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Reliability Analysis And Optimal Maintenance Planning For Repairable Multi-Component Systems Subject To Dependent Competing Risks

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RELIABILITY ANALYSIS AND OPTIMAL MAINTENANCE PLANNING FOR REPAIRABLE MULTI-COMPONENT SYSTEMS SUBJECT TO DEPENDENT COMPETING RISKS

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DEDICATION

To my family

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CHAPTER 1. INTRODUCTION

1.1 Background

Reliability analysis of multi-component repairable systems plays a critical role for system safety and cost reduction. A variety of multi-component systems are subject to competing risks (David and Moeschberger 1978, Meeker and Escobar 1998, Crowder 2010, Hong and Meeker 2014). Under competing risks, only the failure with the smallest latent failure time can be observed. Competing risks theory seeks to provide the statistical relations between all the latent failure times associated with different failure types that cannot be observed directly (Bedford and Alkali 2009). In competing risks systems, different risks can be dependent. For example, consider a vehicle's transmission system in which transmission fluid is used to lubricate the moving parts. The wearing out of the transmission fluid can cause both the clutches and the gears to deteriorate significantly. And failure of either the clutch or the gear can cause the failure of the transmission system. Thus, the clutches and the gears suffer from dependent competing risks. In general, ignoring the failure dependency of multiple components can result in biased predictions of system reliability and non-optimality of the maintenance policy. In addition, in most cases, only the failed component (e.g., either the failed clutch or the failed gear) is repaired, and the repair can be general, including perfect replacement and minimal repair, as well as the situations in between.

There are three prominent aspects which pose as major challenges of building a reliability model and developing maintenance planning for dependent competing risks systems with imperfect repair. First, when the system fails, the failure time data of the un-failed components are right-censored. In other words, only the failure time of the failed component is recorded, while the latent failure times of all the other components cannot be observed due to competing risks. Second, when considering the dependency of competing risks, one component's failure and repair will influence other components' lifetimes. The influence of an imperfect component repair is more complex than a perfect repair that restores the component as good as new. Third, when considering dependent competing risks and general repairs, complex optimization problems arise for new maintenance policies.

1.2 Literature Review

Traditional study on repairable systems mainly focuses on reliability modes for repairable systems with a single component under different repair actions. Kijima and Sumita (1986) and Kijima (1989) suggested two imperfect repair models by introducing the concept of virtual age of repairable systems. Lindqvist, Elvebakk, et al. (2003) proposed a Trend-renewal Process (TRP) to generalize the inhomogeneous and modulated gamma process proposed by Berman (Berman 1981), which can deal with the imperfect repair conditions well. Other imperfect repair models for repairable systems with a single component include the modulated renewal process (Cox 1972), the modulated power law process (Lakey and Rigdon 1992), the arithmetic reduction of age and arithmetic reduction of intensity models (Doyen and Gaudoin 2004), the stochastic general repair model (Guo, Haitao, et al. 2007). A comprehensive review on statistical methods of repairable systems is provided by Lindqvist (2006).

For repairable systems under competing risks, most of the existing research assumes independency of component failure (Pham and Wang 2000, Langseth and Lindqvist 2006, Wang, Chu, et al. 2009, Yang and Chen 2009, Hong and Meeker 2010, Yang and Chen 2010, 2011, Yang, Hong, et al. 2012). Thus, the reliability analysis of the entire system subjected to competing risks can be simplified by analyzing each component independently. The existing reliability models that consider failure dependency assume that when a failure of one component occurs, it will result in a possible shock to the other components with a certain probability (Jhang and Sheu 2000, Scarf and Deara 2002, Satow and Osaki 2003, Zequeira and Bérenguer 2005a, Barros, Berenguer, et al. 2006). Li and Pham (2005) discussed a similar system with component failure dependency, and they assumed a binomial distribution of perfect and minimal repairs with certain probability. Langseth and Lindqvist (2003) developed a model for systems consisting of multiple components associated with failures caused by multiple sources. Shaked and Shanthikumar (1986) developed statistical models and investigated properties of repairable systems with dependent component failures. However, in their work, the parameters estimation approach was not given and the repair actions were not considered.

In the past decades, maintenance study for multi-component system has attracted more and more attention. The main objective of maintenance is to retain or restore a system to perform its required functions satisfactorily. For simple system with a single component, there has been lots of maintenance models based on different assumptions (Lee and Rosenblatt 1989, Bunks, McCarthy, et al. 2000, Grall, Bérenguer, et al. 2002, Marseguerra, Zio, et al. 2002, Wang 2002, Aghezzaf, Jamali, et al. 2007, Peng, Feng, et al. 2011). The definition of multi-component maintenance is defined as: multi component maintenance models are concerned with optimal maintenance policies for a system consisting of several units of machines or many pieces of equipment, which may or may not depend on each other (Cho and Parlar 1991). If there is no dependency, then we can apply the single component maintenance policy on each component of the multi-component system separately. However, the dependency always is not negligible; for example, the down time of the system, which is shared among all components, will cause the economic dependency. When components form a system structurally so that the maintenance of failed component always involves maintenance of other components, it is called structural dependency (Nowakowski and Werbińka 2009). In addition, Murthy introduced three types of stochastic dependency considering failure interaction (Murthy and Nguyen 1985a, b).

Generally, the maintenance can be divided into two main types, i.e., corrective maintenance and preventive maintenance (Nowakowski and Werbińka 2009). Corrective maintenance means the failed component or system will be repaired perfectly immediately after the failure. Due to the limited maintenance resource in reality, immediate repair is difficult to implement, which is the big disadvantage of corrective maintenance. Preventative maintenance is used to describe the maintenance before failure occurs (Valdez-Flores and Feldman 1989). In the literature, block replacement policy is the most well-known preventive maintenance policy (Barlow and Hunter 1960). In such a policy, the components are commonly replaced on periodical intervals or failures (Berg and Epstein 1976). There are various modifications of block replacement policy (Tango 1978, Nakagawa 1986). Scarf and Deara (2002) proposed various block replacement policies considering type I failure interaction as the stochastic dependency, i.e., either component's failure can induce the other's failure in a two-component system. The major drawback of block replacement policy is the waste of components or system replacement even if sometimes it is not necessary. Inspection-based maintenance is another commonly used preventative maintenance model that has been studied intensively (Hosseini, Kerr, et al. 2000, Kallen and van Noortwijk 2005, Zequeira and Bérenguer 2005b, Wang 2009). Under the inspection maintenance, replacement can only be done after the detection of failures on inspections. Thus inspection maintenance policy can avoid the waste of unnecessary replacement in block replacement. Taghipour applied the periodic inspection maintenance for a multi-component system with non-competing risks (Taghipour and Banjevic 2011). Compared with non-competing risks system, the reliability modeling and maintenance planning is more complex as we cannot observe the full failure events due to competing risks. Few studies can be found on the maintenance planning of multi-component systems considering dependent competing risks.

1.3 Research Objectives

In this research, we focus on the repairable multi-component systems under competing risks. The dependency of different component failures is not clear thus we do not make any prior assumption whether different components are independent or not. The key objectives of this research are listed as follows:

1. To establish parametric reliability models to investigate the statistical dependency of multi-component systems under competing risks with imperfect repair.
2. To study optimal inspection-based maintenance planning for multi-component system under dependent competing risks.

1.4 Dissertation Organization

The dissertation consists of three main chapters, preceded by an introduction in the present chapter and followed by a conclusion. CHAPTER 2 presents a statistical model for multi-component repairable systems under dependent competing risks with partially perfect repair assumptions. CHAPTER 3 studies two statistical models considering generally imperfect repair and dependent competing risks for multi-component repairable systems. CHAPTER 4 presents two inspection-based maintenance policies based on the proposed reliability model.

CHAPTER 2. RELIABILITY ANALYSIS OF MULTI-COMPONENT SYSTEMS WITH DEPENDENT COMPETING RISKS UNDER PARTIALLY PERFECT REPAIR

2.1 Data Notation

We consider a competing-risk system consisting of multiple (say K) components. The time scale is the time since installation. Upon each failure, only the failed component is repaired as good as new and the other components are untouched, which is called partially perfect repair. In general, the partially perfect repair is achieved by replacing the failed component with a new one. The successive failure events are recorded by $T_1, T_2, \ldots$, until a predetermined ending time τ. In addition, each event is labeled with a failure type $\Delta_i \in \{0,1,\ldots,K\}$; where $\Delta_i = 0$ indicates there is no failure observed. We use pair (T_i, Δ_i) to represent failure information. An equivalent representation of the failure process is in terms of the marked point process $\{N_k(t); t \geq 0, k = 1,\ldots,K\}$; where k denotes failure type and $N_k(t)$ denotes the cumulative number of failures for component k until time t. We use $N(t) = \sum_{k=1}^{K} N_k(t)$ to denote the total number of failures regardless of failure type until time t. We assume that two failures cannot occur simultaneously, which is a common assumption for repairable systems in the literature. In addition, we assume the repair action is immediate and the repair time is ignored.

2.2 Statistical Modeling for Multiple Dependent Competing Risks under Partially Perfect Repair

In the classical latent failure times model (Prentice, Kalbfleisch, et al. 1978), a single-component system has multiple competing failure types, each of which can cause the system's failure. Each failure type has a failure time, but only the minimum can be observed due to competing risks. Because of unobservable nature, the failure times are also called conceptual or latent failure times, which are generally assumed to follow a joint distribution to capture the dependency of competing risks.

Consider a system that consists of K new components starting to work at time 0. Because the components are under competing risks, the system fails if any component fails, while the failure time of all the other components cannot be observed.

Let $r_k(t)$ be the most recent failure time of component k before time t. The running time of component k at time t since its last replacement, which is defined as age and is denoted as $a_k(t)$, can be calculated as $a_k(t) = t - r_k(t)$. Note that both $r_k(t)$ and $a_k(t)$ are defined as left-continuous functions. Thus, $r_k(t) = \lim_{x \to t^-} r_k(x)$ if a failure occurs at time t, and $r_k(t) = 0$ if no failure occurred by time t. Similarly, $a_k(t) = \lim_{x \to t^-} a_k(x)$.

We use $Z_{k,i}$ to denote the latent age of component k to the system's i^{th} failure. Thus, the random vector $\mathbf{Z}_i = [Z_{1,i}, ..., Z_{K,i}]^T$ represents the latent ages to the system's

i^{th} failure for all components. Due to competing risks, only the minimum of latent ages to failure can be observed. Similar to the classical latent failure times model, we also use a joint distribution F to model $\mathbf{Z}_i$ like the classical latent failure times model if the system is either new or perfectly repaired. The dependency of component failures is captured by the joint distribution F.

The next failure time of the system on or after time t is determined as the minimum value of $\{ r_k(t) + Z_{k,N(t^-)+1},\ k = 1,2,...,K \}$ ($N(t^-)$ denotes its left limit at time t) under the condition that $Z_{k,N(t^-)+1} > a_k(t)$, $k = 1,2,...,K$. Fig. 1 illustrates the first two failures in a competing risks system. Let vector $\mathbf{z}_1 = [z_{1,1},...,z_{K,1}]^T$ be the realization of the latent ages to the first system's failure $\mathbf{Z}_1$. The first failure is determined by the minimal value of $z_{k,1}$; $k = 1,...,K$. Suppose the first failure is due to component l and occurs at time t_1 (Fig. 1 left). Under partially perfect repair assumptions, only component l is replaced, and the most recent failure times are updated as $r_l(t) = t_1$, and $r_j(t) = 0,\ j \neq l$, $j = 1,...,K$. The second failure can be calculated as the minimum value of $\{z_{l,2} + t_1, z_{j,2}; j \neq l, j = 1,...,K\}$ (Fig. 1 right), where $\mathbf{z}_2 = [z_{1,2},...,z_{K,2}]^T$ is the realization of the latent ages to the second system's failure.

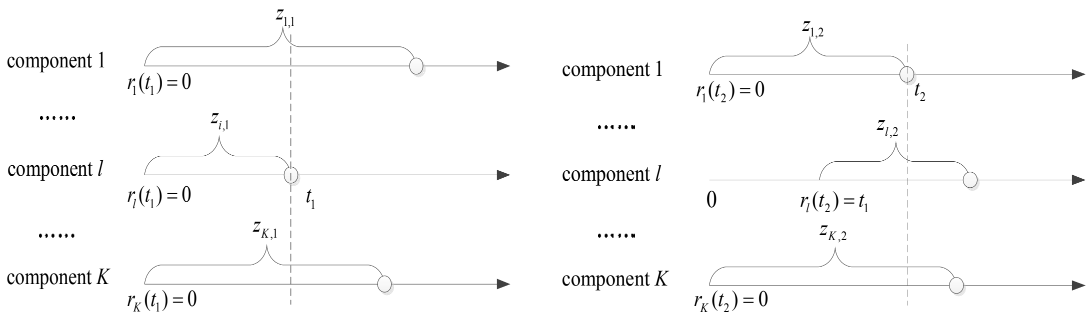

Fig. 1. Illustrations of the first (left), and the second (right) failures in a competing risks system

$Z_{k,i+1}$, the latent age of component k to $(i+1)^{th}$ system's failure, should be larger than the age of component k immediately after time t_i, i.e., $Z_{k,i+1} > a_k\left(t_i^+\right)$ ($a_k\left(t_i^+\right)$ denotes its right limit at time t_i), $\forall\ k \in \{1,...,K\}; i = 1,2,...$. As a result, the random vector $\mathbf{Z}_{i+1} = [Z_{1,i+1},...,Z_{K,i+1}]^T$ is following a truncated distribution of F conditional on the vector of $[a_1\left(t_i^+\right),\ldots,a_K\left(t_i^+\right)]^T$.

2.3 Parametric Forms

The joint distribution of the random vector $\mathbf{Z}_i$ describes the statistical failure mechanism of multiple components, and thus captures their statistical failure dependency. In this section, parametric models are proposed to characterize the joint distribution of $\mathbf{Z}_i$.

As Weibull and lognormal distributions are commonly used as failure time distributions for single-component systems (Barlow and Proschan 1975, Jordan 1978, Prabhakar Murthy, Bulmer, et al. 2004, Pascual, Meeker, et al. 2006), they are separately selected as the marginals of the joint distribution F to illustrate the proposed method in this research. However, other proper univariate distributions can also be applied.

2.3.1 Parametric Forms for Multivariate Lognormal Distribution

When the joint distribution of random vector $\mathbf{Z}_i$ is multivariate lognormal, the joint probability density function (pdf) is calculated as:

$$f(\mathbf{z}_i;\boldsymbol{\mu},\boldsymbol{\Sigma})=\frac{1}{(2\pi)^{K/2}\left|\boldsymbol{\Sigma}\right|^{1/2}}\exp\left\{-\frac{1}{2}\left[\log(\mathbf{z}_i)-\boldsymbol{\mu}\right]^T\boldsymbol{\Sigma}^{-1}\left[\log(\mathbf{z}_i)-\boldsymbol{\mu}\right]\right\} \quad . \tag{1}$$

The model parameters $\boldsymbol{\theta}$ include $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$. $\boldsymbol{\mu}\in\mathbf{R}^K$, and $\boldsymbol{\Sigma}\in\mathbf{R}^{K\times K}$ are the mean vector, and covariance matrix of the multivariate lognormal, respectively.

2.3.2 Parametric Forms for Multivariate Weibull Distribution via Archimedean Copula Function

The cumulative distribution function (cdf) of the Weibull marginal F_k is:

$$F_k(z_{k,i};\boldsymbol{\theta}_k)=1-\exp\left(-\left(\frac{z_{k,i}}{\lambda_k}\right)^{\kappa_k}\right); z_{k,i}\geq 0 \tag{2}$$

where $\kappa_k\in(0,\infty)$ and $\lambda_k\in(0,\infty)$ are called shape and scale parameter, respectively. $\boldsymbol{\theta}_k=[\kappa_k,\lambda_k]^T$ is the parameter vector of the marginal distribution F_k. We can construct the joint distribution from marginal Weibull by using Archimedean copula functions.

The Archimedean family of copulas are frequently used for the construction of multivariate distributions due to their simple forms (Nelsen 2006)

$$C(u_1,\ldots,u_K)=\psi\left[\psi^{-1}(u_1)+\cdots+\psi^{-1}(u_K)\right] \tag{3}$$

where ψ is the generator of the Archimedean copula. Different generators will generate different Archimedean copulas. For example, $\psi(t)=(1+t)^{-1/\lambda}$, and $\psi(t)=\exp(-t^{1/\lambda})$ are generators for Clayton, and Gumbel-Hougaard copulas, respectively.

In this research, the Clayton copula is selected as an example to illustrate the application of the Archimedean copula family in the proposed reliability model. Clayton copula contains one association parameter ρ that relates to the dependency measurement Kendall's tau $\tau_{Kendall}$ (Lindskog, McNeil, et al. 2003), by the relation $\tau_{Kendall}=\rho/(\rho+2)$ (Nelsen 2006).

When the Clayton copula is selected to construct the joint distribution, the dependency of the failure types is captured by the association parameter ρ. The range of the association parameter is $\rho\in[-1,0)\cup(0,\infty)$. The limiting case when $\rho\to 0$ represents the independent situation. In this research, we define the Clayton copula as follows:

$$C_{Clayton}(u_1,\ldots,u_K)=\begin{cases}\left[\max\left(\sum_{k=1}^{K}u_k^{-\rho}-K+1,0\right)\right]^{-1/\rho} & ;\rho\neq 0\\ \prod_{k=1}^{K}u_k & ;\rho=0\end{cases} \tag{4}$$

where $u_k=F_k(z_{k,i};\boldsymbol{\theta}_k);k=1,\ldots,K$. Specifically, the multivariate Weibull cdf $F(\mathbf{z}_i;\boldsymbol{\theta})$

can be obtained by substituting $u_k = 1 - \exp\left(-\left(z_{k,i} / \lambda_k\right)^{\kappa_k}\right)$ into (4), where $\boldsymbol{\theta} = \{\boldsymbol{\theta}_1, ..., \boldsymbol{\theta}_K, \rho\}$.

