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Emerging Scholars
Graphing Effects as Fuzzy Numbers In Meta-Analysis

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Prior to quantitative analyses, meta-analysts often explore descriptive characteristics of effect sizes. A graphic is proposed that treats effect sizes as fuzzy numbers. This plot can provide meta-analysts with such information such as heterogeneity of effects, precision of estimates, possible clusters, and existence of outliers.

Keywords: Meta-analysis, fuzzy numbers, meta-analysis graphics

Meta-Analysis and Graphics

Meta-analysis is the statistical science of analyzing a collection of results from a set of studies with the intention of integrating individual findings (Glass, 1976). Over the past several decades, many fields have not only embraced the practical uses of meta-analysis, but have consistently strived to explore, enhance, and create new methodologies to answer complex research questions. Graphical displays of data in meta-analysis are intrinsic to answering such questions. As meta-analysis has evolved as a science, several graphical approaches have been developed (for overviews and usage suggestions, see Anzures-Cabrera & Higgins, 2010; Bax et al., 2009). The study and introduction of new graphical methods remains active today (e.g., Schild & Voracek, 2015).

The purposes of these graphics vary. Some aim to explore effect-size heterogeneity. Others reveal the possibility of publication bias. Perhaps the most widely used graphic is the forest plot. A forest plot vertically or horizontally "stacks" confidence intervals of collected effects in some predetermined order (e.g., by date, alphabetically). This display has the potential to inform meta-analysts on a variety

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of descriptive qualities that are critical to interpretation including within-study variability, study-to-study variability, presence of outliers, and even differential effects due to moderators. However, forest plots have their weaknesses. Because forest plots stack effect-size confidence intervals, plots with a large number of effects become quite large, and thus can suffer from decreased usability and interpretability. Related to this, large forest plots may cover several pages, further complicating efficient and precise interpretation. As will be shown later, the fuzzy number graphic approach contrasts this by superimposing effects.

When vertically or horizontally stacking effects using forest plots, one must make a decision as to how to order effects. Some orderings can lead to faulty inferences (e.g., false suggestions of effect-size clusters). For example, in some meta-analyses, ordering effects by moderators (e.g., by publication year) may help to explore possible or suspected differential effects. In other meta-analyses, random or non-meaningful effect-size orderings (e.g., by author name) have the potential to erroneously suggest moderating effects or effect-size clusters. The proposed plot automatically orders effects according to magnitude.

The purpose of this article is to describe a new method of graphing effect sizes in meta-analysis using the same study information required for forest plots (i.e., estimates of effect sizes and their variances). The fuzzy number plot may prove useful when attempting to initially describe a collection of effects. To begin, condensed overviews of fuzzy sets and fuzzy-numbers are provided. These central concepts are then developed for the context of meta-analysis. Several examples using existing meta-analyses are provided. The paper concludes with several remarks.

Crisp Sets and Fuzzy Sets

In classical set theory, at the most intuitive level, a set can be described as a collection of elements (Halmos, 1960). An element is either in a set or it is not. In the context of fuzzy set theory, classical sets are often referred to as crisp sets. This terminology is derived from the indicator-like nature of the membership function which defines a crisp set. Let F be the crisp set of elements x from some universal set X. The membership function of X, denoted as μ_F, assigns a membership grade to all elements $x \in X$. In the case of crisp sets, membership functions are deterministic, namely

$$\mu_F = \begin{cases} 1, & x \in F \\ 0, & x \notin F \end{cases} . \quad (1)$$

As shown in (1), an element x is either completely included in or excluded from F.

Moving away from crisp sets, a fuzzy set $\tilde{F}$ containing elements $x \in X$ is also defined by a membership function, which can be presented as a mapping of X to the closed interval [0, 1]. In the case of crisp sets, the membership function is a mapping of X to the finite set {0, 1}. The distinction between mapping to an interval of membership grades and mapping to a finite set of membership grades is critical. The first mapping assigns a membership grade to each element $x \in X$ from the closed interval [0, 1] while the second mapping assigns a membership grade to each element $x \in X$ from the finite set{0, 1}. Succinctly, a fuzzy set $\tilde{F}$ can be expressed as

$$\tilde{F} = \left\{ \left(x, \mu_{\tilde{F}}(x) \right) \mid x \in X, \mu_{\tilde{F}}(x) \in [0,1] \right\} . \quad (2)$$

As $\mu_{\tilde{F}}(x)$ approaches unity, the degree of membership of x in $\tilde{F}$ increases, and as $\mu_{\tilde{F}}(x)$ approaches zero, the degree of membership of x in $\tilde{F}$ decreases (Zadeh, 1965). A membership grade of unity implies an element is completely included in the fuzzy set, while a membership grade of zero implies the element is completely excluded from the fuzzy set. One can consider membership grades for fuzzy sets as numerical specifications as to how well some element $x \in X$ agrees with the imprecise mechanism (the membership function) which formulates the fuzzy set (Negoiță & Ralescu, 1987). Below are two heuristic examples of fuzzy sets.

Magnetic Strength Example

Suppose $\tilde{A}$ is a fuzzy set of strong magnetic field strengths (Gs). What precisely determines a strong magnetic field is a fuzzy concept. In this example, $\mu_{\tilde{A}}$ assigns membership grades to elements a in $\tilde{A}$ such that larger membership grades correspond with stronger magnetic strengths (i.e., greater Gs). As an example, one possible fuzzy set consisting of four elements is $\tilde{A}$ = {(30, 0.2), (74, 0.6), (96, 0.7), (302, 1)}. For the fuzzy set $\tilde{A}$, an element (δ, ϵ) is the paring of a magnetic strength (δ) with its membership grade (ϵ). Although the explicit membership function is not presented here, it is clearly evident that as magnetic strength increases, so does the membership grade.

