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PRODUCT AND PROCESS DESIGN OPTIMIZATION BY QUALITY ENGINEERING

by

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To my wife Wenjuan Lu and my son Rantao Chen

To my mother Shen Dong Xiu and my father Chen Wen Xin

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CHAPTER 1

INTRODUCTION

§ 1.1. Overview

Challenges to improve quality during the "quality revolution" over the past few decades have brought exciting and interesting developments in the methodology of quality design. Japan has shown the world that improving quality leads to improved productivity at no cost increase. Dr. Genichi Taguchi, a Japanese quality control expert, has been credited for his contribution to the Japanese quality engineering. In America, Taguchi's quality control philosophy and methods have attracted extensive attention of quality engineers and statisticians. Many successful applications of Taguchi's method have been published (Bandurek *et al* 1988, Bendell 1988, Kacker and Shoemaker 1986, Lin and Kacker 1985, Phadke 1986, Phadke *et al* 1983). However, much controversy has arisen about the efficiency and the theoretical basis of his engineering and statistical techniques, such as signal-to-noise ratio, minute accumulation analysis, quality loss function, *etc* (Box 1988; Box, Bisgaard and Fung 1988; Nair 1986; Leon, Shoemaker and Kacker 1987; Ryan 1988).

Traditional quality control is concerned with the downstream side of the process, with an emphasis on control charts and inspection schemes. The emphasis now is on moving upstream to the design and development stages of products. The concept of robust design as proposed by Taguchi (Taguchi 1986, Taguchi and Wu 1980, Kacker 1985, Barker 1986) is based on making a system insensitive or robust to manufacturing variation, deterioration over time and environmental disturbance. Philosophically, robust design is an economical way to improve quality, because it builds good quality into products and processes without using high-grade components. Basically, the method consists of three steps (Taguchi 1986, Taguchi and Wu 1980, see also Figure 1.1).

(1) System Design

The objective is to obtain a workable prototype model of the system or the process. Much of the previous and the current effort in the United States is concentrated on this step.

(2) Parameter Design

This is the most important and effective step in the method. In this step, engineers are intended to design a system whose performance is insensitive to variations by selecting the optimal level setting for control factors (with no impact on system cost).

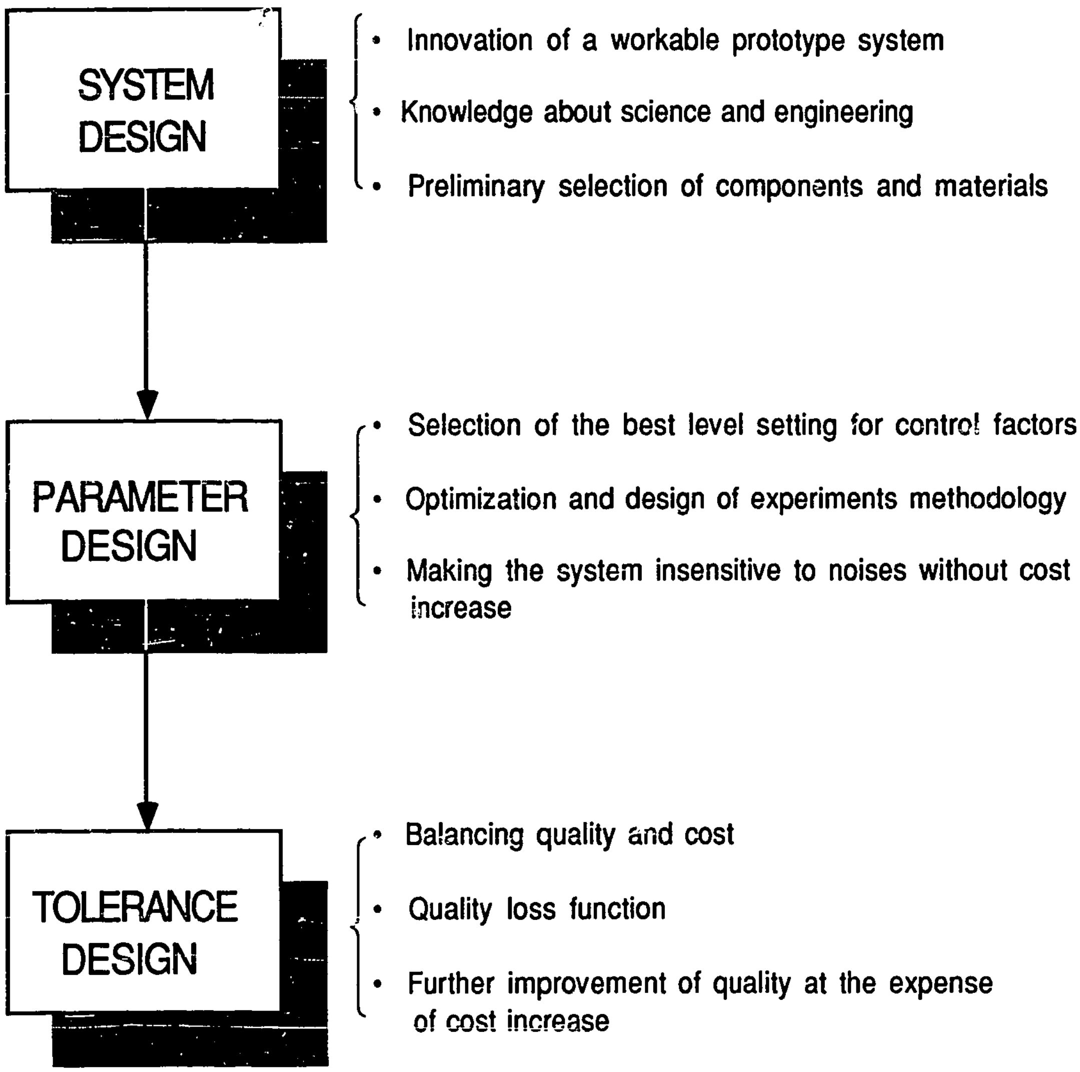


Figure 1.1. Taguchi's robust design procedure.

(3) Tolerance Design

If parameter design cannot achieve the required small performance variation, tolerance design can be used to reduce the variation by tightening tolerance levels. In this step, designers balance the quality loss due to the variation effects and the cost increase due to the control of tolerances.

The discussion in this thesis will primarily focus on parameter design and tolerance design techniques.

Depending on the nature of the target performance, a system can be classified as static or dynamic. The static quality characteristic has a fixed target value. The effort in design and manufacturing is to bring the quality characteristic to the target value. For dynamic characteristics, we do not have a fixed target value. The target is a variable which depends on the intention of the customer. Our effort is to reduce the undesirable deviation from the target function.

Compared with on-line quality control that is the quality control activities on a production line, off-line quality control brings quality control activities into design and development stages. The mission is to design robust systems or to build good quality into products and processes, instead of trying to inspect bad quality out. A classification of various factors for the optimization process is given as follows (bold letters are vector quantities unless otherwise specified):

• *Signal Factor (M)*

For static systems, the target value of the performance is fixed. We do not need a signal factor to adjust the system performance. For dynamic systems, however, a signal factor is used by a user or an operator to dynamically control the system performance. A signal factor is not a design factor and it is not controlled by the design engineer. In the example for calibration of gauge systems, the reference sample value can be considered as a signal factor. Other examples of this factor are the steering angle for the drivability of trucks, the volume control and the tune equalizer for a stereo sound system. For the example in Chapter 4, the input voltage V_i is also a signal factor.

• *Design Factors*

To design a system, we have many design parameters. These parameters are controlled by design engineers and are called design factors. In some cases, the design factors can be partitioned into the *control factors* $\mathbf{Z}$ and the *scaling/leveling factors* $\boldsymbol{R}$. $\mathbf{Z}$ is a vector quantity of the controllable parameters. Each control factor may take one of the several levels which are constrained in design spaces. Parameter design is used to search for the best levels for control factors so that the system performance is less sensitive to noise factors. Since the control factors take discrete values, some adjustable design factors may be needed to be adjusted continuously to achieve desired functional relationships between the signal factor and the performance variable for dynamic systems, in case the desired relationships cannot be obtained by the levels combination of the control factors. **R** is a vector quantity of these continuous design factors. Examples are the gearing ratio in the steering mechanism and the threshold voltage in digital circuits. For static characteristics, these factors can be used to adjust the mean to the target value. For the example in Chapter 4, resistors R_1, R_2 and R_a are the controllable parameters and called control factors. We can select the best levels for these factors to make the amplifier insensitive to the variations. While, resistor R is an adjustable resistor which can be continuously adjusted to achieve the required β value.

- *Noise Factors (e).*

A perfect system is desired to implement the certain task. In the real world, however, manufacturing variation, environmental conditions, wear and deterioration over time may result in deviations of the performance from the target. These undesired and unpredictable impacts are called noise factors. The goal in robust design is to make the system insensitive to these factors. Noise factors include internal noises (such as manufacturing variation and deterioration over time), external noises (such as environmental variations & use conditions).

Engineers are sometimes interested in the effects of some specific factors (**D**), which are called *indicative factors*. The goal is to evaluate the system performance for various values of these factors, rather than to select the best levels for them. For instance, if we need to evaluate the performance of a car under different speeds, we can classify the speed as an indicative factor. This kind of factor is not a control factor, because we cannot reduce the performance sensitivity to noise factors by setting the best levels for indicative factors. Instead, the values of these factors is automatically constrained. It should be indicated that it is not necessary for all factors to exist in every system.

For the cases where a symmetrical quadratic quality loss function (QLF) is appropriate, the expected quality loss is given by (Chen and Kapur 1989)

$$E[L(Y,y_0)] = \delta^2 + \sigma_y^2 \qquad (1.1)$$

where δ is the difference between the target value y_0 and the mean of characteristic Y, and σ_y^2 is the variance of Y. To reduce the expected quality loss, the optimization can be done by minimizing σ_y^2 with the assumption that δ can be adjusted to zero. As a result, a two-step procedure of optimization is proposed to reflect the idea of Taguchi's parameter design using signal-to-noise (SN) ratio:

*(1) Find the optimal levels setting for **Z** to maximize the SN ratio.*

*(2) Adjust **R** to set the mean response to the target value.*

The concept of the two-step optimization was first proposed to explain Taguchi's parameter design by Phadke (1982). To illustrate why the SN ratio is used as a criterion for selecting control factors **Z**, he explains:

Frequently, as the mean decreases the standard deviation also decreases and vice versa. In such cases, if we work in terms of the standard deviation, the optimization cannot be done in two steps, i.e. we cannot minimize the standard deviation first and then bring the mean on target.

This probably is a motive of using the SN ratio for static systems. The goal in design is to minimize the expected quality loss by selecting the best levels setting for **Z** and adjusting **R** in the design space. The basic tools are quality loss function, design of experiments (DOE), orthogonal arrays, statistics and optimization methodology (Barker 1985, Box *et al* 1978, Hicks 1982, Logothetis and Wynn 1989, Montgomery 1984, Ross 1988, Taguchi 1987).

It is obvious that the two-step procedure can simplify the optimization. The first step can be conducted using the classical DOE methodology by assigning the control factors into the inner array and assigning the noise factors as well as the signal factor into the outer array. The second step can be completed by adjustments. However, this two-step procedure results in much discussion. One of the questions addressed is the use of the SN ratio. Based on the research about the SN ratio, Leon *et al* (1987) conclude that the SN ratio may become inefficient for some models and the above two-step procedure may not lead to an optimal solution. For instance, they present a model as follows:

$$Y = \alpha(\mathbf{R},\mathbf{Z}) + \beta(\mathbf{R},\mathbf{Z})M + \varepsilon(\mathbf{Z},\mathbf{e}) \qquad (1.2)$$

For this model, the SN ratio, which is given by $\beta^2(\mathbf{R},\mathbf{Z})/\mathrm{Var}[\varepsilon(\mathbf{Z},\mathbf{e})]$, is not independent of adjustment factors **R**. As a result, the two-step procedure may not lead to the minimal value of the expected quadratic quality loss by using the SN ratio. Instead, they develop the *Performance Measure Independent of Adjustment (PerMIA)* as an optimization criterion to substitute for the SN ratio. The two-step optimization is amended and developed as many two-step procedures. For further details, see Leon *et al* (1987). However, to derive a PerMIA, one must know the model. If an analytical model is unknown, it seems difficult to find a PerMIA. In addition, as revealed in Section 3.4, a performance measure defined only by "independent of adjustment" is not assured to be an efficient optimization criterion without any further explanation or restriction. This issue will be discussed in-depth for dynamic systems later.

One of the conventional approaches to tolerance design is based on Taylor's series (Spence and Soin 1988). The tolerance levels for components are evaluated based on the impacts on the system performance. To conduct tolerance design for components and subsystems, Taguchi uses an equation to transfer the variations in components to the variations in the system performance with the assumption of no interactions between components (Taguchi and Wu

1980; Wu and Moore 1986). In an effort to improve Taguchi's method for tolerance design, D'Errico and Zaino (1988) revise the assignment of the levels of the noise factors to make the simulation of the noise factors more realistic. Regardless of the difficulty in assigning the noise factors into the outer array, D'Errico and Zaino's method can give a more realistic simulation of the noise factors.

§ 1.2. Objective And Outline Of The Research

Product and process design optimization by quality engineering discussed here is related to the parameter design and the tolerance design. Our goal is to investigate, reveal and explore the engineering, mathematical and statistical basis of Taguchi's robust design, to improve and extend the methodology, with an emphasis on dynamic systems and tolerance design.

Taguchi's economic optimization and balancing is evaluated by quality loss functions (QLF). Chapter 2, entitled *quality evaluation system using loss function*, is concerned with various QLF, including the symmetric QLF and the asymmetric QLF. For many systems, the quadratic QLF is a good approximation based on the underlying causes of variations and the attempt to reduce the estimation errors.

In practice, many systems have several quality characteristics rather than a single characteristic. To evaluate quality for such systems, we must develop a multivariate QLF.

In Taguchi's robust design, the SN ratio is used as a criterion for optimization. However, the use of the SN ratio results in much controversy (Box 1988, Box *et al* 1988, Kapur and Chen 1988, Leon *et al* 1987, Phadke and Dehnad 1988). Leon *et al* propose a criterion called *Performance Measure Independent of Adjustment* (PerMIA) as a substitute for the SN ratio. In Chapter 3, we will examine the motivation and the effectiveness of the SN ratio for dynamic systems and make necessary modification of the SN ratio for the model where its use is questionable.

In Chapter 4, we present a generic optimization model for dynamic systems, in order to reduce the undesired effects of noise factors. The dynamic characteristic does not have a fixed target value. The optimization is performed by maximizing the objective function or the quality measure with the parameters of the dynamic function subject to the desired values. A systematic approach to dealing with dynamic systems is also provided in this chapter.

If either or both of the signal factor and the performance of a system is discrete, the system is classified as discrete in nature. A control system is a typical discrete dynamic system, where the performance of the system is on or off. To make the performance of such systems insensitive to noise factors, we can use the approaches similar to those for continuous dynamic characteristics. Procedures of experiments and optimization are given in Chapter 5.

The objective of tolerance design is to balance quality loss due to variations and cost increase due to control of variations. A system may consist of many components and subsystems. Variations in the parameters of these components and subsystems can be transferred to the variations in the system performance. To reduce variations, we can control the tolerances of the components but that will lead to a cost increase. To eliminate the conflict of this, an optimization model is proposed to balance quality and cost. Chapter 6 deals with the method to specify the tolerances for components and subsystems to achieve the above goal.

A machined part to be processed by a manufacturing process may use different quality characteristics before it is processed and after it is processed. For instance, a stamped product manufactured from steel plates may use the dimension as a quality characteristic, but the plates may use hardness and thickness as quality characteristics. The former is named as a higher characteristic and the latter is named as a lower characteristic. In Chapter 7, we present the methods to find tolerances for these characteristics, as well as the tolerances for deterioration characteristics.

In Chapter 8, we give a summary for the contributions of this research to the techniques of Taguchi's quality engineering and we make a recommendation for further research.

CHAPTER 2

QUALITY EVALUATION SYSTEM USING LOSS FUNCTION

§ 2.1. Introduction

The traditional quality evaluation system deals with the conformance to specifications. The system focuses only on the nonconforming units and cost of quality is defined as the cost of nonconformance. A better evaluation system is to evaluate the quality of all the items, both within as well as outside the specifications. We evaluate total population using a quality loss function as the characteristics deviate from an ideal or target value y_0. The following quality evaluation system proposed by Taguchi reflects this idea (Taguchi and Wu 1980, Taguchi 1986, Kacker 1985)

The quality of a product is the (minimum) loss imparted by the product to the society from the time the product is shipped.

This is a more holistic view point of quality that relates quality to cost and loss in dollars, not just to the manufacturer at the time of production, but to the consumer and thus to the whole society. The quality activities must focus on the reduction of this loss.

The undesirable and uncontrollable factors that cause a functional characteristic to deviate from its target value are called noise factors (Figure 2.1). Noises adversely affect quality. However, eliminating noise factors may be expensive. Instead, we can try to reduce the effect of the noise factors. Taguchi's philosophy of robust design is intended to reduce the loss due to variations of the performance from the target value and is based on the quality loss function, the SN ratio, the optimization and design of experiments methodology (Box 1988, Hicks 1982, Kapur and Chen 1988, Leon, *et al* 1987, Montgomery 1984, Taguchi 1987).

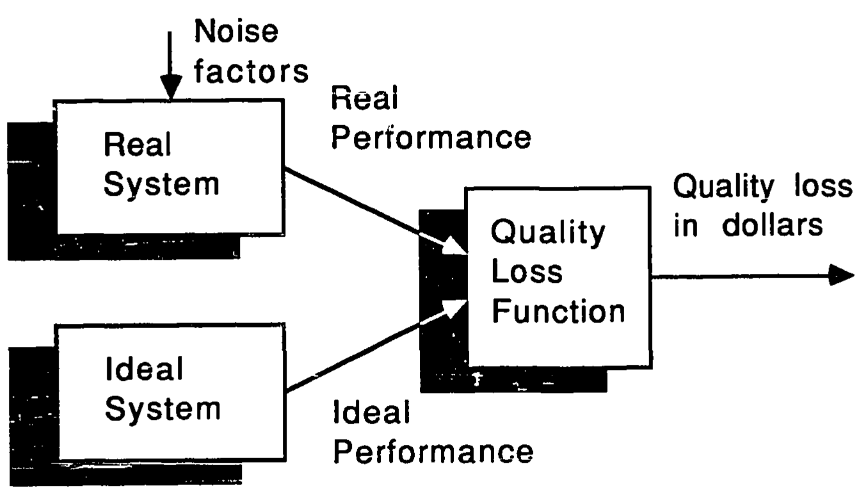


Figure 2.1. QLF transfers deviations to a quality loss in dollars.

The objective of this chapter is to develop various loss functions that can be used to evaluate quality for many systems. We give an overview of the quality evaluation system. We illustrate how to use the loss function to evaluate quality. Various univariate loss functions are presented, including symmetrical and asymmetrical, linear and quadratic loss functions. The reason for using the popular quadratic loss function is also explained. For these loss functions, the expected value and the variance of the quality characteristic is determined to minimize the expected quality loss. Many systems have several quality characteristics and the optimization of these characteristics may conflict with each other. To deal with such systems, we develop the multivariate loss function. An example with two characteristics is given to demonstrate the application of the multivariate loss function.

§ 2.2. Quality Evaluation System

The traditional quality evaluation system is based on specification limits. If the characteristic for the product is within the specification limits, the product is classified as good or conforming and no loss is incurred. Otherwise, the product is nonconforming and results in a certain amount of economic loss, say K_1 or K_2 dollars depending on whether the nonconforming product is to be scrapped or reworked (Figure 2.2). This binary evaluation system is very simplistic. Assume we have two values of the quality characteristic around a specification limit, one within the limit and the other outside the limit. The first one has no loss but the second one causes a certain amount of dollar loss, even if the difference is as small as it can be identified.

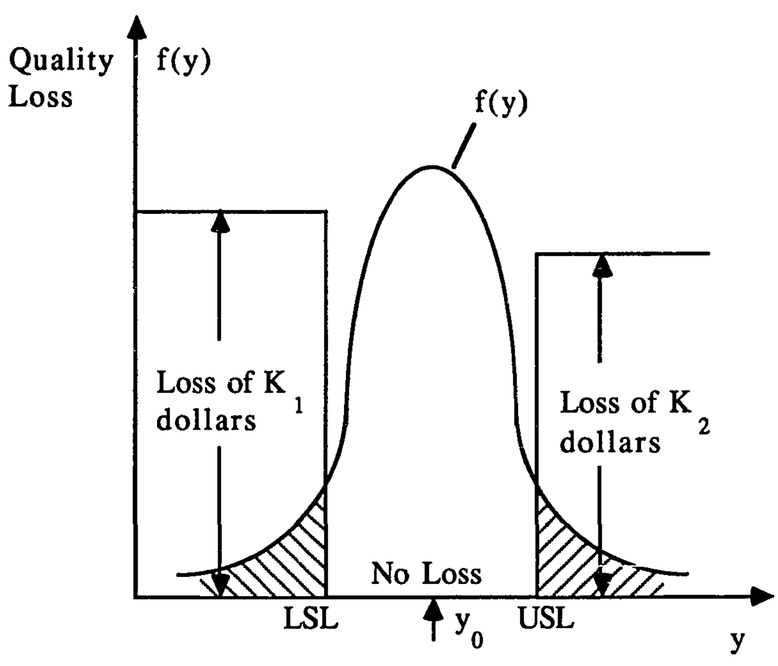


Figure 2.2. Traditional quality evaluation (the shaded area is nonconforming portion)

The quality evaluation system using loss function evaluates quality on a continuous basis. Kapur (1988), Kapur and Wang (1987) illustrate the development of the specification limits using a quality loss function. Actually, the loss function is nothing but a means to transfer the variation of the characteristic to a monetary scale. According to the design goal, the engineer wants to design a system to meet the customer's requirement. However, due to manufacturing and environmental variations, the performance of a real system may deviate from the ideal performance. This results in a quality loss. A loss function (Figure 2.3) can be used to evaluate this quality loss, which is given by

$$\text{Loss} = L(y, y_0) \qquad (2.1)$$

where y is the quality characteristic and y_0 is a target value.

In practice, a product may have several characteristics rather than a single characteristic. If these characteristics are independent of each other and there is no interaction between their effects, then the total quality loss is the sum of the loss caused by each characteristic. But for many real systems, the total quality loss can not be expressed as the sum of the loss for each characteristic. Instead, we must develop a measure to evaluate such systems with multiple characteristics. This is the motivation of the multivariate loss function that we will be discussing later in this chapter.

The manufacturing variation and the environmental variation may adversely affect a system so that the quality characteristic is a random variable. To evaluate quality for the total population, we can use the expected quality loss that is given by

$$E[L] = \begin{cases} \int_{\text{all } y} L(y, y_0) f(y) dy & \text{if } y \text{ is continuous} \\ \sum_{\text{all } i} L(y_i, y_0) p_i & \text{if } y \text{ is discrete} \end{cases} \tag{2.2}$$

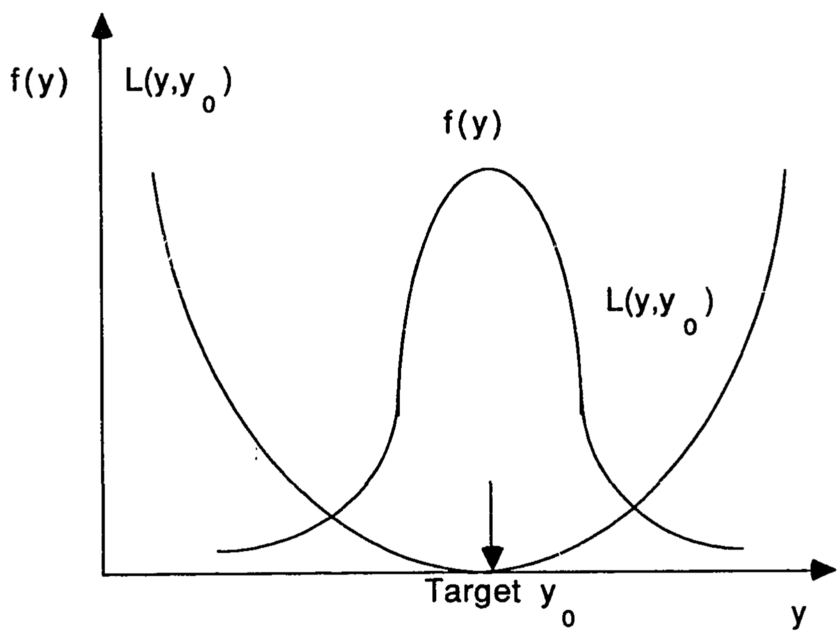


Figure 2.3. Quality evaluation using loss function

In eq. (2.2), f(y) is the probability density function for the random variable Y and p_i is the probability that Y takes value y_i. Quality engineers are interested in the reduction of the expected quality loss by designing the population parameters for Y, such as the location effect or the mean, the dispersion effect or the variance, *etc.* In this chapter, the discussion is based on continuous characteristics, but it is applicable to discrete quality characteristics.

§ 2.3. Quadratic Quality Loss Function

The quadratic loss function is a good approximation for many systems based on the underlying causes of variation. The generic loss function (2.1) can be expanded at y_0 using Taylor's series, with the assumption that it has derivatives up to the fourth order and it is given by

$$L = L(y_o, y_o) + L'(y_o, y_o)(y - y_o) + \frac{L''(y_o, y_o)}{2!}(y - y_o)^2$$

$$+ \frac{L'''(y_o, y_o)}{3!}(y - y_o)^3 + \frac{L^{(4)}(\xi, y_o)}{4!}(y - y_o)^4 \qquad (2.3)$$

where ξ is a value between y_0 and y.

The target y_0 is developed such that the quality loss achieves a minimum at this point. Hence, the first derivative of the loss function at y_0 is zero. Our goal is to investigate the quality loss due to the deviation from y_0 and to minimize this loss. Moreover, in the neighborhood of y_0, the quadratic term in eq. (2.3) can dominate the higher order terms. Thus, the quality loss due to the deviation from y_0 can be estimated by

$$L(y,y_0) \approx K(y-y_0)^2 \qquad (2.4)$$

Since y_0 is a minimum of the loss function, then $K=L''(y_0,y_0)/2>0$. The unknown value of K can be determined by substituting the data of the quality loss at one point. For example, if the deviation Δ_0 from y_0 causes a quality loss A_0, then $A_0=K(\Delta_0)^2$ or $K=A_0/(\Delta_0)^2$. It should be indicated that if the loss function (2.1) is known, it can be used directly.

Since the individual quality characteristic y may be far away from the target value y_0, the higher order terms may become large. It is reasonable to use the expected value of the quality loss, because most products have a quality characteristic close to the target. Let μ_y be the mean of Y and $\delta=y_0-\mu_y$. The expected quality loss is given by

$$E[L(Y,y_0)] \approx \int_{\text{all } y} K(y-y_0)^2 f(y)\,dy = K(\sigma_y^2 + \delta^2) \qquad (2.5)$$

As a result, minimizing $\sigma_y{}^2$ and δ^2 is equivalent to minimizing the expected quality loss. To estimate the error due to the ignored 3rd or higher order terms, we consider the following two cases with the assumption that $y_0=\mu_y$.

(1) $L(y,y_0)$ Is Symmetrical About y_0.

Since $L(y,y_0)$ is symmetrical about y_0, the 3rd order term in eq. (2.3) vanishes. The expected value of the error is given by

$$Er = \left|E[L(Y,y_0)] - K\sigma_y^2\right| = \frac{1}{4!}\left|\int_{\text{all } y} L^{(4)}(\xi,y_0)(y-y_0)^4 f(y)\,dy\right| \le \frac{K_4}{4!}\mu_4 \qquad (2.6)$$

where K_4 is the maximal value of $|L^{(4)}(y,y_0)|$ in the domain of y and μ_4 is the 4th moment of y about the mean. The last inequality uses the fact that $(y-y_0)^4 f(y)$ is greater than zero for all y.

(2) $L(y,y_0)$ Is Asymmetrical About y_0.

