1-1-2012

An Integrated Framework For Configurable Product Assortment Planning

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AN INTEGRATED FRAMEWORK FOR CONFIGURABLE PRODUCT ASSORTMENT PLANNING

by

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DISSERTATION

Submitted to the Graduate School

of Wayne State University,

Detroit, Michigan

in partial fulfillment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

2013

MAJOR: INDUSTRIAL ENGINEERING

Approved by:

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Advisor Date

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DEDICATION

To my family, for all their love and support.

ACKNOWLEDGMENTS

I would like to thank, first and foremost, my advisor, Prof. Ratna Babu Chinnam, for all his continuous support and guidance throughout my Ph.D. His inspiring, and thought provoking directions have significantly improved my academic self. It would not have been possible to complete this Ph.D. without him. I would also like to extend a thank you to Dr. Alper Murat and Dr. Evrim Dalkiran and Dean Pichette for great recommendations and valuable support during my dissertation. I also have a special thank you to Gintaras Puskorius, for his precious assistance and advice during different steps of this work. His insightful comments about my dissertation helped me a lot to shape my research. My gratitude also goes to the other members of my dissertation committee: Dr. Leslie Monplaisir and Dr. Boris Mordukhovich, who served as a strong force of inspiration and encouragement. Last, but not least, I would like to thank my family, in particular my parents, Nematolah and Zahra, who have given me their unconditional love and support. Without them, I would have quit long ago. I also cannot express the level of gratitude and love that I have for my wife, Maryam. Without her encouragement and support this dissertation would never have been done.

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Chapter 1. INTRODUCTION

A manufacturer's assortment is the set of products or product configurations that the company builds and offers to its customers. Assortment planning is a critical strategic decision making process for it considerably affects both the company's sales revenue as well as the cost of its product offering. Over the last hundred years, there have been quite a few different approaches toward planning automotive product assortments, the focus of our dissertation. For example, Henry Ford of Ford Motor Company is famous for offering a single product configuration of the famous Model T. However, over the years, the product offerings and configurations have steadily grown in the U.S. until recent years. Increasing the number of configurations offered to customers makes the manufacturing process more complex and will affect the plant's productivity. Thus, some of the large auto manufacturers have been switching back to control their variety to decrease their operational costs while maintaining their sales and market shares. Variety reduction has been followed by U.S. automakers both in terms of reducing models and cutting the option combinations. For example, Ford Motor Company reduced the ordering complexity of the 2009 F-150 truck by more than 90 percent. (Automotive News. August 18, 2008). There are also studies indicating that a large product variety offering does not necessarily lead to higher sales, even for popular vehicles (see Pil and Holweg, 2004). While there are a number of factors that influence sales, besides product variety, overall, it appears that automakers aren't necessarily driving their strategic decisions regarding product variety based on objective models.

In the past decades, there has been considerable work dedicated to demand aspects of assortment planning (see Kök et al. 2008, for a literature review). Furthermore, some researchers have been interested to study the effect of variety on manufacturing performance (MacDuffie et al. 1996) or its effect on assembly operations (Fisher and Ittner, 1999). The link between product variety and distribution (Zhang et al. 2007) has been studied as well. But very little research has been done that integrally considers demand and supply/manufacturing aspects in planning product assortment for configurable products. It should be noted that most of the studies on assortment planning are developed for non-configurable (like grocery and retail) products (van Ryzin and Mahajan, 1999; Smith and Agrawal, 2000; Gaur and Honhon, 2005; Kok and Fisher, 2007). However, industries involved with configurable products, such as automotive and computer industry require a specific approach to assortment planning that better suits their product line features.

1.1. Research Objective

The primary objective of this research is to develop an integrated framework for assortment planning of configurable products. In doing so, our aim is to develop models that explicitly account for supply, demand, and manufacturing considerations. We mostly focus on the highly complex automotive products to test and validate our models. The research also aims to tackle the related problem of powertrain technology selection in planning assortments, in particular, to support the OEM in addressing federal regulations on vehicle fuel economy expressed through Corporate Average Fuel Economy (CAFE) standards by the U.S. National Highway Traffic Safety Administration (NHTSA) and the U.S. Environmental Pollution Authority (EPA)[1]. Finally, the research also aims to investigate the impact of option/feature bundling in planning product assortments.

[1] See NHTSA Website for more information: http://www.nhtsa.gov/fuel-economy

1.2. Dissertation Organization

In the second chapter, we mainly focus on planning product assortment for a manufacturer of configurable products. We use an exogenous demand model that accounts for product substitution (both assortment based and stock-out based). We also account for product unit cost and overage cost per configuration as well as manufacturing complexity cost and assume that the OEM receives discounts on economies-of-scale. The resulting model is a mixed-integer-nonlinear program that could only be solved for very small size problems and fail to solve real world models. We propose a novel method that uses alternate lower/upper bounds through a modified Branch and Bound procedure for solving the original problem. We compare the results by Bonmin solver and show that (overall) our method is capable of finding better quality solution is a shorter time.

In the third chapter, we keep our attention on assortment planning of automotive products through incorporating environmental issues into decision making. We study the effect of CAFE requirements and carbon footprint on the optimal assortment under different fuel price scenarios. We show that under environmental restrictions, some of the fuel-efficient configurations take a higher share in the fleet compared to conventional vehicles while some other will not be profitable due to small profit margins.

In the fourth chapter, we study feature packaging in assortment planning with a numerical experiment presented from automotive industry. The study entertains a number of packaging rules in order to present product configurations under a a number of series. We show that feature packaging is a powerful tool in reducing product assortment complexity.

Chapter 2. ASSORTMENT PLANNING FOR CONFIGURABLE PRODUCTS

2.1. Introduction

A manufacturer's assortment is the set of products or product configurations that the company builds and offers to its customers. Assortment planning is a critical strategic decision making process for it considerably affects both the company's sales revenue as well as the cost of its product offering. From the marketing perspective, the company seeks to diversify the set of offered configurations in order to match best the customers' preferences. However, this increase in variety leads to a more complex supply and manufacturing system. On the other hand, from the operational point of view, the company seeks to offer fewer configurations to manage manufacturing complexity and benefit from the economies of scale. Thus, "coping with product variety forces a manufacturing firm to confront a fundamental tradeoff: the increased revenue that can result from more variety verses increased costs through the loss of scale economies" (MacDuffie et al. 1996). Pil and Howleg (2004) also emphasize that reducing or delaying variety decreases manufacturing and logistics costs but might also affect design costs and reduce revenue by limiting the offerings in the marketplace.

In the automotive industry, the pioneers decided to limit product variety to manage manufacturing complexity. An extreme example is the case of Henry Ford of Ford Motor Company suggesting and offering a single product configuration (in black) of the famous Model T. However, over the years, the product offerings and configurations have steadily grown in the U.S. until recent years. For example, the number of car models increased from 30 models in 1955 to 84 models in 1973 and then to 142 models in 1989. However, during the same period (1955 to 1989) the average annual sales per model have dropped from 259,000 units to 112,000 units per model (Womack et al. 1990). Increasing the number of configurations offered to customers makes the manufacturing process more complex and affects the productivity of the plant. There are similar consequences for dealers, unable to hold product inventory that is representative of the number of orderable configurations. Thus, some of the major automotive companies are pulling back on variety to decrease their operational costs while maintaining their sales and market shares. “When decisions about variety strategy have needed to be made, the big three auto-makers in U.S. have increasingly chose to reduce variety, partly as a consequence of their determined drive to match or exceed Japanese level of productivity and quality” (Bowman & Kogut, 1995). Variety reduction has been followed by U.S. automakers both in terms of reducing models and cutting the option combinations. In the last couple of years, some automotive companies have started aggressive campaigns for reducing the number of product configurations. For example, Ford Motor Company reduced the ordering complexity (i.e., number of orderable configurations) of the 2009 F-150 truck by more than 90%. As for cars, it planned for the 2010 Ford Focus to have just 150 major combinations, a drop of 95% from the 2008 model. Furthermore, Ford Motor Company announced that most car lines will now have fewer than 1,000 combinations (Automotive News. August 18, 2008). There are also studies suggesting that a large product variety offering does not necessarily lead to higher sales, even for popular vehicles. For example, Pil and Holweg (2004) report the total number of variations offered by key European, American and Japanese automakers for their two best selling products or models in Europe in 2002. The data does not show any considerable correlation between the total number of variations offered and the total sales in Europe. As an example, in the low variety models, they report that Nissan Micra with barely 676 variations achieving sales of 106k sales and Peugeot 206 with 1739 variations with sales of 596k. On the other side, they report that

Fiat Stilo with more than 10 billion offered variations had sales of 173k. Overall, they report that the top automakers in the European market have totally different product variety while not being consistently different in sales. This suggests that while there are a number of factors that influence sales, besides product variety (e.g., product quality, value, brand image), overall, it appears that automakers aren't necessarily driving their strategic decisions regarding product variety based on objective models.

In the past, there has been considerable work dedicated to assortment planning (see Kök et al. 2008, for a literature review). Furthermore, some researchers have been interested to study the effect of variety on manufacturing performance (MacDuffie et al. 1996) or its effect on assembly operations (Fisher and Ittner, 1999). But very little research has been done that integrally considers demand, supply, and manufacturing complexity aspects in planning product assortment, in particular, for configurable products. It should be noted that most of the studies on assortment planning are developed for non-configurable (like grocery and retail) products (van Ryzin and Mahajan, 1999; Smith and Agrawal, 2000; Agrawal and Smith, 2003; Gaur and Honhon, 2006; Maddah and Bish, 2007; Kok and Fisher, 2007; Fisher and Vaidyanathan, 2009). However, industries involved with configurable products, such as automotive and computer industry require a specific approach to assortment planning that better suits their product line features. In this paper, we propose a framework for assortment planning of configurable products with explicit consideration of demand and substitution effects as well as variety-driven costs. We propose a new formulation to calculate stock-out rates while considering manufacturing complexity costs and economies of scales for supplying parts and options. The rest of the paper is organized as follows: section 2 provides a review on most relevant papers in the literature. In section 3 we present problem assumptions and model notation. In section 4 we propose a modified branch and bound procedure to solve MINLP problem through alternate lower/upper MILP problems. Finally the results are presented in section 5.

2.2. Literature Review

In the literature, we face three common approaches to modeling demand: *utility based models*, *locational choice models*, and *exogenous demand models (*Kok et al, 2008). *Utility based models* are more structured and easier to incorporate marketing variables like promotions; however, they usually suffer from the independence of irrelevant attributes (IIA) property (Kok and Fisher 2007). Binary logit, multinomial probit, nested logit, and multinomial logit (MNL) models are some of the utility based models. van Ryzin and Mahajan (1999) formulate an assortment planning problem using the MNL demand model. Maddah and Bish (2004) extended the van Ryzin and Mahajan (1999) model by considering the pricing decisions. Cachon et al. (2005) also made an extension to the model of van Ryzin and Mahajan (1999) by considering consumer search effort. In locational choice models, every customer has one most preferred configuration and she chooses the product that is closest to her ideal preference. Unlike MNL type utility based models, these models allow for different substitution rates even when the initial demand rates are equal. This model has received an increasing attention by researchers in recent years (Gaur and Honhon 2006; Fisher and Vaidyanathan 2009). Finally, e*xogenous demand models* offer more degrees of freedom than the utility based or locational choice models and can accommodate extremely flexible substitution structures (Fisher and Vaidyanathan 2009). Smith and Agrawal (2000) as well as Kok and Fisher (2007) study assortment planning problem under exogenous demand model. In this paper, we employ the exogenous demand model for its flexibility, even though it leads to a more complex formulation for assortment optimization.

In the literature, van Ryzin and Mahajan (1999) were the first to study assortment planning and inventory decisions by using a MNL model of consumer choice. They assume that each product variant carried in the assortment has an identical unit cost, c, and is offered at an identical price, p. They also assume that customers make their choice (if any) when they observe it and they don't switch to a second product if their choice is stocked out. Later on, Mahajan and van Ryzin (2001) study the same problem under dynamic substitution where customers would substitute to their next preferences when the first choice is stocked out. Smith and Agrawal (2000) study assortment planning problem with exogenous demand model and solve an inventory optimization problem that both selects items to stock and the stock levels for each item in the assortment of non-configurable products. They consider inventory policies that reinitialize at the start of each cycle. They also assume a fixed cost for offering each product in the assortment. Our study has some similarities with their work. However, they present an approximation for the demand probabilities corresponding to each product and use the proposed approximations in the assortment planning. Furthermore, in order to solve the assortment optimization problem, they propose solving the problem via total enumeration for small assortments (size n) and suggest a linear approximation for the mixed-integer nonlinear program (MINLP) when the problem size increases to more than 10 items in the assortment. Another study on assortment planning with exogenous demand model is carried by Kök and Fisher (2007) in which they formulate a problem in the context of a supermarket chain. They first show a procedure for estimating the parameters of substitution behavior and demand for the stores' products. Then, they propose a heuristic to solve the assortment planning and inventory problem with one-level stock-out based substitution in the presence of shelf-space constraints. To solve the assortment planning problem, they estimate G_j, the (long run) average gross profit from product j by simulating the replenishment of product j in isolation from other products. Yunes et al. (2007) formulate a method for John Deere & Company to reduce the number of configurations from their product lines without sacrificing profits using a migration list which is a set of acceptable configurations sorted in decreasing order of preference. They initially build and cost out feasible configurations, then, for every customer they build a set of acceptable configurations, sorted in decreasing order of preference, termed migration lists. When the first configuration on a customer's list is not available, she will buy the next available configuration. Finally they propose an optimization procedure to select the products to be offered in the assortment.

Another approach to assortment planning is done through considering locational choice models to estimate demand and substitution behavior. Gaur and Honhon (2006) consider a single-period assortment planning and inventory management problem for a retailer. They represent the consumer choice model with a locational choice model and assume that products are sold at identical exogenous prices and have identical costs due to homogenous quality. They use a newsvendor model with lost sales for excess demand and the excess inventory is salvaged. Later, Honhon, Gaur, and Seshadri (2010) propose an algorithm to determine the optimal assortment and inventory levels under stock-out based substitution for a single period problem assuming that each customer type has a specific preference ordering amongst products and chooses the product with the highest rank according to his type (if any) which is available at the time of purchase. They solve the problem using a dynamic programming formulation. Fisher and Vaidyanathan (2009) propose an algorithm for retail assortment optimization. Their problem is to choose K SKUs from a set of N potential SKUs in a retail category. Although they consider the effect of substitution in the revenue term, our problem will be different from theirs in terms of estimating the demand with an exogenous model as well as considering the effect of variety on the manufacturing and distribution costs.

There are other works on assortment planning that entertain additional decision variables, mainly price, into the modeling. Maddah and Bish (2007) consider a set of product variants differentiated by some attributes such as color, flavor, or size and present a formulation which jointly determines the set of variants to include in the assortment, together with their prices and inventory levels. They use a MNL choice model to represent the consumer choice process and model the inventory setting with a newsvendor formulation. Schon (2010b) formulates an optimal "product-line design" algorithm that jointly addresses assortment and pricing decisions. In her work, the consumers choose among available products according to a general attraction choice model including the MNL, the Brudley-Terry-Luce (BTL), and approximately the first choice model. She assumes that every product uses some resources and using any resource has a specific fixed cost. She proposes a method to transform the standard MINLP formulation of "product line design" problem into a convex mixed-integer program (MIP) and reports solving large problems with thousands of products in a reasonable time. However, our problem will be different than hers in terms of more detailed treatment of variety driven costs, using an exogenous demand model, and explicit consideration of the effect of stock-out based substitution.

2.3. Model Formulation

Suppose that for the product under consideration, $N = (1,2,\cdots,n\}$ denotes the set of potential configurations that can be made available by the original equipment manufacturer (OEM). Assortment planning involves selecting a subset of these configurations for tooling the assembly line and readying the dealers/retailers which maximizes the OEM's profit. We present the major assumptions in this paper as follows:

Market Regions: We assume that the total market can be split into different regions, and $R = (1,2,\cdots,r\}$ denotes the set of regions in the market. Each region could have a particular demand and substitution behavior and the model would determine the optimal product assortment for each specific region.

Discrete Choice: In each region, every potential customer has a favorite or most preferred configuration from the set N.

Demand: The potential demand for each configuration $i \in N$, in each region $r \in R$, d_i^r, is an allocated and known fraction of total market demand, M. We believe that the fractional allocation model is an acceptable approach for modeling demand since some major automakers, including Toyota, have adopted this method for product-mix planning (Iyer et al., 2009).

Demand Uncertainty: The total market demand potential for the particular product under consideration, M, which accounts for competitive products and market conditions, is a discrete random variable with known probability distribution.

Substitution: In case of mismatch between demand and supply at each region r, if the customer cannot find her favorite configuration i, she will decide not to substitute with probability δ_i^r. Else, she will choose configuration j with probability α_{ij}^r. Note that every region has a specific assortment available to customers (not necessarily equal to other region's assortments) and the customers in a region would only substitute to available configurations across the region (and not between regions) in case their favorite configuration stocks out. This assumption is in accordance with dealer transshipments that stand for a considerable proportion (up to 30%) of total sales at some dealers. The substitution probabilities are assumed to be exogenous to our model and are flexibly allowed to take any structure. However, for our numerical experiments and without loss of generality, we consider substitution probabilities to be derived based on price and content similarities between configurations, which is similar to the $a\ priori\ substitutability$ concept discussed by Vaagen et al (2011). Substitution can be broadly divided into *assortment based substitution* (i.e., customer substitutes because preferred configuration is not part of the offered assortment; cannot be ordered either) and *stock-out based substitution* (customer substitutes because the preferred configuration is temporarily out of stock).

Single-Shot Supply: Given the strategic nature of assortment planning, we believe it is reasonable to model the problem as a single-period single-shot supply problem (model assumes no replenishments). This is consistent with the approach taken by others such as Gaur and Honhon (2006).