Child Intelligence Example

Suppose $\tilde{B}$ is a fuzzy set of high-scoring results from the Wechsler Preschool and Primary Scale of Intelligence (Wechsler, 2002). The concept of a high-scoring result is also fuzzy. As with the previous example, $\mu_{\tilde{B}}$ assigns membership grades to elements b in $\tilde{B}$. Larger membership grades correspond with higher intelligence scores. Furthermore, suppose that $\tilde{B} = \{(61, 0.2), (81, 0.3), (111, 0.8), (145, 1)\}$. Similar to the previous example, for the fuzzy set $\tilde{B}$, an element (δ, ϵ) is the paring of an intelligence score (δ) with its membership grade (ϵ). As a child's intelligence score increases, the grade of membership increases.

Confidence Intervals as Fuzzy Sets

One relevant application of fuzzy sets in meta-analysis uses information from effect-size confidence intervals. The very nature of the confidence interval and its underlying notion of precision of estimating a parameter aligns with the ability of fuzzy sets (and later on, fuzzy numbers) to express imprecise beliefs regarding set membership. The pairing of an effect-size estimate and its sample variance provide insight into the precision of an estimate. This notion can also be thought of as representing the degree of fuzziness which exists between the true population parameter (here, an effect size) and the naturally imprecise estimate. Further detail on how to use confidence interval information with fuzzy numbers is presented later.

Select Fuzzy Set Attributes

Before introducing fuzzy numbers, several basic properties of fuzzy sets must be discussed. These definitions are presented in order to move towards defining fuzzy numbers. As will be discussed later, fuzzy numbers are fuzzy sets which satisfy several specifications.

First, the height of a fuzzy set $\tilde{F}$ describes the largest membership grade, or the largest $\mu_{\tilde{F}}(x)$ value, and is denoted as $\text{hgt}(\tilde{F})$. The height of a fuzzy set will necessarily be no larger than unity and will be larger than zero. Related to this, the core of a fuzzy set $\tilde{F}$ is the crisp set of all $x \in X$ having maximal membership grade, denoted as $\text{core}(\tilde{F})$. It is critical to note that, although we are discussing properties of fuzzy sets, $\text{core}(\tilde{F})$ is a crisp set.

The α-cut (alpha cut) of a fuzzy set $\tilde{F}$, denoted as $\text{cut}_{\alpha}\left(\tilde{F}\right)$, is the crisp set of all $x \in X$ for which $\mu_{\tilde{F}}\left(x\right) \geq \alpha$, where $\alpha \in [0, 1]$. Put another way, the α-cut is a crisp set with all elements having a membership grade greater than or equal to some value in the closed interval [0, 1]. When working with α-cuts from fuzzy sets in the context of statistical analysis, it should be noted that the α for determining an α-cut is completely unrelated to the α commonly used to specify Type I error. To avoid confusion, Type I error is denoted here by α'.

The last definition required to introduce the concept of fuzzy numbers is the convexity of a fuzzy set. We say that a fuzzy set $\tilde{F}$ is convex if, for $u, v \in \text{cut}_{\alpha}\left(\tilde{F}\right)$ and all $\alpha \in [0, 1]$, it holds that

$$\phi u + \left(1-\phi\right)v \in \text{cut}_{\alpha}\left(\tilde{F}\right) \forall \phi \in \left[0,1\right] . \tag{3}$$

To describe (3) in another light, a fuzzy set $\tilde{F}$ is convex if all α-cuts of $\tilde{F}$, which are themselves crisp sets, are convex (Zadeh, 1965). From here we proceed to introducing fuzzy numbers.

Fuzzy Numbers

A fuzzy number is a fuzzy set satisfying several requirements. More specifically, the fuzzy number $\tilde{f}$ may be defined from a fuzzy set $\tilde{F}$ if the following properties hold:

1. $\tilde{F}$ is convex
2. $\text{hgt}\left(\tilde{F}\right) = 1$
3. $\left|\text{core}\left(\tilde{F}\right)\right| = 1$
4. $\mu_{\tilde{F}}$ is at least piecewise continuous,

where | | denotes the cardinality of a crisp set. For consistency, we use lower case letters to denote fuzzy numbers and uppercase letters to denote general fuzzy sets. Just as $\tilde{F}$ was defined by its membership function $\mu_{\tilde{F}}$, the fuzzy number $\tilde{f}$ can be defined by its membership function $\mu_{\tilde{f}}$. There are infinitely many possibilities for creating $\tilde{f}$ from $\tilde{F}$ by defining the fuzzy number membership function $\mu_{\tilde{f}}$ in

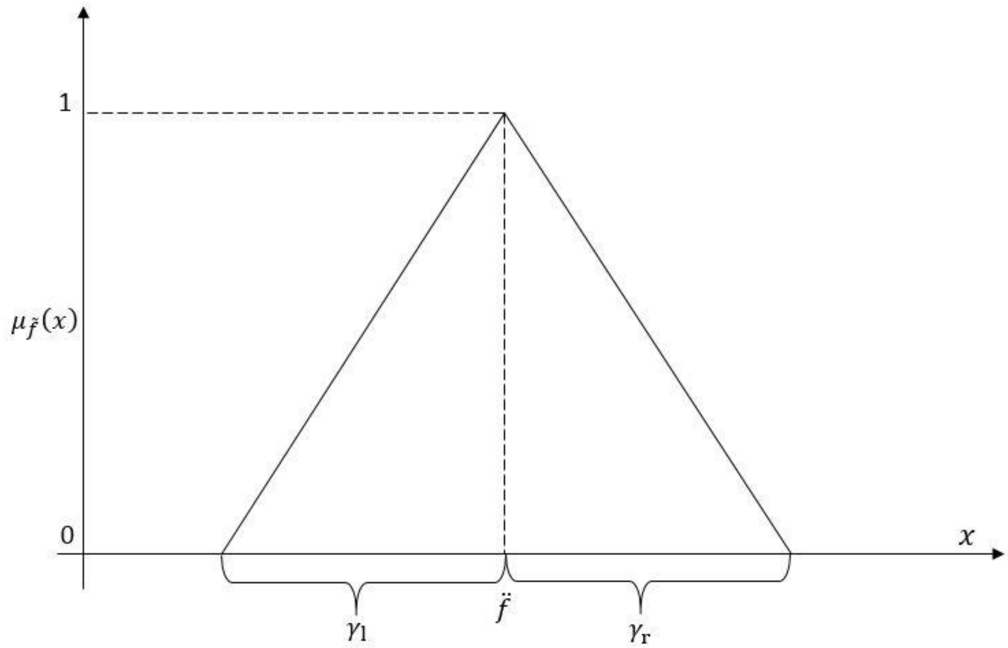

Figure 1. Graphic of single triangular fuzzy number

different ways. A few specific types of fuzzy numbers are common in engineering, soft computing, and other fields. One of these common fuzzy numbers is particularly pertinent for meta-analytic applications, namely the triangular fuzzy number. Although other types of fuzzy numbers are available, using the triangular fuzzy number provides sensible comparability between the membership grade and the confidence interval. Rationale for this choice is discussed later.

