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Demand Modeling And Capacity Planning For Innovative Short Life-Cycle Products

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DEMAND MODELING AND CAPACITY PLANNING FOR INNOVATIVE SHORT LIFE-CYCLE PRODUCTS

by

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DEDICATION

To my parents: Esmaeil and Nasrin

To my sister and brother: Seiran and Siamak

And to my wife: Shavein

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Chapter 1 Motivation

New products are facing shrinking life-cycles due to a rapid rate of innovation, increasing global competition, and fast changing consumer preferences. Such products arise frequently in the electronics (Lee 2010), semiconductors (Mallik and Harker 2004, Kempf, Erhun et al. 2013), toy (Wong, Arlbjørn et al. 2005) and fashion (Christopher, Lowson et al. 2004) industries, where a large proportion of the product mix consists of short life-cycle (SLC) products. A typical demand pattern for such products, as shown in Figure 1.1, is characterized by rapid growth (ramp-up), maturity, and a decline phase (ramp-down).

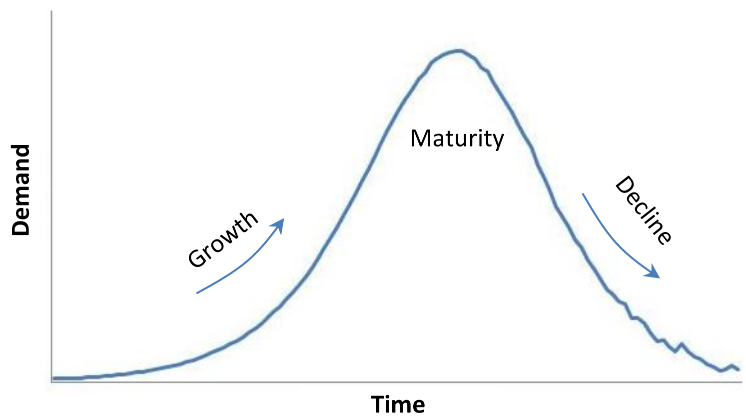

Figure 1.1: Typical demand profile for a short life-cycle product

Although these products provide opportunities for companies to enjoy significant profit margins, inaccurate and unreliable forecasts, due to the lack of history for their demand, can lead to very risky investment decisions. Over investment can lead to poor utilization of production capacity (overage cost) while under investment can lead to product shortages and loss of goodwill and market share (underage cost). In addition to the lack of history, other factors including significant capacity investment costs, long procurement lead-times, high rate of obsolescence, irreversibility of capacity investments, and volatile demand signals make capacity planning for SLC products complicated and challenging.

Capital expenditure can be considered the foremost factor that makes investment decisions for SLC products profoundly risky and challenging. Historically, SLC products in the semiconductor industry suffer from this problem more than other industries. In 2011, it was announced that the main integrated circuit manufacturers alone spent 34-36 billion USD on capital expenditures (Kanellos 2003, AMD 2006, Osborne 2011)[1].

The other issue that makes capacity planning for SLC products more challenging is the long procurement lead-time of new capacity. For example, in the semiconductor industry, the lead-time for procuring new equipment can be up to 16 months (Benavides, Duley et al. 1999, Peng, Erhun et al. 2012).This problem may not be very critical for a manufacturer who faces a predictable demand from the market, but it certainly increases the difficulty of capacity expansion for SLC products. This is due to the fact that in these markets, demand forecasting is not a trivial task and therefore it is often inaccurate.

Another factor that complicates investment in SLC products is irreversibility. Irreversibility, which prevents manufacturers from disinvestment, is the result of technological complexity or from sustaining competitive advantages. As an example, many equipment and machines in the semiconductor industry are custom-made for a specific manufacturer and they cannot be sold to and used by other manufacturers. In some cases, even using them for other types of products in the same company is not possible. Furthermore, the high obsolescence rate of SLC products exacerbates the issue. SLC products, especially in the electronics and the semiconductor industries (see (Aizcorbe 2007) for more detail), are changing rapidly that make the depreciation rate of machines and tools very high. This issue cannot be alleviated by reusing current manufacturing infrastructure in new generations of SLC products since they

[1] Intel and AMD are among the main players in this business. Intel has spent, on average, 5.3 billion USD annually on capital additions to its property, plant, and equipment (Peng, Erhun et al. 2012). Additionally, building a fab costs Intel almost 6 billion USD, up from 100 million USD in 1985 (Kanellos 2003). In another example, AMD spent 2.5 billion USD on building a single plant in 2006 (AMD 2006).

are highly specialized and retrofitting is as costly as procuring new manufacturing capacities (Pangburn and Sundaresan 2009).

These challenges could be coped better by the SLC manufacturers if not for the significant volatility in demand. In most cases, customers send signals to a manufacturer regarding the due date and volume of their final demand. However, these signals are subject to significant volatility since customers' prediction of their final order is a function of other factors that change through time dramatically. Eventually, customers finalize the quantity and due date of their orders, but the time between the finalization and the order due date is shorter than the lead time of capacity procurement significantly (Kempf 2004, Higle and Kempf 2011).

Considering these issues, capacity planning for SLC products has a significant effect on the profitability of a manufacturer and may even endanger the survival of a company (Baljko 1999, Savage 1999, Greek 2000, Singhal and Hendricks 2002). HP (world's largest IT company), Intel (world's largest chip maker), Apple (one of the world's largest consumer electronics company), Taiwan Semiconductor Manufacturing Company (world's largest semiconductor foundry), and Bandai (a global leader in consumer toys) are just few examples of companies that have suffered from this problem. HP lost millions of dollars in unnecessary capacity and excess inventory after a surge in demand for LaserJet printers (Lee, Padmanabhan et al. 1997). Facing shortages of Pentium III processors in 1999, Intel planned to introduce a new production plant in early 2000 (Foremski 1999). However, later that year, Intel could not reach the projected revenues; an incident that the company cites was due to order cancellations and economic slowdowns (Gaither 2001). As for companies suffering from production shortages, Wall Street Journal reported few years ago that Apple is not able to satisfy demand for iPad and iPhone products due to a supply shortage of LCD displays that go into these products (Lee 2010). South Korean LCD-maker LG, the main supplier for Apple, has acknowledged the problem and hoped to have supplies ramped up by the 2nd quarter of 2011. LG Chief Executive stated: "Apple is ordering more and more displays but it isn't something we can respond to quickly. I am not sure whether we can … meet orders from other companies for similar products…" To address the shortage concerns, LG Display said it will invest about $512 million to build a new production line that can produce mobile displays used in iPad and similar products. In another example, in 2011, Qualcomm, AMD, and NVIDIA were affected by a shortage of supply from Taiwan Semiconductor Manufacturing Company for the 28-nanometer Kepler GPUs (Dignan 2012). This shortage led to an increase of 10 percent foundry capacity of Taiwan Semiconductor Manufacturing Company in 2012. Looking into the toy industry, Japanese toy manufacturer Bandai Co. introduced a virtual pet game in 1996 called the Tamagotchi. Although the product was not advertised in the mass media, demand outpaced supply quickly. Bandai decided to expand its manufacturing capacity to produce 2-3 million units per month. Subsequent to expansion, it was met with a sharp decline of demand leading to tremendous unsold inventory resulting in an after-tax loss of 123 million USD in fiscal year 1998 (Higuchi and Troutt 2004).

With these challenges and obstacles, firms might be more cautious in capacity decisions and might attempt to build up inventory before product launch in order to avoid investments during a life-cycle (Ho, Savin et al. 2002, Kumar and Swaminathan 2003). However, this option has some drawbacks, since carrying inventory imposes a cost, while delaying a product launch costs firms in the sense that they lose sales in the short run and may also reduce the market share in the future (Urban, Carter et al. 1986). For example, recently, HP discontinued the production of HP Touchpad and cleared their ample inventory (estimated to be two million tablets) with extensive markdowns that cost HP 400 million USD (O'Flaherty 2011).

1.1 Problem Statement

Since "short" is a relative term in referring to short life-cycle products, we consider the length of capacity procurement lead time in relation to the length of the product life-cycle as a measure for characterizing and labeling a life-cycle. SLC products are those that have a relatively long capacity procurement lead time. This definition helps us to apply this work not only to products with life-cycles measured in quarters but also to other products with longer life-cycles but, based on the length of their capacity procurement lead time, with similar characteristics to SLC products. As an example, two products with the life-cycle of ten years and ten quarters can both can be categorized as SLC when the former has a capacity procurement lead time of three years and the later with a lead time of three quarters.

Moreover, being that "capacity" is a general term in the literature, we limit our discussion in this dissertation to "capital equipment capacity." In addition, it will be assumed that each unit of capacity can be used for producing one unit of a product in each period of a life-cycle.

In our setting, a make-to-order (MTO) middle-echelon manufacturer produces a perishable (non- storable) SLC product that needs significant investment for capacity procurement and maintenance. Additionally, the manufacturer has a forecast of future demand over the course of the product life-cycle. Not to mention, expensive capacity decisions are its main issue that affects profitability. The manufacturer has to bear two types of costs for the capacity: marginal expansion cost and capacity maintenance cost. Marginal expansion cost is a one-time cost that the manufacturer pays at the time of procurement per each unit of new capacity. Maintenance capacity is the cost of maintaining a unit of capacity per period for each unit of installed capacity. We assume that these costs (cost per unit capacity) do not change over the product life-cycle. Any other fixed costs (e.g., cost of setting up the production facility) are assumed to be sunk costs or negligible.

Moreover, the manufacturer is penalized a marginal shortage cost for its inability to meet market demand (this is in addition to the cost of missing revenue from the lost demand). In some industries, this marginal cost is significantly higher than the marginal cost of idle capacity (Peng, Erhun et al. 2012). Measuring this shortage cost is not easy and manufacturers typically assume different values based on the effect of the shortage on their image in a market, their competitors' ability to attract their unmet demand, the effect of unsatisfied orders on future demand, and so on.

Based on these costs, the manufacturer's goal is to maximize the expected discounted (net present value of) profit for the SLC product by balancing over and under expansions of capacity over the course of the product life-cycle. Over-expansion costs are related to capacity expansion and maintenance costs and under-expansion costs are associated with shortage penalty and unrealized revenue of unmet demand.

In terms of the timing of expansion decisions, it is assumed that the manufacturer is able to expand capacity both before launching a product and during the product's life-cycle. However, without loss of generality, we assume that capacity expansion epochs during a life-cycle are limited to a subset of some predetermined epochs. Note that the decision maker may choose to increase or not increase capacity at these predetermined epochs. This assumption is very common in the real world, since expansion decisions are typically made monthly, quarterly, or yearly, depending on the length of the product life-cycle. We also assume that the lead-time for procuring and installing new capacity is fixed and known.

Figure 1.2 illustrates the sequence of events in period s, one of the predetermined expansion periods where expansion is allowed. In this period, based on observed demand and available capacity, the decision maker updates the information regarding the market potential and initializes new capacity procurement if needed. The newly procured capacity can be utilized after a fixed expansion lead-time (L). Production and demand fulfillment decisions are taken after any new expansion decision. We assume that the SLC product is made to order, hence, production does not exceed demand in any period (no finished goods inventory is allowed). We also assume that any unmet demand is lost and will not be backordered.

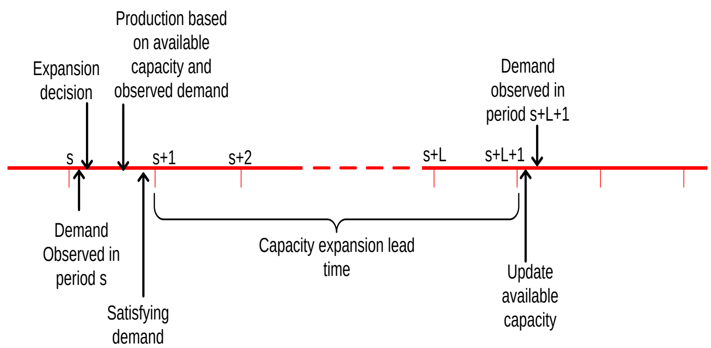

Figure 1.2: Timeline of events for capacity expansion decisions

In addition, product price and shortage penalty are assumed to be monotonically changing (decreasing) during a life-cycle. This assumption makes the model more realistic, since manufacturers are often unable to command prices toward the end of a life-cycle that match the prices at product launch. Other than in the case of a pure monopoly (which is quite rare in the real world), manufacturers have to provide mark downs at the final stages of a life-cycle due to the introduction of new generation products, competitors' entry, etc. Similarly, at the end of a life-cycle, the effect of product shortage on future market demand is less severe than in the beginning of a life-cycle. We assume that the rate of decay in a product's price and shortage cost are the same.

In the main model of this dissertation, we assume that the salvage value of installed capacity is negligible. However, in the extension chapter, we provide an extension to the model that takes into account the salvage value of an installed capacity. Also, in our main model, the manufacturer only produces one product and any installed capacity will be used to manufacture that product. This assumption will be relaxed in the extension chapter where we present a model in which the installed capacity can be employed for producing the next generation version of the current product.

Although the decision maker can pick from different supply modes with different procurement characteristics, a procurement option should be selected before launching a product. It is assumed that the chosen option has unlimited supply and cannot be changed during a life-cycle. This restriction will be removed in the extension chapter in which we entertain the possibility of simultaneously working with a fast but expensive supply supplier as well as a slow but cheaper one (dual mode supply).

As explained earlier, the main challenge of the manufacturer in this setting is capacity planning. An example of expansion decisions for a given (deterministic) SLC demand and specific expansion costs is shown in Figure 1.3. In this work, it is assumed that the time of product entry to the market is fixed and the decision maker will procure initial capacity in such a way to make it available at the first period of a life-cycle. Once the product is launched, the manufacturer only expands capacity in predetermined expansion periods. Due to the fact that available capacity changes dynamically during a life-cycle, we call this strategy *dynamic capacity planning*. Note that based on procurement lead-time and length of a life-cycle and cost of capacity maintenance, initial capacity decision might be the last one.

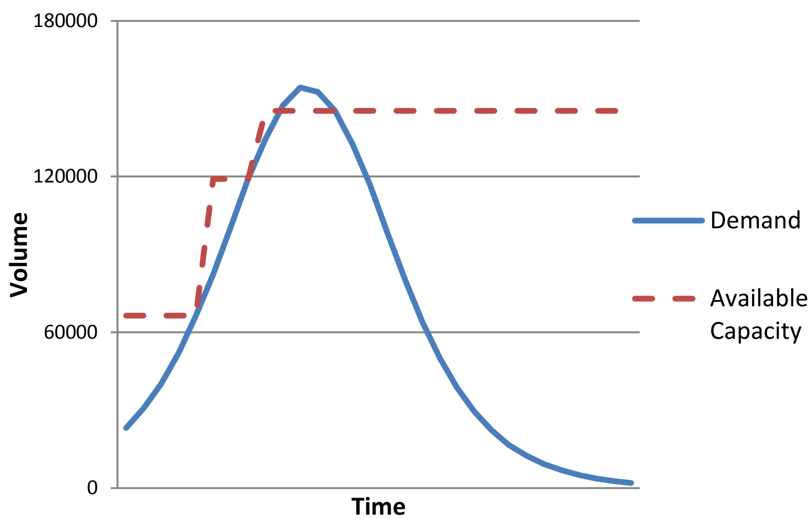

Figure 1.3: Illustration of dynamic capacity planning when the demand is not stochastic

In this dissertation, we propose a novel stochastic capacity expansion model that can be used by a manufacturer in order to optimally determine policies for specifying timing and size of capacity procurement considering the following additional factors:

a. Lead-times for capacity expansion

b. Non-deterministic non-stationary demand and diffusion process

c. Irreversibility (at least partial) in capacity investments

d. Costs associated with lost sales and unutilized capacity

This model attempts to find answers, before and during a life-cycle, for when and how much capacity to expand in a dynamically changing and stochastic environment. The proposed model also can be used for any other product that has a finite life-cycle with stochastic stationary/non-stationary demands. We believe that the proposed model converts the complicated capacity planning problem to a more tractable and efficient one when compared to other classic stochastic methods (e.g., stochastic programming and Markov Decision Processes).

Due to the importance of demand modeling in SLC products, in addition to a model for capacity planning, we also present a new model in the class of stochastic Bass formulations. This stochastic Bass model addresses shortcoming of models from extant literature and will be used for numerical experiments of the proposed capacity expansion model.

This dissertation is organized as follows: in the second chapter, we review the literature for capacity planning and stochastic Bass model and provide a brief overview for the tools and methods that will be used in this work. The proposed stochastic Bass diffusion model will be presented in chapter 3. In chapter 4, the stochastic optimal expansion model, with necessary lemmas, propositions, corollaries and algorithms will be provided. Chapter 5 contains three extensions of the stochastic optimal expansion model. Finally, in chapter 6 the numerical results of the experiments that have been conducted for the model will be presented. In our final chapter, we discuss the implications of our results and future research directions.

Chapter 2 Literature Review

We begin this chapter by reviewing the literature on capacity planning for short life-cycle (SLC) products. Then, we provide an overview of the Bass diffusion model and the major contributions regarding its stochastic versions. Since the proposed stochastic optimal expansion model is inspired by the newsvendor model, a short description of this model is presented at the end of this chapter.

2.1 Capacity Planning

Capacity planning related research can be broadly categorized into two groups: strategic and tactical. At the strategic level, capacity planning involves deciding upon the firm's own capacity investment and that of its supply chain partners. The literature extensively considers settings that model independent decision makers in a supply chain context (Wu, Erkoc et al. 2005). Research in this area employs game-theoretic models addressing issues such as contracting, coordination (Cachon 2003, Bernstein and DeCroix 2004, Armony and Plambeck 2005, Plambeck and Taylor 2005), and risk-associated mechanisms (Birge 2000, Van Mieghem 2003, Ding, Dong et al. 2007). An excellent review of coordinating contracts is provided by Cachon (Cachon 2003). However, in our work, strategic capacity planning is referred to a case at which a decision maker only expands capacity once (before product launch).

2.1.1 Strategic Capacity Planning for Innovative Products

The literature considering strategic capacity expansion for SLC products is limited. Ho et al. (Ho, Savin et al. 2002) proposed a model for capacity expansion and for the timing of the product launch for SLC products. In their work, the Bass diffusion model parameters are assumed to be known and capacity decisions are made before product launch and cannot be changed during the life-cycle. They show that a myopic sales plan is always optimal in their specific setting. In a similar work, Kumar and Swaminathan (Kumar and Swaminathan 2003) showed that the demand for an SLC product is not exclusively exogenous and that the sales plan (myopic or build-up), as well as size of capacity installed, can change the demand curve. Thus, they concluded that the optimal capacity sizing decisions would be quite different depending on whether one used a myopic sales plan or the build-up plan. However, in another work (Shen, Duenyas et al. 2011), it has been shown that the proofs of (Ho, Savin et al. 2002) are not correct and consequently, a myopic sales plan is not an optimal strategy. In all of these works, it is assumed that there is no uncertainty in the demand (Bass model parameters that represent the effect of innovation as well as imitation and the market potential are assumed to be known and fixed). In a very similar work, Yan and Liu (Yan and Liu 2009) modified the Bass diffusion model in order to consider the capacity constraint and derived an optimal capacity level (a one-time decision), an optimal production policy and an optimal sales policy for a manufacturer who does not face any demand uncertainty.

Capacity planning for SLC products in which the decision maker only expands capacity before the product launch was considered by Pangburn and Sundaresan (Pangburn and Sundaresan 2009). However, their model focuses on rapid obsolescence rate of products and an inverse-demand curve has been used for "parsimonious" demand modeling.

2.1.2 Dynamic Capacity Planning

Research on dynamic capacity models with stochastic demand goes back to the work of Manne (Manne 1961), in which he models demand growth using a Brownian motion. The resulting regenerative process leads to uniform capacity augmentation that occurs whenever the demand backlog goes beyond a threshold value. Following Manne's work, many extensions and modifications have been proposed (Giglio 1970, Bean, Higle et al. 1992). In the context of the semiconductor capacity planning, Karabuk and Wu (Karabuk and Wu 2002) considered the coordination between production and marketing in the same corporate organization in order to allocate capacity on a period-by-period basis based on realized uncertainties and acquired capacities. In addition, Vlachos (Vlachos, Georgiadis et al. 2007) presented the development of a system dynamics model for remanufacturing capacity expansion of a reverse supply chain for product recovery.

Although dynamic capacity expansion has been addressed in the literature, very few works have considered the life-cycle of a product and regime switching behavior of a SLC product demand (ramp-up, maturity, and ramp-down). Kamath and Roy (Kamath and Roy 2007), using retail sales data for the capital augmentation decisions, claimed that the information-feedback-based methodology is general enough to be used in designing decision support systems for capacity augmentation in SLC environments. However, their model is a system dynamics model and it does not explicitly address the uncertainty in the demand. In another work (Cantamessa and Valentini 2000), a simple Mixed-Integer Linear programming model is developed to find the optimum initial capacity and fixed capacity augmentation in each period of a new product life-cycle. They, too, assume that the Bass model parameters that represent the effect of innovation, as well as imitation and the market potential are known and fixed.

Angelus and Porteus (Angelus and Porteus 2002) study simultaneous capacity and production planning problem for a SLC product in which the company can invest and disinvest capacity. In their model, it is assumed that there is no lead-time for the capacity expansion and there are no backlogs for unmet demand. They show that the optimal capacity plan can be reduced to a "target interval policy" that is a one-dimensional invest/stay-put/disinvest (ISD) policy. The target interval policy specifies a lower and upper capacity target. In the case when no carry-over inventory is allowed, it was shown that the target interval is an optimal policy. The authors do not consider the complete life-cycle, rather they only consider the one-period-ahead demand distribution. Moreover, they assumed that investment and disinvestment in any period of a life-cycle is possible and that there is no lead-time for capacity investment (or disinvestment). Figure 1 shows an example of an expansion decision in this model.

In summary, to the best of our knowledge, the literature on capacity planning for innovative SLC products is relatively limited. Not only is it limited, but also the few models proposed in the literature have major shortcomings. Regime switching and non-deterministic diffusion of innovative products are critical factors for capacity decisions that are often either ignored (Kamath and Roy 2007) or addressed by questionable assumptions, such as:

a. A decision maker is aware of fixed diffusion parameters and demand behavior is deterministic (Cantamessa and Valentini 2000, Ho, Savin et al. 2002).

b. Investment and disinvestment are possible in any period of a life-cycle with zero lead-time. Thus, the decision maker does not need to consider the total life-cycle (Angelus and Porteus 2002).

c. Demand is a non-decreasing stochastic process (usually geometric Brownian motion with a positive drift). This assumption is very common in capacity planning for semiconductor industry literature (Cakanyildirim, Roundy et al. 2001, Cakanyildirim and Roundy 2002) and totally ignores the fact that the demand of SLC products can be, and usually is, stochastically decreasing after the period of peak demand (Ryan 2004).

2.2 Bass Diffusion Model

For modeling the adoption process of new products and technologies, in particular for SLC products, the marketing science and econometric literature offers different models, including trend curve models (cumulative lognormal (Stapleton 1976), Weibull (Nawaz Sharif and Kabir 1976), extended Logistic (Mahajan, Muller et al. 1990)), linearized trend model (Mansfield 1961, Nawaz Sharif and Kabir 1976), and Non-linear auto-regressive models (Logistic/Mansfield model (Mahajan, Muller et al. 1993), Bass (Bass 2004), non-symmetric responding logistic (Easingwood, Mahajan et al. 1981), Gompertz (Hendry 1972)), just to name a few (see (Mahajan, Muller et al. 1990, Meade and Islam 1998, Peres, Muller et al. 2010) for a complete review). Among all these models, the Bass model has been widely adopted, both in the industry and in academia, and used in many forecasting situations and applications. Moreover, unlike most of the available models that only consider either market innovators (Fourt and Woodlock 1960) or market imitators (Mansfield 1961), the Bass model considers both innovators and imitators at the same time.

The main assumptions of the basic Bass model are (Bass 2004) are as follows: a) the diffusion process is binary, meaning that the potential consumers either adopt or wait to adopt; b)the population of the market potential does not change; c) eventually, the entire potential market will buy the product; d) no repeat purchase is allowed; e) the impact of the word-of-mouth is independent of adoption time; f) there is no supply restriction from the manufacturer; g) price has no effect on the adoption of a new product. However, different extensions of the Bass model have been proposed in which the price has impact on adoption (see (Radas 2006) for the complete review of the extensions).

In the Bass diffusion model, demand at each period is formulated as:

$$b(t|p,q,m) = f(t)m = \frac{\frac{(p+q)^2}{p}e^{-(p+q)t}}{\left(1+\frac{q}{p}e^{-(p+q)t}\right)^2}\,m \tag{2.1}$$

where p and q represent “external influence” and “internal influence”, respectively and m is the potential market population in units. In this formulation, $f(t)$ is the instantaneous rate of product adoption at each period. The cumulative number of products adopted by time t can be formulated as:

$$\sum_{s=1}^{t} b(s|p,q,m) = F(t)\,m = \frac{1-e^{-(p+q)t}}{1-\frac{q}{p}e^{-(p+q)t}}\,m$$

As Equation 2.1 shows, this model describes how a new product is adopted as an interaction between two groups of users. The first group consists of users who are independent of the numbers of previous adopters, while the second group contains users who are buying the product due to the word-of-mouth or other influences from those already using the product. Bass described a force for the first group, represented by *p* (external influence), as the coefficient of innovation, and a force for the second group by *q* (internal influence), as the coefficient of imitation (Figure 2.1).

