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E-Bayesian Estimation of the Parameter of the Logarithmic Series Distribution

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E-Bayesian estimation is introduced to estimate the parameter of logarithmic series distribution. In addition, E-Bayesian, Bayesian and maximum likelihood estimation with through applying mean squared error.

Keywords: Logarithmic series distribution, E-Bayesian estimation, Bayesian estimation, maximum likelihood estimation, means squared error

Introduction

The logarithmic series distribution (LSD) is obtained by expanding the logarithmic function -log(1 – θ) as a power series in. Alternatively, it can also be derived as a limiting case of zero-truncated negative binomial distribution as k decreases to zero. In either case, the logarithmic series distribution is a very useful distribution on the positive integers (Nasiri, 2011). Estimation is an important topic in statistical inference. Bayesian approach is an important approach in the estimation of parameter. A suitable prior distribution plays an effective role in reducing error in the estimation. Therefore, the more the prior information is obtained, the more it affects the posterior.

Lindley and Smith (1972) argued hierarchical prior. E-Bayesian estimation is another method introduced by Han and Ding (2004). Han (2005) applied E-Bayesian estimation for forecast of security investment. He also (2006, 2007) presented hierarchical Bayesian estimation for computing as well as E-Bayesian estimation for transition probability. In this study, maximum likelihood, Bayesian, and E-Bayesian estimations of the parameter of logarithmic series distribution are discussed in detail. This paper considers the maximum likelihood estimation of θ,

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the Bayesian estimation of θ, and the E-Bayesian estimation of θ; by use of a simulation, all estimations will be compared by MSE.

Maximum Likelihood Estimation of *θ*

Let f(x) be the density of the logarithmic series distribution given by

$$\mathrm{f}\left(x\right)=\frac{-1}{\log\left(1-\theta\right)}\bullet\frac{\theta^{x}}{x},\quad x=1,2,3,\ldots,\quad 0<\theta<1 \tag{1}$$

The maximum likelihood estimation of θ in the above distribution is derived by i.i.d observations x_1, x_2,…x_n. Hence, the likelihood function is given by

$$\mathrm{l}\left(\theta\right)=\left(\frac{1}{-\log\left(1-\theta\right)}\right)^{n}\bullet\frac{\theta^{n\bar{x}}}{\prod_{i=1}^{n}x_i}=\left(-\log\left(1-\theta\right)\right)^{-n}\frac{\theta^{n\bar{x}}}{\prod_{i=1}^{n}x_i} \tag{2}$$

Similarly, the logarithm of the likelihood function is given by

$$\mathrm{Log}\,\mathrm{l}\left(\theta\right)=n\bar{x}\log\theta-n\log\left(-\log\left(1-\theta\right)\right)-\sum_{i=1}^{n}\log x_i \tag{3}$$

There are two ways to estimate θ. The first is to apply the “optimum” command in R software, and the second is to take the first order derivative of Log l(θ) over θ and set it equal to zero, as in the following:

$$\frac{d\mathrm{Log}\,\mathrm{l}\left(\theta\right)}{d\theta}=0$$

$$\frac{\bar{x}}{\theta}+\frac{1}{\left(1-\theta\right)\left(\log\left(1-\theta\right)\right)}=0$$

$$\left(1-\theta\right)^{1-\theta}=\mathrm{e}^{\frac{-\theta}{\bar{x}}},\quad 0<\theta<1$$

Assume $\mathrm{g}\left(\theta\right)=\left(1-\theta\right)^{1-\theta}-\mathrm{e}^{\frac{-\theta}{\bar{x}}}$. This equation can be solved via the Newton-Raphson method:

$$\theta_n = \theta_{n-1} - \frac{\mathrm{g}(\theta_{n-1})}{\dot{\mathrm{g}}(\theta_{n-1})} \tag{4}$$

where

$$\dot{\mathrm{g}}(\theta_{n-1}) = (1-\theta)^{1-\theta}\left[-\log(1-\theta)-1\right] + \frac{\mathrm{e}^{\frac{-\theta}{\bar{x}}}}{\bar{x}} ,$$

and Fixed-Point method as

$$\theta_n = \mathrm{h}(\theta_{n-1}) \tag{5}$$

such that

$$\mathrm{h}(\theta) = \bar{x}(\theta-1)\log(1-\theta) .$$

Equations (4) and (5) were solved using the MATLAB software. The “optimum” command was used in R software. There is additional discussion regarding the MLE logarithmic series in Bohning (1983).