A triangular fuzzy number $\tilde{f} = \mathrm{tfn}\left(\gamma_1, \ddot{f}, \gamma_r\right)$, with modal value $\ddot{f}$ and respective left- and right-hand worst-case deviations γ_l and γ_r, is defined by the membership function for all $f \in \tilde{f}$:

$$\mu_{\tilde{f}}\left(f\right) = \begin{cases} \dfrac{f-\gamma_1}{\ddot{f}-\gamma_1}, & \gamma_1 < f \leq \ddot{f} \\ \dfrac{f-\gamma_r}{\ddot{f}-\gamma_r}, & \ddot{f} \leq f < \gamma_r \\ 0, & f \geq \gamma_r \cup f \leq \gamma_1 \end{cases} \tag{4}$$

In (4), $\ddot{f}$ is a unique element with the maximal membership grade of unity. Elements outside the bounds of the worst-case deviations have a membership grade equal to zero, and thus are not in the fuzzy set. All other elements are assigned respective membership grades from the open interval (0, 1) by (4).

Figure 1 shows a graphical representation of a single triangular fuzzy number using notation described above. The vertical axis represents the membership function $\mu_{\tilde{f}}$ while the horizontal axis represents values of $x \in X$. The modal value and worst-case deviations refer to the center and edges of the triangle. There is a symmetry about $\ddot{f}$ in Figure 1, which is a specific type of triangular fuzzy number; asymmetrical triangular fuzzy numbers are also possible.

Fuzzy Numbers for Meta-Analysis

Each primary study in a meta-analysis includes at least one measure of effect, as well as an estimate of effect-size variability. This information is required for a forest plot. I propose to use this same information to create fuzzy numbers from effect sizes as an alternative way to represent the precision of effect-size estimation. For a meta-analysis with a collection of k studies, each with an effect size (T_i) and known variance (v_i), the i^{th} triangular fuzzy number is defined as

$$\tilde{\mathcal{T}}_i = \text{tfn}\left(\gamma_{1i}, \ddot{t}_i, \gamma_{ri}\right) = \text{tfn}\left(T_i - Z_{\alpha'/2}\sqrt{v_i}, T_i, T_i + Z_{\alpha'/2}\sqrt{v_i}\right), \tag{5}$$

where $Z_{\alpha'/2}$ is the critical value of the standard normal distribution with a two-tailed Type I error rate of α' and $i = 1,\ldots, k$. Recall that α is used to denote and α-cut and α' to denote Type I error. In (5), the modal value of the fuzzy number is the effect-size estimate itself (T_i), and both worst-case deviations come from edges of the original confidence interval. The decreasing monotonicity moving outward in both directions from the effect-size estimate (resulting in strictly decreasing membership grades) closely resembles the inherent nature of a confidence interval, and more specifically its width. An argument for the choice of a triangular fuzzy number to represent a confidence interval has also been made by Yao, Su, and Shih (2008). Here, we use information derived using a standard normal confidence interval. The above definitions are valid for other types of confidence intervals as well.

Defining the membership function for (5), we simply revise (4) such that for all $t \in \tilde{\mathcal{T}}$

$$\mu_{\tilde{T}_i}(t) = \begin{cases} \dfrac{t-\gamma_1}{\ddot{t}-\gamma_1}, & \gamma_1 < t \le \ddot{t} \\ \dfrac{t-\gamma_r}{\ddot{t}-\gamma_r}, & \ddot{t} \le t < \gamma_r \\ 0, & t \ge \gamma_r \bigcup t \le \gamma_1 \end{cases}$$

$$= \begin{cases} \dfrac{(t)-\left(T_i - Z_{\alpha'/2}\sqrt{v_i}\right)}{(T_i)-\left(T_i - Z_{\alpha'/2}\sqrt{v_i}\right)}, & \left(T_i - Z_{\alpha'/2}\sqrt{v_i}\right) < t \le T_i \\ \dfrac{(t)-\left(T_i + Z_{\alpha'/2}\sqrt{v_i}\right)}{(T_i)-\left(T_i + Z_{\alpha'/2}\sqrt{v_i}\right)}, & T_i \le t < T_i + Z_{\alpha'/2}\sqrt{v_i} \\ 0, & t \ge T_i + Z_{\alpha'/2}\sqrt{v_i} \bigcup t \le T_i - Z_{\alpha'/2}\sqrt{v_i} \end{cases}$$

$$= \begin{cases} 1+\dfrac{t-T_i}{Z_{\alpha'/2}\sqrt{v_i}}, & \gamma_1 < t \le \ddot{t} \\ 1-\dfrac{t-T_i}{Z_{\alpha'/2}\sqrt{v_i}}, & \ddot{t} \le t < \gamma_r \\ 0, & t \ge T_i + Z_{\alpha'/2}\sqrt{v_i} \bigcup t \le T_i - Z_{\alpha'/2}\sqrt{v_i} \end{cases} \tag{6}$$

Any collection of studies providing ample information to create a forest plot can necessarily be used to create a fuzzy number plot. Treating effect sizes as fuzzy numbers allows for effect-size imprecision to be viewed as fuzziness in effect-size estimation. For some effect-size metrics (e.g., the standardized mean difference), the triangular fuzzy number will be symmetric about $\ddot{t} = T_i$ because $T_i + Z_{\alpha'/2}\sqrt{v_i}$ is without bound on the set of real numbers. For other metrics (e.g., correlation coefficient), there is a possibility for asymmetry due to the natural bounds of the effect-size metric. Furthermore, it is possible to use other measures of variability to define worst-case deviations for triangular fuzzy numbers. One example would be the median absolute deviation. The use of variance in this paper is solely to coincide with confidence interval information found in forest plots.