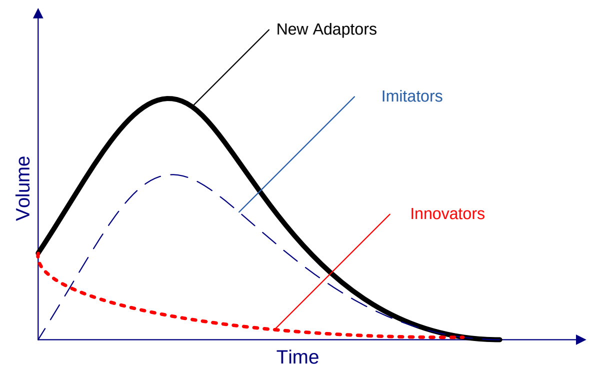

Figure 2.1: An example for a diffusion process

2.2.1 Stochastic Versions of the Bass Diffusion Model

While the Bass diffusion model yields a deterministic market adoption curve, there is a recognized need for a stochastic version of the model (Eliashberg and Chatterjee 1986). In this section, we review the popular stochastic diffusion models from the literature and discuss their shortcomings:

a. Wu and et. al. (2010) proposed an extension of the Bass model in which forecasting is conducted based on demand-leading indicators in a Bayesian framework, combining it with different diffusion models. In their model, future demand in each period follows a Gaussian distribution. Because a Gaussian distribution can yield negative values, this model may cause some problems in simulation-based decision support systems. Moreover, the model needs "leading indicator" variables, which might not be available in certain cases.

b. Boswijk and Franses (2005) proposed a stochastic Bass model that was inspired by a class of stochastic processes used in financial engineering literature for modeling the interest rate. In their model, the cumulative number of adopters at time t, $N(t)$, is a random variable with mean $\bar{N}(t) = E\big(N(t)\big) = m\,F(t)$ and $\bar{n}(t) = E\big(n(t)\big) = m\,f(t)$ where $\bar{n}(t) = \frac{d\bar{N}(t)}{dt}$ and $n(t) =$

$\frac{dN(t)}{dt}$. In their work, two formulations for the stochastic bass model were proposed based the following process:

$$dn(t) = \alpha\big(\bar{n}(t) - n(t)\big)dt + \sigma n(t)^{\gamma} dW(t)$$

where $\alpha > 0$, $\sigma > 0$, $\gamma \geq 0.5$, and $W(t)$ is a standard Brownian motion. The mean reversion component of the model implies that $n(t)$ mean-reverts to $\bar{n}(t)$ in the long run. In addition, they provide estimation procedures for estimating the parameters of their model. However, the authors do not provide any formulation for the distribution of demand at each future period. Moreover, they have assumed that the Bass diffusion parameters are fixed during a life-cycle.

c. Niu (2002) developed a stochastic formulation of the Bass model that is a pure birth model with the following birth rates:

$$\lambda_n = (M - n)\left(p + q\frac{n}{M - 1}\right)$$

In his model, it was shown that when population size goes to infinity, the fraction of customers who have bought a product by time t converges in probability to the fraction of customers in the deterministic Bass Model ($F(t)$):

$$\lim_{M\to\infty} F_M(t) = \lim_{M\to\infty} E(\frac{n}{M}) = F(t)$$

$$F(0) = 0$$

This model also assumes that the Bass model parameters do not change through time and does not provide any distribution for future demands.

d. In another work by Kanniainen and et. al. (2011), it is assumed that future demand can be characterized by a continuous time process, represented logarithmically as the sum of two components: a deterministic Bass function at time t and a stochastic mean-reverting process (X_t) such that $\ln d_t = \ln b(t|p, q, m) + X_t$ and $dX_t = -\kappa X_t d_t + \sigma dW_t$. κ and σ are mean reversion and standard deviation parameters of the process and W_t is a Brownian motion process. They show that the future demand in each period follows a lognormal distribution with the following moments:

$$E_s(S_{t>s}) = b(t)\beta_1(s,t)\beta_2(s,t)$$

$$Var_s(S_{t>s}) = (\beta_2^2(s,t) - 1)\beta_1^2(s,t)\beta_2^2(s,t)b^2(t)$$

where

$$\beta_1(s,t) = \exp\left((\ln d_t - \ln b_t)e^{-\kappa(t-s)}\right)$$

and

$$\beta_2(s,t) = \exp\left(\frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa(t-s)}\right)\right)$$

The overall stochastic demand is essentially the deterministic Bass model with a white noise. This assumption might not be correct, since a white noise cannot be the only reason that makes real demand deviate from what a deterministic Bass model has forecasted. Moreover, in their paper, the forecasting accuracy of the model is not presented.

2.3 Newsvendor Problem

The newsvendor problem (a.k.a. the newsboy problem) pertains to a classical mathematical model in operations management under the category of finite inventory process [REF]. In this model, the aim is to maximize the expected profit by finding an optimal order quantity for a make-to-order product in a single period probabilistic demand framework (retailed has a forecast for the demand distribution). The ordered quantity is stored in a retailer's inventory and will be used for satisfying demand. In this problem, one day, one week, one season, or any other time frame can be considered as one period (Ayhan, Dai et al. 2003).

While the newsvendor model has a number of extensions, in the basic model, the retailer's selling price is *r* and the unit price for product procurement is c_e. The salvage value for each unit unsold at the end of the period is assumed to be c_s, which is assumed to be less than c_e. In case the retailer has to pay to dispose of any unsold product, $c_s < 0$. Let D be the random variable that represents the demand and F be the cumulative distribution function of demand, meaning $\Pr(D < d) = F_D(d)$. In each period, the retailer is looking for an optimal amount order (q) that maximizes the following expected profit function:

$$\Pi(q) = (r - c_s)\,\mathbb{E}(\min(D, q)) - (c_e - c_s)q$$

In order to make this function more intuitive, it can be rewritten as:

$$\Pi(q) = (r - c_e)\,\mathbb{E}(\min(D, q)) - (c_e - c_s)\mathbb{E}(\max(q - D, 0))$$

where $(r - c_e)$ is considered the profit margin of each product sold and $(c_e - c_s)$ is the marginal cost of each unsold product. Although a naive decision maker might suggest an order quantity that is equal to the expected value of the demand, it can be shown that the optimal order quantity, q^*, is not necessarily equal to $\mathbb{E}(D)$. Furthermore, based on the shortage and overage costs, it can be more or less than expected value of demand. It was proven that q^* solves the following equation (assuming *F* is continuous):

$$F(q^*) = \frac{r - c_e}{r - c_s}$$

where $\frac{r-c_e}{r-c_s}$ is called the newsvendor critical fractile. This fractile is the ratio of the cost of being under-stocked and the total costs of being either over-stocked or under-stocked is ($\frac{r-c_e}{(r-c_e)+(c_e-c_s)}$). Finally, it is worth mentioning that in the case of a discrete demand distribution q^* is the smallest q such that:

$$F(q) \geq \frac{r - c_e}{r - c_s}$$

Chapter 3 Stochastic Bass Diffusion Model

While the Bass model (Equation 3.1) yields a deterministic market adoption curve, there is a recognized need for a stochastic version of the model (Chatterjee 1986). Surprisingly, very few papers (Skiadas and Giovanis 1997, Boswijk and Franses 2005, Kanniainen, Makinen et al. 2010) have addressed this issue and the proposed models have not been examined for different products and settings. Moreover, the Bass model parameters in the extant stochastic Bass models are assumed to be fixed during a life-cycle. This assumption ignores the fact that even though diffusion parameters (p and q) might be fixed during a life-cycle and can be estimated by using similar previous products or managerial judgments, market potential (m) might change due to unpredictable exogenous/endogenous factors. Some of these factors include competitors' actions (exogenous), economic circumstances (exogenous) and production/quality issues (endogenous). For example, competitors' actions, e.g. leaving (entry to) a market, can expand (shrink) a firm's market share significantly and may lead to an overwhelming unsatisfied demand (unutilized capacity). On the other hand, macroeconomic factors such as the unemployment rate have a major effect on the demand of many manufacturers and any change in this rate might lead to a realized demand that is different from what was predicted. Another factor includes unpredictable production/quality issues that might occur during a life-cycle. Any defect in a product or shortage in production may cause significant shifts in the potential total market. These three factors are just a few examples that can conceivably change a market potential for the product during its life-cycle.

$$b(t|p,q,m) = f(t)m = \frac{\frac{(p+q)^2}{p} e^{-(p+q)t}}{\left(1+\frac{q}{p} e^{-(p+q)t}\right)^2} m \tag{3.1}$$

In this work, by letting market potential to change during a life-cycle, we propose a new stochastic Bass diffusion model that addresses the shortcoming (fixed market potential) of the available stochastic models in the literature. We have assumed that at each period an estimation of current market potential can be provided by a marketing department or a third party market research company and that this estimation is a valid and a precise estimation for the current market potential. However, we assume that the market potential can change during a product's life-cycle based on a geometric random walk with known parameters:

$$m_t = X_t m_{t-1} \tag{3.2}$$

where X_t's are independent random variables that are not necessarily identically distributed. This is attributed due to the fact that percentage changes in the market potential ($\log \frac{m_t}{m_{t-1}}$) are significantly smaller at the end of a life-cycle compared to that of the beginning of a life-cycle (stochastically decreasing).

Without loss of generality and in order to accommodate the effect of time in X_t, let $X_t = \{\beta^{-\frac{1}{\sqrt[\gamma]{t}}}, 1, \beta^{\frac{1}{\sqrt[\gamma]{t}}}\}$ where β and γ are known parameter to the decision maker. In this formulation, β controls the values of X_t and depends on the newness, as well as the characteristics of a product or the uncertainty of the competitors' actions. A decision maker who predicts less uncertainty for future demand picks a smaller β compared to that of a decision maker who expects a volatile demand. In addition, γ is a parameter that controls the decay of X_t through time and it is assumed to be $\gamma \geq 2$. Larger values of γ decrease the decay speed of X_t through time.

Note that if changes in X_t are not a stochastically decreasing process, i.e. receiving more signal from the market through time does not reduce the uncertainty of the demand, it can be defined as $\{\beta^{-1}, 1, \beta\}$. In this formulation, X_t values are not decreasing through time. However, we believe that this case is not a realistic assumption and $\gamma \geq 2$ makes the model more suitable for real-world applications.

Some examples of X_t paths are illustrated in Figure 1 to Figure 4. These plots illustrate the effect of different γ's and β's through time. In these four figures, a solid line is used for $\beta^{\frac{1}{\sqrt[\gamma]{t}}}$ and a dashed line is used for $\beta^{\frac{-1}{\sqrt[\gamma]{t}}}$. Figure 1 and Figure 2 depict the paths during a 13-period life-cycle without $X_t = 1$, since it is a fixed value. In Figure 1, $\beta = 1.4$ and $\gamma = 2$; in Figure 2, $\beta = 1.4$ and $\gamma = 4$.

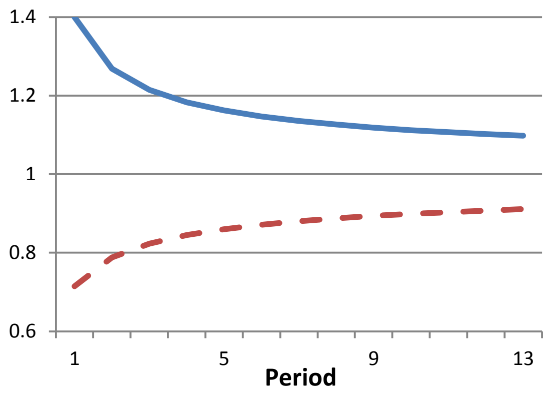

Figure 3.1: Value of X_t during a life-cycle for β=1.4 and γ=2

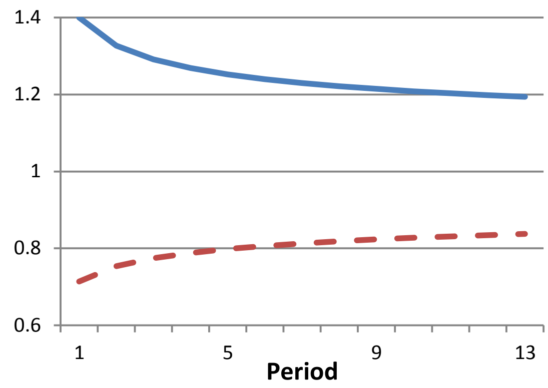

Figure 3.2: Value of X_t during a life-cycle for β=1.4 and γ=4

Similarly, Figures 3 and 4 show the plots for the case in which $\beta = 1.2$.

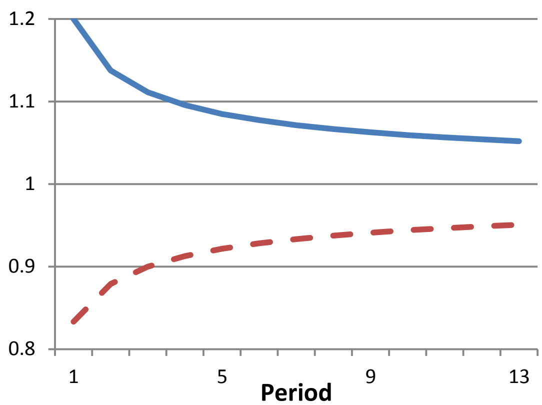

Figure 3.3: Value of X_t during a life-cycle for β=1.2 and γ=2

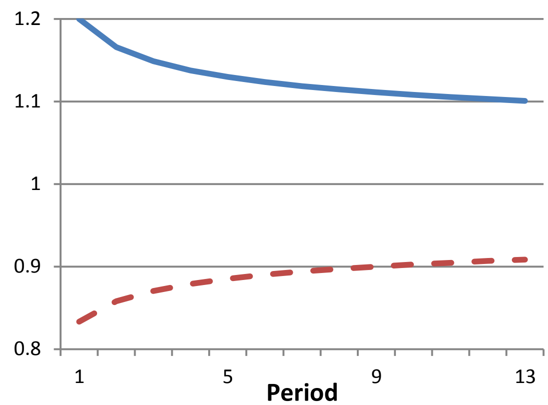

Figure 3.4: Value of X_t during a life-cycle for β=1.2 and γ=4

3.1 Log-normality of Future Demand

By expanding Equation 3.2, the process can be rewritten as $m_k = m_0 \prod_{t=1}^{k} X_t$; consequently, $\log m_k = \log m_0 + \sum_{t=1}^{k} \log X_t$, where m_0 is the current estimation of a market potential that is assumed to be correct. It can be shown that the mean and variance of $\log X_t$ are as follows:

$$E(\log X_t) = \frac{1}{3}\log\beta^{-\frac{1}{\sqrt[\gamma]{t}}} + \frac{1}{3}\log 1 + \frac{1}{3}\log\beta^{\frac{1}{\sqrt[\gamma]{t}}} = 0 \tag{3.3}$$

$$Var(\log X_t) = E(\log^2 X_t) = \frac{1}{3}\left(\log\beta^{-\frac{1}{\sqrt[\gamma]{t}}}\right)^2 + \frac{1}{3}(\log 1)^2 + \frac{1}{3}\left(\log\beta^{\frac{1}{\sqrt[\gamma]{t}}}\right)^2 = \frac{2}{3t^{\frac{2}{\gamma}}}\log^2\beta \tag{3.4}$$

Since the X_t's are not identically distributed, which is apparent from the t in the variance equation, the central limit theorem cannot be used for deriving the distribution of $\log m_k$. However, if it could be proven that the X_t process satisfies the *Lindeberg* condition, it can be shown that $\log m_t$ follows a normal distribution; therefore, m_t has a lognormal distribution in each period of the life-cycle.

In the *Lindeberg* condition, it is stated that when $\{\xi_1, \xi_2 \dots, \xi_t, \dots\}$ are independent random variables with mean 0 and variance σ_t^2 if for every $\epsilon > 0$,

$$\frac{1}{b_t^2}\sum_{j=1}^{t} E\left(\xi_j^2 \mathbf{1}_{|X_j|>\epsilon b_t}\right) \xrightarrow{p} 0 \quad \text{as} \quad t \to \infty$$

then $\frac{S_t}{b_t} \xrightarrow{d} N(0,1)$. In this theorem, $b_n^2 = \sum_{j=1}^{n}\sigma_j^2$ is the variance of $S_n = \xi_1 + \xi_2 + \dots + \xi_n$. In our case, instead of $\{\xi_1, \xi_2 \dots, \xi_t, \dots\}$, the random variables are $\{\log X_1, \log X_2, \dots, \log X_t, \dots\}$ and $b_t^2 = \sum_{j=1}^{t}\sigma_j^2 = \sum_{j=1}^{t}\frac{2}{3j}\log^2\beta = \frac{2}{3}\log^2\beta\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}$. As a result, the *Lindeberg* condition of the problem can be rewritten as:

$$\underbrace{\frac{1}{\frac{2}{3}\log^2\beta\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}}}_{\frac{1}{b_t^2}} \overbrace{\sum_{j=1}^{t} E\left((\log^2 X_j)\mathbf{1}_{\underbrace{|\log X_j|>\epsilon\frac{\sqrt{2}}{\sqrt{3}}(\log\beta)\sqrt{\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}}}_{condition\ A}}\right)}^{\tilde{b}_t^2} \tag{3.5}$$

If it can be shown that there is a finite period ($\tilde{t}$) such that $\left|log\, X_{j>\tilde{t}}\right| \ngtr \epsilon \frac{1\sqrt{2}}{\sqrt{3}}(\log\beta)\sqrt{\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}}$, we can conclude that $\tilde{b}_t^2 = \sum_{j=1}^{n} E\left(\left(\log^2 X_j\right)\mathbf{1}_{|log\, X_j| > \epsilon\frac{1\sqrt{2}}{\sqrt{3}}\log\beta\sqrt{\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}}}\right)$. $\tilde{b}_t^2$ is a restricted total variance based on condition A (in Equation 3.6), which is much smaller than $b_t^2 = \frac{2(1^2)}{3}\log^2\beta\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}$, thus

$$\underbrace{\frac{1}{\frac{2}{3}\log^2\beta\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}}}_{b_t^2}\overbrace{\sum_{j=1}^{t} E\left(\left(\log^2 X_j\right)\mathbf{1}_{\underbrace{|log\, X_j| > \epsilon\frac{\sqrt{2}}{\sqrt{3}}(\log\beta)\sqrt{\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}}}_{condition\ A}}\right)}^{\tilde{b}_t^2} \xrightarrow{p} 0 \tag{3.6}$$

when $t \to \infty$. Based on the definition of X_t, this condition needs to be satisfied for all possible values of X_t, which are $\beta^{\frac{-1}{\gamma\sqrt{t}}}, 1, \beta^{\frac{1}{\gamma\sqrt{t}}}$. In the case of $X_t = 1$, the result is obvious due to the fact that condition A is not satisfied anywhere ($\log 1 = 0 < \epsilon\frac{\sqrt{2}}{\sqrt{3}}(\log 1)\sqrt{\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}}$). However, for $X_t = \beta^{\frac{1}{\gamma\sqrt{t}}}$ condition A can be rewritten as:

$$\left(\frac{1}{t}\right)^{\frac{1}{\gamma}}\log\beta > \epsilon\frac{\sqrt{2}}{\sqrt{3}}(\log\beta)\sqrt{\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}} \tag{3.7}$$

$$\left(\frac{1}{t}\right)^{\frac{1}{\gamma}} > \epsilon\frac{\sqrt{2}}{\sqrt{3}}\sqrt{\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}} \tag{3.8}$$

$$t < \left(\frac{\sqrt{3}}{\epsilon \sqrt{\sum_{j=1}^{t} \frac{1}{j^{\frac{2}{\gamma}}} \sqrt{2}}} \right)^{\gamma} \tag{3.9}$$

The indication is that condition A holds only for those periods that $t < \left(\frac{\sqrt{3}}{\epsilon \sqrt{\sum_{j=1}^{t} \frac{1}{j^{\frac{2}{\gamma}}} \sqrt{2}}} \right)^{\gamma}$. As a result, $\tilde{b}_t^2$ stays fixed after period $\tilde{t}$ where $\tilde{t}$ is the last period such that $\tilde{t} < \left(\frac{\sqrt{3}}{\epsilon \sqrt{\sum_{j=1}^{t} \frac{1}{j^{\frac{2}{\gamma}}} \sqrt{2}}} \right)^{\gamma}$. The same results hold for the case in which $X_t = \beta^{\frac{-1}{\sqrt[\gamma]{t}}}$. Therefore,

$$\lim_{t\to\infty} \tilde{b}_t^2 = \lim_{t\to\infty} \sum_{j=1}^{t} E\left(\left(\log^2 X_j\right) \mathbf{1}_{|log\, X_j| > \epsilon \frac{\sqrt{2}}{\sqrt{3}} (\log \beta) \sqrt{\sum_{j=1}^{t} \frac{1}{j^{\frac{2}{\gamma}}}}} \right) = K < \infty \tag{3.10}$$

Since $\lim_{n\to\infty} \sum_{j=1}^{n} \frac{1}{j^{\frac{2}{\gamma}}} = \infty$ $(\gamma \geq 2)$ and

$$\lim_{t\to\infty} b_t^2 = \lim_{t\to\infty} \frac{2}{3} \log^2 \beta \sum_{j=1}^{t} \frac{1}{j^{\frac{2}{\gamma}}} = \frac{2}{3} \log^2 \beta \lim_{t\to\infty} \sum_{j=1}^{t} \frac{1}{j^{\frac{2}{\gamma}}} = \infty \tag{3.11}$$

Equation 3.5 goes to zero ($\frac{K}{\infty} = 0$) when *t* goes to infinity. Therefore, we have showed that:

$$\log m_k \sim N(\log m_0, \frac{2}{3} (\log^2 \beta) \sum_{j=1}^{k} \frac{1}{j^{\frac{2}{\gamma}}}) \tag{3.12}$$

Based on Equation 3.12 and the definition of a lognormal distribution, m_t is a lognormal distribution with the following mean and variance:

$$E(m_t) = m_0 e^{\frac{1}{3}(\log^2 \beta)\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}} \tag{3.13}$$

$$Var(m_t) = m_0^2 e^{\frac{2}{3}(\log^2 \beta)\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}}\left(e^{\frac{2}{3}(\log^2 \beta)\sum_{j=1}^{t}\frac{1}{j^{\frac{2}{\gamma}}}} - 1\right) \tag{3.14}$$

Note that in each period, after estimating the current market potential, the distributions of the future market potentials are updated. For example, if the market potential at period t_1 is m_1, then the distribution of the future market potential would be:

$$\log m_{k>t_1} \sim N(\log m_{t_1}, \frac{2}{3}(\log^2 \beta)\sum_{j=t_1+1}^{k}\frac{1}{(j-t_1)^{\frac{2}{\gamma}}}) \tag{3.15}$$

At this stage, we can derive the distribution of demand at each period by revisiting the Bass diffusion model and using the properties of a lognormal distribution:

$$b(t|p,q,m_k) \sim \mathrm{logN}\big(E(m_t) + \log f(t)\ , Var(m_t)\big) \tag{3.16}$$

Equation 3.16 implies that the future demand in each period of a life-cycle follows a lognormal distribution. Note that the log-normality of demand is an appealing property in real-world applications due to its non-negativity feature.

3.2 Illustration of Volatility Levels in the Proposed Stochastic Bass Model

In order to show the effect of β and γ on the market potential (*m*), the demand and market potential realizations for different γ's and β's are illustrated in Figure 5 through Figure 12. In each of these figures, five independent demand realizations (D1-D5), along with their associated market potential realizations (m1-m5) are presented. Moreover, each plot contains 95% confidence bands, in both demand and market potential realizations, depicted by dashed lines. *p*, *q*, and the initial market potential of the Bass diffusion model are assumed to be 0.001, 0.35, and two million, respectively. Note that the initial market potential is simply the initial estimation of the market potential and, in each period, this value changes based on a geometric random walk.