2.3.3 Parametric Forms for Multivariate Weibull Distribution via the Gaussian Copula Function

The Gaussian copula is a special copula taking advantage of the pdf of the multivariate normal distribution (Cherubini, Luciano, et al. 2004). Specifically, a Gaussian copula has the form

$$C_{Gauss}(u_1, \ldots, u_K) = \Phi_{\Sigma}[\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_K)] \tag{5}$$

where Φ^{-1} is the inverse of the cdf of the standard normal distribution, and Φ_{Σ} is the cdf of a multivariate normal distribution with zero mean vector, and its covariance matrix equals its correlation matrix. The Gaussian copula density function is given as (Song 2000)

$$c_{Gauss}(u_1, \ldots, u_K) = \frac{1}{|\boldsymbol{\Sigma}|^{1/2}} \exp\left\{ -\frac{1}{2} \begin{bmatrix} \Phi^{-1}(u_1) \\ \vdots \\ \Phi^{-1}(u_K) \end{bmatrix}^T (\boldsymbol{\Sigma}^{-1} - \mathbf{I}) \begin{bmatrix} \Phi^{-1}(u_1) \\ \vdots \\ \Phi^{-1}(u_K) \end{bmatrix} \right\} \tag{6}$$

where $\boldsymbol{\Sigma}$ is the correlation matrix, and $\mathbf{I}$ is the identity matrix.

When applying a Gaussian copula to construct the joint Weibull distribution, the survival function of random vector $\mathbf{Z}_i$ is obtained as

$$S(\mathbf{z}_i;\boldsymbol{\theta}) = \int_{z_{1,i}}^{\infty} \cdots \int_{z_{K,i}}^{\infty} f_{Gauss}(x_{1,i},\ldots,x_{K,i})dx_{1,i}\ldots dx_{K,i} \tag{7}$$

where $f_{Gauss}(\cdot)$ denotes the pdf of the joint Weibull distribution obtained by the chain rule, i.e.,

$$\begin{aligned} f_{Gauss}(z_{1,i},\ldots,z_{K,i}) &= \frac{\partial^K C_{Gauss}}{\partial u_1 \ldots \partial u_K} \frac{du_1}{dz_{1,i}} \cdots \frac{du_K}{dz_{K,i}} \\ &= c_{Gauss}(u_1,\ldots u_K) \cdot f_1(z_{1,i};\boldsymbol{\theta}_1)\ldots f_K(z_{K,i};\boldsymbol{\theta}_K) \end{aligned} \tag{8}$$

and $f_k(\cdot); k = 1,...,K$ denotes the pdf of the marginal for $\mathbf{Z}_{k,i}$.

In the multivariate Weibull distribution constructed via the Gaussian copula, the model parameter $\boldsymbol{\theta} = \{\boldsymbol{\theta}_1,...,\boldsymbol{\theta}_K,\boldsymbol{\Sigma}\}$.

2.4 Parameters Estimation Based on Maximum Likelihood Method

A maximum likelihood (ML) method is developed to estimate model parameters. To implement the ML approach, we first calculate the likelihood function. Suppose the i^{th} failure is due to component *k*, and occurs at time t_i. The latent age to i^{th} system failure of component k is equal to $a_k(t_i)$, while the latent ages of all other components should be larger than $a_j(t_i); j \neq k, j = 1,...,K$. Thus, the unconditional probability to observe failure i is calculated as

$$\begin{aligned} &\Pr(Z_{k,i} = a_k(t_i), Z_{j,i} > a_j(t_i); j \neq k, j = 1,...,K) \\ &= -\left.\frac{\partial S\left(z_{1,i},\ldots,z_{K,i}\right)}{\partial z_{k,i}}\right|_{\{z_{1,i}=a_1(t_i),\ldots,z_{k,i}=a_k(t_i),\ldots,z_{K,i}=a_K(t_i)\}} . \end{aligned} \tag{9}$$

Note that (10) gives the probability at a given time t for a continuous random variable. Although technically $\Pr(X=t)=0$ for a continuous random variable X with pdf $f(t)$, $\Pr(X=t)$ can be interpreted as $\Pr(t \le X \le t+dt) = f(t)dt$, which is proportional to $f(t)$. We ignore dt only for notational convenience in the calculation of the likelihood function for all continuous random variables in the rest of the dissertation.

Equation (9) only accounts for the probability of an observed failure at time t_i regardless of previous failure data. The conditional probability of observing failure i given all the previous $i-1$ failures is solely determined by the ages of all components after the repair action of the $(i-1)^{th}$ failure. Specifically, the likelihood of failure i, $i=1,2,\ldots,N(\tau)$, conditioned on all the previous $i-1$ failures, can be calculated as

$$\mathcal{L}_i = \frac{-\left.\dfrac{\partial S\left(z_{1,i},\ldots,z_{K,i}\right)}{\partial z_{k,i}}\right|_{\left\{z_{1,i}=a_1(t_i),\ldots,z_{k,i}=a_k(t_i),\ldots,z_{K,i}=a_K(t_i)\right\}}}{S\left(a_1(t_{i-1}^+),\ldots,a_K(t_{i-1}^+)\right)} \tag{10}$$

where $S(\cdot)$ is the joint survival function of the latent ages to failures.

For example, consider the first two failures illustrated in Fig. 1. After the repair action of the first failure at time t_1, the age of the failed component l is updated to $a_l(t_1^+)=0$, while the ages of all the other components are given as $a_j(t_1^+)=t_1$,

$j \neq l, j = 1,...,K$. As the second failure is due to component one, and occurs at time t_2, the likelihood of the second failure conditioned on the first failure can be calculated as $\Pr(Z_{1,2} = a_1(t_2), Z_{j,2} > a_j(t_2); j \neq 1, j = 1,...,K \mid Z_{l,2} > 0, Z_{k,2} > t_1; k \neq l, k = 1,...,K)$, where $a_l(t_2) = t_2 - t_1$, and $a_j(t_2) = t_2; j \neq l, j = 1,...,K$.

As there is no failure observed from $t_{N(\tau)}$ to the predetermined ending time τ, the likelihood $\mathcal{L}_{N(\tau)+1}$ can be calculated as

$$\begin{aligned} \mathcal{L}_{N(\tau)+1} &= \frac{\Pr\{Z_{1,N(\tau)+1} > [\tau - r_1(\tau)],..., Z_{K,N(\tau)+1} > [\tau - r_K(\tau)]\}}{S\left(a_1(t^+_{N(\tau)}),..., a_K(t^+_{N(\tau)})\right)} \\ &= \frac{S\left(a_1(t_\tau),..., a_K(t_\tau)\right)}{S\left(a_1(t^+_{N(\tau)}),..., a_K(t^+_{N(\tau)})\right)} \end{aligned} . \tag{11}$$

Combining the results in (10) and (11), the following Proposition 1 can be used to calculate the likelihood function based on the observed failure data.

Proposition 1. Given the observed failure data, the likelihood function can be calculated by

$$\mathcal{L}(\boldsymbol{\theta}) = \prod_{i=1}^{N(\tau)+1} \mathcal{L}_i \tag{12}$$

where $\mathcal{L}_i$ can be calculated based on (10), and (11) for $i = 1,2,\ldots,N(\tau)$, and $i = N(\tau)+1$, respectively.

The estimated model parameters $\hat{\boldsymbol{\theta}}$ are obtained by maximizing $\mathcal{L}(\boldsymbol{\theta})$. Based on the ML theory (Svensson 1990, Casella and Berger 2001), the estimated parameters $\hat{\boldsymbol{\theta}}$ are asymptotically normally distributed under the large sample assumption.

2.5 Hypothesis Testing for Dependency

Based on the proposed reliability model for the competing risks systems, statistical hypothesis testing procedures are developed in this section to determine the component failure dependencies. In Section 2.5.1, a dependency test based on the multivariate lognormal distribution is proposed. Then in Section 2.5.2, a dependency test for the multivariate Weibull distribution derived by the Archimedean copula and the Gaussian copula are discussed, respectively.

2.5.1 A Dependency Test for the Multivariate Lognormal Distribution

In the multivariate lognormal distribution, the dependency information is captured by the correlation matrix $\boldsymbol{\Lambda}$, which can be calculated based on covariance matrix $\boldsymbol{\Sigma}$:

$$\boldsymbol{\Lambda}_{i,j} = \frac{\boldsymbol{\Sigma}_{i,j}}{\sqrt{\boldsymbol{\Sigma}_{i,i} \cdot \boldsymbol{\Sigma}_{j,j}}}; i, j = 1, \ldots, K \tag{13}$$

where $\boldsymbol{\Lambda}_{i,j}$, and $\boldsymbol{\Sigma}_{i,j}$ are the elements of the i^{th} row, and the j^{th} column in $\boldsymbol{\Lambda}$, and $\boldsymbol{\Sigma}$, respectively. Based on the correlation matrix $\boldsymbol{\Lambda}$, a hypothesis testing procedure is developed to determine the statistical dependency among latent ages to failures of components.

$$H_0: \text{component failures } i, j \text{ are statistically independent.}$$
$$H_1: \text{component failures } i, j \text{ are statistically dependent.} \qquad (14)$$

When the asymptotic result is applied, we use a normal approximation to construct the test statistics. The test statistics $W_{i,j}$ are used to test the dependency between component failures i and j, which equals the estimate of correlation $\hat{\mathbf{\Lambda}}_{i,j}$ divided by its estimated standard error $\sqrt{\text{var}(\hat{\mathbf{\Lambda}}_{i,j})}$, i.e.,

$$W_{i,j} = \hat{\mathbf{\Lambda}}_{i,j} / \sqrt{\text{var}(\hat{\mathbf{\Lambda}}_{i,j})}\,. \qquad (15)$$

In hypothesis testing (14), H_0 is rejected if $W_{i,j} > Z_{\alpha/2}$, or $W_{i,j} < Z_{1-\alpha/2}$, where α is the test significance level, and $Z_{\alpha/2}$ is the upper quantile of the standard normal distribution. Based on hypothesis testing (14), pairwise statistical dependencies between different component failures can be tested.

2.5.2 A Dependency Test for the Multivariate Weibull Distribution

In this section, a statistical dependency test for joint distribution constructed via the Archimedean copula is first discussed. The Archimedean copula can capture the overall statistical dependency, which is determined by the global association parameter ρ. We developed the following hypothesis testing procedure to test the overall failure dependency, i.e., to see whether Kendall's tau is equal to zero.

$$\begin{aligned} &H_0: \text{all failure types are statistically independent.} \\ &H_1: \text{not all failure types are statistically independent.} \end{aligned} \tag{16}$$

Here, the asymptotic test statistic $W_{overall}$ is constructed using the estimate of Kendall's tau $\hat{\tau}_{Kendall}$ divided by its estimated standard error $\sqrt{\text{var}(\hat{\tau}_{Kendall})}$. H_0 is rejected if $W_{overall} > Z_{\alpha/2}$ or $W_{overall} < Z_{1-\alpha/2}$. As Kendall's tau is a function of the association parameter ρ, the variance of the estimate of Kendall's tau can be calculated by using the delta method.

$$\begin{aligned} \text{var}(\hat{\tau}_{Kendall}) &= (\frac{d\hat{\tau}_{Kendall}}{d\hat{\rho}})^T \ \text{var}(\hat{\rho})(\frac{d\hat{\tau}_{Kendall}}{d\hat{\rho}})\Big|_{\rho=\hat{\rho}} \\ &= \text{var}(\hat{\rho})\left((\hat{\rho}/(\hat{\rho}+2)^2 - 1/(\hat{\rho}+2))^2\right) \end{aligned} \tag{17}$$

where $\text{var}(\hat{\rho})$ denotes the asymptotic variance of $\hat{\rho}$. Based on (17), the asymptotic test statistic of the overall dependency is given as

$$W_{overall} = \frac{\hat{\tau}_{Kendall} \cdot \hat{\rho}^2}{\sqrt{\text{var}(\hat{\rho})\left((\hat{\rho}/(\hat{\rho}+2)^2 - 1/(\hat{\rho}+2))^2\right)}}. \tag{18}$$

For the multivariate Weibull distribution constructed via the Gaussian copula, the correlation matrix determines the pairwise dependencies of latent ages to component failures. Thus, the test procedure is the same as that discussed for the multivariate lognormal distribution.

The test statistics for Multivariate Weibull via the Gaussian copula has the same form as (15). Thus, hypothesis testing (14) can be used here to test the pairwise statistical dependencies among different failure types.

2.6 Conclusion

In this Chapter, a general statistical reliability model is proposed for repairable multi-component systems considering statistical dependent competing risks under a partially perfect repair assumption. For the reliability analysis of repairable multi-component systems, most of the research in the literature assumes component failure statistical independency. The failure mechanism (marginal distribution) of each component can thus be estimated individually based on its failure data.

In the developed model, copula functions are used to model the joint distribution of component failure times. Specifically, two types of copulas, i.e., the Archimedean copula, and the Gaussian copula, are applied to study the overall dependency, and pairwise dependencies among different components, respectively. Although the copula function method is also applied in the literature to study non-repairable systems, or systems under perfect repair action (replace the whole system when a failure happens), the problem studied in this paper is much more complex than those in the literature. When the whole system is replaced after a failure, the system will have the same failure mechanism as the original one. In contrast, when only the failed component is replaced, replacement of failed component affects the failure mechanism of the other components when considering failure dependency. Thus, the methods in the literature cannot be directly applied.

Under competing risks assumptions, only the failed component is recorded as the latent ages to failures of other components cannot be observed. After a repair action under the partially perfect repair assumption, the failure mechanism of the new system and components will be changed. Thus, for a single repairable system in which the failure data can only be collected from a single realization, model parameters estimation is challenging. To tackle this problem, an ML method is developed in this research, and the ML function is calculated based on conditional probability.

The partially perfect repair action is useful for many complex multi-component engineering systems when only the failed component is repaired, and the repair action is a replacement due to high labor cost.

Hypothesis testing is developed to test the statistical dependency of component failures. The obtained statistical failure dependency provides more accurate information for reliability predictions, which can be used for system maintenance.

The proposed methodology in this Chapter has been published in a journal article (Yang, Zhang, et al. 2013).

CHAPTER 3. RELIABILITY ANALYSIS OF MULTI-COMPONENT SYSTEMS REPAIRABLE SYSTEMS WITH DEPENDENT COMPETING RISKS UNDER IMPERFECT REPAIR

In previous Chapter, statistical model for repairable multi-component systems with partially perfect repair is proposed. However, in most cases, the repair conditions are unknown. Thus, the partially perfect repair assumption may not always hold. Thus, in this Chapter, we extend the model proposed in previous Chapter from partially perfect repair conditions to generally imperfect repair conditions. Specifically, two models are proposed, i.e., the generalized dependent latent age model and the copula-based trend-renewal process model.

3.1 Generalized Dependent Latent Age Model

The generalized dependent latent age model (GDLA) model generalizes partially perfect repair model proposed in CHAPTER 2 by extending Kijima's virtual age models (Kijima and Sumita 1986, Kijima 1989) from repairable single-component systems to multi-component systems.

In this research, we are interested in repairable K-component systems under competing risks. The failures of the same component are regarded as one type of failures. After each failure, the failed component is repaired and other components are untouched. The repair conditions for the failed components are imperfect, including the two extreme cases, i.e., minimal repair and perfect repair. The partially perfect repair condition discussed in CHAPTER 2 is a special case of the imperfect repair conditions.

To deal with generally imperfect repairs of multi-component systems, we first extend Kijima's virtual age models from single-component systems to multi-component systems and then combine it with the partially perfect repair model to develop the GDLA model.

3.1.1 Extended Virtual Ages for Multi-component Systems

Two virtual age models are developed for repairable single-component systems in (Kijima and Sumita 1986, Kijima 1989). Let $t_1, t_2, \ldots$, denote the failure time series ($t_0 = 0$), and let $x_i = t_i - t_{i-1}; i = 1, 2, \ldots$, denote the inter-arrival times of failures. The virtual age is used to describe the system state. Specifically, after the i^{th} repair, the virtual age v_i in Kijima model I and II is defined as $v_i = v_{i-1} + q \cdot x_i$ and $v_i = q \cdot (v_{i-1} + x_i)$ respectively, where $q \in [0,1]$ is the repair effectiveness factor. $q = 0$ and $q = 1$ correspond to the two extreme repair cases, i.e., perfect repair and minimal repair respectively; and $q \in (0,1)$ corresponds to the imperfect repairs.

For the repairable K-component system, we extend the Kijima model II to describe the state of multiple components. Let $t_i; i = 1, 2, \ldots$, denote the observed failure time of the i^{th} failure ($t_0 = 0$), and let $r_l(t); l \in \{1, \ldots, K\}$ be the most recent failure time of component l before time t. The virtual age of component l at time t is defined as:

$$\begin{aligned} &v_l(t) = 0; \text{if } t = 0 \\ &v_l(t) = q_l \cdot v_l(t^-); \text{if } t = r_l(t^+) \text{ and } t > 0 \\ &v_l(t) = v_l(r_l(t)) + (t - r_l(t)); \text{if } t > r_l(t^+) \end{aligned} \tag{19}$$

where $v_l(t)$ is right-continuous and $v_l(t^-)$ denotes its left limit at time t, and q_l represents the repair effectiveness factor of component l. In (19), $t = r_l(t^+)$ indicates that there is a failure from component l which occurs at time t. It is worthwhile to note that the age $a_l(t)$ defined in CHAPTER 2 is left-continuous, compared with the right-continuous virtual age $v_l(t)$.