Costs: Similar to Schon (2011b) that considers a fixed cost for using resources related to products, we consider fixed cost $c_p^{fixed\_part}$ associated with offering part/option p, which accounts for one time costs (share of program design and integration costs in product development phase, validating the production process, warranty, tooling, etc.). Besides, we consider fixed cost $c_j^{fix\_conf}$ associated with introducing configuration j in the assortment, which could be incurred due to several reasons including: integration costs (especially for configurations not built yet) as well as quality and reliability testing, and tooling costs. Note that there are some overhead costs independent of the assortment, like management compensation and staff wages, electricity and heating, costs of leasing or owning the facility, etc. which can be excluded from our model as they will not affect the optimal solution. If they do affect the optimal assortment, the proposed formulation has the flexibility to account for such costs.

We also assume that each configuration j has a manufacturing unit cost c_j^{unit} and that the prices for each product configuration, p_j, are set exogenously and are available $a\ priori$ for product assortment planning. Some papers assume identical variable costs and prices for the product variants (van Ryzin and Mahajan 1999; Gaur and Honhon 2006). However, in the automotive industry, configurations are typically priced differently as a function of the options content. Note that unit cost for configuration j mainly stems from material and labor costs per vehicle. The material cost can be calculated as summation of all the costs related to materials for common parts (like chassis, electrical wiring, etc.) as well as specific parts/options (like power train, seats, sunroof, etc.) used in building the configuration.

Besides fixed and unit costs associated with configurations and contents, we consider another cost factor in our model, the cost of product assortment complexity. The complexity could incur additional costs due to the additional flexibility necessary to support the larger number of configurations in production facilities (e.g., less efficiency in balancing the assembly line and increased station cycle times) and/or to support the larger number of potential choices for parts/options (more manufacturing facilities, more experienced labor force, more inventory storage, more inbound logistics cost, etc.). Based on discussions with couple of Subject Expert Matters (SMEs) from the automotive industry, we also assume that complexity cost is a concave non-decreasing function with regard to size of the assortment.

Finally, we assume that the cost of supplying different parts/options from the suppliers to OEM may be affected by the volume (i.e., economies of scale exist). That is, the company would receive discounts on some option content if purchased in large quantities. The information related to discount policies is exogenous to our model. We assume that the supplier of a specific part/option offers its product under a step-wise non-increasing price and the OEM does not incur any additional cost other than unit purchasing price due to manufacturing or complexity costs of the supplier.

2.4. Modeling Demand

A critical concern for modeling demand in the presence of substitution is to check the significance of stock-out based substitution. Given that vast majority of the customers in the U.S. market purchase vehicles from dealer stock (over 90% according to a major OEM) and not order custom vehicles (which is more common in Europe) and given that most dealers carry very limited stock (tens of vehicles to hundreds at best for any given model) in comparison with the possible number of orderable configurations (that can run into tens to hundreds of thousands and even millions), it is expected that there is significant substitution by customers between configurations in purchasing product. This is not the case with most retail goods at grocery stores and merchandise retailers, where the stock-out rates are typically in the order of few percent (e.g., Kök and Fisher (2007) report the stock-out rates for some product segments in a European grocery chain to be 99%). To this end, we study real data from a collaborating automaker in North America and show that stock-out based substitution is considerably high and should not be ignored. Although there is almost no way for automotive industry to capture the first preference of each customer, it is possible to observe and analyze inventory and sales transactions for an automaker. Using real data from our collaborator for a specific model covering some 9 months of history, we observed that a total of 65,000 vehicles were built and sent to dealers (the dealer order guide can support hundreds of thousands to millions of possible orderable configurations when we account for exterior color and interior trim choices, not counting the dealer installed accessories). Of these vehicles, there exist almost 25,000 distinct configurations in the whole US market. We note that the number of distinct configurations at a dealer varies from one dealer to another, but barely exceeds 200 configurations even at the largest dealers in the nation. The OEM models the U.S. market as roughly 16 distinct "regions" and we studied daily inventory for six different regions. Two of these regions (New York and Detroit) were supplied more than 19,000 vehicles during the 9 month period, representing almost 30% of the total market supply. Daily information regarding stock rates (which we define as total number of available distinct configurations in the "entire region" divided by total number of actual configurations built for the entire U.S. market over the 9 month history) in these six regions are shown in Figure 1.1. The plot reveals that during any day from the studied time period, stock rates are usually less than 10% for New York and Detroit area while for Chicago, Orlando, and Seattle areas the stock rates are no more than 6%. These results suggest that even when we consider a whole region and allow the possibility of dealers exchanging vehicles (so called "transshipments"), configuration stock rates are very low, which suggests that there is a high rate of stock-out and substitution. What we learnt from our OEM collaborator is that no more than 30% of the sales involve transshipments. Given that a small fraction of orderable configurations are actually built and that only a small fraction of these built configurations are carried in any region, and an even tinier fraction carried by the dealer, stock-out based substitution needs careful consideration during assortment planning, at least in the U.S. market.

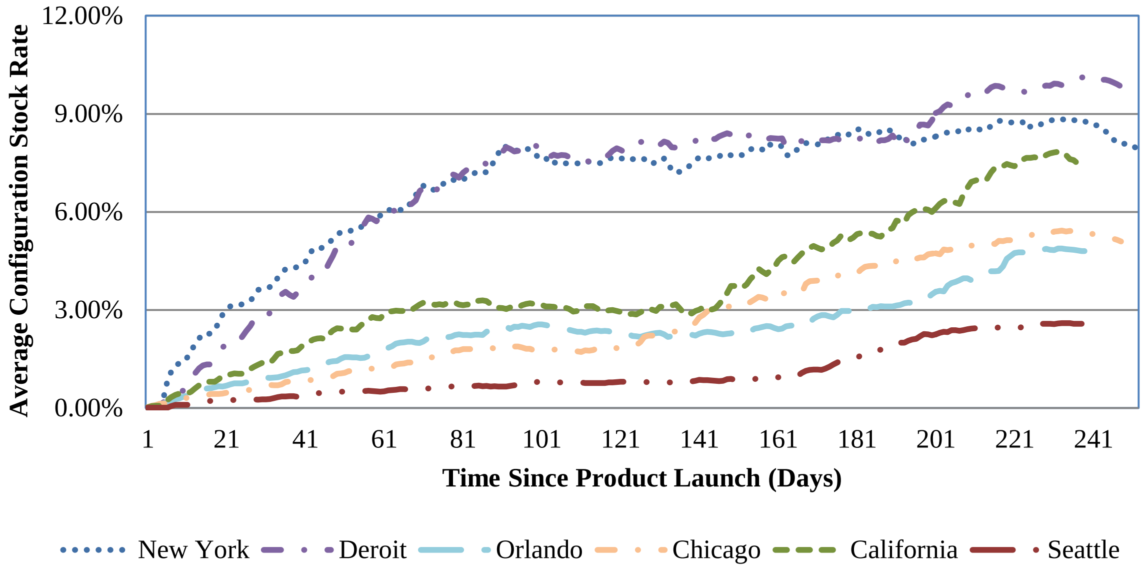

Figure 2.1: Ratio of stocked distinct configurations to number of built configurations in different US regions

After discussion on the importance of stock-out based substitution in modeling demand, we now present our formulation to find demand for each configuration at each region while considering both assortment and stock-out based substitutions. Suppose that $d_{i,r|D}$ is the total primary (original) demand for configuration i in region r. By primary demand, we mean the total demand for each configuration for the entire season, if all the configurations are present in the market (i.e., the number of customers considering the configuration as their most preferred configuration). We have

$$d_{i,r|D} = f_i^r . D \qquad (2\text{-}1)$$

where D is total demand for the product under consideration and f_i^r is the fraction of total demand for configuration i in region r. Then, the observed (effective) demand for configuration i in region r, $E_{i,r|D}$, would be calculated as follows:

$$E_{i,r|D} = d_{i,r|D} + \sum_{\forall j \neq i} d_{i,r|D} \cdot S_{j,r|D} . \alpha_{ij} \tag{2-2}$$

where α_{ij} is the substitution probability for a customer to switch to configuration i after not finding her favorite choice, configuration j . In this formulation, $S_{i,r|D}$ denotes the average percentage of unmet demand for configuration i in region r, given total demand D. Note that for configurations not carried in the assortment, $S_{i,r|D}$, is equal to one. For configurations carried in the assortment, the average unmet demand rate is a number between 0 and 1, calculated as follows:

$$S_{i,r|D} = \begin{Bmatrix} \frac{E_{i,r|D} - Y_{i,r}}{E_{i,r|D}} & if \quad Y_{i,r} \leq E_{i,r|D} \\ 0 & else \end{Bmatrix} \tag{2-3}$$

It should be noted that Hopp and Xu (2008) propose a static approximation for dynamic demand substitution behavior using a fluid network model and suggest an approximation for service rate which measures the ratio of met demand to total demand. More precisely, they define service rate for inventory level y, as $f(y) = min(y/N, 1)$, where N represents the exogenous total number of customer arrivals. Our approximation for stock-out rate is not very different from their approximation for service levels except for the point that we are using effective demand in the denominator instead of using customer arrival rate. One difficulty in calculating our suggested stock-out rate is that calculating $S_{i,r|D}$ requires knowing $E_{i,r|D}$, and hence, $E_{i,r|D}$ is recursively a function of $S_{i,r|D}$. Thus, solving for $S_{i,r|D}$ requires a system of nonlinear equations and makes the optimization problem a discrete nonlinear program (there are several binary variables in our model as well). We will later show a procedure to cope with the nonlinearity of the problem. A summary of notation and definition of all parameters and decision variables is provided in Table 1.4 in Appendix A.

2.5. Mathematical Formulation for Assortment Optimization

We address the assortment planning problem in this section and present the results from our formulation in the next section (Section 2.6). Let $\pi(D,Y)$ be the one-period (newsvendor) profit of OEM, from stocking Y, when demand realization is D. Note that we follow a single period single-shot supply policy and hence, Y is a variable independent of demand scenarios. We have:

$$\pi(D,Y) = \sum_{i=1}^{N}\sum_{r=1}^{R}[(P_i - C_i^{unit}) \cdot (Y_{i,r} - L_{i,r|D}) - C_i^{overage}.L_{i,r|D}]$$

$$-\sum_{i=1}^{N} C_i^{fixed_{conf}}.X_i - \sum_{p=1}^{P} C_p^{fixed_{part}}.P_p$$

$$+\sum_{\forall p\in P}\sum_{\forall l\in L} v_l^p.\tilde{z}_l^p$$

$$+complexity\_cost \quad (2\text{-}6)$$

Then, the expected profit would be:

$$E\Pi(Y) = \textstyle\sum_{D\in D^{total}} \pi(D,Y).f(D) \quad (2\text{-}7)$$

Remember that D^{total} is a discrete random variable representing the total realized demand in the market under study; and $f(D)$ is the corresponding probability. We would like to maximize $E\Pi(\mathrm{Y})$ with respect to some capacity and feasibility constraints. Here, we present the whole optimization problem followed by the feasibility constraints.

$$max_{Y,X}\, E\Pi(Y,X) = \sum_{D\in D^{total}} \left\{ \begin{bmatrix} \sum_{i=1}^{N}\sum_{r=1}^{R} \begin{pmatrix} (P_i - C_i^{unit}) \cdot (Y_{i,r} - L_{i,r|D}) \\ -C_i^{overage} . L_{i,r|D} \end{pmatrix} \\ -\sum_{i=1}^{N} C_i^{fixed_{conf}} . X_i \\ -\sum_{p=1}^{P} C_p^{fixed_{part}} . P_p \\ +\sum_{\forall p\in P}\sum_{\forall l\in L} v_l^p . \tilde{z}_l^p \\ -complexity\_cost \end{bmatrix} . f(D) \right\} \qquad (2\text{-}8)$$

s.t.

$$E_{i,r|D} = d_{i,r|D} + \sum_{\forall j\neq i} d_{j,r|D} \cdot S_{j,r|D} . \alpha_{ji} \qquad \forall i,r,D \qquad (2\text{-}9)$$

$$S_{i,r|D} = \begin{Bmatrix} \frac{E_{i,r|D} - Y_{i,r}}{E_{i,r|D}} & if \quad Y_{i,r} \leq E_{i,r|D} \\ 0 & else \end{Bmatrix}$$

$$= \frac{0.5}{E_{i,r|D}} . \left\{ E_{i,r|D} - Y_{i,r} + \left| E_{i,r|D} - Y_{i,r} \right| \right\} \qquad \forall i,r,D \qquad (2\text{-}10)$$

$$L_{i,r|D} \geq Y_{i,r} - E_{i,r|D} \qquad \forall i,r,D \qquad (2\text{-}11)$$

$$\sum_{\forall r\in R} Y_i^r \leq Y_{max} \cdot X_i \qquad \forall i \qquad (2\text{-}12)$$

$$X_i \leq \sum_{\forall r\in R} Y_{i,r} \qquad \forall i \qquad (2\text{-}13)$$

$$\sum_{\forall r\in R}\sum_{\forall i\in N} Y_{i,r} \leq Y_{max} \qquad (2\text{-}14)$$

$$Z^p = \sum_{\forall i\in N}\{(\sum_{\forall r\in R} Y_i^r) . \beta_i^p\} \qquad \forall p \; in \; part \qquad (2\text{-}15)$$

$$Z^p \leq BigM * P_p \qquad \forall p \; in \; part \qquad (2\text{-}16)$$

$$Z^p = \sum_{\forall l} \tilde{z}_l^p \qquad \forall p \; in \; part \qquad (2\text{-}17)$$

$$\sum_{\forall l} Z\_Binary_l^p = 1 \qquad \forall p \; in \; part \qquad (2\text{-}18)$$

$$\tilde{z}_l^p \leq LP_{l+1}^p \times Z\_Binary_l^p \qquad \forall p,l \qquad (2\text{-}19)$$

$$\tilde{z}_l^p \leq BigM \times Z\_Binary_l^p \qquad \forall p,l \qquad (2\text{-}20)$$

$$\tilde{z}_l^p \geq LP_l^p - BigM * (1 - Z_{Binary}{}_l^p) \qquad \forall p,l \qquad (2\text{-}21)$$

$$\tilde{z}_l^p \leq LP_{l+1}^p + BigM * (1 - Z_{Binary}{}_l^p) \qquad \forall p, l \qquad (2\text{-}22)$$

$$complexity\_cost = ComY_k + \left(\frac{ComY_{k+1} - ComY_k}{ComX_{k+1} - ComX_k}\right) \cdot \left(\sum_{i=1}^{N} X_i - ComX_k\right)$$

$$\forall k \in 1..NbComlevel - 1 \qquad (2\text{-}23)$$

$$\tilde{z}_l^p \in \{0,1\}, Z_{Binary}{}_l^p \in R^+ \qquad \forall p, l$$

$$X_i \in \{0,1\} \qquad \forall i \in N$$

$$Y_i^r \in R^+ \qquad \forall i \in N, \forall r \in R$$

$$Z^p \in N \cup \{0\}, P_p \in \{0,1\} \qquad \forall p \in P$$

$$Y_i^r \in R^+ \qquad \forall i \in N, \forall r \in R$$

$$S_{i,r|D}, L_{i,r|D}, E_{i,r|D} \in R^+ \qquad \forall i \in N, \forall r \in R, \forall D \in demand\_total$$

Equation (2-9) is to calculate the observed (effective) demand for each configuration in each region given the total demand based on scenario D. Equation (2-10) is used for estimating the average percentage of unmet demand (stock-out rate). Equation (2-11) determines the left-over inventory of configuration i in region r. Equations (2-12 and 2-13) are to make sure that there is no production across all regions for a vehicle that is not to be built (i.e., not part of the assortment). Equation (2-14) is to guarantee that total production does not exceed the maximum production limit. Equations (2-15 and 2-16) finds total units of option p required for the production of all configurations. Equations (2-17 and 2-22) are to choose a discount level (for economies-of-scale) for each option in the assortment. Finally, equation (2-23) calculates piece-wise linear manufacturing complexity cost.

2.6. Proposed Methodology

The suggested formulation requires using a MINLP solver to be solved optimally for small size problems and fails to give an optimal solution when we go to real world size problems. We suggest a novel procedure to solve this problem that can further be applied to a large class of MINLP problems (in particular, if the nonlinear function can be proved to be convex or concave). The main idea behind the procedure is to solve alternate linear lower/upper MILP estimators for the original problem and diminish the initial gap of lower/upper estimators by going into a branch and bound tree. We now explain the procedure in more detail.

Alternate Linear Lower/Upper Estimators for Original Problem

In our mathematical formulation, all the terms in the objective function as well as the model constraints are linear except for constraint (2-10), which calculates average rate of stock-outs. Note that this is also a complicating constraint for our model and relaxing it leads to significant improvement in solution time. We show in Appendix B that stock-out rate ($S_{i,r|D}$) is a concave function w.r.t production level ($Y_{i,r}$). Figure 1.2 depicts a schematic diagram of changes in stock-out rate of configuration i w.r.t changes in production level for configuration i when production levels for the rest of the configurations is fixed.

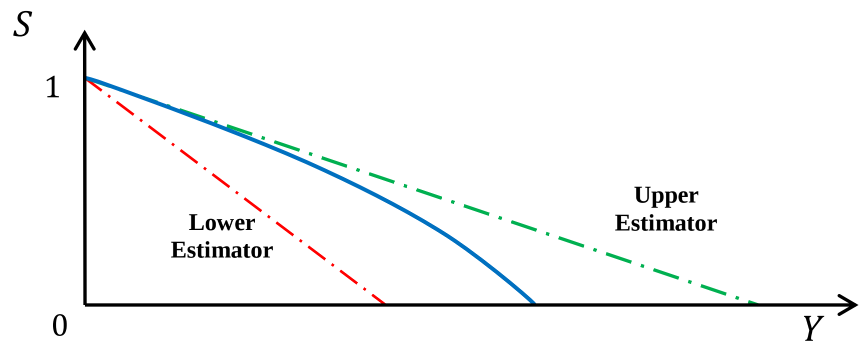

Figure 2.2: Schematic diagram of changes in stock-out rate (S) w.r.t production level (Y) shown in blue. Lower and upper estimators are drawn as red and green dashed lines, respectively.