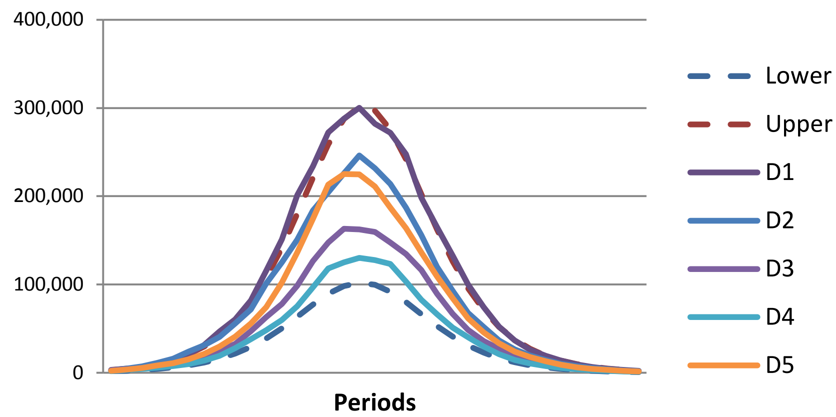

Figure 3.5: Demand realizations for β=1.2 and γ=2

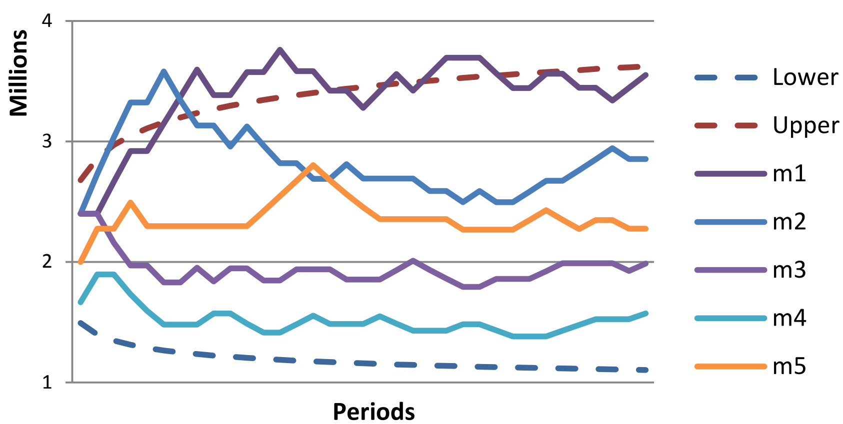

Figure 3.6: Market potential realization for β=1.2 and γ=2

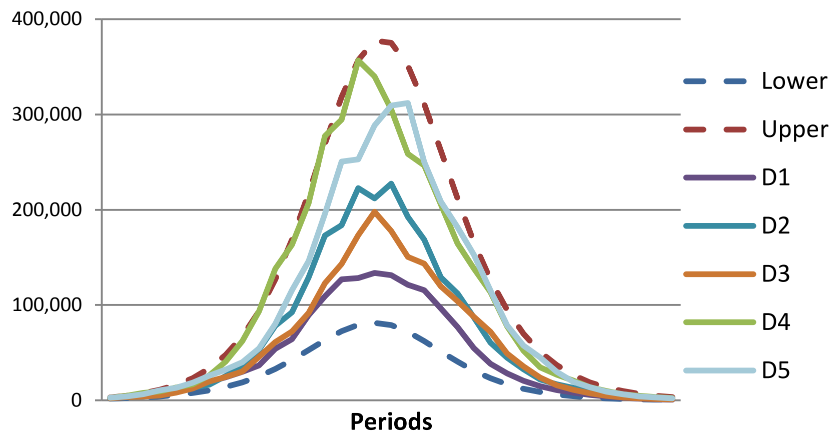

Figure 3.7: Demand realizations for β=1.2 and γ=4

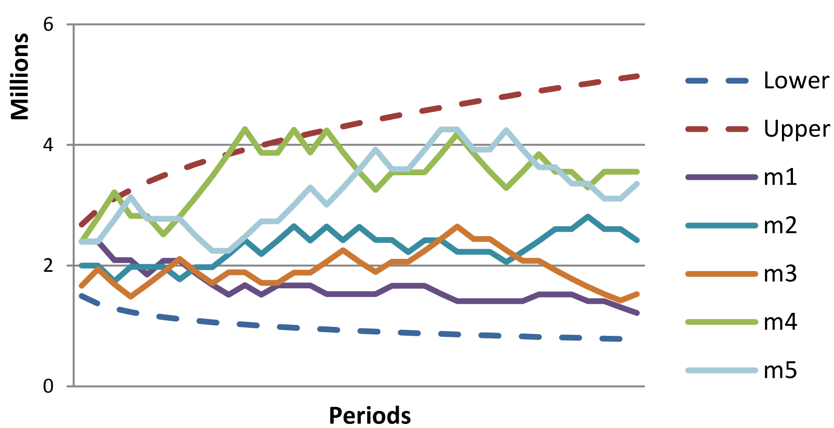

Figure 3.8: Market potential realization for β=1.2 and γ=4

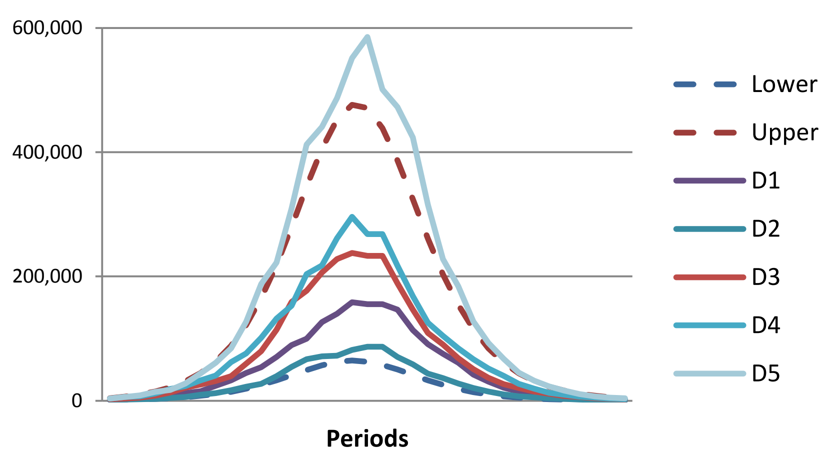

Figure 3.9: Demand realizations for β=1.4 and γ=2

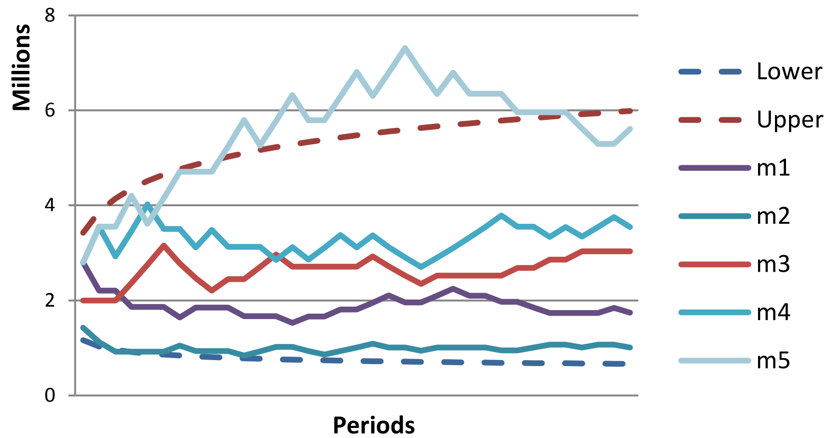

Figure 3.10: Market potential realization for β=1.4 and γ=2

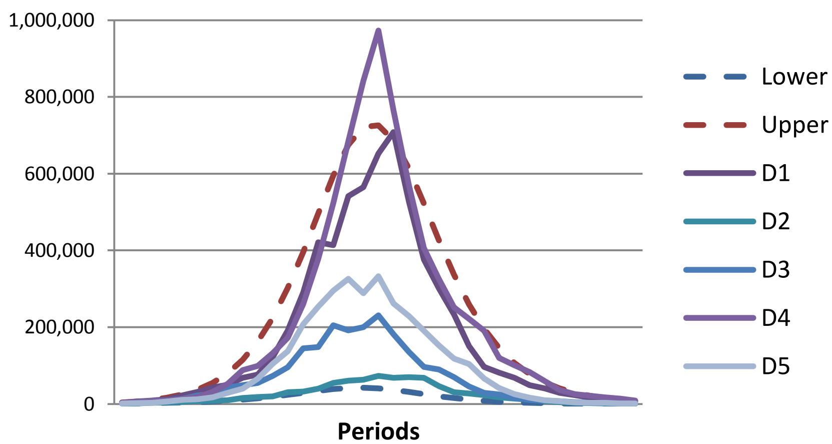

Figure 3.11: Demand realizations for β=1.4 and γ=4

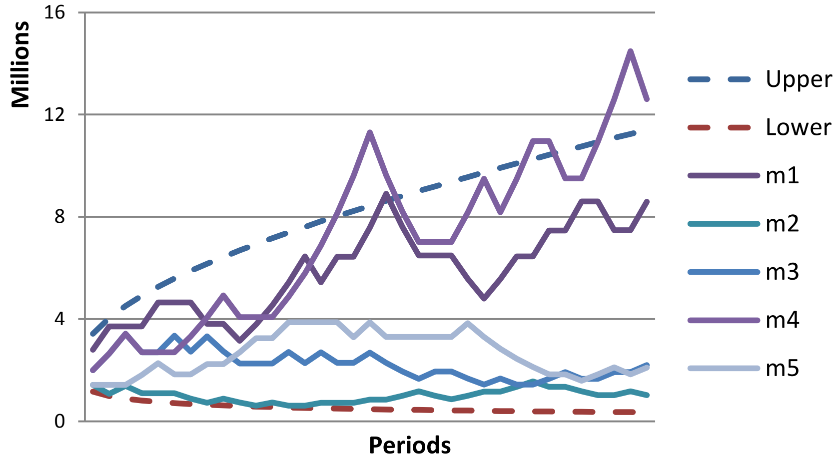

Figure 3.12: Market potential realization for β=1.4 and γ=4

Chapter 4 Optimal Capacity Expansion Model under Stochastic Non-stationary Demand

As discussed earlier, capacity planning for short life-cycle (SLC) products is a non-trivial task and has tremendous impact on the profitability and even survival of a company. It is clear that if the manufacturer is aware of future demand without any uncertainty, capacity expansion decisions are relatively trivial. However, in the real world, decision makers do not have this privilege and often just have a distribution or bounds (upper and lower) on future demand for each period of a product's life-cycle. These characteristics make the dynamic capacity planning problem one of a multistage stochastic optimization that cannot be solved easily and efficiently. Note that the multistage property comes from a fact that the decision maker needs to decide about new capacity procurements for several stages (or epochs) of the life-cycle.

In this chapter, after developing the mathematical formulation of the problem, we first discuss the obstacles for using classical and widely used control methodologies such as Markov Decision Processes (MDP) and stochastic programming for solving this problem (see (Puterman 2009) and (Birge and Louveaux 1997) for more information about MDP and stochastic programming). We then offer an efficient and optimal approach for making these capacity expansion decisions for SLC products. The terminology and notations used in this study are as follows:

- d_s: Stochastic demand for period s (discrete process)
- $\mathcal{F}_s$: Filtration of the stochastic demand process at period s ($\mathcal{F}_s = \{d_1, d_2, \dots, d_s\}$)
- $f_s(.)$: Probability density function of demand at period s
- $F_s(.)$: Cumulative distribution function of demand at period s
- $F_s^c(.)$: Complement of cumulative distribution function of demand distribution ($1 - F_s(.)$)
- μ_s: Expected value of demand at period s

- σ_s^2 : Variance of demand at period s
- K_s: Available capacity at period s
- K_0: Initial Capacity at the beginning of a life-cycle
- y_t: Production amount in period t
- L: Expansion lead-time
- r: Initial product sales price
- r_t: Product sales price at period t
- c_u: Initial marginal shortage penalty of unmet demand
- c_u^t: Marginal shortage penalty at period t
- c_e: Marginal capital expansion cost
- c_h: Marginal holding/maintenance cost of installed capacity
- δ: discounting factor
- α: Depreciation parameter of price and shortage penalty cost ($r_t = re^{-\alpha t}$ and $c_u^t = c_u e^{-\alpha t}$)
- T: Last period of a life-cycle
- $\boldsymbol{T}^s$: Set of life-cycle periods from s to the end of a life-cycle ($\boldsymbol{T}^s = \{s, s+1, \dots, T\}$)
- $\boldsymbol{T}_E^s$: Ordered predetermined periods in which a decision maker can expand capacity ($\boldsymbol{T}_E^s \subset \boldsymbol{T}^s$)
- $\boldsymbol{T}_E^s\{i\}$: *i*-th element of $\boldsymbol{T}_E^s$
- a_s^* : Optimal expansion amount at period s
- $\boldsymbol{a}_s^*$: Optimal expansion vector (policy) for all periods in $\boldsymbol{T}_E^s$
- $\Pi_s(.)$: Profit function for period s and onward periods
- $(x)^+ = \max(x, 0)$

At the end of each period of the SLC product life-cycle, for example s, historical demand information from period 0 (product launch period) to s is available to a decision maker and future demands are unknown. Mathematically speaking, $\mathcal{F}$ is a filtration of the discrete demand process d_s, where $\mathcal{F}_s = \{d_1, d_2, \dots, d_s\}$. We suppose that the decision maker can use $\mathcal{F}_s$ in order to estimate the parameters of demand distribution ($\widehat{\boldsymbol{\Theta}}_{\mathbf{t>s}}$) for each period of the remaining life-cycle. As a result, $\Pr_{\mathrm{t>s}}(d_t = x|\mathcal{F}_s) = f(x|\widehat{\boldsymbol{\Theta}}_{\mathbf{t}})$ and $\Pr_{\mathrm{t>s}}(d_t \leq x|\mathcal{F}_s) = F(x|\widehat{\boldsymbol{\Theta}}_{\mathbf{t}})$. For notation simplicity, instead of $f(x|\widehat{\boldsymbol{\Theta}}_{\mathbf{t}})$ and $F(x|\widehat{\boldsymbol{\Theta}}_{\mathbf{t}})$, we use $f_t(x)$ and $F_t(x)$.

If current period, s, would be a period at which a make-to-order manufacturer can expand capacity ($s \in \boldsymbol{T}_E^s$), s/he needs to have an optimal policy ($\boldsymbol{a}^*_{t\in \boldsymbol{T}_E^s}$) that is a sequence of optimal expansion decisions for current and future periods. In our setting, after receiving demand information at period s, the decision maker updates $f_{t>s}(x)$ and $F_{t>s}(x)$ of the remaining periods of a life-cycle in order to include any available information in deriving the optimal expansion policy ($\boldsymbol{a}^*_{t\in \boldsymbol{T}_E^s}$). Clearly, this policy should be revisited at future expansion periods when more updated (demand) information arrives. Note that $a_s^* = \boldsymbol{a}^*_{t\in \boldsymbol{T}_E^s}\{1\}$ is the only optimal expansion amount that will be ordered from a supplier at period s and it can be used for production at period $s + L + 1$.

After ordering a_s^* at period s, the manufacturer employs the available capacity (K_s) in order to satisfy market demand at this period (d_s). Since there is no inventory, the production amount (y_s) in each period cannot exceed the demand ($y_s = \min(d_s, K_s)$) and the manufactured products are shipped to the customers after production. Note that if a decision maker does not plan to expand capacity at period s, meaning ($s \notin \boldsymbol{T}_E^s$), production amount is the only decision that a manufacturer makes.

As a result, based on this problem setting and a given and fixed cost structure, the decision maker is looking for an optimal expansion policy that maximizes its expected discounted profit for the full product life-cycle that can be formulated as follows:

$$
\begin{aligned}
\mathcal{G} &= \mathbb{E}_{\mathcal{F}_s}\left(\Pi_s\left(\boldsymbol{a}_{t\in \boldsymbol{T}_E^s} \middle| K_s\right)\right) \\
&= \mathbb{E}\left(\sum_{i=1}^{|\boldsymbol{T}_E^s|-1} \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} \delta^{t-s}\left(r\, e^{-\alpha t}\cdot \min\left(d_t,\, K_s+\sum_{j=1}^{i} a_{\tau_j}\right)\right.\right. \\
&\left.\left. - c_u e^{-\alpha t}\left(d_t - K_s - \sum_{j=1}^{i} a_{\tau_j}\right)^{+}\right)\right) \\
&- c_e \sum_{i=1}^{|\boldsymbol{T}_E^s|-1} a_{\tau_j}\,\delta^{\tau_j - s} - c_h \sum_{i=1}^{|\boldsymbol{T}_E^s|-1}\left(\left(K_s+\sum_{j=1}^{i} a_{\tau_j}\right)\sum_{t=\tau_i+L+1}^{\tau_{i+1}+L}\delta^{t-s}\right)
\end{aligned}
\tag{4.1}
$$

where $\tau_i = \boldsymbol{T}_E^s\{i\}$.

The first, second and third terms of the equation correspond to sales revenue, total shortage penalty cost and total capacity expansion/maintenance costs, respectively. In this formulation $\boldsymbol{T}_E^s$ is an ordered set and, for simplification purposes, it is assumed that the last element of the set is $T-L$. Note that any expansion period in this set that leads to a delivery time after the last period of a life-cycle should be omitted from the set.

This problem can be modeled as a stochastic dynamic programming model where the decision epochs are those periods where capacity expansion is allowed. In order to employ common recursive methods for solving this problem, the value function of this problem is:

$$V_s(K_s) = \max_{a_s}\left(-c_e a_s - c_M(K_s + a_s) \sum_{t=s+L+1}^{s'+L} \delta^{t-s} \right.$$
$$+ \sum_{t=s+L+1}^{s'+L} \delta^{t-s}\, e^{-\alpha t}\left(\mathbb{E}_{\mathcal{F}_s}(r\ \min(d_t,\, K_s + a_s) - c_u(d_t\ - K_s - a_s)^{+})\right) \qquad (4.2)$$
$$\left. + \ \delta^{s'-s}\mathbb{E}_{\mathcal{F}_s}\left(V_{s'}(K_s + a_s)\right)\right)$$

where s' is the next available capacity expansion epoch ($s = \boldsymbol{T}_E^s\{1\}$ and $s' = \boldsymbol{T}_E^s\{2\}$). Given that disinvestment is not an option for the decision maker, the optimal policy should only recommend non-negative capacity expansions ($\boldsymbol{a}^*_{t\in T_E^s} \geq 0$ and $\boldsymbol{a}^*_{t\notin T_E^s} = 0$).

It is difficult to solve this multi-stage stochastic optimization problem using the Bellman equation (Equation 4.2) with known approaches. For example, if the decision maker were to employ an MDP approach, at least two variables should be included in the state space of the MDP: a variable for modeling the uncertainty of the demand and another variable for existing capacity. If demand is assumed to follow the proposed stochastic Bass diffusion model (see Chapter 3), the first dimension would be the market potential (Since p and q are assumed to fixed during a life-cycle). In this stochastic Bass model, although the process for market potential is discrete, depending on the number of periods, the size of the market potential set can grow exponentially, leading to computational and dimensionality issues of the state space. Available capacity is the other variable that needs to be in the state space. Based on the granularity of actions (expansion decisions), the action space can explode, as well. These issues might be solved by placing an upper bound on total expansion and employing high granularity for expansion decisions. However, these remedies can lead to suboptimal policies.

Alternatively, the decision maker might decide to use a simulation based solution methodology, e.g. stochastic programming, to derive the optimal expansion policy. However, based on the volatility of the demand and number of expansion periods, this problem might be unsolvable by currently available optimization packages. High volatility of demand leads to a need for more simulation scenarios that consequently increase the number of decision variables and constraints. Moreover, in case of more frequent capacity expansions, the complexity of the stochastic programming model surges considerably.

Another simulation-based methodology that can be used for solving this problem is deriving the optimal policy by optimizing the sample-based expected profit. However, instead of using analytical expression of the expected profit (Equation 4.1), expected profit would be calculated by generating adequate simulation scenarios. In this case, based on the linear structure of the problem, the expected profit can be rewritten as an integer programming (IP) model:

$$\max_{\mathrm{a}_t, y_t} \sum_{w=1}^{W} \sum_{t=s}^{T} \beta^{t-s} \left((r_t + c_t^u) y_{t,w} - c_t^u d_{t,w} - c_M K_{t,w} - c_e a_{t,w} \right)$$

$$s.t.\ K_t = \mathrm{a}_{t-L} + K_{t-1}$$

$$y_{t,w} \leq d_{t,w}$$

$$y_{t,w} \leq K_{t,w}$$

$$\mathrm{a}_{t,w}, y_{t,w} \in \mathbb{Z}^+$$

In this formulation, W is the set of scenarios generated and its size should depend on the level of uncertainty in the demand. Although the problem can be solved in this formulation, it is not an analytical model and it needs substantial number of simulation scenarios in order to shrink the optimality-gap of the solution.

In the next section, we propose an analytical stochastic optimal expansion model that provides the optimal expansion policy in each period of predetermined expansion periods. We show that the model guarantees optimality of the derived expansion policies with respect to the expected discounted profit. We also propose an algorithm that provides an efficient procedure for finding the optimal policy and at the same time avoids computational difficulties of conventional methods.

4.1 Stochastic Optimal Expansion Model

In this section, we present the necessary lemmas and propositions for deriving the optimal expansion policy. Later, we show that the expansion decision at each period can be independent of future expansion decisions if the decision maker is aware of the next period in which expansion is an optimal decision. After that, we show some analytical properties of the model and evaluate the effect of the model's parameters on optimal expansion decisions. At the end, we propose an algorithm in which we employ the presented propositions in order to find the optimal expansion policy. We finish this chapter by illustrating the behavior of the proposed model in a deterministic case, where a decision maker is certain about future demands.

In the following lemma and propositions, without loss of generality, we present the analytical properties of the problem for period s where the decision maker has the demand information of all periods from period 1 to s. Clearly when *s=0*, we are considering the entire planning horizon.

4.1.1 Lemma 1

$\mathcal{G}$ (the expected discounted profit function) is a strictly concave function.

4.1.2 Proof of Lemma 1

Let current period (s) be the first element of $\boldsymbol{T}_E^s$ and $\tau_i = \boldsymbol{T}_E^s\{i\}$. Note that $s = \tau_1$.

$$\mathcal{G} = \mathbb{E}_{\mathcal{F}_s}\left(\sum_{i=1}^{|\boldsymbol{T}_E^s|-1} \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} e^{-\alpha t}\delta^{t-s}\left(r\min\left(d_t, K_s + \sum_{j=1}^{i} a_{\tau_j}\right) - c_u\left(d_t - K_s - \sum_{j=1}^{i} a_{\tau_j}\right)^+\right)\right)$$

$$- c_e \sum_{j=1}^{|\boldsymbol{T}_E^s|-1} a_{\tau_j}\delta^{\tau_j - s} - c_h \sum_{i=1}^{|\boldsymbol{T}_E^s|-1}\left(\left(K_s + \sum_{j=1}^{i} a_{\tau_j}\right)\sum_{t=\tau_i+L+1}^{\tau_{i+1}+L}\delta^{t-s}\right)$$

$$\mathcal{G} = \sum_{i=1}^{|\boldsymbol{T}_E^s|-1} \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} e^{-\alpha t} \delta^{t-s} \left(r\, \mathbb{E}_{\mathcal{F}_s}\left(\min\left(d_t, K_s + \sum_{j=1}^{i} a_{\tau_j} \right) \right) - c_u \mathbb{E}_{\mathcal{F}_s}\left(\left(d_t - K_s - \sum_{j=1}^{i} a_{\tau_j} \right)^{+} \right) \right)$$

$$- c_e \sum_{j=1}^{|\boldsymbol{T}_E^s|-1} a_{\tau_j} \delta^{\tau_j - s} - c_h \sum_{i=1}^{|\boldsymbol{T}_E^s|-1} \left(\left(K_s + \sum_{j=1}^{i} a_{\tau_j} \right) \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} \delta^{t-s} \right)$$

$$\mathcal{G} = \sum_{i=1}^{|\boldsymbol{T}_E^s|-1} \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} e^{-\alpha t} \delta^{t-s} \left(r \left(\int_0^{K_s + \sum_{j=1}^{i} a_{\tau_j}} d_t f_t(d_t) d(d_t) \right.\right.$$

$$\left. + \int_{K_s + \sum_{j=1}^{i} a_{\tau_j}}^{\infty} \left(K_s + \sum_{j=1}^{i} a_{\tau_j} \right) f_t(d_t) d(d_t) \right)$$

$$\left.\left. - c_u \left(\int_0^{K_s + \sum_{j=1}^{i} a_{\tau_j}} 0 f_t(d_t) d(d_t) + \int_{K_s + \sum_{j=1}^{i} a_{\tau_j}}^{\infty} \left(d_t - K_s - \sum_{j=1}^{i} a_{\tau_j} \right) f_t(d_t) d(d_t) \right) \right) \right)$$

$$- c_e \sum_{j=1}^{|\boldsymbol{T}_E^s|-1} a_{\tau_j} \delta^{\tau_j - s} - c_h \sum_{i=1}^{|\boldsymbol{T}_E^s|-1} \left(\left(K_s + \sum_{j=1}^{i} a_{\tau_j} \right) \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} \delta^{t-s} \right)$$

Using $\int_0^{\xi} x\, g(x)\, d(x) = \int_0^{\infty} x\, g(x) d(x) - \int_{\xi}^{\infty} x\, g(x) d(x) = \mathbb{E}(x) - \int_{\xi}^{\infty} x\, g(x) d(x)$ when x is a random variable with PDF of $g(x)$ and by merging some of the arguments, the expected value of profit would be:

$$\mathcal{G} = (r + c_u) \sum_{i=1}^{|\boldsymbol{T}_E^s|-1} \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} e^{-\alpha t} \delta^{t-s} \left(\int_{K_s+\sum_{j=1}^{i} a_{\tau_j}}^{\infty} \left(K_s + \sum_{j=1}^{i} a_{\tau_j} - d_t \right) f_t(d_t) d(d_t) \right)$$

$$+ r \sum_{t=s+L+1}^{T} e^{-\alpha t} \delta^{t-s} \mu_t - c_e \sum_{j=1}^{|\boldsymbol{T}_E^s|-1} a_{\tau_j} \delta^{\tau_j - s} \tag{4.3}$$

$$- c_h \sum_{i=1}^{|\boldsymbol{T}_E^s|-1} \left(\left(K_s + \sum_{j=1}^{i} a_{\tau_j} \right) \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} \delta^{t-s} \right)$$

Based on the Leibniz integral rule:

$$\frac{\partial \mathcal{G}}{\partial a_{\tau_k}} = (r + c_u) \sum_{i=k}^{|\boldsymbol{T}_E^s|-1} \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} e^{-\alpha t} \delta^{t-s} F_t^c \left(K_s + \sum_{j=1}^{i} a_{\tau_j} \right) - c_e \delta^{\tau_k - s} - c_h \sum_{t=\tau_k+L+1}^{T} \delta^{t-s} \tag{4.4}$$

By defining second derivative as $\frac{\partial^2 \mathcal{G}}{\partial a_{\tau_k} \partial a_{\tau_l}}$, the Hessian matrix is:

$$H = \begin{cases} -(r + c_u) \sum_{i=k}^{|\boldsymbol{T}_E^s|-1} \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} e^{-\alpha t} \delta^{t-s} f_t \left(K_s + \sum_{j=1}^{i} a_{\tau_j} \right) & \text{, if } k = l \\ -(r + c_u) \sum_{i=\max(k,l)}^{|\boldsymbol{T}_E^s|-1} \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} e^{-\alpha t} \delta^{t-s} f_t \left(K_s + \sum_{j=1}^{i} a_{\tau_j} \right) & \text{, if } k \neq l \end{cases}$$

Due to the fact that $f_t(.)$ is the PDF of demand at different periods and always a nonnegative value, H is a negative-semidefinite matrix. Therefore, $\mathcal{G}$ is a concave function.

□

As a result, the problem of maximizing $\mathcal{G}$ has a concave objective function and linear non-negativity constraints ($\boldsymbol{a}_{t \in T_E^s} \geq 0$). The following proposition gives the *KKT* conditions to be satisfied by all optimal solutions of the optimization problem.

4.1.3 Proposition 1

Assuming $\boldsymbol{T}_E^s = \{s, s+1, \dots, T-1\}$, the optimal expansion amounts at any period ($a^*_{t \in \boldsymbol{T}_E^s}$) must satisfy the following equation:

$$e^{-\alpha t}\delta^{t+L+1-s}F^{c}_{t+L+1}(K_t+a^*_t)=\frac{c_e(\delta^{t-s}-\delta^{t+1-s})+c_h\delta^{t-s}}{r+c_u}+\mathbf{1}_{a^*_t=0}\lambda'_t-\mathbf{1}_{a^*_{t+1}=0}\lambda'_{t+1}$$

where $\lambda'_t=\frac{\lambda_t}{r+c_u}$ and λ_t are Lagrange multipliers. Consequently, if s would be the only period at which expanding capacity is an optimal decision ($a^*_s>0$ and $a^*_{t\in(\boldsymbol{T}^s_E\setminus s)}=0$), then a^*_s solves the following equation:

$$\sum_{t=s+L+1}^{T}e^{-\alpha t}\delta^{t-s}F^{c}_{t}(K_s+a^*_s)=\frac{c_e-c_h\sum_{t=s+L+1}^{T}\delta^{t-s}}{r+c_u} \tag{4.5}$$

4.1.4 Proof of Proposition 1

Let s and $s+1$ both be an element of $\boldsymbol{T}^s_E$, meaning $(a_{\tau_1}=a_s,a_{\tau_2}=a_{s+1})$. Therefore, based on Equation 4.3 and *KKT* optimality condition for $a_s\geq 0$:

$$\frac{\partial\mathcal{G}}{\partial a_s}-\lambda_{a_s}=0$$

$$(r+c_u)\sum_{i=1}^{|\boldsymbol{T}^s_E|-1}\sum_{t=\tau_i+L+1}^{\tau_{i+1}+L}e^{-\alpha t}\delta^{t-s}F^c_t\left(K_s+\sum_{j=1}^{i}a^*_{\tau_j}\right)-c_e\delta^{s-s}-c_h\sum_{t=s+L+1}^{T}\delta^{t-s}-\lambda_s=0 \tag{4.6}$$

$$\begin{aligned}(r+c_u)&\left(e^{-\alpha t}\delta^{t-s}F^c_{s+L+1}(K_s+a^*_s)+\sum_{i=2}^{|\boldsymbol{T}^s_E|-1}\sum_{t=\tau_i+L+1}^{\tau_{i+1}+L}e^{-\alpha t}\delta^{t-s}F^c_t\left(K_s+\sum_{j=1}^{i}a^*_{\tau_j}\right)\right)-c_e\\&-c_h\sum_{t=s+L+1}^{T}\delta^{t-s}-\lambda_s=0\end{aligned} \tag{4.7}$$

Furthermore, the *KKT* optimality condition for $a_{s+1}\geq 0$ is:

$$\frac{\partial\mathcal{G}}{\partial a_{s+1}}-\lambda_{a_{s+1}}=0$$

$$(r+c_u)\sum_{i=2}^{|\boldsymbol{T}^s_E|-1}\sum_{t=\tau_i+L+1}^{\tau_{i+1}+L}e^{-\alpha t}\delta^{t-s}F^c_t\left(K_s+\sum_{j=1}^{i}a^*_{\tau_j}\right)-c_e\delta^{s+1-s}-c_h\sum_{t=s+L+2}^{T}\delta^{t-s}-\lambda_{s+1}=0 \tag{4.8}$$

Based on complementary slackness ($\lambda_s a_s=0$) and by substituting Equation 4.8 in 4.7:

$$e^{-\alpha t}\delta^{L+1}F^{c}_{s+L+1}(K_s+a_s^*)=\frac{c_e(1-\delta)+c_h\delta^{L+1}}{r+c_u}+\mathbf{1}_{a_s^*=0}\lambda'_s-\mathbf{1}_{a_{s+1}^*=0}\lambda'_{s+1}$$

The generalization of this equation for any period $\tau \geq s$ is:

$$e^{-\alpha\tau}\delta^{\tau+L+1-s}F^{c}_{\tau+L+1}\left(K_s+\sum_{j=s}^{\tau}a_j^*\right)$$
$$=\frac{c_e(\delta^{\tau-s}-\delta^{\tau-s+1})+c_h\delta^{\tau+L+1-s}}{r+c_u}+\mathbf{1}_{a_\tau^*=0}\lambda'_\tau-\mathbf{1}_{a_{s+1}^*=0}\lambda'_{\tau+1},\forall\tau\geq s \tag{4.9}$$

If it is assumed that a_s is the last expansion ($\lambda_s=0$ and $\sum_{j>s}^{T}a_j^*=0$), by summing up Equation 4.9 for all t's we have:

$$\sum_{t=s+L+1}^{T}e^{-\alpha t}\delta^{t-s}F_t^c(K_s+a_s^*)=\frac{c_e+c_h\sum_{t=s+L+1}^{T}\delta^{t-s}}{r+c_u} \tag{4.10}$$

If a decision maker considers s as the last expansion period, the optimal expansion at this period should solve Equation 4.10. Note that in this equation it is assumed that the demand distributions are continuous. In case of discrete distribution, a_s^* is the smallest a_s such that:

$$\sum_{t=s+L+1}^{T}e^{-\alpha t}\delta^{t-s}F_t^c(K_s+a_s)\geq\frac{c_e+c_h\sum_{t=s+L+1}^{T}\delta^{t-s}}{r+c_u}$$

These equations (discrete and continuous) can be easily solved by a root-finding or a nonlinear optimization algorithm combined with a non-negativity constraint ($a_s^*\geq 0$). Note that if a decision maker has no plan to expand capacity after s, these equations provide the optimal expansion amount if there is any (disinvestment would not be the optimal decision). However, if the decision maker is not aware of the optimal timing of the last expansion period, these equations cannot help a decision maker about the timing. As we will discuss later in this chapter, these equations (continuous and discrete cases) are used in one of the steps in the proposed algorithm for finding the optimal expansion policy.

