Synchronicity: An Analysis of Einstein's Halfway Rule

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Abstract

For the modern world to function, Global Positioning System satellites must synchronize to clocks on Earth. This paper examines a concept that underlies GPS systems, namely Albert Einstein's halfway rule—the idea that a line of simultaneity exists between two events in different systems. This essay discusses how Einstein used conventionalist methods to establish ½ as a constant value for σ to take advantage of the property of symmetry.

The current global navigation infrastructure is built upon satellites that contain the Global Positioning System and functions on the basis that these satellites synchronize to clocks on Earth. To ensure this, a method of synchronization exists that involves two clocks and the discovery of a pair of events, one on each clock, that occur simultaneously. The method of determining simultaneity has been a topic of discussion within the mathematical and physical fields for a long time. It was not until the publication of *On the Electrodynamics of Moving Bodies* in 1905 that the physicist Albert Einstein was able to provide the postulates necessary for determining simultaneity. The two postulates of special relativity that were outlined in the 1905 paper helped Einstein develop the halfway rule, which claims that when synchronizing two systems, simultaneity occurs between two points located on each system that are positioned at the midpoint of the total time it takes for the first system to send and receive a signal. The greatest mystery of the halfway rule is that even when disregarding signal symmetry, the results are the same as when symmetry is present. Therefore, the main concept that will be explored throughout this essay is how Einstein used conventionalist methods to deem the value of σ in the halfway rule to be ½.

Conventionalism

Conventionalism is the concept that principles are based on societal agreements and viewpoints when empirical data is insufficient in narrowing down the number of choices available. These viewpoints are not *a priori*, meaning that the knowledge is not dependent on experience, only societal agreements. The most famous case of conventionalism is Poincaré's geometric conventionalism which tackles the issue of determining the universe's geometry. He explains that due to empirical data being insufficient in narrowing down the available choices, there is no such thing as true geometry and the three options of Euclidean, Riemannian, and Lobachevskian are all equally as valid (Heinzmann and Stump). The reason most people choose to use Euclidean geometry in their daily lives is simply due to convention.

Simultaneity

One of Einstein's issues with mechanical physics is that it does not explicitly state to which frame of reference the laws of physics apply to (Einstein and Infeld). Newtonian physics assumes that everybody uses the same frame of reference to solve problems. Additionally, it is assumed that everything must be symmetrical. Both of these issues led to a rise in contradictions in various theories until Einstein published the Theory of Special Relativity. In *On the Electrodynamics of Moving Bodies*, Einstein establishes the two postulates that are the basis of modern relativity:

1) Electrodynamics and mechanics possess no properties corresponding to the idea of absolute rest.
2) Light will always travel with speed c in all directions which is independent of its source.

Both postulates, specifically the second one, play an important role for the basis of the halfway rule in determining simultaneity. The first postulate claims that there is no such thing as absolute rest: objects are always in motion in spacetime. This is important in synchronizing systems; by keeping track of which frame of reference is used to determine the line of simultaneity, there can be a frame of reference where one of the systems is physically moving while the other one is only moving in a time-like manner. The second postulate claims that the speed of light will always travel with speed c regardless of its source, which is important for synchronizing systems by utilizing light as signals to determine simultaneous events. Both postulates will be seen in action later while exploring different lines of simultaneity.

The Halfway Rule: Einstein's Simultaneity

Scenario 1: Two Systems at Rest

To determine when simultaneity occurs, the following experiment can be set up:

> Suppose you have two systems, S_1 and S_2, that are at rest respecting each other. A signal with the speed of light is sent from S_1 at t_1 to S_2 and reaches it at time t_2. The moment S_2 receives that signal, it sends a signal with the same velocity back to S_1 and reaches it at t_3.

In this experiment, the speed of light is chosen to represent the signals because, as supported by the second postulate in the Theory of Special Relativity, light travels at a constant speed. This ensures that the scalar value of the signal sent from the first system is equivalent to the scalar value of the signal sent from the second.