A quadratic loss function is a typical quality loss function. In practice, a loss function may be modeled as asymmetrical about y_0. For instance, a shaft has a dimension of 2.000±0.002 inches for the diameter. An oversized shaft has smaller loss than an undersized shaft even if they have the same absolute value of deviations from the target 2.000 inches, because the oversized shaft can be reworked while the undersized shaft must be scrapped. As a result, $L(y,y_0)$ is asymmetrical about y_0 for this situation. The asymmetric $L(y,y_0)$ can be divided into two parts (Figure 2.4) and given by

$$L(y,y_0) = \begin{cases} L_1(y,y_0) & \text{if } y \le y_0 \\ L_2(y,y_0) & \text{if } y \ge y_0 \end{cases} \tag{2.7}$$

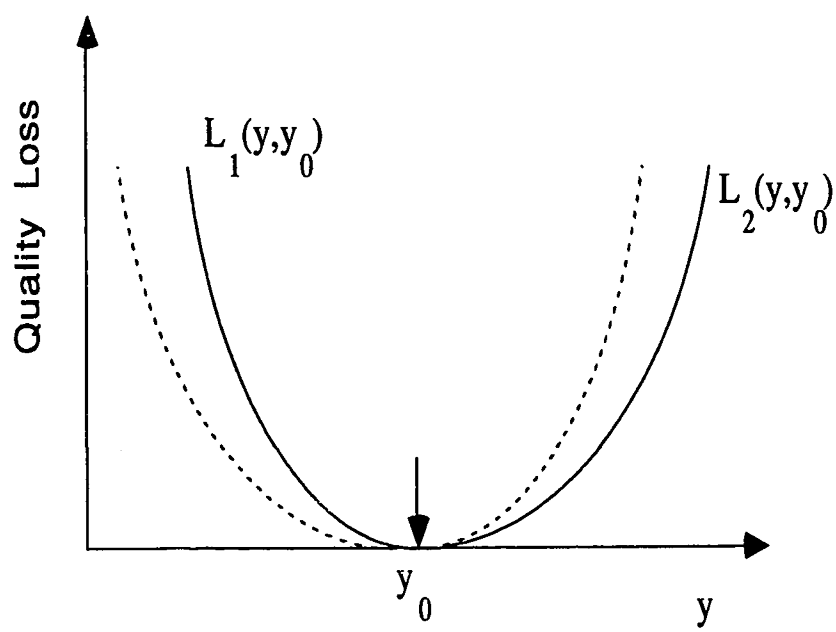


Figure 2.4. $L(y,y_0)$ is asymmetrical about y_0

We extend the definition of $L_1(y,y_0)$ to $y>y_0$ and the definition of $L_2(y,y_0)$ to $y<y_0$ (as the dotted curves in Figure 2.4). We can do this to make $L_1(y,y_0)$ and $L_2(y,y_0)$ symmetrical about y_0. Furthermore, $L_1(y,y_0)$, $L_2(y,y_0)$ can be expanded separately using Taylor's series with the 4th order remainder at y_0:

$$L_i = L_i(y_o,y_o) + L_i'(y_o,y_o)(y-y_o) + \frac{L_i''(y_o,y_o)}{2!}(y-y_o)^2$$
$$+ \frac{L_i'''(y_o,y_o)}{3!}(y-y_o)^3 + \frac{L_i^{(4)}(\xi,y_o)}{4!}(y-y_o)^4 \qquad (i=1,2) \qquad (2.8a)$$

Simplifying eq. (2.8a) as we did for eq. (2.3), we have

$$L(y,y_0) \approx \begin{cases} K_1(y-y_0)^2 & \text{if } y \le y_0 \\ K_2(y-y_0)^2 & \text{if } y \ge y_0 \end{cases} \qquad (2.8)$$

where $K_1=L_1''(y_0,y_0)/2$, $K_2=L_2''(y_0,y_0)/2>0$. The unknown values of K_1 and K_2 can be determined by substituting the data of the quality loss at one point for both $y<y_0$ and $y>y_0$. For example, if a deviation Δ_i from y_0 results in a quality loss A_i $(i=1,2)$, then $A_i=K_i(\Delta_i)^2$ or $K_i=A_i/(\Delta_i)^2$ $(\Delta_1<0,\ \Delta_2>0)$. Since the 3rd order terms for L_1 and L_2 vanish, the expected value of the error for using eq. (2.8) is given by

$$Er = \int_{y \le y_0} \frac{\left|L_1^{(4)}(\xi_1, y_0)\right|}{4!}(y - y_0)^4 f(y)\,dy$$

$$+ \int_{y \ge y_0} \frac{\left|L_2^{(4)}(\xi_2, y_0)\right|}{4!}(y - y_0)^4 f(y)\,dy \le \frac{K_4'}{4!}\mu_4 \qquad (2.9)$$

where $K_4' = \max\limits_{\text{all } y}\left(\left|L_1^{(4)}(y, y_0)\right|, \left|L_2^{(4)}(y, y_0)\right|\right)$

§2.4. Linear Quality Loss Function

In many situations, the quality loss has a linear relationship to the deviation of the quality characteristic from the target value. In other words, the quality loss is proportional to the bias of the quality characteristic from the target y_0. This deviation may result in a different quality loss for $y<y_0$ from that for $y>y_0$. Thus, the loss function is asymmetrical about y_0 and given by

$$L(y, y_0) = \begin{cases} K_1|y - y_0| & \text{if } y < y_0 \\ K_2|y - y_0| & \text{if } y \ge y_0 \end{cases} \qquad (2.10)$$

where K_1, $K_2 \ge 0$ are constants that can be determined in the same way as the quadratic loss function. The loss function is symmetrical about y_0 if $K_1=K_2$. The expected quality loss is given by

$$E[L] = \int_{y<y_0} K_1(y_0 - y)f(y)dy + \int_{y\geq y_0} K_2(y - y_0)f(y)dy \qquad (2.11)$$

We wish to design μ_y to minimize the expected quality loss. This can be achieved as follows: since the expected quality loss is a mean value for the population, E[L] has the same value for graph (i) and graph (ii) in Figure 2.5, provided that the relative positions of $L(y,y_0)$ and f(y) are not changed. This motivates us to fix μ_y first, and then take y_0 as a variable to find y_0* such that $E[L'(y,y_0^*)]$ is a minimum. Afterwards, we move f'(y) to f(y) and $L'(y,y_0)$ to $L(y,y_0)$ so that $y_0^*=y_0$ (Figure 2.5). The corresponding μ_y^* is an optimal solution for μ_y. The advantage to do this is that we can take the explicit derivative of E[L] with respect to y_0 to find y_0* without the knowledge of f(y), because f(y) is not related to y_0.

It can be analytically verified that $E[L(y,y_0)]$ is unchanged if f(y) and $L(y,y_0)$ experience an identical translation. Let $y'=y+(y_0^*-y_0)$. Thus, f(y)=f'(y') as well as $L(y,y_0)=L'(y',y_0^*)$, and we have

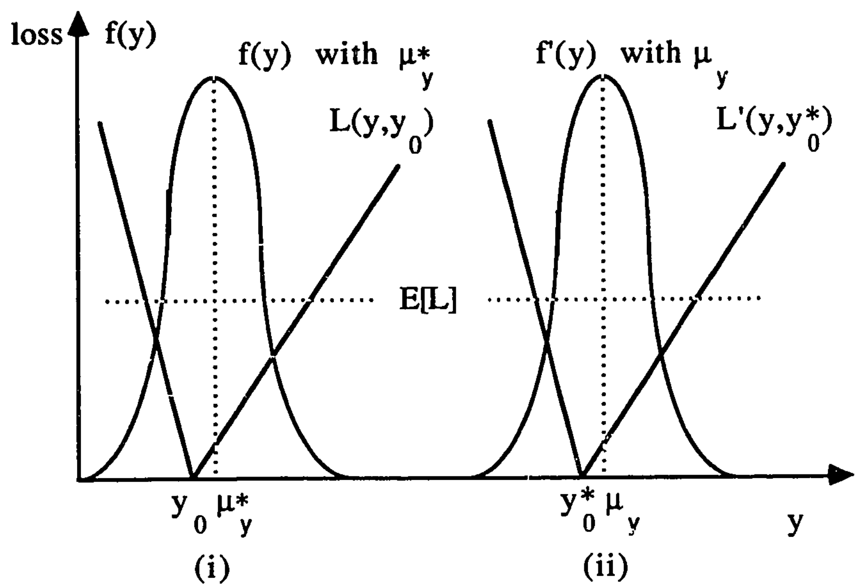


Figure 2.5. E[L] remains unchanged when moving f(y) and $L(y,y_0)$ simultaneously

$$E[L] = \int_{\text{all } y'} L'(y',y_0^*)f'(y')dy' = \int_{\text{all } y'} L(y,y_0)f(y)dy' = \int_{\text{all } y} L(y,y_0)f(y)dy$$

Thus, E[L] is unchanged if f'(y) is moved to f(y) and $L'(y,y_0)$ is moved to $L(y,y_0)$.

Taking the derivative of eq. (2.11) with respect to y_0 and setting it to zero, we have

$$F(y_0^*) = K_2/(K_1+K_2) \tag{2.12}$$

where F(y) is the cumulative distribution function of Y. Setting $y_0^*=y_0$, we can find a μ_y^* by using equation $F(y_0)=K_2/(K_1+K_2)$. μ_y^* can minimize the expected quality loss. If $K_1=K_2$, y_0* is equal to the median of y. Moreover, if f(y) is symmetric about the mean, then the median or y_0 is equal to the mean. Besides, since $(K_1+K_2)f(y_0*)$, the 2nd derivative of $E[L(y,y_0)]$ with respect to y_0, is greater than zero, the solution gives a minimal expected quality loss that is given by

$$E[L(Y,y_0)] = -(K_1+K_2)\int_{y<y_0} y f(y)dy + K_2\mu_y \qquad (2.13)$$

If Y has a normal distribution, then $E[L(Y,y_0)]=(K_1+K_2)\sigma_y^2 f(y_0)$ (see Appendix A).

§2.5. Other Univariate Quality Loss Function

In addition to the linear and the quadratic loss functions, there are other univariate loss functions that can be used in various situations.

(1) "Smaller the Better" Loss Function.

Many quality characteristics have a target value zero and they have the positive values. For these characteristics, engineers usually have an upper specification limit. Examples are wear, degradation, deterioration, shrinkage, noise level, harmful effects, and the level of pollutants, *etc.* The loss function is given by

$$L(y) = Ky^2 \qquad y>0 \tag{2.14}$$

The expected value of the quality loss is given by

$$E[L(Y)] = K(\sigma_y^2 + \mu_y^2) \tag{2.15}$$

To reduce the expected quality loss, we should reduce σ_y and μ_y.

(2) "Larger the Better" Loss Function

The ideal value for this kind of characteristic is infinite. For this characteristic, engineers usually have a lower specification limit. Examples of this are strength, reliability, maintainability, *etc.* The quality loss function is given by

$$L(y) = K/y^2 \qquad y \geq d > 0 \tag{2.16}$$

To estimate the expected quality loss, we expand eq. (2.16) using a Taylor's series at $y=\mu_y$, because most products have a characteristic close to μ_y. Thus, we have

$$L(y) = \frac{K}{\mu_y^2}\left[1 - \frac{2(y-\mu_y)}{\mu_y} + \frac{3(y-\mu_y)^2}{\mu_y^2} - \ldots\ldots\right] \qquad (217)$$

If μ_y is much larger than the absolute value of the $(y-\mu_y)$, the first three terms can dominate the 3rd or higher-order terms. By taking the expected value with the assumption that the 3rd or higher order terms can be ignored, we have

$$E[L(Y)] \approx (K/\mu_y^2)[1+3\sigma_y^2/\mu_y^2] \quad (2.18)$$

Large μ_y and small σ_y^2 can reduce the expected quality loss. If we consider the terms up to the 4th order in expression (2.17), then the expected quality loss is given by

$$E[L(Y)] \approx [K/\mu_y^2][1+3\sigma_y^2/\mu_y^2-4\gamma_1\sigma_y^3/\mu_y^3+5(\gamma_2+3)\sigma_y^4/\mu_y^4] \quad (2.18a)$$

where γ_1 is the skewness and γ_2 is the kurtosis. For the same μ_y and σ_y, positive γ_1 and small γ_2 can reduce the expected quality loss. This is somehow interesting. Several numerical examples in Appendix B can verify this conclusion.

(3) Polynomial Loss Function

If a number of quality loss values are known for various deviations from the target, we can fit a polynomial model

$$L(y,y_o) = \sum_{i=1}^{n} K_i(y-y_o)^i \quad (2.19)$$

If m pairs of y_i and loss value are known, K_i can be determined by either using regression methods (Draper 1981, Gunst and Mason 1980) or solving a system of n simultaneous linear equations if n=m. The expected value of $L(y,y_0)$ is given by

$$E[L(Y,y_0)] = \sum_{i=1}^{n} K_i \sum_{j=0}^{i} \binom{i}{j} \mu_j (-\delta)^{i-j} \qquad (2.20)$$

where μ_j is the jth moment of y about the mean and $\delta = y_0 - \mu_y$. If $\delta = 0$, we have

$$E[L(Y,y_0)] = \sum_{j=1}^{n} K_j \mu_j \qquad (2.21)$$

§ 2.6. Multivariate Loss Function (MLF)

In the real world, a system or a product may have several quality characteristics rather than a single characteristic. For instance, TV sets have picture quality, tone quality, *etc.* Each quality characteristic may cause a quality loss. Moreover, these quality characteristics may be dependent on each other. Therefore, in order to evaluate quality for such systems, we are motivated to develop the multivariate loss function (MLF). Suppose that the multivariate loss function for a system is given by a function

$$\text{Loss} = L(y_1, \ldots y_n; t_1, \ldots t_n) \qquad (2.22)$$

where $y_1, \ldots y_n$ are the quality characteristics of the system and $t_1, \ldots t_n$ are the respective target values (see also Figure 2.6).

In some cases, the loss function can be decomposed as the sum of the quality loss for each characteristic, or

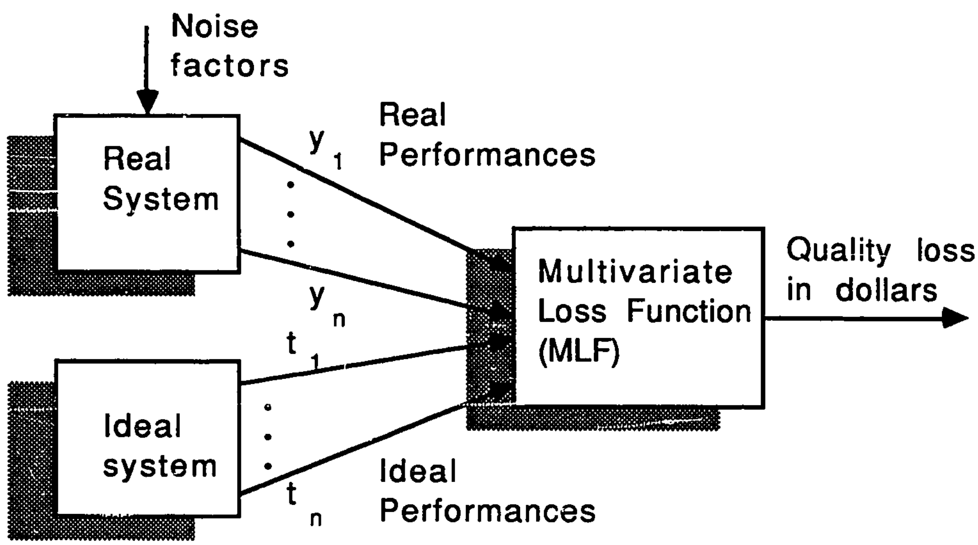


Figure 2.6. MLF transfers multivariate deviations to a loss in dollars

$$L = L_1(y_1, t_1) + \dots + L_n(y_n, t_n) \tag{2.23}$$

The expected value of the total quality loss is the sum of the expected quality loss for each characteristic. Each characteristic can use a univariate loss function to evaluate quality.

For some systems, the total quality loss is the product of the quality loss for each characteristic, or

$$L = \prod_{i=1}^{n} L_i(y_i, t_i) \tag{2.24}$$

In this case, if y_i is independent of y_j (for $i \neq j$), then the expected value of the total quality loss is the product of the expected quality loss for each characteristic. If y_i's are not independent of each other, the expected logarithmic value of the total quality loss can be used by performing logarithmic transformation and taking the expected value of eq. (2.24)

$$E[\log(L)] = E[\log(L_1)] + \cdots + E[\log(L_n)] \qquad (2.25)$$

which can be used as a criterion to evaluate quality for such systems.

(1) Quadratic Multivariate Loss Function

For the generic multivariate loss function, we can rewrite eq. (2.22) using vector notations as follows: (bold symbols are vector quantities unless otherwise specified)

$$L = L(\mathbf{Y},\mathbf{T}) \qquad (2.26)$$

where $\mathbf{Y}=[y_1 \ldots y_n]^T$ is a vector of quality characteristics, $\mathbf{T}=[t_1 \ldots t_n]^T$ is a vector of the target values. Expanding eq. (2.26) as a Taylor's series at $\mathbf{Y}=\mathbf{T}$, we have

$$L= L(\mathbf{T},\mathbf{T})+\nabla L(\mathbf{T},\mathbf{T})(\mathbf{Y}-\mathbf{T})+(\mathbf{Y}-\mathbf{T})^T\mathbf{H}_L(\mathbf{T})(\mathbf{Y}-\mathbf{T})/2 + \ldots \qquad (2.27)$$

where $\mathbf{H}_L(\mathbf{T})$ is the "Hessian matrix" for L, which is the second derivatives of L with respect to $\mathbf{Y}$ at $\mathbf{T}$. Since L has a minimal value at $\mathbf{Y}=\mathbf{T}$, the gradient $\nabla L(\mathbf{T},\mathbf{T})=\mathbf{0}$. Moreover, we are interested in the major variable terms in (2.27). Thus, ignoring the constant terms and the 3rd or higher order terms, we have

$$L \approx (\mathbf{Y}-\mathbf{T})^T \mathbf{H}_L(\mathbf{T},\mathbf{T})(\mathbf{Y}-\mathbf{T})/2 \qquad (2.28)$$

In summation notations, we have

$$L = \sum_{i=1}^{n} \sum_{j=1}^{i} K_{ij}(y_i - t_i)(y_j - t_j) \qquad (2.29)$$

where $K_{ii} = \frac{1}{2}\left(\frac{\partial^2 L}{\partial y_i^2}\right)|(\mathbf{Y}=\mathbf{T}, \forall\, i)$ and $K_{ij} = \left(\frac{\partial^2 L}{\partial y_i \partial y_j}\right)|(\mathbf{Y}=\mathbf{T}, \forall\, i \neq j)$.

K_{ij}'s can be determined by using the regression method (Draper 1981, Johnson and Wichern 1982) or solving a system of linear equations. If m_i is the mean of y_i and δ_i=t_i-m_i, the expected value of L is given by

$$E[L] = \sum_{i=1}^{n} K_{ii}(\sigma_i^2 + \delta_i^2) + \sum_{i=2}^{n} \sum_{j=1}^{i-1} K_{ij}[Cov(y_i, y_j) + \delta_i \delta_j] \qquad (2.30)$$

where σ_i^2 is the variance of y_i , $Cov(y_i, y_j)$ is the covariance of y_i and y_j. If y_i and y_j are independent of each other for all i≠j, then

$$E[L] = \sum_{i=1}^{n} K_{ii}(\sigma_i^2 + \delta_i^2) + \sum_{i=2}^{n} \sum_{j=1}^{i-1} K_{ij}\delta_i \delta_j \qquad (2.31)$$

Furthermore, if the mean is equal to the target for all i, then

$$E[L] = \sum_{i=1}^{n} K_{ii}\sigma_i^2 \qquad (2.32)$$

(2) "Smaller the Better" MLF

A good approximation to the MLF for "the smaller the better" quality characteristics is the following quadratic function

$$L(y_1, y_2, \dots y_n) = \sum_{i=1}^{n} \sum_{j=1}^{i} K_{ij} y_i y_j \tag{2.33}$$

The expected quality loss is given as follows

$$E[L] = \sum_{i=1}^{n} K_{ii}(\sigma_i^2 + m_i^2) + \sum_{i=2}^{n} \sum_{j=1}^{i-1} K_{ij}[Cov(y_i, y_j) + m_i m_j] \tag{2.34}$$

If y_i and y_j are independent for all $i \neq j$, then

$$E[L] = \sum_{i=1}^{n} K_{ii}(\sigma_i^2 + m_i^2) + \sum_{i=2}^{n} \sum_{j=1}^{i-1} K_{ij} m_i m_j \tag{2.35}$$

(3) "Larger the Better" MLF

For a generic multivariate loss function of "larger the better" quality characteristics, we can describe it by

$$L(y_1, \dots y_n) = \sum_{i=1}^{n} \sum_{j=1}^{i} \frac{K_{ij}}{y_i y_j} \tag{2.36}$$

where K_{ij} is constant values ($\forall$ i and j). If $K_{ij}=0$ for all $i \neq j$, the function can be separated into the sum of the quality loss for each quality characteristic. The expected value of the total quality loss is the sum of the expected quality loss for each characteristic. We expand eq.

(2.36) as a Taylor's series at $y_i = m_i$ for all i (m_i is the mean of y_i). Taking its expectation with the assumption that the 3rd or higher order terms can be ignored, we have

$$E[L] = \sum_{i=1}^{n}\sum_{j=1}^{i}\frac{K_{ij}}{m_i m_j} + \sum_{i=1}^{n}\left(\frac{3K_{ii}}{m_i^4} + \sum_{\substack{j=1\\ j\neq i}}^{n}\frac{K_{ij}}{m_i^3 m_j}\right)\sigma_i^2 + \sum_{i=2}^{n}\sum_{j=1}^{i-1}\frac{K_{ij}}{m_i^2 m_j^2}Cov(y_i, y_j) \qquad (2.37)$$

If y_i and y_j are independent for all $i \neq j$, we have

$$E[L] = \sum_{i=1}^{n}\sum_{j=1}^{i}\frac{K_{ij}}{m_i m_j} + \sum_{i=1}^{n}\left(\frac{3K_{ii}}{m_i^4} + \sum_{\substack{j=1\\ j\neq i}}^{n}\frac{K_{ij}}{m_i^3 m_j}\right)\sigma_i^2 \qquad (2.38)$$

(4) Example for Multivariate Loss Function

A high-frequency sine-wave signal generator is an electronic instrument that has two characteristics: voltage V and frequency F. If V, F or both are outside the interval of ±10% of the nominal value (the target value), the instrument must be adjusted or sent for repair. There is a set of statistical data about the repair cost related to the bias of V and F (Table 2.1).

Table 2.1. Cost corresponding to bias of V and F

V bias (%)	0	0	-20	30	-60	40	20	20	-10	-10
F bias (%)	-15	10	0	0	-10	20	-20	-5	-5	5
Cost	45	40	20	20	100	80	70	30	10	12

We can fit a multivariate loss function. The target for F or V is zero. If a quadratic loss function is appropriate, then it is given by

$$L(V,F) = K_{11}V^2 + K_{22}F^2 + K_{21}VF$$

Using the least square method, we can obtain the following normal equations (Draper 1981, Johnson 1982, Gunst and Mason 1980)

$$168.3\ K_{11} + 11.7\ K_{22} + 32.4\ K_{21} = 5.607$$

$$117\ K_{11} + 39.25\ K_{22} + 21.75\ K_{21} = 8.5425$$

$$324\ K_{11} + 21.75\ K_{22} + 117\ K_{21} = 9.29$$

Solving the equations, we have K_{11}=0.028, K_{22}=0.15, K_{21}=-0.025.

To test for the goodness of fit, we can compute the coefficient of determination r^2 or the multiple correlation coefficient r.

Total sum of squares about mean = 13246.8

Regression sum of squares = 12077.49

Residual (error) sum of squares = 1169.315

Thus, r^2=0.912, r=0.955. The model can be considered as "fitted well". If V has a mean m_V=0 as well as a variance σ_V^2=36, F has a mean m_F=0 as well as a variance σ_F^2=9.0, and V & F have a correlation coefficient $\rho_{V,F}$=0.1, the expected quality loss is given by

$$E[L] = 0.028(\sigma_v^2+m_v^2)+0.15(\sigma_F^2+m_F^2)-0.025[cov(V,F)+m_v m_F]$$

$$= 2.31$$

CHAPTER 3

OPTIMIZATION CRITERIA FOR DYNAMIC SYSTEM

§3.1. Dynamic System And Dynamic Characteristic

By "dynamic", here we do not mean the presence of time factor. Instead, it is meant that the ideal performance of a system is not a fixed value and it can be adjusted by a signal factor based on the customer's intention. This kind of system is called dynamic in nature. The performance or the characteristic of a dynamic system is called a dynamic characteristic.

Philosophically, the ideal relationship between the signal factor and the dynamic characteristic is desired by either the customer or the designer of the system. In the real world, however, a random deviation may be present due to manufacturing and environmental variations, use conditions, wear, aging and deterioration over time, *etc.* If y_0 is the dynamic target and M is the signal factor, then y_0 is a function of M for the given values of the design factors, that is

$$y_0 = y_0(M) \qquad (3.1a)$$

In practice, the dynamic system is usually designed as a linear model of the signal factor, because it is easier to be controlled by a user or an operator. Hence, the ideal performance is given by the following linear function

$$y_0(M) = \alpha_0 + \beta_0 M \tag{3.1}$$

where α_0 is the ideal intercept and β_0 is the ideal slope. They reflect a desired relationship between the signal factor and the performance variable.

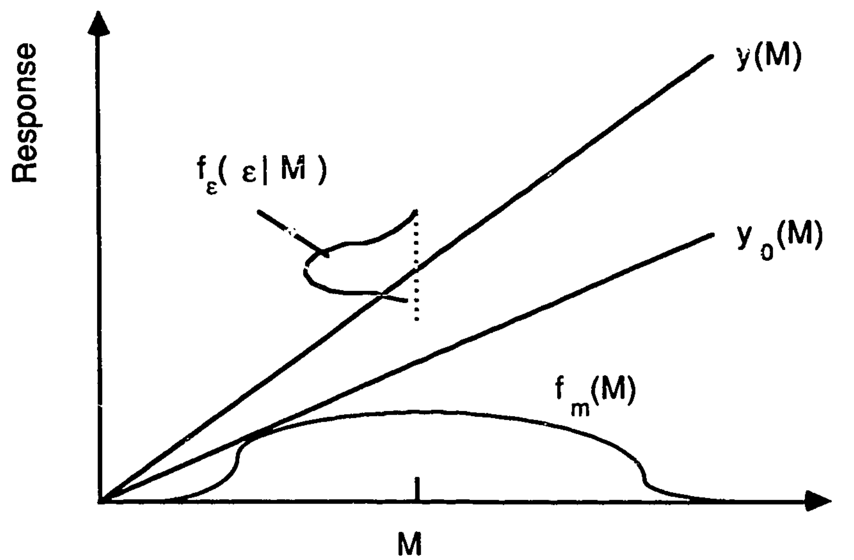
The intercept and the slope for a real system may differ from those of the ideal system due to improper adjustments of α and β. Hence, the model of the real system may be given by

$$y(M) = \alpha + \beta M \tag{3.2a}$$

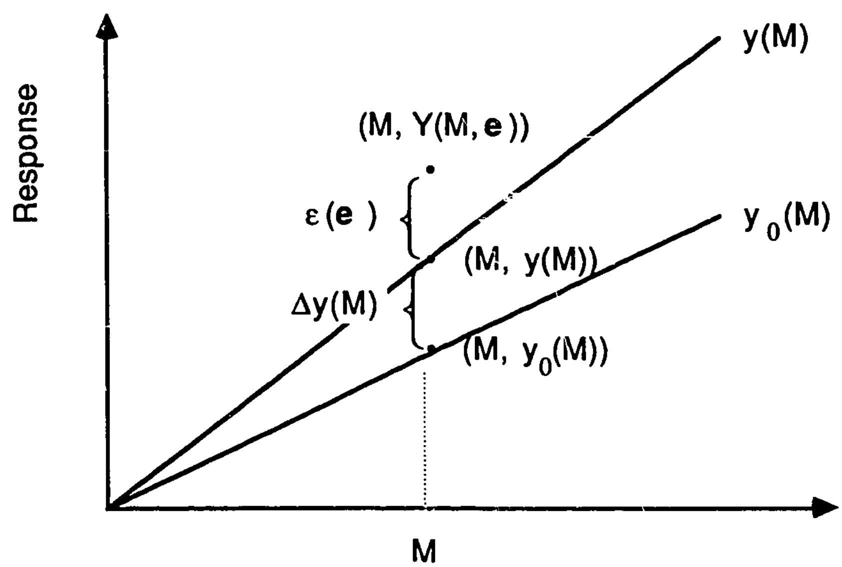
where α and β may differ from α_0 and β_0. Both α and β are controlled by the design factors. The nonlinear effects can be combined into the error term. Due to noise factors **e**, the actual performance can be modeled by

$$Y(M,e) = \alpha + \beta M + \varepsilon(e) \tag{3.2}$$

where $\varepsilon(e)$ is the effect of the noise factor, with a mean zero and a variance σ_ε^2. To understand the relationships between $y_0(M)$, $y(M)$ and $Y(M,e)$, see Figure 3.1.



(a)



(b)

Figure 3.1. The relationships between $y_0(M)$, $y(M)$ and $Y(M,e)$ where $\Delta y(M)=\alpha-\alpha_0+(\beta-\beta_0)M$.

If a quadratic quality loss function is appropriate, the quality loss due to variations from the target is given by

$$L(M,e) = K[Y(M,e)-y_0(M)]^2 \tag{3.3}$$

The expected quality loss is given by the following integration:

$$E[L(M,e)] = \int_{\text{all } M} \int_{\text{all } \varepsilon} K[Y(M,e) - y_0(M)]^2 f_{m,\varepsilon}(M,\varepsilon)\,dM\,d\varepsilon$$

$$= \int_{\text{all } M} \int_{\text{all } \varepsilon} K[\alpha - \alpha_0 + (\beta - \beta_0)M + \varepsilon(e)]^2 f_{m,\varepsilon}(M,\varepsilon)\,dM\,d\varepsilon$$

$$= K\left\{[\alpha - \alpha_0 + (\beta - \beta_0)\mu_M]^2 + (\beta - \beta_0)^2\sigma_M^2\right\} + K\sigma_\varepsilon^2 \tag{3.4}$$

$f_{m,\varepsilon}(M,\varepsilon)$ is the joint density function of M and $\varepsilon(e)$. μ_M is the mean of M and $\sigma_M{}^2$ is the variance. $\sigma_\varepsilon{}^2$ is the variance of the random effect $\varepsilon(e)$. Here for simplicity, $\varepsilon(e)$ is assumed to be independent of M and hence $f_\varepsilon[\varepsilon(e)|M]=f_\varepsilon[\varepsilon(e)]$.

The expected quality loss consists of two parts: the first part is the result of the deviation of the mean performance from the target or the improper adjustments of α and β; the second part is caused by the noise factors. Our goal in design is to minimize this quantity by selecting the best levels setting for control factors **Z** and adjusting scaling/leveling factors **R**.

§3.2. Motivation Of Optimization Criteria

The optimization can be done by selecting control factors **Z** and adjusting scaling/leveling factors **R** simultaneously to minimize E[L(M,e)], but it is an intuitive approach to optimization and the method is not simplified. The selection of **Z** and the adjustment of **R** can be separated as a two-step optimization procedure. If we select **Z** to minimize E[L(M,e)] directly in the first step, the solutions may be obtained by reducing the first part in eq. (3.4) that is supposed to be eliminated by adjusting **R** in the second step. Since **R** may have effects on $\varepsilon(e)$, σ_ε^2 cannot be used as a criterion for the selection of **Z**. When we adjust **R** in the second step, σ_ε^2 may be inflated to a solution that is not optimal.

To eliminate the conflict of this, we can select **Z** to minimize the adjusted quality loss (AQL) in the first step and then adjust **R** to set α and β to the ideal values in the second step. AQL is the quality loss caused by the actual responses with the assumption that the real system has been adjusted to the ideal model.