Bayesian Estimation of *θ*

Let π(*θ*) be prior density of *θ* that has beta distribution:

$$\pi(\theta) = \frac{1}{\mathrm{B}(a,b)}\theta^{a-1}(1-\theta)^{b-1} \tag{6}$$

$$\mathrm{B}(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}, \quad \Gamma(a) = \int_0^\infty t^{a-1}\mathrm{e}^{-t}dt$$

By using i.i.d. observations x_1, x_2,… x_n, the posterior distribution of *θ* was calculated as in the following:

$$\pi(\theta \mid \underline{x}) = \frac{\mathrm{l}(\theta)\pi(\theta)}{\int_0^1 \mathrm{l}(\theta)\pi(\theta)d\theta} = \frac{\left(-\log(1-\theta)\right)^{-n}\theta^{n\bar{x}+a-1}(1-\theta)^{b-1}}{\int_0^1\left(-\log(1-\theta)\right)^{-n}\theta^{n\bar{x}+a-1}(1-\theta)^{b-1}d\theta} \tag{7}$$

where l(θ) is the likelihood function introduced in (2).

Note that $(-\log(1-\theta))^{-n}$ can be expanded as $\theta^{-n}\sum_{m=0}^{\infty}\rho_m(-n)\theta^m$, where $\rho_0(-n) = 1$, $\rho_m(-n) = n\psi_{m-1}(m-n-1)$ for $m \geq 1$, and the coefficients $\psi_m(.)$, are Sterling polynomials given by Castellares and Lemonte (2014).

Consider

$$\left(-\log(1-\theta)\right)^{-n}\theta^{n\bar{x}+a-1}(1-\theta)^{b-1} = \sum_{m=0}^{\infty}\rho_m(-n)\theta^{m+n(\bar{x}-1)+a-1}(1-\theta)^{b-1}$$

and suppose

$$\begin{aligned}\mathrm{A}(a) &= \sum_{m=0}^{\infty}\rho_m(-n)\int_0^1\theta^{m+n(\bar{x}-1)+a-1}(1-\theta)^{b-1}\,d\theta \\ &= \sum_{m=0}^{\infty}\rho_m(-n)\mathrm{B}\left(m+n(\bar{x}-1)+a,b\right)\end{aligned}$$

Then

$$\pi(\theta\,|\,\underline{x}) = \frac{\sum_{m=0}^{\infty}\rho_m(-n)\theta^{m+n(\bar{x}-1)+a-1}(1-\theta)^{b-1}}{\mathrm{A}(a)} \tag{8}$$

The Bayesian estimation of θ under loss function $l(\theta, d) = (d-\theta)^2$ is $\mathrm{E}(\theta\,|\,\underline{x})$,

$$\mathrm{E}(\theta\,|\,\underline{x}) = \frac{\mathrm{A}(a+1)}{\mathrm{A}(a)} \tag{9}$$

$\mathrm{E}(\theta\,|\,\underline{x})$ is computed by numerical methods using R software.