As the repair effectiveness factors are not necessary to be the same for different components, we use a constant vector $\mathbf{q} = [q_1, \ldots, q_K]^T$ to denote the repair effectiveness factor for the entire system. As $q_l \in [0,1]$, the extended virtual age model for multi-component systems can deal with generally imperfect repairs, including the perfect replacement and minimal repairs. Ideally, when the repairs are either perfect or minimal, the repair effectiveness factors can be directly recorded. However, under many situations the repair effectiveness is difficult to be quantified. Thus, it is more reasonable to assume $\mathbf{q} = [q_1, \ldots, q_K]^T$ is an unknown vector which also needs to be estimated from the failure data. The estimated repair effectiveness factors can be further used in the maintenance planning.

3.1.2 Model Building

Based on partially perfect repair reliability model and the extended virtual ages, we propose the GDLA model to deal with both generally imperfect repairs and dependent competing risks.

We use the term initial ages to represent the virtual ages of all components after the system's i^{th} repair, which are denoted as $[v_1(t_i),\ldots,v_K(t_i)]^T$. For $i=0$, $[v_1(t_i),\ldots,v_K(t_i)]^T=[0,...,0]^T$; and for $i=1,2,...$, $[v_1(t_i),\ldots,v_K(t_i)]^T$ can be calculated as a function of repair effectiveness factor $\mathbf{q}=[q_1,...,q_K]^T$ according to (19).

The GDLA model assumes that once a component's virtual age reaches a corresponding threshold, the component fails, which causes the entire system's failure. Thus, the random threshold corresponds to the latent age to failure concept which has been used in CHAPTER 2. We still use $Z_{k,i}$ to denote the latent age of component k to the system's i^{th} failure which is defined in the partially perfect repair reliability model, and the random vector $\mathbf{Z}_i=[Z_{1,i},...,Z_{K,i}]^T$ represents the latent ages to the system's i^{th} failure for all components. Due to competing risks, only the minimum of latent ages to failure can be observed. In the GDLA model, we also use a joint distribution F to model $\mathbf{Z}_i$ like the partially perfect repair model. The dependency of component failures is captured by the joint distribution F. However, if the i^{th} repair is imperfect, the system has non-zero initial ages $[v_1(t_i),\ldots,v_K(t_i)]^T$. Thus, any component's latent age to failure must be larger than its initial age, i.e., $Z_{l,i+1} > v_l(t_i)$, $\forall l \in \{1,...,K\}$. In other words, the latent ages to the $(i+1)^{th}$ system's failure $\mathbf{Z}_{i+1}$ are following a truncated distribution conditional on the initial ages $[v_1(t_i),\ldots,v_K(t_i)]^T$. When the failed component is repaired as good as new, the GDLA model degenerates to the reliability model proposed in CHAPTER 2 under partially perfect repair conditions.

An example that illustrates the extended virtual ages and the proposed GDLA model is given in Fig. 1, where paired (t_i, δ_i) denotes the failure time t_i and the failed component δ_i for the i^{th} failure. The first observed failure comes from component 1 at t_1, since the age of component 1 reaches its latent age to failure $Z_{1,1}$ before the age of component 2 reaches $Z_{2,1}$. The second observed failure comes from component 2 at t_2, since $Z_{2,2}$ is reached ahead of $Z_{1,2}$. The next failure still comes from component 2 as $Z_{2,3}$ is reached first.

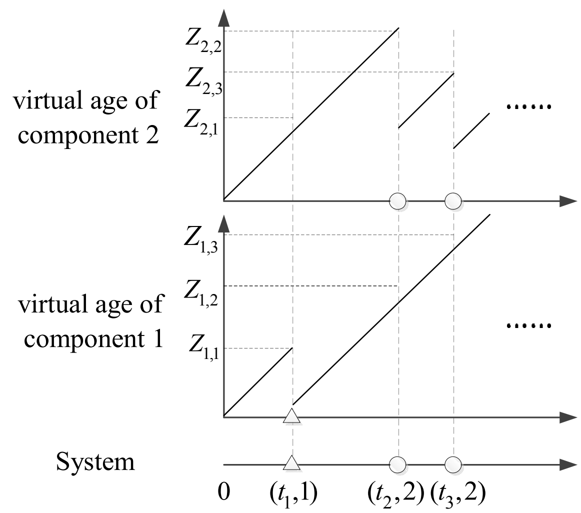

Fig. 2. Illustration of the GDLA model using a two-component repairable system

We construct the joint distribution in the way as the partially perfect repair model. More specifically, the Weibull and lognormal distributions are separately selected as the marginals of the joint distribution F and copula functions are used to construct the joint distribution to illustrate the proposed method in this research.

When the marginal is chosen as lognormal, the multivariate lognormal distribution is generally used as the joint distribution F. The proposed model parameters are denoted by $\boldsymbol{\theta} = \{\mathbf{q}, \boldsymbol{\mu}, \boldsymbol{\Sigma}\}$, where $\boldsymbol{\mu} \in \mathbf{R}^K$ is the mean vector and $\boldsymbol{\Sigma}$ is the $K \times K$ covariance matrix, respectively.

The cdf of the joint Weibull distribution constructed via Gaussian copula is:

$$F_G\left(z_{1,i},\ldots,z_{K,i} \mid \boldsymbol{\theta}\right) = \Phi_{\boldsymbol{\Sigma}}\left(\Phi^{-1}(u_{1,i}),\ldots,\Phi^{-1}(u_{K,i})\right) \tag{20}$$

where $u_{l,i} = 1 - \exp\left(-(z_{l,i}/\lambda_l)^{k_l}\right)$ is the Weibull marginal cdf; $\Phi_{\boldsymbol{\Sigma}}$ is the cdf of a multivariate normal distribution with a zero mean vector requiring its covariance matrix be equal to its correlation matrix; and Φ^{-1} is the inverse cdf of the standard normal distribution. The parameters for the reliability model with the Gaussian copula function and marginal Weibull are denoted by $\boldsymbol{\theta} = \{\mathbf{q}, \boldsymbol{\lambda}, \boldsymbol{\kappa}, \boldsymbol{\Sigma}\}$, where $\boldsymbol{\lambda} = \left[\lambda_1, \ldots, \lambda_K\right]^T$ and $\boldsymbol{\kappa} = \left[\kappa_1, \ldots, \kappa_K\right]^T$ are the scale and shape parameters of Weibull distribution respectively; and $\boldsymbol{\Sigma}$ is the correlation matrix in $\Phi_{\boldsymbol{\Sigma}}$.

3.1.3 Parameter Estimation for the GDLA Model

We estimate the parameters in the GDLA model using the maximum likelihood estimation (MLE) method. The overall likelihood function for n observed failure events equals:

$$\mathcal{L}(\boldsymbol{\theta}) = \prod_{i=1}^{n} \mathcal{L}_i \tag{21}$$

where $\mathcal{L}_i$ denotes the likelihood function of the i^{th} failure conditional on the initial ages of components after the $(i-1)^{th}$ failure and the corresponding repair. Proposition 2 is developed to calculate the likelihood function $\mathcal{L}_i$.

Proposition 2. Suppose the i^{th} failure comes from component j; the initial ages $[v_1(t_{i-1}), \ldots, v_K(t_{i-1})]^T$ can be obtained by (19) as a function of unknown repair effectiveness factor $\mathbf{q} = [q_1, ..., q_K]^T$. Then the likelihood function of the i^{th} failure given can be calculated as:

$$
\begin{aligned}
&\mathcal{L}_i \\
&= \Pr\left\{ Z_{1,i} > v_1\left(t_i^-\right), \ldots, Z_{j,i} = v_j\left(t_i^-\right), \ldots, Z_{K,i} > v_K\left(t_{i-}^-\right) \middle| Z_{1,i} > v_1\left(t_{i-1}\right), \ldots, Z_{K,i} > v_K\left(t_{i-1}\right) \right\} \\
&= \frac{\Pr\left\{ Z_{1,i} > v_1\left(t_i^-\right), \ldots, Z_{j,i} = v_j\left(t_i^-\right), \ldots, Z_{K,i} > v_K\left(t_{i-}^-\right) \right\}}{\Pr\left\{ Z_{1,i} > v_1\left(t_{i-1}\right), \ldots, Z_{j,i} > v_j\left(t_{i-1}\right), \ldots, Z_{K,i} > v_K\left(t_{i-1}\right) \right\}} \\
&= \frac{ -\left. \dfrac{\partial S\left(z_{1,i}, \ldots, z_{K,i}\right)}{\partial z_{j,i}} \right|_{\left\{ z_{1,i} = v_1\left(t_i^-\right), \ldots, z_{j,i} = v_j\left(t_i^-\right), \ldots, z_{K,i} = v_K\left(t_i^-\right) \right\}} }{ S\left(v_1\left(t_{i-1}\right), \ldots, v_j\left(t_{i-1}\right), \ldots, v_K\left(t_{i-1}\right) \right) }
\end{aligned}
\tag{22}
$$

where $S(\cdot)$ is the joint survival function of the latent ages to failure.

Presenting a two-component system as an example, the first failure observed at time 5 came from component 2, and the second failure observed at time 20 came from component 1. Then, the likelihood for the first failure is calculated as $\mathcal{L}_1 = \Pr\left(Z_{1,1} > 5, Z_{2,1} = 5 \middle| Z_{1,1} > 0, Z_{2,1} > 0 \right)$. For the repaired system, the latent ages to the first system's failure for component 1 was exactly 20 given its initial age 5, while that for component 2 was larger than $(15 + q_2 \cdot 5)$ given its initial age $q_2 \cdot 5$. Thus, the likelihood for the second observed failure $\mathcal{L}_2 = \Pr\left(Z_{1,2} = 20, Z_{2,2} > (15 + q_2 \cdot 5) \middle| Z_{1,2} > 5, Z_{2,2} > q_2 \cdot 5 \right)$.

The parameters in the proposed model can be estimated by maximizing the overall likelihood function obtained in (21). Under a large sample assumption, the maximum likelihood estimates are asymptotically normally distributed, and the covariance matrix can be calculated by the inverse of the observed Fisher information matrix (Casella and Berger 2001). Due to the complexity of the likelihood function (21), the analytical maximum likelihood estimate is intractable. To overcome this difficulty, we apply a numerical optimization method, i.e., the simulated annealing algorithm to maximize (21) (Bélisle 1992).

3.1.4 System Reliability Prediction

The reliability of the system after the i^{th} repair can be predicted using the proposed reliability model. Let $T_{\min}$ denote the inter-arrival time between the i^{th} repair and the $(i+1)^{th}$ failure, and let $F_{T_{\min}}$ denote the cdf of the random variable $T_{\min}$. The system reliability at time t since the i^{th} repair is calculated as:

$$R\left(t \mid v_1\left(t_i\right),\ldots,v_K\left(t_i\right)\right)=\frac{S\left[v_1\left(t_i\right)+t,\ldots,v_K\left(t_i\right)+t\right]}{S\left[v_1\left(t_i\right),\ldots,v_K\left(t_i\right)\right]}. \tag{23}$$

where $[v_1\left(t_i\right),\ldots,v_K\left(t_i\right)]^T$ are the initial ages and $S(\cdot)$ is the joint survival function of latent ages to failure. When the joint distribution of latent ages is selected as multivariate lognormal, the joint survival function $S(\cdot)$ is calculated by integrating its pdf.

3.1.5 Simulation Study

To verify the proposed reliability model and parameter estimation method, we conduct a simulation study for a two-component system. The failure data are simulated according to the GDLA model with joint distribution constructed with Gaussian copula and Weibull marginal. We consider failure dependency with different repair effectiveness factors in scenario I and II, whose parameters are listed in Table 1 and Table 2, respectively.

Table 1. Parameter setting in simulation Scenario I

Component	Repair effectiveness factor	Joint distribution (Gaussian copula + Weibull marginal)			
	q	**κ** (shape)	λ (scale)	Mean	Correlation matrix
1	0.20	2.00	3.00	0	1.00 0.50
2	0.20	2.00	3.00	0	0.50 1.00

Table 2. Parameter setting in simulation Scenario II

Component	Repair effectiveness factor	Joint distribution (Gaussian copula + Weibull marginal)			
	q	**κ** (shape)	λ (scale)	Mean	Correlation matrix
1	0.20	2.00	3.00	0	1.00 0.50
2	0.60	2.00	3.00	0	0.50 1.00

In each simulation scenario, we estimate the parameters with failure events n =100, 200, 500 and 1000 respectively. For each n, we conduct the simulations with 1000 replicates to obtain the coverage probabilities for the 95% confidence intervals and mean squared errors (MSE) for the parameter estimates. The simulation results of two scenarios are shown in Fig. 3 and Fig. 4, respectively.

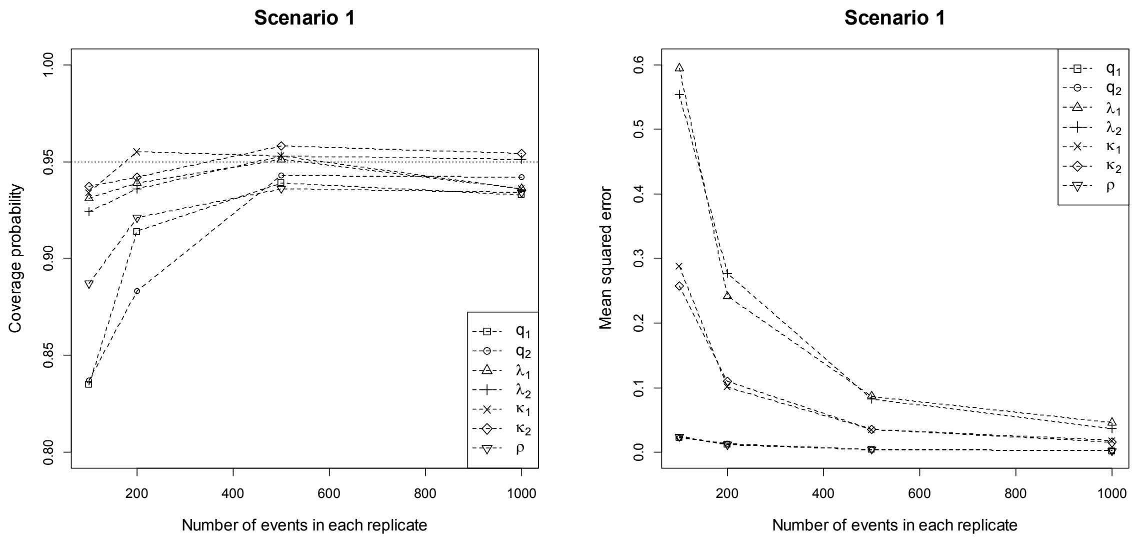

Fig. 3. Simulation results with parameter setting in Table 1

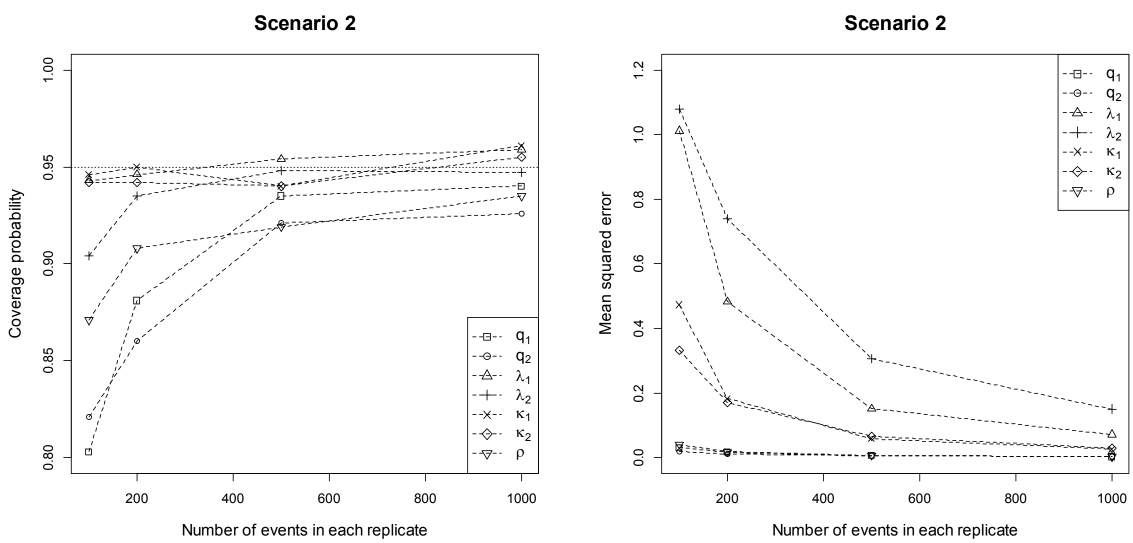

Fig. 4. Simulation results with parameter setting in Table 2

In Fig. 3 and Fig. 4, it can be seen the coverage probabilities for the estimated confidence intervals are approaching 95% and the MSEs are approaching zero as the number of failure/maintenance events increases, which validates the parameter estimation method of proposed reliability model.

3.1.6 Case Study using GDLA Model

The developed GDLA model is applied for a cylinder head assembling cell in a major automobile power-train plant in the United States. The system can be treated as a competing risks system, as several stations are working together, and the failure of any of them would result in the failure of the entire system. In order to illustrate the proposed methodology, we consider the two stations of the system; these two stations are denoted as station 1 and 2 to protect proprietary sensitive information. During the data collection period, the assembling cell adopted run-to-failure policy, i.e., maintenance action was performed when there was a failure. Thus, we recorded the exact failure times. Fig. 5 shows the original failure time data that have been processed to eliminate all the downtime due to failures and maintenances.