Imagine that the real system has been adjusted to be identical with the ideal model. Thus, the adjusted actual response can be modeled by $Y^a(M,e)=(\beta_0/\beta)[Y(M,e)-\alpha]+\alpha_0$ and the AQL is given by

$$E[L(M,e)] = \int_{\text{all } M} \int_{\text{all } \varepsilon} K\left[Y^a(M,e) - y_0(M)\right]^2 f_{m,\varepsilon}(M,\varepsilon)dMd\varepsilon$$

$$= \int_{\text{all } M} \int_{\text{all } \varepsilon} K\left[\frac{\beta_0}{\beta}\varepsilon(e)\right]^2 f_{m,\varepsilon}(M,\varepsilon)dMd\varepsilon$$

$$= K'\frac{\sigma_\varepsilon^2}{\beta^2} \tag{3.5}$$

where $K'=K\beta_0$ is a constant value. We want to select **Z** to minimize AQL or to maximize $\beta^2/\sigma_\varepsilon^2$. This is a motivation of the signal-to-noise ratio from the engineering viewpoint.

For a real system, α, β and σ_ε^2 may be unknown. They can be estimated by performing experiments. If experiments are performed on m levels of M and repeated n times for each level of M, the data is given in Table 3.1.

Table 3.1. The experimental data for the dynamic system.

signal Level	Experimental data			
M_1	Y_{11}	Y_{12}	......	Y_{1n}
M_2	Y_{21}	Y_{22}	......	Y_{2n}
.	.	.		.
.	.	.		.
.	.	.		.
M_m	Y_{m1}	Y_{m2}	......	Y_{mn}

The linear model of Y_{ij} is given by

$$Y_{ij} = \alpha + \beta M_i + \varepsilon_{ij} \tag{3.6}$$

where ε_{ij} is the effect of the noise factors that is an i. i. d. random variable with a mean zero and a variance σ_ε^2. Using the least square method, we can find the maximum likelihood estimators of α and β, which are given by

$$\left.\begin{aligned} \hat{\alpha} &= \overline{Y}_{\bullet\bullet} - \hat{\beta}\overline{M} \\ \hat{\beta} &= \frac{\left[\sum_{i=1}^{m}\sum_{j=1}^{n} Y_{ij}(M_i - \overline{M})\right]}{n\sum_{i=1}^{m}(M_i - \overline{M})^2} \end{aligned}\right\}$$

$\overline{Y}_{\bullet\bullet}$ is the average of the experimental data. $\overline{M}$ is the average of the signal levels. A regression function is given by

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta}M_i \tag{3.7}$$

Since the probability density function of M is unknown, the average quality loss can be used for this set of experiments, that is,

$$QL = K\frac{1}{n\,m}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[Y_{ij} - y_0(M_i)\right]^2 = K\frac{1}{n\,m}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\alpha - \alpha_0 + (\beta - \beta_0)M_i + \varepsilon_{ij}\right]^2$$

$$= \frac{K}{m}\sum_{i=1}^{m}\left[\alpha - \alpha_0 + (\beta - \beta_0)M_i\right]^2 + \frac{K}{n\,m}\sum_{i=1}^{m}\sum_{j=1}^{n}\varepsilon_{ij}^2 + 2K\frac{\sum_{i=1}^{m}\left[\alpha - \alpha_0 + (\beta - \beta_0)M_i\right]\sum_{j=1}^{n}\varepsilon_{ij}}{n\,m} \tag{3.8}$$

The expected value of QL is given by

$$E[QL] = \frac{K}{m}\sum_{i=1}^{m}\left[\alpha - \alpha_0 + (\beta - \beta_0)M_i\right]^2 + K\sigma_\varepsilon^2 \tag{3.9}$$

Similar to eq. (3.4), the first part of E[QL] is a result of the improper adjustments of α and β, and the second part is caused by the noise factors. We would like to reduce this quantity. The objective in using factor **R** is to adjust α and β to α_0 and β_0. The discussion is similar. If the minimization of E[QL] is used directly as an objective for selecting **Z** in the first step, we may get an optimal solution that is given by reducing the first part which is supposed to be eliminated by adjusting **R** in the second step. If the minimization of the second part is used as an objective, it may be inflated as α or β is adjusted to α_0 or β_0 in the second step, because adjusting α and β may have effects on the error terms.

We want to minimize the adjusted quality loss by eliminating the improper effects of α and β in the first step, and later we can adjust α and β to α_0 and β_0 in the second step. Imagine that the expected value of the real system (3.6) has been adjusted to be coincident with the ideal function (3.1). The adjusted response is modeled by

$$Y^a_{ij} = \frac{\beta_0}{\beta}(Y_{ij} - \alpha) + \alpha_0 \tag{3.10}$$

The adjusted quality loss (AQL) is caused by the noise factors with the assumption that the real system has been adjusted to the ideal model, and AQL is given by

$$\begin{aligned}
AQL &= K\frac{1}{n\,m}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[Y^a_{ij} - y_0(M_i)\right]^2 \\
&= K\frac{1}{n\,m}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\beta_0}{\beta}(Y_{ij} - \alpha) + \alpha_0 - y_0(M_i)\right]^2 \\
&= \frac{K\beta_0^2}{nm\beta^2}\sum_{i=1}^{m}\sum_{j=1}^{n}\varepsilon_{ij}^2 = K'\frac{\hat{\sigma}_\varepsilon^2}{\beta^2}
\end{aligned} \tag{3.11}$$

where $K'=K(\beta_0)^2$ is a constant value.

We can select the best levels for control factors in the first step to minimize AQL or to maximize $\beta^2/Var(\varepsilon)$, which is also a motivation of the signal-to-noise ratio from the engineering viewpoint. In the following section, we will give a definition of the signal-to-noise ratio.

§3.3. Signal-To-Noise Ratio For Dynamic Characteristic

In communication theory, generally speaking, a signal-to-noise (SN) ratio is a ratio of the predictable change of the output caused by a signal to the unpredictable change due to noises (Morris 1984). To prevent SN ratio from becoming negative, the square value can be used. The SN ratio (η) for dynamic characteristics is a ratio of the square value of the predictable change of performance caused by a unit change of signal to the expected square value of the unpredictable change caused by noises. In other words, the SN ratio is the ratio of the square value of the predictable change of performance caused by a unit change of the signal factor to the variance of the errors. This is because the mean value of the errors is assumed to be zero and hence the expected square value of the unpredictable change due to the noise is the variance of the error denoted by σ_ε^2.

Thus, the SN ratio for dynamic systems can be defined as the ratio of the square value of the predictable change of performance caused by a unit change of the signal factor to the variance of the error, or

$$\eta = \frac{\beta^2}{\sigma_\varepsilon^2} \tag{3.12}$$

The motivation of this is based on the discussion in last section.

The two-step procedure of optimization is the selection of **Z** to maximize η as the first step and then the adjustment of α and β by using **R** as the second step. This two-step procedure can simplify the optimization.

§3.4. Validity And Limitations Of Signal-To-Noise Ratio

To observe the validity of the SN ratio for dynamic systems, we can consider the following situations:

*(1) **R** has same effects on the signal factor as on the error term*

In this situation, if we adjust **R** to achieve a desired functional relationship between the signal factor and the predictable response, the effects of **R** on the error term will change at the same rate as those of **R** on the predictable response change. The model can be given as follows

$$Y = \alpha(\mathbf{R},\mathbf{Z})+\beta(\mathbf{R},\mathbf{Z})M+\beta(\mathbf{R},\mathbf{Z})\varepsilon'(\mathbf{Z},\mathbf{e}) \tag{3.13}$$

where ε' is a random effect due to **e** and independent of **R**. The gauge system with a sensor and a transforming mechanism is considered to be such a model by Leon *et al* (1987). The sensor transfers signal M to a mechanical signal $M'=\gamma(\mathbf{Z})M+\varepsilon''(\mathbf{Z},e)$, and the mechanism transfers M' to meter reading $Y=\alpha(\mathbf{R},\mathbf{Z})+\beta_1(\mathbf{R},\mathbf{Z})M'$ where $\beta(\mathbf{R},\mathbf{Z})=\beta_1(\mathbf{R},\mathbf{Z})\gamma(\mathbf{Z})$, and $\varepsilon'(\mathbf{Z},e)=\varepsilon''(\mathbf{Z},e)/\gamma(\mathbf{Z})$. For model (3.13), the SN ratio (η) is $\beta^2/Var(\varepsilon)= 1/Var[\varepsilon'(\mathbf{Z},e)]$ which is independent of **R**. Hence, adjusting **R** does not affect η. As a result, the two-step optimization procedure gives the optimal solutions **Z*** and **R*** as follows ($V(\bullet)$ is a variance and equivalent to $Var(\bullet)$ unless otherwise specified):

In step 1, find **Z***:

$$\mathbf{Z}^*= \left\{\mathbf{Z}^*: \eta(\mathbf{Z}^*) = \max_{\mathbf{Z}}[\eta(\mathbf{Z})] \text{ or } V[\varepsilon'(\mathbf{Z}^*,e)] = \min_{\mathbf{Z}}\{V[\varepsilon'(\mathbf{Z},e)]\}\right\}$$

In step 2, find R^*: $R^*=\{R^*: \alpha(R^*,Z^*)=\alpha_0,\ \beta(R^*,Z^*)=\beta_0\}$.

Z* and **R*** can minimize the expected quality loss based on a symmetric quadratic loss function, because

$$V[\varepsilon(\mathbf{R}^*,\mathbf{Z}^*,e)] = \beta_0^2 V[\varepsilon'(\mathbf{Z}^*,e)] = \beta_0^2 \min_{\mathbf{Z}}\{V[\varepsilon'(\mathbf{Z},e)]\}$$

$$= \operatorname*{Min}_{\mathbf{R},\mathbf{Z}}\left\{V[\varepsilon(\mathbf{R},\mathbf{Z},e)]: \alpha(\mathbf{R},\mathbf{Z}) = \alpha_0, \beta(\mathbf{R},\mathbf{Z}) = \beta_0\right\} \qquad (3.14)$$

If we minimize $V[\varepsilon(\mathbf{R},\mathbf{Z},e)]$ directly in the first step and adjust $\beta(\mathbf{R},\mathbf{Z})$ to β_0 in the second step, solutions **R'** and **Z'** can be given as follows:

In step 1, find Z':

$$\mathbf{Z'} = \left\{ \mathbf{Z'} : V[\varepsilon(\mathbf{R},\mathbf{Z'},\mathbf{e})] = \min_{\mathbf{Z}}\{V[\varepsilon(\mathbf{R},\mathbf{Z},\mathbf{e})]\} \right\}$$

In step 2, find R': $\mathbf{R'}=\{\mathbf{R'}: \alpha(\mathbf{R'},\mathbf{Z'})=\alpha_0,\ \beta(\mathbf{R'},\mathbf{Z'})=\beta_0\}$.

Z' and **R'** may not minimize the expected quality loss or the variance of the error, because

$$V[\varepsilon(\mathbf{R'},\mathbf{Z'},\mathbf{e})] = \beta_0^2 V[\varepsilon'(\mathbf{Z'},\mathbf{e})] \geq \beta_0^2 V[\varepsilon'(\mathbf{Z}^*,\mathbf{e})] = V[\varepsilon(\mathbf{R}^*,\mathbf{Z}^*,\mathbf{e})] \quad (3.15)$$

(2) Effects of ***R*** *on the signal factor and the error term are different*

Under this situation, the model is given as follows:

$$\begin{aligned} Y &= \alpha(\mathbf{R},\mathbf{Z})+\beta(\mathbf{R},\mathbf{Z})M+\varepsilon(\mathbf{R},\mathbf{Z},\mathbf{e}) \\ &= \alpha(\mathbf{R},\mathbf{Z})+\beta(\mathbf{R},\mathbf{Z})M+\beta_1(\mathbf{R},\mathbf{Z})\varepsilon'(\mathbf{Z},\mathbf{e}) \end{aligned} \quad (3.16)$$

where β_1 is a function of **R** and **Z**.

This is a generic model for dynamic characteristics, because the impact of **R** on ε and the effect of **e** can usually be decomposed into a product form. Let $f_\beta(\mathbf{R},\mathbf{Z})=\beta(\mathbf{R},\mathbf{Z})/\beta_1(\mathbf{R},\mathbf{Z})$ which is the ratio of the effect of **R** on the signal factor to the effect of **R** on the error term. We can expand it as a Taylor's expansion at $\mathbf{R}_0$ or the initial value of **R** as follows:

$$f_\beta(\mathbf{R},\mathbf{Z}) = f_\beta(\mathbf{R}_0,\mathbf{Z}) + \nabla f_\beta(\mathbf{R}_0,\mathbf{Z})(\mathbf{R}-\mathbf{R}_0) + (\mathbf{R}-\mathbf{R}_0)^T H_{f_\beta}[\xi](\mathbf{R}-\mathbf{R}_0) \quad (3.17)$$

where ξ is a vector quantity between $\mathbf{R}_0$ and $\mathbf{R}$ (bold letters are vector quantities or matrices unless otherwise specified). In practice, the effect of $\mathbf{R}$ on β is approximately proportional to the effect of $\mathbf{R}$ on the random error term ε. In other words, $f_\beta(\mathbf{R}_0,\mathbf{Z})$ is much larger than the other variable terms of the above expansion. Hence,

$$\beta_1(\mathbf{R},\mathbf{Z}) \approx \beta(\mathbf{R},\mathbf{Z})/f_\beta(\mathbf{R}_0,\mathbf{Z}) \tag{3.18}$$

Thus, the SN ratio for this model is given as follows:

$$\eta = \frac{\beta^2(\mathbf{R},\mathbf{Z})}{V[\varepsilon(\mathbf{R},\mathbf{Z},\mathbf{e})]} \approx \frac{\left[f_\beta(\mathbf{R}_0,\mathbf{Z})\right]^2}{V[\varepsilon'(\mathbf{Z},\mathbf{e})]} \tag{3.19}$$

which is independent of $\mathbf{R}$. Hence, adjusting $\mathbf{R}$ does not affect η. As a result, the two-step procedure gives the following optimal solutions $\mathbf{Z}^*$ and $\mathbf{R}^*$:

In step 1, find Z:*

$$\mathbf{Z}^* = \left\{ \mathbf{Z}^* : \eta(\mathbf{Z}^*) = \max_{\mathbf{Z}}[\eta(\mathbf{Z})] \text{ or } \frac{V[\varepsilon'(\mathbf{Z}^*,\mathbf{e})]}{f_\beta^2(\mathbf{R}_0,\mathbf{Z}^*)} = \min_{\mathbf{Z}} \left\{ \frac{V[\varepsilon'(\mathbf{Z},\mathbf{e})]}{f_\beta^2(\mathbf{R}_0,\mathbf{Z})} \right\} \right\}$$

In step 2, find R^*: $R^* = \{R^* : \alpha(R^*,Z^*) = \alpha_0,\ \beta(R^*,Z^*) = \beta_0\}$.

$\mathbf{Z}^*$ and $\mathbf{R}^*$ can minimize the expected quality loss based on a symmetric quadratic loss function, because

$$V[\varepsilon(\mathbf{R}^*,\mathbf{Z}^*,e)] = V\left[\beta_1(\mathbf{R}^*,\mathbf{Z}^*)\varepsilon'(\mathbf{Z}^*,e)\right]$$

$$= V\left[\frac{\beta(\mathbf{R}^*,\mathbf{Z}^*)}{f_\beta(\mathbf{R}_0,\mathbf{Z}^*)}\varepsilon'(\mathbf{Z}^*,e)\right]$$

$$= \beta_0^2 V\left[\frac{\varepsilon'(\mathbf{Z}^*,e)}{f_\beta(\mathbf{R}_0,\mathbf{Z}^*)}\right]$$

$$= \min_{\mathbf{R},\mathbf{Z}}\left\{V[\varepsilon(\mathbf{R},\mathbf{Z},e)]: \alpha(\mathbf{R},\mathbf{Z}) = \alpha_0,\ \beta(\mathbf{R},\mathbf{Z}) = \beta_0\right\} \qquad (3.20)$$

This leads to the minimization of the expected quadratic quality loss.

It should be noted that we can not use Var[$\varepsilon'(\mathbf{Z},e)$] as a criterion to find the optimal solutions of **Z**, although it could be a *Performance Measure Independent of Adjustment factors.* Otherwise, we would obtain the following solutions:

In step 1, find Z':

$$\mathbf{Z}' = \left\{\mathbf{Z}': V[\varepsilon'(\mathbf{Z}',e)] = \min_{\mathbf{Z}}\{V[\varepsilon'(\mathbf{Z},e)]\}\right\}$$

In step 2, find R': R'={R': $\alpha(R',Z')=\alpha_0$, $\beta(R',Z')=\beta_0$}.

Z' and **R'** may not be optimal, because

$$V\left[\beta_1(\mathbf{R}',\mathbf{Z}')\varepsilon'(\mathbf{Z}',e)\right] = \beta_1^2(\mathbf{R}',\mathbf{Z}')\min_{\mathbf{Z}}\{V[\varepsilon'(\mathbf{Z},e)]\}$$

$$\approx \beta_0^2 \frac{\min_{\mathbf{Z}}\{V[\varepsilon'(\mathbf{Z},e)]\}}{f_\beta^2(\mathbf{R}_0,\mathbf{Z}')}$$

$$\geq \beta_0^2 \min_{\mathbf{Z}} \left\{ \frac{V[\varepsilon'(\mathbf{Z},e)]}{f_\beta^2(\mathbf{R}_0,\mathbf{Z})} \right\}$$

$$= \min_{\mathbf{R},\mathbf{Z}} \left\{ V[\varepsilon(\mathbf{R},\mathbf{Z},e)]: \alpha(\mathbf{R},\mathbf{Z}) = \alpha_0, \beta(\mathbf{R},\mathbf{Z}) = \beta_0 \right\} \quad (3.21)$$

It should be observed that if β_1 can not be represented by eq. (3.18), then the optimization problem can not be decomposed as a two-step procedure and it must be solved in a single step, because it is difficult to find a SN ratio which is independent of **R** or an efficient PerMIA. The optimization problem is more complicated. If each scaling/leveling factor can only take one of the several levels, we can assign **R** to the inner array. If **R** can be adjusted continuously, for each level combination of the control factors, we can find a **R*** so that α and β are equal to the desired values. Thus, we can find **Z*** or the best levels setting of the control factors to minimize $V(\varepsilon)$. If the mean performance is not satisfactory, we can finely adjust **R** to reduce the deviations of α and β from the desired values under the best level setting or **Z***. The fine adjustment of **R** will not change the optimal solutions. Thus, the optimization procedure is given as follows: for each levels combination of **Z**, find a **R*** to be such that $\alpha(\mathbf{R}^*,\mathbf{Z})=\alpha_0$ and $\beta(\mathbf{R}^*,\mathbf{Z})=\beta_0$; minimize $V[\varepsilon(\mathbf{R}^*,\mathbf{Z},e)]$ or maximize $\eta(\mathbf{R}^*,\mathbf{Z})$ by selecting the best levels setting for **Z**. That will lead to the minimization of

$V[\varepsilon(\mathbf{R},\mathbf{Z},e)]$ or the expected quality loss or

$$\underset{\mathbf{Z}}{\mathrm{Max}}\{\eta(\mathbf{R}^*,\mathbf{Z}): \alpha(\mathbf{R}^*,\mathbf{Z}) = \alpha_0, \beta(\mathbf{R}^*,\mathbf{Z}) = \beta_0\} = \frac{\beta_0^2}{\underset{\mathbf{Z}}{\mathrm{Min}}\{V[\varepsilon(\mathbf{R}^*,\mathbf{Z},e)]\}} \quad (3.22)$$

(3) R has no effect on the random error term

In this situation, **R** does not appear in the error term of the model, that is,

$$Y = \alpha(\mathbf{R},\mathbf{Z}) + \beta(\mathbf{R},\mathbf{Z})M + \varepsilon(\mathbf{Z},e) \quad (3.23)$$

For this specific model, we can define the SN ratio as $(\beta_0)^2/V[\varepsilon(\mathbf{Z},e)]$ or $1/V[\varepsilon(\mathbf{Z},e)]$. Since β_0 is a constant value and **R** has no effect on $V[\varepsilon(\mathbf{Z},e)]$, maximizing η can minimize the expected quadratic loss and bringing α, β to the desired values does not affect $V[\varepsilon(\mathbf{Z},e)]$.

(4) Scaling/leveling factors (R) do not exist.

If **R** is not available to adjust the predictable performance to the desired value, each level combination of the control factors must give the same α and β which are equal to the desired values α_0 and β_0. Otherwise, this level combination is an infeasible solution for the generic optimization model given in Chapter 4, because the constraint is not satisfied. Eq. (3.2) can be rewritten down as follows

$$Y = \alpha(Z)+\beta(Z)M+\varepsilon(Z,e) \tag{3.24}$$

The optimization can be completed in a single step:

$$\underset{Z}{Max}\{\eta\} \quad \text{or} \quad \underset{Z}{Min}\{Var[\varepsilon(Z,e)]\}$$

with $\alpha(Z)$ and $\beta(Z)$ subject to the desired values. Both criteria lead to the same optimal solution which minimizes the expected quality loss.

§3.5. Signal Factor Has Impacts On Random-Effect Term

For the models discussed early in this chapter, ε is assumed to be independent of M. Under certain circumstances, however, the signal factor may have impacts on the random-effect term. Thus, ε is not independent of M. To ensure the validity of the discussions in last several sections, we observe that the expected quality loss is given by

$$E[L(M,e)] = \int_{all\ \varepsilon}\int_{all\ M} K\left[Y(M,e) - y_0(M)\right]^2 f_{m,\varepsilon}(M,\varepsilon)\,dM\,d\varepsilon$$

$$= \int_{all\ \varepsilon}\int_{all\ M} K\left[\alpha - \alpha_0 + (\beta - \beta_0)M + \varepsilon(M,e)\right]^2 f_{m,\varepsilon}(M,\varepsilon)\,dM\,d\varepsilon$$

$$= K\Big\{\left[\alpha - \alpha_0 + (\beta - \beta_0)\mu_M\right]^2 + (\beta - \beta_0)^2\sigma_M^2 + \sigma_\varepsilon^2 +$$

$$+ \int_{all\ \varepsilon}\int_{all\ M} K(\beta - \beta_0)M\varepsilon(M,e)\,f_{m,\varepsilon}(M,\varepsilon)\,dM\,d\varepsilon\Big\} \tag{3.25}$$

For simplicity, the mean of $\varepsilon(M,e)$ can be assumed to be zero for any value of M, namely $\mu_\varepsilon(M)\equiv 0$ ($\forall M$). As a result, the expected quality loss can be simplified as follows:

$$E[L(M,e)] = K\left\{\left[\alpha - \alpha_0 + (\beta - \beta_0)\mu_M\right]^2 + (\beta - \beta_0)^2\sigma_M^2\right\} + K\sigma_\varepsilon^2 \qquad (3.26)$$

where $\sigma_\varepsilon^2 = \int_{\text{all } \varepsilon} \int_{\text{all } M} \varepsilon^2(M,e) f_{m,\varepsilon}(M,\varepsilon) dM d\varepsilon$

The expected quality loss can be still decomposed into two parts: the first part is the result of the improper adjustment of α and β; the second part is the effect of the noise factors. The discussions in last three sections are applicable here. Since M is not a design factor, its presence in the random-effect term doesn't affect the selection of the levels of the control factors and the SN ratio can be defined in the same way.

So far, we have discussed the optimization criteria for dynamic systems. In next chapter, we will be talking about the optimization model, the use and the computation of the SN ratio.

CHAPTER 4

OPTIMIZATION OF DYNAMIC SYSTEM

§ 4.1. Optimization Model For Dynamic System

Our goal is to investigate the deviation of quality characteristics from the target or the ideal value. Minimizing this deviation can minimize quality loss. We present the generic optimization model for dynamic systems and the approach for development of the signal-to-noise (SN) ratio as a measure for quality improvement. An example is given to illustrate the methods. The SN ratio was originally developed in communication theory. To evaluate the noise content of an amplifier, the SN ratio is defined as the ratio of the information signal at the output to the undesired noise at the output (Morris 1984). The calibration of gauge systems is viewed as a dynamic characteristic and will be discussed later.

Depending upon the nature of the target, quality characteristic can be classified as static or dynamic. For the static characteristic, the target value is fixed. It does not change during the performance period. For the dynamic characteristic, the target value is changing and the performance can be controlled by a signal factor. If either or both of the signal factor and the response variable is discrete, the system is classified as discrete in nature. We will be talking about discrete dynamic systems in the next chapter.

In Figure 4.1, the user sets the signal factor at certain value to achieve the intended performance. Due to the effects of noise factors, the actual performance of the system may deviate from the target value. This results in a quality loss. The model for the behavior of the system may be known or unknown. If there is no sufficient knowledge about the model, we can perform experiments to collect data.

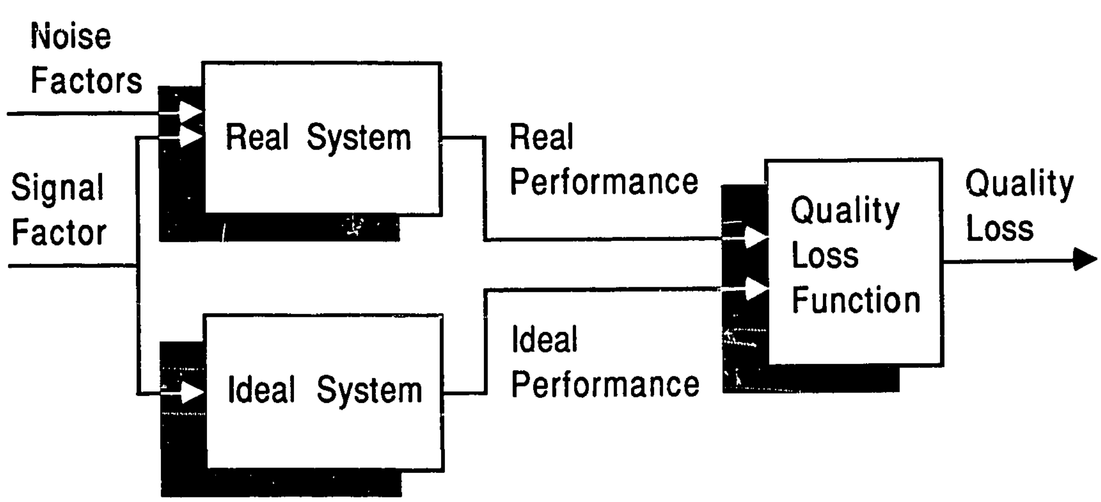


Figure 4.1. A flow chart for the dynamic characteristic

To minimize the expected quality loss given by a quadratic loss function, we must minimize the variance of the error. Thus, a generic optimization model for dynamic systems is given by

$$\underset{\mathbf{R},\mathbf{Z}}{\text{Minimize}}\{\,\text{Var}[\varepsilon(\mathbf{R},\mathbf{Z},e)]\,\}$$

$$\text{Subject to}\begin{cases}\alpha(\mathbf{Z},\mathbf{R})=\alpha_0\\ \beta(\mathbf{Z},\mathbf{R})=\beta_0\end{cases} \tag{4.1}$$

To simplify the optimization, a two-step procedure can be used with the assumption that design factors can be partitioned into control factors **Z** and scaling/leveling factors **R**. In the first step, set $\mathbf{R}=\mathbf{R}_0$, an initial value of **R**, and maximize the SN ratio with **Z** as controllable variables. In the second step, adjust the expected performance to the ideal value, or in other words, adjust α and β to the desired values. However, Leon *et al* (1987) show that the above two-step procedure may not lead to the optimal solution for some specific models. They propose the *performance measure independent of adjustment* (PerMIA) as a substitute for Taguchi's SN ratio. The disadvantage of PerMIA is that one must derive the PerMIA based on the knowledge of the model. Moreover, as the discussion has revealed in case (2) of

Section 3.4, a performance measure defined only by "independent of adjustment" is not sufficient to be an optimization criterion, and it cannot assure the minimization of quality loss if there is not any restriction or further explanation. The slight modification of the SN ratio for the specific models can ensure scientific use of the SN ratio (Chapter 3).

The objective for the first step of the two-step procedure is to maximize η in the domain of the control factors. Hence, the optimization for the selection of **Z** is given by

$$\eta^* = \max_{\mathbf{z}} \{\eta\} \tag{4.2}$$

The SN ratio may become inefficient if it is not independent of **R** (Leon *et al* 1987).

The inner array and the outer array for the optimization of dynamic systems using the SN ratio are given in Figure 4.2. The outer array is used to compute a SN ratio for each level setting of the control factors. The signal factor and the noise factors are assigned to the outer array. The control factors are assigned to the inner array. For each level combination of the control factors or each row of the inner array, we can compute a SN ratio. By performing ANOVA on the SN ratio, we can find the significant effects of the control factors.

Furthermore, we can find the best level setting of the control factors to maximize the SN ratio. To sum up, we give a flow chart to illustrate the generic optimization procedure for dynamic systems in Figure 4.3.