Another important point related to Equation 4.10 is the fact that it can be viewed as a newsvendor solution of the expansion problem. In order to show the newsvendor critical fractile, let discounting factor be negligible, demand be stationary, and r_t and c_u^t be fixed through a life-cycle ($\alpha = 0$). With these assumptions and by letting $t' = s + L$, Equation 4.10 can be rewritten as:

$$(T - t')F^c(K_s + a_s^*) = \frac{c_e + c_h(T - t')}{r + c_u}$$

$$F(K_s + a_s^*) = \frac{(T - t')(r + c_u - c_h) - c_e}{(T - t')(r + c_u)}$$

This shows the critical fractile of the classical newsvendor problem where underage cost is $(T - t')(r + c_u - c_h) - c_e$ and overage cost is $c_e + c_h(T - t')$. In both underage and overage costs, r, c_u, and c_h are multiplied by the number of remaining periods of a life-cycle since they are not one-time costs. On the other hand, marginal expansion cost is a one-time cost and the length of the remaining periods of a life-cycle has no effect on it.

Going back to Equation 4.10, it is clear that finding the optimal expansion decision at s when it is the last expansion period is a trivial task. However, calculating the optimal solution when s is not the last expansion period, even with assuming IID demand and equal intervals between expansions, is not a trivial task. This difficulty is related to the fact that the problem is not of infinite horizon and the state space (K_t) is conditional on the previous expansion decisions. In the next proposition, we address this issue and propose an algorithm in order to find an optimal expansion decision at any period.

4.1.5 Proposition 2

(Separation Property) At the optimal solution, the expansion decision at any period s depends only on the timing of the next expansion and it is independent of all future expansion decisions.

4.1.6 Proof Proposition 2

Let s' be a period after s at which expansion is an optimal decision, meaning $\sum_{k>s}^{s'-1} a_k^* = 0$ and $a_{s'}^* > 0$. Then the optimality equation in Equation 4.9 from $s + L + 1$ until $s' + L$ can be expressed as:

$$e^{-\alpha(s+L+1)}\delta^{L+1}F_{s+L+1}^{c}(K_s+a_s^*)=\frac{c_e(1-\delta)+c_h\delta^{L+1}}{r+c_u}+\mathbf{1}_{a_s^*=0}\lambda_s'-\mathbf{1}_{a_{s+1}^*=0}\lambda_{s+1}'$$

$$e^{-\alpha(s+L+2)}\delta^{L+2}F_{s+L+2}^{c}(K_s+a_s^*)=\frac{c_e(\delta-\delta^2)+c_h\delta^{L+2}}{r+c_u}+\mathbf{1}_{a_{s+1}^*=0}\lambda_{s+1}'-\mathbf{1}_{a_{s+2}^*=0}\lambda_{s+2}'$$

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$$e^{-\alpha(s\prime+L)}\delta^{s'+L-s}F_{s'+L}^{c}(K_s+a_s^*)=\frac{c_e\left(\delta^{s'-s-1}-\delta^{s'-s}\right)\ +c_h\delta^{s'+L+1}}{r+c_u}+\mathbf{1}_{a_{s'-1}^*=0}\lambda_{s'-1}'-\mathbf{1}_{a_{s'}^*=0}\lambda_{s'}'$$

Given that $a_s^*, a_{s'}^* > 0$ and $\lambda_s', \lambda_{s'}' = 0$ and by summing up these equations, a_s^* solves the following equation that is independent of future expansions except the timing of the next expansion:

$$\sum_{t=s+L+1}^{s'+L} e^{-\alpha t}\delta^{t-s}F_t^c(K_s+a_s^*)=\frac{c_e\left(1-\delta^{s'-s}\right)+c_h\sum_{t=s+L+1}^{s'+L}\delta^{t-s}}{r+c_u} \tag{4.11}$$

Note that in case of discrete distributions a_s^* is the smallest a_s such that:

$$\sum_{t=s+L+1}^{s'+L} e^{-\alpha t}\delta^{t-s}F_t^c(K_s+a_s)\geq\frac{c_e\left(1-\delta^{s'-s}\right)+c_h\sum_{t=s+L+1}^{s'+L}\delta^{t-s}}{r+c_u}$$

□

By using the first order condition and the *separation property* that we discussed in Propositions 1 and 2, we demonstrated that expansion at any period is only conditional on the next period in which expansion is an optimal decision. This is a form of *decomposition*.

However, this interpretation is conditional on the timing of these expansion periods. Hence, while the problem is concave (profit is concave subject to affine constraints), through this conditioning, the first order conditions can be satisfied with more than one solution. This important property is the result of the fact that different permutations of expansion periods imposing different affine constraints lead to different optimal solutions.

4.1.7 Corollary 1

At any period s, using Equation 4.11 and without knowledge of the next expansion period, a decision maker is able to determine whether a period is an expansion period or not. Meaning, if there is no solution for a_s^* in $\sum_{t=s+L+1}^{\boldsymbol{T}_E^s\{2\}+L} e^{-\alpha t}\delta^{t-s}F_t^c(K_s+a_s^*)=\frac{c_e\left(1-\delta^{\boldsymbol{T}_E^s\{2\}-s}\right)+c_h\sum_{t=s+L+1}^{\boldsymbol{T}_E^s\{2\}+L}\delta^{t-s}}{r+c_u}$, s is not a period in which expansion is an optimal decision. It may be argued that the equation might have solution for period $\boldsymbol{T}_E^s\{3\}$, which means a_s^* has a solution in the following equation:

$$\sum_{t=s+L+1}^{\boldsymbol{T}_E^s\{3\}+L} e^{-\alpha t}\delta^{t-s}F_t^c(K_s+a_s^*)=\frac{c_e\left(1-\delta^{\boldsymbol{T}_E^s\{3\}-s}\right)+c_h\sum_{t=s+L+1}^{\boldsymbol{T}_E^s\{3\}+L}\delta^{t-s}}{r+c_u}$$

but no solution for a_s^* in $\boldsymbol{T}_E^s\{2\}$ case implies that there is no need for extra capacity for periods between $s+L+1$ and $\boldsymbol{T}_E^s\{2\}+L$ and if any extra capacity would be needed for periods afterwards they can be procured at $\boldsymbol{T}_E^s\{2\}$ epoch. This property helps in discarding some of the combinations of expansion periods and increases the speed of the solution algorithm significantly.

4.1.8 Algorithm 1

As discussed, capacity expansion for SLC products is a trivial task when demand is deterministic and it can be solved by a linear programming model with ease. However, in the stochastic case, it is a multistage stochastic problem that cannot be solved easily and efficiently due to its action and state spaces. In this section, by employing Propositions 3 and 4, we propose an algorithm that has the ability to reduce the problem to a much smaller action space that can be solved in an efficient manner. This algorithm has three main steps and should be repeated in each expansion period since the decision maker's belief might change with each new demand observation.

a. Let $\boldsymbol{\mathcal{T}}_s$ be the power set (collection of all subsets) of $\boldsymbol{T}_E^s$ where each element of $\boldsymbol{\mathcal{T}}_s$ is a subset (permutation) of expansion timings. As an example, if a decision maker decides to expand capacity at periods 10, 30, and 50 and currently the product is at period 10 of the life-cycle ($\boldsymbol{T}_E^{10} = \{10, 30, 50\}$) then $\boldsymbol{\mathcal{T}}_{10} = \{\mathrm{O},\ \{10\}, \{30\}, \{50\}, \{10, 30\} \dots \{10, 30, 50\}\}$, where $|\boldsymbol{\mathcal{T}}_{10}| = 2^{|\boldsymbol{T}_E^s|}$.

b. Use Equations 4.10 and 4.11 in order to calculate the expansion policy ($\boldsymbol{a}_s^*$) for each element of $\boldsymbol{\mathcal{T}}_{10}$. These two equations can be considered as root finding problems or two optimization problems. Solving them as an optimization problem would result in better solutions since there might be multiple solutions for each equation. Given marginal expansion/maintenance costs, the lowest solution is more desirable for a decision maker. Therefore, the optimization formulation of these two equations when the distribution of demand is continuous can be rewritten as:

$$\min a_s$$

$$\text{s.t.} \sum_{t=s+L+1}^{T} e^{-\alpha t} \delta^{t-s} F_t^c(K_s + a_s) - \frac{c_e + c_h \sum_{t=s+L+1}^{T} \delta^{t-s}}{r + c_u} = 0$$

and

$$\min a_s$$

$$\text{s.t.} \sum_{t=s+L+1}^{s'+L} e^{-\alpha t} \delta^{t-s} F_t^c(K_s + a_s) - \frac{c_e\left(1 - \delta^{s'-s}\right) + c_h \sum_{t=s+L+1}^{s'+L} \delta^{t-s}}{r + c_u} = 0$$

In case of discrete distributions, the optimization formulations are as follows:

$$\min a_s$$

$$\text{s.t.} \sum_{t=s+L+1}^{T} e^{-\alpha t} \delta^{t-s} F_t^c(K_s + a_s) - \frac{c_e + c_h \sum_{t=s+L+1}^{T} \delta^{t-s}}{r + c_u} \geq 0$$

and

$$\min a_s$$

$$\text{s.t.} \sum_{t=s+L+1}^{s'+L} e^{-\alpha t}\delta^{t-s}F_t^c(K_s + a_s) - \frac{c_e\left(1-\delta^{s'-s}\right) + c_h \sum_{t=s+L+1}^{s'+L}\delta^{t-s}}{r + c_u} \geq 0$$

For example, in subset {10, 30, 50}, we need to calculate $\boldsymbol{a}_{10} = \{a_{10}, a_{30}, a_{50}\}$. For calculating a_{10}, Equation 4.11 needs to be used where $s = 10$ and $s' = 30$. Then, for a_{30}, again Equation 4.11 would be called where $s = 30$ and $s' = 50$ and $K_{30} = K_{10} + a_{10}$. After these two steps, a_{10} and a_{30} have been calculated. Finally, for a_{50}, Equation 4.10 is used where $s = 50$ and $K_{50} = K_{10} + a_{10} + a_{30}$. Table 4.1 contains all possible subsets of this example and the equations that are necessary for finding the expansion amount. Note that if any of the equations would not be solvable in any element of $\boldsymbol{\mathcal{T}}_{10}$, the associated subset would be discarded and considered as an infeasible solution. At the end of this step, infeasible subsets (permutations) have been discarded and feasible subsets with their associated policies (expansion vectors: $\boldsymbol{a}_s$) are ready to be compared. Let $\boldsymbol{\mathcal{T}}_s^{\boldsymbol{F}}$ be a collection of feasible subsets in $\boldsymbol{\mathcal{T}}_s$. Note that each element of $\boldsymbol{\mathcal{T}}_s^{\boldsymbol{F}}$ has an associated $\boldsymbol{a}_s^i$ where $i = 1 \ldots |\boldsymbol{\mathcal{T}}_s^{\boldsymbol{F}}|$. Each $\boldsymbol{a}_s^i$ contains expansion amounts in the subset's periods.

Subset Number	**Subset**	**Expansion Policy**	**Equations for solving**
1	Empty set	-	-
2	{10}	a_{10}	Equation 4.10
3	{20}	a_{20}	Equation 4.10
4	{30}	a_{30}	Equation 4.10
5	{10, 20}	a_{10} and a_{20}	Equations 4.11 and 4.10
6	{10, 30}	a_{10} and a_{30}	Equations 4.11 and 4.10
7	{20, 30}	a_{20} and a_{30}	Equations 4.11 and 4.10
8	{10, 20, 30}	a_{10}, a_{20} and a_{30}	Equations 4.11, 4.11 and 4.10

Table 4.1: All possible expansion timing combinations for the example setting

c. As a final stage, equation of expected profit in Lemma 1 that provides the expected profits of each subset's expansion policy should be employed for identifying the optimal policy. Therefore, $\boldsymbol{a}_s^* = argmax_i\ \mathbb{E}_{\mathcal{F}_s}(\Pi_s(\boldsymbol{a}_s^i|K_s))$. Note that expected profit equation contains an integration that cannot be solved analytically. As a result it should be solved by a software program that can provide numerical integration methods.

In this algorithm, due to the limited number of expansion periods, the analysis of all possible expansion policies is a much simpler task compared to the multistage stochastic problem.

4.2 Effect of Marginal Expansion Cost and Lead-time on Optimal Expansion Amount

In this part, we provide some insights regarding the effect of marginal expansion cost (c_e) and lead-time on optimal amount of expansion in a period. For simplicity and without loss of generality, we focus on a case in which the current period is the final expansion period. However, the results and insights that are provided in this section can be easily generalized for other scenarios.

Based on the effect of c_e on the expected profit function and since it only exist in one side of Equation 4.10, it is obvious that any changes in this cost can only have indirect effect on the optimal expansion amount in a period. Note that there might be some cases, e.g. when c_e and c_h are completely being dominated by r and c_u, at which changes in c_e would not change an optimal decision.

On the other hand, since procurement lead-time exists in both terms of Equation 4.10, it is not intuitively clear that how an alteration in L affects optimal expansion amount. In order to illustrate this property, we assume that the supplier provides a new procurement option that has a lead-time of L_N and marginal expansion cost of c_e^N. In this option, the manufacturer can receive the capacity one period earlier than the regular one ($L_N = L - 1$) with a higher marginal expansion cost ($c_e^N > c_e$). Note that the manufacturer has to select one of the two supply options and cannot procure from both[2]. It is important to emphasize that here we do not compare the two supply options with respect to the expected profitability. We simply address the effect of procurement lead-time on the optimal expansion amount. Comparing the profit of different procurement options can be accomplished numerically based on Equation 4.3.

As a first step, we are looking for the cost of the new capacity option (let's call this cost η) that makes the manufacturer indifferent (in terms of amount of capacity) to the two procurement options; meaning, for what marginal expansion cost of faster procurement case a_s^* would be the same in both options. Clearly, if $c_e^N > \eta$ ($c_e^N < \eta$), the manufacturer procures less (more) capacity from the faster option comparing to the regular option.

As mentioned, in case of being in the last expansion period, the decision maker uses Equation 4.10 in order to find the optimal expansion amount. If, for notational simplicity, we assume that $\alpha = 0$ and $\delta = 1$ (no price depreciation and no discounting), the optimal expansion amount from the regular option (a_s^*) solves the following equation:

$$\sum_{t=s+L+1}^{T} F_t(K_s + a_s^*) = (T - s - L) - \frac{c_e + c_h(T - s - L)}{r + c_u} \tag{4.12}$$

Similarly, the optimal expansion amount from the shorter lead-time option (z_s^*) solves the following equation:

$$\sum_{t=s+L_N+1}^{T} F_t(K_s + z_s^*) = (T - s - L_N) - \frac{\eta + c_h(T - s - L_N)}{r + c_u} \tag{4.13}$$

where $L_N = L - 1$. As expected, this equation shows that the new capacity from the faster option can be utilized at period $s + L$. Note that we are looking for a cost of marginal expansion from faster option

[2] Procurement from both options (dual source procurement) will be covered in the Chapter 5 of this dissertation.

(η) that leads to the same optimal expansion amount ($a_s^* = z_s^*$). Therefore, Equation 4.13 can be rewritten as:

$$F_{s+L}(K_s + a_s^*) + \sum_{t=s+L+1}^{T} F_t(K_s + a_s^*) = (T - s - L + 1) - \frac{\eta + c_h(T - s - L + 1)}{r + c_u} \tag{4.14}$$

Now, by replacing Equation 4.11 in 4.14:

$$F_{s+L}(K_s + a_s^*) + (T - s - L) - \frac{c_e + c_h(T - s - L)}{r + c_u} = (T - s - L + 1) - \frac{\eta + c_h(T - s - L + 1)}{r + c_u}$$

with some simplifications:

$$1 - F_{s+L}(K_s + a_s^*) = \frac{\eta - c_e + c_h}{r + c_u}$$

That means:

$$\Delta c_e = \eta - c_e = (r + c_u)\big(1 - F_{s+L}(K_s + a_s^*)\big) - c_h \tag{4.15}$$

This equation implies that if a manufacturer pays $c_e + \Delta c_e$ for the faster procurement option, the optimal expansion amount procured from the faster option will not change from what the manufacturer would have procured from the regular option. If the marginal cost of expansion in the faster option would be less than $c_e + \Delta c_e$, the optimal expansion decision from the faster option changes and the decision maker procures more from it compared to the regular option (Figure 4.1). Note that in this figure, ν is a marginal expansion cost for the faster option that leads to zero expansion amount:

$$z_s^* = 0 \quad \Leftrightarrow \quad \nu = (r + c_u)\left(T - s - L_N - \sum_{t=s+L_N+1}^{T} F_t(K_s)\right) - c_h(T - s - L_N)$$

As expected and shown in Equation 4.15, product price (r) and shortage penalty (c_u) increases Δc_e and maintenance cost decreases Δc_e. Although maintenance cost has a linear effect on Δc_e, the effect of product price and shortage penalty depends on the chance of stock-out at period $s+L$. Intuitively, this is true since if demand is less than $K_s + a_s^*$, there is no financial benefit for any extra capacity.

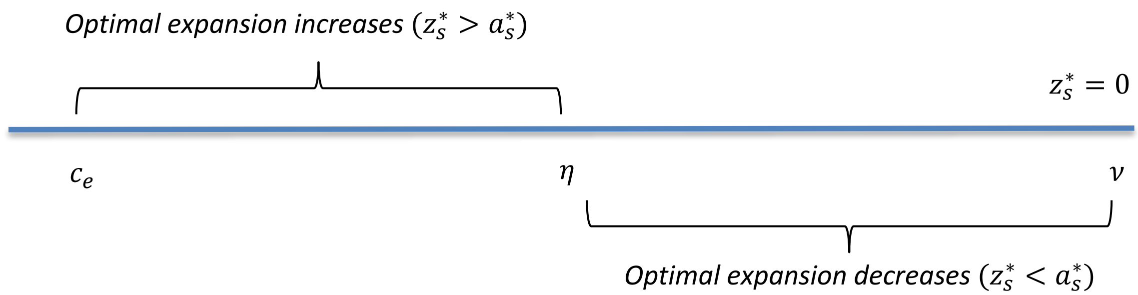

Figure 4.1: Effect of new procurement option in optimal expansion amount

The other issue that needs more elaboration is the effect of demand uncertainty on Δc_e. The main issue that needs to be addressed is how demand uncertainty at period $s+L$ might increase or decrease Δc_e. Intuitively, higher uncertainty should lead to a decrease in Δc_e. However, here we show that this intuition is not always correct and based on the position of $K_s + a_s^*$, to the expected demand at period $s+L$ (μ_{s+L}), Δc_e might increase or decrease for different degrees of certainty.

In order to illustrate this behavior, we consider two scenarios for the demand distribution at period $s+L$. In the first scenario, D_1 is the distribution of demand at this period with mean μ and variance of σ_1^2. In the second scenario, D_2 is the distribution of demand with mean μ and variance of σ_2^2. D_1 and D_2 are both from the same distribution with the same expected value but they have different variances ($\sigma_1^2 > \sigma_2^2$), meaning that in the scenario one the decision maker has less certainty about demand at period $s+L$. For illustrating the effect of uncertainty, we need to consider two cases: $K_s + a_s^* > \mu$ and $K_s + a_s^* < \mu$:

a. In case of $K_s + a_s^* > \mu$, based on the property of cumulative probability distribution functions that is illustrated in Figure 4.2, it can be shown that $F^{D_1}(K_s + a_s^*) < F^{D_2}(K_s + a_s^*)$. As a result, Δc_e in the scenario of D_2 (less uncertainty) would be less than Δc_e of D_1 scenario.

This property implies that, in case of $K_s + a_s^* > \mu$, Δc_e for a decision maker who is more uncertain about demand at period $s+L$ is greater than the case of less uncertain decision maker. In terms of η, we have showed that $c_e < \eta_{D_1} < \eta_{D_2}$ when $K_s + a_s^* > \mu$.

b. On the other hand, when $K_s + a_s^* < \mu_t$, as shown in Figure 4.2, $F^{D_1}(K_s + a_s^*) > F^{D_2}(K_s + a_s^*)$. This property leads to a higher Δc_e in the scenario of D_2 (less uncertainty) compared to D_1 scenario. This property implies that $c_e < \eta_{D_2} < \eta_{D_1}$.

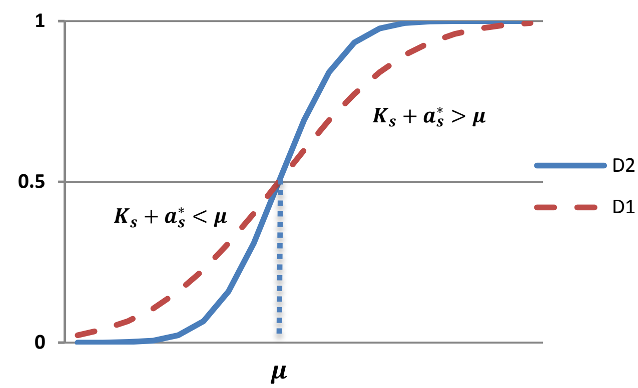

Figure 4.2: CDF of two distributions of demand: D_1 and D_2

So far, we have discussed the effect of one-period shorter lead-time procurement option and its cost on optimal expansion amount. Here, we generalize the idea in order to cover the case in which the decision maker has a procurement option with n-period shorter lead-time compared to a regular option. Obviously, this shorter lead-time comes with a higher cost that the manufacturer needs to bear (c_e^N). Similar to the one-period shorter lead-time case, we are looking for a cost of this faster procurement option that does not change the optimal procurement amount of the regular option. Let this cost be η_n.

Again, by assuming no price depreciation and no discounting, a_s^* should solve the following equation for the option that has a lead-time of n-period shorter than L:

$$\sum_{t=s+L-n+1}^{s+L} F_t(K_s + a_s^*) + \sum_{t=s+L+1}^{T} F_t(K_s + a_s^*) = (T - s - L + n) - \frac{\eta_n + c_h(T - s - L + n)}{r + c_u} \tag{4.16}$$

By replacing Equation 4.12 in Equation 4.16, we have:

$$\sum_{t=s+L-n+1}^{s+L} F_t(K_s + a_s^*) + (T - s - L) - \frac{c_e + c_h(T - s - L)}{r + c_u} = (T - s - L + n) - \frac{\eta_n + c_h(T - s - L + n)}{r + c_u} \tag{4.17}$$

With some simplification, the final equation would be:

$$\Delta c_e = \eta_n - c_e = (r + c_u)\left(n - \sum_{t=s+L-n+1}^{s+L} F_t(K_s + a_s^*)\right) - nc_h \tag{4.18}$$

It is very important to re-emphasize that, so far, we have not discussed the effect of new procurement option on the profitability. We only considered the effect of shorter lead-time with more expensive cost on a_s^*. Therefore, Δc_e does not provide any insight for a manufacturer about the financial superiority (profitability) of one procurement option to the other one in either side of Δc_e. It simply indicates the amount of change in the expansion cost for the shorter lead-time that does not make the optimal expansion amount different from the regular option.

Although, what is truly interesting for a decision maker is the selection of optimal procurement option, comparing the options with respect to their expected profit is not a trivial task. The reason is related to the fact that there is no closed form solution for a_s^* and what we have is an equality equation (Equation 4.12) for a_s^*. Therefore, expected profit equation (Equation 4.3) of the two procurement options can only be compared numerically but not analytically. However, if it can be proved that the two procurement options has the same optimal expansion amount (as we discussed in Corollary 6), the analytical comparison of the expected profits of the two procurement options can be accomplished.