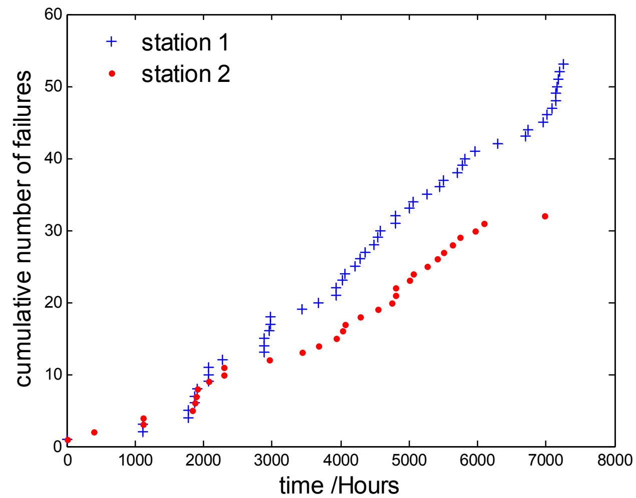

Fig. 5. Failure data of two stations from an assembling cell

We first estimate the parameters of the GDLA model using the collected failure data. The repairs of each component keep the same although the repair effectiveness are difficult to be quantified. The marginal distribution in the proposed model is chosen as lognormal and Weibull, respectively. When the marginal is lognormal, a bivariate lognormal is directly used as the joint distribution. Table 3 lists the estimated parameters $\hat{\boldsymbol{\theta}} = \{\hat{\mathbf{q}}, \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}}\}$ by maximizing the overall likelihood in (21).

Table 3. Estimated parameters and the standard errors when using bivariate lognormal distribution

failure type	Parameter estimates				Standard errors			
	$\hat{\mathbf{q}}$	$\hat{\boldsymbol{\mu}}$	$\hat{\boldsymbol{\Sigma}}$		$\hat{\mathbf{q}}$	$\hat{\boldsymbol{\mu}}$	$\hat{\boldsymbol{\Sigma}}$	
1	0.007	2.48	4.93	2.11	0.02	0.68	1.59	1.02
2	0.031	4.15	2.11	2.93	0.04	0.60	1.02	1.06

When Weibull is chosen as the marginal, a Gaussian copula with Weibull marginal is applied. Using the MLE method, the estimated parameter $\hat{\boldsymbol{\theta}} = \{\hat{\mathbf{q}}, \hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\kappa}}, \hat{\boldsymbol{\Sigma}}\}$ and standard errors are listed in Table 4.

Table 4. Estimated parameters and the standard errors when using bivariate Weibull constructed via Gaussian copula

failure type	Parameter estimates					Standard errors				
	$\hat{\mathbf{q}}$	$\hat{\boldsymbol{\lambda}}$	$\hat{\boldsymbol{\kappa}}$	$\hat{\boldsymbol{\Sigma}}$		$\hat{\mathbf{q}}$	$\hat{\boldsymbol{\lambda}}$	$\hat{\boldsymbol{\kappa}}$	$\hat{\boldsymbol{\Sigma}}$	
1	9.64e-13	49.00	0.55	1.00	0.39	1.14e-9	1.93	0.07	-	0.13
2	1.39e-07	143.60	0.69	0.39	1.00	2.39e-5	4.28	0.11	0.13	-

In the literature, log-likelihood or Akaike information criterion (AIC) values are commonly used to evaluate the model fitting (Akaike 1974, Akaike 1980). As both the bivariate lognormal and the Gaussian copula have the same numbers of parameters, the comparisons using log-likelihood values and AIC values are consistent. The log-likelihood values for a bivariate lognormal and a Gaussian copula with Weibull marginals are -514.05 and -502.69, respectively. As a larger log-likelihood value indicates better modeling fitting, the reliability model based on the Gaussian copula with Weibull marginals is selected. Since the estimated repair effectiveness factors are quite close to zero, the repairs of failed components can be treated as perfect.

Based on the estimated parameters, the system reliability that depends on the elapsed time and both components' initial ages since last repair can be calculated using (23). Fig.

6 (I) and (II) show the calculated system reliability after the i^{th} repair under two special cases, i.e., component 2 and component 1 were replaced, respectively.

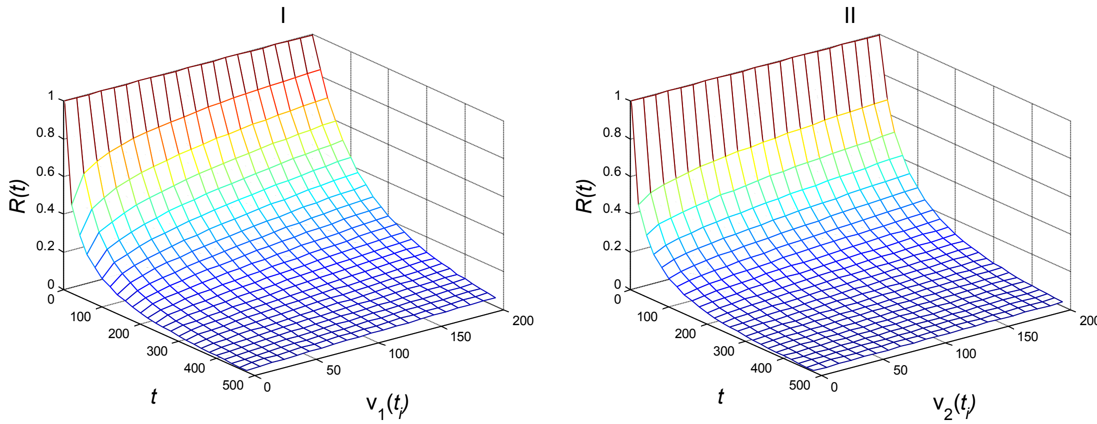

Fig. 6. System reliabilities after a repair vs. time and initial ages (I: the initial age of component 2 is zero; II: the initial age of component 1 is zero).

3.2 Copula-based Trend-renewal Process Model

In this section, we propose a copula-based Trend-Renewal process (CTP) model to analyze the multiple-component repairable systems under dependent competing risks. The failed component is subject to general repair actions, including perfect and minimal repairs as well as situations in between. In the GDLA model, the repair conditions are quantitatively described by Repair effectiveness factor. While in the CTP model, the repair conditions are modeled by the trend functions.

3.2.1 Trend-renewal Process Model for A Single Component

The trend-renewal process (TRP) model (Lindqvist, Elvebakk, et al. 2003) is a statistical model to model the single-component repairable system under general system repair actions from perfect to minimal, in which both perfect and minimal repairs are included as two extreme cases. The basic idea of the TRP model is to apply a trend function $\Lambda(t)$ to transform the original failure times into a new time domain so that the transformed failure times can be modeled by a renewal process following a renewal distribution.

3.2.2 A General Reliability Model for Imperfect Component Repair Actions

In this section, the TRP model is extended for systems consisting of multiple components that can fail dependently. The partially perfect repair reliability model developed in CHAPTER 2 is further extended for systems with imperfect component repair actions with time transformation.

In the partially perfect repair reliability model, the system failures are only determined by the last failure times of all the components because the repair actions of failed components are assumed to be perfect. When the repair action is imperfect, however, the component failures are affected by the effect of imperfect component repair accumulated from the all the repair history, which are coupled with the effect of failure dependency of other components in a complex manner.

To overcome this difficulty, we propose a multiple transformation procedure in this research by applying the TRP model to transform the failure times of individual components to separate transformed time domains in which the effect of imperfect repair can be eliminated. Specifically, as shown in Fig. 7, the failure times of component k (denoted by $T_{k,1}$, $T_{k,2}$,…) are transformed into the k^{th} transformed time domain using a trend function $\Lambda_k(\cdot)$. Based on the properties of the TRP model, the transformed failure times of component k, $\Lambda_k(T_{k,1})$, $\Lambda_k(T_{k,2})$ … are following a renewal process characterized by a renewal distribution F_k.

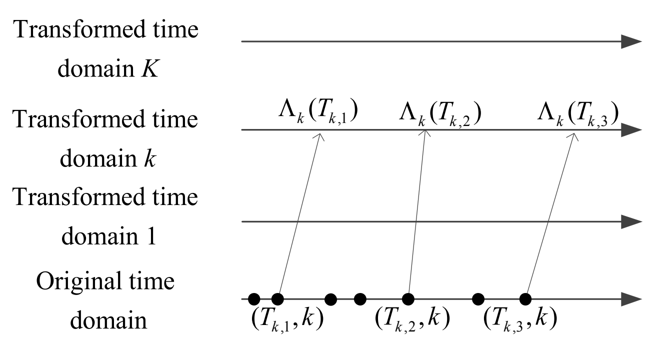

Fig. 7. Illustration of the multiple transformation procedure based on different trend functions for different failure types

According to the properties of the renewal process, the accumulated effect of imperfect repair in the original time domain is eliminated in the transformed domain. Thus, the failures time in the transformed time domains can be modeled in a similar way to capture component failure dependency. We assume component $k; k = 1,...,K$ has a latent age to failure, defined as a random variable and is denoted by $V_{k,i}$, in the k^{th} transformed time domain after the $(i-1)^{th}$ system failure and the corresponding repair action. When considering the first system failure from installation, $\mathbf{V}_1 = [V_{1,1},...,V_{K,1}]'$ is modeled by a joint distribution F whose marginal distribution equals F_k, the renewal distribution of the k^{th} component in the transformed time domain. Let $a_k(t)$ and $r_k(t)$ be the component age and the most recent failure time for component k in the original time domain, respectively. Let $b_k(t)$ denote the age of component k in the k^{th} transformed time domain. When the i^{th} system failure occurs at time point t_i, the failed component can be treated as perfectly replaced in the corresponding transformed time domain according to the properties of renewal process. Hence, $b_k(t) = \Lambda_k(t) - \Lambda_k(r_k(t))$. Because components may have a none-zero age right after a system failure and the corresponding repair, the $(i+1)^{th}$ latent age to failure of component k in the k^{th} transformed time domain, denoted by $V_{k,i+1}$, should be larger than $b_k\left(t_i^+\right)$, i.e., $V_{k,i+1} > b_k\left(t_i^+\right)$, $\forall k \in \{1,...,K\}$. As a result, the random vector $\mathbf{V}_{i+1} = [V_{1,i+1},...,V_{K,i+1}]'$ are following a truncated distribution of F conditional on the vector of $[b_1\left(t_i^+\right),\ldots,b_K\left(t_i^+\right)]^T$. It can be seen that the partially perfect repair model is a special case of the general CTP model, when the trend function is the identity function, i.e., $\Lambda_k(t) = t$.

3.2.3 Parametric Forms

The proposed copula-based TRP model is determined by the trend function and the joint distribution F. In this research, the copula functions are used to build the joint distribution F based on the marginal distributions. Thus, the model parameters include those from the trend function, the marginal distribution, and the copula function.

3.2.3.1 Trend Function

The power law relationship, which is generally used in the trend function of the TRP model, is also used in the multiple transformation procedure. In particular, the power law intensity function $\lambda_k(\cdot)$ for failure type k has the following form:

$$\lambda_k(t;\boldsymbol{\theta}_{k,\lambda}) = \frac{\beta_k}{\eta_k}\left(\frac{t}{\eta_k}\right)^{\beta_k - 1} \tag{24}$$

where $\boldsymbol{\theta}_{k,\lambda} = [\beta_k, \eta_k]'$ is the parameter vector of intensity function $\lambda_k(\cdot)$. We use $\boldsymbol{\theta}_{\lambda} = \left\{\boldsymbol{\theta}_{1,\lambda}, ..., \boldsymbol{\theta}_{K,\lambda}\right\}$ to denote the parameters in all trend functions.

3.2.3.2 Renewal Distribution

We choose Weibull distribution (2) as the marginal distribution of the joint distribution F. However, other distributions can also be applied in the model. The joint distribution is constructed by using copula functions that are introduced in section 2.3.2 and 2.3.3.

Similar to the traditional TRP model for single-component systems, in our model the marginal expectations are restricted to one in order to reduce the degree of freedom of the model, because if a trend function is multiplied by a constant then we can modify the corresponding marginal distribution accordingly by scaling the time. In practice, we add a constraint that $\lambda_k \cdot \Gamma(1+1/\kappa_k)=1; k=1,...,K$, where $\Gamma(\cdot)$ is the gamma function.

3.2.4 Parameter Estimation and Statistical Inference

3.2.4.1 Construction of Likelihood Function

The maximum likelihood (ML) approach is used to estimate the model parameters, including those in the joint distribution and those in the trend functions. To implement the ML approach, the likelihood function is firstly calculated.

Let $\mathcal{F}_{t^-} = \left\{(T_1,\Delta_1),...,\left(T_{N(t^-)},\Delta_{N(t^-)}\right)\right\}$, which contains all the paired failure times and failure types until, but not include, time t. Thus, the whole dataset can be denoted by $\mathcal{F}_\tau = \mathcal{F}_{\tau^-} \cup \{(\tau,0)\}$. Given the failure data set $\mathcal{F}_\tau$, the likelihood function can be decomposed according to the conditional probability as follows:

$$\mathcal{L}(\boldsymbol{\theta} \mid \mathcal{F}_\tau) = \prod_{i=1}^{N(\tau)+1} \mathcal{L}_i \tag{25}$$

where $\mathcal{L}$ denotes the conditional likelihood function of failure i given all previous failures. The parameter set $\boldsymbol{\theta} = \{\boldsymbol{\theta}_\lambda, \boldsymbol{\theta}_F\}$ denotes all parameters in our model, where $\boldsymbol{\theta}_\lambda$ and $\boldsymbol{\theta}_F$ are parameters in trend functions and those in joint distributions, which are defined in Section 3.2.3. Specifically,

$$\mathcal{L}_i = \begin{cases} \Pr(T_1 = t_1, \Delta_1 = \delta_1) \text{ for } i=1 \\ \Pr(T_i = t_i, \Delta_i = \delta_i \mid T_j = t_j, \Delta_j = \delta_j; j=1,...,i-1) \text{ for } i=2,...,N(\tau) \\ \Pr(T_i = \tau, \Delta_i = 0 \mid T_j = t_j, \Delta_j = \delta_j; j=1,...,N(\tau)) \text{ for } i = N(\tau)+1 \end{cases}. \tag{26}$$

In equation (26), we slightly abuse the notation of $\Pr(X=x)$. Theoretically,

$\Pr(X = x)$ is zero when X is a continuous random variable. Here we interpret $\Pr(X = x)$ as $\Pr(x < X \le x + dx) = f(x)dx$, which is proportional to the density $f(x)$. For convenience of notation, we ignore dx in the likelihood functions.

Due to the cumulative effect of imperfect repair and component failure dependency, $\mathcal{L}_i$ in (26) is difficult to be calculated in the original time domain as it dependents on the entire failure history. To overcome this difficulty, we calculate the likelihood function in the transformed time domains, in which the cumulative effects of imperfect repair are eliminated so that $\mathcal{L}_i$ only depends on the most recent components' failures. However, it needs to be noted that the likelihood function in (26) is constrained by the sequence of the failures happened in the original time domain. This constraint needs to be taken into account in the transformed domains correspondingly.

The following Proposition 3 shows the calculation of $\mathcal{L}_i$ in the transformed time domains. The detailed proof of Proposition 3 is available in Appendix 3.

Proposition 3: the conditional likelihood function of failure i given all previous failures, i.e., $\mathcal{L}_i$, is given as follows,

$$\mathcal{L}_i = \frac{\left\{ -\frac{\partial S(v_{1,i}, \cdots, v_{\delta_i,i}, \cdots, v_{K,i})}{\partial v_{\delta_i,i}} \Big|_{\mathbf{v}_i = [b_1(t_i), \cdots, b_K(t_i)]'} \right\} \lambda_{\delta_i}(t_i)}{S[b_1(t_{i-1}^+), \cdots, b_K(t_{i-1}^+)]} \tag{27}$$

where $S(\cdot)$ denotes the survival function of $\mathbf{V}_1$; $\lambda_{\delta_i}(t_i)$ is the derivative of trend function that is used to transform the probability density from the original time domain to a transformed domain; and $b_k(t)=\Lambda_k(t)-\Lambda_k(r_k(t))$.

When $i=N(\tau)+1$, as there is no failure observed from $t_{N(\tau)}$ to the predetermined ending time τ, the conditional probability can be calculated as:

$$\mathcal{L}_{N(\tau)+1}=\frac{S[b_1(\tau),\cdots,b_K(\tau)]}{S\{b_1[t^+_{N(\tau)}],\cdots,b_K[t^+_{N(\tau)}]\}}. \quad (28)$$

3.2.4.2 Maximization of Likelihood Function

Model parameters can be estimated by maximizing the likelihood function obtained in the previous subsection. In practice, however, several issues need to be addressed.

When the Gaussian copula is used, two constraints exist: (a) the covariance matrix needs to be positive definite; and (b) the covariance matrix is equal to its correlation matrix. Constraint (b) is satisfied by directly fixing the diagonal elements of the covariance matrix as one. To satisfy constraint (a), we apply the nearest correlation matrix method (Higham 2002). Specifically, at each iteration of the optimization process, the estimated correlation matrix is approximated by the nearest correlation matrix that is positive definite. In practice, the correlation matrix only needs to be approximated by the nearest correlation matrix at the first several iterations. After a number of iterations, the output correlation matrix will automatically become positive definite as the estimated correlation matrix converge to the real correlation matrix.

The first order derivative of the survival function in (27) needs to be evaluated many times during the process of optimizing the likelihood function. In order to speed up the parameter estimation process, we evaluate the first order derivative of survival function of Gaussian copula as follows:

$$-\frac{\partial S(v_{1,i},\ldots,v_{k,i},\ldots,v_{K,i})}{\partial v_{k,i}} = f_k(v_{k,i}) S_{Normal}(\gamma_1,\ldots,\gamma_j,\ldots,\gamma_K); j \neq k \tag{29}$$

where $f_k(v_{k,i})$ is the pdf of random variable $v_{k,i}$. In this paper, we consider $f_k(v_{k,i})$ as Weibull marginal distribution. However, equation (29) still holds for other marginal distributions. Here, $S_{Normal}(\cdot)$ is the survival function of a $K-1$ dimensional multivariate normal, and $\gamma_j = \Phi^{-1}(u_j)$, where u_j denotes the cumulative density of the j^{th} marginal. The proof of (29) is given in Appendix 4.