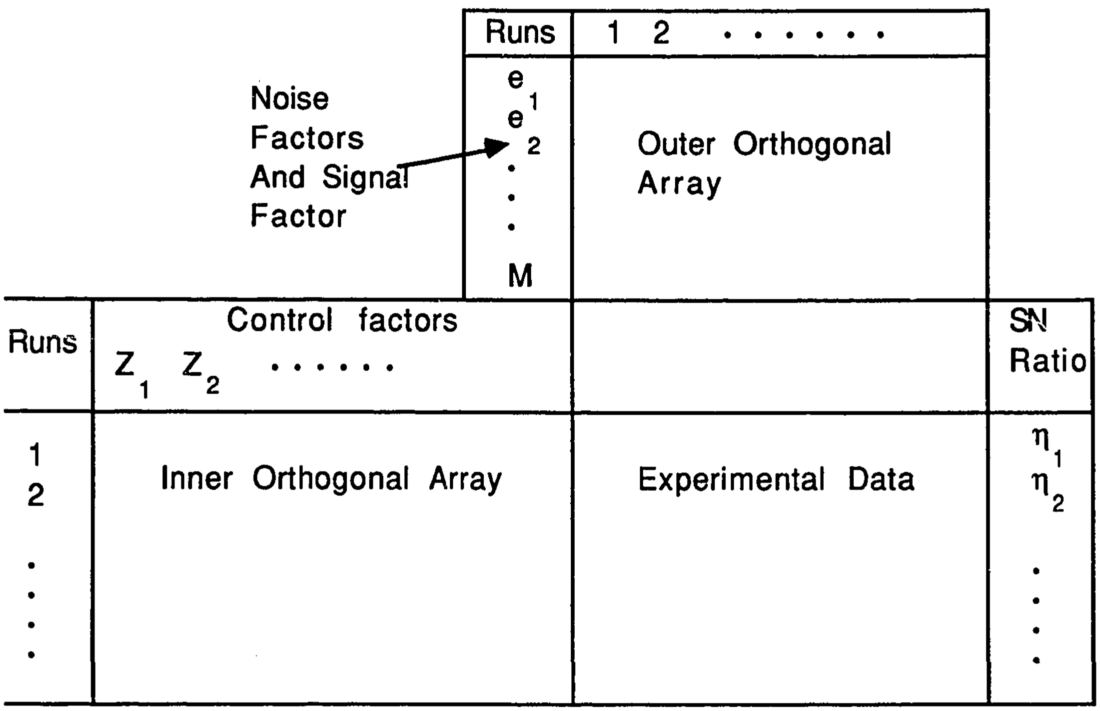


Figure 4.2. Inner & outer orthogonal arrays for dynamic characteristic.

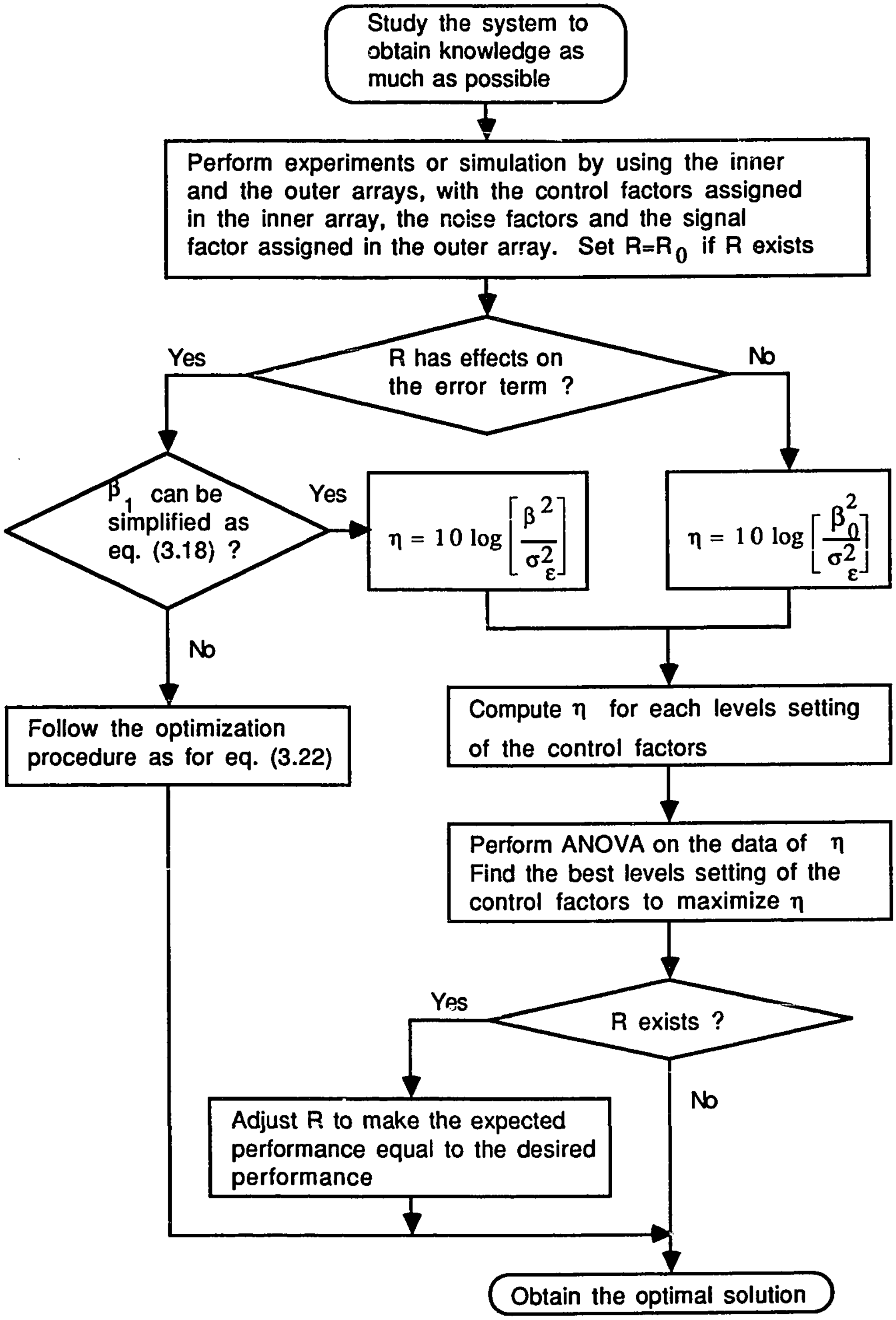


Figure 4.3. A flow chart for the optimization of dynamic systems

§ 4.2. Computation Of SN Ratio For Dynamic System

The calibration of gauge systems can be viewed as a dynamic characteristic problem. The reference samples to be tested are the signals. The reading on the meter or the tester is the response characteristic. This example is used to present the development of the SN ratio for dynamic systems.

The calibration with an equal replication number for each signal level is referred to as the balanced calibration. Otherwise, the calibration is called unbalanced. The balanced calibration is only a special case of the unbalanced calibration. We will discuss the unbalanced calibration. Assume that the response is a linear function of the signal factor and for the ith signal level M_i (i=1,2 ... k) and the jth replication (j=1,2 ... r_i), the response is Y_{ij} and the random error is ε_{ij} that is an i. i. d. random variable with a mean zero and a variance σ_ε^2. The linear function (3.2) can be rewritten as follows:

$$Y_{ij} = m + \beta(M_i-\overline{M}) + \varepsilon_{ij} \qquad (4.3)$$

where $\overline{M}=(r_1M_1+r_2M_2+ \cdots +r_kM_k)/(r_1+r_2+ \cdots +r_k)$, $m=\alpha+\beta\overline{M}$. The maximum likelihood estimators of parameters m and β are given as follows

$$\left.\begin{aligned}\hat{m} &= \left[\sum_{i=1}^{k}\sum_{j=1}^{r_i} Y_{ij}\right] \Big/ \left[\sum_{i=1}^{k} r_i\right] \\ \hat{\beta} &= \frac{1}{r}\left[\sum_{i=1}^{k}\sum_{j=1}^{r_i} Y_{ij}(M_i - \overline{M})\right]\end{aligned}\right\} \qquad (4.4)$$

where $r = \sum_{i=1}^{k} r_i(M_i - \overline{M})^2$

Since the predictable change of the response Y due to a unit change of the signal factor is β, the SN ratio for model (4.3) is given by (in decibel value)

$$\eta = 10 \log(\beta^2/\sigma_\varepsilon^2) \qquad (4.5)$$

We can show that $\hat{\beta}^2$ is not an unbiased estimator of β^2. Instead, $E[\hat{\beta}^2]=\beta^2+\sigma_\varepsilon^2/r$ (see also Appendix C). In order to find the SN ratio by performing ANOVA, we compute the total corrected sum of squares

$$SS_T = \sum_{i=1}^{k}\sum_{j=1}^{r_i} Y_{ij}^2 - \left(\sum_{i=1}^{k}\sum_{j=1}^{r_i} Y_{ij}\right)^2 \Big/ \left[\sum_{i=1}^{k} r_i\right] \qquad \left(\text{d.f.}= \sum_{i=1}^{k} r_i - 1\right) \qquad (4.6)$$

To compute the sum of squares for the 1st order variation, we would like to remove the unknown constant m from the expression. Hence, we use $u_i=M_i-\overline{M}$ ($i=1, 2, \cdots k$) as the coefficients of a linear contrast. Multiplying eq. (4.3) by u_i and taking the sum for all i and j, we have

$$\sum_{i=1}^{k}\sum_{j=1}^{r_i} u_i Y_{ij} = \beta \sum_{i=1}^{k} r_i u_i^2 + \sum_{i=1}^{k}\sum_{j=1}^{r_i} u_i \varepsilon_{ij} \tag{4.7}$$

This is a linear contrast. The sum of squares for this linear contrast is given by (see Appendix D)

$$SS_\beta = \frac{\left[\sum_{i=1}^{k}\sum_{j=1}^{r_i} u_i Y_{ij}\right]^2}{\left[\sum_{i=1}^{k} r_i u_i^2\right]} = \frac{1}{r}\left[\sum_{i=1}^{k} (M_i - \overline{M}) Y_{i\bullet}\right]^2 \qquad (\text{d.f.} = 1) \tag{4.8}$$

where $Y_{i\bullet}$ is the sum of all readings Y_{ij} for the given signal level i. Obviously, $SS_\beta = r\hat{\beta}^2$. The sum of squares for the error is given by

$$SS_e = SS_T - SS_\beta \qquad \left(\text{d.f.} = \sum_{i=1}^{k} r_i - 2\right) \tag{4.9}$$

Since $E[\hat{\beta}^2] = \beta^2 + \sigma_\varepsilon^2/r$, then $E[MS_\beta] = E[r\hat{\beta}^2] = r\beta^2 + \sigma_\varepsilon^2$. Also, we can show that $E[MS_e] = \sigma_\varepsilon^2$ (Appendix C) where MS_e is the mean sum of squares for the error. Thus, $\beta^2 \approx (MS_\beta - MS_e)/r$ and the SN ratio can be estimated by

$$\eta = 10 \log\left\{\frac{MS_\beta - MS_e}{r\, MS_e}\right\} = 10 \log\left\{\frac{r\,\hat{\beta}^2 - MS_e}{r\, MS_e}\right\} \tag{4.10}$$

Eq. (4.6) through eq. (4.10) can be used to compute the SN ratio for linear dynamic systems.

If the intercept of a linear dynamic system is zero, then the model is called proportional equation calibration and given by

$$Y_{ij} = \beta M_i + \varepsilon_{ij} \qquad (i=1, \ldots k;\ j=1, \ldots r_i) \qquad (4.11)$$

Again we can use eq. (4.6) through eq. (4.10) to compute the SN ratio, with $\overline{M}=0$.

The confidence interval for the expected value of the real performance or the response of the true model can be derived with the assumption that the error term is normally distributed with a mean zero and a variance σ_ε^2. For the ith level of the signal factor and the jth replication (i=1,2, ... a; j=1,2, ... n), the actual response is given by

$$Y_{ij} = \alpha + \beta M_i + \varepsilon_{ij} \qquad (4.12)$$

For the ith signal level M_i, $Y_i=\alpha+\beta M_i$ is the mean response value of the system. The estimated value of the response is given by $\overline{Y}_{i\bullet}=(Y_{i1}+Y_{i2}+ \cdots + Y_{in})/n$. $\overline{Y}_{i\bullet}-Y_i=(\varepsilon_{i1}+\varepsilon_{i2}+ \cdots + \varepsilon_{in})/n$ is normally distributed with a mean zero, a variance σ_ε^2/n. Hence, $n(\overline{Y}_{i\bullet}-Y_i)^2/\sigma_\varepsilon^2$ has a Chi-square distribution with one degree of freedom. Moreover, $f_e MS_e/\sigma_\varepsilon^2$ has a Chi-square distribution with f_e degrees of freedom, where f_e is the degree of freedom for the errors. Consequently, $n(\overline{Y}_{i\bullet}-Y_i)^2/MS_e$ is distributed by a $F(1,f_e)$ distribution. Thus, the confidence interval for $Y_i=\alpha+\beta M_i$ is given by

$$\overline{Y}_{i\bullet}-\sqrt{F_{\alpha,1,f_e}\frac{MS_e}{n}}\leq\ Y_i\ \leq\overline{Y}_{i\bullet}+\sqrt{F_{\alpha,1,f_e}\frac{MS_e}{n}} \qquad (4.13)$$

§ 4.3. Example

A noninverting amplifier consists of an operational amplifier and several resistors (Figure 4.4, Savant *et al* 1987). The output voltage V_o is a linear function of the input voltage V_i, which is given as follows:

$$V_o=\left(1+\frac{R}{R_a}\right)\left(\frac{R_2}{R_1+R_2}\right)V_i$$

In this linear model, $\alpha=0$ and $\beta=(1+R/R_a)R_2/(R_1+R_2)$. R is an adjustable resistor. If the ideal function is $V_o=100V_i$, then $\alpha_0=0$ & $\beta_0=100$. Each resistor has two levels (Table 4.1). As suggested by Taguchi (Taguchi 1986, Taguchi & Wu 1980), lower-priced components should be used in selecting the best levels setting for control factors. Assume that the standard deviation for the lower-priced components except R is 1.67% of the nominal value. As a result, three levels can be used to simulate the noise factors associated with the resistors: zero and ±2.04% away from the nominal value (Table 4.1), because $\sqrt{(3/2)}\sigma$ is equal to 2.04% of the nominal value. V_i is considered as the signal factor whose domain is defined as a range of 0.00 to 0.10 volt.

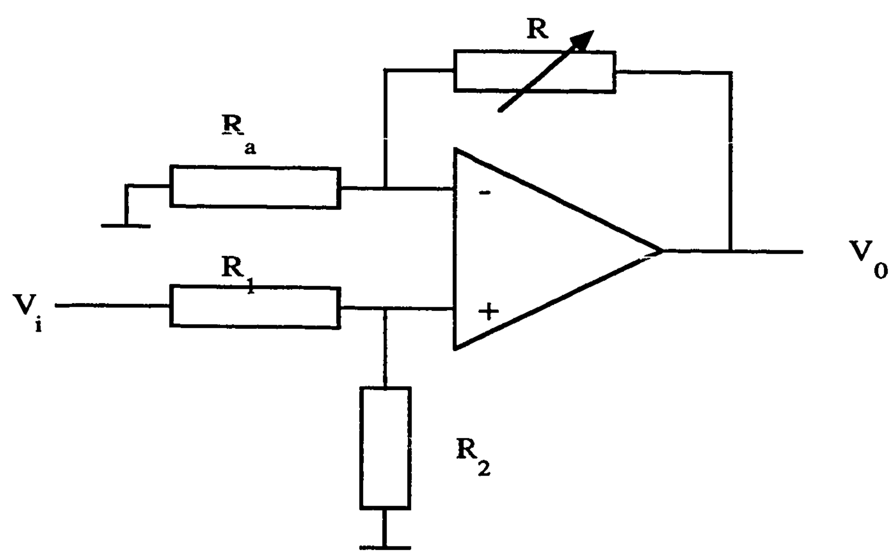


Figure 4.4. A noninverting amplifier.

We use this example to demonstrate the two-step procedure for optimization and the use of SN ratio. We want to select the best level setting for R_1, R_2, R_a and adjust R to minimize the expected quality loss. In the first step, we maximize the SN ratio with R_1, R_2 and R_a as the variables. In the second step, we adjust R to achieve $\beta=100$. We can show that the SN ratio for this model is independent of the adjustable factor R. Although signal factor V_i has effects on the error term, it doesn't affect the selection of the levels of R_1, R_2 and R_a (Appendix E).

In the first step, we use $L_8(2^7)$ as the inner array and $L_9(3^4)$ as the outer array. The noise factors for resistors can be found in Table 4.1. If we set V_i at 3 levels: 0.01, 0.04, 0.07 and start with an initial value R_0=150 KΩ, the simulation data is given in Figure 4.5. In Table 4.2, the SN ratio for each level combination is computed by using eq. (4.6) through eq. (4.10). The ANOVA of the SN ratio is given in Table 4.3.

Signal factor and noise factors

Run	1	2	3	4	5	6	7	8	9
V_i 1	1	1	1	2	2	2	3	3	3
R'_1 2	1	2	3	1	2	3	1	2	3
R'_2 3	1	2	3	2	3	1	3	1	2
R'_a 4	1	2	3	3	1	2	2	3	1

Run \ Col	R_1	R_2	$R_1 x R_2$	R_a	$R_1 x R_a$	$R_2 x R_a$		V_0									η
	1	2	3	4	5	6	7										
1	1	1	1	1	1	1	1	1.47	1.44	1.41	5.64	5.88	5.74	10.1	9.86	10.3	61.09
2	1	1	1	2	2	2	2	0.74	0.72	0.71	2.84	2.96	2.89	5.08	4.96	5.17	61.14
3	1	2	2	1	1	2	2	1.50	1.47	1.44	5.78	6.02	5.89	10.3	10.1	10.5	61.10
4	1	2	2	2	2	1	1	0.76	0.74	0.73	2.91	3.03	2.96	5.20	5.09	5.29	61.17
5	2	1	2	1	2	1	2	1.40	1.37	1.35	5.39	5.62	5.47	9.64	9.40	9.79	61.00
6	2	1	2	2	1	2	1	0.71	0.69	0.68	2.71	2.83	2.75	4.85	4.73	4.93	61.05
7	2	2	1	1	2	2	1	1.47	1.44	1.41	5.64	5.88	5.74	10.1	9.86	10.3	61.09
8	2	2	1	2	1	1	2	0.74	0.72	0.71	2.84	2.96	2.89	5.08	4.96	5.17	61.14

Figure 4.5. Simulation data using the inner array and the outer array

Table 4.1. The nominal values and the noise factors associated with R_1, R_2, R_a

Factor	Level	Nominal Value (kΩ)	Noise Factors Level 1	Level 2	Level 3
R_1	1	5	4.898	5.000	5.102
	2	10	9.796	10.00	10.204
R_2	1	100	97.96	100.00	102.04
	2	200	195.92	200.00	204.08
R_a	1	1	0.9796	1.000	1.0204
	2	2	1.9592	2.000	2.0408

Table 4.2. The computation of the SN ratio

Run	SS_T	SS_β	SS_e	MS_e	η (db)
1	111.845	111.733	0.112	0.016	61.09
2	28.329	28.301	0.028	4.03×10^{-3}	61.14
3	117.363	117.245	0.118	0.017	61.10
4	29.729	29.699	0.029	4.20×10^{-3}	61.17
5	101.910	101.805	0.105	0.015	61.00
6	25.819	25.792	0.026	3.75×10^{-3}	61.05
7	111.845	111.733	0.112	0.016	61.09
8	28.329	28.301	0.028	4.03×10^{-3}	61.14

Since the significant factors are the effects of R_1, R_2, R_a and the interaction of R_1 and R_2, the model is given by

$$Y_{ijk} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + \varepsilon_{ijk}$$

where μ is the overall mean and it is estimated by $\overline{Y}_{\bullet\bullet\bullet}$;

τ_i is the effect of R_1 and it is estimated by $\overline{Y}_{i\bullet\bullet} - \overline{Y}_{\bullet\bullet\bullet}$;

β_j is the effect of R_2 and it is estimated by $\overline{Y}_{\bullet j\bullet} - \overline{Y}_{\bullet\bullet\bullet}$;

γ_k is the effect of R_a and it is estimated by $\overline{Y}_{\bullet\bullet k} - \overline{Y}_{\bullet\bullet\bullet}$;

$(\tau\beta)_{ij}$ is the effect of the interaction of R_1 and R_2 and it is estimated by $\overline{Y}_{ij\bullet} - \overline{Y}_{i\bullet\bullet} - \overline{Y}_{\bullet j\bullet} + \overline{Y}_{\bullet\bullet\bullet}$ (i=1,2; j=1,2; k=1,2).

As a result, the estimated value of Y_{ijk} is given by

$$Y_{ijk} = \overline{Y}_{ij\bullet} + \overline{Y}_{\bullet\bullet k} - \overline{Y}_{\bullet\bullet\bullet}$$

Hence, the estimates of the SN ratios for various level combinations of R_1, R_2, and R_a are given in Table 4.4. The best levels setting for control factors is: R_1=5, R_2=200 and R_a=2, because these parameters can maximize the SN ratio. Adjusting R to 203 kΩ, we can achieve β=100 or V_0=100V_i.

Table 4.3. ANOVA on the SN ratio using the inner array

Source	SS	df	MS	F_0
R_1	6.38×10^{-3}	1	6.38×10^{-3}	157.5 ***
R_2	6.38×10^{-3}	1	6.38×10^{-3}	157.5 ***
R_a	5.51×10^{-3}	1	5.51×10^{-3}	136.0 ***
$R_1 x R_2$	2.11×10^{-3}	1	2.11×10^{-3}	52.1 ***
$R_1 x R_a$	4.05×10^{-5}	1	4.05×10^{-5}	
$R_2 x R_a$	4.05×10^{-5}	1	4.05×10^{-5}	
Errors	4.05×10^{-5}	1	4.05×10^{-5}	
(Errors)	(1.22×10^{-4})	(3)	(4.05×10^{-5})	
Total	0.0205	7	2.93×10^{-3}	

*** Significant at 1%.

Table 4.4. SN ratios for various level combinations of R_1, R_2, R_a

	R_2=100		R_2=200	
	R_a=1	R_a=2	R_a=1	R_a=2
R_1=5	61.09	61.14	61.11	61.16*
R_1=10	61.00	61.05	61.09	61.14

CHAPTER 5

OPTIMIZATION MEASURES FOR DISCRETE DYNAMIC SYSTEM

§5.1. Discrete Dynamic Characteristic

The optimization of the continuous dynamic characteristic has been discussed for quality improvement in last chapter. Depending on the nature of the signal factor and the performance, dynamic characteristic problems can be further classified as four types (Taguchi and Phadke 1984, Kapur and Chen 1988). If either or both of the signal factor and the performance is discrete, we classify the quality characteristic as discrete in nature. The control system is a typical discrete dynamic system.

In general, a discrete dynamic characteristic problem can be transferred to a continuous problem. The approach developed in last two chapters can be applied to the discrete dynamic characteristic. This is the topic in this chapter. We will be developing the SN ratio for several discrete dynamic systems. The optimization procedure is similar to the continuous cases and is performed on levels settings of the design factors.

§5.2. SN Ratio For Sensory And Reliability Test

As a coin is inserted in a slot of an automatic vending machine, a product should emerge as a response. If not, the machine fails to perform the intended function. For this system, the response is a measure of performance, and feeding the coin is a signal. Let Y be the response and

$$Y = \begin{cases} 0 & \text{if a product does not emerge} \\ 1 & \text{if a product emerges} \end{cases}$$

In order to test the reliability of the vending machine, we insert the coin n times, and the point estimator for the reliability p is given by

$$R = (Y_1+Y_2+ \ldots\ldots +Y_n)/n \qquad (5.1)$$

The sum of squares for this linear equation is given by

$$SS_R = R^2/\{n(1/n)^2\} = nR^2 \qquad (\text{d.f.}=1) \qquad (5.2)$$

If p is the population mean, then E[nR]=np, Var[nR]=np(1-p), because $nR=Y_1+Y_2+ \ldots +Y_n$ has a binomial distribution. As a result, we have

$$E[(nR)^2] = Var[nR]+\{E[nR]\}^2 = np(1-p)+(np)^2 \qquad (5.3)$$

as well as

$$E[SS_R] = (1/n)E[(nR)^2] = p(1-p) + np^2 \qquad (5.4)$$

The total sum of squares is given by

$$SS_T = \sum_{i=1}^{n} Y_i^2 = \sum_{i=1}^{n} Y_i \qquad (5.5)$$

and $SS_e = SS_T - SS_R = nR(1-R)$. Thus, we have

$$E[SS_e] = E(SS_T) - E(SS_R) = (n-1)p(1-p) \qquad (5.6)$$

The degree of freedom for SS_e is n-1. The ANOVA table for this problem can be developed as Table 5.1.

Obviously, the change of the output due to a unit change of the signal is the population mean p and the variance of the error is p(1-p). Thus, the SN ratio for this problem is given by

$$\eta = \frac{p^2}{p(1-p)} \qquad (5.7)$$

Table 5.1. ANOVA for sensory and reliability test

Source	df	SS	MS	EMS
R	1	nR^2	nR^2	$p(1-p)+np^2$
error	n-1	$nR-nR^2$	$nR(1-R)/(n-1)$	$p(1-p)$
Total	n	nR		

It is observed in Table 5.1 that η can be estimated by

$$\frac{\frac{1}{n}(MS_R - MS_e)}{MS_e} = \frac{R}{1-R}\left[1 - \frac{1}{nR}\right] \approx \frac{R}{1-R} \qquad (5.8a)$$

or in decibel, we have

$$\eta = -10 \log\left(\frac{1}{R} - 1\right) \qquad (5.8)$$

The logarithmic expression (5.8) is called the *decibel unit for R* (Taguchi 1987). This is also called omega transformation.

For the sensory and reliability test of the machine, we can insert the coin n times, and record Y_1, Y_2, ... Y_n to calculate R. The objective is to maximize the SN ratio given by eq. (5.8) over the design space of the machine.

Maximizing η leads to the maximization of R. Hence, η is consistent with R as a measure of performance which is used in the field of reliability. If a machine has a SN ratio η and countermeasure A can obtain a SN ratio gain of η_A, the SN ratio for the machine after adopting countermeasure A is $\eta+\eta_A$ (Taguchi 1987).

§5.3. On-Off Control System

An on-off control system has a continuous signal and a discrete response. It can be divided into two types depending on the presence or the absence of hysteresis.

(1) Absence Of Hysteresis.

The control system will turn on a switch as a characteristic value is greater than or equal to a critical value Y_c. Otherwise, it will turn off the switch. That is

$$\text{Switch} = \begin{cases} \text{on} \quad (1) & \text{if the characteristic value} \geq Y_c \\ \text{off} \quad (0) & \text{if the characteristic value} < Y_c \end{cases}$$

Y_c is controlled by a certain factor which can be classified as a signal factor. Assume that the critical value has a linear relationship with the signal factor (nonlinear terms can be combined with the errors). Thus, for the ith signal level and the jth replication, we have the function $Y_c = m + \beta(M_i - \overline{M}) + \varepsilon_{ij}$. The analysis is similar to the continuous dynamic systems in last chapter. Setting different levels for the signal factor, we can collect performance data by measuring the characteristic value when the switch is turned on or off. Thus, the SN ratio can be computed by using eq. (4.6) through eq. (4.10).

(2) The Presence Of Hysteresis

To design a stable system, hysteresis is often introduced to prevent the system from oscillating. The hysteresis makes the control switch turn on or off at different critical values. That is,

$$\text{Switch} = \begin{cases} \text{is turned on } (0 \rightarrow 1), \\ \quad \text{if the characteristic increases to } Y_{C_1}; \\ \\ \text{is turned off } (1 \rightarrow 0), \\ \quad \text{if the characteristic decreases to } Y_{C_0}. \end{cases}$$

Obviously, unless Y_{C1} is greater than Y_{C0}, the system would not work properly. The critical values Y_{C1} and Y_{C0} are controlled by the signal factor.

One example of this is an air-conditioner. The air-conditioner can be set at a particular temperature by turning the control switch (signal factor), say 80 °F. Due to the hysteresis, the power is not turned on ($0 \rightarrow 1$) until the temperature increases to a quantity larger than 80 °F (Y_{C1}). When the switch is on, it is not turned off ($1 \rightarrow 0$) until the temperature drops to a value lower than 80 °F (Y_{C0}) (Figure 5.1). As a result, the system has two SN ratios, one for shift $0 \rightarrow 1$ and the other for shift $1 \rightarrow 0$. Which one is taken as the optimization criterion depends on the nature of the practical problem.

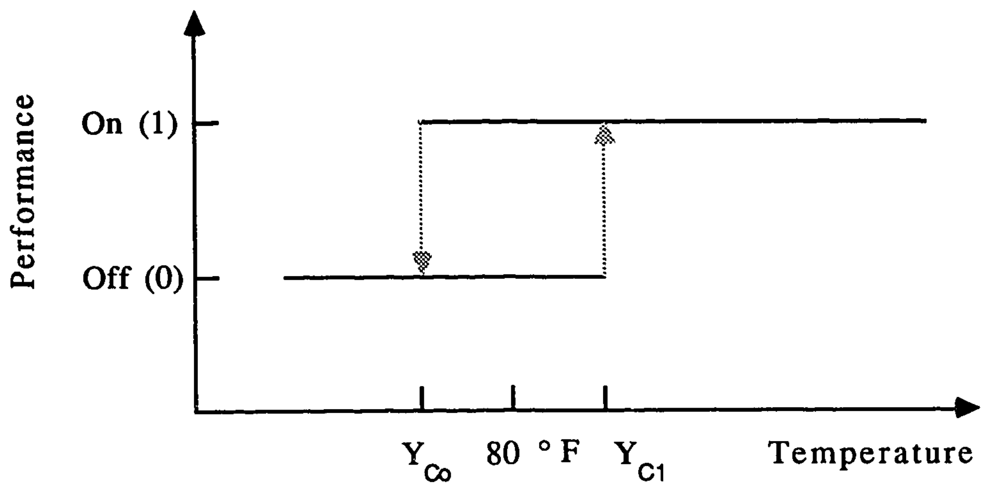


Figure 5.1. Temperature control with hysteresis

§5.4. Two-Type Error System

In hypothesis testing, if the hypothesis is really true but it is rejected, a type-one error has been committed. If the hypothesis is accepted when it is not true, then a type-two error has been made. In digital communication systems, it is possible to mistake input "1" for output "0" and also to mistake input "0" for output "1". This is called a two-type-error system.

For the data given in Table 5.2, n_0 0's are sent as inputs but n_{00} 0's and n_{01} 1's are received as outputs, and n_1 1's are sent as inputs but n_{11} 1's and n_{10} 0's are received as outputs.