The halfway rule states that t_2, the time at which S_2 received the signal, is the halfway point between t_3 and t_1. Let's label that midpoint as event e. Therefore, we can summarize as

$$e = t_1 + \tfrac{1}{2}(t_3 - t_1)$$

where $(t_3 - t_1)$ is the overall time it takes for the whole experiment to occur. To determine what events are simultaneous to event e, a simple line can be drawn straight across as such:

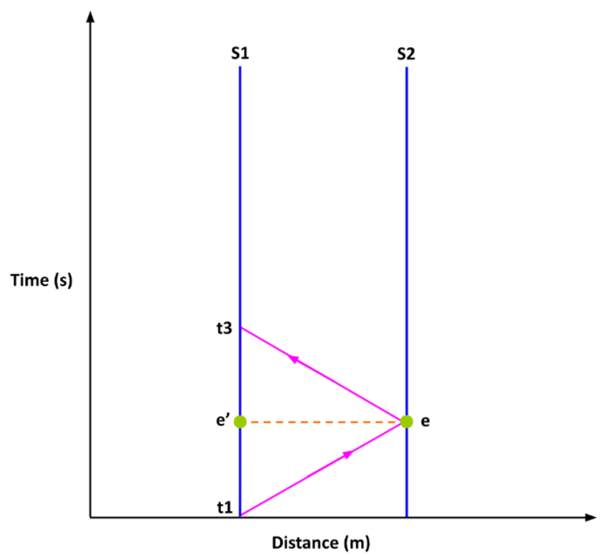

Figure 1. Two inertial systems at rest regarding each other.

Any event along the dotted line in *Figure 1* will be simultaneous to event e. Therefore, e' is the event along the path of S_1 that is simultaneous to event e on the path of S_2.

Scenario 2: One System at Rest, One System in Motion

Let's look at another scenario where system S_3 is moving away from S_2 as it sends a signal. The signal from t_1 to t_2 will travel the same amount of time as in *Figure 2*, but the second signal will travel longer on the way back, which is due to S_3 moving away from S_2. Therefore, the second signal takes more time and distance to reach S_3. This can be demonstrated by the following diagram:

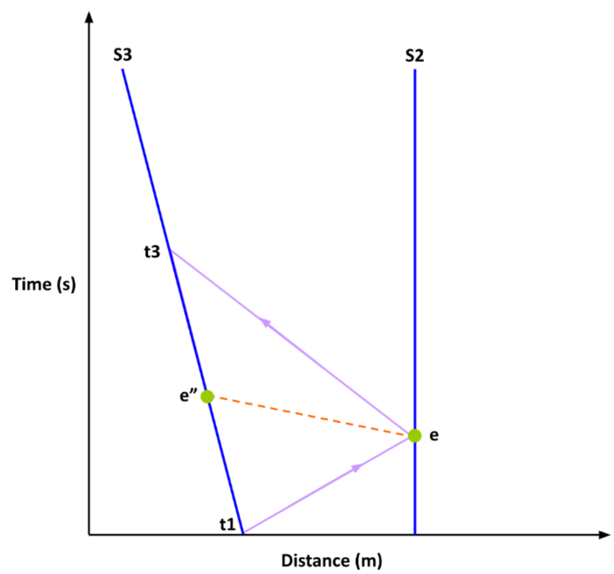

Figure 2. Two inertial systems with one moving away respecting the other.

As can be seen, the second signal has a longer path to travel to get back to S_3 which in turn makes $(t_3 - t_1)$ larger and causes the line of simultaneity to shift. The new event on S_3 that is simultaneous to *e* can be labeled as *e''*.

Comparing Scenarios

To better demonstrate what difference motion makes in determining simultaneity, we can visually combine *Figure 1* and *Figure 2* into *Figure 3*. This will allow us to notice the difference in the lines of simultaneity and the relationship between events *e*, *e'*, and *e''*:

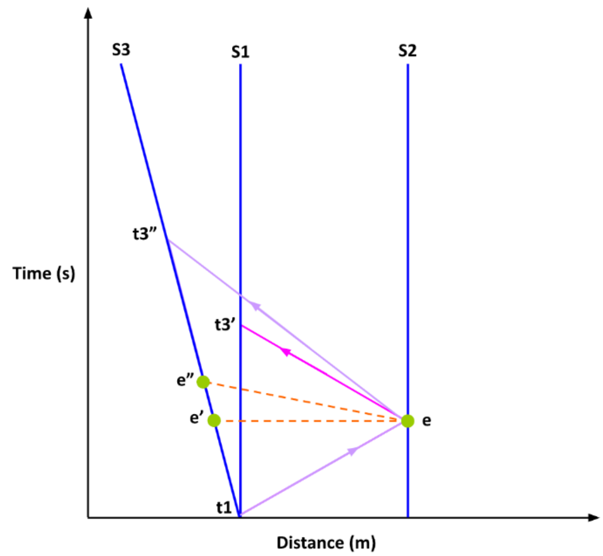

Figure 3. Two systems at rest with a third in motion relevant to them.