Under a large-sample assumption, the ML estimate $\hat{\boldsymbol{\theta}}$ is asymptotically normally distributed based on ML theory (Casella and Berger 2001). Thus, the asymptotic covariance $\hat{\boldsymbol{\Sigma}}_{\hat{\boldsymbol{\theta}}}$ for $\hat{\boldsymbol{\theta}}$ can be calculated from the observed Fisher information matrix $\mathbf{I}(\hat{\boldsymbol{\theta}})$, i.e., $\hat{\boldsymbol{\Sigma}}_{\hat{\boldsymbol{\theta}}} = \mathbf{I}(\hat{\boldsymbol{\theta}})^{-1}$, where $\mathbf{I}(\hat{\boldsymbol{\theta}})$ is the negative of the Hessian matrix $\mathbf{H}(\boldsymbol{\theta})$ evaluated at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$, i.e., $\mathbf{I}(\hat{\boldsymbol{\theta}}) = -\left.\frac{\partial^2 \log(\mathcal{L}(\boldsymbol{\theta}))}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^T}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}$.

3.2.5 Statistical Hypothesis Test

Dependency information of different component failures is important for the maintenance of complex systems and their design. In addition, distinguishing subsets of components which fail independently can simplify the reliability model and efficiently reduce the parameter dimension.

In this section, we propose hypothesis testing procedures to determine the dependency among different component failures based on the proposed model. The likelihood ratio test statistic is calculated as follows,

$$D = -2 \cdot \ln\left(\frac{\sup\{\mathcal{L}_n\}}{\sup\{\mathcal{L}_f\}}\right) \tag{30}$$

where $\mathcal{L}_n$ indicates the likelihood values for the null model; and $\mathcal{L}_f$ indicates the likelihood values for the full model that includes both the null and the alternative models.

The likelihood ratio test statistic in (30) follows a χ^2 distribution with the degrees of freedom ω; where ω is the difference between the number of parameters in the full model and that in the null model. The null and alternative hypotheses, in practice, depend on the specific copula function that is used to construct the reliability model. In this research, hypothesis tests for statistical model via Clayton copula and that via Gaussian copula are developed in Sections 3.2.5.1 and 3.2.5.2, respectively.

3.2.5.1 Hypothesis Test for Clayton Copula

When the Clayton copula is selected to construct the reliability model, the overall dependency among all failure types can be tested using the following hypothesis test.

H_0: all failure types are independent (31)

H_1 : not all failure types are independent.

Hypothesis test (31) can be tested based on the likelihood ratio test statistic that is defined in (30). In (30), $\sup\{\mathcal{L}_f\}$ can be obtained by maximizing (26), while $\sup\{\mathcal{L}_n\}$ can be obtained by maximizing (26) with a constraint that the association parameter ρ is fixed as zero.

3.2.5.2 Hypothesis Test for Gaussian Copula

When the Gaussian copula is selected to construct the reliability model, the pairwise dependency among all failure types can be tested. The following test is proposed:

H_0 : failure types i, j are independent.

H_1 : failure types i, j are dependent . (32)

Similar to hypothesis test (31), hypothesis test (32) can be tested based on the likelihood ratio test statistic that is defined in (30). Under this situation, $\sup\{\mathcal{L}_f\}$ can be obtained by maximizing (26), while $\sup\{\mathcal{L}_n\}$ can be obtained by maximizing (26) with a constraint that the correlation for stations i, j in the Gaussian copula is fixed as zero.

3.2.6 Simulation Study

A comprehensive simulation study is conducted to verify the developed model. We consider five scenarios. Scenario I-IV are used to examine the effect of different copula functions, different degree of dependency, different marginal distributions, and different trend functions, respectively; while Scenario V is used to verify the parameter estimation method. To keep the setting simple, we consider a two-component system for Scenario I-IV and a three-component system for Scenario V. For each scenario, the failure data are simulated based on the proposed CTP reliability model. The detailed procedure for data simulation is described in Appendix 5.

3.2.6.1 Parameter Setting

1) Scenario I: examine the effect of form of copula.

We use the Weibull marginal distribution and the power law trend function with increasing trend. We consider two copula functions with moderate dependency: Gaussian copula and Clayton copula. The parameters of the copula functions are chosen such that the copula functions have the same overall dependency. When the Gaussian and Clayton copulas are chosen, the parameters are listed in Table 5 and Table 6, respectively.

Table 5. Parameter setting in simulation Scenario I (Gaussian copula)

Component	Trend function		Joint distribution (Gaussian copula + Weibull marginal)			
	β	η	κ (shape)	λ (scale)	Mean	Correlation matrix
1	1.200	1.000	2.000	1.128	0	1.000 0.500
2	1.200	1.000	2.000	1.128	0	0.500 1.000

Table 6. Parameter setting in simulation Scenario I (Clayton copula)

Component	Trend function		Joint distribution (Clayton copula + Weibull marginal)		
	β	η	κ (shape)	λ (scale)	Association parameter
1	1.200	1.000	2.000	1.128	1.000
2	1.200	1.000	2.000	1.128	

2) Scenario II: examine the effect of dependency in copula.

We use the Weibull marginal distribution, power law trend function with increasing trend, and the Gaussian copula. By choosing different values of the copula, we consider three situations: component failure independency, moderate failure dependency, and strong failure dependency. For the moderate dependency case, the simulation parameters setting are the same as listed in Table 5. For independency and the strong dependency cases, we set the values of the correlation coefficients to be 0 and 0.9, respectively, while all other parameters are the same as those in Table 1.

3) Scenario III: Examine the effect of marginal distribution:

We use the power law trend function with increasing trend and Gaussian copula. We consider two marginal distributions: the Weibull and the lognormal distribution. For the Weibull case, the parameters are the same as listed in Table 5. The parameters for the lognormal distribution case are listed in Table 7.

Table 7. Parameter setting in simulation Scenario III (lognormal marginal)

Component	Trend function		Joint distribution (Gaussian copula + lognormal marginal)				
	$\boldsymbol{\beta}$	$\boldsymbol{\eta}$	$\boldsymbol{\mu}$	$\boldsymbol{\sigma}$	Mean	Correlation matrix	
1	1.200	1.000	-0.125	0.500	0	1.000	0.500
2	1.200	1.000	-0.125	0.500	0	0.500	1.000

4) Scenario IV: examine the effect of trend function:

We use the Weibull marginal distribution and Gaussian copula function. We consider three situations of the power law trend function: increasing trend, constant, or decreasing trend. For increasing trend function case, the parameters are the same as those in Table 5. For constant and decreasing trend functions, we set $\boldsymbol{\beta}=[1.0,1.0]'$ and $\boldsymbol{\beta}=[0.8,0.8]'$, respectively, while all the other parameters are the same as those in Table 1.

5) Scenario V: validate the parameter estimation method:

We use the Weibull marginal distribution, Gaussian copula function, and the power law trend function with increasing trend. A three-component system is considered, and the parameters are given in Table 8.

Table 8. Parameter setting in simulation Scenario V

Component	Trend function		Joint distribution (Gaussian copula + Weibull marginal)					
	β	**η**	**κ** (shape)	λ (scale)	Mean	Correlation matrix		
1	1.200	1.000	2.000	1.128	0	1.000	0.100	0.400
2	1.200	1.000	2.000	1.128	0	0.100	1.000	0.800
3	1.200	1.000	2.000	1.128	0	0.400	0.800	1.000

3.2.6.2 Parameter Estimation

In the simulation study, we vary the value of stopping time τ to obtain different values of the expected number of events. We think it is more informative to show the number of events, instead of the value of τ. We consider four different numbers of events for each scenario, i.e., 100, 200, 500 and 1000 respectively.

To evaluate the performance of the parameter estimation method, we calculate both the MSEs of estimators and the coverage probabilities for the 95% confidence intervals based on 1000 replicates under each parameter setting. Fig. 8 – Fig. 15 plot the MSEs (left) and coverage probabilities (right). From Fig. 8 - Fig. 15, we can see that when the sample size is large enough, the MSEs are approaching to zero, and the coverage probabilities of 95% confidence intervals for the unknown parameters are approaching 95%. Thus, the estimators of the parameters perform well.

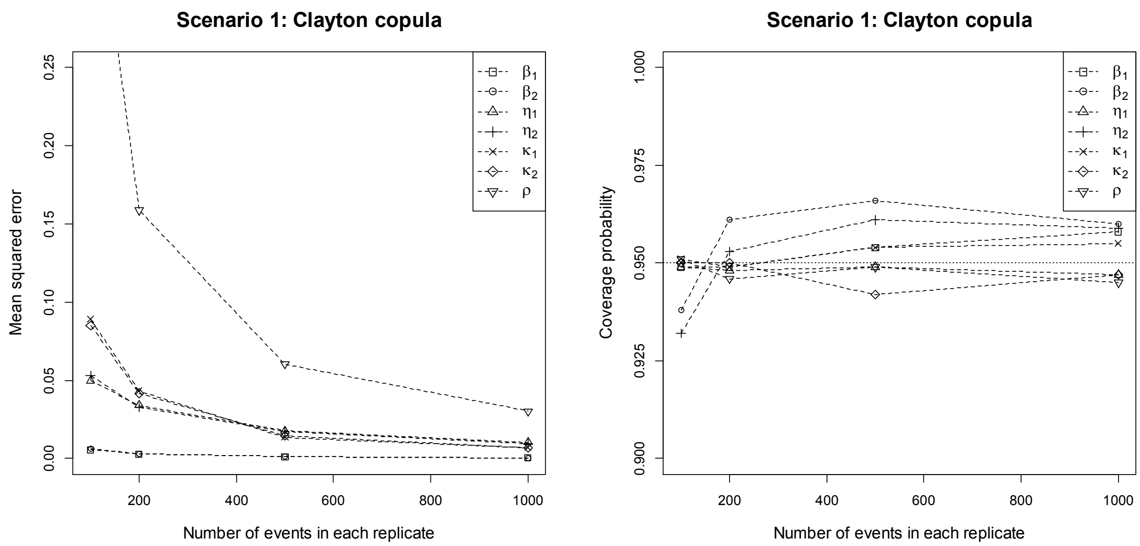

Fig. 8. Simulation results for scenario 1 with Clayton copula

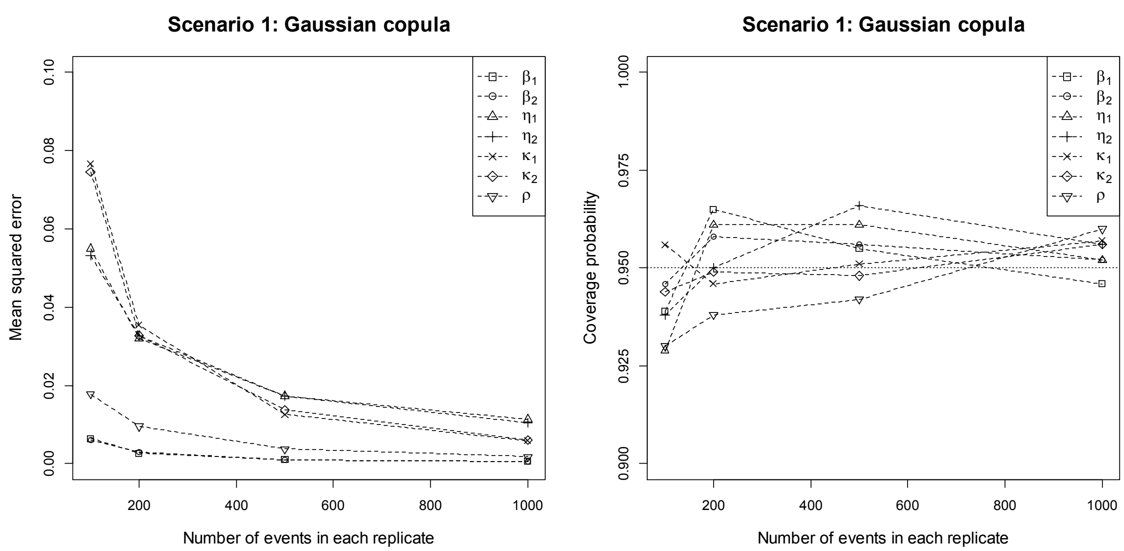

Fig. 9. Simulation results for scenario 1 with Gaussian copula.

