulting cost of a typical bridge superstructure reinforced with FRP may range from 25-75% higher than its steel-reinforced equivalent, reduced maintenance costs can be expected to result in a net cost savings near 20 years of service life. Moreover, considering a 50 to 75 year time span, total life-cycle costs for typical FRP-reinforced bridges were generally predicted to be one-half or less of their steel-reinforced alternatives (Eamon et al. 2012).

Although the use of FRP reinforcement may have long-term economic advantages, the brittle tensile failures associated with traditional FRP bars and the resulting relatively non-ductile response of concrete flexural members reinforced with them is clearly undesirable from a safety perspective. Fundamental advancements have been made by various researchers in the last two decades in the area, however, and numerous FRP bar designs with ductility rivaling that of steel have been achieved (Tamuzs and Tepfers 1995; Harris et al. 1998; Bakis et al. 2001; Belarbi et al. 2001; Cheung and Tsang 2010; Won et al. 2007; Cui and Tao 2009; Wierschem and Andrawes 2010). In each of these cases, ductility is achieved by use of the hybrid concept. Here, the reinforcing bars are made of not one, but multiple fiber types, where each type of fiber has a different ultimate failure strain. When the bar is overloaded in tension, the fibers with the lowest ultimate strain (generally the most stiff) rupture first. As load is further increased, the fibers with the next lowest ultimate strain rupture, and so on, until all materials in the bar fail. This incremental failure reduces stiffness as the materials rupture, and if material properties and their volume fractions are selected properly, the remaining fibers will maintain the applied load until the desired level of ductility is reached. Figure 1 presents analytical stress-strain curves generated for a set of ductile FRP bar designs. The general response shown in the figure closely resembles the discontinuous stress-strain behavior experimentally determined by the various researchers noted above. Experimental results have similarly shown that a significant ductile response similar to, and sometimes exceeding that of, a steel-reinforced concrete flexural member can be achieved with ductile FRP reinforcement (Harris et al. 1998; Cheung and Tsang 2010).

Ductile hybrid FRP (DHFRP) reinforced members have the potential to eliminate corrosion damage, decrease structure life cycle costs, as well as provide significant ductility. However, it is currently unclear what, if any, design advantage DHFRP reinforcement may have over traditional FRP bars, as resistance factors (ϕ) for the use of these bars in flexural members have not been developed. The importance of this concern can be illustrated by comparing steel-reinforced concrete flexural member design strength requirements per *ACI 318-11* (2011) (or *AASHTO LRFD* (2010)), to that required for FRP-reinforced sections, per *ACI 440.1R-06* (2006) (or *AASHTO GRFP* (2009)).

For the steel-reinforced section, to provide a sufficiently ductile failure, *ACI 318* requires that flexural members are under-reinforced to prevent a much less ductile failure caused by concrete crushing. When such members are designed to an adequate level of ductility, as specified by strain in the extreme layer of tension steel equaling a value of 0.005 or greater while maximum compressive strain in the concrete is 0.003, the resistance factor is taken as 0.90.

To account for uncertainties affecting flexural capacity such the yield strength of steel reinforcement, compressive strength of concrete, as well as geometric section properties, the strength reduction factor was derived by considering random variable statistical parameters specifically determined for steel-reinforced flexural members, in order to provide the target level of safety (Szerszen and Nowak 2003).

In contrast, for a concrete flexural member reinforced with FRP, *ACI 440.1R* is considered, where both under-reinforced as well as over-reinforced flexural failure modes are permitted. For the former, moment capacity is governed by FRP rupture in tension, while for the latter, capacity is governed by crushing of the concrete compressive block. The corresponding resistance factor varies with failure mode, and is linearly interpolated between 0.55 (for tension-controlled sections) and 0.65 (compression-controlled), as a function of reinforcement ratio. However, the resistance factor of 0.55 given for tension-controlled sections is lower than that actually needed to provide the target reliability level with regard to flexural capacity; it was lowered further to account for the lack of ductility associated with FRP bar failure (Shield et al. 2011).