As we have showed in Corollary 6, let η be the marginal expansion cost of the one-period faster procurement option that leads to the same optimal expansion amount (a_s^*) as the regular procurement option. As a result, the expected profit of the expansion decision at period s for the faster optimal expansion is (assuming no price depreciation and no discounting):

$$\begin{aligned}\mathcal{G}_{fast} = (r+c_u) \sum_{t=s+L}^{T} \int_{K_s+a_s}^{\infty} (K_s + a_s - d_t) f_t(d_t) d(d_t) + r \sum_{t=s+L}^{T} \mu_t \; - \eta \; a_s \\ - c_h(K_s + a_s)(T - s - L + 1)\end{aligned} \tag{4.19}$$

On the other hand, the expected profit of expansion decision at period s for the regular procurement option is:

$$\begin{aligned}\mathcal{G}_{slow} = (r+c_u) \sum_{t=s+L+1}^{T} \int_{K_s+a_s}^{\infty} (K_s + a_s - d_t) f_t(d_t) d(d_t) + r \sum_{t=s+L+1}^{T} \mu_t \; - c_e \; a_s \\ - c_h(K_s + a_s)(T - s - L)\end{aligned} \tag{4.20}$$

Since we have showed that the same a_s^* is the optimal point of both Equations 4.19 and 4.20, these two expected profits can be compared in their optimal points. As a first step we concentrate on an extra cost of the faster procurement option that makes the decision maker indifferent to both options with respect to the expected profitability:

$$\begin{aligned}(r+c_u) \sum_{t=s+L}^{T} \int_{K_s+a_s^*}^{\infty} (K_s + a_s^* - d_t) f_t(d_t) d(d_t) + r \sum_{t=s+L}^{T} \mu_t \; - \eta \; a_s^* - c_h(K_s + a_s^*)(T - s - L + 1) \\ = (r+c_u) \sum_{t=s+L+1}^{T} \int_{K_s+a_s^*}^{\infty} (K_s + a_s^* - d_t) f_t(d_t) d(d_t) + r \sum_{t=s+L+1}^{T} \mu_t \; - c_e \; a_s^* \\ - c_h(K_s + a_s^*)(T - s - L)\end{aligned}$$

This equation can be simplified as:

$$\eta - c_e = \frac{\overbrace{r\,\mu_{s+L} - c_h(K_s + a_s^*) + (r + c_u)\int_{K_s+a_s^*}^{\infty}(K_s + a_s^* - d_{s+L}) f_{s+L}(d_{s+L}) d(d_{s+L})}^{\epsilon}}{a_s^*} \tag{4.21}$$

Equation 4.21 implies that if the difference of η and c_e would be ϵ (the right side of the equation), the decision maker would be indifferent to the two options with respect to expected profit. If $\eta - c_e$ is less (more) than ϵ, then the faster (regular) option should be the choice of the decision maker.

Similarly, this result can be generalized for a faster option that has *n* period shorter lead-time compared to the regular option:

$$\mathcal{G}_{fast} = (r + c_u) \sum_{t=s+L-n+1}^{T} \int_{K_s+a_s}^{\infty} (K_s + a_s - d_t) f_t(d_t) d(d_t) + r \sum_{t=s+L-n+1}^{T} \mu_t - \eta_n\, a_s - c_h(K_s + a_s)(T - s - L + n + 2) \tag{4.22}$$

Similar to the one-period shorter lead-time:

$$(r + c_u) \sum_{t=s+L-n+1}^{T} \int_{K_s+a_s^*}^{\infty} (K_s + a_s^* - d_t) f_t(d_t) d(d_t) + r \sum_{t=s+L-n+1}^{T} \mu_t - \eta_n\, a_s^* - c_h(K_s + a_s^*)(T - s - L + n + 2)$$
$$= (r + c_u) \sum_{t=s+L+1}^{T} \int_{K_s+a_s^*}^{\infty} (K_s + a_s^* - d_t) f_t(d_t) d(d_t) + r \sum_{t=s+L+1}^{T} \mu_t - c_e\, a_s^* - c_h(K_s + a_s^*)(T - s - L)$$

Finally this equation can be rewritten as:

$$\eta_n - c_e = \frac{r\,\sum_{t=s+L-n+1}^{s+L} \mu_t - n\, c_h(K_s + a_s^*)}{a_s^*} + \frac{(r + c_u)\sum_{t=s+L-n+1}^{s+L} \int_{K_s+a_s^*}^{\infty} (K_s + a_s^* - d_t) f_t(d_t) d(d_t)}{a_s^*} \tag{4.23}$$

Equation 4.23 provides a level at which the decision maker would be indifferent to the two procurement options. However, as discussed, these results are based on an assumption that the optimal expansion amounts for both procurement options are the same. Although this assumption is very restrictive, it is not possible to relax it since there is no closed form solution for optimal expansion amount. Without a closed form solution of optimal expansion amount, expected profits of different procurement options are not comparable.

4.3 Model Behavior in Deterministic Case

As discussed, if a decision maker has complete information regarding the future demands, there is no need for any stochastic model and an IP model can be employed for deriving the optimal policy. However, in this section we provide a modified version of the proposed expansion model that can be used for deriving the policy in case of complete knowledge of the future demand.

Based on Algorithm 6, in order to find the optimal policy, the subsets of all possible expansion timing combinations should be evaluated. After that, for each subset, Equations 4.10 and 4.11 will be used in order to calculate the expansion vector associated to that subset. The expansion vector with the highest expected profit would be the optimal policy. In the deterministic case, however, these two equations should be modified in order to consider the fact that the variance of demand distributions in each period is zero. Zero variance converts any distribution to a degenerate distribution. A degenerate distribution is the probability distribution of a random variable that only takes a single value. Since, the CDF of demand at each period can be rewritten as $F_t(x) = \begin{cases} 1 & \text{, if } x \geq d_t \\ 0 & \text{, if } x < d_t \end{cases}$. Therefore, Equations 4.10 and 4.11 can be rewritten as:

$$\min a_s$$

$$\text{s.t.} \quad \frac{(T-s-L+1)(c_e + c_h \sum_{t=s+L+1}^{T} \delta^{t-s})}{r + c_u} - \sum_{t=s+L+1}^{T} e^{-\alpha t} \delta^{t-s} \mathbf{1}_{(K_s + a_s \geq d_t)} = 0$$

and

$$\min a_s$$

$$\text{s.t. } \frac{(s'-s-1)\left(c_e\left(1-\delta^{s'-s}\right)+c_h \sum_{t=s+L+1}^{s'+L} \delta^{t-s}\right)}{r+c_u} - \sum_{t=s+L+1}^{s'+L} e^{-\alpha t}\delta^{t-s}\mathbf{1}_{(K_s+a_s\geq d_t)} = 0$$

Note that once the optimal policy is calculated, there is no need to proceed further since the life-cycle demand does not change.

Chapter 5 Extensions of the Stochastic Optimal Expansion Model

In this chapter, we present some extensions of the proposed stochastic expansion model that can be used for solving similar SLC capacity planning problems with more complexity. The first extension tackles a problem in which a manufacturer is able to procure capacity from two different supply modes: A fast mode (short lead-time) with expensive marginal cost and a slower mode (longer lead-time) with less expensive marginal cost. The second extension extends the model in a way that augments its capabilities in order to consider the future generations of a product. At both extensions, as a first step, we provide the detail for a case in which the manufacturer is planning to expand capacity only one time during a life-cycle. Later, we provide the formulation for more than one expansion period. Finally, in the last extension, we make provisions for considering salvage value of capacity into the main model of Chapter 3 (single supply mode-single generation).

5.1 Dual Mode Sourcing

Some manufacturers, like Intel (Peng, Erhun et al. 2012), employ a dual-mode equipment procurement strategy in order to hedge against any unexpected surge in demand. In this strategy, they procure equipment from their supplier using two supply modes with complementary lead-times and prices: a base mode with longer lead-time (L_B) and less expensive price (c_e^B) compared to a flexible mode that is more expensive (c_e^F) but has a shorter lead-time (L_F). We let expansion amounts from base and flexible modes be a_s^B and a_s^F, respectively.

Although shorter lead-times are always more favorable, higher marginal expansion costs prevents decision makers to procure only from flexible mode. Therefore, the challenge in this case is optimizing profit by balancing between base and flexible modes. This problem is handled in "execution" stage in (Peng, Erhun et al. 2012) by scenario generation and stochastic programming.

In this extension, we assume that the decision maker plans to expand capacity only two times: One time before product launch and another one during the life-cycle of the product that is predetermined and fixed. The base mode is used to procure capacity before launching a product and it is based on the belief of a decision maker (e.g., initial market potential estimate of the stochastic Bass model) and the next expansion period that s/he will expand capacity again. During a life-cycle, however, a decision maker can use both base and flexible modes for procurement. In the second part of this extension, we propose an algorithm for the multi-period dual source expansion scenario and generalize results for that case.

Note that in the following proposition, the product is in the market and the decision maker needs to decide for optimal mix of capacity based on the available (initial) capacity.

5.1.1 Proposition 1 (Single Period)

(Single Period) If an optimal solution contains procurement from both base and flexible sources, the procurement amount from flexible source is independent of the amount from base source.

5.1.2 Proof of Proposition 1

The proof is similar to the proof of Lemma 1 from Chapter 3. Let current period, s, be the first and only element of $\boldsymbol{T}_E^s$.

$$\begin{aligned}
\mathcal{G} = \mathbb{E}_{\mathcal{F}_s}&(\Pi_s(a_s^F, a_s^B | K_s)) \\
&= \mathbb{E}_{\mathcal{F}_s}\left(r \left(\sum_{t=s+L_F+1}^{s+L_B} e^{-\alpha t}\delta^{t-s} \min(d_t, K_s + a_s^F) \right.\right. \\
&\left. + \sum_{t=s+L_B+1}^{T} e^{-\alpha t}\delta^{t-s} \min(d_t, K_s + a_s^F + a_s^B) \right) \\
&- c_u \left(\sum_{t=s+L_F+1}^{s+L_B} e^{-\alpha t}\delta^{t-s} (d_t - K_s - a_s^F)^+ \right. \\
&\left.\left. + \sum_{t=s+L_B+1}^{T} e^{-\alpha t}\delta^{t-s} (d_t - K_s - a_s^F - a_s^B)^+ \right)\right) - c_e^F a_s^F - c_e^B a_s^B \\
&- c_h \left((K_s + a_s^F) \sum_{t=s+L_F+1}^{s+L_B} \delta^{t-s} + (K_s + a_s^F + a_s^B) \sum_{t=s+L_B+1}^{T} \delta^{t-s} \right)
\end{aligned} \tag{5.1}$$

$$\begin{aligned}
\mathcal{G} = r &\sum_{s+L_F+1}^{T} e^{-\alpha t}\delta^{t-s} \mu_t \\
&+ (r + c_u) \left(\sum_{t=s+L_F+1}^{s+L_B} e^{-\alpha t}\delta^{t-s} \int_{K_s + a_s^F}^{\infty} (K_s + a_s^F - d_t) f_t(d_t) d(d_t) \right. \\
&\left. + \sum_{t=s+L_B+1}^{T} e^{-\alpha t}\delta^{t-s} \int_{K_s + a_s^F + a_s^B}^{\infty} (K_s + a_s^F + a_s^B - d_t) f_t(d_t) d(d_t) \right) \\
&- c_e^F a_s^F - c_e^B a_s^B \\
&- c_h \left((K_s + a_s^F) \sum_{t=s+L_F+1}^{s+L_B} \delta^{t-s} + (K_s + a_s^F + a_s^B) \sum_{t=s+L_B+1}^{T} \delta^{t-s} \right)
\end{aligned} \tag{5.2}$$

Based on Leibniz integral rule and the first order optimality condition ($\frac{\partial \mathcal{G}}{\partial a_s^F} = 0$ and $\frac{\partial \mathcal{G}}{\partial a_s^B} = 0$), $a_s^{F^*}$ and $a_s^{B^*}$ solve the following equations:

$$(r + c_u)\left(\sum_{t=s+L_F+1}^{s+L_B} e^{-\alpha t}\delta^{t-s}F_t^c\left(K_s + a_s^{F^*}\right) + \sum_{t=s+L_B+1}^{T} e^{-\alpha t}\delta^{t-s}F_t^c\left(K_s + a_s^{F^*} + a_s^{B^*}\right)\right) = c_e^F + c_h \sum_{t=s+L_F+1}^{T} \delta^{t-s} \quad (5.3)$$

$$(r + c_u) \sum_{t=s+L_B+1}^{T} e^{-\alpha t}\delta^{t-s}F_t^c\left(K_s + a_s^{F^*} + a_s^{B^*}\right) = c_e^B + c_h \sum_{t=s+L_B+1}^{T} \delta^{t-s} \quad (5.4)$$

Assuming that the optimal solution contains both sources, part of Equation 5.3 can be replaced by Equation 5.4. As a result, Equation 5.3 can be rewritten as:

$$\sum_{t=s+L_F+1}^{s+L_B} e^{-\alpha t}\delta^{t-s}F_t^c\left(K_s + a_s^{F^*}\right) = \frac{(c_e^F - c_e^B) + c_h \sum_{t=L_F}^{L_B} \delta^t}{r + c_u} \quad (5.5)$$

□

Note that Proposition 1 of this chapter only provides the necessary conditions for obtaining the optimal expansion amounts from base and flexible modes given the assumption that procurement from both modes is the optimal decision. It means, if procurement from one of the modes would not be an optimal decision, both equations (Equation 5.4 and 5.5) would not be solvable together. However, it would not be clear that procurement from which mode(s) causes infeasibility. Following algorithm provides necessary steps for finding the optimal procurement amounts in this setting.

5.1.3 Algorithm 1 (Single Period)

Follow the following steps at period *s* in order to find the optimal policy:

a. Use Equations 5.4 and 5.5 in order to find the optimal mix of procurement from both modes ($a_s^{F^*}$ and $a_s^{B^*}$). If these equations are solvable together, there is no need for extra steps and optimal mix would be $a_s^{F^*}$ and $a_s^{B^*}$. If any of the equations (or both) is not solvable go the next step.

b. Consider the following three cases and obtain procurement amount and expected profit for each of them:

i. Assume that procurement from the base mode is the only option. Use Equation 4.3 and Equation 4.10 in order to find the optimal capacity procurement from the base mode and its expected profit.

ii. Similar to step (i), assume that procurement from the flexible mode is the only option; find the optimal capacity procurement amount from the flexible mode and its associated expected profit.

iii. Assume that no expansion is the optimal decision and calculate the expected profit for this case as well.

c. Now compare the expected profit of case (i), (ii), and (iii) of the previous step and select the case with the highest expected profit as the optimal decision.

In the previous proposition, we presented the equations that $a_s^{F^*}$ and $a_s^{B^*}$ need to solve when the decision maker chooses to expand capacity only once during a product's life-cycle. The following proposition expands the results to a case in which capacity procurement is planned to occur twice during a product's life-cycle and in both expansion periods both modes are available for the decision maker.

5.1.4 Proposition 2 (Multiple Periods)

If a decision maker plans to expand capacity at period *s* and *s'* in a dual source mode, optimal expansion values ($a_s^{F^*}, a_s^{B^*}, a_{s'}^{F^*}, a_{s'}^{B^*}$) can be obtained by solving Equations 5.6 to 5.9 consequently:

$$\sum_{s+L_F+1}^{s+L_B} e^{-\alpha t}\delta^{t-s}F^c(K_s + a_s^{F^*}) = \frac{(c_e^F - c_e^B) + c_M \sum_{t=s+L_F+1}^{s+L_B}\delta^{t-s}}{r + c_u} \tag{5.6}$$

$$\sum_{s+L_B+1}^{s'+L_F} e^{-\alpha t}\delta^{t-s}F^c(K_s + a_s^{F^*} + a_s^{B^*}) = \frac{(c_e^B - c_e^F) + c_M \sum_{t=s+L_B+1}^{s'+L_F+1}\delta^{t-s}}{r + c_u} \tag{5.7}$$

$$\sum_{s'+L_F+1}^{s'+L_B} e^{-\alpha t}\delta^{t-s}F^c(K_s + a_s^{F^*} + a_s^{B^*} + a_{s'}^{F^*}) = \frac{(c_e^F - c_e^B) + \ c_M \sum_{t=s'+L_F+1}^{s'+L_B}\delta^{t-s}}{r + c_u} \tag{5.8}$$

$$\sum_{s'+L_B+1}^{T} e^{-\alpha t}\delta^{t-s}F^c(K_s + a_s^{F^*} + a_s^{B^*} + a_{s'}^{F^*} + a_{s'}^{B^*}) = \frac{c_e^B + c_M \sum_{t=s'+L_B+1}^{T}\delta^{t-s}}{r + c_u} \tag{5.9}$$

5.1.5 Proof of Proposition 2

Let $A_1 = K_s + a_s^F$, $A_2 = K_s + a_s^F + a_s^B$, $A_3 = K_s + a_s^F + a_s^B + a_{s'}^F$, and $A_4 = K_s + a_s^F + a_s^B + a_{s'}^F + a_{s'}^B$:

$$\mathcal{G} = \mathbb{E}\left(\Pi\left(a_s^B, a_s^F, a_{s'}^B, a_{s'}^F \mid K_s\right)\right)$$

$$= r\left(\sum_{s+L_F+1}^{s+L_B} e^{-\alpha t}\delta^{t-s}\min(d_t, A_1) + \sum_{s+L_B+1}^{s'+L_F} e^{-\alpha t}\delta^{t-s}\min(d_t, A_2)\right.$$

$$\left. + \sum_{s'+L_F+1}^{s'+L_B} e^{-\alpha t}\delta^{t-s}\min(d_t, A_3) + \sum_{s'+L_B+1}^{T} e^{-\alpha t}\delta^{t-s}\min(d_t, A_4)\right)$$

$$-c_u e^{-\alpha t}\left(\sum_{s+L_F+1}^{s+L_B} e^{-\alpha t}\delta^{t-s}(d_t - A_1)^+ + \sum_{s+L_B+1}^{s'+L_F} e^{-\alpha t}\delta^{t-s}(d_t - A_2)^+\right.$$

$$\left. + \sum_{s'+L_F+1}^{s'+L_B} e^{-\alpha t}\delta^{t-s}(d_t - A_3)^+ + \sum_{s'+L_B+1}^{T} e^{-\alpha t}\delta^{t-s}(d_t - A_4)^+\right)$$

$$-c_e^F\left(a_s^F + a_{s'}^F\right) - c_e(a_s + a_{s'})$$

$$- c_M\left(A_1 \sum_{s+L_F+1}^{s+L_B} \delta^{t-s} + A_2 \sum_{s+L_B+1}^{s'+L_F} \delta^{t-s} + A_3 \sum_{s'+L_F+1}^{s'+L_B} \delta^{t-s}\right.$$

$$\left. + A_4 \sum_{s'+L_B+1}^{T} \delta^{t-s}\right)$$

$$= r\left(\sum_{s+L_F+1}^{s+L_B} e^{-\alpha t}\delta^{t-s}\left(\int_0^{A_1} d_t f(d_t)d(d_t) + \int_{A_1}^{\infty} A_1 f(d_t)d(d_t)\right)\right.$$

$$+ \sum_{s+L_B+1}^{s'+L_F} e^{-\alpha t}\delta^{t-s}\left(\int_0^{A_2} d_t f(d_t)d(d_t) + \int_{A_2}^{\infty} A_2 f(d_t)d(d_t)\right)$$

$$+ \sum_{s'+L_F+1}^{s'+L_B} e^{-\alpha t}\delta^{t-s}\left(\int_0^{A_3} d_t f(d_t)d(d_t) + \int_{A_3}^{\infty} A_3 f(d_t)d(d_t)\right)$$

$$\left. + \sum_{s'+L_B+1}^{T} e^{-\alpha t}\delta^{t-s}\left(\int_0^{A_4} d_t f(d_t)d(d_t) + \int_{A_4}^{\infty} A_4 f(d_t)d(d_t)\right)\right)$$

$$-c_u\left(\sum_{s+L_F+1}^{s+L_B} e^{-\alpha t}\delta^{t-s}\int_{A_1}^{\infty}(A_1 - d_t)f(d_t)d(d_t) + \sum_{s+L_B+1}^{s'+L_F} e^{-\alpha t}\delta^{t-s}\int_{A_2}^{\infty}(A_2 - d_t)f(d_t)d(d_t)\right.$$

$$+ \sum_{s'+L_F+1}^{s'+L_B} e^{-\alpha t}\delta^{t-s}\int_{A_3}^{\infty}(A_3 - d_t)f(d_t)d(d_t)$$

$$\left. + \sum_{s'+L_B+1}^{T} e^{-\alpha t}\delta^{t-s}\int_{A_4}^{\infty}(A_4 - d_t)f(d_t)d(d_t)\right) - C$$

$$= r \sum_{t=s+L_F+1}^{T} e^{-\alpha t} \delta^{t-s} \mu_t$$

$$+ (r + c_u)\left(\sum_{s+L_F+1}^{s+L_B} e^{-\alpha t} \delta^{t-s} \int_{A_1}^{\infty} (A_1 - d_t) f(d_t) d(d_t) \right.$$

$$+ \sum_{s+L_B+1}^{s'+L_F} e^{-\alpha t} \delta^{t-s} \int_{A_2}^{\infty} (A_2 - d_t) f(d_t) d(d_t)$$

$$+ \sum_{s'+L_F+1}^{s'+L_B} e^{-\alpha t} \delta^{t-s} \int_{A_3}^{\infty} (A_3 - d_t) f(d_t) d(d_t)$$

$$\left. + \sum_{s'+L_B+1}^{T} e^{-\alpha t} \delta^{t-s} \int_{A_4}^{\infty} (A_4 - d_t) f(d_t) d(d_t) \right) - C$$

After taking the expectations, the result would be straight forward:

$$\frac{\partial \mathcal{G}}{\partial a_{s'}^{B}} = (r + c_u)\left(\sum_{s'+L_B+1}^{T} e^{-\alpha t} \delta^{t-s} F^c(A_4) \right) - c_e^B - c_M \sum_{s'+L_B+1}^{T} \delta^{t-s} \tag{5.10}$$

$$\frac{\partial \mathcal{G}}{\partial a_{s'}^{F}} = (r + c_u)\left(\sum_{s'+L_F+1}^{s'+L_B} e^{-\alpha t} \delta^{t-s} F^c(A_3) + \sum_{s'+L_B+1}^{T} e^{-\alpha t} \delta^{t-s} F^c(A_4) \right) - c_e^F - c_M \sum_{s'+L_F+1}^{T} \delta^{t-s} \tag{5.11}$$

$$\frac{\partial \mathcal{G}}{\partial a_{s}^{B}} = (r + c_u)\left(\sum_{s+L_B+1}^{s'+L_F} e^{-\alpha t} \delta^{t-s} F^c(A_2) + \sum_{s'+L_F+1}^{s'+L_B} e^{-\alpha t} \delta^{t-s} F^c(A_3) + \sum_{s'+L_B+1}^{T} e^{-\alpha t} \delta^{t-s} F^c(A_4) \right) - c_e^B - c_M \sum_{s+L_B+1}^{T} \delta^{t-s} \tag{5.12}$$

$$\frac{\partial \mathcal{G}}{\partial a_s^F} = (r + c_u)\left(\sum_{s+L_F+1}^{s+L_B} e^{-\alpha t}\delta^{t-s}F^c(A_1) + \sum_{s+L_B+1}^{s'+L_F} e^{-\alpha t}\delta^{t-s}F^c(A_2)\right.$$
$$\left. + \sum_{s'+L_F+1}^{s'+L_B} e^{-\alpha t}\delta^{t-s}F^c(A_3) + \sum_{s'+L_B+1}^{T} e^{-\alpha t}\delta^{t-s}F^c(A_4)\right) - c_e^F \tag{5.13}$$
$$- c_M \sum_{s+L_F+1}^{T} \delta^{t-s}$$

By first order optimality condition, Equations 5.10 to 5.14 should be equal to zero in order to find the optimal values. $a_s^{F^*}$ that is obtained by solving Equation 5.10 should be used in equation 5.11 for finding $a_s^{B^*}$. This procedure should be followed until all optimal values are computed.

□

Note that Proposition 3 assumes that procurement from both modes in both expansion periods are in the optimal solution. If any of the four expansion amounts would not be in the optimal solution, the four equations of Proposition 4 will not be solvable together. The following algorithm provides the steps that need to be followed when equations of Proposition 3 are not solvable.

5.1.6 Algorithm 2 (Multiple Periods)

(Multi-period dual expansion model) Similar to Algorithm 1 of Chapter 3, the following steps should be taken in order to compute the optimal expansion policy:

a. Let $\boldsymbol{\mathcal{T}}_s$ be the power set (collection of all subsets) of $\boldsymbol{T}_E^s$ where each element of $\boldsymbol{\mathcal{T}}_s$ is a subset (permutation) of expansion timings. Note that in each period, a decision maker has to consider two sources for the expansions. As an example, if a decision maker decides to expand capacity at period 10 and 30 and currently the product is at period 10 of the life-cycle ($\boldsymbol{T}_E^{10} = \{10_F, 10_B, 30_F, 30_B\}$), then

$\boldsymbol{\mathcal{T}}_s = \{0, \{10_F\}, \{30_F\}, \{10_B\}, \{30_B\}, \{10_F, 30_F\}, \ldots\}$, where $|\boldsymbol{\mathcal{T}}_{10}| = 2^{|T_E^{10}|}$

b. Use Proposition 3 and develop necessary equations for each subset in $\mathcal{T}_{10}$. Note that each combination, based on periods and modes, needs its specific equations and it is not possible to write a general formulation for all possible cases. For example, Proposition 3 can be used for a combination where the decision maker expands capacity in periods 10 and 30 from both sources. Note that if any of the equations would not be solvable in any subset (permutation), the associated subset would be discarded and considered as an infeasible solution. At the end of this step, infeasible subsets (permutations) have been discarded and feasible subsets with their associated policies (expansion vectors: $\boldsymbol{a}_s^F$ and $\boldsymbol{a}_s^B$) are ready to be compared. Let $\boldsymbol{\mathcal{T}}_s^F$ be a collection of feasible subsets in $\boldsymbol{\mathcal{T}}_s$. Note that each element of $\boldsymbol{\mathcal{T}}_s^F$ has an associated $\boldsymbol{a}_s^i$ where $i = 1, \ldots, |\boldsymbol{\mathcal{T}}_s^F|$. Each $\boldsymbol{a}_s^i$ contains expansion amounts in the subset's periods.

c. As a final stage, each expansion policy should be plugged in the expected profit equation ($\mathcal{G}$) in order to identify the optimal policy. Therefore $\boldsymbol{a}_s^* = argmax_i \ \mathbb{E}_{\mathcal{F}_s}(\Pi_s(a_s^i|K_s))$.

5.2 Multi-generation Products

After launching a new product in a market, many manufacturers continue improving their initial products and introduce the improved versions as new generations to the market. This strategy is very common in electronics and semiconductor industries including smartphones, tablets, processors and etc. In these industries, new generations are introduced in the market when the current generations of the product are still available, which leads to a cannibalization of the current generation.

Unless a manufacturer considers dramatic changes in a new generation, the installed capacity for a current generation might be used for the next generation. As a result, considering solely the demand for the current generation of a product might lead to a procurement policy that is far from optimal. In this section, we generalize the proposed model for a case in which the decision maker is planning for a product with two generations. However, the results can be considered as a framework for other scenarios in which the product has more than two generations.