E-Bayesian Estimation

Let the prior distribution of θ be given as:

$$\pi(\theta\,|\,a,b) = \frac{1}{\mathrm{B}(a,b)}\theta^{a-1}(1-\theta)^{b-1}, \quad 0<\theta<1 \tag{10}$$

where *a* and *b* are super parameters. According to Han (1997) *a* and *b* should be selected to guarantee $\pi(\theta| a, b)$ is a decreasing function of θ. Therefore, we applied one order derivative of $\pi(\theta| a, b)$ over θ to obtain

$$\frac{d\pi(\theta|a,b)}{d\theta}=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{a-2}(1-\theta)^{b-2}\left[(a-1)(1-\theta)-(b-1)\theta\right]$$

Because $a > 0$, $b > 0$, and $0 < \theta < 1$, then $0 < a \leq 1$, $b > 1$ result in $\frac{d\pi(\theta|a,b)}{d\theta}<0$. Thus, $\pi(\theta| a, b)$ is a decreasing function of θ given $0 < a \leq 1$, $b > 1$.

As *b* grows larger, the tail of the beta density function grows thinner. However, as far as the robustness of Bayesian estimation is concerned (Berger, 1985), the thinner-tailed prior distribution often leads to the worse robustness of the Bayesian estimate. Accordingly, *b* should not be too big; it is better to be selected below the given upper bound *c* ($c > 1$) (see Han & Ding, 2004). All in all, the super parameters *a* and *b* were selected to be in the ranges $0 < a \leq 1$ and $1 < b \leq c$.

Let $a = 1$ and *b* have density function given by the following:

$$\mathrm{g}(b)=\frac{1}{c-1},\quad 1<b\leq c \tag{11}$$

Hence, the prior distribution is given by

$$\pi(\theta|a=1,b)=\pi(\theta|b)=b(1-\theta)^{b-1},\quad 0<\theta<1$$
$$\pi(\theta,b)=\pi(\theta|b)\mathrm{g}(b)=\frac{b}{c-1}(1-\theta)^{b-1},\quad 0<\theta<1,\quad 1<b\leq c \tag{12}$$

If the prior distribution is named $\pi_E(\theta)$, it is calculated as

$$\pi_E(\theta)=\frac{1}{c-1}\int_1^c b(1-\theta)^{b-1}\,db \tag{13}$$

then $\pi(\theta|\underline{x})=\frac{l(\theta)\pi_E(\theta)}{\int_0^1 l(\theta)\pi_E(\theta)d\theta}$.

Consider

$$\sum_{m=0}^{\infty} \rho_m(-n)\theta^{m+n(\bar{x}-1)+a-1} \int_1^c b(1-\theta)^{b-1}\, db$$

and suppose $\mathrm{B}(a) = \sum_{m=0}^{\infty} \left(\rho_m(-n) \int_1^c \mathrm{B}(m+n(\bar{x}-1)+a, b)\, db \right)$. Then

$$\pi_E(\theta \mid \underline{x}) = \frac{\sum_{m=0}^{\infty} \rho_m(-n)\theta^{m+n(\bar{x}-1)+a-1} \int_1^c b(1-\theta)^{b-1}\, db}{\mathrm{B}(a)} \tag{14}$$

is the posterior distribution of θ and, under loss function $l(\theta, d) = (d - \theta)^2$,the Expected Bayesian (E-Bayesian) estimation is given as

$$\mathrm{E}(\theta \mid \underline{x}) = \frac{\mathrm{B}(a+1)}{\mathrm{B}(a)} \tag{15}$$

$\mathrm{E}(\theta \mid \underline{x})$ is computed by numerical methods using R software.

Simulation

The simulation logarithmic series distribution is applied and the MSE among these three estimations are compared. The sample sizes chosen are n = 10 (10)50, 100 from the logarithmic series distribution and then the above sampling is repeated 1000 times. In all the tables below, a = 1, b = 1.

Table 1. MSE of MLE, Bayesian, and E-Bayesian estimation for c = 1.1, θ = 0.2

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE
10	0.1754406	0.02280529	0.268888	0.015603929	0.2670986	0.015603929
20	0.1848736	0.01274284	0.235789	0.009822050	0.2349044	0.009710344
30	0.1888380	0.00885097	0.223687	0.007274929	0.2231012	0.007219458
40	0.1915576	0.00665076	0.217965	0.005719935	0.2175264	0.005686960
50	0.1951806	0.00536783	0.216286	0.004821819	0.2159331	0.004798442
100	0.1968546	0.00251246	0.207595	0.002366069	0.2074195	0.002360269