Using *ACI 440.1R* as a guide for the design of FRP-reinforced sections, the two allowed failure modes (tension- and compression-controlled) theoretically apply to DHFRP as well. If DHFRP bars are used in an over-reinforced design, the resistance factor is appropriately taken as 0.65, just as for the case of an over-reinforced beam with brittle FRP or steel, as the uncertainties associated with the concrete, rather than the reinforcement, control the flexural failure and hence reliability. However, it only makes sense to use DHFRP in an under-reinforced member, where bar ductility could actually be utilized in the case of an overload. As *ACI 440.1R* does not provide a resistance factor specifically for DHFRP-reinforced sections, and the existing tension-controlled resistance factor of 0.55 was set assuming brittle bar behavior, the design strength of a DHFRP-reinforced section may be unnecessarily penalized. It would be similarly inappropriate to adopt the resistance factor of 0.9 found in *ACI 318* for steel-reinforced, tension-controlled members, as that factor was derived based on random variable statistical parameters only applicable to steel reinforcement, such as uncertainties in yield strength and bar size.

Given this concern, the objective of this study is to determine potential resistance factors for tension-controlled, DHFRP-reinforced concrete flexural members. The results may be used to estimate what design advantage, in terms of potential increase in resistance factor, might be obtained by using DHFRP in the place of non-ductile FRP bars. Appropriate resistance factors can be determined with a reliability-based calibration process, where resistance factors are determined such that tension-controlled, DHFRP-reinforced flexural members meet the existing target reliability level established for ductile (i.e. steel-reinforced) flexural members, as designed according to *ACI 318* or *AASHTO LRFD*. In this process, appropriate constraints on section behavior are imposed such that a sufficient level of ductility is maintained

Reliability analysis as well as reliability-based calibration has been conducted for non-ductile FRP bars in reinforced concrete members (Shield et al. 2011; Ribeiro and Diniz 2012), and a significant body of work exists examining the reliability of externally-bonded, non-ductile FRP used to strengthen reinforced concrete beams (Plevris et al. 1995; Okeil et al. 2002; Monti and Santini 2002; Zureick et al. 2006; Atadero and Karbhari 2008; Wang et al. 2010; Wieghaus and Atadero 2011; Ceci et al. 2012).

The typical process used for reliability-based calibration is well established. This involves selecting representative designs for consideration; establishing a probabilistic model by identifying the limit state function, the relevant random variables, and their statistical parameters;

selecting a target reliability index; evaluating the reliability of the cases considered; and adjusting resistance factors such that the target level is met, which is often an iterative process. Each of these tasks is described below.

DHFRP bars considered

Numerous DHFRP schemes have been developed, with regard to constituent material properties, number of materials, and construction technique. A conceptual diagram of a DHFRP bar with four materials is given in Figure 2, where the materials, each composed of a different fiber type, are separated into concentric layers.

Although many material combinations are feasible for strength, once other practical design constraints are imposed on bar performance, particularly to ensure adequate ductility, the range of possible arrangements decreases considerably. In this study, five generic bar layouts were considered, for a representative range of configurations that can achieve the required performance results in terms of strength and ductility. Here, 2, 3, and 4-material continuous fiber bars are considered (designated *B1*, *B2*, and *B3*, respectively), the most prevalent type, as well as less common alternative schemes including a bar of 2 continuous and 2 randomly-dispersed, chopped-fiber materials (*B4*), as well as a 4-material bar with 1 continuous layer, 2 chopped fiber layers, and a small steel core (8mm diameter), as proposed by Cheung and Tsang (2010) (*B5*).

Material volume fractions for each bar are given in Table 1, while pertinent material properties, Young's modulus (E) and ultimate strain (ε_u), are given in Table 2. For schemes *B4* and *B5*, bars that include randomly dispersed chopped fiber layers, fiber length is taken as 6 mm, with 65% fiber and 35% resin in the layer. Although other DHFRP bar configurations are possible, the generic configurations considered in this study are meant to represent a selection of reasonable possibilities guided by suggestions in the literature, and adjusted to minimize bar costs. In general, once bar material types and number of layers were chosen, the required volume fractions were then determined in order to meet both strength and ductility requirements, as described below.