There are obvious similarities among typical confidence intervals, probability values, and fuzzy numbers. However, the two representations of effects differ in several important ways. First, there is an intrinsic uncertainty due to randomness and uncertainty due to fuzziness. Among other things, this concerns definitions of subset domains. Fuzzy set theory replaces typical σ-algebra domains by the universe of discourse; see Aliev, Fazlollahi, and Aliev (2004) for a more elaborate discussion. Second, a membership grade is not the same as a probability value. One reason for this distinction is that probabilities must exist in the closed interval [0, 1]. This is not always the case for fuzzy membership functions. Also, it need not be the case that the summation of all membership grades is unity (for a detailed discussion, see Singpurwalla & Booker, 2004). Furthermore, in the absence of fuzzy set theory (i.e., simply plotting confidence intervals in the same manner of fuzzy numbers), the metric of the vertical axis is unclear. Using fuzzy set theory, an established and interpretable membership grade is assigned to the vertical axis.

Aggregate Fuzzy Number

It may prove desirable to compute and plot some aggregate fuzzy number measure, similarly to the common practice of plotting weighted means on forest plots. One could use typical fixed-effect or random-effects weighted means as fuzzy numbers using the same formulas discussed above. Parallel to how fuzzy numbers are constructed from fuzzy effects in (5), one could use components from fixed- or random-effects confidence intervals for means. The weighted mean estimate would be the modal value and worst-case deviations would be the edges of the confidence interval for the weighted mean.

Alternatively, one could plot a mean fuzzy number $\bar{\tilde{\mathcal{T}}}$ following Buckley (1985):

$$\begin{aligned}\bar{\tilde{\mathcal{T}}} &= \text{tfn}\left(\gamma_1^m, \ddot{t}^m, \gamma_r^m\right)\\ &= k^{-1}\bullet\left[\tilde{\mathcal{T}}_1 \oplus \tilde{\mathcal{T}}_2 \oplus \cdots \oplus \tilde{\mathcal{T}}_k\right]\\ &= \text{tfn}\left(k^{-1}\sum_{i=1}^{k}\gamma_{li}, k^{-1}\sum_{i=1}^{k}\ddot{t}_i, k^{-1}\sum_{i=1}^{k}\gamma_{ri}\right)\end{aligned} \tag{7}$$

Where k is the number of studies in the meta-analysis, $\bullet$ is scalar multiplication and $\oplus$ is fuzzy addition (Hanss, 2005). In the context of meta-analysis, (7) can be calculated as

$$\bar{\tilde{\mathcal{T}}} = \text{tfn}\left(k^{-1}\sum_{i=1}^{k}\left[T_i - Z_{\alpha'/2}\sqrt{v_i}\right], k^{-1}\sum_{i=1}^{k}T_i, k^{-1}\sum_{i=1}^{k}\left[T_i + Z_{\alpha'/2}\sqrt{v_i}\right]\right). \tag{8}$$

This aggregate measure is calculated as a direct function of fuzzy numbers, which is different than calculating a fuzzy number representing a fixed- or random-effects weighted mean. Typical fixed- and random-effects means in meta-analysis are inverse-variance weighted, so that studies with higher precision are afforded more weight when determining an average. The method presented in (8) averages each component of the collection of fuzzy numbers. This result is more similar to a "typical study result" than to a weighted mean. The endpoints of the fuzzy mean are not to be directly compared to those of a confidence interval. While statistical significance can be assessed using confidence intervals, this is not valid for fuzzy numbers. The fuzzy mean is essentially a representation of the fuzziness or uncertainty of a typical study.

Examples of Fuzzy Number Plots

Three examples of fuzzy number plots from published meta-analyses are shown and discussed. Forest plots with effects ordered by their magnitudes are also provided for comparison. When plotting fuzzy numbers, the degree of color shading provides a simple interpretation such that more dense (i.e., darker) shading corresponds with more fuzzy number overlap. All fuzzy number plots in this paper were produced using basic R (R Core Team, 2013) procedures along with the *FuzzyNumbers* package (Gagolewski, 2013). Forest plots were produced using the *metafor* package (Viechtbauer, 2010). R code for producing fuzzy number plots is provided in the appendix.

Exercise Training and Depressive Symptoms: A Large Number of Effects

The first example comes from a meta-analysis of the effects of exercise training on select depressive symptoms for patients with a chronic illness (Herring, Puetz, O'Connor, & Dishman, 2012). To quantify treatment effects, standardized mean differences (d) were calculated. These effects represented the mean difference between an exercise condition and a comparison condition on several mental and physical health outcomes. In total, 167 effect sizes were obtained from 90 studies. Figure 2 provides the fuzzy number plot for this data, while Figure 3 provides the forest plot for the same data.

Recall that the notion of fuzziness is represented by the vertical axis (membership grade) and the width of the triangle associated with each effect size. A wider triangle denotes a fuzzier estimate. What is immediately noticeable from Figure 2 is that the vast majority of effect-size point estimates are positive, indicating a reduction in depressive symptoms in the exercise condition (compared to the non-exercise condition). While most point estimates (i.e., circles falling on the vertical line where the membership grade is equal to unity) were positive, a dense cluster of effects falls in the interval [0, 0.75]. In this example, the center of the fuzzy mean $\bar{\tilde{\mathcal{T}}}_1 = \mathrm{tfn}(-0.19, -0.35, 0.89)$ is located in the approximate center of the clustered effects. Last, several large effects ($d > 1$) may be potential outliers.