As shown, there is a distinctive difference between the lines of simultaneity, which is due to Einstein's claim that simultaneity is relative to a certain frame of reference. *Figure 3* consists of two frames of reference; two systems at rest to each other, and one system in motion relative to a system at rest.

The comparison shown in *Figure 3* demonstrates how the physical motion of a system can alter the line of simultaneity, therefore, causing two different events to be simultaneous. Another interesting feature of *Figure 3* is the logical argument that event *e* is simultaneous with event *e'*, event *e* is simultaneous with *e"*, but events *e'* and *e"* are not simultaneous. This result establishes the claim that properties of simultaneity are not translational between frames of reference.

Why ½?

To understand why Einstein chose ½ to be the fraction represented in the halfway rule, various possible scenarios must be explored to understand why he used that value. First, let's replace the constant of ½ with the Greek letter σ. The generalized form of the halfway rule is

$$e = t_1 + \sigma(t_3 - t_1).$$

The first criterion is that σ must not be equal to 0. This is because if σ was 0, the equation would look like

$$e = t_1 + 0(t_3 - t_1) \text{ or simply } e = t_1.$$

This is not a possible answer because it means that the time it takes for a signal to travel from time t_1 and arrive at time t_2, where event e is located, is instantaneous, and as a result, $t_1 = t_2$. The only way this would be possible is if the signal was traveling faster than the speed of light, which cannot be the case due to the Theory of Special Relativity. Therefore, σ cannot equal zero.

The second criterion is that σ cannot be a negative number. To understand why, let's assume that σ is a negative number. This would give the equation

$$e = t_1 - (t_3 - t_1).$$

The consequence of having a negative sign in the equation would lead to a phenomenon where the signal would travel back in time. This is due to the value of $(t_3 - t_1)$ being subtracted from t_1, where event e would occur before the signal is sent from t_1. This scenario is impossible because events from the future cannot affect the past, therefore, $-\sigma$ is an invalid frame of reference.

The final criterion is that σ cannot be greater than 1. To demonstrate why, let's assume that $\sigma = 2$. This would generate the equation

$$e = t_1 + 2(t_3 - t_1).$$

What this equation indicates is that twice the value of $(t_3 - t_1)$ is being added to t_1 which leads to $e > t_3$, in which t_3 occurs before its signal is even sent from event e. Once again, this is considered traveling backward in time because the sender of that signal, event e, is sending it from the future back to the past. Therefore, σ having a value greater than 1 is an invalid frame of reference.

Considering the three scenarios above, the rule $0 < \sigma < 1$ can be established. The question that remains is the reasoning behind σ equaling ½. It's not simply because Einstein deemed it so; when σ is any other fraction other than ½, experimental results are the same; there's a line of simultaneity generated halfway between t_1 and t_3. Hence further exploration is required to understand this outcome.

When $\sigma \neq \frac{1}{2}$

Experimental results claim that as long as $0 < \sigma < 1$ is true, the value of σ is irrelevant because the result will still be a pair of signals sent from t_1 to t_2 and from t_2 to t_3. Moreover, event e will still end up as the halfway point which marks the line of simultaneity. Let's explore a situation where $\sigma \neq$ ½.

When $\sigma \neq$ ½ the two light vectors are not equal in length and can be assumed that one signal is favored over the other. However, if each signal's speed is c, there is no favoritism because the only difference between the signals is the time taken and distance traveled to get from one point to another. It only seems like one is favored over the other because the scenario is being perceived from the wrong frame of reference.

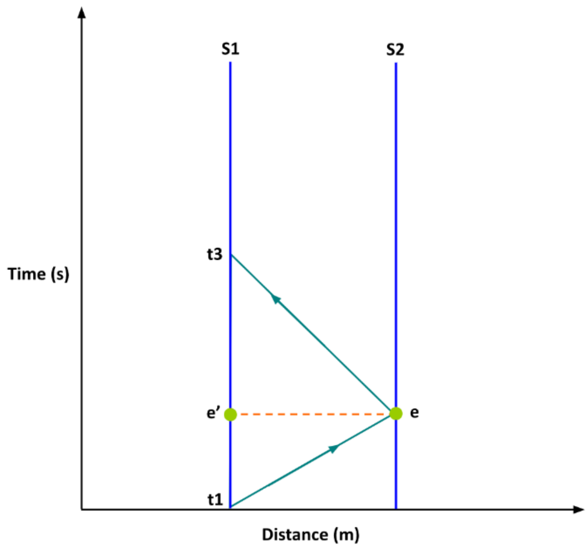

Figure 4. Two systems at rest to each other with uneven signals.