Table 2. MSE of MLE, Bayesian, and E-Bayesian estimation for c = 1.5, θ = 0.2

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE
10	0.1754406	0.02280529	0.268888	0.015603929	0.2600561	0.013984288
20	0.1848736	0.01274284	0.235789	0.009822050	0.2313332	0.009281856
30	0.1888380	0.00885097	0.223687	0.007274929	0.2207135	0.007003590
40	0.1915576	0.00665076	0.217965	0.005719935	0.2157289	0.005557730
50	0.1951806	0.00536783	0.216286	0.004821819	0.2144824	0.004706188
100	0.1968546	0.00251246	0.207595	0.002366069	0.2066942	0.002337212

Table 3. MSE of MLE, Bayesian, and E-Bayesian estimation for c = 2, θ = 0.2

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE
10	0.1754406	0.02280529	0.268888	0.015603929	0.2517434	0.012635242
20	0.1848736	0.01274284	0.235789	0.009822050	0.2269419	0.008802810
30	0.1888380	0.00885097	0.223687	0.007274929	0.2177276	0.006756307
40	0.1915576	0.00665076	0.217965	0.005719935	0.2134603	0.005407923
50	0.1951806	0.00536783	0.216286	0.004821819	0.2126412	0.004597898
100	0.1968546	0.00251246	0.207595	0.002366069	0.2057618	0.002309824

Table 4. MSE of MLE, Bayesian, and E-Bayesian estimation for c = 3, θ = 0.2

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE
10	0.1754406	0.02280529	0.268888	0.015603929	0.2370938	0.010660721
20	0.1848736	0.01274284	0.235789	0.009822050	0.2187640	0.008047442
30	0.1888380	0.00885097	0.223687	0.007274929	0.2120299	0.006352388
40	0.1915576	0.00665076	0.217965	0.005719935	0.2090714	0.005159055
50	0.1951806	0.00536783	0.216286	0.004821819	0.2090494	0.004414182
100	0.1968546	0.00251246	0.207595	0.002366069	0.2039068	0.002262655

Table 5. MSE of MLE, Bayesian, and E-Bayesian estimation for c = 5, θ = 0.2

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE
10	0.1754406	0.02280529	0.268888	0.015603929	0.2145302	0.008617966
20	0.1848736	0.01274284	0.235789	0.009822050	0.2051204	0.007182372
30	0.1888380	0.00885097	0.223687	0.007274929	0.2021512	0.005863618
40	0.1915576	0.00665076	0.217965	0.005719935	0.2012877	0.004851086
50	0.1951806	0.00536783	0.216286	0.004821819	0.2025892	0.004176082
100	0.1968546	0.00251246	0.207595	0.002366069	0.2004542	0.002201122