Table 5.2. The data of input vs output

	Output		
Input	0	1	Total
0	n_{00}	n_{01}	n_0
1	n_{10}	n_{11}	n_1
Total	r_0	r_1	n

Let y_i (y_i=1 or 0; i=1,2, ... n_1) represent the output when input is "1", and x_j (x_j=0 or 1; j=1,2, ... n_0) represent the output when input is "0". To see how accurately the input signals can be identified in the output data, we consider the following contrast

$$L = \frac{1}{n_1}\sum_{i=1}^{n_1} y_i - \frac{1}{n_0}\sum_{j=1}^{n_0} x_j = \left(\frac{n_{11}}{n_1}\right) - \left(\frac{n_{01}}{n_0}\right) \qquad (5.9)$$

which represents the significance between the response to input "1" and the response to input "0". The larger the contrast L is, the more accurate the system is. The sum of squares for this linear contrast is given by

$$SS_L = \left[\frac{n_{11}}{n_1} - \frac{n_{01}}{n_0}\right]^2 / \left[\sum_{i=1}^{n_1}\left(\frac{1}{n_1}\right)^2 + \sum_{j=1}^{n_0}\left(\frac{1}{n_0}\right)^2\right]$$

$$= \frac{(n_{00}n_{11} - n_{01}n_{10})^2}{n_0 n_1 n} \tag{5.10}$$

For input "0", the sum of squares for the error is $n_{01}[1-(n_{01}/n_0)]$ (binomial distribution) and for input "1", the sum of squares for the error is $n_{11}[1-(n_{11}/n_1)]$. The sum of both terms is given by

$$SS_e = n_{01}[1-(n_{01}/n_0)] + n_{11}[1-(n_{11}/n_1)] \tag{5.11}$$

The total sum of squares is given by

$$SS_T = SS_L + SS_e = r_1 - r_1^2/n \tag{5.12}$$

By comparison with SS_L, $MS_e=SS_e/(n-2)$ is so small that $SS_L-MS_e \approx SS_L$. The proportion of the signals in the total output is termed a percent contribution ratio ρ $(0<\rho<1)$

$$\rho = (SS_L-MS_e)/SS_T \approx (n_{00}n_{11} - n_{01}n_{10})^2/(r_0 r_1 n_0 n_1) \tag{5.13}$$

A large value of ρ is desired. The perfect system has ρ approaching one. Similar to the sensory and reliability testing, the SN ratio for this system can be computed in terms of ρ rather than R:

$$\eta = -10\log(\frac{1}{\rho} - 1) \tag{5.14}$$

Table 5.3. The data of input vs output in terms of p and q.

	Output		
Input	0	1	Total
0	1-p	p	1
1	q	1-q	1
Total	1-p+q	1+p-q	2

If the data of input and output is given as $p=n_{01}/n_0$ as well as $q=n_{10}/n_1$ (Table 5.3), then by substituting p and q, we have

$$\rho = (1-p-q)^2/\{(1-p+q)(1+p-q)\} \qquad (5.15)$$

The SN ratio can be computed using eq.(5.14). If p=q=0, we substitute $1/(2n_0)$ for p and $1/(2n_1)$ for q, to prevent the SN ratio from being infinite.

In practice, it is impossible for a variable to have an exact "0" or "1" value. The variable to represent "0" or "1" is a random variable with a distribution around 0 or 1 (Figure 5.2). If an input value is greater than a threshold value, it is classified as 1. If the input value is smaller than the threshold value, the input is received as 0.

Let p be the probability that the input is 0 but it is greater than the threshold value. Let q be the probability that the input is 1 but it is smaller than the threshold value. In communication systems, it is desirable to have p=q. Taguchi (1987) suggests using an omega transformation to adjust p=q based on the fact that if the decibel unit for p is increased (or decreased) by K db, then the decibel unit for q will decrease (or increase) by K db. If p' and q' are the probabilities after adjustment (Figure 5.3), by increasing the decibel unit of p by K db and reducing the decibel unit of q by K db, we have

$$-10\log\left(\frac{1}{p}-1\right)+K=-10\log\left(\frac{1}{q}-1\right)-K \qquad (5.16)$$

or $K=-10\log\sqrt{\left(\frac{1}{q}-1\right)/\left(\frac{1}{p}-1\right)}$

Thus, the decibel unit for p' and q' after adjusting by K db is given by

$$-10\log\left(\frac{1}{p'}-1\right)=-10\log\left(\frac{1}{q'}-1\right)=-10\log\sqrt{\left(\frac{1}{p}-1\right)\left(\frac{1}{q}-1\right)} \qquad (5.17)$$

Consequently, $\rho'=(1-2p')^2$ and the SN ratio is given by

$$\eta=-10\log\left(\frac{1}{\rho'}-1\right) \qquad (5.18)$$

Again, the system can be optimized using η as a measure of the performance of the system.

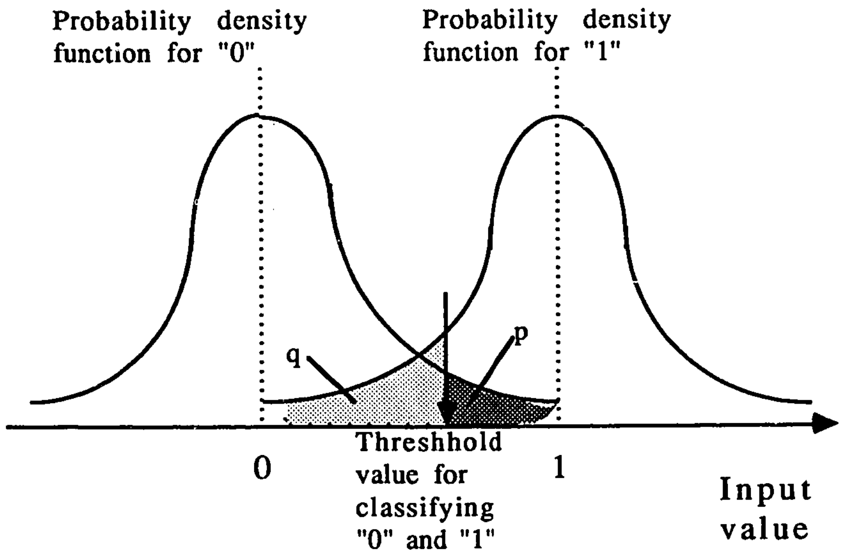


Figure 5.2. Distribution of the input variable for p≠q

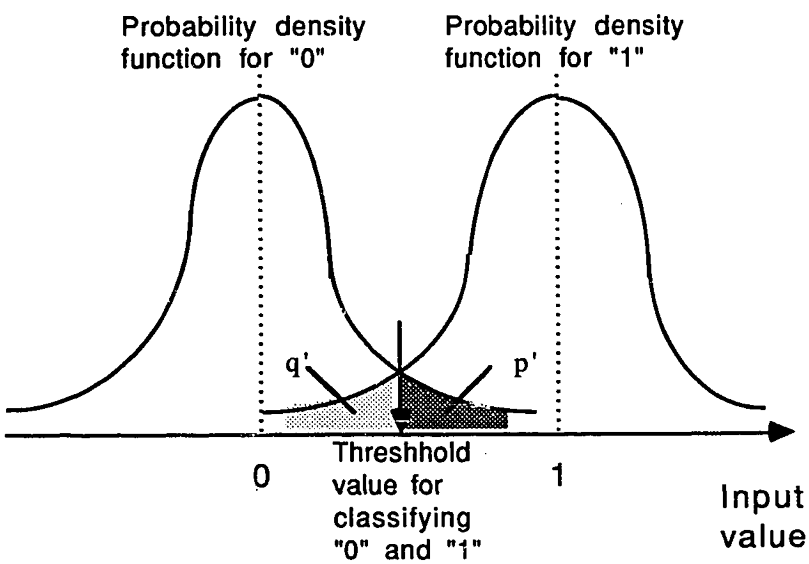


Figure 5.3. Distribution of the input variable for p'=q'

CHAPTER 6

TOLERANCE DESIGN OPTIMIZATION FOR COMPONENTS AND SUBSYSTEMS

§6.1. Introduction

In Taguchi's parameter design, the experiment or the computer simulation is conducted to search for the optimal level setting for design factors so that the system performance is less sensitive to the noise factors without cost increase. However, if the system variations can not meet the quality requirement, the components or subsystems must be upgraded to balance the quality loss due to variations and the cost due to control of tolerances. To do this, Taguchi uses an equation to transfer the variations of the components to the system variations with the assumption of no interaction between the components (Taguchi and Wu 1980; Wu and Moore 1986). This equation can be understood intuitively but no theoretical basis is given and interactions between components are not considered.

In this chapter, we will be presenting the optimization model for tolerance design. Also, we will be discussing the theoretical basis of the variation transmission equation (VTE) which transfers the variations of the components to the system variations, regardless of the significance of the interactions between components. Based on the quality loss function (QLF) and the VTE, we can perform the economic analysis for tolerance design to optimize the total cost which consists of the quality loss due to variations from the target and the cost due to control of the tolerances.

There are many conventional approaches for tolerance design (Spence and Soin 1988), one of which is based on the Taylor's series. Assume that a system consists of n components. A characteristic Y of the system is a function of the parameters of these components and Y is given by

$$Y = f(X_1, X_2, \cdots, X_n) \tag{6.1}$$

where X_1, X_2,$\cdots$ X_n are the parameters of the components. By considering Taylor's series representation of eq. (6.1) truncated after the first order term at the point (μ_1, μ_2, $\cdots$ μ_n), the variance of Y can be found to be

$$\sigma_y^2 = E(Y^2) - [E(Y)]^2 \approx \sum_{i=1}^{n} \sum_{j=1}^{n} \left[\frac{\partial f(\mu_1, \ldots \mu_n)}{\partial x_i}\right]\left[\frac{\partial f(\mu_1, \ldots \mu_n)}{\partial x_j}\right] Cov(X_i, X_j) \tag{6.2}$$

where μ_i is the mean of X_i ($\forall$i), and Cov(X_i, X_j) is the covariance of X_i and X_j. In practice, X_1, X_2, $\cdots$ X_n are independent random variables.

Hence σ_y^2 can be reduced to

$$\sigma_y^2 \approx \sum_{i=1}^{n}\left[\frac{\partial f(\mu_1,\mu_2,\ldots\mu_n)}{\partial x_i}\right]^2 \sigma_i^2 \qquad (6.3)$$

where σ_i^2 is the variance of X_i ($\forall i$). This conventional method has a good mathematical explanation, but the disadvantage is that we need an analytical model and the computation of derivatives. Moreover, it may lead to a less accurate result, because the higher order derivative terms are not considered.

The parameter of a component may have a random variation from the nominal value due to noise factors. This variation results in a deviation of the system characteristic from the target value y_0 and a quality loss is incurred. The traditional quality evaluation systems focus on the conformity to the specifications, while QLF evaluates quality in terms of the deviation from the target value. The quadratic QLF is a good approximation for many systems based on the underlying causes of variation and the attempt to reduce the error of the estimation (Chen and Kapur 1989). Eq. (2.4) can be rewritten as follows:

$$\text{Loss} = K(y-y_0)^2 \qquad (6.4)$$

where K can be determined as follows: if the deviation Δ_0 from y_0 causes a quality loss A_0, then $A_0=K(\Delta_0)^2$ or $K=A_0/(\Delta_0)^2$. The expected quality loss can be obtained by taking the expectation of eq. (6.4).

$$E[Loss] = K(\sigma_y^2 + \delta^2) \tag{6.5}$$

where δ is the difference between y_0 and the mean of Y. The quality design can be conducted by making the performance robust to noise factors or reducing σ_y^2 and then adjusting the mean of Y to y_0. σ_y^2 is affected by the variations of the components or subsystems.

The objective of tolerance design is to decide the tolerances of the components or the subsystems to balance the quality loss due to variations and the cost due to controlling the tolerances of the components. We will limit our approach for the system where only linear and/or quadratic effects of the components are significant. However, the approach can be extended to the cubic or higher order effects.

§6.2. Simulation Of The Noise Factors

It is well known in design of experiments (DOE) that the responses on three levels of a factor can evaluate the linear effect and the quadratic effect of the factor. The linear effect of a component can be evaluated by the difference between the response values for the high level and the low level of the component. The quadratic effect can be evaluated by contrasting the response values among the three levels of the component (Hicks 1982; Montgomery 1984). Thus, to evaluate the linear and the quadratic effects of a component on the system, we must assign three levels to the component to represent the component from the nominal value, or the noise factor associated with this component.

Since the actual performance of experiments may be expensive, it is inefficient to set too many levels to represent the noise factors. Assume that the nominal or the mean parameter of the component is μ and the standard deviation of the noise factor associated with this component is σ. For the noise factor with a symmetrical distribution, we can assign the following three levels to the component:

$$\left.\begin{aligned} &\text{the 1st level} = \mu - h\sigma \\ &\text{the 2nd level} = \mu \\ &\text{the 3rd level} = \mu + h\sigma \end{aligned}\right\} \qquad (6.6)$$

where h is a constant value. To perform experiments or simulations, we assign each component with these three levels to an orthogonal array (either full factorial or partial factorial). These three levels appear at the same number of times in the orthogonal array. Thus, it is reasonably implied that under this discrete distribution, the parameter of the component has an equal probability 1/3 to take each value of the above three levels.

To make the mean and the variance of this discrete distribution equal to those of the true distribution of the noise factor, Taguchi suggests using $h=\sqrt{(3/2)}$ (Kacker 1985; Taguchi 1986; Taguchi and Phadke 1984). D'Errico and Zaino (1988) propose other selections of the noise levels such as $h=\sqrt{3}$ to give a better approximation to the true distribution. Under the assumption that the component has a probability 1/6 to take the value of the 1st level or the 3rd level, a probability 4/6 to take the value of the 2nd level, this discrete distribution can match up to the 5th moment of the true distribution (D'Errico and Zaino 1988). However, it is difficult to assign the three levels to an orthogonal array to conduct experiments or simulations to imply that the probabilities are 1/6, 4/6, 1/6 respectively rather than 1/3, 1/3, 1/3 at the three levels. In other words, if these three levels appear in the orthogonal array the same number of times, it will be implied that the probability is 1/3 at each level and the variance of this discrete distribution is $2\sigma^2$ rather than σ^2, which is not what we want. The example discussed later in this chapter has revealed the fact that $\sqrt{3}$ is not a good value for h. If the three levels appear in the orthogonal array at different number of times, the number of experiments will be significantly increased. Our objective in simulating noise factors is to make the distribution of Y as close as possible to the true one or the moments of Y as close as possible to those of the true distribution.

In Taguchi's parameter design, experiments are first performed on low-priced components with large variations to determine the robust parameter level setting. If the parameter design can not meet the quality requirement in terms of the variance of the response variable, we must upgrade the tolerances or variation levels of the components. We will derive the variation transmission equation (VTE) which makes it possible to estimate the variance of Y if the variances of the components are known. For simplicity, we consider a two-component system. The response Y is given by

$$Y = f(A,B) \tag{6.7}$$

where A is the parameter of component A, and B is the parameter of component B.

Model (6.7) may be known or unknown. For the unknown model, we can perform experiments to collect data. Let μ_A or μ_B be the nominal value of component A or component B respectively, and $\sigma_A{}^2$ or $\sigma_B{}^2$ denote the variance of A or B respectively. In the following two sections, we will be discussing VTE.

§6.3. VTE For Linear-Effect Model

For the linear model, the components have significant linear effects on Y. Since the levels of noise factors associated with A and B are fixed in terms of eq. (6.6), the model is a fixed model (Hicks 1982; Montgomery 1984). Assume the model is given as follows for A at the ith level, B at the jth level and the kth repetition:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \qquad (6.8)$$

where α_i is the effect of A at the ith level and it may be represented by $k_A(A_i-\mu_A)$; β_j is the effect of B at the jth level and it may be represented by $k_B(B_j-\mu_B)$; $(\alpha\beta)_{ij}$ is the interaction between the effect of A at the ith level and the effect of B at the jth level, and it may be represented by $\gamma\alpha_i\beta_j$ (Montgomery 1984); ε_{ijk} is the experimental error assumed to be normally distributed with a mean zero and a variance σ_ε^2; k_A, k_B and γ are constants; A_i and B_j are the values of A at the ith level and B at the jth level respectively.

Let a and b be the numbers of the levels for the noise factors associated with A and B. Let n be the number of repetitions for each combination of the levels. The levels for the noise factors associated with A and B are given in terms of eq. (6.6). Hence a=b=3. The total sum of squares for Y can be partitioned as the sums of the squares for the effects of the components.

$$SS_T = SS_\alpha + SS_\beta + SS_{\alpha x \beta} + SS_e \qquad (6.9)$$

Since the model is a fixed model (Hicks 1982; Montgomery 1984), the expected mean square (EMS) for each sum of squares can be found by substituting A_i and B_j (i, j=1,2,3) into the following equations:

$$EMS_\alpha = \sigma_\varepsilon^2 + \frac{bn\sum_{i=1}^{a}\left[k_A(A_i - \mu_A)\right]^2}{(a-1)} = \sigma_\varepsilon^2 + bnk_A^2h^2\sigma_A^2$$

$$EMS_\beta = \sigma_\varepsilon^2 + \frac{an\sum_{j=1}^{b}\left[k_B(B_j - \mu_B)\right]^2}{(b-1)} = \sigma_\varepsilon^2 + ank_B^2h^2\sigma_B^2$$

$$EMS_{\alpha x \beta} = \sigma_\varepsilon^2 + \frac{n \sum_{i=1}^{a} \sum_{j=1}^{b} \left[\gamma k_A k_B (A_i - \mu_A)(B_j - \mu_B) \right]^2}{(a-1)(b-1)} = \sigma_\varepsilon^2 + n\gamma^2 k_A^2 k_B^2 h^4 \sigma_A^2 \sigma_B^2$$

$$EMS_e = \sigma_\varepsilon{}^2$$

These EMS values can be estimated by MS values in the ANOVA table. Since the three levels of the component are used to simulate the noise factors associated with the component, then the variance of Y can be estimated by MS_T and also $EMS_T = \sigma_y{}^2$. As a result, by taking the expected value of eq. (6.9), we have

$$EMS_T = \frac{(a-1)bnk_A^2 h^2 \sigma_A^2 + (b-1)ank_B^2 h^2 \sigma_B^2 + (a-1)(b-1)n\gamma k_A^2 k_B^2 h^2 \sigma_A^2 \sigma_B^2}{(abn-1)} + \sigma_\varepsilon^2 \qquad (6.10)$$

Let ρ be the percent contribution ratio defined as follows:

$$\rho_A = \frac{(a-1)bnk_A^2 h^2 \sigma_A^2}{(abn-1)EMS_T}$$

$$\rho_B = \frac{(b-1)ank_B^2 h^2 \sigma_B^2}{(abn-1)EMS_T}$$

$$\rho_{AxB} = \frac{(a-1)(b-1)n\gamma k_A^2 k_B^2 h^4 \sigma_A^2 \sigma_B^2}{(abn-1)EMS_T}$$

$$\rho_e = \sigma_\varepsilon{}^2 / \sigma_y{}^2 = 1 - \rho_A - \rho_B - \rho_{AxB}$$

Referred to the expressions for the EMS values, we can observe that ρ_A, ρ_B and ρ_{AxB} can be estimated by the following equations:

$$\hat{\rho}_A = \frac{(a-1)(MS_\alpha - MS_e)}{SS_T}$$

$$\hat{\rho}_B = \frac{(b-1)(MS_\beta - MS_e)}{SS_T}$$

$$\hat{\rho}_{AxB} = \frac{(a-1)(b-1)(MS_{\alpha x\beta} - MS_e)}{SS_T}$$

We can perform experiments and ANOVA on the responses for a set of tolerances of the components and estimate the above contribution ratios. Let the notations with a prime such as EMS_T', σ_A', σ_B' represent the existing or present values. We can estimate EMS_T for any other values of σ_A^2 and σ_B^2 by using the following VTE which is derived from eq. (6.10):

$$EMS_T = EMS'_T\left[\rho_A\left(\frac{\sigma_A^2}{\sigma'^2_A}\right) + \rho_B\left(\frac{\sigma_B^2}{\sigma'^2_B}\right) + \rho_{AxB}\left(\frac{\sigma_A^2\sigma_B^2}{\sigma'^2_A\sigma'^2_B}\right) + \rho_e\right] \quad (6.11)$$

EMS_T can be considered as σ_y^2, because levels given by eq. (6.6) are used to simulate the noise factors. For the system with three or more components, the VTE can be derived in the same way.

§6.4. VTE For Nonlinear-Effect Model

The problem arising in practice usually is not so simple as the linear-effect model mentioned above. Instead, the nonlinear effects of the factors are significant where the nonlinear model must be used. Although we limit our discussion to the linear and quadratic effects, the method can be applied to the model with the cubic or higher order effects, as long as we set appropriate noise levels for components and choose appropriate factorial coefficients.

First, we consider a two-component system (6.7) and then we extend it to a n-component system. Assume the levels for simulation of noises are given by eq. (6.6). Hence a=b=3. For A at the ith noise level, B at the jth noise level and for the kth repetition, the response for the quadratic model is given by

$$Y_{ijk} = m + k_{A_l} A_i + k_{A_q} A_i^2 + k_{B_l} B_j + k_{B_q} B_j^2 + k_{lxl} A_i B_j$$

$$+ k_{lxq} A_i B_j^2 + k_{qxl} A_i^2 B_j + k_{qxq} A_i^2 B_j^2 + \varepsilon_{ijk} \qquad (6.12)$$

where m and $k_{Al}, \cdots k_{qxq}$ are constant values depending on the model. The total sum of squares for Y can be decomposed into SS_A, SS_B and

SS_{AxB} that can be further decomposed as follows:

$$SS_T = SS_{Al} + SS_{Aq} + SS_{Bl} + SS_{Bq} + SS_{lxl} + SS_{lxq} + SS_{qxl} + SS_{qxq} + SS_e \qquad (6.13)$$

SS_{Al} is the sum of squares for the linear effect of A and it can be computed by a contrast $C_{Al} = \mathbf{L}_{Al}\mathbf{Y}$. Here $\mathbf{L}_{Al}$ is the row vector of the coefficients of the contrast for the linear effect of A and it can be found in the first row of Table 6.1. **Y** is a vector of the response totals $Y_{ij\bullet}$ for each level combination of A and B (similar notations will be used in the context and bold letters are vector quantities unless otherwise specified). The sum of squares for this linear contrast is given as follows (Appendix D):

$$SS_{A_l} = \frac{\left(\mathbf{L}_{A_l}\mathbf{Y}\right)^2}{\mathbf{L}_{A_l}\mathbf{L}_{A_l}^T} = \frac{\left[\sum_{j=1}^{3}(Y_{3j\bullet} - Y_{1j\bullet})\right]^2}{6n} \qquad (6.14)$$

Using eq. (6.12) and eq. (6.6) as well as taking the expectation of eq. (6.14), we can find (For mathematical details, see Appendix F)

$$EMS_{A_l} = F_{A_l}\sigma_A^2 + \sigma_\varepsilon^2 \qquad (6.15)$$

where $F_{A_l} = 6nh^2\left[k_{A_l} + 2k_{A_q}\mu_A + k_{lxl}\mu_B + 2k_{qxl}\mu_A\mu_B + (k_{lxq} + 2k_{qxq}\mu_A)\left(\mu_B^2 + \frac{2h^2\sigma_B^2}{3}\right)\right]^2$

Table 6.1. Coefficients for a 3^2 Factorial (Hicks 1982)

Factors	Level Combination *									SS for coefficients $LL^T = n\sum_i\sum_j c_{ij}^2$
	11	12	13	21	22	23	31	32	33	
A_l	-1	-1	-1	0	0	0	+1	+1	+1	6n
A_q	+1	+1	+1	-2	-2	-2	+1	+1	+1	18n
B_l	-1	0	+1	-1	0	+1	-1	0	+1	6n
B_q	+1	-2	+1	+1	-2	+1	+1	-2	+1	18n
$A_l x B_l$	+1	0	-1	0	0	0	-1	0	+1	4n
$A_l x B_q$	-1	+2	-1	0	0	0	+1	-2	+1	12n
$A_q x B_l$	-1	0	+1	+2	0	-2	-1	0	+1	12n
$A_q x B_q$	+1	-2	+1	-2	+4	-2	+1	-2	+1	36n

*(* use the same coefficient for each repetition of the same level combination; Repetition Number=n for each level combination)*

SS_{Bl} is the sum of squares for the linear effect of B and it can be computed by a contrast $C_{Bl}=\mathbf{L}_{Bl}\mathbf{Y}$. Similarly, the expected value of MS_{Bl} can be given by

$$EMS_{B_l} = F_{B_l}\sigma_B^2 + \sigma_\varepsilon^2 \qquad (6.16)$$

where $F_{B_l} = 6nh^2\left[k_{B_l} + 2k_{B_q}\mu_B + k_{lxl}\mu_A + 2k_{lxq}\mu_A\mu_B + \left(k_{qxl} + 2k_{qxq}\mu_B\right)\left(\mu_A^2 + \frac{2h^2\sigma_A^2}{3}\right)\right]$

SS_{Aq} is the sum of squares for the quadratic effect of A and it can be computed by a contrast $C_{Aq}=\mathbf{L}_{Aq}\mathbf{Y}$. Here $\mathbf{L}_{Aq}$ is the row vector of the coefficients of the contrast for the quadratic effect of A and it can be found in the second row of Table 6.1. The sum of squares for this contrast is given by

$$SS_{A_q} = \frac{\left[\sum_{j=1}^{3}(Y_{3j\cdot} + Y_{1j\cdot} - 2Y_{2j\cdot})\right]^2}{18n} \tag{6.17}$$

Again, using eq. (6.12) and eq. (6.6) as well as taking the expectation of eq. (6.17), we can find (Appendix F)

$$EMS_{Aq} = F_{Aq}\sigma_A{}^4 + \sigma_\varepsilon{}^2 \tag{6.18}$$

$$\text{where } F_{Aq} = 2nh^4\left[k_{Aq} + k_{qxl}\mu_B + k_{qxq}\left(\mu_B^2 + \frac{2h^2\sigma_B^2}{3}\right)\right]^2.$$

Similarly, the expected value of MS_{Bq} can be given by

$$EMS_{Bq} = F_{Bq}\sigma_B{}^4 + \sigma_\varepsilon{}^2 \tag{6.19}$$

$$\text{where } F_{Bq} = 2nh^4\left[k_{Bq} + k_{lxq}\mu_A + k_{qxq}\left(\mu_A^2 + \frac{2h^2\sigma_A^2}{3}\right)\right]^2.$$

SS_{lxl} is the sum of squares for the interaction between the linear effect of A and the linear effect of B. It can be computed by a contrast $C_{lxl}=L_{lxl}Y$. Here L_{lxl} is the row vector of the coefficients of the contrast for the interaction $A_l x B_l$ and it can be found in the 5th row of Table 6.1. The sum of squares for this linear contrast is given by

$$SS_{lxl} = \frac{\left[Y_{11\bullet} + Y_{33\bullet} - Y_{13\bullet} - Y_{31\bullet}\right]^2}{4n} \qquad (6.20)$$

Using eq. (6.12) and eq. (6.6) as well as taking the expectation of eq. (6.20), we can find (Appendix F)

$$EMS_{lxl} = F_{lxl}\,\sigma_A^2\,\sigma_B^2 + \sigma_\varepsilon^2 \qquad (6.21)$$

where $F_{lxl}=4nh^4[k_{lxl}+2(k_{lxq}\mu_B+k_{qxl}\mu_A)+4k_{qxq}\mu_A\mu_B]^2$.

SS_{lxq} is the sum of squares for the interaction between the linear effect of A and the quadratic effect of B. It can be computed by a contrast $C_{lxq}=\mathbf{L}_{lxq}\mathbf{Y}$. Here $\mathbf{L}_{lxq}$ is the row vector of the coefficients of the contrast for interaction $A_l x B_q$. Similarly, we have (Appendix F)

$$EMS_{lxq} = F_{lxq}\,\sigma_A^2\,\sigma_B^4 + \sigma_\varepsilon^2 \qquad (6.22)$$

where $F_{lxq}=(4nh^6/3)(k_{lxq}+2k_{qxq}\,\mu_A)^2$. And

$$EMS_{qxl} = F_{qxl}\,\sigma_A^4\,\sigma_B^2 + \sigma_\varepsilon^2 \qquad (6.23)$$

where $F_{qxl}=(4nh^6/3)(k_{qxl}+2k_{qxq}\,\mu_B)^2$. And

$$EMS_{qxq} = F_{qxq}\,\sigma_A^4\,\sigma_B^4 + \sigma_\varepsilon^2 \qquad (6.24)$$

where $F_{qxq}=(4nh^8/9)(k_{qxq})^2$.

The corresponding ANOVA is given in Table 6.2. In practice, σ_A or σ_B, which represents the noise level associated with A or B, is much smaller than μ_A or μ_B. Thus, F_{Al}, ··· F_{Bq}, F_{lxl}, ··· F_{qxq} can be considered as independent of σ_A and σ_B. In other words, the changes of σ_A and σ_B do not affect these F-coefficients.