In a scenario where two systems are at rest with each other (*Figure 4*), if you have one signal "favored" by having $\sigma \neq \frac{1}{2}$ and the signal is with speed c, then the time taken for it to travel from t_1 to t_2 is not equivalent to the time taken for the second signal to travel from t_2 to t_3. Looking at it from this perspective, it only seems like the two signals are not equal. However, for every two events connected by a line of simultaneity, there will be a frame of reference from which they will be perceived to be the halfway mark between t_1 and t_3. In this scenario, rotating the space-time axis of *Figure 5* a certain number of degrees will generate a scenario where this is true for events e and e'.

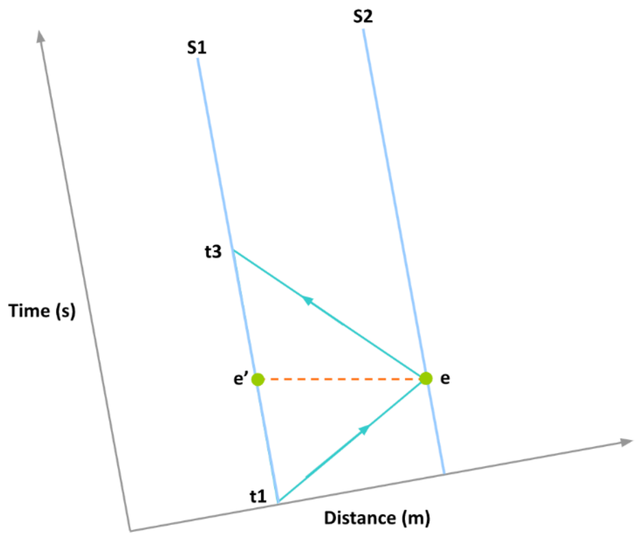

Figure 5. Two systems at rest to each other with a rotated space-time axis.

The result of this rotation is *Figure 5*. Importantly, the events *e* and *e'* in *Figure 5* are the same events from *Figure 4* at the same distance and angle (0°), illustrated by superimposing those events as shown in *Figure 6*:

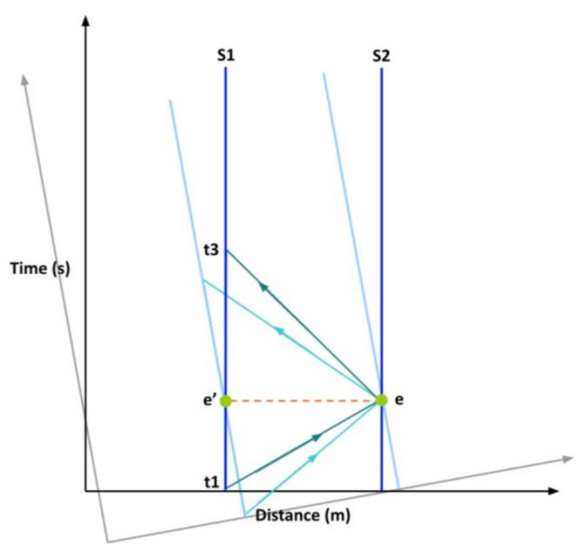

Figure 6. Two space-time axes sharing the same line of simultaneity and events e and e'.

By rotating the space-time axis a specific number of degrees, the frame of reference for events e and e' are changed and their positions are the halfway point between the newly rotated t_1 and t_3. This corresponds with Einstein's experimental findings which show that applying the halfway rule when $\sigma \neq ½$ does not affect the result.

Conclusion

Exploring the boundaries of the halfway rule by entertaining the idea of $\sigma \neq ½$ led to the observation that regardless of what the value of σ is, there will always be a frame of reference from which it will be equal to ½. It is possible to perform synchronization experiments from the same frame of reference where $\sigma \neq ½$, but that will make the calculations more complex due to the absence of symmetry in the light signals. Considering the facts above, it can be claimed that Einstein chose σ to be equal to ½ simply out of convention.

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