Deterministic Analysis

For bars composed of fiber and resin only (i.e. no steel), as with *B1-B4*, DHFRP-reinforced concrete flexural member moment capacity can be determined as:

$$M_c = \left[d \; - K_2 \frac{\varepsilon_{f_1}}{K_1 \cdot f_c' \cdot b} \left(\sum_{i=1}^{n} v_{f_i} E_{f_i} + v_m E_m \right) \left(\sum_{i=1}^{n} v_{f_i} + v_m \right) A_T \right] \cdot$$

$$\left[\varepsilon_{f_1} \left(\sum_{i=1}^{n} v_{f_i} E_{f_i} + v_{f_m} E_{f_m} \right) \left(\sum_{i=1}^{n} v_{f_i} + v_m \right) A_T \right] \qquad (1)$$

Equation (1) provides the moment capacity at the point where the first FRP material in the bar fails, as governed by the lowest ultimate fiber strain. As shown in Tables 1 and 2, for the schemes considered, this first material failure is IMCF-II for *B1-B3* and IMCF-I for *B4* and *B5*. For design as well as capacity analysis, M_c is taken as the nominal moment capacity M_n. The contribution to flexural capacity from the concrete tensile strength is ignored in the expression, which is insignificant for the schemes considered.

In eq. (1), the first term in square brackets is the distance between the tensile reinforcement centroid and that of the concrete compressive block. The second term in square brackets represents the tensile force in the reinforcement at first material failure. In both terms, $\sum_{i=1}^{n} v_{f_i} E_{f_i} = v_{f_1} E_{f_1} + v_{f_2} E_{f_2} + \cdots + v_{f_n} E_{f_n}$, where n is the number of material layers; v_{f_i} and E_{f_i}

are the volume fraction and Young's modulus of fiber in layer *i*, respectively; f_c' is the concrete compressive strength; E_m and v_m are the Young's modulus and volume fraction of the resin; ε_{f_1} is the failure strain of the first fiber layer to fail; K_1 and K_2 define the shape of the concrete compression block in Hognestad's parabolic stress-strain model (Hognestad 1952), where K_1 is the ratio of average concrete stress to maximum stress in the block and K_2 defines the location of the compressive block centroid; A_T is the total tensile reinforcement area; *d* is the distance from the centroid of tension reinforcement to the extreme compression fiber in the beam; and *b* is the width of the concrete compression block. For scheme *B5*, with a steel core, the resistance moment can be similarly developed as:

$$M_c = \left[d - K_2 \frac{\left(\sum_{i=1}^{n} v_{f_i} + v_s + v_m \right) A_T}{K_1 \cdot f_c' \cdot b} \left\{ \left(\sum_{i=1}^{n} v_{f_i} E_{f_i} + v_m E_m \right) \varepsilon_{f_1} + v_s \cdot f_y \right\} \right] \cdot$$

$$K_1 \cdot f_c' \cdot b \left[\frac{\left(\sum_{i=1}^{n} v_{f_i} + v_s + v_m \right) A_T}{K_1 \cdot f_c' \cdot b} \left\{ \left(\sum_{i=1}^{n} v_{f_i} E_{f_i} + v_m E_m \right) \varepsilon_{f_1} + v_s \cdot f_y \right\} \right] \quad (2)$$

where the second square bracketed term represents the compressive force in the concrete at first material failure; v_s is the volume fraction of steel; and f_y is the yield stress of steel. For schemes where chopped fiber layers are used (*B4, B5*), the effective modulus of the fibers, E_{fi}, must be reduced to account for non-continuity. This can be calculated as $E_{fi} = \eta_{LE} \cdot \eta_{OE} \cdot E_f . v_f$, where η_{LE} accounts for reductions due to fiber length and η_{OE} accounts for fibers that are misaligned with the direction of load (i.e. fibers not oriented parallel to the bar). Various fiber properties affect η_{LE}, including fiber length and diameter, mean distance between fibers, packing geometry, fiber modulus, and resin shear modulus (Cox 1952). The chopped fibers considered have a resulting η_{LE} from 0.98-0.99. η_{OE} is a function of the distribution of fiber orientation, and can be shown to be $\frac{3}{8}$ for randomly dispersed, in-plane fibers (Krenchel 1964). Note that it is assumed that the bars are sand-coated or ribbed for adequate bond (Bank 2006). Using Whitney's stress block to determine ultimate capacity would result in no significant difference from that found with the nonlinear Hognestad model considered. However, the Hognestad model was used as it allows evaluation of section moment-curvature behavior at load levels below ultimate, which is needed to evaluate section ductility.