Table 6.2. ANOVA For Two-Component System

Source	df	SS	MS	EMS *	Estimated ρ	ω
A_l	1	SS_{A_l}	MS_{Al}	$\sigma_e^2+F_{Al}\sigma_A^2$	$(SS_{Al}-MS_e)/SS_T$	ρ_{Al}/σ_A^2
A_q	1	SS_{Aq}	MS_{Aq}	$\sigma_e^2+F_{Aq}\sigma_A^4$	$(SS_{Aq}-MS_e)/SS_T$	ρ_{Aq}/σ_A^4
B_l	1	SS_{B_l}	MS_{Bl}	$\sigma_e^2+F_{Bl}\sigma_B^2$	$(SS_{Bl}-MS_e)/SS_T$	ρ_{Bl}/σ_B^2
B_q	1	SS_{Bq}	MS_{Bq}	$\sigma_e^2+F_{Bq}\sigma_B^2$	$(SS_{Bq}-MS_e)/SS_T$	ρ_{Bq}/σ_B^4
A_lxB_l	1	SS_{lxl}	MS_{lxl}	$\sigma_e^2+F_{lxl}\sigma_A^2\sigma_B^2$	$(SS_{lxl}-MS_e)/SS_T$	$\rho_{lxl}/\sigma_A^2\sigma_B^2$
A_lxB_q	1	SS_{lxq}	MS_{lxq}	$\sigma_e^2+F_{lxq}\sigma_A^2\sigma_B^4$	$(SS_{lxq}-MS_e)/SS_T$	$\rho_{lxq}/\sigma_A^2\sigma_B^4$
A_qxB_l	1	SS_{qxl}	MS_{qxl}	$\sigma_e^2+F_{qxl}\sigma_A^4\sigma_B^2$	$(SS_{qxl}-MS_e)/SS_T$	$\rho_{qxl}/\sigma_A^4\sigma_B^2$
A_qxB_q	1	SS_{qxq}	MS_{qxq}	$\sigma_e^2+F_{qxq}\sigma_A^4\sigma_B^4$	$(SS_{qxq}-MS_e)/SS_T$	$\rho_{qxq}/\sigma_A^4\sigma_B^4$
Error	9(n-1)	SS_e	MS_e	σ_e^2	1-Σ above ratios	ρ_e
Total	9n-1	SS_T	MS_T**	σ_y^2 **	1.00	

*(* F_{Al} ,···F_{qxq} are constants related to the specific model;*
***Since the levels of the components are used to simulate the noise factors, then MS_T can be taken as the estimator of σ_y^2, while for a general ANOVA, MS_T has no meaning)*

For the existing tolerances σ_A' and σ_B', we can estimate the percent contribution ratios ρ_{Al}, $\cdots$ ρ_{qxq} and EMS_T'. For other values of σ_A and σ_B, EMS_T can be found by using the following VTE (6.25) that is derived by taking the expectation of eq. (6.13) and substituting the EMS values:

$$EMS_T = EMS'_T\Big[\omega_{A_l}\sigma_A^2 + \omega_{A_q}\sigma_A^4 + \omega_{B_l}\sigma_B^2 + \omega_{B_q}\sigma_B^4 + \omega_{lxl}\sigma_A^2\sigma_B^2$$
$$+ \omega_{lxq}\sigma_A^2\sigma_B^4 + \omega_{qxl}\sigma_A^4\sigma_B^2 + \omega_{qxq}\sigma_A^4\sigma_B^4 + \rho_e\Big] \qquad (6.25)$$

where $\omega_{Al} = \rho_{Al}/\sigma_A'^2$, $\cdots$ $\omega_{qxq}=\rho_{qxq}/(\sigma_A'^4\sigma_B'^4)$. These ω-values are given in Table 6.2. The insignificant terms can be combined with the error.

Finally, it is observed that ω_{Al} and ω_{Aq} contain $\mu_B^2+(2h^2/3)\sigma_B^2$ or $\mu_B^2+\sigma_B^2$ if $h=\sqrt{(3/2)}$, and also ω_{Bl} and ω_{Bq} contain $\mu_A^2+(2h^2/3)\sigma_A^2$. As a result, the changes of σ_A and σ_B may affect ω_{Al}, ω_{Aq}, ω_{Bl} and ω_{Bq}, unless σ_A or σ_B is much smaller than the mean value μ_A or μ_B.

If all the interactions are insignificant, the percent contribution ratios for these interactions are zeros. The VTE is reduced to the model without interactions, which is used in Taguchi's tolerance design (Taguchi 1986; Taguchi and Wu 1980).

The method can be applied to the system with more than two components, say, for a system with n components and with the assumption that the interactions between three or more components are insignificant, the VTE can be derived as follows:

As we consider the sum of squares for an effect of a component or an interaction between two components, the responses under different levels of other components not included in this effect or interaction are considered as repetitions. For instance, for SS_{AqxBq}, the responses under the different levels of other components except A and B are considered as repetitions and the sum of the responses is a response total $Y_{ij\bullet}$. Thus, it can be shown that the VTE is given by

$$EMS_T = EMS'_T\left[\omega_{\ell_1}\sigma_1^2 + \ldots \omega_{q_n}\sigma_n^2 + \omega_{\ell_1 x \ell_2}\sigma_1^2\sigma_2^2 + \ldots \omega_{q_{n-1} x q_n}\sigma_{n-1}^4\sigma_n^4 + \rho_e\right] \qquad (6.26)$$

where $\sigma_1 \ldots \sigma_n$ are the tolerance levels of component 1 through component n, and $\omega_{\ell_1} \ldots \omega_{q_{n-1} x q_n}$ are ω-values which are defined in a similar way to the ω-values in eq. (6.25). These ω-values can be obtained from the ANOVA table which is based on a set of tolerances $\sigma'_1 \cdots \sigma'_n$. Using eq. (6.26), we can estimate EMS_T for any values of $\sigma_1 \cdots \sigma_n$. If true $\sigma_y'^2$ is available, substituting $\sigma_y'^2$ into (6.26) for EMS'_T, we will have a better estimation of σ_y^2 than EMS_T.

§6.5. Optimization Model For Tolerance Design

The goal of tolerance design is to minimize the total cost which consists of quality loss and the cost increase due to control of tolerances for the components and the subsystems. For a system with n components, the tolerance control cost for the ith component is a function of its tolerance level or the standard deviation σ_i, and is modeled by $d_i(\sigma_i)=c_i/\sigma_i^{a_i}$ (Figure 6.1) where c_i and a_i are positive constant values.

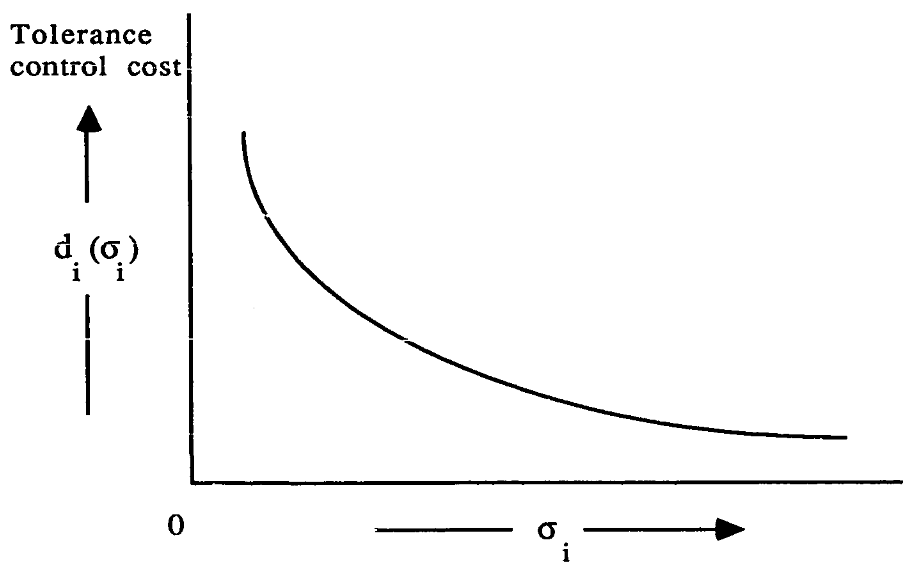


Figure 6.1. Tolerance control cost vs tolerance σ_i

If EMS_T can be considered as $\sigma_y{}^2$, the following optimization model can be obtained (see also figure 6.2):

$$\underset{\sigma_1,\sigma_2,\ldots\sigma_n}{\text{Minimize}}\left\{TC=\sum_{i=1}^{n}\frac{c_i}{\sigma_i^{a_i}}+K\sigma_y^2\right\}$$

$$\text{Subject to}\begin{cases}\sigma_y^2=VTE(\sigma_1^2,\sigma_2^2,\ldots\sigma_n^2)\le V\\ \sigma_1,\sigma_2,\ldots\sigma_n>0\end{cases}\qquad(6.27)$$

where V is the maximum value for $\sigma_y{}^2$ based on the requirements of the customer.

This is a nonlinear programming (NLP) problem. Many NLP algorithms can be used to solve this problem (Bazaraa and Shetty 1979, Mangasarian 1969, Martos 1975). However, since the model consists of posynomials, geometric programming (GP) can be applied to this problem (Duffin *et al* 1967). In next section, an example is given to numerically illustrate the tolerance design methodology for components and subsystems. For this example, we formulate the optimization model and use the GP method to solve the problem.

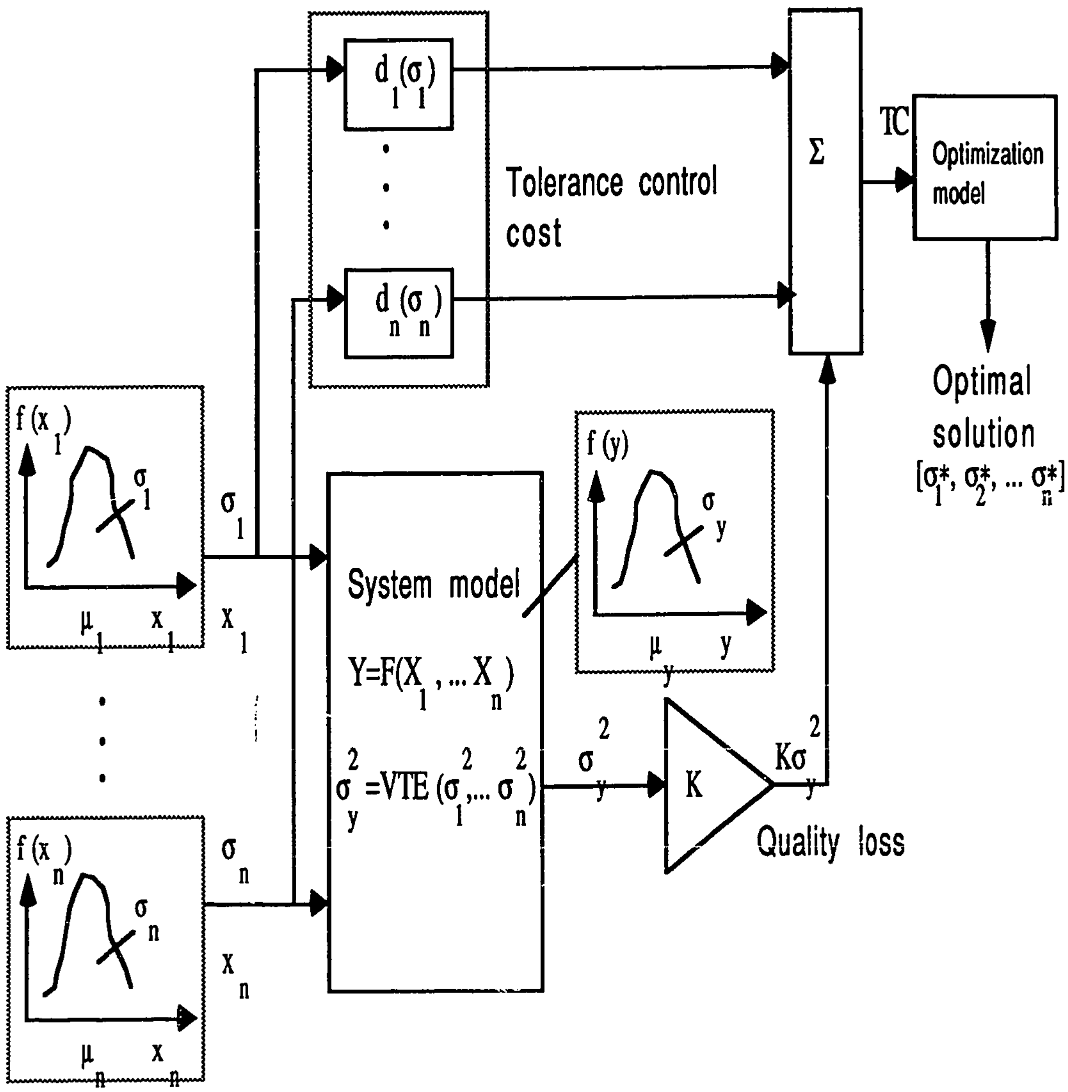


Figure 6.2. Optimization for tolerance design

§6.6. Example For Tolerance Design

Let us consider a hypothetical RLC circuit where the response variable is the current given by

$$Y=\frac{110}{\sqrt{R^2+\left(120\pi L-\frac{1}{120\pi C}\right)^2}} \tag{6.28}$$

Suppose the target value of Y is 2.00. The quality loss function (QLF) is given by

$$\text{Loss} = K(Y\text{-}2.00)^2 \tag{6.29}$$

where K=250. Using this example, we will demonstrate the concept of tolerance design for the components based on QLF, VTE, DOE and GP methodology.

Assume that the cost functions for tolerance control are given by

$$\left.\begin{aligned} d_R(\sigma_R) &= 0.80/\sigma_R \\ d_L(\sigma_L) &= 0.017/\sigma_L \\ d_C(\sigma_C) &= 6.0\text{x}10^{-6}/\sigma_C \end{aligned}\right\} \tag{6.30}$$

Suppose the nominal values of R, L and C after parameter design are given by: R=40, L=0.17 and C=100x10^{-6}. We investigate the linear and the quadratic effects of the components as well as the interactions. For σ_R'=2.666, σ_L'=0.01133 and σ_C'=6.67x10^{-6}, we can use eq. (6.6) for the noise representations for each component with $h=\sqrt{(3/2)}$. Thus, the noise levels associated with R, L and C are given in Table 6.3.

The three-way layout for the response variable Y for different noise level combinations is given by Table 6.4 (Generally, this layout and the ANOVA may be available from the parameter design step if parameter design is done before the tolerance design).

Table 6.3. The noise levels associated with R, L and C

Component	1st level	2nd level	3rd level
R	R_1=36.735	R_2=40.000	R_3=40.265
L	L_1=0.1561	L_2=0.1700	L_3=0.1839
C	$C_1=91.83\times10^{-6}$	$C_2=100\times10^{-6}$	$C_3=108.17\times10^{-6}$

Table 6.4. The responses for different noise level combinations

	L_1			L_2			L_3		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
R_1	2.32	2.25	2.19	2.16	2.09	2.04	2.01	1.95	1.90
R_2	2.20	2.14	2.09	2.06	2.00	1.96	1.93	1.88	1.83
R_3	2.09	2.04	1.99	1.97	1.92	1.88	1.86	1.81	1.77

For these responses, we can perform ANOVA to find the significant effects and the significant interactions (see Table 6.5). The insignificant terms have been combined with the error. We illustrate the computation of SS_{L1xC1} as follows (see also Table 6.6):

The contrast of $L_l x C_l$ is $\sum_i \sum_j c_{ij} Y_{ij\cdot}$. The sum of squares for this contrast is given by

$$SS_{L_l x C_l} = \frac{(\sum_i \sum_j c_{ij} Y_{ij\cdot})^2}{n \sum_i \sum_j c_{ij}^2} = \frac{(6.61 - 6.27 - 5.8 + 5.5)^2}{3\,[1^2 + (-1)^2 + (-1)^2 + 1^2]} = 0.00013$$

The VTE for any σ_R, σ_L and σ_C is given by

$$EMS_T = EMS_T' \{0.03855\sigma_R^2 + 1.98x10^{-6}\sigma_R^4 + 4803\sigma_L^2 + 18205\sigma_L^4 + 2.23x10^9\sigma_C^2 + 1.52x10^{17}\sigma_C^4 + 8.66\sigma_R^2\sigma_L^2 + 3.16x10^6\sigma_R^2\sigma_C^2 + 3.5x10^{10}\sigma_L^2\sigma_C^2 + 0.0005\} \quad (6.31)$$

Since Table 6.4 is obtained by simulating the noise factors associated with the components, EMS_T' can be considered as $\sigma_y'^2$, and estimated by MS_T' in Table 6.5, which is 0.019464.

Our goal is to minimize the total cost consisting of tolerance control and expected quality losses due to variations from target. The optimization problem can be modeled by (6.32) as follows:

Table 6.5. ANOVA for the responses (subscript l represents the linear effect; subscript q represents the quadratic effect)

Source	SS	df	MS	F_0	ρ	ω
R_l	0.13869	1	0.13869	18565 ***	0.2740	0.03855
R_q	0.00007	1	0.00007	9.37 ***	0.0001	1.98×10^{-6}
L_l	0.31205	1	0.31205	41770 ***	0.6166	4803.0
L_q	0.00015	1	0.00015	20.1 ***	0.0003	18205.0
C_l	0.05014	1	0.05014	6712 ***	0.0991	2.23×10^{9}
C_q	0.00015	1	0.00015	20.1 ***	0.0003	1.516×10^{17}
$R_l x L_l$	0.00403	1	0.00403	539 ***	0.0079	8.660
$R_l x C_l$	0.00053	1	0.00053	70.9 ***	0.0010	3.16×10^{6}
$L_l x C_l$	0.00013	1	0.00013	17.4 ***	0.0002	3.50×10^{10}
Errors	.000127	17	.00000747		0.0005	0.0005
Total	0.506067	26	0.019464			1.0000

**** Significant at the level 0.01.*

Table 6.6. Illustration of the computation of SS_{LlxCl}

Levels	11	12	13	21	22	23	31	32	33	$LL^T = n\Sigma\Sigma c_{ij}^2$
c_{ij}*	+1	0	-1	0	0	0	-1	0	+1	4n
$Y_{ij\bullet}$**	6.61	6.43	6.27	6.19	6.01	5.88	5.88	5.64	5.50	

* *c_{ij} is from Table 1*

** *$Y_{ij\bullet}$ is the sum of the responses for the three levels of R, namely, the three levels of R can be considered as repetitions, then n=3.*

$$\underset{\sigma_R, \sigma_L, \sigma_C}{\text{Minimize}}\left\{TC = d_R(\sigma_R) + d_L(\sigma_L) + d_C(\sigma_C) + K^*VTE\left(\sigma_R^2, \sigma_L^2, \sigma_C^2\right)\right\}$$

with subject to $\sigma_y^2 = VTE\left(\sigma_R^2, \sigma_L^2, \sigma_C^2\right) \leq V$ and $\sigma_R, \sigma_L, \sigma_C > 0$ (6.32)

where V is the maximum allowable value for σ_y^2, which is 0.00100.

Using the geometric programming and the computer program given in Kuester and Mize (1973), we obtain the solutions as follows:

$$\sigma_R^* = 0.459, \quad \sigma_L^* = 0.00255, \quad \sigma_C^* = 2.325 \times 10^{-6}$$

For this set of tolerances, the estimated value of EMS_T given by VTE is 0.00100.

It should be observed that if the tolerance control cost for each component is not given by a function like (6.30), but by several discrete values associated with different tolerance levels or grades, we can minimize the total cost by balancing the expected quality loss and the cost due to the tolerance control.

Taking the first order approximation for the Taylor's series of eq. (6.28) and in terms of eq. (6.3), the variance of Y is given by

$$\sigma_y^2 \approx 7.10 \times 10^{-4} \sigma_R^2 + 88.9 \sigma_L^2 + 4.40 \times 10^7 \sigma_C^2 \qquad (6.33)$$

also, if we ignore the quadratic terms and the interactions in the VTE (6.31), we have

$$\sigma_y^2 \approx 7.50\text{x}10^{-4}\,\sigma_R^2 + 93.5\sigma_L^2 + 4.34\text{x}10^7\sigma_C^2 \qquad (6.34)$$

The values of the respective coefficients in eq. (6.33) and eq. (6.34) are close to each other.

To find a realistic value of h to represent the effect of the noise factors, we performed computer simulation for different h values to find the true values of μ_Y and σ_y^2. The results indicate that for $h=\sqrt{(3/2)}$, MS_T is very close to the σ_y^2 given by the Monte Carlo simulation (see Figure 6.3) and h has very small effects on the mean of Y. As a result, $h=\sqrt{(3/2)}$ is suggested if the noise factors in the outer array are represented by three levels.

It is interesting to observe that the MS_T for $h=\sqrt{3}$ is about twice as large as the σ_y^2 given by Monte Carlo simulation. As discussed before, $h=\sqrt{3}$ gives a variance of the noise factor two times the true variance. In this example, the linear effects dominate the variable part of the response, or the contribution ratios of the linear effects in Table 6.5 are much larger than the other contribution ratios. Thus, twice the variances of the components leads to twice the variance of Y.

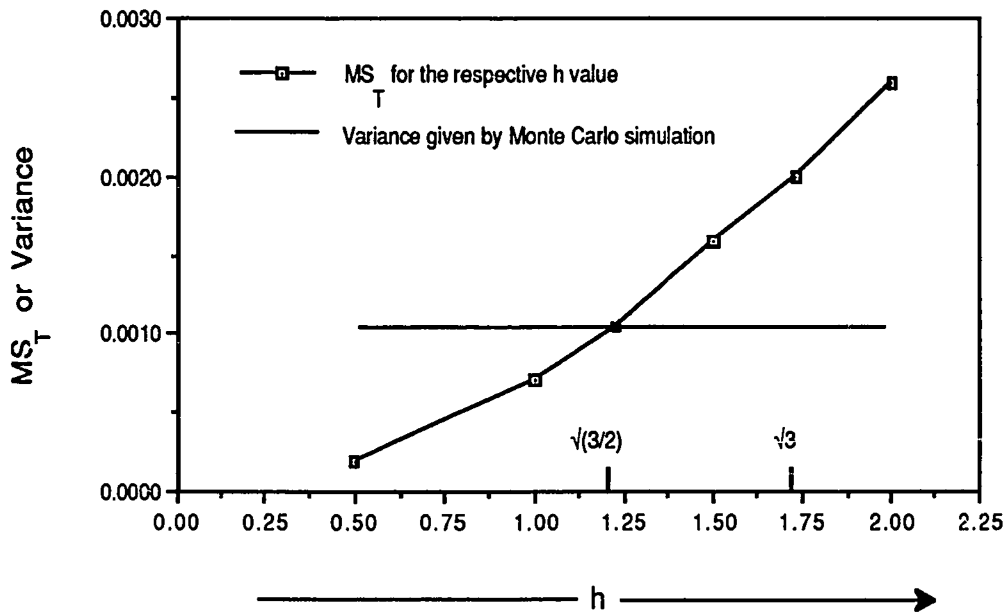


Figure 6.3. *The comparison of σ_y^2 and MS_T for various h values (The result of the Monte Carlo simulation is based on 2000 times of simulation and the assumption that all noise factors are normally distributed).*

CHAPTER 7

TOLERANCE DESIGN TO BALANCE QUALITY AND COST

§7.1. Optimization Model For Tolerances Design

Tolerance design is performed to balance quality loss due to variations from the target and the cost due to control of tolerances (precision or allowances). In last chapter, we present the approaches to tolerance design for components and subsystems. We also develop the variation transmission equation (VTE) which transfers variations in the parameters of components to the variations in the system performance. Our goal is to balance the quality loss due to variations of the components and the cost for controlling tolerances.

In this chapter, we present an optimization model and the tolerance design for the quality characteristic of a system as well as the lower-level characteristic, even if the higher-level characteristic has a nonlinear relationship with the lower-level characteristic. We will be discussing tolerance design for the deterioration characteristic over time. In the context, we give scientific illustrations as the proofs for the optimal property of these approaches.

The objective of tolerance design is to minimize the total cost which consists of quality loss due to variations (QLV) from the target and the cost increase (CI) due to control of tolerances. That is,

$$\underset{\Delta}{\text{Minimize}}\{QLV + CI\} \qquad (7.1)$$

where Δ is the tolerance level.

Traditionally, QLV can be considered as a quadratic function of Δ or $K_q\Delta^2$, which can be derived from a quadratic QLF with the assumption that the process mean has been adjusted to the target. CI can be considered as a negative power function of Δ or K_C/Δ^a ($a>0$, Krishnamoorthi 1989). K_q and K_C are constant values. The objective of tolerance design is to find an optimal solution Δ^* (see Figure 7.1).

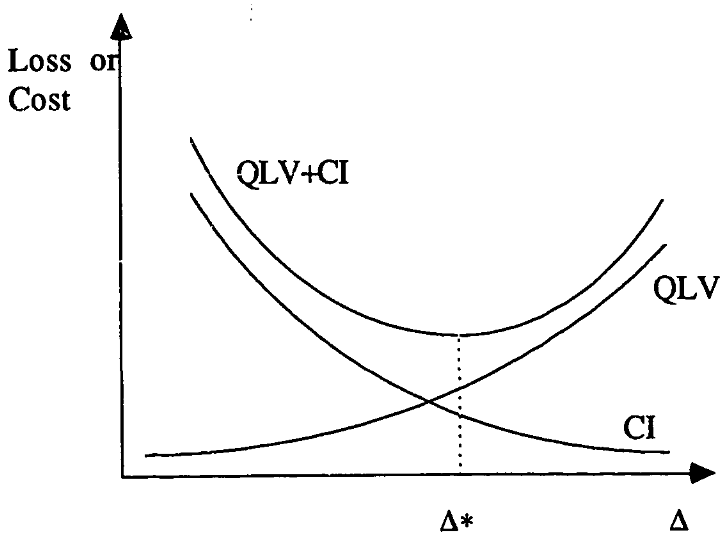


Figure 7.1. Balance of quality loss due to variations and cost increase due to control of tolerances.

If both QLV function and CI function are known, we can find the optimal solution Δ^* by taking the derivative of QLV+CI with respect to Δ and setting it to zero. As a result, we have

$$\Delta^* = \left(\frac{aK_C}{2K_q}\right)^{\frac{1}{a+2}}$$

If a=2, then $\Delta^*=(K_C/K_q)^{1/4}$, which is the point where $QLV=CI=\sqrt{(K_C K_q)}$.

For many systems, we know the cost related to scrap or rework of a nonconforming product, rather than a cost function associated with the tolerance levels. In the following sections, we will be discussing tolerance design based on the quality loss function and the cost for scrap or rework of a nonconforming unit.

§7.2. Manufacturer's Tolerances For Shipping

The tolerance Δ for a characteristic y of a product for shipping can be designed based on the quality loss function and the cost for scrap (or rework, degradation) of a nonconforming product. Assume that the quadratic loss function (2.4) is appropriate where K can be determined by $A_0/(\Delta_0)^2$ (see Section 2.3), and A is the cost increase for repairing or replacing a defective item before shipping. The manufacturer's tolerance Δ can be given by finding an economic equilibrium point or letting $A=K(\Delta)^2$. As a result, we have

$$\Delta = \sqrt{(A/K)} = \sqrt{(A/A_0)}\Delta_0 \qquad (7.2)$$

A is smaller than A_0. Hence, Δ is smaller than Δ_0.

To see why Δ is a best tolerance for y, look at Figure 7.2. Since a product with a y outside manufacturer's tolerance is not shipped, the value of QLV is zero for y outside the tolerance. However, a defective item results in a loss of A for the manufacturer, as indicated in Figure 7.2 (b). By adding these two parts, we obtain the adjusted loss function or QLV+CI which is the solid curve in Figure 7.2 (c), denoted by $L^0(y,y_0)$.

If we have a tolerance Δ^+ which is larger than Δ, as indicated in Figure 7.3, we can obtain the adjusted loss function as the solid curve in Figure 7.3 (c), denoted by $L^+(y,y_0)$. Obviously, $L^+(y,y_0) \geq L^0(y,y_0) \geq 0$ for all y. Thus, we have $E[L^+(Y,y_0)]>E[L^0(Y,y_0)]$. As a result, Δ is a better tolerance than Δ^+.

If we have a tolerance Δ^- which is smaller than Δ, as indicated in Figure 7.4, similarly, the adjusted loss function is given by the solid curve in Figure 7.4 (c), denoted by $L^-(y,y_0)$. Obviously, $L^-(y,y_0) \geq$

$L^0(y,y_0) \geq 0$. Consequently $E[L^-(Y,y_0)] > E[L^0(Y,y_0)]$. Thus, Δ is a better tolerance than Δ^-. In conclusion, Δ is the best tolerance for Y, because for this tolerance, the expected quality loss achieves a minimum with the tolerance level as a variable.

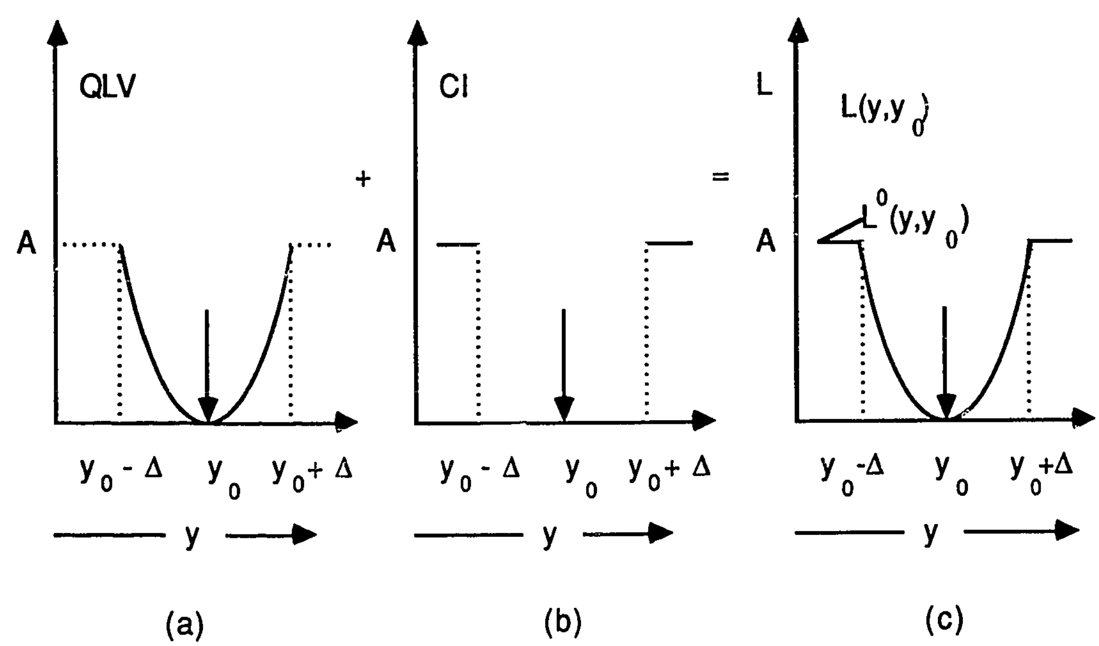


Figure 7.2. Best manufacturer's tolerance Δ.