We need to derive linear lower/upper estimators for the original problem that could be solved as an alternate to the original problem. We show in Appendix C that objective function monotonously changes w.r.t changes in $S_{i,r|D}$. This implies that if we replace constraint (2-10) with any lower bound for $S_{i,r|D}$, that should provide a true lower bound for the original problem. The same procedure would be stated for a true upper bound for the original problem. We now focus on finding valid lower/upper bounds for $S_{i,r|D}$. To that end, we suggest the following bounds:

Lower Bound

The basic idea behind finding the lower bound for stock-out based substitution ($S_{i,r|D}$) is to replace it with assortment-based substitution as follows:

$$S_{i,r|D}^{L1} = 1 - X_i \quad \text{(2-24)}$$

Using this lower bound for estimation of effective demand is similar to the lower bound suggested by Smith and Agrawal (2000). Moreover, we are interested to have a set of lower/upper bounds with diminishing gap. We will later present another set of dynamic lower bounds when discussing the Branch and Bound procedure, which could converge to zero gap given close enough lower/upper production levels.

Upper Bound

We derive a different set of upper bounds for our model. First, consider constraint (2-10), we then replace $E_{i,r|D}$ with its equivalent form on constraint (2-9) and rewrite it as follows:

$$S_{i,r|D} = \begin{cases} \frac{d_{i,r|D} + \sum_{\forall j \neq i} d_{j,r|D} \cdot S_{j,r|D} . \alpha_{ji} - Y_{i,r}}{d_{i,r|D} + \sum_{\forall j \neq i} d_{j,r|D} \cdot S_{j,r|D} . \alpha_{ji}} & if \quad Y_{i,r} \leq E_{i,r|D} \\ 0 & else \end{cases} \tag{2-25}$$

We now replace $S_{j,r|D}$ in the numerator as well as the denominator with the highest value it could get (one) and that gives a *linear* upper bound for constraint (2-10).

$$S_{i,r|D}^{U1} = \begin{cases} \frac{d_{i,r|D} + \sum_{\forall j \neq i} d_{j,r|D} . 1 . \alpha_{ji} - Y_{i,r}}{d_{i,r|D} + \sum_{\forall j \neq i} d_{j,r|D} . 1 . \alpha_{ji}} & if \quad Y_{i,r} \leq E_{i,r|D} \\ 0 & else \end{cases} \tag{2-26}$$

Another bound for (2-25) is derived by replacing $S_{j,r|D}$ in the denominator with a lower value, $1 - X_j$. Remember that X_j is a binary decision variable equal to one when configuration j is on the assortment and zero otherwise. The new bound is as follows:

$$S_{i,r|D}^{U2} = \begin{cases} \frac{d_{i,r|D} + \sum_{\forall j \neq i} d_{j,r|D} \cdot S_{j,r|D} . \alpha_{ji} - Y_{i,r}}{d_{i,r|D} + \sum_{\forall j \neq i} d_{j,r|D} \cdot (1 - X_j) . \alpha_{ji}} & if \quad Y_{i,r} \leq E_{i,r|D} \\ 0 & else \end{cases}$$

$$or \begin{cases} S_{i,r|D}^{U2} \times \left(d_{i,r|D} + \sum_{\forall j \neq i} d_{j,r|D} \cdot (1 - X_j) . \alpha_{ji}\right) & if \quad Y_{i,r} \leq E_{i,r|D} \\ \quad = d_{i,r|D} + \sum_{\forall j \neq i} d_{j,r|D} \cdot S_{j,r|D} . \alpha_{ji} - Y_{i,r} & \\ S_{i,r|D}^{U2} = 0 & else \end{cases} \tag{2-27}$$

Since we have a continuous decision variable multiplied by a binary variable, we can use linearization techniques to convert it into a MILP. The final linearized version of upper bound problem is presented in Appendix E.

The reason for using these two bounds is that the first bound is straight-forward to calculate but not very tight, versus the second one which is much more tight but is more expensive. So, we suggest to start with the first set of bounds and add new cuts derived from the second bounds into the branch and bound tree.

2.7. Modified Branch and Bound Procedure for Solving MINLP Problem

We suggest using a Modified Branch and Bound (M-BnB) procedure which is able to tackle mixed-integer nonlinear programs. The idea is to use problem relaxations through using lower/upper bounds instead of relaxing binary (integer) variables, which is a common step in regular Branch and Bound procedure. After solving lower/upper relaxations (estimators) at the root node, we would stop branching if the gap between the lower and upper relaxations is less than a predetermined optimality gap. If not, we would branch to further nodes until we can prune all the nodes by either bound or optimality gap. A flowchart of procedure is depicted in Figure 2.3.

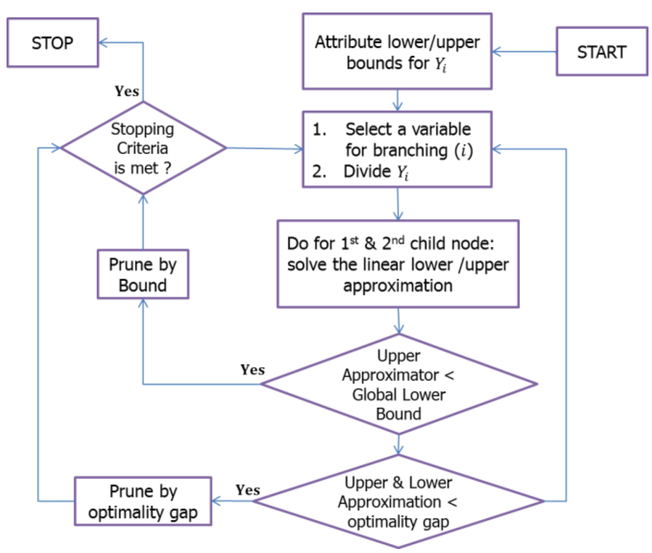

Figure 2.3: Flowchart of proposed Modified Branch and Bound (M-BnB) algorithm

Branching Strategy

We choose to pick a decision variable for branching that could help in diminishing global lower/upper bounds. This could be done through choosing production level variable $Y_{i,r}$ as the branching variable and adding new lower/upper bounds derived for that end. Note that the continuous random variable $Y_{i,r}$ takes a lower value of zero. To find a maximum value for $Y_{i,r}$, we can pick maximum production level or to have a better bound, we can find it through forcing all of the configurations not to be in the assortment except for configuration i and solve the problem. The obtained production level for configuration i will be an upper value for $Y_{i,r}$.

Dynamic Bounds

We now discuss dynamic bounds, which are calculated and updated for each particular node of the Branch and Bound tree. Remember that the main motivation behind these bounds is that the previously mentioned lower/upper bounds are not changing from their initial values at the root node. Hence, we employ these dynamic bounds to be able to diminish the gap between global lower/upper bounds. The dynamic lower/upper bounds are derived this way respectively:

$$S_{i,r|D}^{L2} = \begin{Bmatrix} \frac{d_{i,r|D} + \sum_{\forall j \neq i} d_{j,r|D} \cdot S_{j,r|D}^{L2} \cdot \alpha_{ji} - Y_{i,r}^{U}}{d_{i,r|D} + \sum_{\forall j \neq i} d_{j,r|D} \cdot S_{j,r|D}^{L2} \cdot \alpha_{ji}} & if \;\; Y_{i,r}^{U} \leq E_{i,r|D} \\ 0 & else \end{Bmatrix} \tag{2-28}$$

$$S_{i,r|D}^{U3} = \begin{Bmatrix} \frac{d_{i,r|D} + \sum_{\forall j \neq i} d_{j,r|D} \cdot S_{j,r|D}^{U3} \cdot \alpha_{ji} - Y_{i,r}^{L}}{d_{i,r|D} + \sum_{\forall j \neq i} d_{j,r|D} \cdot S_{j,r|D}^{U3} \cdot \alpha_{ji}} & if \;\; Y_{i,r}^{L} \leq E_{i,r|D} \\ 0 & else \end{Bmatrix} \tag{2-29}$$

where $Y_{i,r}^{L}, Y_{i,r}^{U}$ represent the lower and upper bounds for $Y_{i,r}$, respectively, and are parameters with defined values before solving each node. Note that as values for $Y_{i,r}^{L}$ and $Y_{i,r}^{U}$ come closer to each other, dynamic lower and upper bounds would also converge to closer values.

2.8. Experimental Results

Numerical Experiment

We generated a set of examples with 36 potential configurations and generated an artificial dataset for cost of each part/option and based on that cost, we derive the unit cost associated with each configuration. We consider a profit margin between 15% - 30% and add it to the unit cost in order to derive the selling price. We then consider the overage cost of left-over inventory to be between 5%-12% of unit cost of a configuration. We also generate artificial data related to other parameters like substitution probabilities (α_{ij}), demand fractions (f_i^r), fixed costs of offering configuration ($C_i^{fixed_{conf}}$), and fixed costs of offering an option ($C_p^{fixed_{part}}$), as well as manufacturing complexity costs. It should be noted that to derive our artificial data, we relied on viewpoints of several Subject Expert Matters from a North-American OEM to make sure that data is consistent with the real-world situation. All generated data is presented in Appendix D. The above formulation was coded in ILog-Cplex 11.0. Table 2.1 shows the descriptive statistics on running time as well as optimal objective function for different problem settings. For each problem setting, we generate 10 random replicates by changing only one parameter at a time by a factor of +/- 20% and return average, minimum, maximum and standard deviation of solution time and the objective function. As can be seen from Table 2.1, solution times steadily increase by increasing the overall substitution probability. One reason might be that calculating observed demand using equation (2-9) is expensive and as we have more nonzero terms in substitution probability matrix, we expect more expensive calculations and hence, larger solution times. Another interesting fact is that on average total profit increases by 12.2% when substitution probabilities sum to 0.95 compared to the scenario with more picky customers whose substitution probabilities only sum to 0.45. This amount of change in total profit shows the importance of incorporating substitution effects into assortment planning models.

Table 2.1: Descriptive statistics regarding solution time and objective function for 70 different experiments.

				Solution Time				Objective	
Experiments	**Demand fraction multiplied by**	**Profit margin multiplied by**	**Substitution Sum to**	**Min**	**Max**	**Average**	**St. dev**	**Average**	**St. dev**
1-10	1	1	Rand (0.45)	97.9	263.4	151.4	50.8	3,353,344.6	11,180.1
11-20	1	1	Rand (0.7)	419.1	613.7	513.7	66.0	3,437,651.4	5,848.6
21-30	1	1	Rand (0.95)	916.7	2,822.0	1,612.5	566.7	3,653,366.9	35,320.6
31-40	1	Rand (0.4:0.6)	0.7	159.1	408.8	285.6	93.8	731,324.4	77,520.1
41-50	1	Rand (0.8:1.2)	0.7	185.8	771.7	416.9	190.7	3,464,968.0	146,383.4
51-60	1	Rand (1.6:2.4)	0.7	105.2	318.1	198.3	74.2	10,079,854.5	350,091.8
61-70	Rand (0.8:1.2)	1	0.7	182.2	650.0	353.8	130.3	3,497,166.7	777,017.8

We can also take a more detailed look at the result in terms of optimal values of some variables like assortment size, production levels, and number of chosen parts/options. For this end, we run an experiment (21) with all parameters presented in Appendix D. Figure 2.4 shows the production fractions as well as the primary demand fractions for 36 configurations when substitution probabilities are assumed to be high (sum to 0.95). As you can see from the graph, there are some configurations missing in the optimal assortment and there is not a similar trend between primary demand fractions and production volume fractions. Also, some of the popular configurations are missing in the optimal assortment due to high substitution rates, which makes it profitable to satisfy demand for some configurations through other configurations. Figure 2.5 shows similar information for the scenario in which substitution probabilities are moderate (sum to 0.70). In this scenario and due to smaller substitution effects, the production fractions have a more similar trend toward primary demand fractions even though they are still different. This observation supports the idea that optimal assortment decisions are highly dependent to substitutability of customers in case of miss-match of supply and demand.

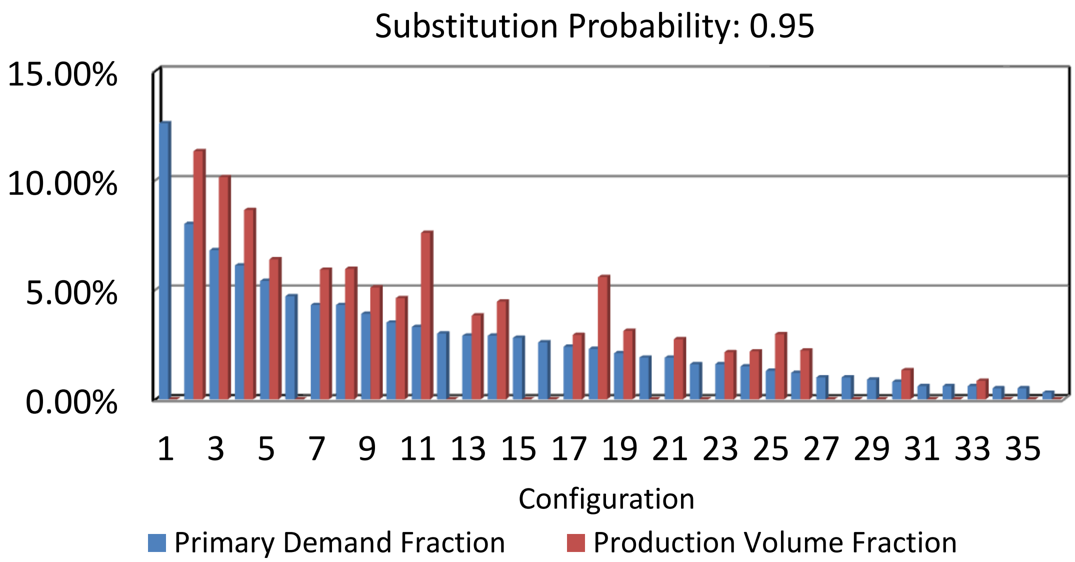

Figure 2.4: Production and primary demand fractions for configurations under high substitution rate

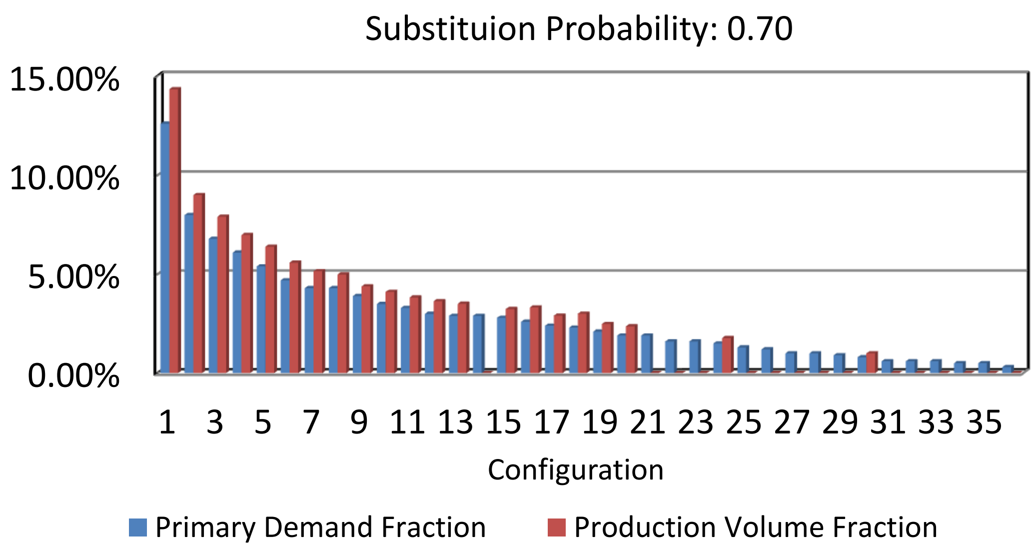

Figure 2.5: Production and primary demand fractions for configurations under moderate substitution rate

Sensitivity Analysis

We perform an analysis to assess the impact of assortment size on overall profit. For this end, we choose an experiment with nominal values (experiment 21) and run it with different limits on the size of the assortment. That is, each time we made the assortment size to be equal to a pre-set value. As expected and shown on Figure 2.6, the overall profits seem to take on a concave form as a function of the size of the assortment. Initially the profits increase with the assortment size to capture additional demand. However, too high an assortment seems to compromise the production economies-of-scale as well as incurring complexity costs), at least for the given setting and cost parameters. One observation is that total profit only drops less than 1.85% when assortment size varies from 19 to 24 which suggests that even though configurations are likely to differ considerably from one assortment size to another, total profit is relatively robust to assortment size variation by carrying optimal configuration.