In this study, ductility index μ_ϕ is evaluated from the moment-curvature response with (Naaman and Jeong, 1995):

$$\mu_\phi = \frac{\phi_u}{\phi_y} = \frac{1}{2}\left(\frac{E_{total}}{E_{elastic}} + 1\right) \quad (3)$$

where ϕ_u is ultimate curvature and ϕ_y is yield curvature (i.e. curvature at first DHFRP material failure), while E_{total} is computed as the area under the load displacement or moment-curvature diagram and E_{elastic} is the area corresponding to the elastic deformation.

In this study, a minimum ductility index of 3.0 is specified for flexural member performance, which represents a lower limit similar to that of corresponding steel-reinforced sections (Maghsoudi and Bengar 2010; Shin et al. 2010). As noted earlier, the source of DHFRP bar ductility results from non-simultaneous material failures, such that after a material fails, the remaining materials have the capacity to carry the tension force until the final material fails, to produce the desired ductility level. Correspondingly, before the desired level of ductility is reached, each bar material must fail before the concrete crushes in compression, which is assumed to occur at an ultimate strain of $\varepsilon_{cu} = 0.003$. Enforcing these constraints results in bar performance such that subsequent peaks on the stress-strain diagram do not decrease as bar strain increases (Figure 1), as well as DHFRP-reinforced concrete sections that have reinforcement strain ε_t significantly higher (approximately $0.02 < \varepsilon_t < 0.04$) when the concrete compressive block crushes than that required for tension controlled, steel-reinforced sections ($\varepsilon_t \geq 0.005$) according to *ACI 318*.

To evaluate the ductility of a DHFRP-reinforced section, the load deflection or moment-curvature function is needed; in this study, moment-curvature is considered. Before the section cracks, moment capacity is calculated based on elastic section properties with $M_{cr} = \dfrac{f_r I_g}{y_t}$, where f_r is the modulus of rupture of the concrete, I_g is the uncracked section moment of inertia, and y_t is the distance from the centroid of the section to the extreme tension fiber. The concrete stress-strain behavior for cracked sections is developed based on the modified Hognestad model [33], and the corresponding resisting moment is then determined from: $M = C_c\left(d - K_2\, c\right)$, where C_c is the compressive force in the concrete; d is the distance from the top of the concrete compression block to the reinforcement centroid; and c is the distance from the top of the concrete compression block to the section neutral axis. Curvature ϕ_c is then calculated from $\phi_c = \dfrac{\varepsilon_c}{c}$, where ε_c is the concrete strain at the top of the compression block. For development of the moment-curvature diagram, it is assumed that once the failure strain of a particular bar layer is reached, the entire layer throughout the bar length immediately loses all force carrying capability. This conservative assumption results in non-smooth moment-curvature diagrams as shown in Figure 3. At peak moments on the diagrams, two different capacity values are theoretically associated with the same curvature. This occurs because once the stiffest existing material in the bar fails, the stiffness of the cracked section decreases and less moment is required to deform the beam the same amount. Experimental results of DHFRP-reinforced beams have developed somewhat smoother curves, closer to that found by drawing a line between the moment peaks and excluding the capacity drops as shown in Figure 3 (Cheung and Tsang 2010; Harris et al. 1998). However, including the theoretically low-capacity points produces conservative ductility indices, and thus this method is used to enforce the ductility constraint imposed in the analysis.