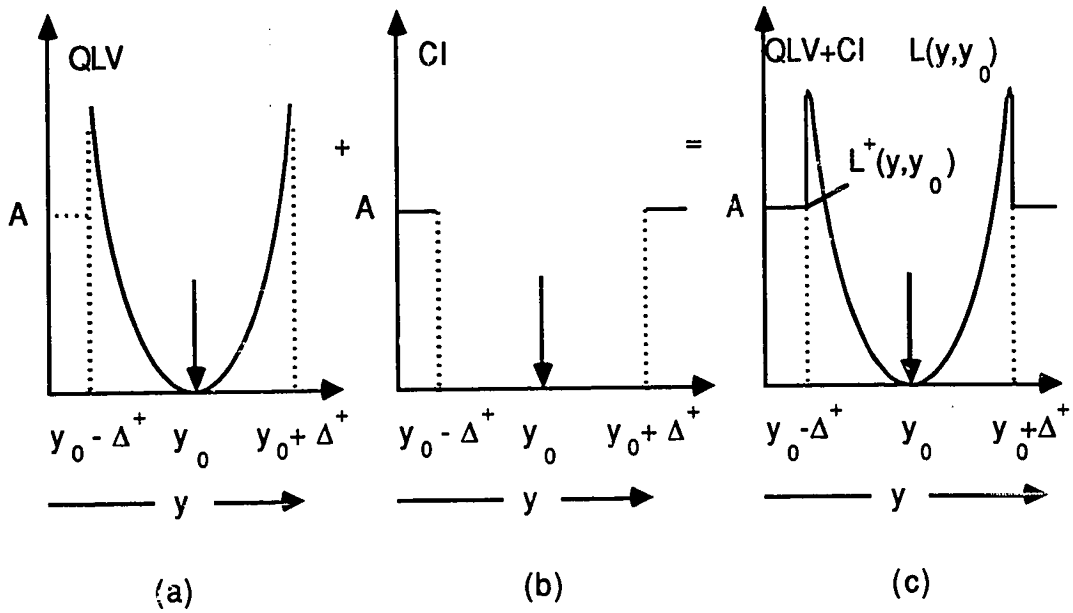


Figure 7.3. Tolerance Δ^+ is larger than Δ.

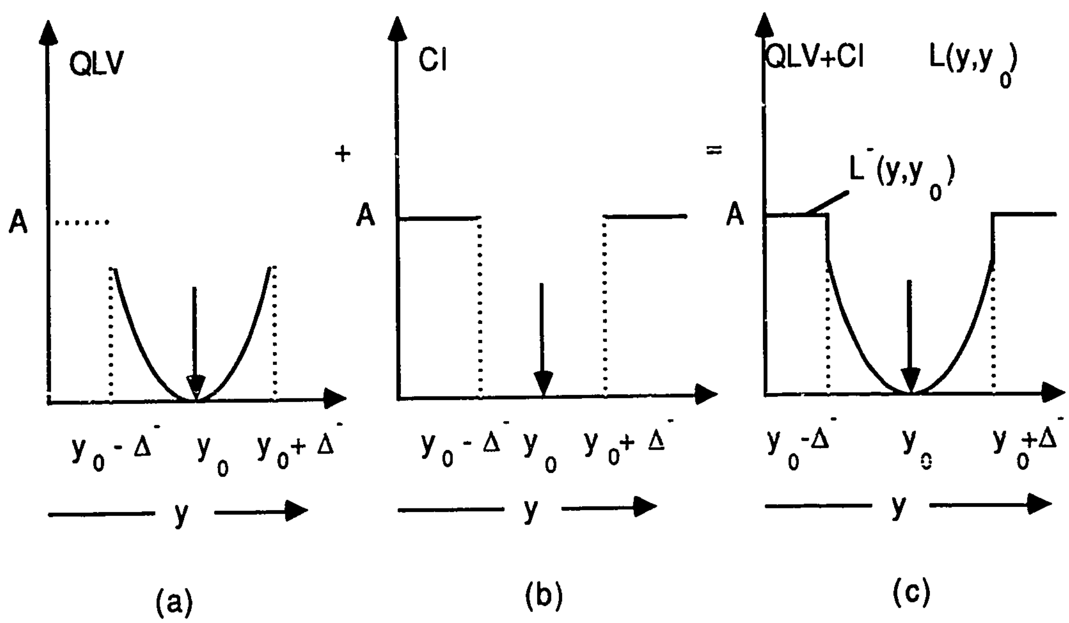


Figure 7.4. Tolerance Δ^- is smaller than Δ.

§7.3. Tolerance Design For Lower-level Characteristic

A product may use different quality characteristics for various manufacturing processes. For instance, a stamped component made from steel plates uses the dimension as a characteristic, while plates use hardness or thickness as a characteristic. The characteristic of a product before processing is called lower-level characteristic and the characteristic after processing is called higher-level characteristic which is affected by the lower-level characteristic (Taguchi 1986).

The tolerance for a lower-level characteristic is specified at the economic equilibrium point where the quality loss for a lower-level characteristic outside its tolerance is equal to the quality loss based on the quality loss function for the higher-level characteristic. In other words, there is no economic difference no matter whether or not we replace the product with a lower-level characteristic outside the tolerance. Suppose that

y is a higher-level characteristic and y_0 is the target value;

x is a lower-level characteristic and x_0 is the target value;

Δ_x or Δ_{x1} and Δ_{x2} are the tolerances for x.

A_x is the cost for scrap (rework or degradation) of a unit if x is outside the tolerances before processing.

A manufacturing process (Figure 7.5) transfers the lower-level characteristic x to the higher-level characteristic y by a function y=P(x). In practice, this is a one-to-one mapping. Hence, y=P(x) is strictly increasing or strictly decreasing. Thus, $x=P^{-1}(y)$, the inverse function of P(x), is a monotone increasing or decreasing function.

If a quality loss function for x is known, the tolerance of x can be obtained by finding the economic equilibrium point in the same way as in last section. However, if we do not have a loss function for x, the tolerance of x can be developed based on $L(y,y_0)$, the loss function of y, which is assumed to be a quadratic function and given by eq. (2.4).

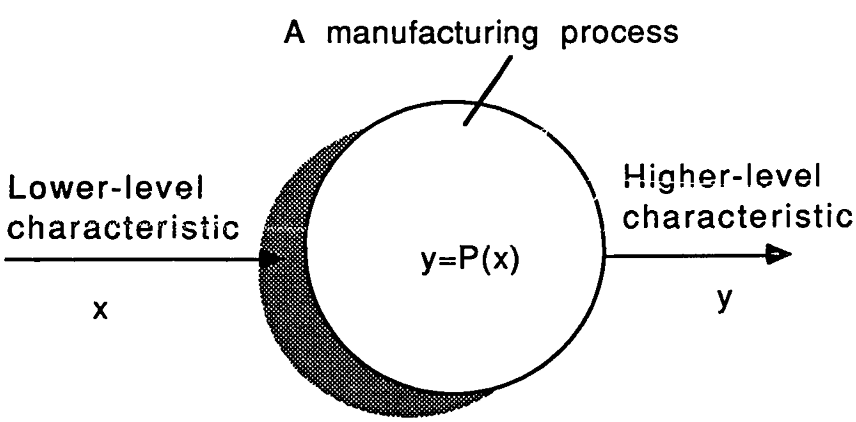


Figure 7.5. y=P(x) transfers the lower-level characteristic x to the higher-level characteristic y.

Due to the nonlinear effect of P(x), we may have to develop the asymmetrical tolerance limits for x. For the lower-side tolerance of x, we can find an economic equilibrium point by $A_x=K[P(x_0-\Delta_{x1})-y_0]^2$. If y is an increasing function of x, then $P(x_0-\Delta_{x1})<y_0$ and we have $y_0-P(x_0-\Delta_{x1})=\sqrt{(A_x/K)}$. By simplifying, we can find the lower-side tolerance limit for x as follows:

$$\Delta_{x1} = x_0 - P^{-1}[y_0-\sqrt{(A_x/K)}] \qquad (7.3)$$

For the higher-side tolerance of x, we can find an economic equilibrium point by $A_x=K[P(x_0+\Delta_{x2})-y_0]^2$. Similarly, we can obtain the higher-side tolerance of x as follows:

$$\Delta_{x2} = P^{-1}[y_0+\sqrt{(A_x/K)}] - x_0 \qquad (7.4)$$

To see why Δ_{x1} and Δ_{x2} are the best tolerances for x, look at Figure 7.5 and consider Δ_{x2} first. Function y=P(x) transfers x value to y value and the quality loss is evaluated by $L(y,y_0)=K(y-y_0)^2$ on the deviation of y from the target value. Since a product with a x value outside $x_0+\Delta_{x2}$ is scrapped or degraded with a loss of A_x, the adjusted quality loss function is given by the solid curve in Figure 7.6, denoted by L^0.

If a tolerance Δ^-_{x2} is smaller than Δ_{x2}, the corresponding adjusted loss function is denoted by L^- (see Figure 7.6). Obviously, $L^- \geq L^0$. Thus, Δ_{x2} is a better tolerance than Δ^-_{x2}. If a tolerance Δ^+_{x2} is larger than Δ_{x2}, the corresponding adjusted loss function is denoted by L^+ (Figure 7.6). It is obviously that $L^+ \geq L^0$. Thus, Δ_{x2} is a better tolerance than Δ^+_{x2}. In conclusion, Δ_{x2} is a best higher-side tolerance for x. Based on a similar discussion, we can conclude that Δ_{x1} is a best lower-side tolerance for x.

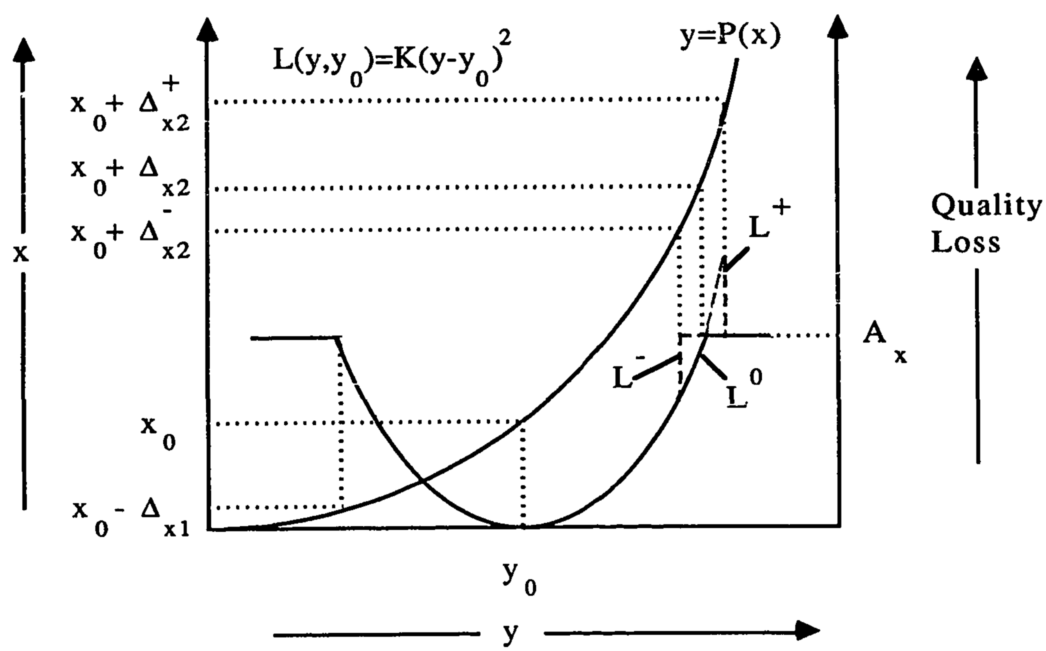


Figure 7.6. The tolerances for the lower-level characteristic x.

Here we take a cubic function as an example. Suppose that we have $y=P(x)=y_0+b(x-x_0)^3$ where b is a constant. Then

$$x = P^{-1}(y) = \sqrt[3]{[(y-y_0)/b]}+x_0$$

Hence,

$$\Delta_{x1} = x_0 - P^{-1}[y_0-\sqrt{(A_x/K)}]$$

$$= \sqrt[6]{(A_x/K)}/\sqrt[3]{b}$$

$$\Delta_{x2} = P^{-1}[y_0+\sqrt{(A_x/K)}] - x_0$$

$$= \sqrt[6]{(A_x/K)}/\sqrt[3]{b}$$

For this example, $\Delta_{x1}=\Delta_{x2}$. If K is given by $A_0/\Delta_0{}^2$, then we have $\Delta_{x1}=\Delta_{x2}=\sqrt[6]{(A_x/A_0)}\sqrt[3]{(\Delta_0/b)}$.

Especially for a linear system or $y= y_0+b(x-x_0)^3$, we have

$$\Delta_{x1}=\Delta_{x2} = \sqrt{(A_x/A_0)}(\Delta_0/b) \qquad (7.5)$$

Furthermore, if $x=g(z)$ where z is another lower-level characteristic, the tolerance for z can be determined in a similar way.

§7.4. Tolerance Design For Deterioration Characteristic

The effect of deterioration can cause the quality characteristic to deviate from target as time increases. For instance, the resistance of a resistor will increase due to aging of the material. In general, the deterioration linearly depends on time. On an average, for a system that has lasted for t units of time, its quality characteristic is $Y(t)=Y(0)+\beta t$ where β is the deterioration rate and $Y(0)$ is the value of Y at t=0 or at the time of shipping.

Assume that a nonconforming unit results in a loss of A_β due to β outside tolerances (to be scrapped, degraded or reworked). Since the quality loss is a quadratic function of the deviation from the target, then the quality loss during one unit of time at time t is given by $L_1[Y(t),y_0]=k[Y(t)-y_0]^2$, where k is a constant value which is equal to the quality loss due to one unit of deviation for one unit of time. Hence, the expected value is given by

$$E\{L_1[Y(t),y_0]\} = E\{\ k[Y(0)+\beta t-y_0]^2\}$$

$$= k[\ \sigma_y^2 + \delta^2 - 2\delta\beta t + (\beta t)^2\ ] \qquad (7.6a)$$

where $\delta=y_0-\mu_y$ or the difference between the target and the mean of Y at the time of shipping. Taking the derivative of $E\{L_1[Y(t),y_0]\}$ with respect to δ and setting it to zero, we can obtain $\delta^*=\beta T/2$. Since the second derivative of $E\{L_1[Y(t),y_0]\}$ is 2k (>0), then δ^* gives a minimal value of $E\{L_1[Y(t),y_0]\}$. The target value of β is zero. Thus, the ideal value of δ is zero. If δ is designed to be zero, we have

$$E\{L_1[Y(t),y_0]\} = k\sigma_y{}^2 + k(\beta t)^2 \tag{7.6}$$

The 1st term is independent of t and is caused by manufacturing variations. The 2nd term is caused by the deterioration over time.

(1) Characteristic Y Can Not Be Reset BY Customers

Examples of this are the resistance of a resistor and the output voltage of a power circuit in a TV set. In this case, once the product is shipped, Y can not be reset. If T is the design life time, then the quality loss for the whole life time is given by

$$\begin{aligned} QLV &= \int_0^T k\left[\sigma_y^2 + (\beta t)^2\right] dt \\ &= kT\left[\sigma_y^2 + \frac{1}{3}(\beta T)^2\right] \\ &= K\sigma_y^2 + \frac{K}{3}(\beta T)^2 \end{aligned} \tag{7.7}$$

where K=kT is equal to the quality loss due to one unit of deviation for T units of time.

We are discussing the quality loss due to deviation or $K(\beta T)^2/3$. By finding the economic equilibrium point or $A_\beta = K(\beta^* T)^2/3$, we can obtain the manufacturer's tolerance for β before shipping:

$$\Delta_\beta = |\beta^*| = \frac{1}{T}\sqrt{\frac{3A_\beta}{K}} = \sqrt{\frac{3A_\beta}{A_0}}\,\frac{\Delta_0}{T} \tag{7.8}$$

where $K=A_0/\Delta_0{}^2$ and A_0 is the quality loss if Y is outside customer's tolerance $y_0 \pm \Delta_0$. Any product with a $|\beta|$ greater than Δ_β needs reworking, degrading or scrapping.

(2) Characteristic Y Can Be Reset By Customers

An example of this is the time of a watch or a clock. In this case, a customer can adjust Y to the target at a certain cost a_0, if Y is outside customer's tolerance $y_0 \pm \Delta_0$. The quality loss due to variations from the target for one unit of time is given by

$$QLV_1 = \int_0^1 k\left[\sigma_y^2 + (\beta t)^2\right]dt$$

$$= k\sigma_y^2 + \frac{k}{3}\beta^2 \tag{7.9 a}$$

where $k=a_0/\Delta_0{}^2$ (see Section 2.3). The quality loss due to variations for life time T is given by

$$QLV = T \cdot QLV_1$$

$$= K\sigma_y{}^2 + K\beta^2/3 \tag{7.9}$$

where K=kT is equal to the quality loss due to a unit of deviation for T units of time.

We are discussing the quality loss due to deviation or $K\beta^2/3$. By finding the economic equilibrium point or $A_\beta=K(\beta^*)^2/3$, we can obtain the manufacturer's tolerance for β before shipping:

$$\Delta_\beta = |\beta^*| = \sqrt{\frac{3A_\beta}{K}} = \sqrt{\frac{3A_\beta}{A_0}}\,\Delta_0 \tag{7.10}$$

where K is given by $A_0/(\Delta_0)^2$ and $A_0=a_0T$. Any product with a $|\beta|$ greater than Δ_β needs reworking, degrading or scrapping.

To see why the equilibrium point gives a good tolerance for deterioration rate β, look at Figure 7.7. In Figure 7.7 (a), the tolerance is Δ_β. Any product with a $|\beta|$ greater than Δ_β will be adjusted, degraded or scrapped at a cost of A_β. Thus, the quality loss due to deterioration is given by $L(\beta)$ as the solid curve in Figure 7.7 (a). If a tolerance for β is larger than Δ_β such as $\Delta_{\beta+}$ in Figure 7.7 (b), the quality loss due to deterioration is given by $L^+(\beta)$. Obviously, $L(\beta)$ is smaller than $L^+(\beta)$. Hence, Δ_β is a better tolerance than $\Delta_{\beta+}$. If a tolerance for β is smaller than Δ_β such as $\Delta_{\beta-}$ in Figure 7.7 (c), the quality loss due to deterioration is given by $L^-(\beta)$. It is obvious that

$L(\beta)$ is smaller than $L^-(\beta)$. Hence, Δ_β is a better tolerance than $\Delta_{\beta-}$. In conclusion, Δ_β is the best tolerance of the deterioration characteristic.

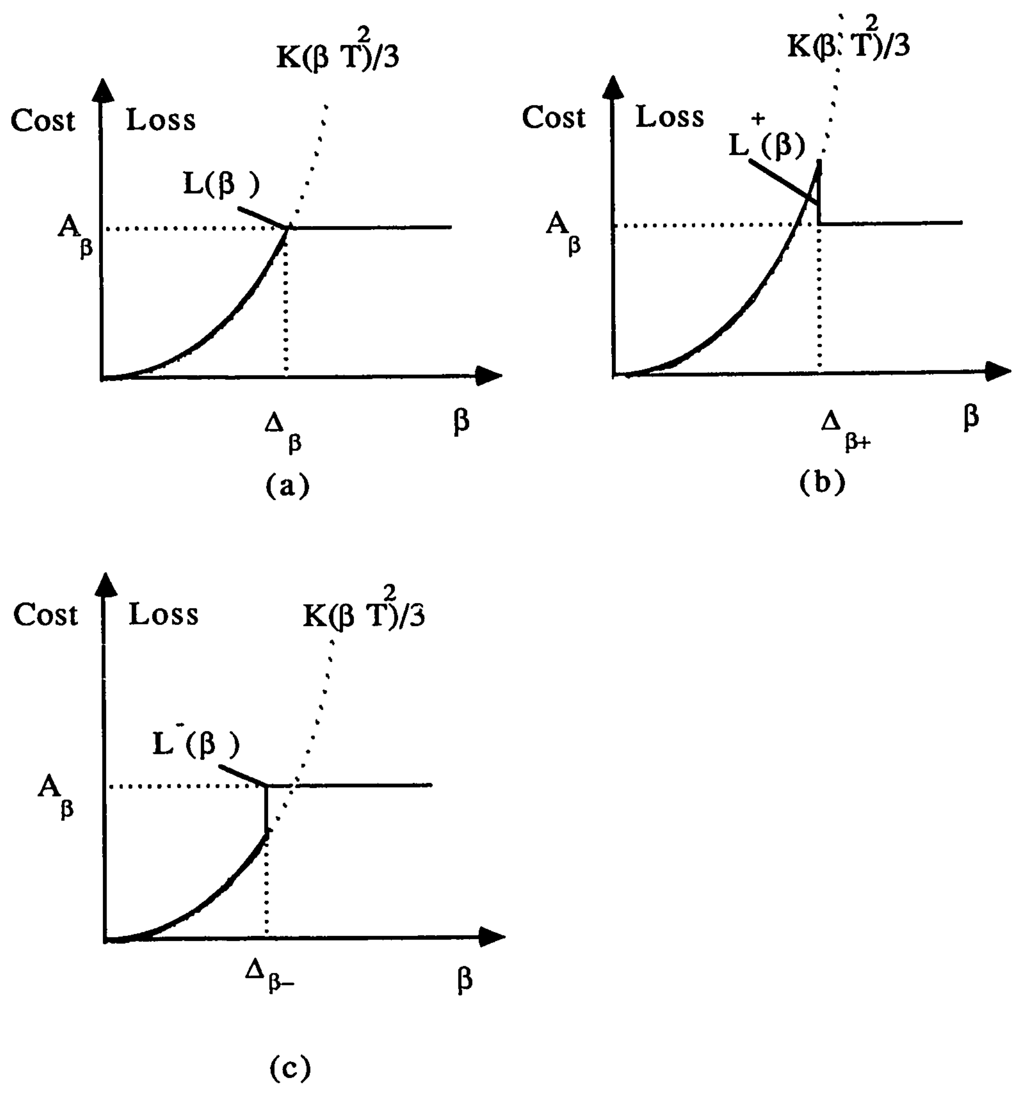


Figure 7.7. An explanation of the best tolerance Δ_β for β.

CHAPTER 8

SUMMARY AND RECOMMENDATION FOR FURTHER RESEARCH

§8.1. Summary And Contribution Of This Research

Traditional quality control is concerned with the downstream side of a process. The goal is to inspect bad quality out and eliminate or remove the assignable sources of variations. Quality engineers and the management today have realized that a better way for quality improvement is to build good quality into products and processes by design engineering. The objective of the robust design proposed by Taguchi is to design a system whose performance is less sensitive to manufacturing and environmental variations, deterioration over time and changes in use conditions. As a result, the effects of these noise factors can be reduced by finding the better values for the design factors. This research is regarding the improvement and extension in some aspects of Taguchi's quality engineering methodology, with an emphasis on the optimization of dynamic systems and tolerance design.

Quality loss due to variations from the target value can be evaluated by a loss function. Various quality loss functions (QLF) are presented in Chapter 2, including the symmetric QLF and the asymmetric QLF. Based on the underlying causes of variations and the desire to reduce the error of estimation, the quadratic QLF is a good approximation as a quality measure. QLF is nothing but a means to transfer the variation to a loss value in dollars so as to quantify the quality loss. Many systems have several quality characteristics. To evaluate quality for such systems, we develop multivariate loss functions which may be used as a measure for the optimization of the multivariate systems.

The goal of robust design for dynamic systems is to minimize deviations of the real system from the ideal model. The optimization can be done by finding the best values for **Z** and **R** simultaneously to minimize the expected quality loss. This is possible but sometimes difficult or even impossible to be implemented, especially for the cases where an analytical model is unknown. The optimization can be simplified by decomposing the selection of **Z** and the adjustment of **R** into a two-step procedure. In the first step, if we select **Z** to minimize the expected quality loss, we may find a solution of **Z** by reducing the first part in eq. (3.4) that is supposed to be eliminated by adjusting **R** in the second step. The variation part or the second part in eq. (3.4) could be used as an optimization criterion for the selection of **Z**. However, the variation part may be inflated as we are adjusting **R** to set the first part to zero in the second step, because the adjustment of **R** may have effects on the variation part. The conflict of this can be eliminated by using an efficient SN ratio as a criterion for the selection of **Z**.

The validity and limitations of the SN ratio have been examined for various models. Necessary modifications of the SN ratio are suggested for the specific models. A generic model and a systematic approach to dealing with dynamic systems are provided. For discrete dynamic characteristics, we present the development of the SN ratio. As long as a SN ratio has been developed for discrete dynamic characteristics, the optimization can be performed in a procedure similar to the continuous dynamic characteristics.

The objective of tolerance design is to balance quality loss due to variations and cost due to control of variations by specifying best tolerance levels for products and components. If a system consists of many components, variations in the parameters of the components will be transferred to the variations of the system performances by a transfer function. As a result, a quality loss is incurred. To reduce this quality loss, we can control the tolerances of the components. However, that results in an increase in cost. To balance this, an optimization model is proposed based on the variation transmission equation that is developed in Chapter 6, regardless of the significance of the interactions between components. In Chapter 7, we present the approaches to tolerance design for deterioration characteristics as well as lower-level quality characteristics, regardless of a nonlinear relationship between the higher-level characteristic and the lower-level characteristic. The illustrations given in Chapter 7 can convince us of the efficiency of the tolerance design.

§8.2. Recommendation For Further Research

In practice, many systems have several quality characteristics rather than a single characteristic. Under certain circumstances, the quality of such systems may be evaluated by measuring these characteristics separately. However, if the optimization of these characteristics conflicts with each other or if they are not independent of each other, the multiple objective decision making (Bell *et al* 1977, Hwang and Yoon 1981) may be used to determine the best levels setting of the control factors. We have proposed the multivariate loss function (MLF) as a quality measure that combines the multiple attributes into a single criterion. If a suitable MLF can be developed, it is possible to use the MLF as a substitute for the optimization criteria. Unfortunately, a systematic approach is not available. Anyway, it seems to be valuable to investigate the application of the multiple decision making in the optimization of such systems. That may make a contribution to the extension of robust design for more realistic and sophisticated systems.

Although We have investigated the optimization criteria for dynamic systems, the optimization criterion for static systems is also very important and worth further research. If a criterion is not efficient, the optimization model as well as the method is meaningless.

The optimization model (6.27) for tolerance design can be solved by either geometric programming or nonlinear programming. However, due to the structure of the model, it is possible to develop a simpler algorithm to solve the model. Also, this is a worthwhile further research.

APPENDIX A

If a random variable Y has a normal distribution, then we have

$$\int_{y \le y_0} y f(y) dy = \int_{-\infty}^{y_0} \frac{y}{\sqrt{2\pi}\,\sigma_y} e^{-\frac{(y-\mu_y)^2}{2\sigma_y^2}} dy$$

$$= \int_{-\infty}^{y_0} \frac{(y-\mu_y)}{\sqrt{2\pi}\,\sigma_y} e^{-\frac{(y-\mu_y)^2}{2\sigma_y^2}} dy + \int_{-\infty}^{y_0} \frac{\mu_y}{\sqrt{2\pi}\,\sigma_y} e^{-\frac{(y-\mu_y)^2}{2\sigma_y^2}} dy$$

$$= -\frac{\sigma_y^2}{\sqrt{2\pi}\,\sigma_y} e^{-\frac{(y_0-\mu_y)^2}{2\sigma_y^2}} + \mu_y F(y_0) = -\sigma_y^2 f(y_0) + \mu_y F(y_0) \qquad \text{(A. 1)}$$

Thus, the expected quality loss given by eq. (2.13) for the linear loss function can be simplified as follows:

$$E[L(Y, y_0)] = -(K_1 + K_2) \int_{y \le y_0} y f(y) dy + K_2 \mu_y$$

$$= -(K_1 + K_2)\left[-\sigma_y^2 f(y_0) + \mu_y F(y_0)\right] + K_2 \mu_y$$

$$= (K_1 + K_2)\sigma_y^2 f(y_0) \qquad \text{(A.2)}$$

where f(y) is the probability density function of Y and F(y) is the cumulative distribution function of Y.

APPENDIX B

The objective of this appendix is to verify the conclusion that positive skewness γ_1 and small kurtosis γ_2 can reduce the expected quality loss for "the larger the better" QLF, if μ_y and σ_y remain unchanged. For simplicity, a few discrete examples are used as the numerical illustration of this.

B.1. Discrete Case One

Random variables Y_1, Y_2 have the probability density functions $f_1(y_1)$ and $f_2(y_2)$ respectively (Figure B.1). They have the same values of the mean and the variance. However, Y_1 has a γ_1=0.579 but Y_2 has a γ_1=-0.579. According to eq. (2.2), the expected quality loss for Y_1 is given by

$$E[L(Y_1)] = K\left(\frac{0.5}{7^2} + \frac{0.3}{8^2} + \frac{0.2}{9^2}\right) = 0.01736K$$

The expected quality loss for Y_2 is given by

$$E[L(Y_2)] = K\left(\frac{0.2}{6.4^2} + \frac{0.3}{7.4^2} + \frac{0.5}{8.4^2}\right) = 0.01745K$$

Thus, $E[L(Y_1)]<E[L(Y_2)]$. The conclusion for this example is correct.

B.2. Discrete Case Two

Similarly, if two random variables Y_1, Y_2 have the distributions with the same values of the mean and the variance as indicated in Figure B.2 , the expected values of the quality loss for them in terms of eq. (2.2) are given respectively as follows:

$$E[L(Y_1)] = K\left(\frac{0.3}{3^2} + \frac{0.4}{4^2} + \frac{0.2}{5^2} + \frac{0.1}{6^2}\right) = 0.0691K$$

$$E[L(Y_2)] = K\left(\frac{0.1}{2.2^2} + \frac{0.2}{3.2^2} + \frac{0.4}{4.2^2} + \frac{0.3}{5.2^2}\right) = 0.074K$$

Thus, $E[L(Y_1)] < E[L(Y_2)]$. The conclusion for this example is correct.

B.3. Discrete Case Three

If two random variables Y_1, Y_2 have the distributions with the same values of the mean and the variance as indicated in Figure B.3, the expected values of quality loss for them are given respectively as follows:

$$E[L(Y_1)] = K\left(\frac{0.2}{2^2} + \frac{0.3}{3^2} + \frac{0.2}{4^2} + \frac{0.2}{5^2} + \frac{0.1}{6^2}\right) = 0.1066K$$

$$E[L(Y_2)] = K\left(\frac{0.1}{1.4^2} + \frac{0.2}{2.4^2} + \frac{0.2}{3.4^2} + \frac{0.3}{4.4^2} + \frac{0.2}{5.4^2}\right) = 0.1254K$$

Thus, $E[L(Y_1)] < E[L(Y_2)]$. The conclusion for this example is correct.