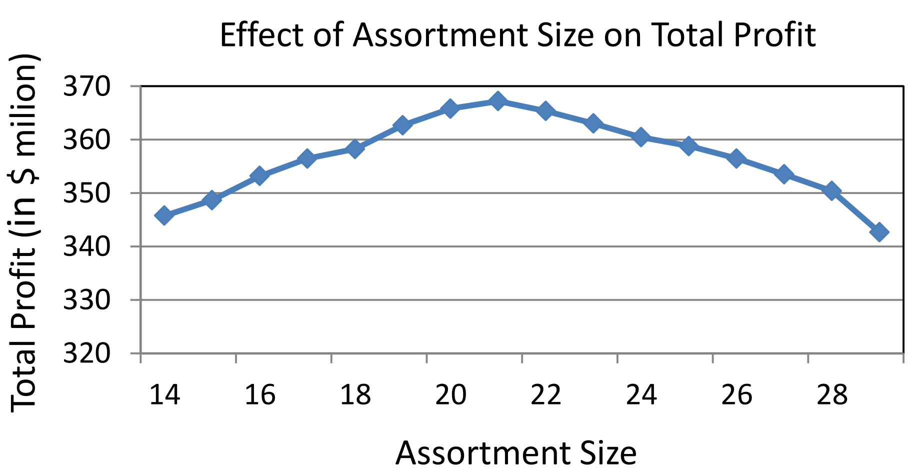

Figure 2.6: Effect of assortment size on total profit (in $ millions)

Another observation is the impact of economies-of-scale on optimal assortment. Figure 2.7 shows two scenarios regarding the optimal assortment where effect of economies-of-scale is included / excluded in the model. As can be seen, although 'assortment size' is equal for this particular experiment, the optimal assortment is different in those two scenarios which suggests necessity of incorporating economies-of-scales in assortment planning for automotive product.

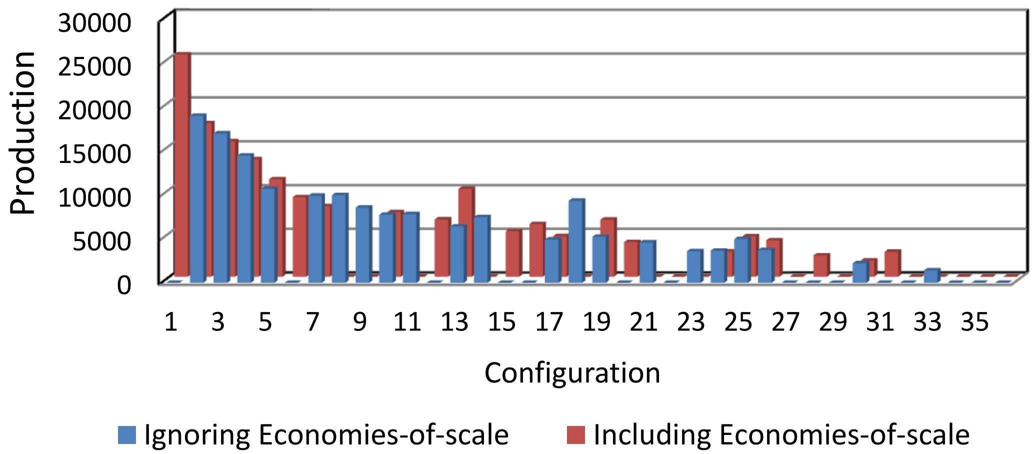

Figure 2.7: Effect of economies-of-scale on optimal assortment

2.9. Analyzing Modified Branch and Bound (M-BnB) Method

We now compare the modified branch and bound procedure with commercial Mixed-integer nonlinear programming solvers in terms of solution time and quality. We choose two experiments from each of the seven scenarios with maximum and minimum solution time and solve them with AMPL Bonmin solver. As the maximum solution time with M-BnB method is less than an hour, we set a 3,600 seconds time limit to run each of the experiments with AMPL as well as a 1% optimality tolerance gap for both methods. The results are shown on Table 2.2. As can be seen for our current experiment, M-BnB method surpasses Bonmin in terms of solution time for all the selected experiments while in terms of objective function it returns a better solution in 11 out of 14 experiments with two other experiments returning the same objective value.

Table 2.2: Comparison of M-BnB method with AMPL Bonmin solver over the randomized experiments

		M-BnB Method				AMPL			
Experiment #	**# of Potential Configurations**	**Solution Time (Seconds)**	**Best Solution Objective**	**Best Upper Bound**	**Optimality Gap**	**Solution Time (Seconds)**	**Best Solution Objective**	**Best Upper Bound**	**Optimality Gap**
1	36	**98**	**3,356,212**	3,356,212	0.00%	3,600	3,307,110	3,957,950	19.68%
10		**263**	**3,336,379**	3,336,379	0.00%	3,600	3,297,830	3,979,380	20.67%
12		**614**	**3,441,529**	3,441,529	0.00%	3,600	3,408,290	4,065,200	19.27%
18		**419**	**3,436,629**	3,440,065	0.10%	3,600	3,389,310	4,070,370	20.09%
27		**2,822**	**3,624,690**	3,624,690	0.00%	3,600	3,600,020	4,225,270	17.37%
30		**917**	**3,743,492**	3,743,492	0.00%	3,600	3,678,530	4,323,430	17.53%
32		**160**	690,337	691,027	0.10%	3,600	**690,338**	1,009,460	46.23%
33		**409**	**629,244**	629,244	0.00%	3,600	622,356	922,820	48.28%
46		**186**	**3,367,229**	3,367,229	0.00%	3,600	**3,367,229**	3,957,230	17.52%
50		**772**	**3,547,129**	3,548,962	0.05%	3,600	3,548,070	4,182,540	17.88%
54		**105**	**9,337,718**	9,339,337	0.02%	3,600	9,271,380	10,029,700	8.18%
59		**318**	**10,679,400**	10,679,400	0.00%	3,600	10,575,800	11,303,300	6.88%
66		**650**	**3,022,360**	3,022,360	0.00%	3,600	**3,022,360**	3,561,040	17.82%
68		**182**	**4,972,590**	4,972,590	0.00%	3,600	4,949,060	5,527,450	11.69%

Table 2.3: Comparison of M-BnB method with AMPL Bonmin solver over different problem sizes

	M-BnB Method				AMPL			
# of Potential Configurations	**Solution Time (Seconds)**	**Best Solution Objective**	**Best Upper Bound**	**Optimality Gap**	**Solution Time (Seconds)**	**Best Solution Objective**	**Best Upper Bound**	**Optimality Gap**
18	**35**	**4,443,795**	4,443,795	0.00%	236	**4,443,795**	4,443,795	0.00%
36	**558**	**3,447,201**	3,447,201	0.00%	80,000	3,446,770	3,662,750	5.90%
72	**17,431**	**2,686,846**	2,686,846	0.00%	80,000	2,952,180	4,273,080	30.91%

We would also like to test efficiency and effectiveness of the algorithm for dealing with different problem sizes. For this end, we select 3 groups of problems with 18, 36, and 72 potential configurations and compare the result with AMPL output in Table 2.3. As can be seen, M-BnB method returns optimal solutions in all three problem instances while AMPL is not able to return a 1% optimality solution in two out of three scenarios even after more than 22 hours. However, it also turns out that this problem suffers from curse of dimensionality and we suggest using some sort of heuristic algorithms when problem dimensions explode.

2.10. Appendix

Appendix A:

Table 2.4: Problem parameters, sets and decision variables

Sets	
$i \in 1, \dots, N$	Set of all potential configurations
$r \in 1, \dots, R$	Set of all market regions
$p \in 1, \dots, P$	denote the set of all options required for assortment production,
$l \in 1, \dots, L$	Set of all discount levels for an option.
$k \in NbComlevel$	Set of all levels piecewise linear parts of manufacturing complexity cost
D^{total}	Set of discrete random variables representing total demand in the market.

Parameters	
$d_{i,r\|D}$	Primary (original) demand for configuration i in region r given D as total demand for the product under consideration
f_i^r	Fraction of total demand for configuration i in region r.
α_{ij}	Substitution probability of switching to configuration i after not finding the favorite choice, configuration j
p_i	Selling price of configuration i,
$c_i^{variable}$	Variable cost of every unit of configuration i
C_i^{fixed}	Fixed cost of configuration i , if carried in the market.
$C_i^{overage}$	Overage cost of configuration i if not sold.
v_l^p	be the amount of discount for each unit of option p if purchased at quantity level l
β_i^p	BOM parameter equal to one if configuration i requires option p and zero otherwise.
LP_l^p	The lower-point of quantity for level l of option p.
$BigM$	A sufficiently large number
Y_{max}	maximum production capacity of the OEM
$f(D)$	Probability mass function of D^{total}
$ComY_k$	Total manufacturing complexity cost for kth level of complexity
$ComX_k$	Assortment size at kth level of complexity

Decision Variables	
Y_i^r	Number of vehicles of configuration i supplied to the region r of the market.
X_i	A binary decision variable equal to one if configuration i is built, and zero otherwise.
$\mathcal{L}_{i\|D,Y}^r$	Left-over inventory of configuration i in region r, given demand is D, and the supply vector is Y.
$E_{i,r\|D}$	Effective demand for configuration i in region r, given D as total demand
$S_{i,r\|D}$	Average percentage of unmet demand for configuration i in region r, given total

	demand, D.
P_p	A binary decision variable equal to one if option p is required, and zero otherwise.
Z^p	Total required units of option p for the production of all configurations
$\tilde{z}_l^p$	Total required units of option p purchased at discount level l
$Z\_Binary_l^p$	A binary decision variable equal to one if option p is purchased at discount level l, and zero otherwise.

Appendix B:

Theorem: $S_{i,r|D}$ is concave w.r.t. Y_i^r.

Consider these functions:

$$E_{i,r|D} = d_{i,r|D} + \sum_{\forall j\neq i} d_{j,r|D} \cdot S_{j,r|D}.\alpha_{ji} \qquad \forall i,r,D \qquad \text{(B-1)}$$

$$S_{i,r|D} = \frac{d_{i,r|D}+\sum_{\forall j\neq i} d_{j,r|D}\cdot S_{j,r|D}.\alpha_{ji}-Y_i^r}{d_{i,r|D}+\sum_{\forall j\neq i} d_{j,r|D}\cdot S_{j,r|D}.\alpha_{ji}} \text{ if } 0 \le Y_i^r \le E_{i,r|D} \qquad \forall i,r,D \qquad \text{(B-2)}$$

We show the proof for $0 \le Y_i^r \le E_{i,r|D}$ and re-write equation (2) as follows:

$$Y_i^r = d_{i,r|D} + \sum_{\forall j\neq i}^{N} d_{j,r|D} \cdot S_{j,r|D}.\alpha_{ji} - S_{i,r|D}.d_{i,r|D} - \sum_{\forall j\neq i}^{N} d_{j,r|D} \cdot S_{j,r|D}.S_{i,r|D}.\alpha_{ji} \quad \forall i,r,D \text{ (B-3)}$$

We write the Hessian Matrix as follows:

$$H = \frac{\partial^2 Y_1}{\partial S.\partial S} = \begin{bmatrix} 0 & -d_{2,r|D}\cdot\alpha_{21} & -d_{3,r|D}\cdot\alpha_{31} & \cdots & -d_{N,r|D}\cdot\alpha_{N1} \\ \vdots & 0 & 0 & \cdots & 0 \\ -d_{N,r|D}\cdot\alpha_{N1} & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$H = \frac{\partial^2 Y_2}{\partial S.\partial S} = \begin{bmatrix} 0 & -d_{1,r|D}\cdot\alpha_{12} & 0 & \cdots & 0 \\ -d_{1,r|D}\cdot\alpha_{12} & 0 & -d_{3,r|D}\cdot\alpha_{32} & \cdots \quad \vdots & -d_{N,r|D}\cdot\alpha_{N2} \\ 0 & -d_{N,r|D}\cdot\alpha_{N2} & 0 & \cdots & 0 \end{bmatrix}$$

$$H = \frac{\partial^2 Y_i}{\partial S.\partial S} = \begin{bmatrix} 0 & 0 & 0 & -d_{1,r|D}\cdot\alpha_{1i} & \cdots & 0 \\ -d_{1,r|D}\cdot\alpha_{1i} & -d_{2,r|D}\cdot\alpha_{2i} & \cdots\ 0 & -d_{2,r|D}\cdot\alpha_{2,i+1} & \dots & -d_{N,r|D}\cdot\alpha_{Ni} \\ 0 & 0 & 0 & -d_{N,r|D}\cdot\alpha_{Ni} & \cdots & 0 \end{bmatrix}$$

All the diagonal elements of the hessian matrix are zero and the other elements are either zero or non-positive. In order to prove that H is a negative semi-definite matrix and hence, Y(s) is concave, we need to show this:

$s.H.s \leq 0 \qquad \forall s$

We have assumed $0 \leq Y_i^r \ \leq E_{i,r|D}$ which implies that $0 \leq S_{i,r|D} \ \leq 1$, so since all elements of H are non-positive and $s \geq 0$, => $s.H.s \leq 0$

We showed that Y_i^r is concave w.r.t $S_{i,r|D}$. If Y is a concave decreasing function w.r.t. S, its inverse would also be a decreasing concave function (Mrsevic, 2008). This implies that $S_{i,r|D}$ is also concave w.r.t Y_i^r . To prove that Y_i^r is decreasing w.r.t. $S_{i,r|D}$, we can re-write equation (B-2) as follows:

$$Y_i^r = (d_{i,r|D} + \sum_{\forall j \neq i}^{N} d_{j,r|D} \cdot S_{j,r|D}.\alpha_{ji}).(1 - S_{i,r|D}) \qquad \forall \mathrm{i,r,D} \quad \text{(B-4)}$$

It is clear that increasing value of $S_{i,r|D}$ leads to decreased value of Y_i^r.

Appendix C:

We show that the objective function of the original problem will always get smaller (greater) if we replace constraint (2-3) with the under-estimator (over-estimator). The first observation is the value of effective demand ($E_{i,r|D}$) which is a function of $S_{i,r|D}$ with all positive coefficients. So it is obvious that:

$$E^{l}_{i,r|D} \leq E_{i,r|D} \leq E^{u}_{i,r|D}$$

Where $E^{l}_{i,r|D}$ ($E^{u}_{i,r|D}$) is the value of effective demand if we replace $S_{i,r|D}$ by $S^{l}_{i,r|D}$ ($S^{u}_{i,r|D}$). Notice that the objective function has only two terms that are affected by the value of effective demand. Those two terms are $(P_i - C_i^{Variable}) \cdot (Y_i^r - L_{i,r|D})$ and $(-C_i^{overage}) \cdot (L_{i,r|D})$. We need to check the effect of effective demand on both of them:

1- We can re-arrange constraint (2-11) as follows:

$$E_{i,r|D} \geq Y_i^r - L_{i,r|D} \quad \text{(C-1)}$$

Which shows that $E_{i,r|D}$ is an upper bound for the sales volume $(Y_i^r - L_{i,r|D})$ in the objective function. Since the problem is maximization and the coefficient of the sales volume $(P_i - C_i^{Variable}) \geq 0$, we see that decreasing (increasing) the value of effective demand makes the value of $(P_i - C_i^{Variable}) \cdot (Y_i^r - L_{i,r|D})$ smaller (larger).

2- The other term in the objective which might be affected by the change in effective demand value is the value of left-over inventory ($L_{i,r|D}$). Left-over inventory volume increases when we decrease the effective demand. Since this terms is in the objective function with a non-positive coefficient ($-C_i^{overage} \leq 0$), any decrease (increase) in the value of effective demand makes the value of $(-C_i^{overage}) \cdot L_{i,r|D}$ smaller (larger).

From 1 & 2 we see that if we replace constraint (2-3) with the proposed under-estimator (over-estimator), the objective function will be always smaller (larger).

Appendix D: Data input used in numerical experiment with 36 configurations

Table 2.5: Total demand and corresponding probabilities in different scenarios

	Demand Scenario 1	Demand Scenario 2
Total demand	150,000	180,000
Probability	0.40	0.60

Table 2.6: Selling price, unit and overage cost, and primary demand fraction per configuration

Configuration ID	Selling price ($1,000)	Unit cost ($1,000)	Overage cost ($1,000)	Primary demand fraction
1	21.8	18.1	1.5	0.126
2	23.8	18.3	2.2	0.08
3	21.2	17.6	1.5	0.068
4	21	17.5	1.4	0.061
5	21.3	17.7	1.5	0.054
6	20.7	17.2	1.4	0.047
7	24.4	18.7	2.3	0.043
8	21.4	17.8	1.5	0.043
9	21.2	17.6	1.5	0.039
10	21.5	17.9	1.5	0.035
11	19.6	17	0.9	0.033

12	20.9	17.4	1.4	0.03
13	20.7	17.2	1.4	0.029
14	24	18.4	2.3	0.029
15	24.5	18.8	2.3	0.028
16	19.3	16.7	0.9	0.026
17	23.7	18.2	2.2	0.024
18	21.5	17.9	1.5	0.023
19	20.7	17.2	1.4	0.021
20	20.9	17.4	1.4	0.019
21	24.1	18.5	2.3	0.019
22	19.5	16.9	0.9	0.016
23	21.5	17.9	1.5	0.016
24	24	18.4	2.3	0.015
25	21	17.5	1.4	0.013
26	20.8	17.3	1.4	0.012
27	21.8	18.1	1.5	0.01
28	21	17.5	1.4	0.01
29	19.6	17	0.9	0.009
30	24.1	18.5	2.3	0.008
31	19.4	16.8	0.9	0.006
32	19.6	17	0.9	0.006
33	23.7	18.2	2.2	0.006
34	19.1	16.6	0.9	0.005
35	21.6	18	1.5	0.005
36	21.3	17.7	1.5	0.003

Table 2.7: Bill of Materials

Configuration ID	Part 1	Part 2	Part 3	Part 4	Part 5	Part 6	Part 7	Part 8	Part 9	Part 10
1	0	0	0	1	0	0	1	0	0	1
2	0	0	0	1	0	0	0	1	0	1
3	0	0	0	1	0	0	1	0	0	0
4	0	1	0	0	0	0	1	0	0	1
5	0	0	1	0	0	0	1	0	0	1
6	1	0	0	0	0	0	1	0	0	1
7	0	0	0	0	0	1	1	0	0	1
8	0	0	0	1	0	0	0	1	0	0
9	0	1	0	0	0	0	0	1	0	1
10	0	0	1	0	0	0	0	1	0	1
11	0	1	0	0	0	0	1	0	0	0
12	1	0	0	0	0	0	0	1	0	1
13	0	0	1	0	0	0	1	0	0	0
14	0	0	0	0	1	0	1	0	0	1
15	0	0	0	0	0	1	0	1	0	1
16	1	0	0	0	0	0	1	0	0	0
17	0	0	0	0	0	1	1	0	0	0
18	0	0	0	1	0	0	0	0	1	1
19	0	1	0	0	0	0	0	1	0	0
20	0	0	1	0	0	0	0	1	0	0