Flexural Members Considered

Resistance factors for flexural components in two typical reinforced concrete member applications are considered; a bridge deck and a building floor beam. For the bridge deck, three continuous slab geometries with f_c'= 31 MPa (4500 psi) spanning over girder spacings of 1.8, 2.7, and 3 m (6, 9, and 10 ft), were considered, as shown in Figure 4. Corresponding slab thicknesses were from 200-250 mm (8-10 in), with a 13 mm (0.5 in) wearing surface and allowance for a 65 mm (2.5 in) future wearing surface. The DHFRP bars were placed in the top and bottom of the slab, with diameters of 22 mm (7/8 in) for bars *B1-B4* and 19 mm (3/4 in) for bar *B5*. Although *AASHTO GFRP* (2009) allows a minimum of 19 mm (¾ in) cover for a slab reinforced with composite bars, cover was taken as 25 mm (1 in), as used in two FRP-reinforced bridge decks built in Wisconsin (Berg et al., 2006; Bank et al., 2006). The deck is designed as tension-controlled for positive and negative moments using the equivalent strip method according to the *AASHTO LRFD Specifications* (2010), where the governing design equation is: $\phi M_n = \gamma_{DC} M_{DC} + \gamma_{DW} M_{DW} + 1.75 M_{LL+IM}$. Here, M_{DC} and M_{DW} are moments caused by the weight of the slab and wearing surface, respectively; γ_{DC} and γ_{DW} are load factors that may vary from 1.25 to 0.9, and 1.5 to 0.65, respectively, to maximize load effect; and M_{LL+IM} is the live load moment caused by two 72 kN (16 kip) truck wheel loads on the slab, in addition to a design impact factor of 1.33. The selection of resistance factor ϕ used for design is discussed in the next section. An environmental factor to account for material degradation is taken as $C_E = 0.9$, as recommended in *ACI 440.1R* for carbon FRP bars, as the outer material of the DHFRP bars considered is carbon. The reinforcement ratios for the slabs varied from 0.003-0.009, with the highest values associated with the greatest girder spacings as well as the bars with chopped fiber layers (*B4, B5*), as expected. Slab ductility indices varied from approximately 3-4 when reinforced with bars *B1, B2*, and *B5*, and from 5-6 when reinforced with bars *B3* and *B4*. The upper range of these ductility indices are higher than equivalent slabs reinforced with steel, which is a result of the large post 'yield' (i.e. after first material failure) deformations of DHFRP-reinforced slabs, which are typically greater than those of corresponding steel-reinforced sections.

For the building beam, three span lengths of 6, 7.6, and 9.1 m (20, 25, and 30 ft), were considered, with f_c' = 38 MPa (5500 psi). A rectangular, simple-span beam was considered for analysis; T-beams and continuous members were found to have no significant effect on reliability results. The beam width was approximately 405 mm (16 in) for the 6 m span and 510 mm (20 in) for the longer spans. Beam height was selected to satisfy the minimum recommendation given in *ACI-440.1R* for non-prestressed FRP-reinforced beams (1/10 of span length for simply supported beams). The beam is designed to satisfy the same strength requirements for tension-controlled members reinforced with steel bars. The relevant flexural design equation is $\phi M_n = 1.2 M_{DL} + 1.6 M_{LL}$, where M_{DL} and M_{LL} are the dead and live load moments, respectively. The beam was loaded with a dead load to total load (*D/(D+L)*) ratio of approximately 0.5. Decreasing this ratio did not change results, while increasing this ratio beyond 0.5 generally resulted in slight decreases in reliability, as similar to the results found by Szersen and Nowak (2003) for steel-reinforced beams. Resulting DHFRP reinforcement ratios were from 0.004-0.005, with the same bar diameters used as for the bridge deck. Ductility indices ranged from approximately 3-3.4 for beams reinforced with bars *B1* and *B2*, and from approximately 5-6 when reinforced with bars *B3, B4*, and *B5*.

Reliability Analysis

Resistance random variables (RVs) pertinent for moment capacity analysis of DHFRP-reinforced concrete members include those that account for variations in material and resin properties such as volume fractions (v), moduli of elasticity (E), and failure strains (ε_{f_1}); concrete compressive strength (f_c'); reinforcement depth (d); and the professional factor (P), which is the ratio of actual section capacity to the capacity predicted by analysis. Additional RVs specific to select cases include beam width (b) for building beams, and steel core yield strength (σ_y) for cases that consider bar *B5*. RV statistical parameters relevant to this study are coefficient of variation, V, bias factor λ (ratio of mean to nominal value), and distribution type, and are given in Table 3. To maintain consistency with previous code calibration efforts, load and resistance RVs for building beam cases are taken from Nowak and Szerszen (2003), as used to calibrate the *ACI 318* Code; bridge deck RVs are taken from Nowak (1999), as used for the *AASHTO LRFD* Code calibration; and FRP RV data are taken from Shield et al. (2011), as used for the *ACI 440.1R* calibration, as well as from Eamon and Rais-Rohani (2008). For the bridge slab, load RVs include dead load of the slab (*DS*), wearing surface (*DW*), and parapets (*DP*), and truck wheel live load (*LL*); while for the building beam, load RVs are dead load (*DL*) and transient live load (50-year maximum). These values are shown in Table 4.