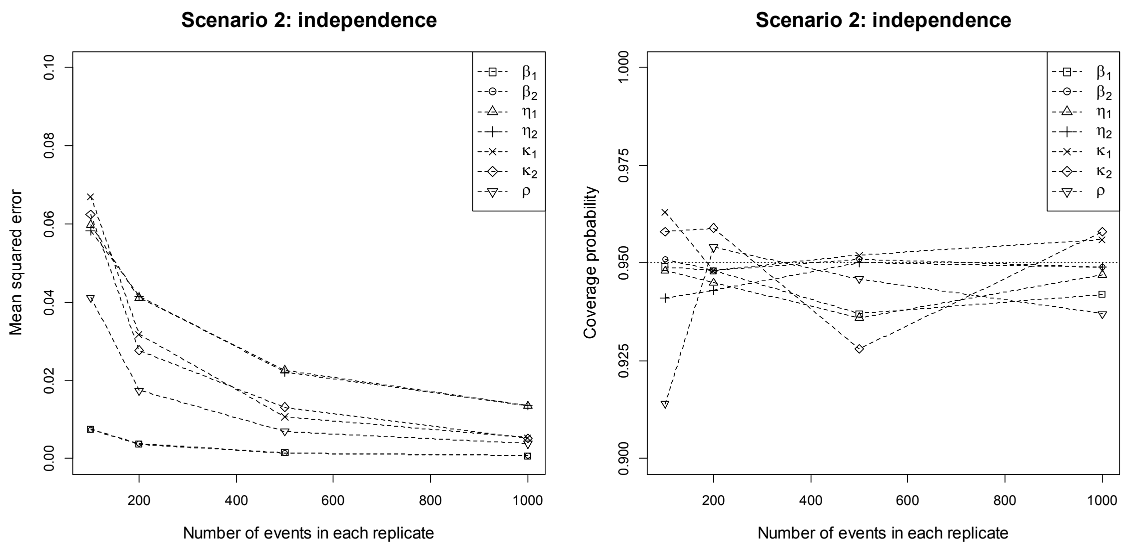

Fig. 10. Simulation results for scenario 2 with independent failures

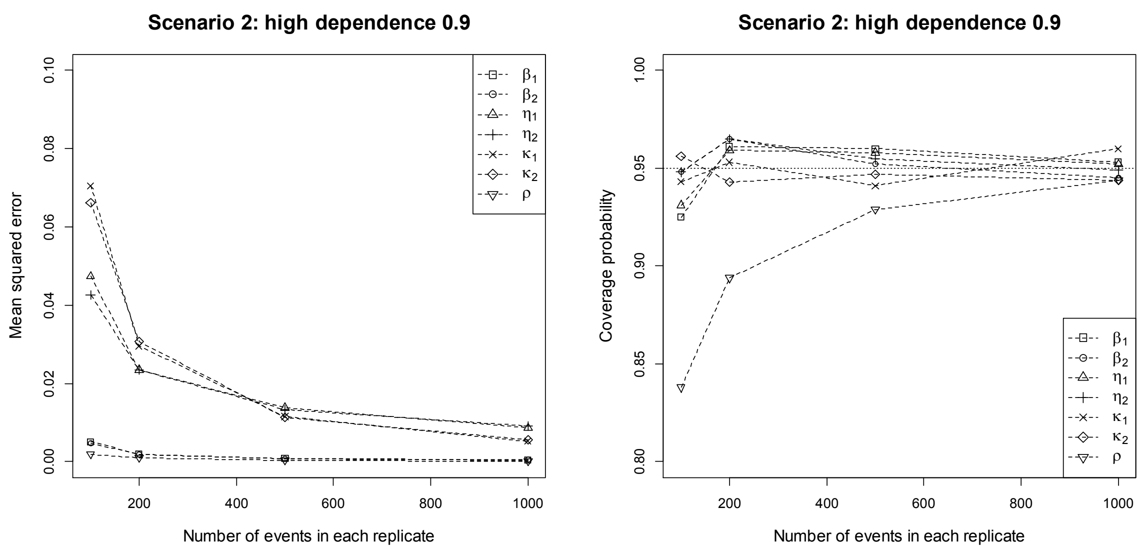

Fig. 11. Simulation results for scenario 2 with high dependency

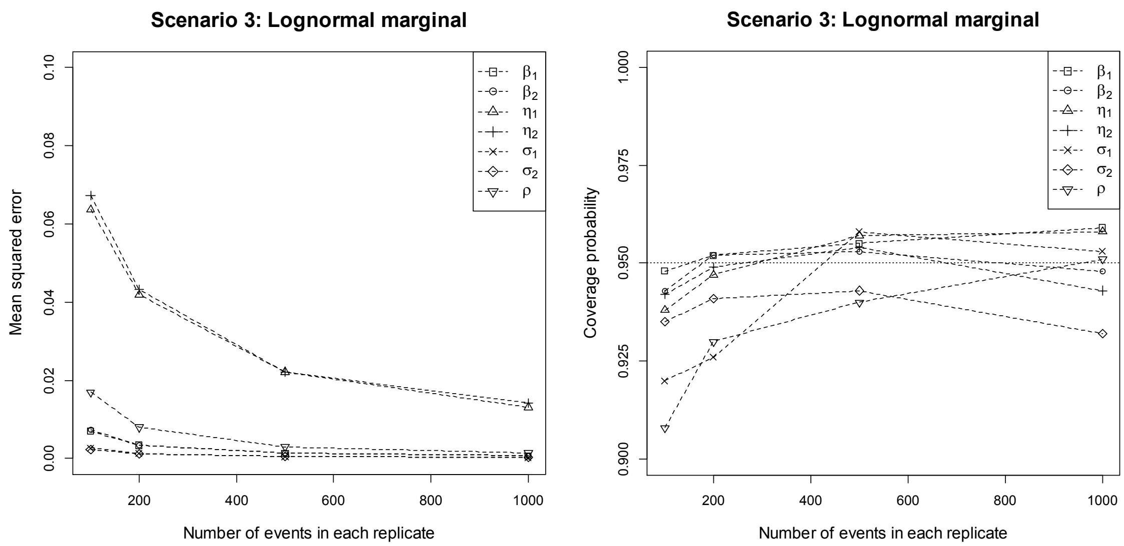

Fig. 12. Simulation results for scenario 3 with lognormal marginal

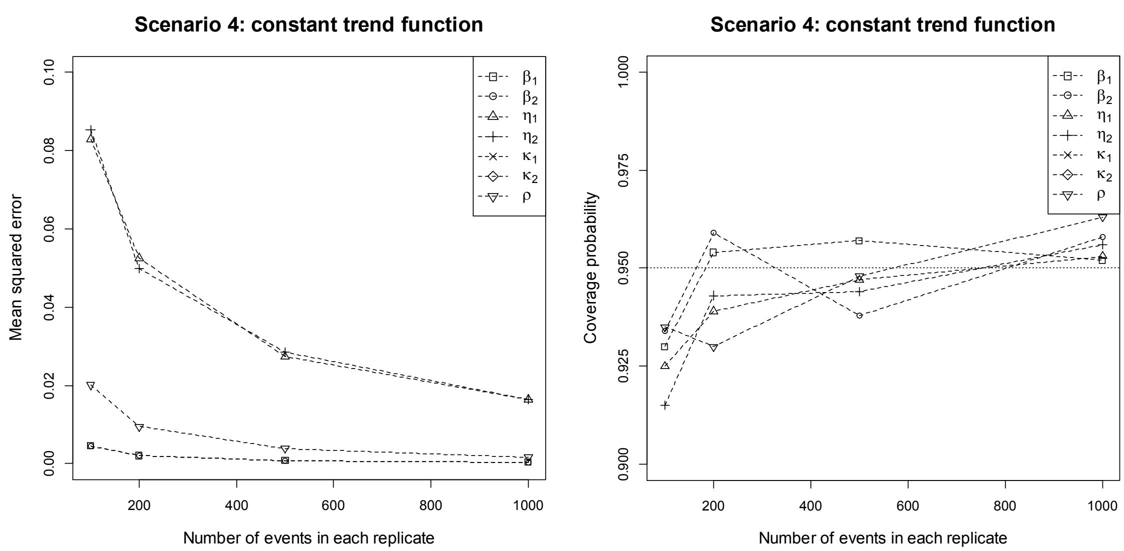

Fig. 13. Simulation results for scenario 4 with constant trend function

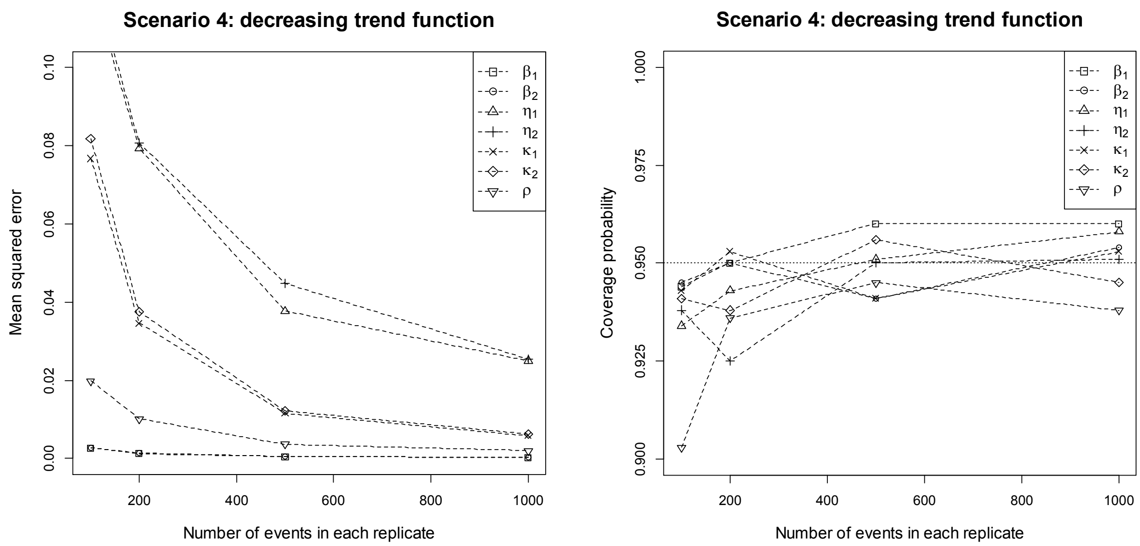

Fig. 14. Simulation results for scenario 4 with decreasing trend functions

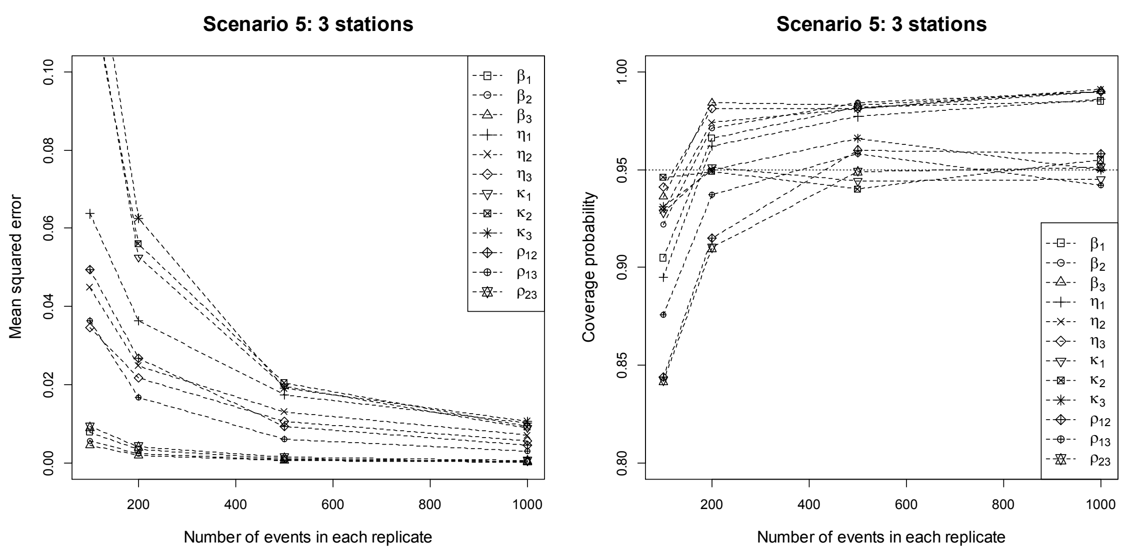

Fig. 15. Simulation results for scenario 5 with 3 stations

3.2.6.3 Case Study

In this case study, we apply the proposed CTP model for the assembling cell data. In addition to the two stations' failure history data used in section 3.1.6, we further another stations' failure history. And these three stations are denoted by stations A, B, and C in this section.

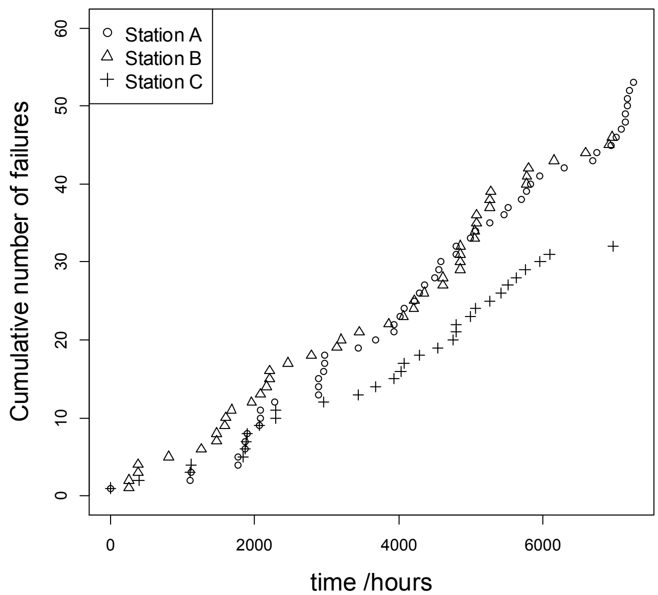

Fig. 16. Failure data from stations A, B and C

We first apply the developed method to the three-station assembling process. The overall likelihood is obtained by substituting (27) and (28) into (26). The parameters are estimated by maximizing the likelihood function. When applying the Clayton copula, the estimated parameters and the standard errors are listed in the following Table 9.

Table 9. Parameter estimates and standard errors (values in the bracket) when choosing the Clayton copula

failure type	Trend function		Joint distribution		
	$\hat{\boldsymbol{\beta}}$	$\hat{\boldsymbol{\eta}}$	$\hat{\boldsymbol{\kappa}}$	$\hat{\boldsymbol{\lambda}}$	Association parameter ρ
A	1.17(0.27)	254.73(208.22)	0.64(0.07)	0.72(0.09)	
B	1.22(0.32)	387.44(312.43)	0.52(0.07)	0.54(0.12)	0.0001(0.003)
C	0.98(0.23)	223.27(190.84)	0.74(0.11)	0.83(0.10)	

When applying the Gaussian copula to obtain the joint distribution, the estimated parameters and the corresponding standard errors are listed in Table 10.

Table 10. Parameter estimates and standard errors (values in the bracket) when choosing the Gaussian copula

failure type	Trend function		Joint distribution				
	$\hat{\boldsymbol{\beta}}$	$\hat{\boldsymbol{\eta}}$	$\hat{\boldsymbol{\kappa}}$	$\hat{\boldsymbol{\lambda}}$	$\hat{\boldsymbol{\Sigma}}$		
A	1.26(0.29)	192.24(160.78)	0.53(0.08)	0.56(0.14)	1.00	0.11(0.17)	0.40(0.13)
B	1.24(0.31)	348.52(301.17)	0.50(0.08)	0.50(0.15)	-	1.00	0.01(0.23)
C	1.03(0.27)	208.62(179.38)	0.69(0.12)	0.78(0.14)	-	-	1.00

Hypothesis tests are applied in the case study to examine the failure dependency structure of the stations. When applying hypothesis test (31) to test the overall failure dependency, $\sup\{\mathcal{L}_n\}$ equals -768.4394 and $\sup\{\mathcal{L}_f\}$ equals -768.4388. Based on (30), test statistic D is calculated as 0.001, As D follows a chi-square distribution with degree of freedom, $10-9=1$, the p-value is obtained as 0.97. Thus, H_0 in hypothesis test (31) cannot be rejected, which indicates that all failure types are independent.

When applying hypothesis test (32) on the case study to test the pairwise dependency, Table 11 lists the maximum log-likelihood values for the full model and those for the null model. Based on (30), test statistics D are calculated, and the corresponding p-values are listed in Table 12.

Table 11. Maximum log-likelihood values for pair-wise dependency tests

Full model	Null model		
	Independent A, B	Independent A, C	Independent B, C
-764.991	-765.183	-767.993	-765.016

Table 12. p-values for dependency test from Gaussian copula

Station	A	B	C
A	-	0.54	0.01
B	-	-	0.82
C	-	-	-

From Table 12, it can be seen that Stations (A, B) and Stations (B, C) are independent while Stations (A, C) are dependent. By consulting the process engineers, it is found that the same part of the working piece is processed in both stations A and C. As the tools in station A degrades, the assembled part, called part I, tends to have a large deviation to the desired position. When the working piece reaches station C, as the assembled part contact part I, the large position deviation of part I can apply an undesired force or vibration to the assembling tool in station C, which accelerates its degeneration.

By comparing the log-likelihood values in Table 11, we can see there is significant improvement when considering the station failure dependency. However, the reliability model via the Clayton copula shows overall station failure dependency. This can be explained as two out of three pairs' stations show independency in the reliability model via the Gaussian copula. In practice, when the pairwise dependency is more interesting, Gaussian copula is preferred rather than Clayton copula to construct the reliability model.

3.3 Conclusion

In this chapter, two reliability models for multi-component systems subject to competing risks considering imperfect repair conditions are proposed based on the partially perfect repair model proposed in CHAPTER 2.

First, we initially propose a generalized dependent latent age model for repairable multi-component systems under dependent competing risks. In the proposed model, after an imperfect repair the initial ages of components change so that the new latent ages to failure follow a truncated joint distribution conditional on the initial ages. The dependency of component failures is then captured by the joint distribution which can be constructed via copula functions. In the meanwhile, the imperfect repairs are quantified by the repair effectiveness factors. The parameters in the proposed model are estimated using the MLE method.

Second, a CTP model which is an extension of traditional trend-renewal process model from single-component system to multi-component systems, to deal with imperfect component repair from perfect to minimal, and to capture the dependency among different failure types. Specifically, we extend the TRP model for single-component systems to competing-risk systems by transforming original failure times into new time domains for each component respectively. Then, the dependency of different component failures is captured by a joint distribution established from marginal in the transformed time domains. The model parameters are estimated using the ML method. The dependency is further examined by the suggested hypothesis tests.

For both proposed reliability models, simulation studies and case studies are conducted for verification and illustration.

The presented GDLA model has been accepted for publication (Zhang and Yang 2014).

CHAPTER 4. INSPECTION-BASED OPTIMAL MAINTENANCE PLANNING

4.1 Developed Maintenance Policies

Based on the GDLA reliability model proposed in section 3.1, we develop inspection-based maintenance policies at both the system level (MP I) and the component level (MP II) for repairable multi-component systems under dependent competing risks. Inspection-based maintenance is a commonly used maintenance model in the literature (Nakagawa 1984, Scarf 1997, Chen, Chen, et al. 2003, Wang, Chu, et al. 2009, Wang and Pham 2011). Under an inspection-based maintenance policy, replacement can be implemented only after the detection of a failure upon inspections. Specifically, periodical inspections are scheduled at discrete times $i\omega; i = 1, \ldots, n$ in the developed policies. We assume the time of inspection is negligible and all failures can be detected by inspections correctly.

The developed MP I assumes that once a failure is detected the entire system will be replaced perfectly. In contrast, only the failed component will be repaired imperfectly with certain repair effectiveness factor once a failure is detected in MP II. We assume the repair effectiveness factor is fixed during the entire operation period. Fig. 17 illustrates how we implement MP I (left) and MP II (right) based on the GDLA model for a simple two-component system. The entire system is immediately replaced once failures are detected on inspections at Fig. 17 (left), and the ages of both components in the renewed system return to zero. At Fig. 17 (right), only the component with a detected failure is imperfectly repaired and its initial age is reduced according to the repair effectiveness factor.

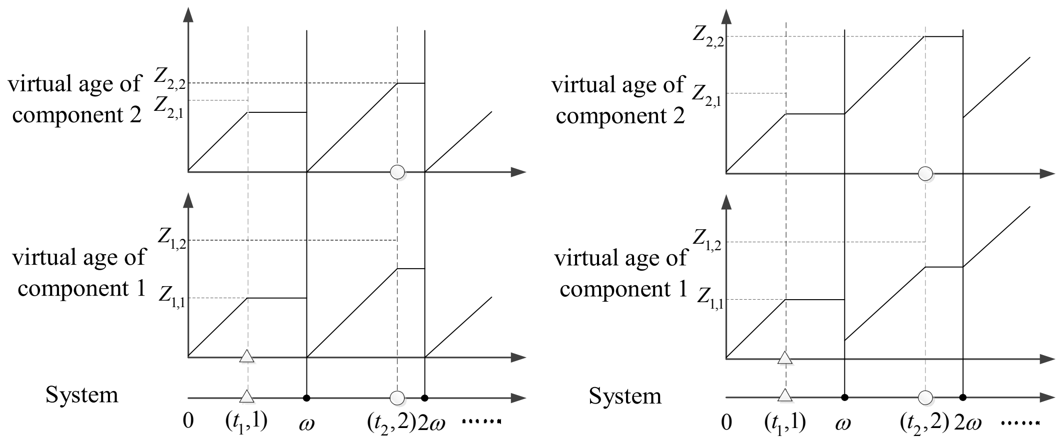

Fig. 17. MP I (left) and MP II (right) for a two-component system

4.2 Optimization of Maintenance Policies

We use the average long run cost rate denoted by $r(\omega)$ as the maintenance planning criteria, since it is generally used in the literature (Yeh 1988, Grall, Dieulle, et al. 2002, Li and Pham 2005). The objective of maintenance planning is to find the optimal inspection interval ω_0 so that the average long run cost rate is minimized. Let $C(t)$ denote the cumulative maintenance cost until time t. The average long run cost rate can be calculated as $r(\omega)=\lim_{t\to\infty}\{E[C(t)]/t\}$. The costs incurred in periodic inspections include the single inspection cost C_I, the downtime cost per unit time C_D, and the repair cost $C_{R,l}$ for component $l; l = 1,...,K$. The repair cost $C_{R,l} = C_{M,l} + C_{L,l}$, where $C_{M,l}$ is the material cost and $C_{L,l}$ is the labor cost. For MP I, $C_{M,l} = C_l$, where C_l is the cost of a new component l. For MP II, the material cost depends on both the price of a new component and the repair effectiveness factor according to the relation $C_{M,l} = (1-q_l)\cdot C_l$. We also assume the labor cost does not change with respect to different repair effectiveness factors. In addition, we assume the repair cost C_R for the entire system equals the summation of costs to repair each component, i.e., $C_R = \sum_{l=1}^{K} C_{R,l}$.