21	0	0	0	0	1	0	0	1	0	1
22	1	0	0	0	0	0	0	1	0	0
23	0	0	0	0	1	0	1	0	0	0
24	0	0	0	0	0	1	0	1	0	0
25	0	0	0	1	0	0	0	0	1	0
26	0	1	0	0	0	0	0	0	1	1
27	0	0	0	0	1	0	0	1	0	0
28	0	0	1	0	0	0	0	0	1	1
29	1	0	0	0	0	0	0	0	1	1
30	0	0	0	0	0	1	0	0	1	1
31	0	1	0	0	0	0	0	0	1	0
32	0	0	1	0	0	0	0	0	1	0
33	0	0	0	0	1	0	0	0	1	1
34	1	0	0	0	0	0	0	0	1	0
35	0	0	0	0	0	1	0	0	1	0
36	0	0	0	0	1	0	0	0	1	0

Table 2.8: Fixed cost and discount information per part/option

Part ID	Fixed cost ($1,000)	discount	Minimum purchase (to receive discount)
1	27,000	0	-
2	30,000	200	45,000
3	33,000	220	45,000
4	38,250	0	-
5	41,250	275	45,000
6	45,000	0	-
7	6,750	45	80,000
8	9,000	60	80,000
9	4,500	0	-
10	6,000	0	-

Table 2.9: production and manufacturing complexity cost

Assortment size	0	7	14	21	28	36
Manufacturing complexity cost	0	100,000	250,000	450,000	800,000	1,500,000

Table 2.10: Substitution probabilities from missing configurations (rows) to available ones (columns)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	0.12	0.14	0.07	-	-	0.05	0.05	-	-	-	-	-	0.04	-	-	-	0.16	-	-	-	-	-	-	0.07	-	-	-	-	-	-	-	-	-	-	-
2	0.17	-	0.07	-	-	-	-	0.14	0.07	-	-	-	-	-	0.05	-	-	0.11	-	-	0.04	-	-	-	0.05	-	-	-	-	-	-	-	-	-	-	-
3	0.15	0.06	-	-	-	-	-	0.12	-	-	0.06	-	-	-	-	-	0.05	0.07	-	-	-	-	0.04	-	0.15	-	-	-	-	-	-	-	-	-	-	-
4	0.06	-	-	-	0.05	0.09	-	-	0.10	-	0.11	-	-	-	-	0.04	-	-	0.04	-	-	-	-	-	-	0.12	-	-	0.04	-	0.05	-	-	-	-	-
5	0.04	-	-	0.08	-	-	-	-	-	0.09	0.03	-	0.10	0.07	-	-	-	-	-	0.04	-	-	-	-	-	0.04	-	0.12	-	-	-	0.05	0.03	-	-	-
6	-	-	-	0.13	-	-	-	-	0.05	-	0.06	0.08	-	0.05	-	0.09	-	-	-	-	-	0.03	-	-	-	0.06	-	-	0.10	-	-	-	-	0.04	-	-
7	0.07	-	-	-	0.05	-	-	-	-	-	-	-	-	0.11	0.08	-	0.09	-	-	-	0.04	-	0.04	-	-	-	-	-	-	0.11	-	-	0.05	-	0.04	-
8	0.08	0.15	0.17	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.05	0.06	-	-	-	-	0.05	0.10	-	0.04	-	-	-	-	-	-	-	-	-
9	-	0.06	-	0.13	-	0.04	-	-	-	0.05	0.06	0.09	-	-	-	-	-	-	0.11	-	-	0.04	-	-	-	0.08	-	-	-	-	0.03	-	-	-	-	-
10	-	0.04	-	0.04	0.13	-	-	-	0.08	-	-	-	0.05	0.04	-	-	-	-	0.03	0.10	0.07	-	-	-	-	-	-	0.08	-	-	-	0.03	-	-	-	-
11	-	-	0.06	0.12	-	0.04	-	-	0.04	-	-	-	0.05	-	-	0.08	-	-	0.09	-	-	-	-	-	-	0.06	-	-	-	-	0.12	-	-	0.04	-	-
12	-	-	-	0.07	-	0.11	-	-	0.14	-	-	-	-	-	-	0.05	-	-	0.06	-	0.06	0.09	-	-	-	0.04	-	-	0.07	-	-	-	-	-	-	-
13	-	-	0.03	0.03	0.11	-	-	-	-	0.04	0.07	-	-	0.03	-	-	-	-	-	0.08	-	-	0.07	-	-	-	-	0.05	-	-	0.03	0.11	-	-	-	0.03
14	-	-	-	-	0.11	-	0.03	-	-	0.04	-	-	0.05	-	-	-	-	-	-	-	0.09	-	0.10	-	-	-	0.04	0.05	-	-	-	-	0.12	-	-	0.05
15	0.04	0.07	-	-	-	-	0.12	-	-	0.05	-	-	-	0.06	-	-	0.05	-	-	-	0.10	-	-	0.09	-	-	0.04	-	-	0.07	-	-	-	-	-	-
16	-	-	-	0.06	-	0.10	-	-	-	-	0.13	0.04	-	-	-	-	-	-	0.05	-	-	0.08	0.05	-	-	-	-	-	0.05	-	0.06	-	-	0.10	-	-
17	-	-	0.07	-	-	-	0.10	-	-	-	-	-	0.04	0.05	0.04	-	-	-	-	-	-	-	0.10	0.08	-	-	0.04	-	-	0.05	-	-	-	-	0.10	0.05
18	0.18	0.09	0.08	0.04	-	-	-	0.04	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.13	0.06	-	-	-	0.05	-	-	0.04	-	-	-
19	-	-	-	0.06	-	-	-	0.06	0.12	-	0.13	0.04	-	-	-	0.04	-	-	-	0.05	-	0.08	-	-	-	0.04	-	-	-	-	0.08	-	-	-	-	-
20	-	-	-	-	0.06	-	-	0.03	0.03	0.10	0.04	-	0.12	-	-	-	-	-	0.07	-	0.03	-	0.03	-	-	-	0.06	0.03	-	-	-	0.07	-	-	-	-
21	-	-	-	-	0.06	-	-	-	-	0.11	-	-	-	0.13	0.03	-	-	-	-	0.05	-	-	0.05	-	-	-	0.10	0.04	-	-	-	-	0.08	-	-	0.03
22	-	-	-	0.03	-	0.05	-	-	0.06	-	0.07	0.10	-	-	-	0.11	-	-	0.13	-	-	-	-	-	-	-	0.05	-	-	-	0.04	-	-	0.07	-	-
23	-	-	-	-	0.05	-	-	-	-	-	-	-	0.11	0.11	-	-	0.03	-	-	0.04	0.04	-	-	-	-	-	0.09	-	-	-	-	0.05	0.05	-	-	0.11
24	-	-	0.04	-	-	-	0.05	0.07	-	-	-	-	-	-	0.10	-	0.11	-	-	0.05	0.05	-	0.05	-	-	-	0.10	-	-	-	-	-	-	-	0.07	-
25	0.09	0.04	0.18	-	-	-	-	0.09	-	-	-	-	-	-	-	-	-	0.15	-	-	-	-	-	-	-	-	-	-	-	-	0.06	-	-	-	0.05	0.04
26	0.03	-	-	0.14	-	0.05	-	-	0.07	-	0.06	-	-	-	-	-	-	0.06	-	-	-	-	-	-	-	-	-	0.05	0.08	-	0.11	-	-	0.04	-	-
27	-	-	-	-	-	-	-	-	-	0.05	-	-	0.06	0.06	-	-	-	-	-	0.11	0.11	-	0.12	0.03	-	-	-	-	-	-	-	0.03	0.04	-	-	0.08
28	-	-	-	0.05	0.14	-	-	-	-	0.07	-	-	0.06	0.04	-	-	-	0.04	-	-	-	-	-	-	-	0.08	-	-	-	-	0.03	0.11	0.07	-	-	-
29	-	-	-	0.08	-	0.12	-	-	0.04	-	0.03	0.06	-	-	-	0.05	-	-	-	-	-	-	-	-	-	0.13	-	-	-	-	0.06	-	0.05	0.09	-	-
30	0.04	-	-	-	-	-	0.12	-	-	-	-	-	-	0.06	0.06	-	0.05	0.07	-	-	-	-	-	-	-	-	-	0.05	-	-	-	-	0.10	-	0.09	0.04
31	-	-	-	0.07	-	-	-	-	-	-	0.15	-	-	-	-	0.05	-	-	0.07	-	-	-	-	-	0.06	0.12	-	-	0.04	-	-	0.05	-	0.09	-	-
32	-	-	-	-	0.06	-	-	-	-	-	0.04	-	0.13	-	-	-	-	-	-	0.06	-	-	0.04	-	0.03	0.03	-	0.11	-	-	0.07	-	0.03	-	-	0.07
33	-	-	-	-	0.07	-	-	-	-	0.03	-	-	-	0.15	-	-	-	-	-	-	0.07	-	0.06	-	-	-	-	0.12	-	0.03	-	0.05	-	-	-	0.11
34	-	-	-	0.03	-	0.05	-	-	-	-	0.07	-	-	-	-	0.11	-	-	0.03	-	-	0.05	-	-	-	0.06	-	-	0.09	-	0.12	-	-	-	-	0.05
35	-	-	0.04	-	-	-	0.06	-	-	-	-	-	-	-	-	-	0.12	-	-	-	-	-	0.06	0.06	0.07	-	-	-	-	0.10	-	0.04	0.05	-	-	0.10
36	-	-	-	-	-	-	-	-	-	-	-	-	0.07	0.07	-	-	-	-	-	-	0.03	-	0.14	-	-	-	0.07	0.05	-	-	-	0.11	0.12	-	0.03	-

Chapter 3. ASSORTMENT PLANNING OF CONFIGURABLE PRODUCTS: Considerations for Economic and Environmental Impacts on Technology Selection

3.1. Introduction

Besides economic objectives to maximize profit, there are other factors affecting the final assortment of an OEM. Environmental issues are important driving forces that impact the automotive industry due to increasingly strict governmental regulations and increasing social expectations (Geffen and Rthenberg, 2000; Koplin et al. 2007). In the U.S., the main federal regulations on vehicle fuel economy have been expressed through Corporate Average Fuel Economy (CAFE) standards by National Highway Traffic Safety Administration (NHTSA) and the Environmental Pollution Authority (EPA)[2]. CAFE is the sales weighted average fuel economy, expressed in miles per gallon (mpg), of a manufacturer's fleet of passenger cars or light trucks with a gross vehicle weight rating of 8,500 lbs. or less, manufactured for sale in the U.S., for any given model year. The automakers are subject to financial penalties for not meeting CAFE standards. In 2011, the penalty for failing to meet the standards was $5.50 per tenth of a MPG for each tenth under the target value times the total volume of vehicles manufactured. According to NHTSA, most European manufacturers regularly pay CAFE civil penalties ranging from less than $1 million to more than $20 million annually while Asian and American manufacturers have not paid a civil penalty in recent years. Instead of CAFE requirements, some countries including European states have imposed taxation policy on gasoline and diesel prices. This policy has been considered as one of the best ways to fiscally control the amount of energy consumption and emissions from the transportation sector (Steenberghen and Lopez, 2008). This policy often involves significantly increasing fuel price (van Vliet et. al 2010) and motivates

[2] See NHTSA Website for more information: http://www.nhtsa.gov/fuel-economy

customer's evolution toward more fuel efficient vehicles. This dynamic will be considered in our model implicitly through the impact of vehicle price on primary demand fractions for distinct configurations. Another important measure for automakers to decide on their optimal assortment could be the total carbon emissions produced during the vehicle's life cycle.

In this Chapter, while limiting our discussion/scope to the automotive industry, we aim to develop configurable product assortment planning models that explicitly account for both demand and supply issues while entering environmental concerns into decision making. In the past decades, there has been considerable work dedicated to demand aspects of assortment planning (see Kök et al. 2008, for a literature review). Furthermore, some researchers have been interested in studying the effect of variety on manufacturing performance (MacDuffie et al. 1996) or its effect on assembly operations (Fisher and Ittner, 1999). But very little research has been done that integrally considers demand and supply/manufacturing aspects in planning product assortment, in particular, for configurable products. It should also be noted that most of the studies on assortment planning are developed for non-configurable products, e.g., grocery and retail products (van Ryzin and Mahajan, 1999; Agrawal and Smith, 2003; Kok and Fisher, 2007; Fisher and Vaidyanathan, 2009). The automotive industry, with its highly configurable products and unique federal and state regulations requires a specific approach to product assortment planning that better suits its product line features. The contribution of the Chapter is to propose an objective decision support system for assortment planning of automotive products by exploiting exogenous demand models. Moreover, and to best of our knowledge, this is the first work on product assortment planning that takes environmental issues into consideration.

The rest of the Chapter is organized as follows: Section 2 reviews the relevant literature; Section 3 discusses the problem setting in more detail and the main assumptions behind our model. Methodology and problem formulation are discussed in Section 4. Section 5 reports the results from a number of experiments. Finally, we conclude in Section 6.

3.2. Literature Review

Section 2.2 has provided a detailed review of the assortment planning literature. This section extends that review by examining literature that addresses environmental impacts through product technology selection.

As mentioned earlier, environmental concerns have not been explicitly studied in the literature for assortment planning problems. However, Goldberg (1998) studies the effects of CAFE standards on automobile prices and sales and the expected environmental effects of CAFE standards. He claims that policies oriented towards shifting the mixture of the new car fleet towards more fuel efficient vehicles are promising and CAFE provides incentives for manufacturers to develop more fuel efficient vehicles. Michalek et al. (2004) study the impact of fuel efficiency and emission policy on optimal vehicle design decisions in an oligopoly market. They evaluate several policy scenarios for the small car market, including CAFE standards, carbon dioxide (CO_2) emissions taxes, and diesel technology quotas. The results show that imposing CO_2 taxes on producers for expected life cycle emissions results in diminishing returns on fuel efficiency improvement as the taxes increase, while CAFE standards lead to higher average fuel efficiency per regulatory dollar. Although their model decides on design parameters (such as engine size), prices, and production volumes, it is different from our approach on assortment planning by considering no substitution effects. Recently, Hoen et al. (2010) study the effect of carbon emission regulations on transport mode selection in supply chains. Their results suggest that introducing a constraint on emissions is a more powerful tool for policymakers in reducing emissions compared to introducing an emission cost for freight transport via a direct emission tax or a market mechanism. In this paper, similar to results of Hoen et al (2010), we constrain the maximum emissions allowed by the automotive manufacturer rather than introducing an emission cost.

3.3. Assumptions

Suppose that for the product under consideration, $N = \{1,\ldots, I\}$ denotes the set of potential configurations that can be made available by the OEM. Assortment planning involves selecting a subset of these configurations for tooling the assembly line and readying the suppliers and dealers/retailers. Due to the long lead times associated with engineering parts/options and their integration into the vehicle as well as lead times associated with supply chain readiness, the assortment decisions have to be made well up front, often several years in advance of product launch. This forces the planners to make a number of assumptions. The major assumptions behind our assortment planning model are as follows, categorized into assumptions related to demand, supply, costs, and environmental issues.

1. Demand

    a. Assumption (A1). We assume that the target market needs to be split into different regions, and $R = \{1,\ldots, r\}$ denotes the set of regions in the market. This is less of an assumption and more of a model feature. Each region is expected to have its distinct product configuration demand and substitution behavior (e.g., colder northern states generally exhibit less demand for convertible models and have higher demand for features such as heated seats and engine block heaters). From our conversations with SMEs, this is how OEMs model the market.

b. Assumption (A2). Given the long lead times associated with assortment planning and the significant uncertainty in fuel prices and their impact on product demand, we consider different potential scenarios for market fuel price. A probability is associated with each of the scenarios and the associated product demand mix (i.e., the demand for the different configurations). We impose no restrictions on the structure of the relationship between fuel price and the product demand mix within these scenarios.

b. Assumption (A3). We assume that every potential customer has a favorite (most preferred) configuration from the set N. Under fuel price scenario $f \in FP$, the potential demand for each configuration $i \in N$, at each region $r \in R$, is a known fraction of total market demand, M. Note that total market demand (as well as demand mix for different technologies) could take different values across fuel price scenarios. If the customer cannot find her favorite configuration i, she will decide not to substitute with probability $\delta_i^r(s)$, else, will choose configuration j with probability α_{ji}^r. We assume that customers in a region would only substitute to available configurations across the region (and not between regions) in case their favorite configuration is missing.

2. Supply

a. Assumption (A4). Although there are multiple market realization scenarios (in terms of fuel price and associated demand), given the long lead times involved for product development and supply chain readiness, the OEM has to decide on a unique product configuration assortment upfront (strategic planning process). The model also has to decide upfront the planned production volumes for each configuration, so that the supply chain can install the necessary manufacturing capacity for producing the parts/options content. While the OEMs can try to adjust these production capacities after launching the product and observing the actual realized demand, this is often a very lengthy and expensive process for highly engineered and complex products such as automobiles and could take several months to a year. OEMs can also rely on tactics such as price rebates to alter demand. These reactionary decisions are more tactical in nature and outside the scope of our strategic assortment planning model. This being a strategic model, we model the production supply and consumption setting as a single-period problem, also known as the newsboy or news-vendor model, where the manufacturer would supply the regions with product configurations at the beginning of the time period; we do not explicitly model the on-going replenishment process with dealers ordering and the OEM trying to fulfill the orders.

b. Assumption (A5). We allow economies-of-scale for the OEM in purchasing parts/options from the suppliers as a function of purchase quantity. That is, the OEM could receive discounts on some parts/options if purchased in large quantities. We assume that the information related to discounts is exogenous to our model and purchasing cost is assumed to follow a step-wise non-increasing function as a function of purchase quantity; we assume an all-unit quantity discount model. If parts are shared across models, the step-wise function is expected to capture the incremental price benefits from using the part within the product under consideration. This is again less of an assumption and more of a flexible model feature.