This example shows how the fuzzy plot reveals some simple yet valuable descriptive features when initially describing a collection of effects. Several large effects appear to be divergent from the rest of the data, which is an indicator of possible outliers. These effects do not stand out as prominently as in Figure 3. This attribute, as well as the clustering of effects around 0 to 0.75, is more easily seen with the fuzzy number plot (Figure 2) compared to the respective forest plot (Figure 3). Furthermore, the sheer size of the forest plot is impractical. To include this forest plot in a relatively small area of a single publication page, effect sizes would need

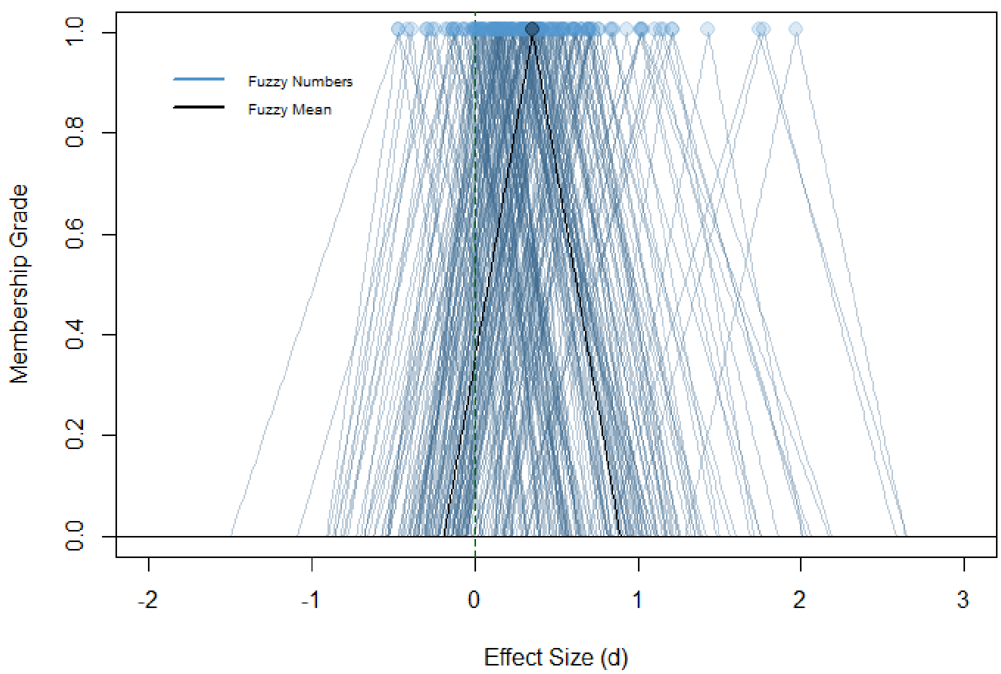

Figure 2. Fuzzy number plot of exercise training and depressive symptoms data

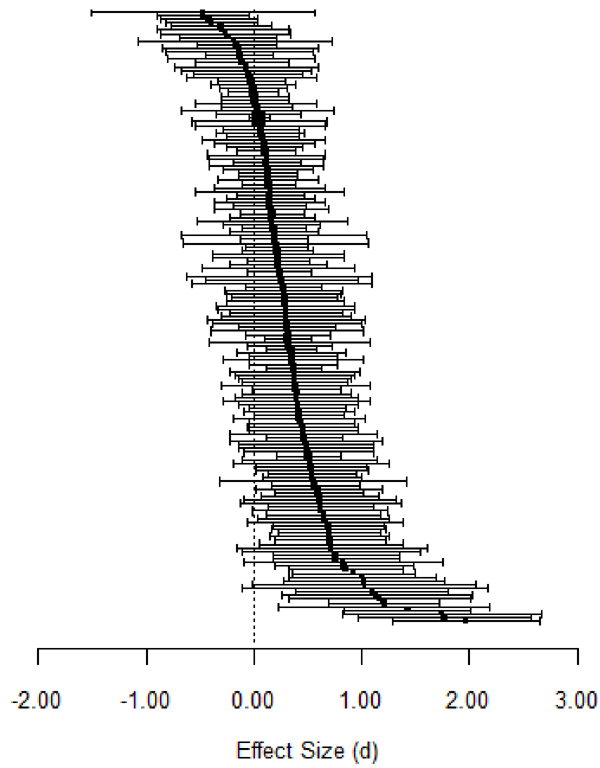

Figure 3. Forest plot of exercise training and depressive symptoms data

to be graphed very close together. Consequently, the quality of interpretation can be diminished. On the other hand, increasing the size of the forest plot so that effects are not forced so close together would require more journal pages. This is not the case for the fuzzy number plot. The superimposition of effects resolves this issue.

Positive Psychology Interventions for Well-Being: The Presence of Moderators

The second example stems from a meta-analysis (Sin & Lyubomirsky, 2009) which synthesized bivariate correlations (*r*) from studies analyzing the effect of positive psychology interventions for well-being in depressive symptoms. This meta-analysis collected 42 effect sizes from 37 studies. The original meta-analysis included more effects and studies. Although most studies in the meta-analysis were moderately recent, some date back to the 1970s and 1980s. The example in this

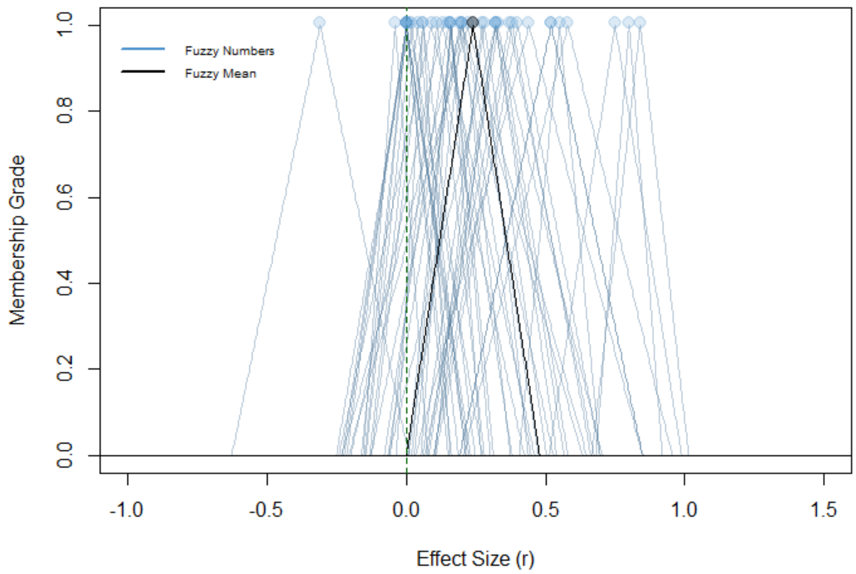

Figure 4. Fuzzy number plot of psychology interventions data

paper excluded all effects before 1990 ($N = 7$). Effects were plotted using both the novel fuzzy number plot (Figure 4) and the traditional forest plot (Figure 5). This example also illustrates the capability of fuzzy number plots to explore possible moderators.