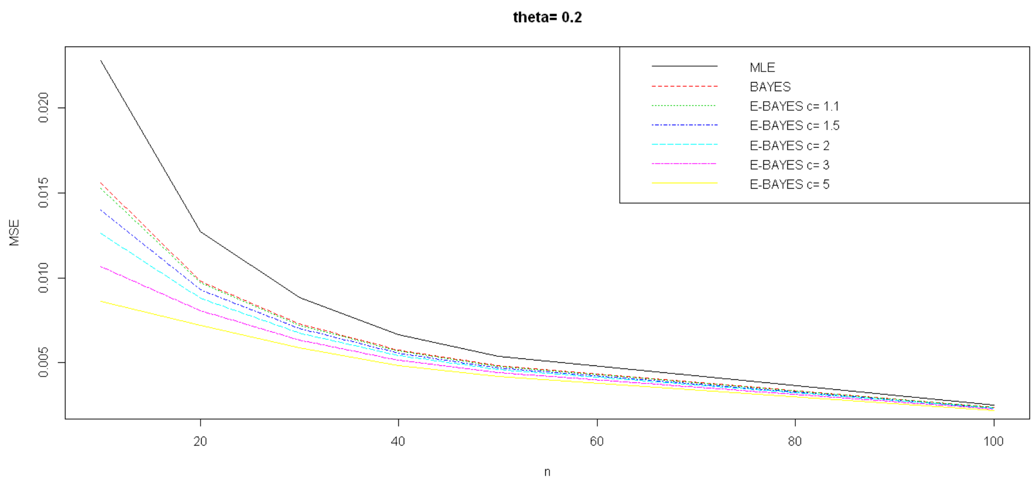

Figure 1. MSE of MLE, Bayesian, and E-Bayesian estimation for $\theta = 0.2$

According to Tables 1-5 and Figure 1 below, if θ is close to zero, then the E-Bayesian estimator will be better than the others. Furthermore, the E-Bayesian estimator for big c is better than for that of small c.

Table 6. MSE of MLE, Bayesian, and E-Bayesian estimation for $c = 1.1$, $\theta = 0.5$

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**
10	0.4493031	0.036068680	0.4727567	0.020631200	0.4703712	0.020689950
20	0.4774472	0.015412300	0.4856296	0.011751570	0.4842737	0.011769950
30	0.4822146	0.011401902	0.4872720	0.009491975	0.4863317	0.009503848
40	0.4862952	0.007998191	0.4897325	0.006975903	0.4890090	0.006984277
50	0.4899989	0.006229734	0.4925262	0.005593061	0.4919384	0.005597817
100	0.4917995	0.003252503	0.4929799	0.003078263	0.4926775	0.003081360

Table 7. MSE of MLE, Bayesian, and E-Bayesian estimation for $c = 1.5$, $\theta = 0.5$

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**
10	0.4493031	0.036068680	0.4727567	0.020631200	0.4611687	0.021062690
20	0.4774472	0.015412300	0.4856296	0.011751570	0.4789428	0.011893800
30	0.4822146	0.011401902	0.4872720	0.009491975	0.4826069	0.009577576
40	0.4862952	0.007998191	0.4897325	0.006975903	0.4861318	0.007033491
50	0.4899989	0.006229734	0.4925262	0.005593061	0.4895955	0.005627388
100	0.4917995	0.003252503	0.4929799	0.003078263	0.4914670	0.003097037

Table 8. MSE of MLE, Bayesian, and E-Bayesian estimation for c = 2, θ = 0.5

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**
10	0.4493031	0.036068680	0.4727567	0.020631200	0.4506494	0.021772010
20	0.4774472	0.015412300	0.4856296	0.011751570	0.4726720	0.012145060
30	0.4822146	0.011401902	0.4872720	0.009491975	0.4781747	0.009721149
40	0.4862952	0.007998191	0.4897325	0.006975903	0.4826866	0.007126098
50	0.4899989	0.006229734	0.4925262	0.005593061	0.4867802	0.005685544
100	0.4917995	0.003252503	0.4929799	0.003078263	0.4900001	0.003122489

Table 9. MSE of MLE, Bayesian, and E-Bayesian estimation for c = 3, θ = 0.5

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**
10	0.4493031	0.036068680	0.4727567	0.020631200	0.4329851	0.023664110
20	0.4774472	0.015412300	0.4856296	0.011751570	0.4617896	0.012865210
30	0.4822146	0.011401902	0.4872720	0.009491975	0.4703766	0.010129876
40	0.4862952	0.007998191	0.4897325	0.006975903	0.4765777	0.007386216
50	0.4899989	0.006229734	0.4925262	0.005593061	0.4817677	0.005854256
100	0.4917995	0.003252503	0.4929799	0.003078263	0.4873595	0.003187286

Table 10. MSE of MLE, Bayesian, and E-Bayesian estimation for c = 5, θ = 0.5

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**
10	0.4493031	0.036068680	0.4727567	0.020631200	0.4081765	0.027960910
20	0.4774472	0.015412300	0.4856296	0.011751570	0.4460818	0.014636860
30	0.4822146	0.011401902	0.4872720	0.009491975	0.4590000	0.011149126
40	0.4862952	0.007998191	0.4897325	0.006975903	0.4676159	0.008033505
50	0.4899989	0.006229734	0.4925262	0.005593061	0.4744028	0.006284713
100	0.4917995	0.003252503	0.4929799	0.003078263	0.4834468	0.003338421