In this two-generation setting, it will be assumed that the launch and termination periods for both generations are fixed and known to the decision maker. We let period t_1^L and t_2^L be the launch periods of the first and second generation respectively. In addition, t_1^T and t_2^T are the termination periods of the first and second generation products (Figure 1). t_1^L can be set to zero in order to be considered as the starting point of the product introduction to the market.

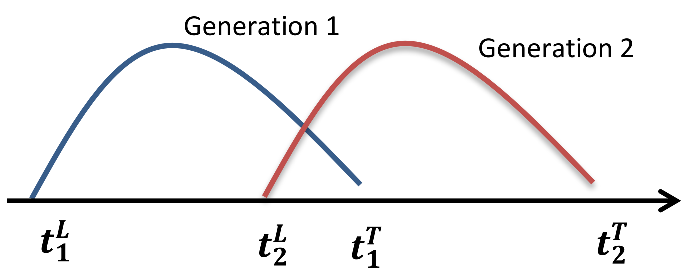

Figure 5.1: Launch and termination timings of a product with two generations

In addition to the notations that we have introduced in Chapter 4, that are used here for the first generation, following notations will be used for the second generation:

- $\tilde{d}_t$: Stochastic demand of the second generation at period t
- $\tilde{f}_t(.)$: Probability density function of the second generation demand at period t
- $\tilde{F}_t(.)$: Cumulative distribution function of the second generation demand at period t
- $\tilde{\mu}_t$: Expected value of the second generation demand at period t
- $\tilde{\sigma}_t^2$: Variance of the second generation demand at period t
- $\tilde{r}$: Initial product sales price for the product in the second generation
- $\tilde{r}_t$: Product sales price for the second generation at period t
- $\tilde{c}_u$: Initial marginal penalty of unmet demand for the second generation

- $\tilde{c}_u^t$: Marginal shortage penalty for the second generation at period t

Note that marginal expansion cost (c_e) of the single generation model is slightly different from a multi-generation setting. In the multi generation model, a cost that is associated to the reconfiguration of an installed capacity, for producing new generations, should also be included in this price. Since, the time of reconfiguration depends on the manufacturer's production policy it is not trivial to predict precisely when the installed capacity will be reconfigured for producing new generation of a product. As a result, in this section we assume that the reconfiguration cost occurs at the time of capacity procurement.

As a first step, we concentrate on a setting in which the decision maker only expands capacity once during life-cycles of the two generations. Later, we generalize the result to a case where the new capacity procurement is planned to occur twice. Providing a general formulation for any scenario (more than two expansion periods) is not trivial since different sequences of expansion periods and launching/termination periods need different formulations. However, we believe that these two cases can be generalized for any scenario where the decision maker is planning to expand capacity more than two times.

For the case in which the decision maker plans to expand capacity only once, we assume that the first generation product is in period s of its life-cycle and based on the remaining periods of the first generation life and complete life-cycle of the second generation product, the decision maker needs to decide on the amount of new capacity that should be procured. In this period, the amount of available capacity is K_s and the manufacturer can utilize the new capacity after L periods ($-L \leq s < t_2^L \leq t_1^T < t_2^T$). Note that if $s = -L$, the decision is considered as the initial capacity. Since holding capacity is not free (due to maintenance cost), there is no reason to expand capacity before $-L$. Figure 2 illustrates the sequence of events/timings.

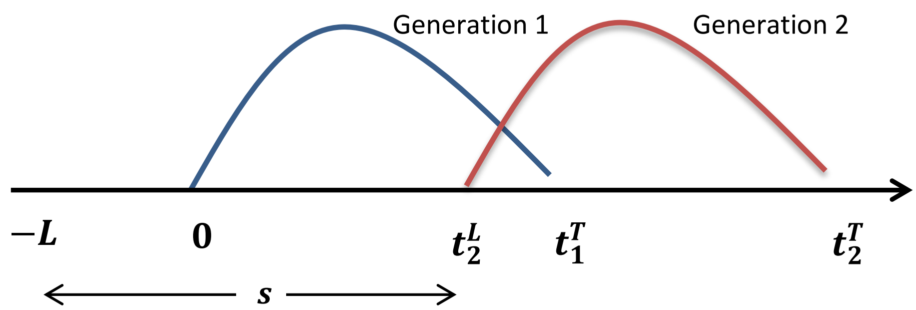

Figure 5.2: Sequence of timing of events in a case that a decision maker expands capacity only once after launching a product

5.2.1 Proposition 3 (Single Period)

If s (where $-L \le s < t_2^L \le t_1^T < t_2^T$) would be the last expansion period, the optimal expansion decision at this period should satisfy the following equation:

$$(r + c_u) \sum_{t=s+L+1}^{t_1^T} e^{-\alpha t} \delta^{t-s} F_t^c(K_s + a_s^*) + (\tilde{r} + \tilde{c}_u) \sum_{t=t_2^L}^{t_2^T} e^{-\alpha(t-t_1)} \delta^{t-s} \tilde{F}_t^c(K_s + a_s^*) = c_e + c_h \sum_{t=s+L+1}^{t_2^T} \delta^{t-s} \quad (5.14)$$

5.2.2 Proof of Proposition 3

Similar to the Proposition 1 of Chapter 3, the expected profit of an expansion policy can be written as:

$$\mathcal{G} = \mathbb{E}_{\mathcal{F}_s}\left(\sum_{t=s+L+1}^{t_1^T} e^{-\alpha t} \delta^{t-s} \big((r + c_u) \min(d_t, K_s + a_s) - c_u d_t \big) + \sum_{t=t_2^L}^{t_2^T} e^{-\alpha(t-t_1)} \delta^{t-s} \left((\tilde{r} + \tilde{c}_u) \min(\tilde{d}_t, K_s + a_s) - \tilde{c}_u \tilde{d}_t \right) \right) - c_e a_s - c_h (K_s + a_s) \sum_{t=s+L+1}^{t_2^T} \delta^{t-s} \quad (5.15)$$

$$\mathcal{G} = \sum_{t=s+L+1}^{t_1^T} e^{-\alpha t}\delta^{t-s}\left(r\,\mu_t + (r+c_u)\int_{K_s+a_s}^{\infty}(K_s+a_s-d_t)f_t(d_t)d(d_t)\right) + \sum_{t=t_2^L}^{t_2^T} e^{-\alpha(t-t_1)}\delta^{t-s}\Bigg(\tilde{r}\,\tilde{\mu}_t + (\tilde{r}+\tilde{c}_u)\int_{K_s+a_s}^{\infty}\big(K_s+a_s-\tilde{d}_t\big)\tilde{f}_t\big(\tilde{d}_t\big)d\big(\tilde{d}_t\big)\Bigg) - c_e a_s - c_h(K_s+a_s)\sum_{t=s+L+1}^{t_2^T}\delta^{t-s} \tag{5.16}$$

The first and second derivatives of $\mathcal{G}$ are:

$$\frac{\partial \mathcal{G}}{\partial a_s} = (r+c_u)\sum_{t=s+L+1}^{t_1^T} e^{-\alpha t}\delta^{t-s}F_t^c(K_s+a_s) + (\tilde{r}+\tilde{c}_u)\sum_{t=t_2^L}^{t_2^T} e^{-\alpha(t-t_1)}\delta^{t-s}\tilde{F}_t^c(K_s+a_s) - c_e - c_h\sum_{t=s+L+1}^{t_2^T}\delta^{t-s} \tag{5.17}$$

$$\frac{\partial^2 \mathcal{G}}{\partial a_s^2} = -\left((r+c_u)\sum_{t=s+L+1}^{t_1^T} e^{-\alpha t}\delta^{t-s}f_t(K_s+a_s) + (\tilde{r}+\tilde{c}_u)\sum_{t=t_2^L}^{t_2^T} e^{-\alpha(t-t_1)}\delta^{t-s}\tilde{f}_t(K_s+a_s)\right) < 0 \tag{5.18}$$

We have showed that the expected value of the profit function is strictly concave. Therefore, based on the first order optimality condition:

$$(r + c_u) \sum_{t=s+L+1}^{t_1^T} e^{-\alpha t} \delta^{t-s} F_t^c(K_s + a_s^*) \; + \; (\tilde{r} + \tilde{c}_u) \sum_{t=t_2^L}^{t_2^T} e^{-\alpha(t-t_1)} \delta^{t-s} \tilde{F}_t^c(K_s + a_s^*)$$
$$= c_e + c_h \sum_{t=s+L+1}^{t_2^T} \delta^{t-s} \tag{5.19}$$

□

5.2.3 Corollary 1

If initial product price and shortage penalty for both generations would be the same, optimal expansion decision solves the following equation:

$$\sum_{t=s+L+1}^{t_1^T} e^{-\alpha t} \delta^{t-s} F_t^c(K_s + a_s^*) \; + \; \sum_{t=t_2^L}^{t_2^T} e^{-\alpha(t-t_1)} \delta^{t-s} \tilde{F}_t^c(K_s + a_s^*)$$
$$= \frac{c_e + c_h \sum_{t=s+L+1}^{t_2^T} \delta^{t-s}}{(r + c_u)} \tag{5.20}$$

□

5.2.4 Corollary 2

The result of Proposition 5 can be easily generalized for a multiple-generation scenario. Let us assume that the manufacturer is planning to launch G generations of a product and period s would be the only expansion period during the first generation. t_g^L and t_g^T are launch and termination periods of generation g. Also, r^g and c_u^g are initial product price and shortage penalty for generation g of a product, where $g = 1, 2, \dots, G$. In addition, $(F_t^g)^c$ is the complement of CDF for demand of generation g at period t. It can be shown that a_s^* solves the following equation:

$$(r^1 + c_u^1) \sum_{t=s+L+1}^{t_1^T} e^{-\alpha t} \delta^{t-s} F_t^{1^c}(K_s + a_s^*)$$

$$+ \sum_{g=2}^{G} \left(\left(r^g + c_u^g\right) \sum_{t=t_g^L}^{t_g^T} e^{-\alpha\left(t-t_g^L\right)} \delta^{t-s} F_t^{g^c}(K_s + a_s^*) \right) \tag{5.21}$$

$$= c_e + c_h \sum_{t=s+L+1}^{t_G^T} \delta^{t-s}$$

□

Proposition 5 provides an equation that a_s^* has to solve when s is the final expansion period. In the following proposition, we generalize this results to a case where the decision maker, in addition to period s, plans to procure new capacity in another period (s') as well. Here, we let s' to be $t_1 - L - 1$, meaning that $a_{s'}$ will be utilized at the first period of the launching period of the second generation. Note that this is just an example that makes the formulation easier to read and any other expansion timings can be modeled in the same way.

5.2.5 Proposition 4 (Multiple Periods)

If s and s', where $-L < s < s' < t_2^L \leq t_1^T < t_2^T$, would be the two expansion periods, the optimal expansion decisions at these periods should satisfy the following equations:

$$\sum_{t=s+L+1}^{t_2^L-1} e^{-\alpha t} \delta^{t-s} F_t^c(K_s + a_s^*) = \frac{c_e\left(1 - \delta^{\left(t_2^L - L - 1\right) - s}\right) + c_h \left(\sum_{t=s+L+1}^{t_2^L-1} \delta^{t-s}\right)}{r + c_u} \tag{5.22}$$

$$(r+c_u)\sum_{t=t_2^L}^{t_1^T} e^{-\alpha t}\delta^{t-s}F_t^c\left(K_s+a_s^*+a_{s'}^*\right)$$

$$+(\tilde{r}+\tilde{c}_u)\sum_{t=t_2^L}^{t_2^T} e^{-\alpha\left(t-t_2^L\right)}\delta^{t-s}\tilde{F}_t^c\left(K_s+a_s^*+a_{s'}^*\right) \tag{5.23}$$

$$= c_e\delta^{\left(t_2^L-L-1\right)-s}+c_h\left(\sum_{t=t_2^L}^{t_2^T}\delta^{t-s}\right)$$

5.2.6 Proof of Proposition 4

Similar to the previous proofs, it can be shown that expected profit can be written as:

$$\mathcal{G}=\sum_{t=s+L+1}^{t_2^L-1} e^{-\alpha t}\delta^{t-s}\left(r\,\mu_t+(r+c_u)\int_{K_s+a_s}^{\infty}(K_s+a_s-d_t)f_t(d_t)d(d_t)\right)$$

$$+\sum_{t=t_2^L}^{t_1^T} e^{-\alpha t}\delta^{t-s}\left(r\,\mu_t\right.$$

$$\left.+(r+c_u)\int_{K_s+a_s+a_{s'}}^{\infty}(K_s+a_s+a_{s'}-d_t)f_t(d_t)d(d_t)\right)$$

$$+\sum_{t=t_2^L}^{t_2^T} e^{-\alpha\left(t-t_2^L\right)}\delta^{t-s}\left(\tilde{r}\,\tilde{\mu}_t\right. \tag{5.24}$$

$$\left.+(\tilde{r}+\tilde{c}_u)\int_{K_s+a_s+a_{s'}}^{\infty}\left(K_s+a_s+a_{s'}-\tilde{d}_t\right)\tilde{f}_t\left(\tilde{d}_t\right)d\left(\tilde{d}_t\right)\right)$$

$$-c_e\left(a_s-\delta^{\left(t_2^L-L-1\right)-s}a_{s'}\right)$$

$$-c_h\left((K_s+a_s)\sum_{t=s+L+1}^{t_2^L-1}\delta^{t-s}+\ (K_s+a_s+a_{s'})\sum_{t=t_2^L}^{t_2^T}\delta^{t-s}\right)$$

$$\frac{\partial \mathcal{G}}{\partial a_s} = (r + c_u)\left(\sum_{t=s+L+1}^{t_2^L - 1} e^{-\alpha t}\delta^{t-s}F_t^c(K_s + a_s) + \sum_{t=t_2^L}^{t_1^T} e^{-\alpha t}\delta^{t-s}F_t^c(K_s + a_s + a_{s'})\right)$$
$$+ (\tilde{r} + \tilde{c}_u)\sum_{t=t_2^L}^{t_3} e^{-\alpha(t-t_2^L)}\delta^{t-s}\tilde{F}_t^c(K_s + a_s + a_{s'}) - c_e$$
$$- c_h\left(\sum_{t=s+L+1}^{t_2^T} \delta^{t-s}\right) \quad (5.25)$$

$$\frac{\partial \mathcal{G}}{\partial a_{s'}} = (r + c_u)\sum_{t=t_2^L}^{t_1^T} e^{-\alpha t}\delta^{t-s}F_t^c(K_s + a_s + a_{s'})$$
$$+ (\tilde{r} + \tilde{c}_u)\sum_{t=t_2^L}^{t_2^T} e^{-\alpha(t-t_2^L)}\delta^{t-s}\tilde{F}_t^c(K_s + a_s + a_{s'}) - c_e\delta^{(t_2^L - L - 1)-s}$$
$$- c_h\left(\sum_{t=t_2^L}^{t_2^T} \delta^{t-s}\right) \quad (5.26)$$

As a result, at optimal solution:

$$(r + c_u)\left(\sum_{t=s+L+1}^{t_2^L - 1} e^{-\alpha t}\delta^{t-s}F_t^c(K_s + a_s^*) + \sum_{t=t_2^L}^{t_1^T} e^{-\alpha t}\delta^{t-s}F_t^c\left(K_s + a_s^* + a_{s'}^*\right)\right)$$
$$+ (\tilde{r} + \tilde{c}_u)\sum_{t=t_1}^{t_2^T} e^{-\alpha(t-t_2^L)}\delta^{t-s}\tilde{F}_t^c\left(K_s + a_s^* + a_{s'}^*\right) \quad (5.27)$$
$$= c_e + c_h\left(\sum_{t=s+L+1}^{t_2^T} \delta^{t-s}\right)$$

and

$$(r + c_u) \sum_{t=t_2^L}^{t_1^T} e^{-\alpha t} \delta^{t-s} F_t^c\left(K_s + a_s^* + a_{s'}^*\right)$$

$$+ (\tilde{r} + \tilde{c}_u) \sum_{t=t_2^L}^{t_2^T} e^{-\alpha(t-t_2^L)} \delta^{t-s} \tilde{F}_t^c\left(K_s + a_s^* + a_{s'}^*\right) \tag{5.28}$$

$$= c_e \delta^{(t_2^L - L - 1) - s} + c_h \left(\sum_{t=t_2^L}^{t_2^T} \delta^{t-s} \right)$$

By replacing Equation 5.28 in Equation 5.27, we have:

$$\sum_{t=s+L+1}^{t_2^L - 1} e^{-\alpha t} \delta^{t-s} F_t^c(K_s + a_s^*) = \frac{c_e\left(1 - \delta^{(t_2^L - L - 1) - s}\right) + c_h \left(\sum_{t=s+L+1}^{t_2^L - 1} \delta^{t-s} \right)}{r + c_u} \tag{5.29}$$

□

5.3 Impact of Capacity Salvage Value

As we discussed in the introduction, irreversibility of investments is one of the main issues in the capacity planning for SLC products due to negligible salvage values of the highly customized tools and machines with very high obsolescence rates. However, there might be some cases where a manufacturer might be able to recover some of its investment by reselling its installed used capacity. In this section, we modify the model presented in Chapter 3 in order to include the salvage value into the formulation of the problem. Note that this extension only extends the main model of the dissertation in which a manufacturer only procures capacity for one generation of a product and from one supply option.

If we let c_s be the unit salvage value of an installed capacity that can be sold at the end of a life-cycle, the expected value of profit function (Equation 4.3) can be rewritten as:

$$\begin{aligned}\mathcal{G} = \ & \mathbb{E}_{\mathcal{F}_s}\left(\Pi_s\left(\boldsymbol{a}_{t\in\boldsymbol{T}_E^s}|K_s\right)\right) \\ & = (r \\ & + c_u) \sum_{i=1}^{|\boldsymbol{T}_E^s|} \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} e^{-\alpha t}\delta^{t-s}\left(\int_{K_s+\sum_{j=1}^{i} a_{\tau_j}}^{\infty}\left(K_s + \sum_{j=1}^{i} a_{\tau_j}\right.\right. \\ & \left.\left. - d_t\right) f_t(d_t) d(d_t)\right) + r \sum_{t=s+L+1}^{T} e^{-\alpha t}\delta^{t-s}\mu_t \\ & + c_s\delta^{T-s}\left(K_s + \sum_{j=1}^{|\boldsymbol{T}_E^s|} a_{\tau_j}\right) - c_e \sum_{j=1}^{|\boldsymbol{T}_E^s|} a_{\tau_j}\delta^{\tau_j - s} \\ & - c_h \sum_{i=1}^{|\boldsymbol{T}_E^s|}\left(\left(K_s + \sum_{j=1}^{i} a_{\tau_j}\right) \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} \delta^{t-s}\right)\end{aligned} \tag{5.30}$$

Based on Leibniz integral rule, the first derivative of the expected profit with respect to expansion decisions is:

$$\begin{aligned}\frac{\partial \mathcal{G}}{\partial a_{\tau_k}} = (r + c_u) \sum_{i=k}^{|\boldsymbol{T}_E^s|} \sum_{t=\tau_i+L+1}^{\tau_{i+1}+L} e^{-\alpha t}\delta^{t-s} F_t^c\left(K_s + \sum_{j=1}^{i} a_{\tau_j}\right) - c_e\delta^{\tau_k - s} - c_h \sum_{t=\tau_k+L+1}^{T} \delta^{t-s} \\ + c_s\delta^{T-s}\end{aligned} \tag{5.31}$$

As a result, if a_s would be the last expansion period of a life-cycle, the optimal expansion amount at this period (a_s^*) solves the following equation:

$$\sum_{t=s+L+1}^{T} e^{-\alpha t}\delta^{t-s} F_t^c(K_s + a_s^*) = \frac{c_e - c_s\delta^{T-s} + c_h \sum_{t=s+L+1}^{T} \delta^{t-s}}{r + c_u} \tag{5.32}$$

Note that this equation is very similar to Equation 4.10. Assuming no discounting ($\delta = 0$), it is obvious that if a manufacturer would be able to recover all his investment ($c_e = c_s$), the optimal expansion would only depend on maintenance cost, underage penalty cost and product price.

Although Equation 5.32 has a slight difference with the similar equation of the main model (no salvage value), it can be shown that in case of multiple expansion periods, salvage value has no effect on the equation that optimal expansion of period s should solve (Equation 4.11). If we let s' be the next expansion period at which expanding capacity is an optimal decision, it can be proved that optimal expansion at period s (a_s^*) should solve the following equation:

$$\sum_{t=s+L+1}^{s'+L} e^{-\alpha t} \delta^{t-s} F_t^c(K_s + a_s^*) = \frac{c_e\left(1 - \delta^{s'-s}\right) + c_h \sum_{t=s+L+1}^{s'+L} \delta^{t-s}}{r + c_u} \tag{5.33}$$

That is exactly the same equation of the main model in Chapter 4.

Chapter 6 Numerical Experiments

In this chapter, we conduct sensitivity analysis for factors that affect the optimal policies and performance of the proposed capacity expansion models from Chapters 4 and 5, through simulation. The factors that can potentially affect an expansion policy are usually imposed by the environment and should be studied individually or in combination with each other. Here, we focus on the factors that are related to a market, which a manufacturer operates in, or the type of procurement that it has to procure capacity from.

Diffusion speed, demand volatility and underage penalty cost are among those factors that vary from one industry/market to another and have a major impact on expansion policies. In contrast, marginal capacity cost and procurement lead-time are specific to the technology that a manufacturer employs and the supply option that it is procured from.

Diffusion Speed	p	q
Slow	0.001	0.25
Medium	0.001	0.35
Fast	0.004	0.4

Table 6.1: Bass model parameters for three diffusion speeds

For producing different demand scenarios with different diffusion speeds, the proposed stochastic Bass diffusion model of Chapter 3 is used. In this model that is based on the Bass model, p and q are the parameters that control the diffusion of a product. Three sets of diffusion parameters (p and q) are defined in order to illustrate the behavior of the model in three different diffusion speeds named slow, medium and fast (Table 6.1). Initial market potential is assumed to be two million customers. Note that these parameters are simply selected as an example and they can take any value with respect to different products. In order to show the adoption processes for these three levels, Figure 6.1 illustrates the demand volumes at each period with very low volatility in market potential.

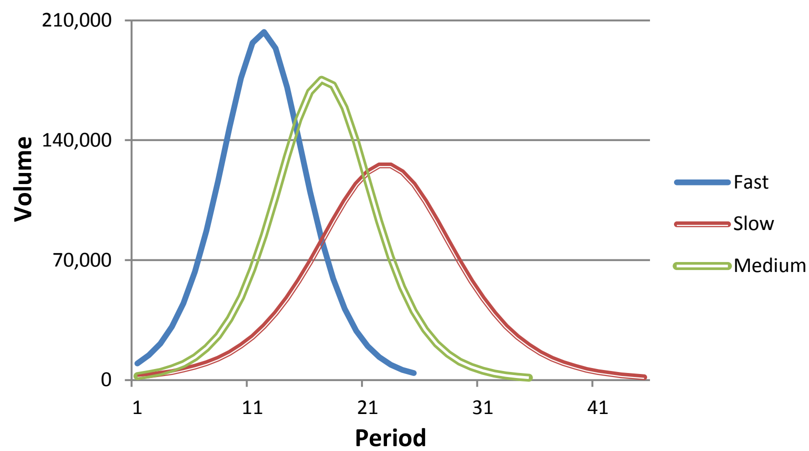

Figure 6.1: Demand curves in three diffusion speeds (no volatility)

The other important factor is the volatility of demand. Volatility is the level of uncertainty that a decision maker has in regards to future demand. As explained in Chapter 3, demand uncertainty is modeled with two parameters in the proposed stochastic Bass diffusion model: β and γ. β is the factor that controls the changes of the market potential in each period and γ is the decay factor of β through a life-cycle. Based on the numerical examples that are presented in Chapter 3, three levels for β and one level for γ are selected. These values provide different levels of volatility and unpredictability in demand that mimic well some of the real world examples.

Volatility Level	β	γ
Low	1.2	4
Medium	1.3	4
High	1.4	4

Table 6.2: Demand volatility levels

The last factor, that is considered to be related to a market that a manufacturer operates in, is shortage penalty cost/unit (c_u). As discussed in Chapter 4, in addition to missing revenue, manufacturers are penalized for not being able to meet market demand. In this chapter, we present the simulation results for three levels of the initial values of this penalty cost/unit: \$20, \$30, and \$40. Note that this cost is not fixed and decreases exponentially during a life-cycle by a decay factor (α).

Marginal capacity cost and procurement lead-time are among those factors that are specific to a supply option that the manufacturer procures the capacity from. We consider two different types of procurement options based on their lead-time length and marginal expansion cost. First option has a longer lead-time with a cheaper capacity cost (\$200/unit capacity, 4 period lead-time) compared to an option with shorter lead-time but more expensive expansion cost (\$225/unit capacity, 1 period lead-time). Note that in the proposed model of Chapter 4, the decision maker only procures capacity from a specific supply option and cannot switch between different options once a product is launched. As a result, in our experiments, the decision maker selects one of the options before a product launch and continues procuring from it until the end of a life-cycle.

In all experiments, the initial price is assumed to be \$40 and it is monotonically decreasing throughout the life-cycle ($r_t = r_0 e^{-\alpha t}$). The decay factor (α) for both product price and underage penalty cost is the same and it is picked in a way to provide half of the initial values at the end of a life-cycle with 35 periods (*T* = 35).

Moreover, we assume that the decision maker expands capacity in those periods that are multiples of the expansion gap. In addition, s/he procures new capacity in period $-L$ and the first period of a life-cycle. As an example, if the lead-time would be three periods and the expansion gap is assumed to be 6 periods, the decision maker procures new capacity in the following periods: $\{-3, 1, 6, 12, 18\ \ldots\ \}$. Period zero is assumed to be the last period before launching a product. Finally, it is assumed that a decision maker can expand capacity every three periods. Discounting factor is assumed to be negligible. The list of different levels of parameters and costs are presented in the following table:

Parameter	Level(s)
r_0	\$40
c_u	\$20, \$30, and \$40
c_h	\$0.5
c_e and L	(\$200, 4 periods) and (\$225, 1 periods)
α	.0025
Expansion gap	Every 3 periods
δ	1

Table 6.3: Parameters and settings for simulation experiments

Note that each combination of these values (Tables 6.1, 6.2, and 6.3) is considered as one simulation setting. In the following sections, we have generated adequate number of demand scenarios (by using the stochastic Bass diffusion model of Chapter 3) for each simulation setting and used the proposed expansion model for capacity planning. At the end of each simulation scenario, we have collected the performance-related statistics including life-cycle profit, total sales, total lost sales, total installed capacity, and others. Average of the profits across all simulation experiment setting replications will be the measure for performance comparison.

6.1 Sensitivity Analysis

In this section, first we illustrate the effect of shortage penalty cost (c_u) combined with diffusion speeds and volatility levels on the performance of the proposed expansion model. Then, by considering the interaction of procurement lead-time and marginal expansion cost, we present the circumstances at which faster-but-expensive procurement option is more profitable for a manufacturer.