Both the fuzzy number and forest plots clearly show a preponderance of positive effects, indicating that positive psychological interventions were associated with higher states of well-being. The fuzzy number plot shows several clusters of effects. Although one of these clusters is located close to the fuzzy mean $\bar{\tilde{\mathcal{T}}}_2 = \text{tfn}\left(-0, 0.24, 0.48\right)$ and shows a moderately large effect, there is also a discernible cluster of effects around zero.

To illustrate how potential moderators can be graphed on a fuzzy number plot, we again utilized the same data from Sin and Lyubomirsky (2009). One of the coded moderators in the original meta-analysis was whether physiological interventions were: 1) Individually Administered; 2) Group Administered; or 3) Self-Administered. To demonstrate the salience of this moderator, Figure 6 displays a revised fuzzy number plot with color-coded fuzzy numbers. Figure 6 suggests that those interventions administered by psychological professionals had more of an effect than interventions which were self-administered by the client.

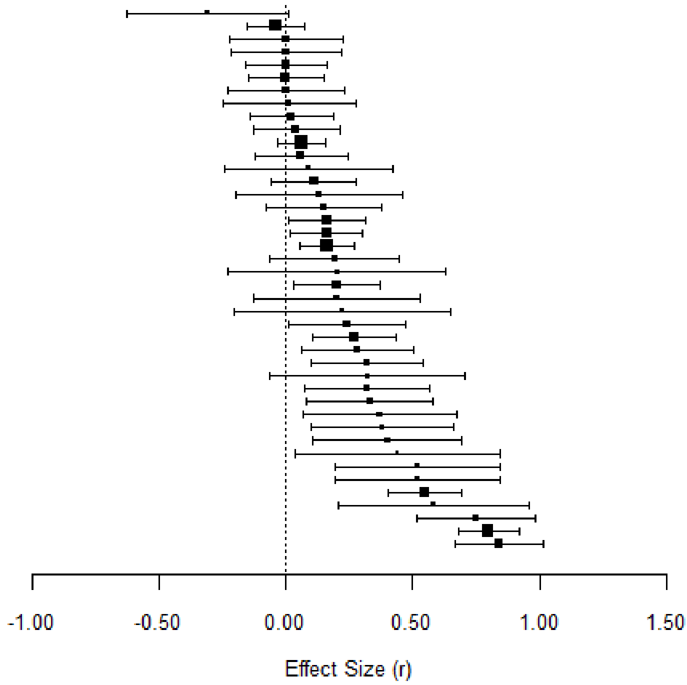

Figure 5. Forest plot of psychology interventions data

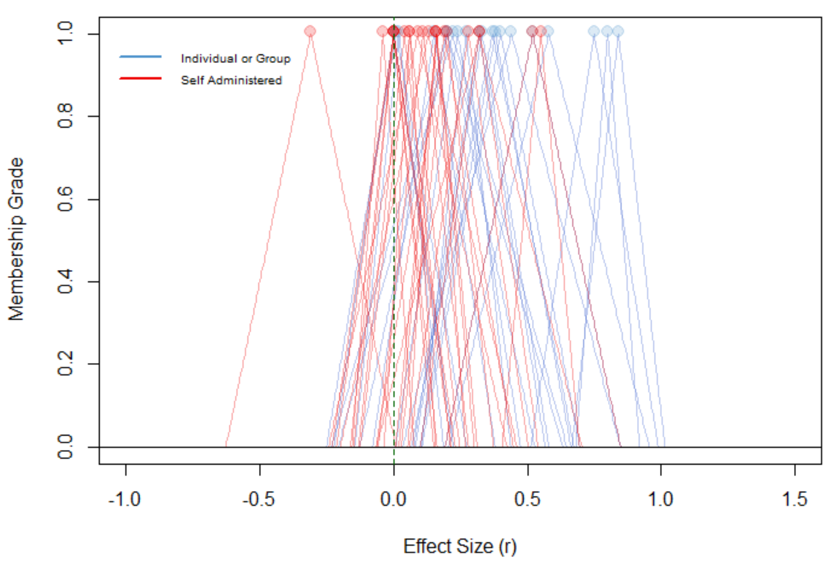

Figure 6. Fuzzy number plot of psychology interventions data with moderator

Alcohol Consumption and Coronary Heart Disease: The Presence of Outliers

The third example was taken from a published meta-analysis on alcohol consumption and its effect on select biological markers known to be associated with adult risk of coronary heart disease (Brien, Ronksley, Turner, Mukamal, & Ghali, 2011). These authors state that they “systematically reviewed the effect of experimentally manipulated alcohol consumption (alcohol use versus a period of no alcohol use) on the circulating concentrations of selected cellular and molecular biological markers of atherothrombotic conditions associated with increased coronary heart disease risk in adults without pre-existing cardiovascular disease” (Brien et al., 2011, p. 1). Analyses were reasonably extensive; over a dozen different biomarker means were examined. Here, we focus on results of one biomarker, high density lipoprotein cholesterol (HDLC).