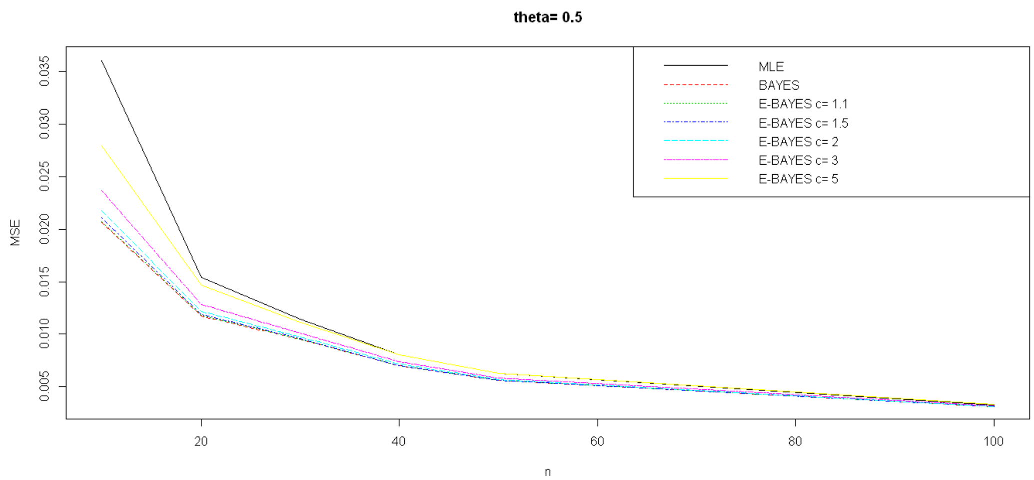

Figure 2. MSE of MLE, Bayesian, and E-Bayesian estimation for $\theta = 0.5$

According to Tables 6-10 and Figure 2 above, if θ is equal to 0.5, then the Bayes estimator will be better than the others.

Table 11. MSE of MLE, Bayesian, and E-Bayesian estimation for $c = 1.1$, $\theta = 0.8$

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**
10	0.7565883	0.016160921	0.7342053	0.016276950	0.7320425	0.016636190
20	0.7771566	0.006733548	0.7635732	0.007190623	0.7623963	0.007302307
30	0.7853821	0.004234158	0.7757980	0.004486893	0.7749946	0.004538776
40	0.7908123	0.003149407	0.7834313	0.003279836	0.7828241	0.003307885
50	0.7891018	0.002431975	0.7831037	0.002570948	0.7826102	0.002592876
100	0.7962759	0.000991319	0.7931716	0.001020993	0.7929747	0.001032422

Table 12. MSE of MLE, Bayesian, and E-Bayesian estimation for $c = 1.5$, $\theta = 0.8$

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**	$\hat{\theta}$	**MSE**
10	0.7565883	0.016160920	0.7342053	0.016276950	0.7240587	0.018094940
20	0.7771566	0.006733548	0.7635732	0.007190623	0.7580292	0.007758596
30	0.7853821	0.004234158	0.7757980	0.004486893	0.7720108	0.004751512
40	0.7908123	0.003149407	0.7834313	0.003279836	0.7805691	0.003423867
50	0.7891018	0.002431975	0.7831037	0.002570948	0.7807726	0.002682152
100	0.7962759	0.000991319	0.7931716	0.001020993	0.7919732	0.001041411