For reliability analysis, the relevant limit state is: $g = M - M_a$. Failure is defined when flexural member moment capacity M exceeds maximum applied moment on the flexural member M_a (i.e. when $g \leq 0$). M is defined as M_cP, where M_c is the moment capacity of the section, as given by eqs. 1 or 2, as appropriate for the DHFRP bar type used, and corresponds to the first peak of the moment-curvature diagram in Figure 3. M is a function of the resistance RVs given in Table 3, while P is the professional factor given in Table 3. M_a is the applied moment effect, as a function of the dead and live load RVs given in Table 4. Note for consistency with existing code calibration efforts, the reliability analysis in this study is similarly based on strength, although serviceability limits typically govern design. Probability of failure p_f of the limit state for each case considered was calculated with Monte Carlo simulation, then transformed to reliability index β with $\beta = -\Phi^{-1}(p_f)$. As the MCS procedure progressed, the number of simulations was increased until β converged, which occurred close to 2×10^6 simulations for most cases.

For each flexural member considered, a resistance factor ϕ is determined for design that is required for the reliability index of the member to meet the minimum target of 3.5, as used for the *AASHTO LRFD* as well as *ACI-318* Code calibrations (Nowak 1999; Szerszen and Nowak 2003). It was found that designs with DHFRP bars using the resistance factor of 0.55 specified by *ACI 440.1R* for tension controlled sections resulted in reliability indices near 3.9, an overdesign from the target of 3.5 for ductile sections. This finding is close to that found for tension-controlled designs considering non-ductile FRP, for which Shield et al. (2011) estimated reliability indices from 3.5 to 4.8 when using $\phi = 0.55$. For *ACI 440.1R*, however, ϕ was not increased due to the non-ductile failures anticipated with traditional FRP.

Results

The results are presented in Tables 5 and 6, which show the final reliability indices and corresponding resistance factors needed. For both the bridge deck as well as the building beam, to achieve the target reliability index of 3.5, it appears that ϕ can be increased to approximately 0.61-0.64, above the current value of 0.55 specified for non-ductile FRP bars. For comparison, the calculations were repeated to determine resistance factors needed to obtain higher reliability indices of 3.75 and 4.0. Similar results were found to those in Tables 5 and 6, but with a typical ϕ of 0.58 (with a range of 0.57-0.60 for all cases) required for a reliability index of 3.75, and a ϕ of 0.52 (with a range of 0.51-0.53 for all cases) required for a reliability index of 4.0.

Although the resistance factor range of 0.61-0.64 is above that specified for non-ductile bars, it is still much below the 0.90 for tension controlled, steel-reinforced members needed to produce the same target reliability index of 3.5. Clearly then, if designed with the same reduction factor, a steel-reinforced member would be significantly more safe than one using FRP. It should be note that this discrepancy in safety level is not related to ductility, and is observed only from consideration of moment capacity. It is a direct result of the different levels of uncertainty inherent in steel as opposed to DHFRP (or FRP) reinforcement, as described by the different statistical parameters of critical reinforcement random variables.

Here there are three important differences in variability: reinforcement geometry, reinforcement stiffness, and analytical prediction of moment capacity. With regard to reinforcement geometry, the variation in cross-sectional area of steel reinforcement is negligible, and is generally taken as deterministic in reliability analysis (Nowak and Szerszen 2003). However, for DHFRP bars, variation in material volume fractions has a significant effect on reliability. Similarly, as variations in reinforcement elastic modulus do not affect the capacity calculation of steel-reinforced sections, these uncertainties do affect DHFRP-reinforced section moment capacity, as per eqs. 1 and 2. Finally, the professional factor P considered for FRP reinforced sections ($\overline{P} = 0.89$, V_P = 0.16) not only has a lower mean value but much greater variability than P used for steel-reinforced members ($\overline{P} = 1.02$, V_P = 0.06) (Shield et al. 2011; Nowak and Szerszen 2003). Each of these changes in RV statistical parameters serves to lower the reliability of DHFRP-reinforced sections.