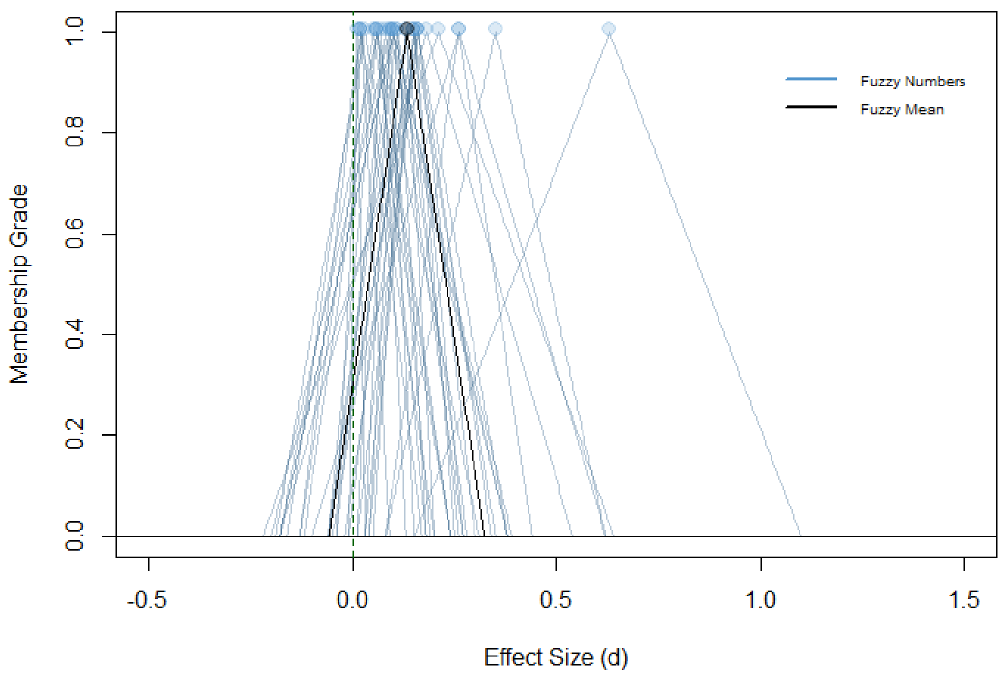

Figure 7. Fuzzy number plot of cholesterol biomarker data

Thirty-two mean differences comparing average concentration of HDLC biomarkers after alcohol consumption to the average concentration of the HDLC biomarker with no alcohol consumption were extracted. The original meta-analysis had 33 effects. However, an effect from one study had such a large variance (1.78) compared to the remaining effects, we decided that its exclusion would produce a more interpretable graphic. As with the previous example, effects were plotted using both the fuzzy number plot (Figure 7) and a traditional forest plot (Figure 8).

What is immediately noticeable from Figure 7 is the overwhelming amount of positive effect-size point estimates corresponding with a positive relationship between alcohol consumption and the HDLC biomarker. In addition, a possible outlier appears at $d = 0.63$. This extreme value also appears to have more fuzziness compared to other effects, as indicated by the larger width of the triangle. A dense cluster of effects is seen between approximately $d = 0$ and $d = 0.25$. In the approximate center of this cluster is the fuzzy mean $\bar{\tilde{\mathcal{T}}}_3 = \text{tfn}\left(-0.06, 0.13, 0.32\right)$. This graphical representation suggests a fairly weak overall effect, if any at all.

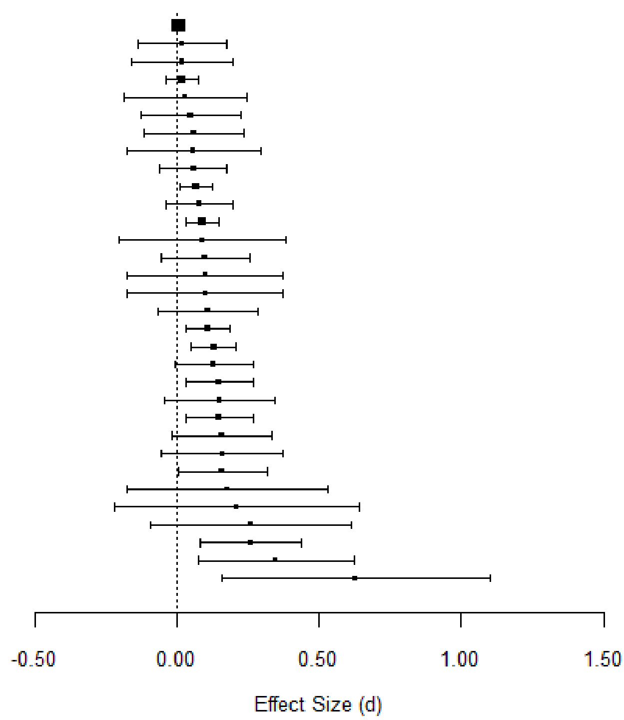

Figure 8. Forest plot of cholesterol biomarker data

Concluding Remarks

A novel method for graphing effects was proposed for describing study information in meta-analysis by way of treating and plotting effect sizes as fuzzy numbers. A brief introduction to fuzzy set theory was provided and extensions to meta-analysis were explained. Plots using data from both fuzzy number plots and forest plots were illustrated and discussed in the context of three previously published meta-analyses.

Treating effects as fuzzy numbers allows the meta-analyst to use the same information required for the common forest plot but provides several advantages. For example, plotting effects as fuzzy numbers is likely to increase usability and interpretability in situations where a large number of effects are to be meta-analyzed. In such situations, fuzzy number plots will use less publication page space than forest plots. Last, as was demonstrated, fuzzy number plots can be used to explore possible moderators and outliers at initial stages of a meta-analysis.

There are some possible limitations to the method of plotting effects as fuzzy numbers. In select instances, it may be challenging to differentiate within a group of very small effects or within a group of very large effects. Also, compared to the state-of-the-art forest plots, creating fuzzy number plots for meta-analysis involves slightly more computation on the user's end. However, R code has been provided in the appendix for this very reason.

References

Aliev, R. A., Fazlollahi, B., & Aliev, R. R. (2004). *Soft computing and its applications in business and economics*. Studies in fuzziness and soft computing (Vol. 157). Berlin, Germany: Springer.