6.1.1 Diffusion Speed, volatility, and underage penalty cost

As discussed, missing market demand has some consequences for manufacturers that in some cases can be measured and monetized in order to be included in the investment decisions. Figure 6.2 illustrates the effect of three levels of shortage cost that are $20 (solid lines), $30 (dashed lines), and $40 (dot-dashed lines) combined with different diffusion speeds and volatility levels. In this figure, marginal expansion cost is $200/unit capacity.

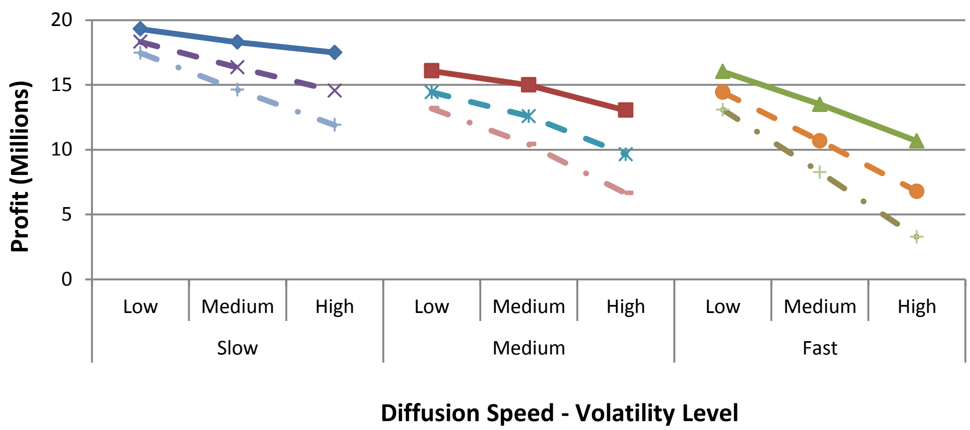

Figure 6.2: Average profit for shortage cost of $20 (solid lines), $30 (dashed lines), and $40 (dot-dashed lines)

As it is clear in Figure 6.2, increasing diffusion speeds has a negative effect on the profitability of a product. In each volatility level, slow diffusion products have higher average life-cycle profits compared to the other two diffusion speeds. The reason is related to a fact that in slow diffusion speeds a manufacturer has more opportunities to adjust the available capacity compared to the faster diffusion speeds. In slow diffusion case, the time between launching a product and the period of the peak in a demand is longer than the other two diffusion speeds and the decision maker is able to modify/improve previous policies more frequently. In products with medium and fast diffusion speeds, on the contrary, the decision maker has very few opportunities for procuring new capacities and any miscalculation or mistake in an expansion policy leads to more severe consequences (increases with the level of uncertainty in the stochastic Bass demand diffusion model).

In terms of the volatility effect, it is obvious that a manufacturer can achieve a higher profit in a low volatile environment compared to a turbulent market. In case of low volatility, the decision maker can procure enough capacity with a high level of certainty and avoid any costly under- or over-expansions. On the other hand, it is again an intuitive expectation that in a volatile environment aligning capacity expansions with a stochastic demand is more challenging and consequently the chance of misalignment is higher.

In addition, this figure shows that a high underage cost intensifies the effect of volatility levels on the average profit that a manufacturer can achieve in a setting. The differences of average profits are insignificant when the volatility is low and more profound when the volatility is high. Steeper slope of the dot-dashed lines (the highest shortage cost) in Figure 6.2 is the result of this relationship. Since in a low volatile environment the decision maker can procure enough capacity without any significant risk, level of shortage penalty cost cannot affect the profitability so much as the high volatility setting.

6.1.2 Interaction of expansion cost and lead-time

As explained in Chapter 4, the proposed expansion model assumes that marginal expansion cost (c_e) and procurement lead-time (L) are fixed during a life-cycle and a manufacturer cannot switch among different supply options once the product is launched. However, before launching a product the decision maker might have different options with respect to procurement lead-time and marginal expansion cost and s/he would be able to pick among them. In this section, by presenting results from related simulation experiments, we provide some insights on the superiority of different procurement options (based on their profitability) under different circumstances.

Here we assume that the decision maker has two options before launching a SLC product: 1) Fast-but-expensive option ($L = 1$ period and $c_e = \$225$/unit capacity); 2) Slow-but-cheap option ($L = 4$ periods and $c_e = \$200$/unit capacity). In the following figures, the fast-but-expensive option is showed in dashed lines and the slow-but-cheap option is showed in solid lines.

Figure 6.3 and Figure 6.4 illustrate the average profit of the two procurement options under different diffusion speeds and volatility levels when $c_u = 20$ and $c_u = 40$. As it is obvious from Figure 6.3 (low shortage penalty cost), cheaper procurement option leads to higher average profits under all settings when compared to the more expensive but faster option. However, the differences between two options shrink as volatility level increases, especially when the product has a fast diffusion speed.

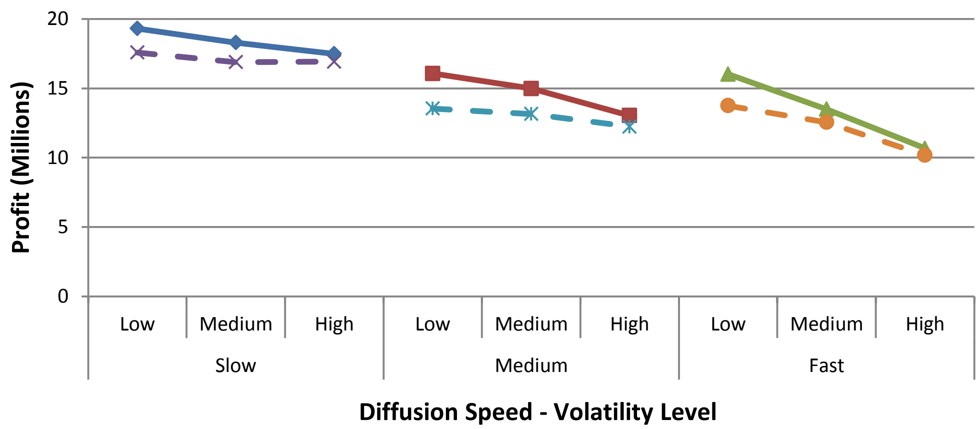

Figure 6.3: Average profit for a slow-but-cheap procurement option (solid lines) and fast-but-expensive option (dashed lines) when c_u=$20

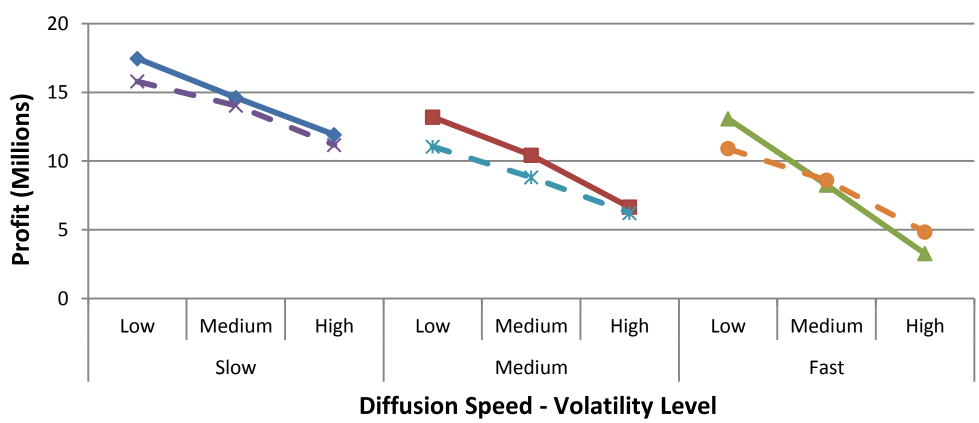

Figure 6.4: Average profit for a slow-but-cheap procurement option (solid lines) and fast-but-expensive option (dashed lines) when c_u=$40

When shortage penalty cost is higher (Figure 6.4), this shrinkage is more visible and there is clearly a cutoff point in fast diffusion speed at which the fast-but-expensive option is more profitable than the slow option.

This cutoff point implies that a manufacturer has more incentive to switch to the faster option when s/he operates in a higher volatile environment and has a product with a faster diffusion speed. In products with fast diffusion speeds, decision makers have very few opportunities to expand capacity, and, as a result faster procurement options are more valuable for them. In addition, when the volatility is high, capacity planning is riskier since any initial demand estimations might be completely unreliable. In this case, fast-but-expensive option gives the manufacturer an ability to postpone the procurement as much as possible. The extra cost that the manufacturer has to pay for the faster delivery can be considered the cost of “postponement option.”

6.2 Benchmark Comparison

Up to this point, we only discussed the behavior and the performance of the proposed model under a variety of circumstances. In order to show the benefit of using the proposed expansion model, we compare its performances with a benchmark model that is certainty equivalent controller (CEC).

6.2.1 Certainty Equivalent Controller

It was discussed in Chapter 4 that deriving an optimal policy for a capacity planning problem under stochastic non-stationary demand is a challenging task. In these situations, a decision maker often tries to settle for a suboptimal control model that provides a reasonable balance between convenient implementation and adequate performance (Bertsekas 2005).

CEC is one of the suboptimal control schemes that use the available information in order to fix the uncertain quantities with some values. Incorporating the fixed values, the decision maker applies at each stage the control that would be optimal (Bertsekas 2005). In each period, s/he needs to rerun this deterministic model with new information.

Here, we assume that the expected values of the demand are used for replacing the uncertain demand values. This replacement converts the problem to a deterministic dynamic programming model and the derived policy from this model is a suboptimal expansion vector. The objective function and the state transition equation of this deterministic dynamic programming model are defined as follows:

$$\max_{\mathbf{a}_t} \sum_{t=s}^{T} \delta^{t-s}(r_t \min(\mu_t, K_t) - c_t^u(\mu_t - \min(\mu_t, K_t)) - c_M K_t - c_e a_t)$$

$$s.t.\ K_t = \mathrm{a}_{t-L} + K_{t-1}$$

$$\mathbf{a}_t \geq 0$$

This formulation can be converted to a linear programming problem:

$$\max_{\mathrm{a}_t, y_t} \sum_{t=s}^{T} \beta^{t-s}((r_t + c_t^u) y_t - c_t^u \mu_t - c_M K_t - c_e a_t)$$

$$s.t.\ K_t = \mathrm{a}_{t-L} + K_{t-1}$$

$$y_t \leq \mu_t$$

$$y_t \leq K_t$$

$$\mathbf{a}_{\{t: t \in T_E^s\}} \geq 0$$

$$\mathbf{a}_{\{t: t \notin \boldsymbol{T}_E^s\}} = 0$$

As mentioned, the CEC model only considers the expected value of demand and ignores the distributions of demand in future periods. Therefore, CEC performs well when the volatility of a demand is low and performs poorly in case of highly volatile demand. Although in a volatile environment CEC will certainly provide a poor performance, in some cases in which the volatility of the demand is extremely severe, CEC and the proposed expansion model might have non-differentiable performances. This issue will be discussed later in detail.

6.3 Policy Comparison

In this section, we present policy comparisons of the proposed stochastic model (SM) and the linear programming formulation of the CEC model (LP). Figure 6.5, Figure 6.6, and Figure 6.7 illustrate the expansion policies of SM and LP for the three different volatility scenarios. In each figure, one demand realization is illustrated with a solid line and available production capacities, which are calculated by SM and LP, are showed with two different dashed lines. In addition, each plot reports the profit that can be achieved by employing each model. Note that the reported profits are only for one realization of a demand and therefore not adequate for comparing the proposed stochastic expansion model and CEC model. In this section, the simulations are based on a setting in which the expansion gap is six periods, diffusion speed is medium, $c_e = \$200$, $c_u = \$30$ and $c_M = \$0.5$.

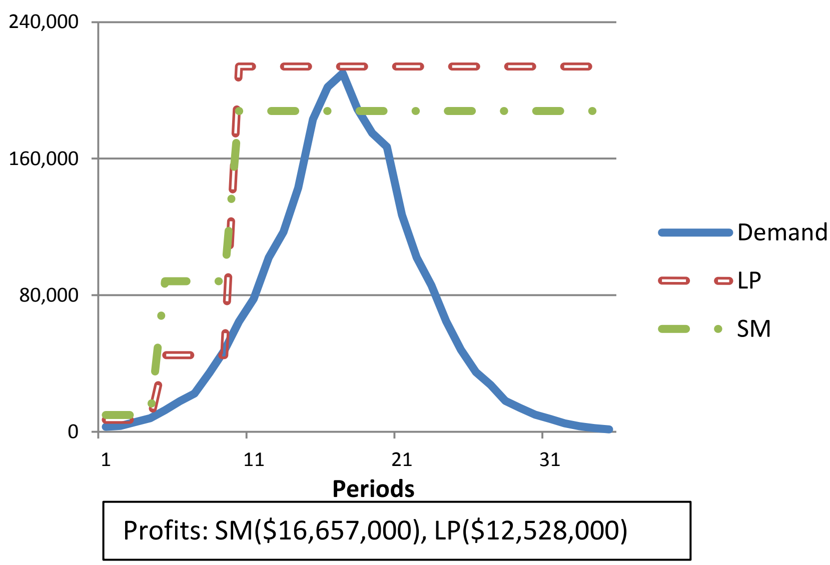

Figure 6.5: Demand and available capacities of SM and LP under low demand volatility

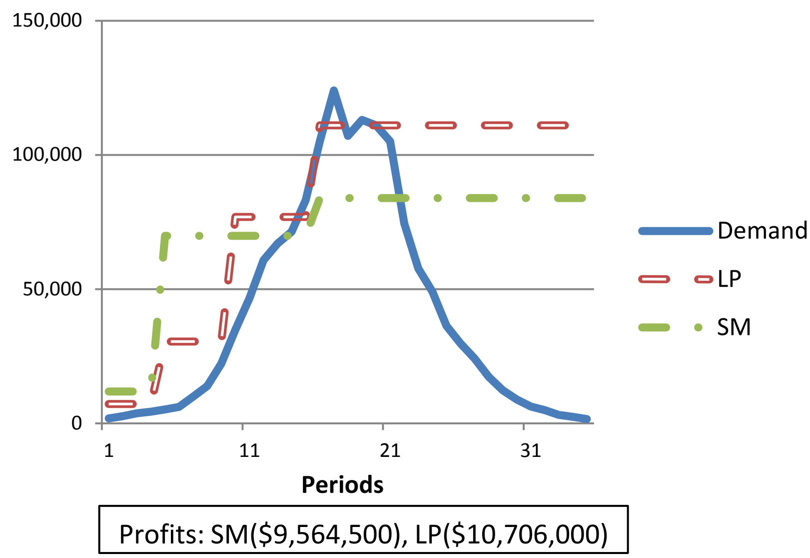

Figure 6.6: Demand and available capacities of SM and LP under medium demand volatility

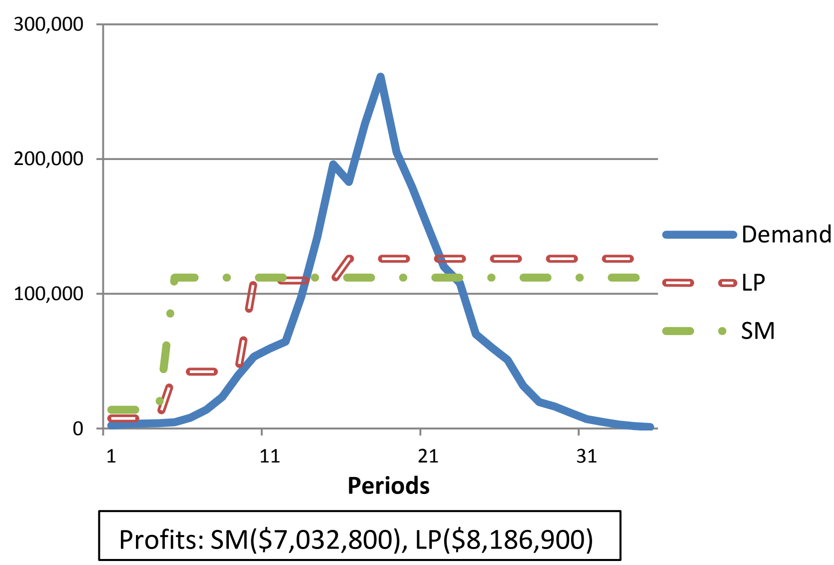

Figure 6.7: Demand and available capacities of SM and LP under high demand volatility

As can be seen from the plots, the expansion policies calculated by the stochastic expansion model (SM) and CEC model (LP) have systematic differences in different stages of a life-cycle. SM is expanding more than LP in the early periods of a life-cycle and LP ends up with higher installed capacity in all three plots. In order to compare the installed capacities in both models, Figure 6.8, Figure 6.9, and Figure 6.10 depict the average installed capacity for each of the three volatility cases. Note that previously we showed the installed capacity in both models for only one realization of demand, but in the following plots (Figure 6.8, Figure 6.9, and Figure 6.10) we are illustrating the average installed capacity based on 5000 realizations of the stochastic Bass model in each volatility case.

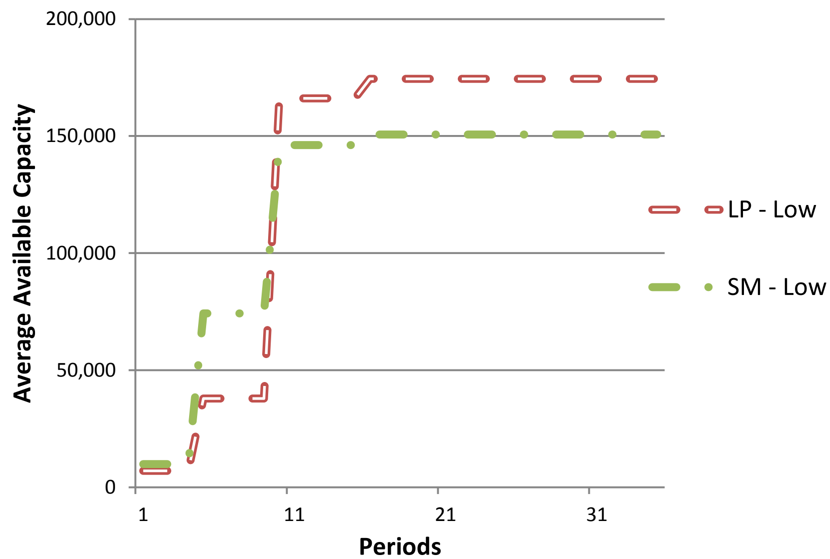

Figure 6.8: Average installed capacity calculated by SM and LP when demand has low volatility

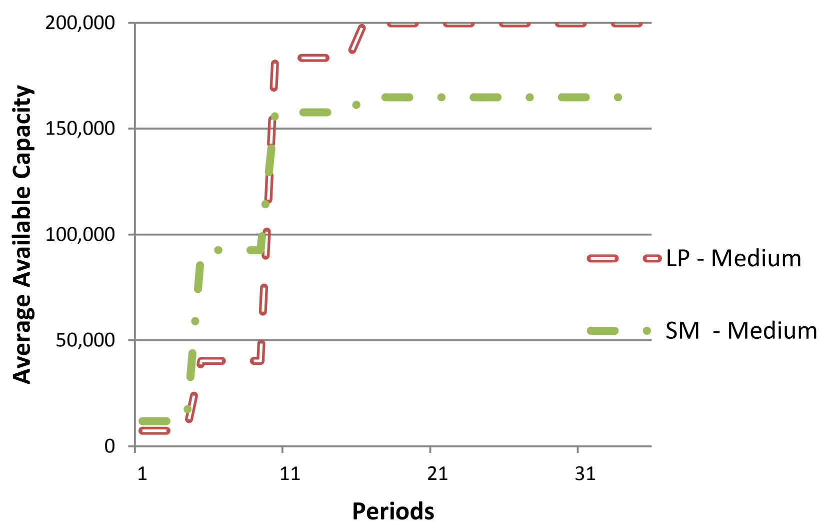

Figure 6.9: Average installed capacity calculated by SM and LP when demand has medium volatility

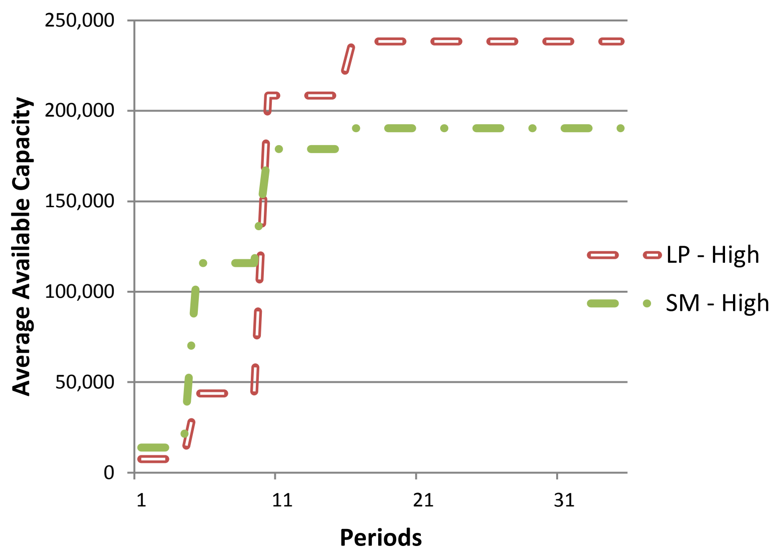

Figure 6.10: Average installed capacity calculated by SM and LP when demand has high volatility

These plots show that our initial observation about installed capacity in these two models is correct, and clearly, the proposed stochastic model installs more capacity at the initial stages of a life-cycle and less towards the end of a life-cycle compared to the deterministic model of CEC.

In order to be able to compare the different policies under different volatility levels, Figure 6.11 and Figure 6.12 present the average differences of the two models through a life-cycle. Figure 6.11 depicts the difference of available capacity in each period of a life-cycle for the three different volatility settings: low, medium and high. Each line in this plot is associated to a volatility level in demand and represents the average of the difference between installed capacity in the proposed stochastic model and the CEC deterministic model (5000 realizations). Figure 6.12 shows this difference relative to the installed capacity of the proposed stochastic model.

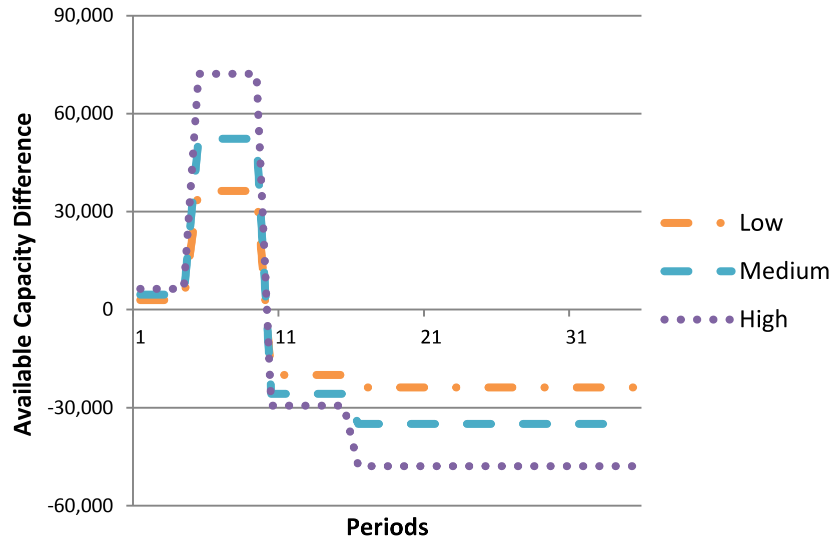

Figure 6.11: Average differences of installed capacity by the two models (SM and LP) through a life-cycle

As it is obvious from Figure 6.11, the proposed stochastic expansion model expands more than CEC model in the first two expansions of the life-cycle (since the difference is positive) and expands less in the last two expansion periods. Moreover, the gap between two models widens by with the level of volatility. Meaning, the difference between the two policies is larger when the demand has higher volatility and is smaller otherwise.

This behavior can be explained by looking at the mechanics behind the proposed stochastic expansion model. In our model, not only one point (expected value in the CEC model) of the demand distributions but also the complete information of the distributions is considered for deriving the optimal expansion policy. This feature leads to a more aggressive policy (comparing to the CEC model) at the initial stages of a life-cycle since the model sees an opportunity of receiving demand volumes more than what it is expected (expected value of the distribution). Note that since expansion decisions are not one-time decisions, this aggressive policy can be converted to a more conservative policy (again comparing to the CEC model) later in a life-cycle. Meaning, the proposed expansion model expands more at the beginning since there is a (significant) chance of receiving more orders than the expected values and considering a fact that, if it would not be the case (receiving orders less than the expected values), the model will have a chance to balance out this aggressive policy with more conservative subsequent expansions.

In the later expansion decisions, however, there will not be any opportunity to adjust expansion decisions and the model adopts a more conservative policy.

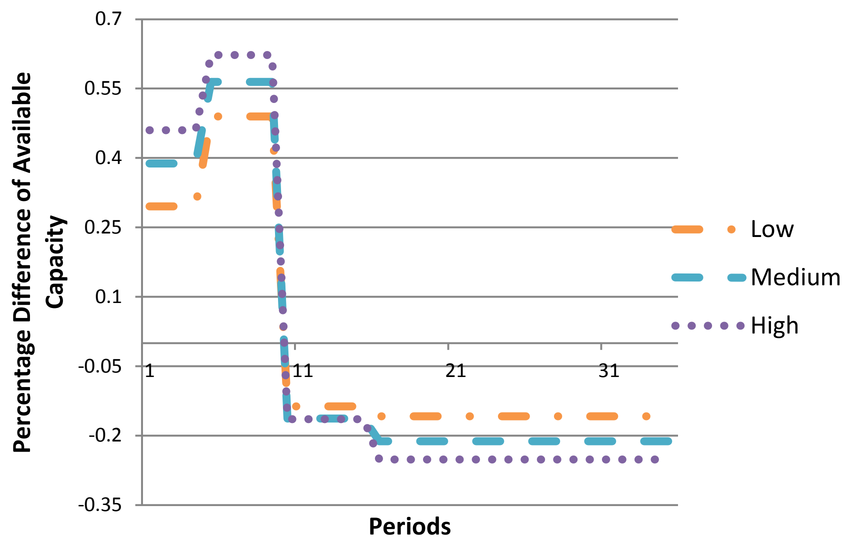

Figure 6.12: Relative average differences of installed capacity by the two models (SM and LP) through a life-cycle

In summary, at early stages of a life-cycle it is more profitable to emphasize on “What if the demand will be more than the expected values?” compared to the final stages at which it is more profitable to emphasize on “What if the demand is less than the expected values?” Note that although different cost structures might change the level of capacity expansion difference between the two models, the direction of the differences is probably valid for most settings.