Conclusions

Resistance factors needed for tension controlled DHFRP-reinforced flexural members to meet *ACI 318* and *AAASTHO LRFD* target reliability levels were estimated with a reliability analysis and calibration process. Using the models considered, it was found that the resistance factor has the potential to be increased above the currently specified value of 0.55 for tension controlled members reinforced with non-ductile FRP bars. Recall that the range of resistance factors specified by *ACI 440.1R* for non-ductile FRP bars (0.55-0.65) varies due to the expected failure mode, where compression controlled failures correspond to ϕ=0.65 and tension-controlled FRP-reinforced beams are given ϕ=0.55. The use of DHFRP bars, however, practically only concerns tension-controlled failures. That is, it is appealing to use DHFRP bars only in tension-controlled members, where bar ductility could be taken advantage of in a failure. In this case, rather than specifying a resistance factor of 0.55, the findings of this study suggest that ϕ might be reasonably increased to a value between 0.61-0.64. This is not a large increase, but is a significant difference, particularly given the relatively high cost of FRP. If DHFRP bars are used in compression-controlled sections, the properties of the DHFRP bars do not significantly affect the reliability analysis, and it can be demonstrated that ϕ should appropriately remain unaltered at 0.65, as specified for all concrete-controlled failures. Thus, using DHFRP bars, a linear interpolation might be made between 0.65 for compression controlled failures to a value between (0.61-0.64) for tension-controlled failures, based on reinforcement ratio, rather than from 0.65 to 0.55 as for non-ductile bars. The specific choice of an appropriate ϕ value for a tension-controlled condition (i.e. perhaps between 0.61-0.64) is an issue in need of further study, as this directly effects the target reliability index. In this study, the target index of 3.5 for steel-reinforced beams was considered as a baseline for comparison due to the ability of DHFRP-reinforced sections to meet ductility indices and tensile strains similar to steel-reinforced beams at section ultimate capacity. However, it can be argued that due to other performance differences between DHFRP and steel (such as, the inability of the DHFRP-reinforced section to behave in a ductile manner for more than a single overload, which is clearly disadvantageous for cyclic forces), a different target level may be deemed appropriate. Raising this target level would provide the need for a lower range of resistance factors.

Although strength and ductility requirements can be addressed, an additional consideration with the use of DHFRP, as well as non-ductile FRP bars, is cracked section stiffness for cost-effective bar configurations. As the effective elastic modulus of DHFRP reinforcement is lower than that of steel, deeper sections as well as higher concrete strengths are generally required to simultaneously meet strength, ductility, as well as deflection constraints. For the girder spacings considered, this required minimum bridge deck thicknesses from 200-

250 mm (8-10 in). For building beams, it is suggested that *ACI 440.1R* recommendations are used to establish minimum depths.

The results of this study suggest that an increase in resistance factor for tension controlled, DHFRP-reinforced flexural members may be warranted. However, for better statistical quantification, additional experimental research data is desirable for DHFRP-specific random variables, including fabrication and manufacturing variations as well as professional factor. A larger database of experimental results would allow greater refinement of the results found in this study.

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Table 1. Material Volume Fractions in Considered DHFRP Bars

Bar Number:	*B1*	*B2*	*B3*	*B4*	*B5*
No. of Layers:	2	3	4	4	4
IMCF-I*	-	-	-	0.189	0.147
IMCF- II	0.29	0.20	0.20	-	-
SMCF-I	-	0.06	0.07	-	-
SMCF-II*	-	-	-	0.0875	0.063
AKF-I	-	-	-	0.04	0.10
AKF-II	0.29	0.25	0.10	-	-
EGF	-	-	0.17	0.06	-
Steel	-	-	-	-	0.20
Resin	0.42	0.49	0.64	0.6235	0.49