4.2.1 Optimization of MP I

Under MP I, the entire system will be replaced once a failure is detected on inspections. Thus, the renewal process can be used to model the failure and maintenance process (Pham and Wang 1996, Li and Pham 2005, Ross 2006). According to the renewal theory, the average long run cost rate is calculated as:

$$r(\omega) = \lim_{t\to\infty}\left\{E\left[C(t)\right]/t\right\} = \frac{E(C_L)}{E(L)} \tag{33}$$

where L is the length of a renewal cycle which is defined as time interval from the beginning to the first replacement or the time interval between two consecutive system replacements; C_L is the overall maintenance cost in one renewal cycle; and $E(L)$ and $E(C_L)$ denote the expectation of L and C_L respectively.

As the system can only be replaced on inspections, the length of a renewal cycle L has to be $i\omega; i = 1, 2, \ldots$. Let $E(N_I)$ denote the expected number of inspections in one renewal cycle; then $E(L) = \omega \cdot E(N_I)$. Generally the number of inspections i required in a renewal cycle is constrained by $(i-1)\omega < T_{\min} \leq i\omega$, where $T_{\min}$ is the inter-arrival time between a replacement and the next failure, which was defined in Section 2.4. We calculate the expected number of inspections required in one renewal cycle as follows:

$$
\begin{aligned}
E(N_I) &= \sum_{i=1}^{\infty} i \cdot \Pr(N_I = i) \\
&= \sum_{i=1}^{\infty} i \cdot \Pr((i-1)\omega < T_{\min} \leq i\omega) \\
&= \sum_{i=1}^{\infty} i \cdot \left[F_{T_{\min}}(i\omega) - F_{T_{\min}}((i-1)\omega) \right]
\end{aligned}
\tag{34}
$$

where $F_{T_{\min}}(t) = 1 - R(t \mid v_1(t_i) = 0, \ldots, v_K(t_i) = 0)$ can be calculated from (23). Based on (34), the expected length of a renewal cycle is obtained as:

$$
E(L) = \sum_{i=1}^{\infty} i\omega \cdot \left[F_{T_{\min}}(i\omega) - F_{T_{\min}}((i-1)\omega) \right] \quad . \tag{35}
$$

Given the inspection number i in one renewal cycle, the downtime η equals $i\omega - T_{\min}$. Hence the expected downtime in one renewal cycle can be calculated as follows:

$$\begin{aligned}
E(\eta) &= \sum_{i=1}^{\infty} E(\eta | N_I = i) \cdot \Pr(N_I = i) \\
&= \sum_{i=1}^{\infty} E(i\omega - T_{\min} | N_I = i) \cdot \Pr((i-1)\omega < T_{\min} \le i\omega) \\
&= \sum_{i=1}^{\infty} \int_{(i-1)\omega}^{i\omega} (i\omega - t) \frac{f_{T_{\min}}(t)}{F_{T_{\min}}(i\omega) - F_{T_{\min}}((i-1)\omega)} dt \cdot \left[F_{T_{\min}}(i\omega) - F_{T_{\min}}((i-1)\omega) \right] \\
&= \sum_{i=1}^{\infty} \int_{(i-1)\omega}^{i\omega} (i\omega - t)\, dF_{T_{\min}}(t)
\end{aligned} \quad (36)$$

Because the maintenance cost C_L in a renewal cycle equals $E(N_I) \cdot C_I + E(\eta) \cdot C_D + C_R$, the average long run cost rate in (33) can be rewritten as:

$$\lim_{t \to \infty} \left\{ E\left[C(t) \right] / t \right\} = \frac{E(C_L)}{E(L)} = \frac{E(N_I) \cdot C_I + E(\eta) \cdot C_D + C_R}{E(L)}. \quad (37)$$

Substituting (34), (35) and (36) into (37), the objective function is further written as a function of the inspection interval ω as follows:

$$r(\omega) = \frac{\sum_{i=1}^{\infty} i \left[F_{T_{\min}}(i\omega) - F_{T_{\min}}((i-1)\omega) \right] \cdot C_I + \sum_{i=1}^{\infty} \int_{(i-1)\omega}^{i\omega} (i\omega - t)\, dF_{T_{\min}}(t) \cdot C_D + C_R}{\sum_{i=1}^{\infty} i\omega \left[F_{T_{\min}}(i\omega) - F_{T_{\min}}((i-1)\omega) \right]}. \quad (38)$$

It is challenging to obtain a closed-form solution of the optimal inspection interval ω_0 that can minimize (38). Instead, we calculate the optimal inspection interval of MP I using numerical optimization methods, i.e., a simulated annealing method (Van Laarhoven and Aarts 1987, Bélisle 1992).

4.2.2 Optimization of MP II

Under MP II, as only the failed component is repaired, the entire system cannot be modeled by a regenerative process, making the optimization of MP II more challenging than that of MP I. To overcome this difficulty, we propose an iterative simulation-based optimization method as illustrated in Fig. 18.

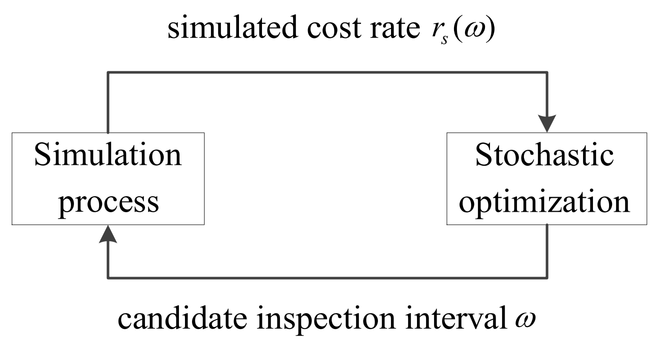

Fig. 18. Simulation-based optimization method with stochastic approximation

In the developed simulation-based optimization method, we utilize a simulation process to approximate the objective function $r(\omega)$ instead of evaluating the exact objective function. Specifically, a designed simulation process will simulate the average cost rate $r_s(\omega)$ given the inspection interval ω in a finite time horizon $[0,T]$. When T goes to infinite, the expectation of the simulated cost rate $E[r_s(\omega)]$ is approaching the true average long run cost rate $r(\omega)$. The simulated cost rate $r_s(\omega)$ will be fed to an optimization method as input. Considering that the simulated $r_s(\omega)$ is a random variable, a deterministic optimization method cannot be applied. In this research, we apply the finite difference stochastic approximation (FDSA) algorithm (Fu, Glover, et al. 2005) as the optimization method to deal with the random input $r_s(\omega)$. Specifically, the FDSA algorithm would stochastically estimate the minimum of a function, which is the expectation of the simulated cost rates in our problem. The FDSA algorithm will update the candidate inspection interval and then pass it to the simulation process. Under appropriate conditions, the FDSA algorithm can convergence in probability (Spall 2003). This iterative simulation-based optimization process will stop when the maximum number of iterations is reached.

To simulate the average cost rate $r_s(\omega)$ on a finite time horizon $[0,T]$ within the simulation process, first the useful simulation statistics such as the number of replacements are initialized. Second, a random vector of the dependent latent ages conditional on their initial ages is generated to determine the next failure time; then the required number of inspections needed to detect the next failure is calculated. Afterwards, the initial ages will be updated according to MP II. Step two will be repeated until the whole time horizon is spanned by inspections. The detailed simulation process is described in the following algorithm:

Algorithm 1:

1. Let $i=0$. Initialize the cumulative number of component failures of each component $N_{R,l}=0; l=1,\ldots,K$, and the cumulative downtime $T_D=0$; set the initial ages $[v_1(t_i),\ldots,v_K(t_i)]^T=\mathbf{0}$;

2. Let $i = i + 1$. Generate the latent ages to the i^{th} failure $\left[Z_{1,i},\ldots,Z_{K,i}\right]^T \sim F$ given $Z_{l,i} > v_l\left(t_{i-1}\right); l = 1,\ldots,K$; solve $k_1 \in N^+$ according to $\left(k_1 - 1\right)\omega < \min\left(Z_{1,i} - v_1\left(t_{i-1}\right),\ldots,Z_{K,i} - v_K\left(t_{i-1}\right)\right) \leq k_1\omega$; if $\min\left(Z_{1,i} - v_1\left(t_{i-1}\right),\ldots,Z_{K,i} - v_K\left(t_{i-1}\right)\right) = Z_{l,i} - v_l\left(t_{i-1}\right)$, then $N_{R,l} = N_{R,l} + 1$, $T_D = T_D + [k_1\omega - \left(Z_{l,i} - v_l\left(t_{i-1}\right)\right)]$, $v_l\left(t_i\right) = Z_{l,i} \cdot q_l$ and $v_j\left(t_i\right) = v_j\left(t_{i-1}\right) + \left(Z_{l,i} - v_l\left(t_{i-1}\right)\right), j \neq l$;
3. Repeat step 2 until $\sum_i k_i\omega \geq T$; and then calculate $r_s\left(\omega\right) = \left(\sum_i k_i \cdot C_I + \sum_{l=1}^K N_{R,l} \cdot C_{R,l} + T_D \cdot C_D\right) / T$.

An important step in Algorithm 1 is to generate the random latent ages conditional on the initial ages. When F is chosen as a multivariate lognormal distribution, it can be conducted by sampling from a multivariate normal distribution (Robert 1995), and then taking the logarithm of the samples. If F is a general joint distribution constructed by a Gaussian copula, we developed algorithm 2 to generate the latent ages to failure conditional on the initial ages, which is given in Appendix 1.

The simulated cost rate obtained from Algorithm 1 is passed on to the FDSA algorithm. The FDSA algorithm mimics the gradient descent algorithm for deterministic optimization problems, i.e., during each step the gradient of the objective function is estimated in order to find the local minimum. Specifically, the iterative procedure of FDSA algorithm is described in algorithm 3 in Appendix 2.

4.3 Case Study

In this section, we apply the developed maintenance policies for the two stations in the assembling cell introduced in section 3.1.6, where the model parameters have been estimated based on the developed GDLA model.

4.3.1 Optimal Maintenance Policies

The cost parameter setting used in the maintenance planning is listed in Table 13.

Table 13. Cost parameter setting in the maintenance

Inspection cost	Downtime cost per unit time	Repair cost of each component	
		New component cost	Labor cost
C_I=\$10	C_D=\$50	$C_1 = C_2$=\$80	$C_{L,1} = C_{L,2}$=\$20

1) Optimal solution for MP I:

To minimize the objective function $r(\omega)$ given in (38), we apply the simulated annealing method to obtain the optimal inspection interval ω_0=4.63 and optimal average long run cost rate $r(\omega_0)$= 8.07. The expected number of inspections in one renewal cycle $E(N_I)$ given in (34) equals 12.03, and the expected downtime $E(\eta)$ given in (36) equals 2.59.

2) Optimal solution for MP II:

Under most situations including the problem in this case study, only limited number of repair effectiveness factors can be used for the maintenance. For example, the materials or parts used in the repairs for the failed component can have different qualities due to the manufacturers and prices, which may result in different levels of repair effectiveness. In the case study, both the estimated repair effectiveness factors for the two components are greatly close to zero. Thus, we select zero as one level of repair effectiveness. In addition 0.2 is selected as another level which corresponds to imperfect repair. In total, we have two levels of repair effectiveness and four combinations of these two levels for the two-component system.

To obtain the optimal solution for MP II, we apply the simulation-based optimization method on a finite time horizon [0, 50000]. Table 14 lists the optimal inspection intervals and cost rates obtained for four combinations of repair effectiveness, which are optimized by using the simulation-based optimization approach.

Table 14. Optimal MP II results with four combinations of repair effectiveness levels

Repair effectiveness factors	$q_1 = 0, q_2 = 0$	$q_1 = 0, q_2 = 0.2$	$q_1 = 0.2, q_2 = 0$	$q_1 = 0.2, q_2 = 0.2$
Optimal inspection interval	6.47	6.91	7.41	7.70
Optimal average long run cost rate	\$4.06	\$3.73	\$3.41	\$3.21

Comparing with the optimal cost rate of MP I, MP II that considers different repair effectiveness factors has result of smaller average long run cost rates. Within these four combinations of repair effectiveness in MP II, $[q_1, q_1]' = [0.2, 0.2]'$ leads to smallest optimal average long run cost rate.

4.3.2 Comparison of Maintenance Planning Results with and without Considering Failure Dependency

In order to verify the necessity of considering failure dependency in the proposed method, we apply the MP I for a two-component system. First we simulate 1000 failure events using the parameters which are near the estimated parameters in the case study. Then the parameters of the proposed model are estimated with and without failure dependency. When we estimate parameters without dependency, the correlation coefficient of the Gaussian copula is fixed to zero. The real parameters used in the data simulation and estimated parameters with and without dependency are listed in Table 15.

Table 15. Real and estimated parameters with and without considering failure dependency

	Real parameters used in simulation	Estimated parameters with failure dependency	Estimated parameters without failure dependency
Shape parameters	$[0.55\ 0.70]'$	$[0.57\ 0.66]'$	$[0.69\ 0.80]'$
Scale parameters	$[50\ 140]'$	$[52\ 124]'$	$[119\ 264]'$
Correlation matrix of Gaussian copula	$\begin{bmatrix} 1.00 & 0.40 \\ 0.40 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.41 \\ 0.41 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.00 \\ 0.00 & 1.00 \end{bmatrix}$

We apply MP I for these three sets of parameters and obtain the optimal inspection intervals listed in Table 16.

Table 16. Optimal inspection intervals for different parameter sets

	For real parameters used in simulation	For estimated parameters with failure dependency	For estimated parameters without failure dependency
Optimal Inspection interval	4.65	4.59	5.82

From Table 16, the optimized inspection interval that was obtained using parameters estimated with failure dependency is more close to the optimal interval obtained using real parameters. When the inspection intervals are 4.59 and 5.82, the corresponding cost rates calculated from real parameters used in simulation equal \$8.01 and \$8.11, respectively. Thus, by considering the failure dependency, the optimal maintenance planning is more economical than that without considering failure dependency based on the proposed methodology.

4.4 Conclusion

Based on the proposed GDLA model in section 3.1, we develop both system level and component level inspection-based maintenance policies for multi-component systems under dependent competing risks. The developed maintenance policies utilize the dependency information and repair effectiveness factors estimated from the reliability model. An analytical optimization method is given using the renewal theory for the system level maintenance policy. Due to the complex characteristics of both the failure process and maintenance process, we propose a simulation-based optimization approach to find the optimal solution for component level maintenance policy. A case study is conducted to apply the proposed methodology to a multi-component system in an automobile power-train plant.

As conducted in the paper, the first concern to select the appropriate maintenance policy from the two proposed policies is the average long run cost rate. Thus, the maintenance decision maker would choose the best maintenance policy which provides lower average long run cost rate. For the case study, MP II is better than MP I regarding the average long run cost rate. However, under some specific situations the multi-component system has structural correlation. As a result it would be much more difficult to replace a single component rather than to replace the entire system. A simple example is the union formed by a bicycle chain and a cassette, which should always be replaced as an entirety (Nicolai and Dekker 2008).

The presented methodology in this Chapter has been accepted for publication (Zhang and Yang 2014).

CHAPTER 5. GENERAL CONCLUSIONS

In this dissertation, we developed general statistical methodology for reliability modelling of multi-component systems subject to dependent competing risks with different repair conditions. Based on one of the proposed model, optimal maintenance planning is also studied.

In CHAPTER 2, a general statistical reliability model is proposed for repairable multi-component systems considering statistical dependent competing risks under a partially perfect repair assumption. Maximum likelihood estimation of model parameters is developed. Based on the proposed reliability model, hypothesis tests for components' failure dependency are established.

In CHAPTER 3, we proposed two statistical reliability models, i.e., GDLA model and CTP model for repairable multi-component systems considering both statistical dependent competing risks and the generally imperfect repair conditions. For both models, maximum likelihood estimation methods are developed to estimate the model parameters. Simulation studies are conducted to assess the performance of both models. And case studies using failure history data collected from an automobile plant are given for illustration. It is worthwhile to note that the partially perfect reliability model proposed in CHAPTER 2 can be treated as extreme cases for both GDLA model and CTP model.

In CHAPTER 4, based on the proposed GDLA reliability model, both system and component level periodic inspection-based maintenance polices are considered for repairable multi-component systems that are subject to dependent competing risks. Under the system level maintenance policy, the entire system is restored to as good as new once a failure is detected. While under the component level maintenance policy, only the failed component is repaired imperfectly. We obtain the optimal solution of the system level policy by using renewal theory. The optimal solution of the component level policy, however, cannot be obtained analytically, due to its complex failure and repair characteristics. We developed a simulation-based optimization approach with stochastic approximation to solve the optimization problem for the component level policy. The developed methods are illustrated by using a cylinder head assembling cell that consists of multiple stations.

APPENDIX 1. Generation of Latent Ages to Failure from Truncated Distribution Constructed via Gaussian Copula

Conditional on the initial ages $[v_1(t_{i-1}),\ldots,v_K(t_{i-1})]^T$, the following algorithm can be used to generate the latent ages to failure $\left[Z_{1,i},\ldots,Z_{K,i}\right]^T$ from joint distribution F.