Anzures-Cabrera, J., & Higgins, J. P. T. (2010). Graphical displays for meta-analysis: An overview with suggestions for practice. *Research Synthesis Methods, 1*(1), 66-80. doi: 10.1002/jrsm.6

Bax, L., Ikeda, N., Fukui, N., Yaju, Y., Tsuruta, H., & Moons, K. G. M. (2009). More than numbers: The power of graphs in meta-analysis. *American Journal of Epidemiology, 169*(2), 249-255. doi: 10.1093/aje/kwn340

Brien, S., Ronksley, P., Turner, B., Mukamal, K., & Ghali, W. (2011). Effect of alcohol consumption on biological markers associated with risk of coronary heart disease: Systematic review and meta-analysis of interventional studies. *BMJ, 342*(d636). doi: 10.1136/bmj.d636

Buckley, J. J. (1985). Ranking alternatives using fuzzy numbers. *Fuzzy Sets and Systems, 15*(1), 21-32. doi: 10.1016/0165-0114(85)90013-2

Gagolewski, M. (2013). *Fuzzynumbers package: Tools to deal with fuzzy numbers in R* [Computer software manual]. Retrieved from http://www.rexamine.com/resources/fuzzynumbers/

Glass, G. V. (1976). Primary, secondary, and meta-analysis of research. *Educational Researcher, 5*(10), 3-8. Available from http://www.jstor.org/stable/1174772

Halmos, P. R. (1960). *Naive set theory*. New York, NY: Springer.

Hanss, M. (2005). *Applied fuzzy arithmetic: An introduction with engineering applications*. New York, NY: Springer-Verlag.

Herring, M. P., Puetz, T. W., O'Connor, P. J., & Dishman, R. K. (2012). Effect of exercise training on depressive symptoms among patients with a chronic illness: A systematic review and meta-analysis of randomized controlled trials. *Archives of Internal Medicine, 172*(2), 101-111. doi: 10.1001/archinternmed.2011.696.

Negoiță, C. V., & Ralescu, A. L. (1987). *Simulation, knowledge-based computing, and fuzzy statistics*. New York, NY: Van Nostrand Reinhold.

R Core Team. (2013). *R: A language and environment for statistical computing* [Computer software manual]. Retrieved from http://www.R-project.org

Schild, A. H. E., & Voracek, M. (2015). Finding your way out of the forest without a trail of bread crumbs: Development and evaluation of two novel displays of forest plots. *Research Synthesis Methods, 6*(1), 74-86. doi: 10.1002/jrsm.1125

Sin, N. L., & Lyubomirsky, S. (2009). Enhancing well-being and alleviating depressive symptoms with positive psychology interventions: A practice-friendly meta-analysis. *Journal of Clinical Psychology, 65*(5), 467-487. doi: 10.1002/jclp.20593

Singpurwalla, N. D., & Booker, J. M. (2004). Membership functions and probability measures of fuzzy sets. *Journal of the American Statistical Association, 99*(467), 867-877. doi: 10.1198/016214504000001196

Viechtbauer, W. (2010). Conducting meta-analyses in R with the metafor package. *Journal of Statistical Software, 36*(3), 1-48. Retrieved from http://www.jstatsoft.org/article/view/v036i03

Wechsler, D. (2002). *Wechsler preschool and primary scale of intelligence* (3rd ed.). San Antonio, TX: Psychological Corporation.

Yao, L. J.-S., Su, J.-S., & Shih, T.-S. (2008). Fuzzy system reliability analysis using triangular fuzzy numbers based on statistical data. *Journal of Information Science and Engineering*, *24*(5), 1521-1535. Retrieved from http://www.iis.sinica.edu.tw/page/jise/2008/200809_15.html

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338-353. doi: 10.1016/S0019-9958(65)90241-X

Appendix

R Code for Fuzzy Number Plot

```
library(FuzzyNumbers)

## Data is a data set with k rows (one for each effect size) and three
columns: effect-size measures, left-hand worst case deviations, right-
hand worst case deviations
## T    is a column of effect-size measure
## LH   is a column of left-hand worst case deviation
## RH   is a column of right-hand worst case deviation

# Color code from mages' blog:
# http://www.magesblog.com/2013/04/how-to-change-alpha-value-of-colours-
in.html
add.alpha <- function(col, alpha=1){
  if(missing(col))
  stop("Please provide a vector of colours.")
  apply(sapply(col, col2rgb)/255, 2,
  function(x)
  rgb(x[1], x[2], x[3], alpha=alpha))}

DataM <- sum(Data$T)/length(Data$T)
DataL <- sum(Data$LH)/length(Data$T)
DataR <- sum(Data$RH)/length(Data$T)

D<-list()
for(i in 1:length(Data$T)){
  fuzzynameS<-paste('a',i,sep='')
  D[[i]]<- PiecewiseLinearFuzzyNumber(Data$LH[i], Data$T[i], Data$T[i],
Data$RH[i])}

MeanData <- PiecewiseLinearFuzzyNumber(DataL, DataM, DataM, DataR)
Dpointest<-rep(1.011,length(Data$T))
DpointestL<-rep(0,length(FPData$T))
DpointestR<-rep(0,length(FPData$T))

plot(D[[1]], type='l', lty=1, col=rgb(54/255, 100/255, 139/255,
alpha=.4),
     xlab='Effect Size (T)', ylab="Membership Grade")


for(i in 2:length(D)){
  plot(D[[i]], type='l', col=rgb(54/255, 100/255, 139/255, alpha=.4),
lty=1, add=TRUE)}

plot(MeanData, type='l', col='black', lwd=1, add=TRUE)
```

```
abline(h=0, lwd=1)
abline(v=0, lwd=1, lty=2, col="darkgreen")
points(x=Data$T, y=Dpointest, type="p", pch=20,
       col=add.alpha("steelblue3", alpha=0.2)[Ppointest], cex=2)
points(x=DataM, y=Dpointest[1], type="p", pch=20,
       col=add.alpha("black", alpha=0.35), cex=2)
legend(-.52, 1.1, lty=c(1,1), c("Fuzzy Numbers","Fuzzy Mean"),
       lwd=c(2,2), col=c("steelblue3", "black"), cex=.6, bty="n")
```