3. Costs

a. Assumption (A6). We assume that each potential configuration j has a variable production and supply cost $c_j^{variable}$. We assume that prices for each product configuration, p_j, are set exogenously and are available a *priori* for product assortment planning. In addition, we assume that each configuration j has a fixed cost c_j^{fixed}, if selected in the final assortment. The fixed cost can be attributable to factors such as incremental design, engineering, testing, quality, warranty, and service costs from adding the configuration to the assortment.

4. Environmental Factors

   a. Assumption (A7). We assume that the OEM will target a specific average fuel economy (AFE) for each model in the aim of meeting the CAFE or similar requirements for the overall company across all models.

   b. Assumption (A8). We assume that the effect of any tax and excise policy on fuel prices (e.g., policies encouraging diesel vehicles in Europe) and/or financial incentives for purchasing fuel efficient vehicles (e.g., recent subsidies for electric cars in the U.S.) would only affect potential demand for each configuration within these regions and does not incur any cost to the OEM.

   c. Assumption (A9). We limit modeling of greenhouse gas (GHG) emissions to just product use emissions from tailpipe and upstream fuel supply chain GHG emissions. The data we are using in our numerical experiments is derived from www.fueleconomy.gov and those estimates include CO_2, methane, and nitrous oxide emitted from all steps in the use of a fuel, from production and refining to distribution and final use—vehicle manufacture is excluded. Methane and nitrous oxide emissions are converted into a CO_2 equivalent. Tailpipe emissions and upstream emissions—those that occur prior to the fuel being used in the vehicle—are displayed. We also assume that all vehicle units sold will be used for the same number of years and the emissions limit is in average tons/year. Future studies should account for complete product life-cycle emissions, including manufacturing, supply chain, and distribution (i.e., Scope 1, 2, and 3) and other emissions besides CO2.

Furthermore, we assume that the regulations affecting the assortment decisions do not change during the planning horizon. We also assume that the fuel prices will remain constant during the planning horizon based on one of the fuel price scenarios. There is also no explicit consideration for supply chain decisions in the model.

3.4. Methodology

In this section, we present our framework to model assortment planning for automotive products. The mathematical model seeks to maximize OEM's profit subjected to feasibility and environmental constraints. Before presenting the model's objective function, it is necessary that we introduce demand modeling structure based on an exogenous demand model. Assume that $d_{i,r,f}$ is the primary (first-choice) demand for configuration i in region r under fuel price scenario f. The observed demand for configuration i in region r under fuel price scenario f would be introduced and calculated as follows:

$$E_{i,r,f} = d_{i,r,f} + \sum_{\forall j \neq i} d_{j,r,f} \cdot (1 - X_j). \alpha_{ji} \quad (3\text{-}1)$$

where α_{ji} is the substitution probability for a customer to switch to configuration i after not finding her favorite choice, configuration j ; X_j is a binary decision variable equal to one if configuration i is a part of the final assortment (i.e., built), and zero otherwise. We can now introduce the mathematical model to find the optimal assortment. A summary of notation and definition of all parameters and decision variables is provided in Appendix A.

Let $\pi(f, Y)$ be the one-period (newsvendor) profit for the OEM, from stocking assortment Y, when fuel price realization is f. We have:

$$\pi(f,Y) = \sum_{i=1}^{N}\sum_{r=1}^{R}\left[(P_i - C_i^{Variable})\cdot(Y_i^r - \mathcal{L}_{i,r,f}) - C_i^{overage}.\mathcal{L}_{i,f}^r\right]$$

$$-\sum_{i=1}^{N} C_i^{fixed}.X_i^{config} - \sum_{p=1}^{P} C_p^{fixed\_part}.X_p^{part} + \sum_{\forall p\in P}\sum_{\forall l\in L} v_l^p.\tilde{z}_l^p \qquad (3\text{-}2)$$

This profit function consists of revenue associated with sales minus any fixed cost associated with offering configurations as well as parts/options. The last term in equation (3-2) is the saving due to the economies-of-scale. Then, the expected profit would be:

$$E\Pi(Y) = \sum_{f\in FP}\pi(f,Y).P(f) \qquad (3\text{-}3)$$

Remember that FP represents the set of fuel price scenarios and $P(f)$ is the corresponding probability. We would like to maximize $E\Pi(\mathrm{Y})$ with respect to capacity and feasibility constraints. The complete formulation for the optimization problem is presented in Appendix B.

3.5. Numerical Experiment

In consultation with several subject-matter-experts (SMEs) from the U.S. automotive industry, we generated a set of hypothetical product assortment planning problems generally representative of the mid-size sedan segment in the U.S. These problems carried 120 potential product configurations for consideration.[4] The SMEs also provided guidance in generating cost data of each part/option, and subsequently, deriving the unit costs associated with each configuration. The profit margins were set between 15%-30% of the unit cost (with 15% for cheapest configurations and 30% for most expensive ones) and added to the unit cost to compute

[4] It is typical for OEMs to limit the strategic assortment planning activity to key vehicle part/option content (e.g., body styles, engines, transmissions) to limit data collection and model formulation complexity and avoid considering relatively simple/cheap accessories such a floor mats and most other dealer installed content. Colors also often finalized much later.

the selling price for each configuration[5]. The overage cost of left-over inventory at the end of the selling period is assumed to be between 4%-12% of the unit cost of a configuration (4% for cheapest configurations and 12% for most expensive ones). Other data/parameters such as substitution probabilities (α_{ij}), primary demand fractions ($d_{i,r,f}$), fixed costs of offering configuration ($C_i^{fixed\_conf}$), and fixed costs of offering options ($C_p^{fixed_{part}}$) were also generated in consultation with SMEs. In total, the 120 potential configurations employ 15 different parts/options which are grouped into powertrain technology choices (with 10 different types consisting of 6 gasoline engine choices, 3 diesel engine choices, and one hybrid engine choice), three body style choices (sedan, two-door coupe, and hatchback), sunroof option, and finally a satellite radio option. While the sunroof and satellite options are truly optional (meaning that the customer can select a configuration without these options), powertrain and body style choices are choices (e.g., customer cannot select a configuration without a powertrain). Also, we assume three different market scenarios based on low, medium, and high but realistic prices of fuel. In each of these scenarios, the customers exhibit different demand for configurations with different levels of fuel economy. Given that the assortment planning problem is a strategic problem and the OEM cannot easily change the product assortment in response to changes in fuel prices (though customers tend to react fast to big swings in fuel prices as noticed during the last decade), hence, the model aims to find a robust yet optimal assortment that best maximizes the expected profit across all possible fuel-price / demand scenarios. In deriving the settings for our artificial experiments, we not only relied on the viewpoints of several SMEs from the North-American OEMs but also the official U.S. government source for 'fuel economy information'[6] to

[5] In this study, we assume that the prices are fixed and do not change as a function of the fuel prices encountered in the market. This assumption has to be relaxed in a future study.
[6] Data is gathered from http://www.fueleconomy.gov

make sure that the data employed is consistent with the real-world situation. All generated data are presented in detail in Appendix C.

Based on powertrain technology, we categorize the whole set of configurations into conventional, diesel, and hybrid vehicles. Each of these configurations has its specific fuel economy and product use emission footprint, which could affect the optimal assortment through either average fuel economy (AFE) requirement or maximum allowed product use emission footprint constraints. Figure 1 shows the primary demand fractions for different vehicles (based on technology class) under different scenarios. As expected for the U.S. market, demand for conventional powertrain technologies (i.e., with gasoline engines) is the highest while there is much less demand for diesel and hybrid technologies.[7] As evident from the figure, the demand for hybrid and diesel technologies is assumed to increase with higher fuel prices, for their higher fuel efficiency.

Table 3.1 shows average profit margin, average overage cost, fuel efficiency in miles-per-gallon (MPG), and greenhouse gas (GHG) emissions for the different technologies. As is currently the case, hybrid technologies are least profitable models but offer the highest fuel efficiency and lowest GHG emissions while the 3.5 liter automatic all-wheel drive (AWD) powertrain technology builds the most profitable configurations that return the lowest fuel efficiency and release highest GHG emissions. Our goal is to maximize total expected profit while satisfying AFE requirements and emissions constraints by identifying the supply amount (if any) of each configuration at each region. We used ILOG-CPLEX 11.0 to run the proposed mathematical model.

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[7] This is very different from other markets such as Europe where diesel powertrains carry a large market-share in many vehicle segments.

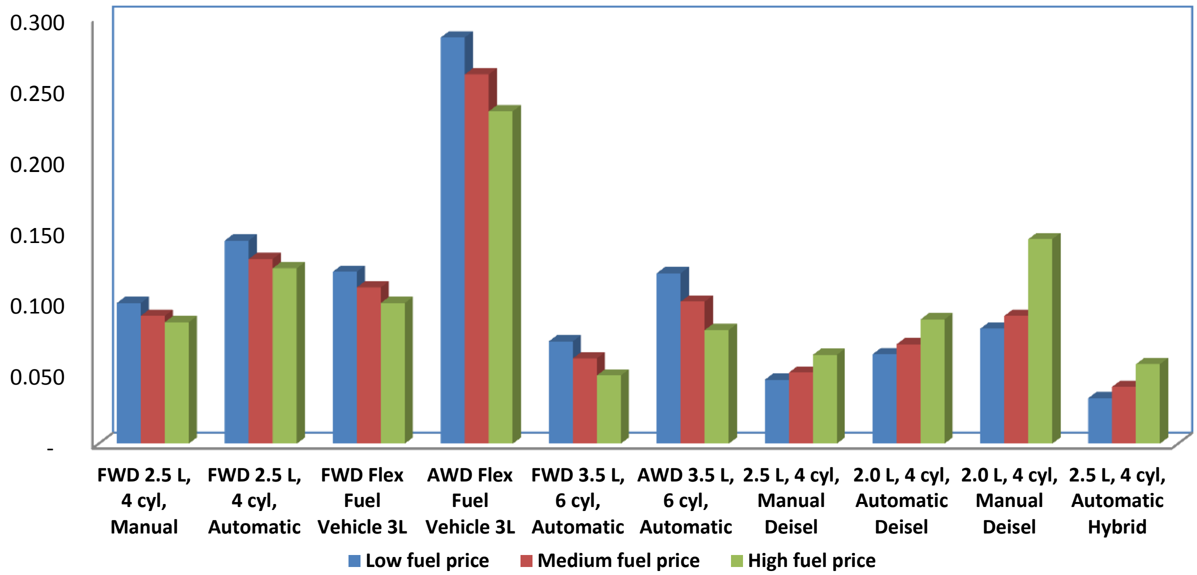

Figure 3.1: Demand fraction for ten different powertrain technologies under different fuel prices.

We first investigate the effect of environmental constraints on the optimal assortment. Figure 3.2 shows the optimal solution for the problem under different AFE levels. First, it shows the optimal solution when there is no AFE constraint, and then, by considering AFE requirements under different levels.

The first counter-intuitive observation is that production shares for different technologies overall look different from their primary demand fractions. As an example, although the all-wheel-drive (AWD) 3.0 liter powertrain has the highest primary demand fraction across all technologies in all fuel price scenarios, this technology is only selected under two AFE requirement levels (No AFE & AFE = 25) and it has zero production for the other cases. The inconsistency between primary demand fraction and production share is mainly a result of environmental restrictions, product substitutions, and economies-of-scale. Another unexpected observation is seen by comparing 2.0 liter (2.0L) Automatic Diesel technology with FWD 3.0L, FWD 3.5L Automatic, and AWD 3.5L Automatic. The diesel technology is getting much higher production share in all AFE levels (in particular when there is no AFE requirement) compared to those conventional technologies even-though the average profit margin is at least 20% less for the Diesel technology. The exact reason behind this observation is not clear for us; however, we do suspect that the effect of product substitution is an important factor in determining the optimal share for each configuration.

In addition, one could observe that some particular technologies are not profitable in most AFE scenarios (e.g., 2.5L Manual Diesel, AWD 3.0L, and 2.5L hybrids). Since there are many factors affecting the optimal assortment solution (substitution effects, option fixed costs, economies-of-scale, etc.), it is not straightforward to predict the behavior of the optimal solution. However, one might consider the fact that 2.5 Manual Diesel has a lower MPG w.r.t other diesel technologies, and hence, is not getting a share in the optimal solution.

One intuitive observation is that the optimal solution gives a higher share to some of the fuel efficient technologies when we consider environmental requirements. Also, one might observe that 3.5L FWD Automatic technologies get smaller production share w.r.t 3.5L AWD Automatic technologies in most AFE scenarios (except AFE = 28) even-though they have a better fuel economy, which is possibly the result of having lower average profit margin. Another observation is that hybrid technologies seem to be non-profitable (at least with our current settings) due to low profit margins even-though they are very fuel-efficient. This particular experiment suggests diesel technologies to be a dominant alternative for hybrid technologies in order to achieve higher AFE levels.

Table 3.1: Average margin, overage cost, MPG, and GHG emissions for different powertrain technologies

	Technology	Average margin ($)	Average overage cost ($)	MPG	Greenhouse Gas Emissions (tons/year)
Conventional	2.5 L, 4 cyl, Manual , FWD	2,500	1,200	25*	7.3*
	2.5 L, 4 cyl, Automatic, FWD	3,000	1,400	26*	7.1*
	FFV 3.0 L, 6 cyl, Automatic, FWD	3,000	1,400	23*	8.0*
	FFV 3.0 L, 6 cyl, Automatic, AWD	2,800	1,300	20*	9.1*
	3.5 L, 6 cyl, Automatic, FWD	3,000	1,400	21*	8.7*
	3.5 L, 6 cyl, Automatic, AWD	3,100	1,500	19*	9.6*
Diesel	2.5 L, 4 cyl, Manual Diesel	2,600	1,500	26**	7.1**
	2.0 L, 4 cyl, Automatic Diesel	2,500	1,500	33**	6.3**
	2.0 L, 5 cyl, Manual Diesel	2,400	1,600	34**	5.9**
Hybrid	2.5 L, 4 cyl, Automatic Hybrid	1,800	1,300	39*	4.7*

* Numbers are from http://www.fueleconomy.gov
** Numbers are generated based on data for similar class vehicles

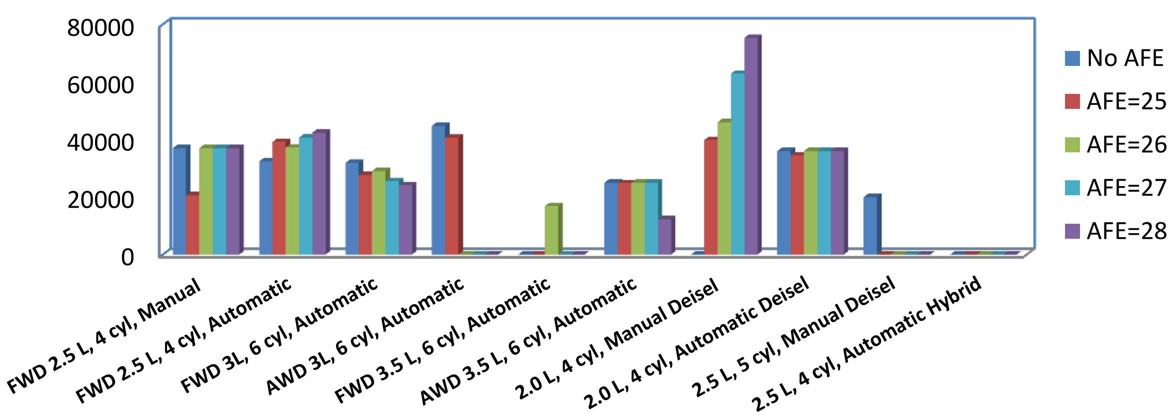

Figure 3.2: Production levels for different powertrain technologies under different AFE requirements

In terms of profit, it can be seen from Figure 3 that satisfying AFE requirements is reducing OEM’s profit from 0.77% to 6.91% for different AFE requirements. This is a considerable share of profit and suggests need for potential investment in developing fuel efficient technologies at lower prices. Finally, Figure 4 shows amount of average annual GHG emissions under different AFE levels, which steadily reduces when satisfying higher AFE levels. This diagram can be a good indicator of pollution savings while forcing automakers to satisfy AFE requirement. Considering two extreme AFE levels (AFE=28 and No AFE) in Figures 3 and 4, the net effect is a reduction of GHG emissions by about 200,000 U.S. tons (almost one U.S. tons per vehicle) costs almost 69 million dollars for the automakers (which could be a potential limit on any subsidies spent on GHG emission reduction in the auto industry).