Another point that is worth mentioning is the variability of expansion decisions. Figure 6.13 depicts the standard deviations of the available capacity in the proposed expansion model and the CEC model. As is clear from the figure, under all levels of volatility (Low, Medium, and High), the CEC model has a larger standard deviation for installed capacities compared to the proposed expansion model. The reason is related to the fact that the CEC model is a "point-based" deterministic optimization model (based on expected values) and any changes in the expected values of future demands can potentially change the optimal solution significantly. In contrast, the proposed expansion model is based on the distributions, and unless the distributions of future demands experience significant shifts, optimal expansion decisions have less variability.

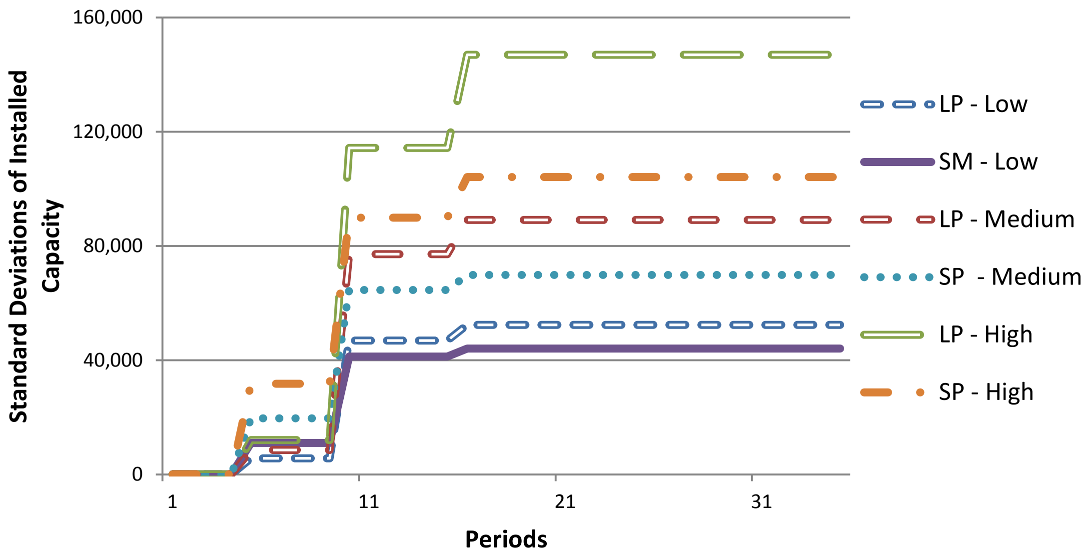

Figure 6.13: Standard Deviations of Installed Capacities in the CEC model (LP) and the stochastic optimal model (SM)

So far we have examined the differences in the policies of CEC model and the proposed stochastic optimal expansion model. In this section, we present the performance (average life-cycle profit) of the proposed model accompanied by the CEC model in order to illustrate the advantages of using the proposed model under different settings

Figure 6.14 and Figure 6.16 show this average for the CEC (dashed lines) and the proposed expansion models (solid lines). Each figure is based on a specific cost structure. In addition, Figure 6.15 and Figure 6.17 show the ratios of the average profits of the CEC model to the average profits of the proposed expansion model. Each of these plots is associated to a cost structure used in Figure 6.14 and Figure 6.16. Note that each point in Figure 6.15 is the ratio of the associated average profit in the CEC model to the associated average profit of the proposed expansion model in Figure 6.14. The same relationship is valid for Figure 6.16 and Figure 6.17.

The setting of Figure 6.14 and Figure 6.15 are based a cost structure in which $c_u = \$30$, $c_e = \$200$, $c_m = \$0.5$ and a lead-time of 4 periods. Figure 6.16 and Figure 6.17 are based on the same setting except underage penalty cost that is $c_u = \$40$. Note that in both settings the gap between expansion decisions is three periods.

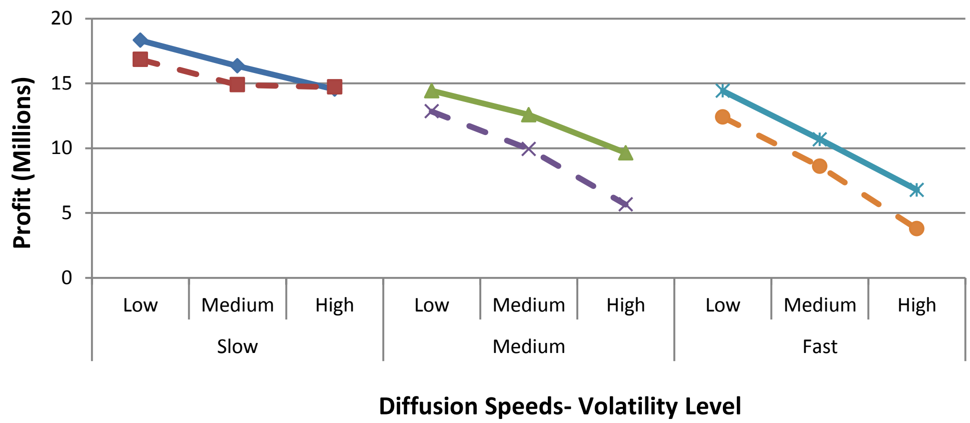

Figure 6.14: Average life-cycle profits for the proposed model (solid line) and CEC (dashed line) when *c_u=\$30, c_e=\$200, c_m=\$0.5* and *L=4*

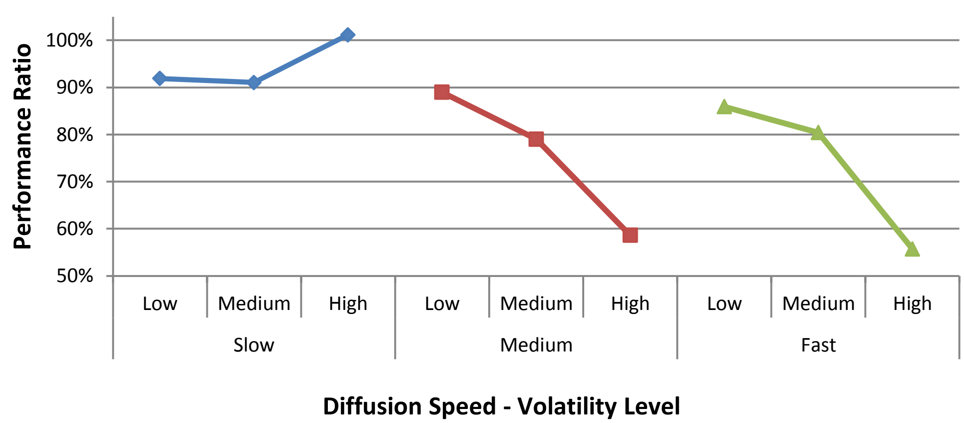

Figure 6.15: Performance ratio of the CEC model in comparison to the proposed model when *c_u=\$30, c_e=\$200, c_m=\$0.5* and *L=4*

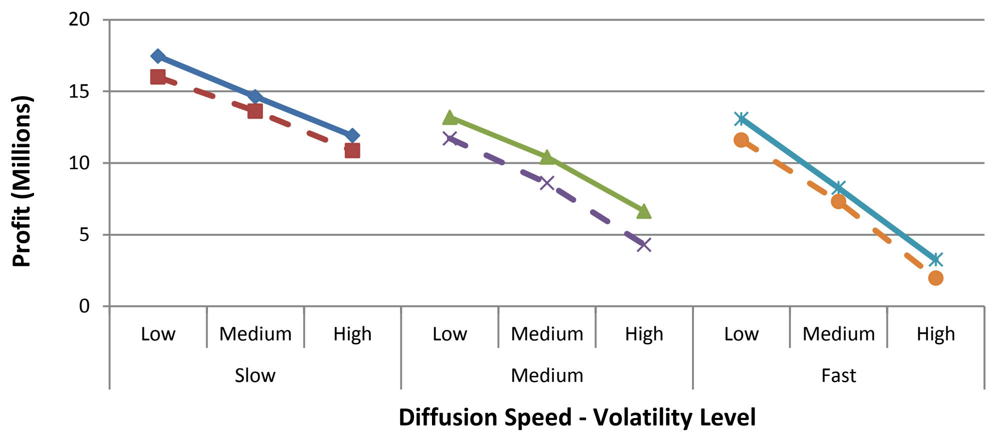

Figure 6.16: Average life-cycle profits comparison of the proposed model (solid line) and CEC (dashed line) when c_u=$40, c_e=$200, c_m=$0.5 and L=4

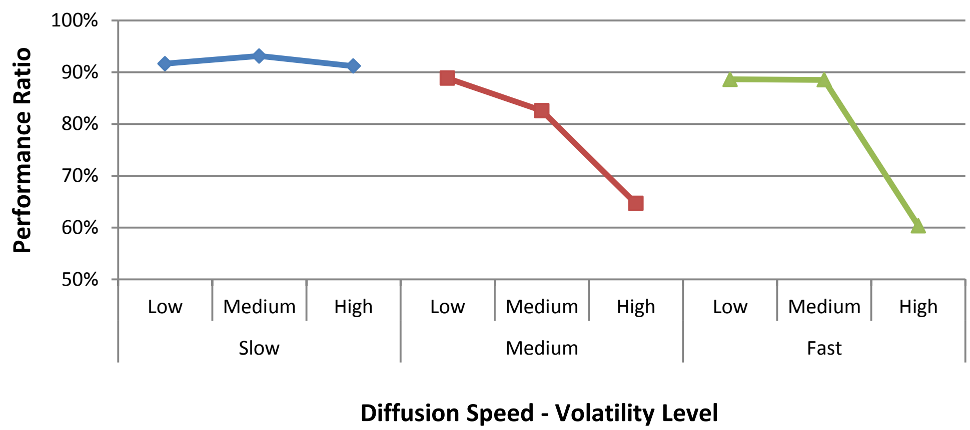

Figure 6.17: Performance ratio of CEC in comparison to the proposed model for the setting with c_u=$40, c_e=$200, c_m=$0.5 and L=4

The first observation in these figures is the effect of volatility on the performance ratios. As discussed in detail, the proposed expansion model considers the distribution of future demands and the CEC formulation is a deterministic optimization model that only considers the expected value of future demands. As a result, the proposed model outperforms the CEC model in any setting, especially when the level of volatility is high. When a product has a high volatility level, deviation from any expected path is more likely. Consequently, the CEC model that uses the expected values of the future demand performs poorly in high volatile environments. However, in the products with a slow diffusion speed, the volatility level has a less pronounced effect on the performance ratio and in some cases (low c_u) the CEC model might even perform slightly better than the proposed expansion model. This result is related to a fact that in the products with a slow diffusion speed, a decision maker has abundant number of opportunities to adjust expansion policies and any non-optimal policy can be improved in future periods. Moreover, in the proposed stochastic Bass model, magnitude of the changes in the market potential decreases through a life-cycle. This property helps the CEC model to pass the very volatile initial stages and, since it is a low diffusion product, to adjust any miscalculation when the market potential reaches a more stable state.

6.4 Dual Sourcing Extension

In Chapter 5, in which we covered the extensions to the proposed expansion model of Chapter 4, three different extensions with necessary algorithms were presented. One of the extensions considers a scenario in which a decision maker has two procurement modes (base and flexible) during a life-cycle and can procure from any of them without any restriction. In this section, we assume that the base mode has a lead-time of four periods with $c_e = \$150$. On the hand, the flexible mode has a one-period lead-time with a marginal expansion cost of \$225 (50 percent more than the base mode). In this section capacity maintenance cost is assumed to be 0.5 dollars per unit per period. Moreover, we concentrate on a case in which a decision maker uses the base mode for the initial capacity procurement (before product launch) and plans to expand capacity only once during a life-cycle, which is on the fourth period. In this section, the ratios of the procured capacity from flexible mode to the total procured capacity will be reported and discussed.

Figure 6.18 shows this ratio for the two different shortage penalty costs combined with different diffusion speeds and volatility levels. In this figure, solid and dashed lines represent those settings in which $c_u = \$40$ and $c_u = \$20$, respectively.

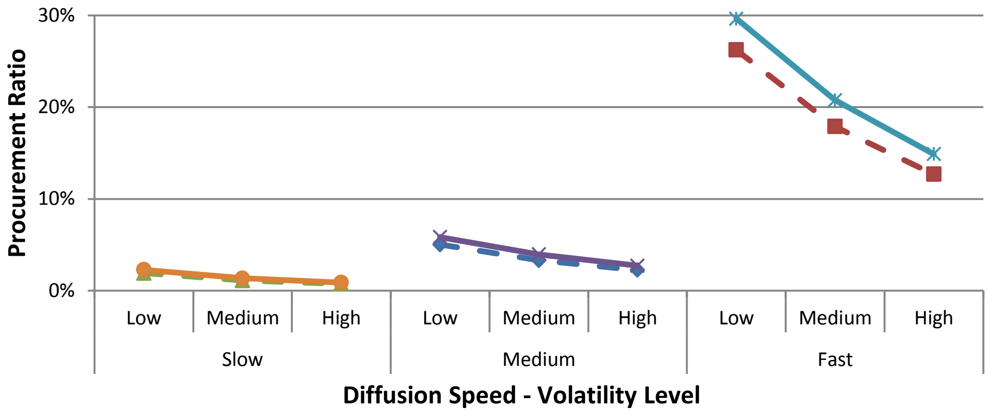

Figure 6.18: Ratio of the procured capacity from flexible mode to the total procured capacity for two shortage penalty costs: $20 (dashed lines) and $40 (solid lines)

The effect of diffusion speed and volatility level on these ratios is very interesting. As it is clear from Figure 6.18, diffusion speed of a product has a direct effect on the amount of procurement capacity from the flexible mode. This intuitive point is related to the fact that a manufacturer with a fast diffusion product receives large volume of demand in a very short time (tall demand curve). As a result, using flexible mode is more profitable for them compared to a case where demand has a flatter curve (slow diffusion speed) and the decision maker has adequate time to use the base (cheaper) mode for capacity procurement.

The other interesting observation is the effect of uncertainty on this ratio. Since the ratios decrease when the volatility levels increase, it is more profitable for a manufacturer in case of volatile environment to procure more from the base mode instead of the flexible mode. This interesting result is the consequence of the fact that decision makers are not willing to bear the extra cost of the flexible mode for a demand that they have less certainty about.

Figure 6.18 also illustrates the effect of increasing shortage penalty cost on the procurement amount from the flexible mode. As expected, a higher stock-out cost increases the ratio and makes it more profitable for a decision maker to procure more from a flexible mode.

On the hand, Figure 6.19 depicts the effect of the marginal expansion cost for the flexible mode on the procurement amounts. In this figure, two marginal expansion costs for the flexible mode are presented: $195 that is 30% more than the base mode (solid lines) and $225 that is 50% more than the base mode (dashed lines).

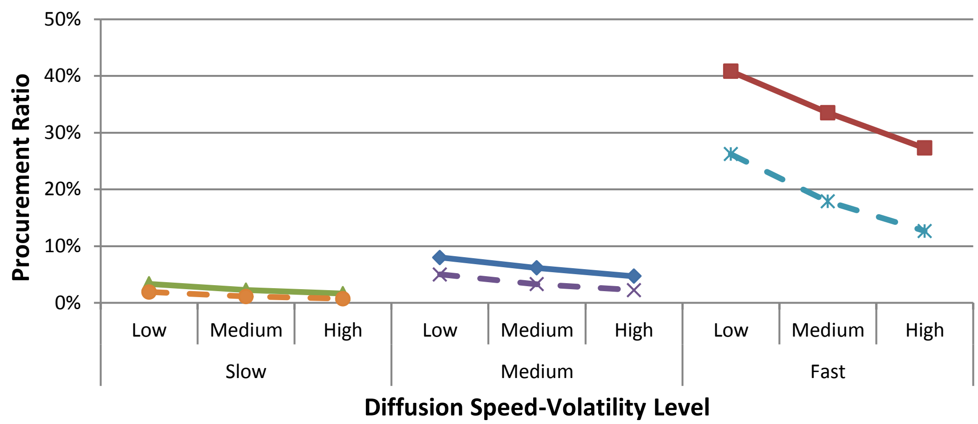

Figure 6.19: Ratio of the procured capacity from flexible mode to the total procured capacity for two marginal expansion costs of flexible mode: $195 (solid lines) and $225 (dashed lines)

Although a cheaper flexible mode increased the amount of procurements from flexible mode in all settings, its effect on a product with a fast diffusion speeds is more profound compared to slow and medium diffusion speeds. Since a decision maker in a fast diffusion product has very short time to recover its investment, it is intuitively reasonable for him to procure more from a fast procurement mode.

Chapter 7 Conclusion and Future Research

In this dissertation, we focus on demand modeling and capacity planning for innovative short life-cycle (SLC) products.

As a first step, we developed a new model in the class of stochastic Bass formulations that addresses the shortcomings of the extant works in the literature. The proposed model considers the common fact that the market potential for a product is not fixed and might change during its life-cycle due to exogenous (e.g., economic- or competitors-related) or endogenous (e.g., quality-related) factors. Allowing this parameter (market potential in the Bass model) to follow a geometric random walk, we have showed that the future demand of a product in each period follows a lognormal distribution with a specific mean and variance.

As a second step, we developed a novel stochastic optimal capacity expansion model that can be used by a make-to-order manufacturer, who faces stochastic stationary/non-stationary demand, in order to optimally determine policies for specifying the size of capacity installation and augmentation. In addition to the cost of expansion decisions, the proposed risk-neutral expansion model considers procurement lead-times, irreversibility of investments, and the costs associated with lost sales and unutilized capacity. We provide necessary and sufficient conditions for the derived optimal policy. We then present an exact solution method, which is more efficient than classical recursive methods.

Additionally, three extensions of the proposed expansion model that can address more complicated settings are presented. The first extension increases the capability of the model in order to tackle capacity planning for a multi-sourcing scenario. Multi-sourcing is a case in which the manufacturer can procure capacity from two supply modes whose marginal expansion costs and lead-times are complementary. The second extension addresses a scenario in which an installed capacity can be used for producing future generations of a product. The last extension accounts for salvage value of the installed capacity in the model and provides the necessary and sufficient conditions for the optimal policy.

Finally, using the proposed stochastic Bass model, we present the results and managerial insights gathered from numerical experiments that have been conducted for the stochastic optimal expansion models. The main insights and results of the simulations can be summarized as follows:

a. ***Effect of product diffusion speed***: In this work, ample number of simulations for three levels of diffusion speed (slow, medium, and fast) have been conducted. Based on the results, it was observed that increasing diffusion speed has a negative impact on the profitability of a product since a decision maker would have less opportunities for adjusting the available capacity in order to respond to demand volatilities. In products with fast diffusion speeds, since the demand curve is relatively peaked, any miscalculation or mistake in an expansion policy cannot be revised and it can lead to more severe consequences. In contrast, in products with a relatively slow diffusion, a manufacturer has more opportunities to adjust the available capacity since the time between launching a product and the period of the peak in a demand is longer and any mistake or miscalculation can be improved and revised in the later stages of a life-cycle.

b. ***Effect of demand volatility***: It is obvious that a manufacturer can achieve a higher profit in a low volatile environment compared to a turbulent market. In case of low volatility, the decision maker can procure enough capacity with a high level of certainty and avoid any costly under- or over- expansions. Contrastingly, in a highly volatile environment, aligning capacity expansions with stochastic demand is more challenging and consequently the chance of misalignment is higher. In addition, we have observed that higher underage costs intensify the effect of volatility levels on the average profit that a manufacturer can achieve.

c. ***Comparison of derived optimal policies and performance of the optimal expansion model with sub-optimal policies recommended by a certainly equivalent controller (CEC) model:*** In this work, we have compared the optimal policies that are derived from the proposed optimal expansion model to a model in which a decision maker converts the stochastic optimization problem to a deterministic one by replacing random variables/parameters by their expected values. Based on the numerical experiments, it was observed that the proposed stochastic expansion model expands more than the CEC model at the initial stages of a life-cycle and expands less in the later periods. The gap between installed capacities of the two models widens by the level of volatility. With respect to the performance (average life-cycle profit) of the two models, it is clear that in most cases the proposed expansion model outperforms the CEC model. The difference between the performances of the two models is more significant for the settings in which demand is highly volatile. The reason is related to the fact that the proposed expansion model considers the distribution of future demands and the CEC formulation is a deterministic optimization model that only considers the expected value of future demands. When a product has a high demand volatility level, deviation from any expected path is more likely. Consequently, the CEC model that uses the expected values of the future demand performs poorly in highly volatile environments. However, under scenarios with extreme volatility, it was observed that the performances of the both models are very close to each other.

d. ***Supplier mode selection and procurement mix:*** We have studied the supply relationship management from two different perspectives. In the first case, we address a problem in which the manufacturer is involved in negotiating and selecting the best procurement option (based on cost and lead time) among a menu of options that are provided by a supplier. This selection and negotiation process happens before launching a product and the selected option cannot be changed during a life-cycle. In the second case, we investigate the optimal procurement mix through a life-cycle when a decision maker has two procurement options (flexible option that is fast but more expensive and base case option that is slow but less expensive).

i. ***Case 1 (supply option selection)***: In the numerical experiments, we have studied the tradeoff between marginal capacity cost and procurement lead-time and investigated the superiority of one procurement option to another under different settings. Note that in this setting we assume that a procurement option should be selected before launching the product and the manufacturer is not able to switch to another one during a life-cycle. Based on the results, we have showed that if the manufacturer has two procurement options (before launching), fast-but-expensive and slow-but-cheap, s/he has more incentive to select the faster option when facing a higher volatile environment with a product that has a faster diffusion speed. In products with fast diffusion speeds, decision makers have very few opportunities to expand capacity, and, as a result, faster procurement options are more valuable to them. In addition, when the volatility is high, capacity planning is riskier since any initial demand estimations might be completely unreliable. In this case, fast-but-expensive option gives the manufacturer an ability to postpone the procurement as much as possible. The extra cost that the manufacturer has to pay for the faster delivery can be considered the cost of "postponement option." Additionally, we also have observed that higher marginal shortage penalty cost makes fast-but-expensive option more advantageous.

ii. ***Case 2 (Procurement mix: dual mode sourcing):*** Finally, we have studied the dual-mode sourcing option in which a decision maker can procure from two procurement modes with complementary marginal expansion costs and lead-times (flexible option that is fast but more expensive and base case option that is slow but less expensive). Using an extension of the proposed expansion model, we investigate the optimal mix of capacity procurement from the two supply modes under different settings. Based on the numerical experiments, it was shown that diffusion speed of a product has a direct effect on the amount of procured capacity from the flexible mode. This intuitive result is related to the fact that the manufacturer with a fast diffusion product receives large volume of demand in a very short time. As a result, using flexible (faster) mode is more profitable for them compared to a case where the demand profile has a flatter curve (slow diffusion speed) and the decision maker has adequate time to use the base (cheaper) mode for capacity procurement. The other interesting observation is the effect of uncertainty on procurement mix. We have observed that, in cases of a highly volatile environment, it is more profitable for a manufacturer to procure more from the base mode instead of the flexible mode. This interesting result is the consequence of the fact that decision makers are not willing to bear the extra cost of the flexible mode for a demand that they have less certainty about. Moreover, as expected, a higher marginal shortage penalty cost makes it more profitable for a decision maker to rely more on the flexible mode.

7.1 Future Research

In this section, we identify promising directions for future research for the two main models of this work: The stochastic Bass diffusion model and the proposed stochastic optimal expansion model. In the proposed stochastic Bass model, changes in market potential are completely random and occurrence probability of different exogenous and endogenous factors cannot affect these changes. A very interesting future research might be a Markovian switching model that can provide a framework for a decision maker to define different regimes and their probabilities for the exogenous and endogenous factors. These regimes can have different volatility levels for market potential. Moreover, due to the lack of access to relevant demand data, we have not examined the prediction power of the proposed model on a real-world product. It will be interesting if this test would be conducted for different industries and macroeconomic circumstances.

With respect to the optimal expansion model, we have not considered possible restrictions of suppliers' capacity for the fulfillments of a manufacturer's orders. However, suppliers' production capacity is not unlimited and in most cases the manufacturer has to bear a part of this investment cost, e.g. capacity reservation fee. One possible future research direction would be addressing this issue by including suppliers' capacity restriction and reservation fees in the optimal expansion model.

Additionally, we have assumed that a manufacturer, who uses the proposed stochastic optimal expansion model, is not able to store any product (a make-to-order manufacturer) and as a result it does not produce more than demand at any period. One extension to this model can be relaxing this restriction and enhancing the capabilities of the model by considering possibility of storage.

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ABSTRACT

DEMAND MODELING AND CAPACITY PLANNING FOR INNOVATIVE SHORT-LIFE CYCLE PRODUCTS

by

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Major: Industrial Engineering

Degree: Doctor of Philosophy

This dissertation focuses on demand modeling and capacity planning for innovative short life-cycle products. We first developed a new model in the class of stochastic Bass formulations that addresses the shortcomings of models from the extant literature. The proposed model considers the common fact that the market potential of a product is not fixed and might change during a life-cycle due to exogenous (e.g., economic- or competitors-related) or endogenous (e.g., quality-related) factors. Allowing this parameter (market potential in the Bass model) to follow a geometric random walk, we have showed that the future demand of a product in each period follows a lognormal distribution with specific mean and variance.

We also developed a novel stochastic capacity expansion model that can be used by a make-to-order manufacturer, who faces stochastic stationary/non-stationary demand, in order to optimally determine policies for specifying the sizes of capacity procurement. In addition to the cost of expansion decisions, the proposed risk-neutral expansion model considers procurement lead-times, irreversibility of investments, and the costs associated with lost sales and unutilized capacity. We provide necessary and sufficient conditions for the derived optimal policy. We then present an exact solution method, which is more efficient than classical recursive methods.

Additionally, three extensions of the proposed expansion model that can address more complicated settings are presented. The first extension increases the capability of the model in order to tackle capacity planning for a multi-sourcing scenario. Multi-sourcing is a case in which the manufacturer can procure capacity from two supply modes whose marginal expansion costs and lead-times are complementary. The second extension addresses a scenario in which an installed capacity can be used for producing future generations of a product. The last extension accounts for salvage value of the installed capacity in the model and provides the necessary and sufficient conditions for the optimal policy.

Finally, using the proposed stochastic Bass model, we present the results and managerial insights gathered from numerical experiments that have been conducted for the stochastic capacity expansion models.

AUTOBIOGRAPHICAL STATEMENT

Saman Alaniazar holds B.Sc. in Industrial Management from Alameh Tabatabaie University (Iran) and M.Sc. in Information Technology Management from University of Tehran (Iran). Since 2007, Saman has been studying at Wayne State University (WSU) for a Master's degree in Applied Mathematics (2011) and a PhD degree in Industrial Engineering. During this period, he has worked as a graduate research assistant in the Industrial and Systems Engineering department at WSU.

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Saman's main research interests lie in supply chain management in the areas of operational and financial risk management. In particular, he studied risk management and forecasting models for manufacturers that produce short life-cycle products.