B.4. Discrete Case Four

In this example, we want to verify that the smaller γ_2 can reduce the expected quality loss. Assume that two random variables Y_1 and Y_2 have the discrete distribution functions as in Figure B.4 (a) and Figure B.4 (b) respectively. They have the same values of the mean and the variance. However, Y_1 has a γ_2=1.167, but Y_2 has a γ_2=-1.5. The expected values of the quality loss for them are given as follows:

$$E[L(Y_1)] = K\left(\frac{0.12}{7^2} + \frac{0.76}{8^2} + \frac{0.12}{9^2}\right) = 0.0158055K$$

$$E[L(Y_2)] = K\left(\frac{(1/3)}{7.4^2} + \frac{(1/3)}{8^2} + \frac{(1/3)}{8.6^2}\right) = 0.0158024K$$

Thus, $E[L(Y_1)]>E[L(Y_2)]$. The conclusion for this example is correct or the smaller γ_2 has a smaller expected quality loss if other parameters of the distribution remain unchanged.

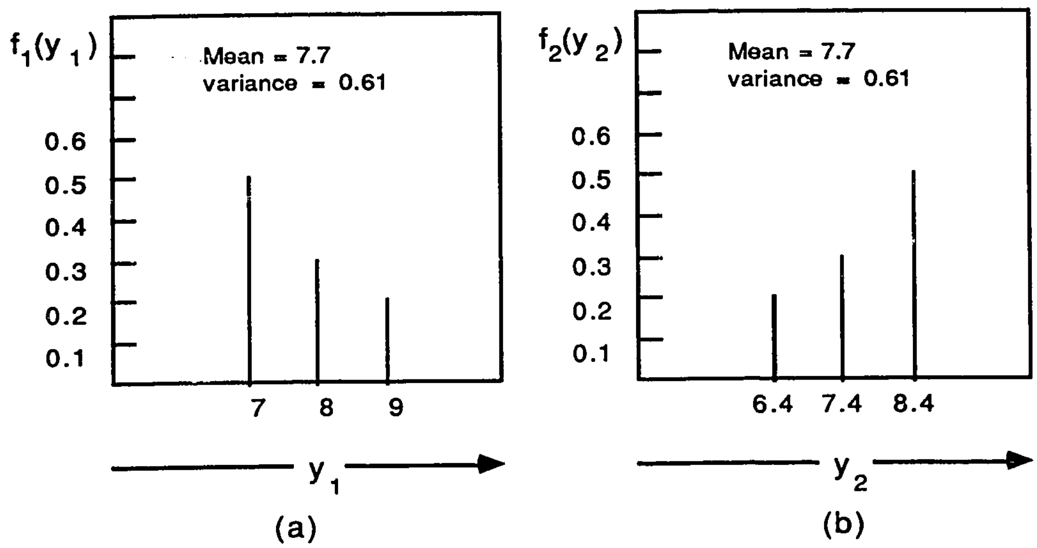


Figure B.1. *Distributions of discrete random variables Y_1 and Y_2.*
(a) Positive skewness, $\gamma_1 = 0.579$.
(b) Negative skewness, $\gamma_1 = -0.579$.

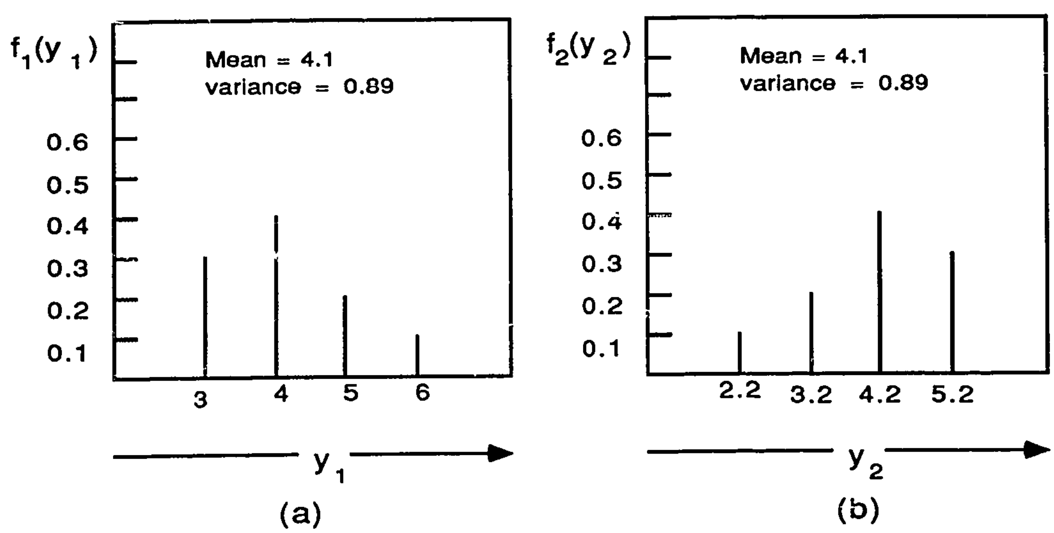


Figure B.2. *Distributions of discrete random variables Y_1 and Y_2.*
(a) Positive skewness, $\gamma_1 = 0.5145$.
(b) Negative skewness, $\gamma_1 = -0.5145$.

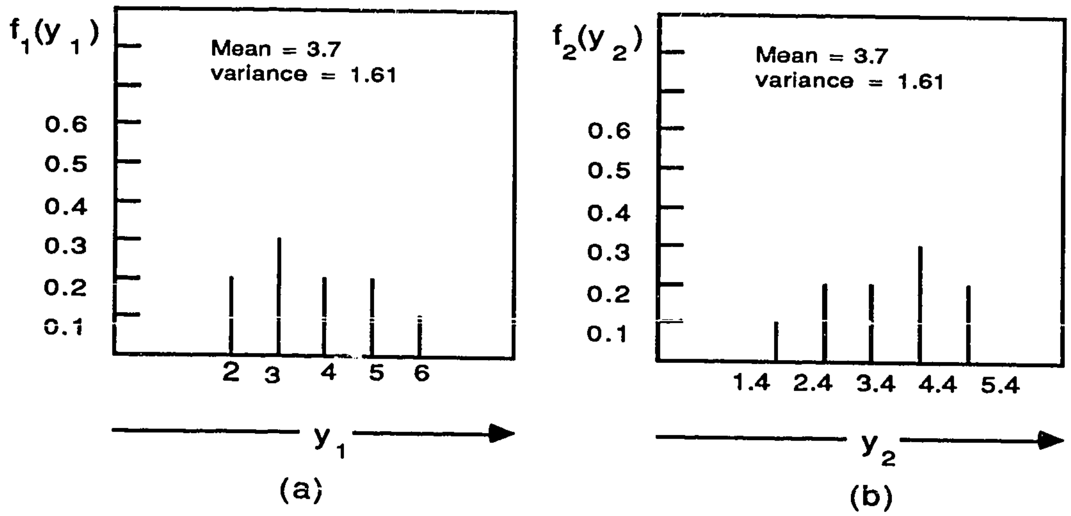


Figure B.3. *Distributions of discrete random variables Y_1 and Y_2.*
(a) Positive skewness, $\gamma_1 = 0.282$.
(b) Negative skewness, $\gamma_1 = -0.282$.

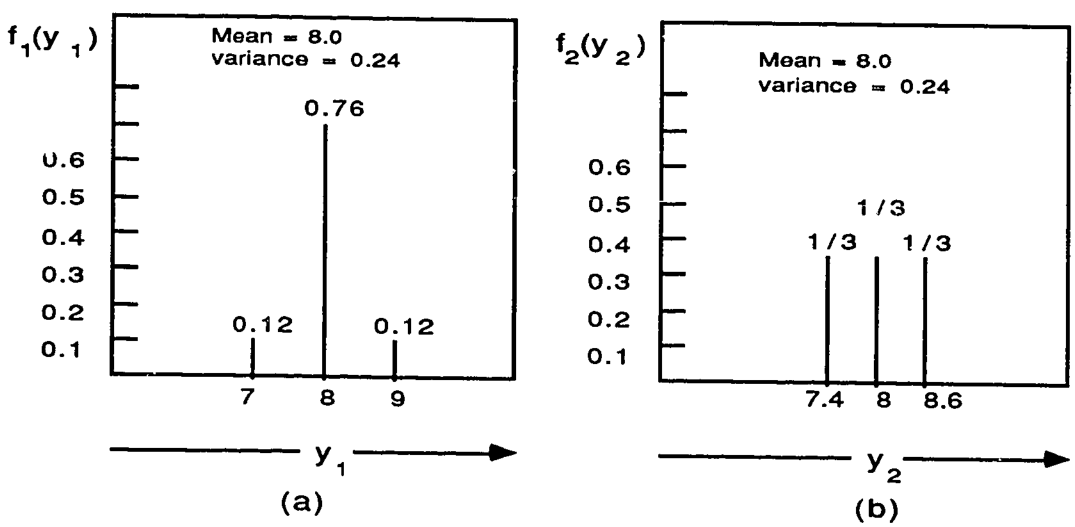


Figure B.4. *Distributions of discrete random variables Y_1 and Y_2.*
(a) Large kurtosis, $\gamma_2 = 1.167$.
(b) Small kurtosis, $\gamma_2 = -1.500$.

APPENDIX C

We want to show that the expected value of $\hat{\beta}^2$ is not equal to β^2. Since

$$\hat{\beta} = \frac{\sum_{i=1}^{k} \sum_{j=1}^{r_i} Y_{ij}(M_i - \overline{M})}{\sum_{i=1}^{k} r_i (M_i - \overline{M})^2}$$

by substituting eq. (4.3) into the above equation, we have

$$\hat{\beta}^2 = \left\{ \frac{\sum_{i=1}^{k} \sum_{j=1}^{r_i} \left(m + \beta(M_i - \overline{M}) + \varepsilon_{ij} \right) \left(M_i - \overline{M} \right)}{\sum_{i=1}^{k} r_i (M_i - \overline{M})^2} \right\}^2$$

$$= \beta^2 + \frac{\left[\sum_{i=1}^{k} (M_i - \overline{M}) \sum_{j=1}^{r_i} \varepsilon_{ij} \right]^2}{\left[\sum_{i=1}^{k} r_i (M_i - \overline{M})^2 \right]^2} + 2\beta \frac{\sum_{i=1}^{k} (M_i - \overline{M}) \sum_{j=1}^{r_i} \varepsilon_{ij}}{\left[\sum_{i=1}^{k} r_i (M_i - \overline{M})^2 \right]^2} \tag{C.1}$$

Taking the expected value of eq. (C.1), we have

$$E\left[\hat{\beta}^2\right] = \beta^2 + \frac{\sigma_\varepsilon^2}{\sum_{i=1}^{k} r_i (M_i - \overline{M})^2} = \beta^2 + \sigma_\varepsilon^2 / r \tag{C.2}$$

Since $MS_\beta = r\hat{\beta}^2$, we have

$$E[MS_\beta] = r\beta^2 + \sigma_\varepsilon^2 \tag{C.3}$$

Let $\bar{\varepsilon}_{\bullet\bullet} = \dfrac{\sum_{i=1}^{k} \sum_{j=1}^{r_i} \varepsilon_{ij}}{\sum_{i=1}^{k} r_i}$. Thus, $E\left[(\bar{\varepsilon}_{\bullet\bullet})^2\right] = \sigma_\varepsilon^2 / \sum_{i=1}^{k} r_i$ and $E\left[\varepsilon_{ij}\,\bar{\varepsilon}_{\bullet\bullet}\right] = \sigma_\varepsilon^2 / \sum_{i=1}^{k} r_i$.

Thus, the expected value of the total sum of squares is given by

$$E\left[SS_T\right] = E\left[\sum_{i=1}^{k} \sum_{j=1}^{r_i} (Y_{ij} - \bar{Y}_{\bullet\bullet})^2\right]$$

$$= E\left[\sum_{i=1}^{k} \sum_{j=1}^{r_i} \left(\beta(M_i - \bar{M}) + \varepsilon_{ij} - \bar{\varepsilon}_{\bullet\bullet}\right)^2\right]$$

$$= \beta^2 \sum_{i=1}^{k} r_i (M_i - \bar{M})^2 + \sigma_\varepsilon^2 \left(\sum_{i=1}^{k} r_i - 1\right) \qquad \text{(C.4)}$$

and

$$E[SS_e] = E\left[SS_T\right] - E\left[SS_\beta\right] = \sigma_\varepsilon^2 \left(\sum_{i=1}^{k} r_i - 2\right) \qquad \text{(C.5)}$$

or $E[MS_e] = \sigma_\varepsilon{}^2$. It should be noted that the degree of freedom for SS_e is

$\sum_{i=1}^{k} r_i - 2$ and $r = \sum_{i=1}^{k} r_i (M_i - \bar{M})^2$.

APPENDIX D

Assume that a model is given by

$$Y_{ij} = \mu_i + \varepsilon_{ij} \qquad (i=1, 2, \cdots m;\ j=1, 2, \cdots n) \qquad (D.1)$$

where μ_i is the mean of Y_{ij} (for any j) and ε_{ij} is an i. i. d. random error that is assumed to have a normal distribution with a mean zero and a variance σ_ε^2. We want to show that the sum of squares for a linear contrast L is given by the following equation:

$$SS_L = L^2/(nD) \qquad (D.2)$$

$L = \sum_{i=1}^{m} C_i Y_{i\bullet}$ is a Linear contrast. C_i is the coefficient of this linear contrast where $\sum_{i=1}^{m} C_i = 0$, $Y_{i\bullet} = \sum_{j=1}^{n} Y_{ij}$ and $D = \sum_{i=1}^{m} C_i^2$.

Obviously, L has a normal distribution with a mean of $n(C_1\mu_1 + \cdots + C_m\mu_m)$ and a variance of $nD\sigma_\varepsilon^2$. If $(C_1\mu_1 + \cdots + C_m\mu_m)=0$ or this linear contrast is insignificant, then L has a mean of zero. Consequently, $L^2/(nD\sigma_\varepsilon^2)=SS_L/\sigma_\varepsilon^2=MS_L/\sigma_\varepsilon^2$ has a Chi-square distribution with one degree of freedom. While, since $MS_e/\sigma_\varepsilon^2$ has a Chi-square distribution with a degree of freedom f_e (f_e is the degree of freedom for SS_e), then

$F_0=MS_L/MS_e$ is a F statistic that has a distribution of $F(1, f_e)$.

If $(C_1\mu_1+ \cdots +C_m\mu_m)\neq 0$ or L is a significant linear contrast, the expected value of MS_L can be found by

$$E[MS_L] = E\left[\frac{L^2}{nD}\right] = E\left[\frac{\left(n\sum_{i=1}^{m} C_i\mu_i + \sum_{i=1}^{m}\sum_{j=1}^{n} C_i\varepsilon_{ij}\right)^2}{nD}\right]$$

$$= \frac{\left(n\sum_{i=1}^{m} C_i\mu_i\right)^2}{nD} + \sigma_\varepsilon^2 \qquad \text{(D.3)}$$

In contrast, $E[MS_e]=\sigma_\varepsilon{}^2$. As a result, $F_0= MS_L/MS_e$ tends to be greater than one. Intuitively it can be observed that the more significant L is or the larger $(C_1\mu_1+ \cdots +C_m\mu_m)$ is, the greater F_0 tends to be. Hence, F_0 can be used to do F test for this linear contrast L and the sum of squares for L is given by eq. (D.2).

APPENDIX E

For very small x, $1/(1+x) \approx 1-x$. Since a manufacturing deviation is much smaller than the nominal value of a resistor such as e_1/R_1, e_2/R_2 or e_a/R_a is much smaller than one, the actual response variable for model (4.14) can be simplified. By considering the noise factors associated with the resistors, we have the actual response value of the output voltage that is given as follows:

$$V_o = \left[1 + \frac{R}{R_a + e_a}\right]\left[\frac{R_2 + e_2}{(R_1 + R_2) + (e_1 + e_2)}\right]V_i$$

$$\approx \left[1 + \frac{R}{R_a}\left(1 - \frac{e_a}{R_a}\right)\right]\left[\frac{R_2}{R_1 + R_2}\right]\left[1 + \frac{e_2}{R_2}\right]\left[1 - \frac{e_1 + e_2}{R_1 + R_2}\right]V_i \qquad \text{(E.1)}$$

where R_1, R_2 or R_a is the nominal value of the resistor and e_1, e_2 or e_a is the respective deviation from the nominal value.

Eq. (E.1) can be expanded and simplified as a linear dynamic model. Since the product term of two or more ratios of the deviation to the nominal value [such as $(e_1/R_1)(e_2/R_2)$] is much smaller than the other terms, these product terms can be ignored. Hence, eq. (E.1) can be simplified as follows:

$$V_o = \left(1+\frac{R}{R_a}\right)\left(\frac{R_2}{R_1+R_2}\right)V_i + \left(\frac{R_2}{R_1+R_2}\right)\left[\left(1+\frac{R}{R_a}\right)\left(\frac{e_2}{R_2}-\frac{e_1+e_2}{R_1+R_2}\right)-\frac{Re_a}{R_a^2}\right]V_i$$

$$= \beta V_i + \varepsilon(V_i, e_1, e_2, e_a) \qquad \text{(E.2)}$$

where ε is not independent of the signal factor V_i. In addition, the variance of ε is given by

$$Var[\varepsilon] = \left(\frac{R_2}{R_1+R_2}\right)^2\left[\left(1+\frac{R}{R_a}\right)^2\left(\frac{\sigma^2_{e_2}}{R_2^2}+\frac{\sigma^2_{e_1}+\sigma^2_{e_2}}{(R_1+R_2)^2}-\frac{2\sigma^2_{e_2}}{R_2(R_1+R_2)}\right)+\frac{R^2\sigma^2_{e_a}}{R_a^4}\right]$$

$$x\left(\mu^2_{V_i}+\sigma^2_{V_i}\right) \qquad \text{(E.3)}$$

where μ_{Vi} and $\sigma_{Vi}{}^2$ are the mean value and the variance of V_i.

Since R/R_a is much larger than one, the SN ratio is given by

$$\eta = \frac{\beta^2}{Var[\varepsilon]}$$

$$\approx \left[\frac{\sigma^2_{e_1}}{(R_1+R_2)^2}+\frac{R_1^2\sigma^2_{e_2}}{R_2^2(R_1+R_2)^2}+\frac{\sigma^2_{e_a}}{R_a^2}\right]^{-1}\left(\mu^2_{V_i}+\sigma^2_{V_i}\right)^{-1} \qquad \text{(E.4)}$$

which is independent of adjustable factor R. Adjusting R has no effect on η. In addition, for any levels setting of R_1, R_2 and R_a, $(\mu_{Vi}{}^2+\sigma_{Vi}{}^2)$ is a constant value. It has no effect on the selection of the levels of R_1, R_2 as well as R_a.

APPENDIX F

To derive eq. (6.15), we substitute eq. (6.12) into eq. (6.14) and thus, we have

$$SS_{A_1} = \left[\sum_{j=1}^{3}\left(Y_{3j\cdot} - Y_{1j\cdot}\right)\right]^2 / 6n = \Big[3n\Big(k_{A_1}(A_3 - A_1) + k_{A_q}(A_3^2 - A_1^2)\Big) +$$

$$n(A_3 - A_1)\sum_{j=1}^{3}(k_{1x1}B_j + k_{1xq}B_j^2) + n(A_3^2 - A_1^2)\sum_{j=1}^{3}(k_{qx1}B_j + k_{qxq}B_j^2)$$

$$+ \sum_{j=1}^{3}\sum_{k=1}^{n}(\varepsilon_{3jk} - \varepsilon_{1jk})\Big]^2 / 6n \qquad \text{(F.1)}$$

It is observed by considering eq. (6.6) that

$$\begin{cases} A_3 - A_1 = 2h\sigma_A \\ A_3^2 - A_1^2 = 4h\mu_A\sigma_A \\ \sum_{j=1}^{3} B_j = 3\mu_B \text{ and } \sum_{j=1}^{3} B_j^2 = 3\mu_B^2 + 2h^2\sigma_B^2 \end{cases}$$

Substituting these equations into eq. (F.1) and taking the expectation, we have

$$E\left[SS_{A_1}\right] = 6nh^2\left[k_{A_1} + 2k_{A_q}\mu_A + k_{lxl}\mu_B + 2k_{qxl}\mu_A\mu_B + (k_{lxq} + 2k_{qxq}\mu_A)\right.$$

$$\left.\times\left(\mu_B^2 + \frac{2h^2\sigma_B^2}{3}\right)\right]^2 \sigma_A^2 + E\left[\left(\sum_{j=1}^{3}\sum_{k=1}^{n}(\varepsilon_{3jk} - \varepsilon_{1jk})\right)^2 / 6n\right]$$

$$= F_{A_1}\sigma_A^2 + \sigma_\varepsilon^2 \qquad \text{(F.2)}$$

Similarly, it can be shown that eq. (6.16) is true.

To derive eq. (6.18), we substitute eq. (6.12) into eq. (6.17) and thus, we have

$$SS_{A_q} = \left[\sum_{j=1}^{3}\left(Y_{3j\cdot} + Y_{1j\cdot} - 2Y_{2j\cdot}\right)\right]^2 / 18n$$

$$= \left[3n\left(k_{A_1}(A_3 + A_1 - 2A_2) + k_{A_q}(A_3^2 + A_1^2 - 2A_2^2)\right)\right.$$

$$+ n(A_3 + A_1 - 2A_2)\sum_{j=1}^{3}(k_{lxl}B_j + k_{lxq}B_j^2) + n(A_3^2 + A_1^2 - 2A_2^2)$$

$$\left.\times\sum_{j=1}^{3}(k_{qxl}B_j + k_{qxq}B_j^2) + \sum_{j=1}^{3}\sum_{k=1}^{n}(\varepsilon_{3jk} + \varepsilon_{1jk} - 2\varepsilon_{2jk})\right]^2 / 18n \qquad \text{(F.3)}$$

It is observed by considering eq. (6.6) that

$$\begin{cases} A_3 + A_1 - 2A_2 = 0 \\ A_3^2 + A_1^2 - 2A_2^2 = 2h^2\sigma_A^2 \end{cases}$$

As a result, by taking the expected value of eq. (F.3), we have

$$E\left[SS_{A_q}\right] = 2n\,h^4\left[k_{A_q} + k_{qxl}\mu_B + k_{qxq}\left(\mu_B^2 + \frac{2h^2\sigma_B^2}{3}\right)\right]^2 \sigma_A^4$$

$$+ E\left[\left(\sum_{j=1}^{3}\sum_{k=1}^{n}(\varepsilon_{3jk} + \varepsilon_{1jk} - 2\varepsilon_{2jk})\right)^2 / 18n\right] = F_{A_q}\sigma_A^4 + \sigma_\varepsilon^2 \qquad \text{(F.4)}$$

Similarly, it can be shown that eq. (6.19) is true.

To derive eq. (6.21), we substitute eq. (6.12) into eq. (6.20) and thus, we have

$$SS_{lxl} = \left[Y_{11\bullet} + Y_{33\bullet} - Y_{13\bullet} - Y_{31\bullet}\right]^2 / 4n$$

$$= \left[n(A_3 - A_1)\left(k_{lxl}(B_3 - B_1) + k_{lxq}(B_3^2 - B_1^2)\right) + n\,k_{qxl}(A_3^2 - A_1^2)(B_3 - B_1)\right.$$

$$\left. + n\,k_{qxq}(A_3^2 - A_1^2)(B_3^2 - B_1^2) + \sum_{k=1}^{n}(\varepsilon_{11k} + \varepsilon_{33k} - \varepsilon_{13k} - \varepsilon_{31k})\right]^2 / 4n \qquad \text{(F.5)}$$

It is observed by considering eq. (6.6) that

$$\begin{cases} (A_3 - A_1)(B_3 - B_1) = 4h^2\sigma_A\sigma_B \\ (A_3 - A_1)(B_3^2 - B_1^2) = 8h^2\mu_B\sigma_A\sigma_B \\ (A_3^2 - A_1^2)(B_3 - B_1) = 8h^2\mu_A\sigma_A\sigma_B \\ (A_3^2 - A_1^2)(B_3^2 - B_1^2) = 16h^2\mu_A\mu_B\sigma_A\sigma_B \end{cases}$$

As a result, by taking the expected value of eq. (F.5), we have

$$E\left[SS_{lxl}\right] = 4nh^4\left[k_{lxl} + 2k_{lxq}\mu_B + 2k_{qxl}\mu_A + 4k_{qxq}\mu_A\mu_B\right]^2\sigma_A^2\sigma_B^2$$

$$+ E\left[\left(\sum_{k=1}^{n}(\varepsilon_{11k} + \varepsilon_{33k} - \varepsilon_{13k} - \varepsilon_{31k})\right)^2 / 4n\right] = F_{lxl}\sigma_A^2\sigma_B^2 + \sigma_\varepsilon^2 \qquad \text{(F.6)}$$

To derive eq. (6.22), we substitute eq. (6.12) into SS_{lxq} and thus, we have

$$SS_{lxq} = \left[(2Y_{32\bullet} - Y_{31\bullet} - Y_{33\bullet}) - (2Y_{12\bullet} - Y_{11\bullet} - Y_{13\bullet})\right]^2 / 12n$$

$$= \left[n\left(k_{lxl}(A_3 - A_1) + k_{qxl}(A_3^2 - A_1^2)\right)(2B_2 - B_1 - B_3)\right.$$

$$+ n\left(k_{lxq}(A_3 - A_1) + k_{qxq}(A_3^2 - A_1^2)\right)(2B_2^2 - B_1^2 - B_3^2)$$

$$\left. + \sum_{k=1}^{n}(2\varepsilon_{32k} - \varepsilon_{31k} - \varepsilon_{33k} + \varepsilon_{11k} + \varepsilon_{13k} - 2\varepsilon_{12k})\right]^2 / 12n \qquad \text{(F.7)}$$

It is observed by considering eq. (6.6) that

$$\begin{cases} 2B_2 - B_1 - B_3 = 0 \\ 2B_2^2 - B_1^2 - B_3^2 = -2h^2\sigma_B^2 \end{cases}$$

As a result, by taking the expected value of eq. (F.7), we have

$$E\left[SS_{lxq}\right] = (4nh^6/3)\left[k_{lxq} + 2k_{qxq}\mu_A\right]^2 \sigma_A^2\sigma_B^4 + \sigma_\varepsilon^2$$

$$= F_{lxq}\sigma_A^2\sigma_B^4 + \sigma_\varepsilon^2 \qquad \text{(F.8)}$$

Similarly, it can be shown that eq. (6.23) is true.

To derive eq. (6.24), we substitute eq. (6.12) into SS_{qxq} and thus, we have

$$SS_{qxq} = \frac{\left[Y_{11\cdot} - 2Y_{12\cdot} + Y_{13\cdot} - 2Y_{21\cdot} + 4Y_{22\cdot} - 2Y_{23\cdot} + Y_{31\cdot} - 2Y_{32\cdot} + Y_{33\cdot}\right]^2}{36n}$$

$$= \Big[nk_{qxq}(2A_2^2 - A_3^2 - A_1^2)(2B_2^2 - B_1^2 - B_3^2) + \sum_{k=1}^{n}(\varepsilon_{11k} - 2\varepsilon_{12k} + \varepsilon_{13k} - 2\varepsilon_{21k}$$

$$+ 4\varepsilon_{22k} - 2\varepsilon_{23k} + \varepsilon_{31k} - 2\varepsilon_{32k} + \varepsilon_{33k})\Big]^2 / 36n \qquad \text{(F.9)}$$

It is observed by considering eq. (6.6) that

$$\begin{cases} 2A_2^2 - A_1^2 - A_3^2 = -2h^2\sigma_A^2 \\ 2B_2^2 - B_1^2 - B_3^2 = -2h^2\sigma_B^2 \end{cases}$$

As a result, by taking the expected value of eq. (F.9), we have

$$E\left[SS_{qxq}\right] = (4nh^8/9)\, k_{qxq}^2\sigma_A^4\sigma_B^4 + \sigma_\varepsilon^2$$

$$= F_{qxq}\sigma_A^2\sigma_B^4 + \sigma_\varepsilon^2 \qquad \text{(F.10)}$$

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ABSTRACT

PRODUCT AND PROCESS DESIGN OPTIMIZATION BY QUALITY ENGINEERING

by
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This research is concerned with product and process design optimization by quality engineering based on the work of Dr Taguchi, with emphasis on the optimization of dynamic systems and tolerance design. Various quality loss functions are presented in this thesis which can be used for quality evaluation. The goal of robust design for dynamic systems is to reduce the deviations of quality characteristics for the real system from an ideal target which can change based on the requirements of the customer. The optimization can be simplified by decomposing the selection of control factors **Z** and the adjustment of scaling/leveling factors **R** into a two-step procedure. The first step is selecting levels for factors **Z** to maximize the signal-to-noise (SN) ratio that is supposed to be independent of

the adjustment of factors **R**. The second step is used to adjust the real system to a desired model. A systematic approach to optimization is provided for dynamic systems. The motivation of the SN ratio is given and the validity of the SN ratio is examined for various systems. However, for the specific models where the use of the SN ratio is questionable, the necessary modification is suggested. In addition, discrete dynamic characteristics are discussed. The objective of tolerance design is to balance quality loss due to variations and cost increase due to control of variations. Based on the variation transmission equation developed in this thesis, the best tolerance levels are specified for components and subsystems. The tolerance design approach is presented for quality characteristics which may deteriorate over time. Also, a method is presented to develop the tolerances for lower-level quality characteristics based on the tolerances for higher-level quality characteristics, to reflect the voice of the customer. Illustrations are given to demonstrate the efficiency of the tolerance design methodology.

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"Quality evaluation system using loss function", *Proceedings of 1989 International Industrial Engineering Conference*, Toronto, Canada, May 1989.

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