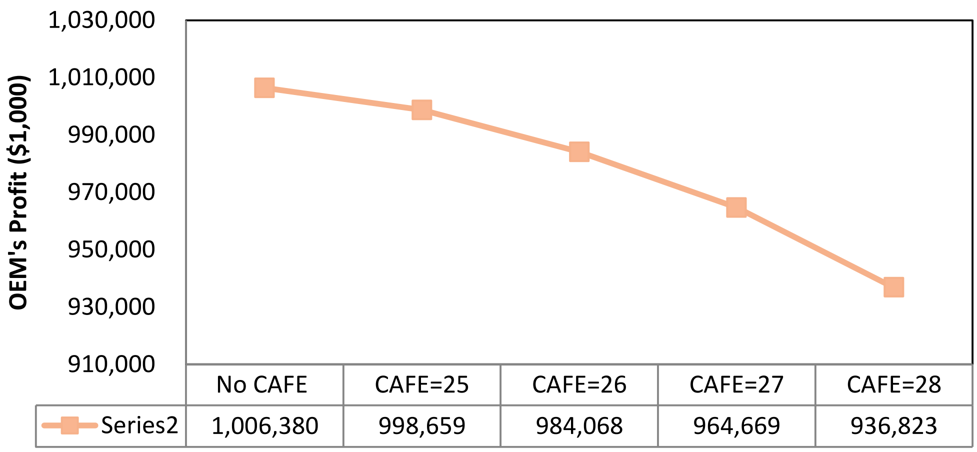

Figure 3.3: Total OEM’s profit under different CAFE requirements

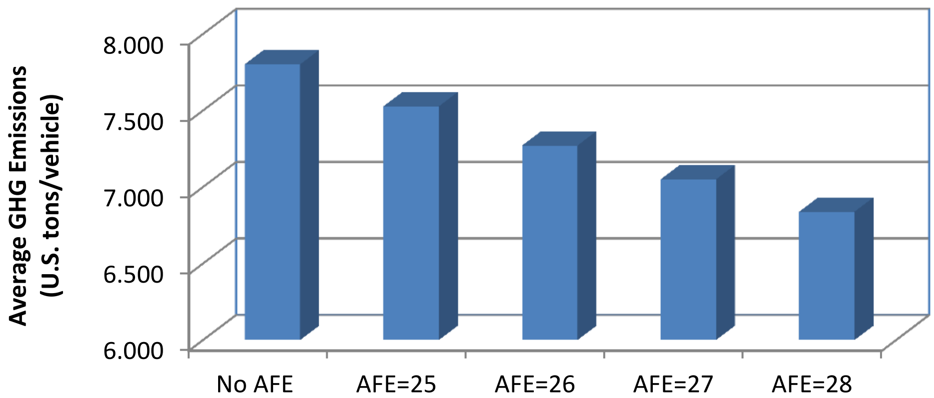

Figure 3.4: Average Annual GHG Emissions (U.S. tons/vehicle) under Different AFE Requirements

3.6. Conclusion

We present a modeling framework for assortment planning of automotive products using an exogenous demand model. The objective is to maximize OEM's profit while meeting a set of environmental constraints. The numerical experiments presented suggest that production rates have a different trend w.r.t. primary demand fractions due to product substitution effects, profit margin differences, and environmental constraints. Also, we show that hybrid technologies are not being selected in the optimal assortment solution because of low profit margins while 2.0 L manual Diesel technologies have been selected as a reliable alternative to help meet CAFE requirements.

3.7. Appendix

Appendix A: Notation

Table 3.2: Problem parameters, Sets, and Decision variables

Sets	
$i \in 1,\ldots,N$	Set of all potential configurations,
$r \in 1,\ldots,R$	Set of all market regions,
$p \in 1,\ldots,P$	denote the set of all options required for assortment production,
$l \in 1,\ldots,L$	Set of all discount levels for an option.
$f \in 1,\ldots,FP$	Set of all scenarios on fuel price.

Parameters	
$\mathrm{d_{i,r,f}}$	Primary (first-choice) demand for configuration i in region r under fuel price scenario f when facing a full assortment
$\mathrm{FuelEC_i}$	fuel economy (calculated based on miles per gallon) of configuration i
CAFE_Standard	CAFE standard regulated by US federal government
$\mathrm{LCCE_i}$	Life cycle carbon equivalent emission of configuration i
Max_LCCE	Maximum level of total carbon equivalent emission allowed for the whole assortment
$\mathrm{\alpha_{ij}}$	Substitution probability of switching to configuration i after not finding the favorite choice, configuration j
$\mathrm{p_i}$	Selling price of configuration i
$\mathrm{c_i^{variable}}$	Variable cost of every unit of configuration i
$\mathrm{C_i^{fixed}}$	Fixed cost of configuration i , if carried in the market.
$\mathrm{C_i^{overage}}$	Overage cost of configuration i if not sold.
$\mathrm{v_l^p}$	Amount of discount for each unit of option p if purchased at quantity level l
$\mathrm{\beta_i^p}$	Bill of material parameter equal to one if configuration i requires option p and zero otherwise.
$\mathrm{LP_l^p}$	The lower-point of quantity for level l of option p.
$BigM$	A sufficiently large number
$\mathrm{Y_{max}}$	Maximum production capacity of the OEM
P(f)	Probability mass function for fuel price scenario f

Decision Variables	
Y_i^r	Number of vehicles of configuration i supplied to the region r of the market.
X_i^{config}	A binary decision variable equal to one if configuration i is built, and zero otherwise.
X_p^{part}	A binary decision variable equal to one if option p is required, and zero otherwise.
Z^p	Total required units of option p for the production of all configurations
$\tilde{z}_l^p$	Total required units of option p purchased at discount level l
$Z\_Binary_l^p$	A binary decision variable equal to one if option p is purchased at discount level l, and zero otherwise.
$L_{i,r,f\|Y}$	Left-over inventory of configuration i in region r , under fuel price scenario f, given supply vector is Y.
$E_{i,r,f}$	Effective demand for configuration i in region r, under fuel price scenario f

Appendix B: Mathematical Formulation

$$max_{Y,X}\, E\Pi(\boldsymbol{Y},\boldsymbol{X}) = \sum_{f\in FP}\{\sum_{i=1}^{N}\sum_{r=1}^{R}[(P_i - C_i^{Variable})\cdot(Y_i^r - L_{i,r,f}) - C_i^{overage}.L_{i,r,f}] - \sum_{i=1}^{N} C_i^{fixed}.X_i^{config} - \sum_{p=1}^{P} C_p^{fixed\_part}.X_p^{part} + \sum_{\forall p\in P}\sum_{\forall l\in L} v_l^p.\tilde{z}_l^p\}.P(f) \quad (3\text{-}4)$$

s.t.

$$E_{i,r,f} = d_{i,r,f} + \sum_{\forall j\neq i} d_{j,r,f}\cdot\left(1 - X_j\right).\alpha_{ij}.\frac{N}{\sum_{\forall k} X_k}\,\forall i,r,f \quad (3\text{-}5)$$

$$\mathcal{L}_{i,r,f} = (Y_i^r - E_{i,r,f})^+ \qquad \forall i,r,f \quad (3\text{-}6)$$

$$\sum_{\forall r\in R} Y_i^r \leq Y_{max}\cdot X_i^{config} \qquad \forall i \quad (3\text{-}7)$$

$$X_i^{config} \leq \sum_{\forall r\in R} Y_i^r \qquad \forall i \quad (3\text{-}8)$$

$$\sum_{\forall r\in R}\sum_{\forall i\in N} Y_i^r \leq Y_{max} \quad (3\text{-}9)$$

$$\sum_{\forall i\in N} X_i^{config} \leq X_{max} \quad (3\text{-}10)$$

$$\sum_{\forall i\in N}\sum_{\forall r\in R} Y_i^r \geq CAFE\_Standard * \sum_{\forall i\in N}\frac{\sum_{\forall r\in R} Y_i^r}{FuelEC_i} \quad (3\text{-}11)$$

$$\sum_{\forall i\in N}(LCCE_i.\sum_{\forall r\in R} Y_i^r) \leq Max_{LCCE} * \sum_{\forall i\in N}\sum_{\forall r\in R} Y_i^r \quad (3-12)$$

$$Z^p = \sum_{\forall i\in N}\{(\sum_{\forall r\in R} Y_i^r).\beta_i^p\} \qquad \forall p\ in\ part \quad (3\text{-}13)$$

$$Z^p \leq \mathrm{BigM} * \mathrm{X}_{\mathrm{p}}^{\mathrm{part}} \qquad \forall p\ in\ part \quad (3\text{-}14)$$

$$Z^p = \sum_{\forall l}\tilde{z}_l^p \qquad \forall p\ in\ part \quad (3\text{-}15)$$

$$\sum_{\forall l} Z\_Binary_l^p = 1 \qquad \forall p\ in\ part \quad (3\text{-}16)$$

$$\tilde{z}_l^p \leq LP_{l+1}^p \times Z\_Binary_l^p \qquad \forall p,l \quad (3\text{-}17)$$

$$\tilde{z}_l^p \leq LP_l^p \times BigM \qquad \forall p,l \quad (3\text{-}18)$$

$$\tilde{z}_l^p \geq LP_l^p - BigM * (1 - Z_{Binary}{}_l^p) \qquad \forall p, l \qquad (3\text{-}19)$$

$$\tilde{z}_l^p \leq LP_{l+1}^p + BigM * (1 - Z_{Binary}{}_l^p) \qquad \forall p, l \qquad (3\text{-}20)$$

$$X_i^{config} \in \{0,1\} \qquad \forall i \in N \qquad (3\text{-}21)$$

$$Z^p \in \mathbb{R}^+,\ X_p^{part} \in \{0,1\} \qquad \forall p \in P \qquad (3\text{-}22)$$

$$Y_i^r \in \mathbb{R}^+ \qquad \forall i \in N, \forall r \in R \qquad (3\text{-}23)$$

$$\mathcal{L}_{i,r,f}\,, d_{i,r,f}^E \in \mathbb{R}^+ \qquad \forall i \in N, \forall r \in R\,, \forall f \in FP \qquad (3\text{-}24)$$

Constraint (3-5) captures effective demand for configuration i in region r under fuel price scenario fp. This constrain consists of original demand ($d_{i,r,f}$) plus any demand as a result of substitution from missing products. Constraint (3-6) determines the left-over inventory of configuration i in region r, constraint (3-7) is to guarantee that total production does not exceed the maximum production limit. Constraint (3-8) is to ensure that there is no production for a vehicle that is not to be built. Constraints (3-9 and 3-10) are optional and used to limit the total production/maximum number of configurations supplied to the market. Constraint (3-11) is to ensure that the assortment satisfies the OEM's target average fuel economy (*AFE*) for the model in support of meeting overall CAFE requirement for the whole company; is the linearized form of this formulation:

$$\frac{\sum_{\forall i \in N} \sum_{\forall r \in R} Y_i^r}{\sum_{\forall i \in N} \frac{\sum_{\forall r \in R} Y_i^r}{FuelEC_i}} \geq AFE \qquad (3\text{-}25)$$

Constraint (3-12) is to ensure that the average annual tail-pipe and upstream fuel CO_2 emissions in the assortment do not exceed a predetermined threshold. Constraints (3-13 and 3-14) find total units of option p required for the production of all configurations. Constraints (3-15 to 3-20) are required to determine the discount level from economies-of-scale for each option in the assortment. Equations (3-21to 3-24) declare the variable types.

Appendix C: Data Employed for Numerical Experiments

Table 3.3: Total demand and corresponding probabilities in different scenarios

	Fuel price Scenario 1	Fuel price Scenario 2	Fuel price Scenario 3
Total demand	200,000	200,000	200,000
Probability	0.2	0.45	0.35

Table 3.4: Selling price, unit and overage cost, and primary demand fraction per configuration

Configuration ID	Technology type	Selling price ($1,000)	Unit cost ($1,000)	Overage cost ($1,000)	Primary demand fraction in fuel price scenario 1	Primary demand fraction in fuel price scenario 2	Primary demand fraction in fuel price scenario 3
1	2.5 L, 4 cyl, Manual , FWD	17.6	14.6	1.8	0.0241	0.0219	0.0208
2		18.2	15.1	1.9	0.0145	0.0132	0.0125
3		15.5	13.4	0.7	0.0130	0.0118	0.0112
4		15.6	13.5	0.7	0.0097	0.0088	0.0083
5		16	13.9	0.7	0.0080	0.0073	0.0069
6		17.6	14.6	1.8	0.0078	0.0071	0.0067
7		18.6	15.5	1.9	0.0052	0.0047	0.0045
8		20.4	17	2.1	0.0048	0.0044	0.0042
9		15.1	13.1	0.7	0.0043	0.0039	0.0037
10		15.9	13.8	0.7	0.0032	0.0029	0.0028
11		16	13.9	0.7	0.0026	0.0024	0.0022
12		16.1	14	0.7	0.0017	0.0016	0.0015
13	2.5 L, 4 cyl, Automatic, FWD	22.8	17.5	2.1	0.0349	0.0317	0.0301
14		14.2	12.3	0.7	0.0209	0.0190	0.0181
15		17.3	14.4	1.8	0.0188	0.0171	0.0162
16		19.2	16	2	0.0139	0.0127	0.0120
17		18	15	1.8	0.0116	0.0106	0.0100
18		14.7	12.7	0.7	0.0113	0.0102	0.0097
19		15.5	13.4	0.7	0.0075	0.0068	0.0065
20		22.8	17.5	2.1	0.0070	0.0063	0.0060
21		15.7	13.6	0.7	0.0063	0.0057	0.0054
22		17.2	14.3	1.8	0.0046	0.0042	0.0040
23		15.6	13.5	0.7	0.0038	0.0034	0.0032
24		17.2	14.3	1.8	0.0025	0.0023	0.0022
25	FFV 3.0 L,	23.4	18	2.2	0.0295	0.0268	0.0241

26	6 cyl, Automatic, FWD	19	15.8	1.9	0.0177	0.0161	0.0145
27		13.7	11.9	0.6	0.0159	0.0144	0.0130
28		14.5	12.6	0.7	0.0118	0.0107	0.0097
29		16	13.9	0.7	0.0098	0.0089	0.0080
30		14.7	12.7	0.7	0.0095	0.0087	0.0078
31		23.4	18	2.2	0.0064	0.0058	0.0052
32		14.8	12.8	0.7	0.0059	0.0054	0.0048
33		18.6	15.5	1.9	0.0053	0.0048	0.0043
34		25.4	19.5	2.4	0.0039	0.0036	0.0032
35		19.6	16.3	2	0.0032	0.0029	0.0026
36		15.1	13.1	0.7	0.0021	0.0019	0.0017
37	FFV 3.0 L, 6 cyl, Automatic, AWD	20.4	17	2.1	0.0697	0.0634	0.0570
38		18.6	15.5	1.9	0.0418	0.0380	0.0342
39		15.1	13.1	0.7	0.0375	0.0341	0.0307
40		15.2	13.2	0.7	0.0279	0.0254	0.0228
41		15.9	13.8	0.7	0.0232	0.0211	0.0190
42		17.7	14.7	1.8	0.0225	0.0205	0.0184
43		17.8	14.8	1.8	0.0150	0.0137	0.0123
44		15.9	13.8	0.7	0.0139	0.0127	0.0114
45		19.5	16.2	2	0.0125	0.0114	0.0102
46		14.2	12.3	0.7	0.0093	0.0085	0.0076
47		24.1	18.5	2.3	0.0075	0.0068	0.0061
48		19.6	16.3	2	0.0050	0.0046	0.0041
49	3.5 L, 6 cyl, Automatic, FWD	14.3	12.4	0.7	0.0176	0.0146	0.0117
50		22.8	17.5	2.1	0.0105	0.0088	0.0070
51		14.2	12.3	0.7	0.0095	0.0079	0.0063
52		15.1	13.1	0.7	0.0070	0.0059	0.0047
53		20.2	16.8	2.1	0.0059	0.0049	0.0039
54		15.2	13.2	0.7	0.0057	0.0047	0.0038
55		26	20	2.4	0.0038	0.0032	0.0025
56		18	15	1.8	0.0035	0.0029	0.0023
57		14.7	12.7	0.7	0.0032	0.0026	0.0021
58		20.1	16.7	2.1	0.0023	0.0020	0.0016
59		13.3	11.5	0.6	0.0019	0.0016	0.0013
60		20.2	16.8	2.1	0.0013	0.0011	0.0008
61	3.5 L, 6 cyl, Automatic, AWD	15.7	13.6	0.7	0.0293	0.0244	0.0195
62		17.2	14.3	1.8	0.0176	0.0146	0.0117
63		23.8	18.3	2.2	0.0158	0.0131	0.0105
64		23.4	18	2.2	0.0117	0.0098	0.0078
65		18.3	15.2	1.9	0.0098	0.0081	0.0065
66		17.1	14.2	1.8	0.0095	0.0079	0.0063
67		13.7	11.9	0.6	0.0063	0.0053	0.0042
68		19	15.8	1.9	0.0059	0.0049	0.0039
69		17.2	14.3	1.8	0.0053	0.0044	0.0035
70		13.7	11.9	0.6	0.0039	0.0033	0.0026
71		14.5	12.6	0.7	0.0032	0.0026	0.0021
72		20.1	16.7	2.1	0.0021	0.0018	0.0014
73	2.5 L, 4 cyl, Manual	14.1	12.8	0.7	0.0197	0.0219	0.0351
74		14.9	13.5	0.7	0.0118	0.0132	0.0211