*Chopped fiber layers

Table 2. DHFRP Bar Material Properties

Label	Material	E GPa (ksi)	ε_u
IMCF-I	IM-Carbon Fiber Type I	650 (94000)	0.0045
IMCF-II	IM-Carbon Fiber Type II	400 (58000)	0.0050
SMCF-I	SM-Carbon Fiber Type I	238 (34500)	0.0150
SMCF-II	SM-Carbon Fiber Type II	230 (33400)	0.0150
AKF-I	Aramid Kevlar-49 Fiber Type I	125 (18000)	0.0250
AKF-II	Aramid Kevlar-49 Fiber Type II	102 (15000)	0.0250
EGF	E-Glass fiber	74 (11000)	0.0440
Steel	Steel, Grade 60	200 (29000)	0.0021*
Resin	Epoxy	3.5 (540)	0.0600

*Yield strain is given for steel, not ultimate.

Table 3. Resistance Random Variables

RV*	Description	V	λ
$v_{\text{IM-Carbon}}$	Volume fraction of IM-Carbon	0.05	1.00
$v_{\text{SM-Carbon}}$	Volume fraction of SM-Carbon	0.05	1.00
$v_{\text{Kevlar}-49}$	Volume fraction of Kevlar-49	0.05	1.00
$v_{E-Glass}$	Volume fraction of E-Glass	0.05	1.00
v_{Steel}	Volume fraction of steel	0.05	1.00
v_{resin}	Volume fraction of resin	0.05	1.00
$E_{\text{IM-Carbon}}$	Modulus of elasticity of IM-Carbon	0.08	1.04
$E_{\text{SM}-\text{Carbon}}$	Modulus of elasticity of SM-Carbon	0.08	1.04
$E_{\text{Kevlar}-49}$	Modulus of elasticity of Kevlar-49	0.08	1.04
$E_{E-glass}$	Modulus of elasticity of E-glass	0.08	1.04
E_{resin}	Modulus of elasticity of resin	0.08	1.04
σ_y	Yield strength of steel	0.06	1.14
ε_{f_1}	Failure Strain of IM-Carbon	0.05	1.20
f'_c	Compressive strength of concrete	0.10	1.14
d	Depth of reinforcement	0.04	0.99
b	Building beam width	0.04	1.01
P	Professional factor	0.16	0.89

*All distributions are normal except steel yield strength, which is lognormal.

Table 4. Load Random Variables

RV*	Description	V	λ
Bridge Slab			
DS	Dead load, slab	0.10	1.05
DW	Dead load, wearing surface	0.25	1.00
DP	Dead load, parapet	0.10	1.05
LL	Truck wheel load	0.18	1.20
Building Beam			
DL	Dead load	0.10	1.00
LL	Live load	0.18	1.00

*All RVs are normal except building live load, which is extreme type I.

Table 5. Resistance Factors for Bridge Deck

Beam Spacing (m)	*B1*		*B2*		*B3*		*B4*		*B5*	
+/- Moment	β	ϕ	β	ϕ	β	ϕ	β	ϕ	β	ϕ
1.8 (+M)	3.50	0.64	3.51	0.63	3.51	0.63	3.52	0.63	3.50	0.63
1.8 (- M)	3.50	0.63	3.50	0.61	3.50	0.61	3.51	0.64	3.50	0.61
2.7 (+M)	3.50	0.64	3.51	0.63	3.50	0.63	3.52	0.62	3.50	0.61
2.7 (- M)	3.51	0.63	3.50	0.63	3.50	0.61	3.50	0.61	3.51	0.62
3.0 (+M)	3.52	0.63	3.50	0.62	3.50	0.62	3.51	0.61	3.50	0.63
3.0 (- M)	3.50	0.63	3.52	0.63	3.50	0.62	3.52	0.64	3.51	0.62

Table 6. Resistance Factors for Building Beam

	B1		*B2*		*B3*		*B4*		*B5*	
span (m)	β	ϕ	β	ϕ	β	ϕ	β	ϕ	β	ϕ
6.0	3.50	0.60	3.50	0.61	3.50	0.61	3.50	0.62	3.52	0.62
7.6	3.50	0.62	3.50	0.63	3.52	0.63	3.51	0.63	3.50	0.65
9.1	3.51	0.64	3.50	0.63	3.50	0.64	3.50	0.63	3.52	0.63