Algorithm 2:

set $\left[Z_{1,i},\ldots,Z_{K,i}\right]^T = \mathbf{0}$;

while $\exists l \in \{1,\ldots,K\}, Z_{l,i} < v_l(t_{i-1})$:

generate $[U_1,\ldots,U_K]^T$ from $\mathcal{N}(0,\mathbf{\Sigma})$;

$\forall l \in \{1,\ldots,K\}, p_l = \Phi(\mathrm{U}_l), \mathrm{Z}_l = F^{-1}(p_l,\boldsymbol{\theta}_l)$;

end

return $\left[Z_{1,i},\ldots,Z_{K,i}\right]^T$

where $\mathbf{\Sigma}$ is the correlation matrix in the Gaussian copula function; $\Phi(\cdot)$ denotes the standard normal cdf; and $F^{-1}(\cdot)$ denotes the inverse cdf of the marginal distribution for component l with parameters $\boldsymbol{\theta}_l$.

APPENDIX 2. FDSA Algorithm Applied in MP II for Optimization

Algorithm 3:

1. Initialize $\hat{\omega}_1$;

2. Calculate the estimate of the gradient of $E[r_s(\hat{\omega}_i)]$, i.e.,

$$\hat{g}_i(\hat{\omega}_i) = \frac{r_s\left(\hat{\omega}_i + c/(i+1)^{\gamma}\right) - r_s\left(\hat{\omega}_i - c/(i+1)^{\gamma}\right)}{2c/(i+1)^{\gamma}}$$

and update $\hat{\omega}_i$ according to

$$\hat{\omega}_{i+1} = \hat{\omega}_i - \frac{a}{(i+1+A)^{\alpha}}\hat{g}_i(\hat{\omega}_i)$$

3. Repeat step 2 until the maximum number of iterations is reached

where a, A, c, α and γ are constants, which should be selected to satisfy certain conditions so that $\hat{\omega}_{i+1}$ will almost surely converge to the optimal solution (Spall 2003).

APPENDIX 3. Proof of Proposition 3

Note that $V_{k,i}$ is defined as the latent age to failure of component k after the $(i-1)^{th}$ failures in the k^{th} transformed time domain. Let $W_{k,i}$ be the corresponding random variable for the age to failure of component k after the $(i-1)^{th}$ failure in the original time domain. Suppose that the i^{th} system failure occurs at time point t_i in the original time domain. Because $r_k(t_i)$ is left continuous, $r_k(t_i) = r_k(t_{i-1}^+)$. Thus,

$$V_{k,i} = \Lambda_k[W_{k,i} + r_k(t_i)] - \Lambda_k[r_k(t_i)] = \Lambda_k[W_{k,i} + r_k(t_{i-1}^+)] - \Lambda_k[r_k(t_{i-1}^+)]$$

As $b_k(t_i) = \Lambda_k(t_i) - \Lambda_k[r_k(t_i)]$, and $b_k(t_i)$ is also left continuous. Thus,

$$b_k(t_{i-1}^+) = \Lambda_k[a_k(t_{i-1}^+) + r_k(t_{i-1}^+)] - \Lambda_k[r_k(t_{i-1}^+)] = \Lambda_k(t_{i-1}) - \Lambda_k[r_k(t_i)].$$

Note that

$$\begin{aligned} W_{k,i} > a_k(t_i) &\Leftrightarrow W_{k,i} + r_k(t_i) > a_k(t_i) + r_k(t_i) \\ &\Leftrightarrow \Lambda_k[W_{k,i} + r_k(t_i)] > \Lambda_k[a_k(t_i) + r_k(t_i)] \\ &\Leftrightarrow \Lambda_k[W_{k,i} + r_k(t_i)] - \Lambda_k[r_k(t_i)] > \Lambda_k[a_k(t_i) + r_k(t_i)] - \Lambda_k[r_k(t_i)] \\ &\Leftrightarrow V_{k,i} > b_k(t_i). \end{aligned} \tag{39}$$

Similarly,

$$W_{k,i} > a_k(t_{i-1}^+) \Leftrightarrow V_{k,i} > b_k(t_{i-1}^+)\,. \tag{40}$$

In addition,

$$\begin{aligned} W_{k,i} = a_k(t_i) &\Leftrightarrow a_k(t_i) \le W_{k,i} < a_k(t_i) + dt \\ &\Leftrightarrow a_k(t_i) + r_k(t_i) \le W_{k,i} + r_k(t_i) < a_k(t_i) + r_k(t_i) + dt \\ &\Leftrightarrow \Lambda_k[a_k(t_i) + r_k(t_i)] \le \Lambda_k[W_{k,i} + r_k(t_i)] < \Lambda_k[a_k(t_i) + r_k(t_i) + dt] \\ &\Leftrightarrow \Lambda_k[a_k(t_i) + r_k(t_i)] - \Lambda_k[r_k(t_i)] \le \Lambda_k[W_{k,i} + r_k(t_i)] - \Lambda_k[r_k(t_i)] \\ &\quad < \Lambda_k[a_k(t_i) + r_k(t_i) + dt] - \Lambda_k[r_k(t_i)] \\ &\Leftrightarrow b_k(t_i) \le V_{k,i} < \Lambda_k[a_k(t_i) + r_k(t_i)] + \lambda_k[a_k(t_i) + r_k(t_i)]dt - \Lambda_k[r_k(t_i)] \\ &\Leftrightarrow b_k(t_i) \le V_{k,i} < b_k(t_i) + \lambda_k(t_i)dt. \end{aligned} \quad (41)$$

From (26), we have

$$\begin{aligned} \mathcal{L}_i &= \Pr(T_i = t_i, \Delta_i = \delta_i \mid T_j = t_j, \Delta_j = \delta_j; j = 1, \cdots, i-1) \qquad ; i = 1, \cdots, N(\tau) \\ &= \Pr[W_{\delta_i,i} = a_{\delta_i}(t_i), W_{l,i} > a_l(t_i); l \ne \delta_i \mid W_{k,i} > a_k(t_{i-1}^+); k = 1, \cdots, K] \\ &= \frac{\Pr[W_{\delta_i,i} = a_{\delta_i}(t_i), W_{l,i} > a_l(t_i); l \ne \delta_i]}{\Pr[W_{k,i} > a_k(t_{i-1}^+); k = 1, \cdots, K]} \end{aligned} \quad (42)$$

Substituting (39), (40), and (41) into (42), we obtain

$$\begin{aligned} \mathcal{L}_i &= \frac{\Pr[b_{\delta_i}(t_i) \le V_{\delta_i,i} < b_{\delta_i}(t_i) + \lambda_{\delta_i}(t_i)dt, V_{l,i} > b_l(t_i); l \ne \delta_i]}{\Pr[V_{k,i} > b_k(t_{i-1}^+); k = 1, \cdots, K]} \\ &= \frac{\left\{ -\dfrac{\partial S(v_{1,i}, \cdots, v_{\delta_i,i}, \cdots, v_{K,i})}{\partial v_{\delta_i,i}} \Big|_{\mathbf{v}_i = [b_1(t_i), \cdots, b_K(t_i)]'} \right\} \lambda_{\delta_i}(t_i)}{S[b_1(t_{i-1}^+), \cdots, b_K(t_{i-1}^+)]} \end{aligned}$$

where $\mathbf{v}_i = (v_{1,i}, ..., v_{K,i})'$; and $S(\cdot)$ denotes the survival function of $\mathbf{V}_1$.

When $i = N(\tau) + 1$, the conditional probability can be calculated as:

$$\begin{aligned} \mathcal{L}_{N(\tau)+1} &= \Pr[T_{N(\tau)} = \tau, \Delta_i = 0 \mid T_j = t_j, \Delta_j = \delta_j; j = 1, \cdots, N(\tau)] \\ &= \Pr\left\{ W_{k,N(\tau)} > a_k(\tau); k = 1, \cdots, K \mid W_{k,N(\tau)} > a_k[t_{N(\tau)}^+]; k = 1, \cdots, K \right\} \end{aligned} \quad (43)$$

Substituting (39) and (40) into (43), we obtain

$$\begin{aligned}\mathcal{L}_{N(\tau)+1} &= \frac{\Pr[V_{k,N(\tau)} > b_k(\tau); k = 1, \cdots, K]}{\Pr\{V_{k,N(\tau)} > b_k[t^+_{N(\tau)}]; k = 1, \cdots, K\}} \\ &= \frac{S\left(b_1(\tau), \cdots, b_K(\tau)\right)}{S\left(b_1[t^+_{N(\tau)}], \cdots, b_K[t^+_{N(\tau)}]\right)}.\end{aligned}$$

.

APPENDIX 4. Proof of Equation 29

We use γ_j to denote $\Phi^{-1}(u_j)$, i.e., $u_j = \Phi(\gamma_j)$. Based on (5), the Gaussian copula function can be written as $C_{Gauss}(u_{1,},...,u_K) = \Phi_\Sigma(\gamma_1,...,\gamma_K)$. Thus, the pdf of Gaussian copula becomes:

$$
\begin{aligned}
f_{Gauss}(v_1,...,v_K;\boldsymbol{\theta}_F) &= \frac{\partial^K C_{Gauss}}{\partial\gamma_1...\partial\gamma_K}\left(\frac{d\gamma_1}{dv_1}...\frac{d\gamma_K}{dv_K}\right) \\
&= \phi(\gamma_1,...,\gamma_K)\left(\frac{d\gamma_1}{dv_1}...\frac{d\gamma_K}{dv_K}\right),
\end{aligned}
$$

where $\phi(\cdot)$ denotes the pdf of multivariate normal distribution Φ_Σ. In particular, we use $\boldsymbol{\Sigma}_{1,1}$ and $\boldsymbol{\Sigma}_{1,2}$ to denote the covariance of $[\gamma_1,...,\gamma_j,...,\gamma_K]', j \neq i$ and the covariance between $[\gamma_1,...,\gamma_j,...,\gamma_K]' j \neq i$ and γ_i, respectively. Here $[\gamma_1,...,\gamma_j,...,\gamma_K]' j \neq i$ is the vector without γ_i . By using the result in Eaton (1983), the pdf of multivariate normal distribution can be calculated by conditional probability, i.e., $\phi(\gamma_1,...,\gamma_K) = g(\gamma_i)\cdot h(\gamma_1,...,\gamma_j,...,\gamma_K); j \neq i$, where $g(\cdot)$ denotes the standard normal pdf, and $h(\cdot)$ denotes a $K-1$ dimensional multivariate normal with a mean vector of $\Sigma_{1,2}\cdot\gamma_i$ and a covariance vector of $\boldsymbol{\Sigma}_{1,1} - \boldsymbol{\Sigma}_{1,2}\cdot\boldsymbol{\Sigma}_{1,2}^T$. Thus, the first order partial derivative of the survival function becomes:

$$
\begin{aligned}
&-\frac{\partial S(v_1,...,v_i,...,v_K)}{\partial v_i} \\
&= \int_{v_K}^{\infty}...\int_{v_j}^{\infty}...\int_{v_1}^{\infty}\left\{f_{Gauss}(v_1,...,v_K;\boldsymbol{\theta}_F)\right\}dv_1...dv_j...dv_K;\, j\neq i \\
&= \int_{v_K}^{\infty}...\int_{v_j}^{\infty}...\int_{v_1}^{\infty}\left\{g(\gamma_i)h(\gamma_1,...,\gamma_j,...,\gamma_K)\left(\frac{d\gamma_1}{dv_1}....\frac{d\gamma_K}{dv_K}\right)\right\}dv_1...dv_j...dv_K;\, j\neq i \\
&= \left\{g(\gamma_i)\frac{d\gamma_i}{dv_i}\right\}\left\{\int_{v_K}^{\infty}...\int_{v_j}^{\infty}...\int_{v_1}^{\infty}\left\{h(\gamma_1,...,\gamma_j,...,\gamma_K)\left(\frac{d\gamma_1}{dv_1}....\frac{d\gamma_j}{dv_j}...\frac{d\gamma_K}{dv_K}\right)\right\}dv_1...dv_j...dv_K;\, j\neq i\right\} \\
&= \left\{g(\gamma_i)\left(\frac{du_i}{d\gamma_i}\right)^{-1}f_i(v_i)\right\}\left\{\int_{v_K}^{\infty}...\int_{v_j}^{\infty}...\int_{v_1}^{\infty}h(\gamma_1,...,\gamma_j,...,\gamma_K)d\gamma_1...d\gamma_j...d\gamma_K;\, j\neq i\right\} \\
&= \left\{g(\gamma_i)\left(g(\gamma_i)\right)^{-1}f_i(v_i)\right\}S_{Normal}(\gamma_1,...,\gamma_j,...,\gamma_K);\, j\neq i \\
&= f_i(v_i)S_{Normal}(\gamma_1,...,\gamma_j,...,\gamma_K);\, j\neq i
\end{aligned}
$$

where $f_i(\cdot)$ denotes the i^{th} marginal distribution in the Gaussian copula; $S_{Normal}(\gamma_1,...,\gamma_j,...,\gamma_K);\, j\neq i$, is the survival function of a multivariate normal distribution whose pdf is $h(\gamma_1,...,\gamma_j,...,\gamma_K);\, j\neq i$.

APPENDIX 5. Procedure to Simulate the Failure Data of A K-component System Based on the Proposed CTP Model

In this simulation procedure, we generate the failure data set $\{(T_1,\Delta_1),(T_2,\Delta_2),...,(T_N,\Delta_N)\}$; where N is the total number of failures. We use $T_i^o=[T_{i,1}^o,...,T_{i,n_i}^o]'$ and $T_i^t=[T_{i,1}^t,...,T_{i,n_i}^t]'$ to denote the failure times of component $i; i\in\{1,...,K\}$ in the original time domain and those in the corresponding transformed time domain, respectively; where n_i denotes the number of failures for component i. In addition, we use m_i to denote the number of generated failures of component i. At the beginning of the simulation procedure, we initialize $T_{i,1}^o=0$, $T_{i,1}^t=0$ and $m_i=0$ for all i. The detailed simulation procedure is listed as follows:

Step 1. Generate the first failure event:

a. Generate the first latent age vector $\mathbf{V}_1=[V_{1,1},...,V_{K,1}]'$ from the joint distribution constructed by copula function, which is in the transformed time domain.

b. Determine which component contributes to the first failure by transforming $\mathbf{V}_1=[V_{1,1},...,V_{K,1}]^T$ back to the original time domain. If $\Lambda_i^{-1}\left(V_{i,1}\right)=\min\left\{\Lambda_1^{-1}\left(V_{1,1}\right),...,\Lambda_K^{-1}\left(V_{K,1}\right)\right\}$, we update $T_{i,1}^o=\Lambda_i^{-1}\left(V_{i,1}\right)$, $T_{i,1}^t=V_{i,1}$ and $m_i=m_i+1$, where $\Lambda_i^{-1}(\cdot)$ denotes the inverse of $\Lambda_i(\cdot)$. As a result, $(T_1,\Delta_1)=(T_{i,1}^o,i)$.

Step 2. Generate the j^{th} failure event based on the failure history until the $(j-1)^{th}$ failure:

a. Generate the j^{th} latent age vector $\mathbf{V}_j = [V_{1,j},...,V_{K,j}]'$ from the joint distribution in the transformed time domain until $\Lambda_i^{-1}\left(V_{i,j} + T_{i,m_i}^t\right) \geq T_{i-1}$ for all i ; $i \in \{1,..,K\}$.

b. If $\Lambda_i^{-1}\left(V_{i,j} + T_{i,m_i}^t\right) = \min\left\{\Lambda_1^{-1}\left(V_{1,j} + T_{1,m_1}^t\right),...,\Lambda_K^{-1}\left(V_{K,j} + T_{K,m_K}^t\right)\right\}$ we update $T_{i,j}^t = V_{i,j} + T_{i,m_i}^t$, $T_{i,j}^o = \Lambda_i^{-1}\left(V_{i,j} + T_{i,m_i}^t\right)$, $m_i = m_i + 1$, and $(T_j, \Delta_j) = (T_{i,j}^o, i)$.

Repeat step 2 until all the N failures are generated.

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ABSTRACT

RELIABILITY ANALYSIS AND OPTIMAL MAINTENANCE PLANNING FOR REPAIRABLE MULTI-COMPONENT SYSTEMS SUBJECT TO DEPENDENT COMPETING RISKS

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Modern engineering systems generally consist of multiple components that interact in a complex manner. Reliability analysis of multi-component repairable systems plays a critical role for system safety and cost reduction. Establishing reliability models and scheduling optimal maintenance plans for multi-component repairable systems, however, is still a big challenge when considering the dependency of component failures. Existing models commonly make prior assumptions, without statistical verification, as to whether different component failures are independent or not. In this dissertation, data-driven systematic methodologies to characterize component failure dependency of complex systems are proposed. In CHAPTER 2, a parametric reliability model is proposed to capture the statistical dependency among different component failures under partially

perfect repair assumption. Based on the proposed model, statistical hypothesis tests are developed to test the dependency of component failures. In CHAPTER 3, two reliability models for multi-component systems with dependent competing risks under imperfect assumptions are proposed, i.e., generalized dependent latent age model and copula-based trend-renewal process model. The generalized dependent latent age model generalizes the partially perfect repair model by involving the extended virtual age concept. And the copula-based trend renewal process model utilizes multiple trend functions to transform the failure times from original time domain to a transformed time domain, in which the repair conditions can be treated as partially perfect. Parameter estimation methods for both models are developed. In CHAPTER 4, based on the generalized dependent latent age model, two periodic inspection-based maintenance polices are developed for a multi-component repairable system subject to dependent competing risks. The first maintenance policy assumes all the components are restored to as good as new once a failure detected, i.e., the whole system is replaced. The second maintenance policy considers the partially perfect repair, i.e., only the failed component can be replaced after detection of failures. Both the maintenance policies are optimized with the aim to minimize the expected average maintenance cost per unit time. The developed methodologies are demonstrated by using applications of real engineering systems.

AUTOBIOGRAPHICAL STATEMENT

Nailong Zhang was born in Heilongjiang, China. He received his B.Eng. degree in Mechanical Engineering from Harbin Institute of Technology, Harbin, China, in 2009. His research interests include statistical methods in reliability engineering and data analysis.

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