Table 13. MSE of MLE, Bayesian, and E-Bayesian estimation for c = 2, θ = 0.8

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE
10	0.7565883	0.016160920	0.7342053	0.016276950	0.7156517	0.019879590
20	0.7771566	0.006733548	0.7635732	0.007190623	0.7534337	0.008318997
30	0.7853821	0.004234158	0.7757980	0.004486893	0.7688793	0.005013464
40	0.7908123	0.003149407	0.7834313	0.003279836	0.7782094	0.003568182
50	0.7891018	0.002431975	0.7831037	0.002570948	0.7788405	0.002790142
100	0.7962759	0.000991319	0.7931716	0.001020993	0.7910271	0.001068188

Table 14. MSE of MLE, Bayesian, and E-Bayesian estimation for c = 3, θ = 0.8

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE
10	0.7565883	0.016160920	0.7342053	0.016276950	0.7034408	0.023040870
20	0.7771566	0.006733548	0.7635732	0.007190623	0.7469579	0.009294010
30	0.7853821	0.004234158	0.7757980	0.004486893	0.7645488	0.005464724
40	0.7908123	0.003149407	0.7834313	0.003279836	0.7749910	0.003818176
50	0.7891018	0.002431975	0.7831037	0.002570948	0.7762116	0.002971987
100	0.7962759	0.000991319	0.7931716	0.001020993	0.7897346	0.001107678

Table 15. MSE of MLE, Bayesian, and E-Bayesian estimation for c = 5, θ = 0.8

	MLE		BAYES		E-BAYES	
n	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE	$\hat{\theta}$	MSE
10	0.7565883	0.016160921	0.7342053	0.016276950	0.6907121	0.027424820
20	0.7771566	0.006733548	0.7635732	0.007190623	0.7410871	0.010504326
30	0.7853821	0.004234158	0.7757980	0.004486893	0.7609354	0.005989118
40	0.7908123	0.003149407	0.7834313	0.003279836	0.7724500	0.004101652
50	0.7891018	0.002431975	0.7831037	0.002570948	0.7741743	0.003164136
100	0.7962759	0.000991319	0.7931716	0.001020993	0.7888172	0.001145513

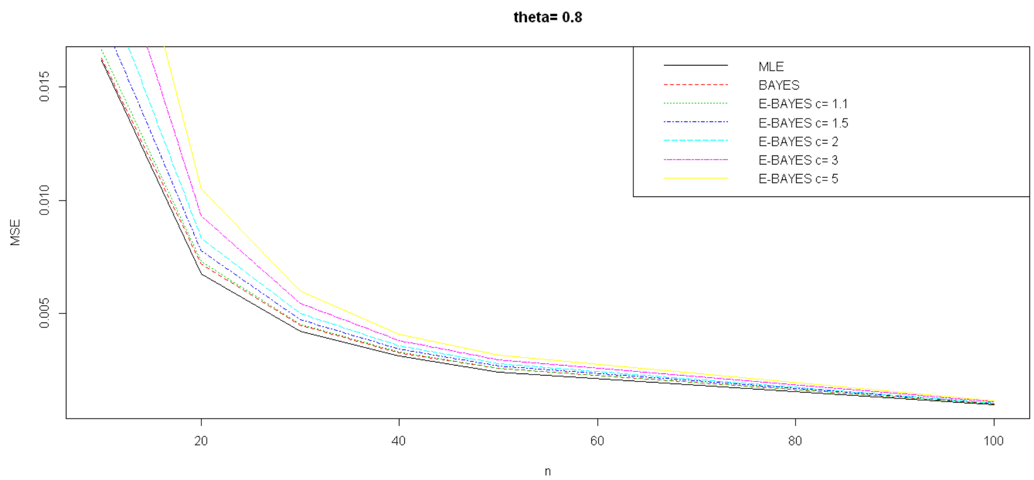

Figure 3. MSE of MLE, Bayesian, and E-Bayesian estimation for $\theta = 0.8$

According to Tables 11-15 and Figure 3 above, if θ is close to 1, then the maximum likelihood estimator will be better than the others.

Conclusion

The comparison among the three estimators revealed that with increasing sample size, all three estimators come together and as a result, the error rate is reduced. However, in the small samples according to the value of θ is superior to any of the rest, of the figures and tables is shown.

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