75	Diesel	14	12.7	0.7	0.0106	0.0118	0.0189
76		21.5	17.2	2.1	0.0079	0.0088	0.0140
77		17.3	15	1.8	0.0066	0.0073	0.0117
78		12.3	11.1	0.6	0.0064	0.0071	0.0113
79		21.7	17.3	2.1	0.0043	0.0047	0.0076
80		24.4	19.5	2.4	0.0039	0.0044	0.0070
81		18.8	16.3	2	0.0035	0.0039	0.0063
82		14.5	13.1	0.7	0.0026	0.0029	0.0047
83		21.5	17.2	2.1	0.0021	0.0024	0.0038
84		13.2	12	0.6	0.0014	0.0016	0.0025
85	2.0 L, 4 cyl, Automatic Diesel	17	14.7	1.8	0.0154	0.0171	0.0213
86		23.4	18.7	2.3	0.0092	0.0102	0.0128
87		23.5	18.8	2.3	0.0083	0.0092	0.0115
88		15.2	13.8	0.7	0.0061	0.0068	0.0085
89		17.9	15.5	1.9	0.0051	0.0057	0.0071
90		18.7	16.2	2	0.0050	0.0055	0.0069
91		17	14.7	1.8	0.0033	0.0037	0.0046
92		13.6	12.3	0.7	0.0031	0.0034	0.0043
93		13.7	12.4	0.7	0.0028	0.0031	0.0038
94		19.4	16.8	2.1	0.0020	0.0023	0.0028
95		15.4	14	0.7	0.0017	0.0018	0.0023
96		14.3	13	0.7	0.0011	0.0012	0.0015
97	2.0 L, 5 cyl, Manual Diesel	22.2	17.7	2.2	0.0110	0.0122	0.0152
98		17.9	15.5	1.9	0.0066	0.0073	0.0091
99		12.8	11.6	0.6	0.0059	0.0066	0.0082
100		19.3	16.7	2.1	0.0044	0.0049	0.0061
101		12.7	11.5	0.6	0.0037	0.0041	0.0051
102		18.4	16	2	0.0035	0.0039	0.0049
103		22.9	18.3	2.2	0.0024	0.0026	0.0033
104		24	19.2	2.4	0.0022	0.0024	0.0030
105		16.4	14.2	1.8	0.0020	0.0022	0.0027
106		13.1	11.9	0.6	0.0015	0.0016	0.0020
107		18.4	16	2	0.0012	0.0013	0.0016
108		14.9	13.5	0.7	0.0008	0.0009	0.0011
109	2.5 L, 4 cyl, Automatic Hybrid	20.2	17.5	1.6	0.0078	0.0098	0.0137
110		19.8	17.2	1.6	0.0047	0.0059	0.0082
111		16.5	15	1.4	0.0042	0.0053	0.0074
112		14.2	13.5	0.6	0.0031	0.0039	0.0055
113		11.7	11.1	0.5	0.0026	0.0033	0.0046
114		18.2	16.5	1.5	0.0025	0.0032	0.0044
115		21.6	18.7	1.7	0.0017	0.0021	0.0029
116		17.1	15.5	1.4	0.0016	0.0020	0.0027
117		20.7	18	1.7	0.0014	0.0018	0.0025
118		13.7	13	0.6	0.0010	0.0013	0.0018
119		17.6	16	1.5	0.0008	0.0011	0.0015
120		20.2	17.5	1.6	0.0006	0.0007	0.0010

Table 3.5: Bill of materials

Configuration ID	Technology type	4 Doors Body	2 Doors Body	Hatch Back Body	Sunroof	Satellite radio
1	2.5 L, 4 cyl, Manual , FWD	1	0	0	1	1
2		0	1	0	1	1
3		1	0	0	0	1
4		0	0	1	1	1
5		1	0	0	1	0
6		0	1	0	0	1
7		0	0	1	0	1
8		0	1	0	1	0
9		1	0	0	0	0
10		0	0	1	1	0
11		0	1	0	0	0
12		0	0	1	0	0
13	2.5 L, 4 cyl, Automatic, FWD	1	0	0	1	1
14		0	1	0	1	1
15		1	0	0	0	1
16		0	0	1	1	1
17		1	0	0	1	0
18		0	1	0	0	1
19		0	0	1	0	1
20		0	1	0	1	0
21		1	0	0	0	0
22		0	0	1	1	0
23		0	1	0	0	0
24		0	0	1	0	0
25	FFV 3.0 L, 6 cyl, Automatic, FWD	1	0	0	1	1
26		0	1	0	1	1
27		1	0	0	0	1
28		0	0	1	1	1
29		1	0	0	1	0
30		0	1	0	0	1
31		0	0	1	0	1
32		0	1	0	1	0
33		1	0	0	0	0
34		0	0	1	1	0
35		0	1	0	0	0
36		0	0	1	0	0
37	FFV 3.0 L, 6 cyl, Automatic, AWD	1	0	0	1	1
38		0	1	0	1	1

39		1	0	0	0	1
40		0	0	1	1	1
41		1	0	0	1	0
42		0	1	0	0	1
43		0	0	1	0	1
44		0	1	0	1	0
45		1	0	0	0	0
46		0	0	1	1	0
47		0	1	0	0	0
48		0	0	1	0	0
49		1	0	0	1	1
50		0	1	0	1	1
51		1	0	0	0	1
52		0	0	1	1	1
53		1	0	0	1	0
54	3.5 L, 6 cyl, Automatic, FWD	0	1	0	0	1
55		0	0	1	0	1
56		0	1	0	1	0
57		1	0	0	0	0
58		0	0	1	1	0
59		0	1	0	0	0
60		0	0	1	0	0
61		1	0	0	1	1
62		0	1	0	1	1
63		1	0	0	0	1
64		0	0	1	1	1
65		1	0	0	1	0
66	3.5 L, 6 cyl, Automatic, AWD	0	1	0	0	1
67		0	0	1	0	1
68		0	1	0	1	0
69		1	0	0	0	0
70		0	0	1	1	0
71		0	1	0	0	0
72		0	0	1	0	0
73		1	0	0	1	1
74		0	1	0	1	1
75		1	0	0	0	1
76		0	0	1	1	1
77		1	0	0	1	0
78	2.5 L, 4 cyl, Manual Diesel	0	1	0	0	1
79		0	0	1	0	1
80		0	1	0	1	0
81		1	0	0	0	0
82		0	0	1	1	0
83		0	1	0	0	0
84		0	0	1	0	0
85	2.0 L, 4 cyl, Automatic Diesel	1	0	0	1	1
86		0	1	0	1	1
87		1	0	0	0	1

88		0	0	1	1	1
89		1	0	0	1	0
90		0	1	0	0	1
91		0	0	1	0	1
92		0	1	0	1	0
93		1	0	0	0	0
94		0	0	1	1	0
95		0	1	0	0	0
96		0	0	1	0	0
97	2.0 L, 5 cyl, Manual Diesel	1	0	0	1	1
98		0	1	0	1	1
99		1	0	0	0	1
100		0	0	1	1	1
101		1	0	0	1	0
102		0	1	0	0	1
103		0	0	1	0	1
104		0	1	0	1	0
105		1	0	0	0	0
106		0	0	1	1	0
107		0	1	0	0	0
108		0	0	1	0	0
109	2.5 L, 4 cyl, Automatic Hybrid	1	0	0	1	1
110		0	1	0	1	1
111		1	0	0	0	1
112		0	0	1	1	1
113		1	0	0	1	0
114		0	1	0	0	1
115		0	0	1	0	1
116		0	1	0	1	0
117		1	0	0	0	0
118		0	0	1	1	0
119		0	1	0	0	0
120		0	0	1	0	0

Table 3.6: Fixed cost and discount information per part/option

Part ID	Part/option Name	Fixed cost ($1,000)	Minimum purchase (to receive discount)	Discount
1	2.5 L, 4 cyl, Manual , FWD	10,800	37,000	180
2	2.5 L, 4 cyl, Automatic, FWD	12,000	50,000	200
3	FFV 3.0 L, 6 cyl, Automatic, FWD	13,200	40,000	220
4	FFV 3.0 L, 6 cyl, Automatic, AWD	15,300	16,000	255
5	3.5 L, 6 cyl, Automatic, FWD	16,500	12,000	275
6	3.5 L, 6 cyl, Automatic, AWD	18,000	25,000	300
7	2.5 L, 4 cyl, Manual Diesel	22,500	46,000	375
8	2.0 L, 4 cyl, Automatic Diesel	24,000	36,000	400
9	2.0 L, 5 cyl, Manual Diesel	25,500	20,000	425
10	2.5 L, 4 cyl, Automatic Hybrid	30,000	20,000	500
11	4 Doors Body	4,500	95,000	75
12	2 Doors Body	6,000	72,000	100
13	Hatch Back Body	4,500	60,000	75
14	Sunroof	3,600	130,000	60
15	Satellite radio	2,400	135,000	40

Chapter 4. Packaging Product Features for Automotive Assortments

4.1. Introduction

Building on the models presented in earlier chapters, here, we once again limit our discussion/experiments to the automotive industry and develop product assortment planning models that explicitly consider “feature packaging” rules in decision making. In the past decades, there has been considerable work dedicated to demand aspects of assortment planning (see Kök et al. 2008, for a literature review). Furthermore, some researchers have been interested in studying the effect of variety on manufacturing performance (MacDuffie et al. 1996) or its effect on assembly operations (Fisher and Ittner, 1999). There has also been some research on feature bundling and its effect on consumer behavior (Johnson et al 1999, Hamilton and Koukova 2008). But to the best of our knowledge, there is not yet a comprehensive approach that considers feature packaging in planning product assortments for configurable products, the focus of this Chapter.

The rest of the Chapter is organized as follows: Section 2 reviews the most relevant literature; Section 3 discusses the problem setting and the main assumptions behind our model in more detail. Problem formulation is presented in Section 4 while a number of numerical experiments are discussed in Section 5. Finally, Section 6 gives some conclusions.

4.2. Literature Review

In terms of research on product bundling, we can see two different approaches to bundling in the literature. Venkatesh and Mahajan (2009) describe these major types of methodological approaches as design oriented and pricing oriented approaches. The first approach is mainly used to identify the set of products which should go into the bundle while the latter one “typically assumes a product portfolio and proposes the prices at which the individual items and/or bundles should be offered”. In this paper we use packaging as a tool to identify the features that should be grouped together to form a bundle for consumers. Hanson and Martin (1990) present a mathematical programming formulation to determine the bundle configurations and prices to maximize profit. Venkatesh and Mahajan (1993) propose an approach to optimally price a bundle of products or services to maximize firms’ profit. They consider a multi-criteria decision-making situation and find the optimal price for a given bundle under different bundling strategies. Johnson et, al. (1999) study the effects of price bundling on how consumers evaluate product offerings. Later, Stremersch and Tellis (2002) present a set of strategies for bundling and discuss that price bundling and product bundling are independent strategies allowing firms to mix and match them in order to meet consumer demand better. Bitran and Ferrer (2007) consider the problem to find the optimal selection of the bundle’s component and the optimal price to offer it. They assume that demand is modeled using a multinomial logit model (MNL) and is a function of the bundle price and product attributes. Gurler et al (2009) study optimal bundling and pricing decisions for perishable products with limited stocks and discuss how to form bundles and how to price them in order to maximize expected profit.

4.3. Assumptions

In order to manage product assortment complexity, the OEMs usually build and offer the optimal assortment to the dealers under a particular representation where all product configurations are grouped into a number of series. In this representation, each individual series consist of a number of base parts/options which are decided up-front for that series (like a fixed powertrain choice, a fixed body type choice, etc.) besides a number of packages and/or stand-alone options that the customer can choose from. The packages usually include more than one option and the customer should either choose a configuration containing all options in the package or choose a configuration with none of the options in the package. Note that the content of the packages and stand-alone choices (as well as the size of the packages) is not set prior to solving the model. We mention the rules/requirements that are used in this study to return optimal packages as well as the optimal assortment. These rules/requirements are based on our discussion with several Subject Expert Matters from North-American auto-makers and are as following:

1- If a package in series X consists of options A and B, all the configurations in this series, should have either both options A and B or neither of them installed.

2- We consider upper and lower limits on the size of the packages, however the formulation is capable of handling any arbitrary combination of number of packages and any package size

3- The price of configurations within a particular series is subject to be within a given range. (for example S-series are subject to take lower price range compared to premium series)

4- There is a lower and upper limit on the price of any package for different series.

5- The model allows assigning an option to more than one package within a particular series.

6- There are some exclusive rules by automakers that prohibit offering some of the options in some particular series (for example 2.0L EcoBoost Automatic AWD and BLIS are never offered for basic series like series S). These exclusive rules are exogenous data to our model.

Now suppose that for the product under consideration, $N = \{1,\ldots, n\}$ denotes the set of potential configurations that can be made available by the OEM. Assortment planning involves selecting a subset of these configurations for tooling the assembly line and readying the dealers/retailers which maximizes OEM's profit. We present the rest of assumptions in this paper as follows:

Every potential customer has a favorite configuration from the set N. The potential demand for each configuration $i \in N$, d_i, is an allocated and known fraction of total market demand, M. We believe that the fractional allocation model is an acceptable approach for modeling demand since some major automakers, including Toyota, have adopted this method for mix planning (Iyer et all, 2009). We assume that if the customer cannot find her favorite configuration i, she will decide not to substitute with probability δ_i. Else, she will choose configuration j with probability α_{ij}. The substitution probabilities are assumed to be exogenous to our model and are flexibly allowed to take any structure. However, for our numerical experiments and without loss of generality, we consider substitution probabilities to be derived based on price and content similarities between configurations which is similar to the $a\ priory\ substitutability$ concept discussed by Vaagen et al (2011). We also consider fixed cost $c_p^{fixed\_part}$ associated with offering part/option p as well as $c_j^{fix\_conf}$ associated with fixed cost of introducing configuration j in the assortment. We assume that each configuration j has a manufacturing unit cost c_j^{unit} and that the prices for each product configuration, p_j, are set exogenously and are available $a\ priory$ for product assortment planning. We also consider complexity cost factor in our model which could incur additional costs due to larger number of configurations (less efficiency in balancing assembly line and increased TACT time, etc.) and/or larger number of potential choices per parts/options (more manufacturing facilities, more experienced labor forces, etc.). In this paper and based on discussions with couple of Subject Expert Matters in automotive industry, we assume that complexity cost is a concave non-decreasing function with regard to size of the assortment. We also assume that the cost of supplying different parts/options from the suppliers to OEM may be affected by the volume. That is, the company would receive discounts on some options if purchased in large quantities. The information related to discount policies is exogenous to our model and the supplier offers its products under All-Unit-Discount policy and the OEM does not incur any additional cost other than unit purchasing price due to manufacturing or complexity costs of supplier.

4.4. Problem Formulation

In this section, we present our framework to model assortment planning for automotive products. The mathematical model seeks to maximize OEM's profit subjected to feasibility constraints and packaging requirements. Before presenting the model's objective function, it is necessary that we introduce demand modeling structure based on an exogenous demand model. Assume that d_i is the primary (first-choice) demand for configuration i. The observed demand for configuration i would be introduced and calculated as follows:

$$E_i = d_i + \sum_{\forall j \neq i} d_j \cdot (1 - X_j).\alpha_{ji} \tag{4-1}$$

where α_{ji} is the substitution probability for a customer to switch to configuration i after not finding her favorite choice, configuration j. After calculating the effective demand for each configuration, we can build our model to find the optimal assortment. Let $\pi(Y)$ be the one-period (newsvendor) profit for the OEM, from stocking assortment Y, We have:

$$\pi(Y) = \sum_{i=1}^{N} \{(SP_i - C_i^{Variable}) \cdot (Y_i - \mathcal{L}_i) - C_i^{overage}.\mathcal{L}_i\}$$

$$-\sum_{i=1}^{N} C_i^{fixed}.X_i - \sum_{p=1}^{P} C_p^{fixed_{part}}.P_p$$

$$-Complexity\_Cost + \sum_{\forall p \in P} \sum_{\forall l \in L} v_l^p . \tilde{z}_l^p \qquad (4\text{-}2)$$

This profit function consists of revenue associated with sales minus complexity cost and any fixed cost associated with offering configurations as well as parts/options. The last term in equation (4-2) is the discounts received due to the economies-of-scale. We would like to maximize $\pi(Y)$ with respect to capacity and feasibility constraints and packaging requirements. Here, we present the whole optimization problem followed by the feasibility constraints.

$$\max_{Y,X} E\Pi(Y,X) = \sum_{i=1}^{N} \{(SP_i - C_i^{Variable}) \cdot (Y_i - L_i) - C_i^{overage} . L_i\}$$

$$-\sum_{i=1}^{N} C_i^{fixed} . X_i - \sum_{p=1}^{P} C_p^{fixed_{part}} . P_p$$

$$-Complexity_{Cost} + \sum_{\forall p \in P} \sum_{\forall l \in L} v_l^p . \tilde{z}_l^p \qquad (4\text{-}3)$$

s.t.

$E_i = d_i + \sum_{\forall j \neq i} d_j \cdot (1 - X_j) . \alpha_{ji}$ ∀i in Configuration   (4-4)

$\mathcal{L}_i \geq Y_i - E_i$ ∀i in Configuration   (4-5)

$\sum_{\forall r \in R} Y_i \leq Y_{max} \cdot X_i$ ∀i in Configuration   (4-6)

$X_i \leq \sum_{\forall r \in R} Y_i$ ∀i in Configuration   (4-7)

$\sum_{\forall r \in R} \sum_{\forall i \in N} Y_i \leq Y_{max}$ (4-8)

$Z^p = \sum_{\forall i \in N} \{(\sum_{\forall r \in R} Y_i) . \beta_i^p\}$ ∀p in part   (4-9)

$Z^p \leq BigM * P_p$ ∀p in part   (4-10)

$Z^p = \sum_{\forall l} \tilde{z}_l^p$ ∀p in part   (4-11)

$\sum_{\forall l} Z\_Binary_l^p = 1$ ∀p in part   (4-12)

$$\tilde{z}_l^p \leq LP_{l+1}^p \times Z\_Binary_l^p \qquad \forall p, l \qquad (4\text{-}13)$$

$$\tilde{z}_l^p \leq BigM \times Z\_Binary_l^p \qquad \forall p, l \qquad (4\text{-}14)$$

$$\tilde{z}_l^p \geq LP_l^p - BigM * (1 - Z_{Binary_l^p}) \qquad \forall p, l \qquad (4\text{-}15)$$

$$\tilde{z}_l^p \leq LP_{l+1}^p + BigM * (1 - Z_{Binary_l^p}) \qquad \forall p, l \qquad (4\text{-}16)$$

$$Complexity\_Cost = ComY_k + \left(\frac{ComY_{k+1} - ComY_k}{ComX_{k+1} - ComX_k}\right) * \left(\sum_{i=1}^{N} X_i - ComX_k\right)$$

$$\forall k \in 1..NbComlevel - 1 \qquad (4\text{-}17)$$

$$\sum_{\forall s \in S} X\_Series_{i,s} \leq 1 \qquad \forall i \text{ in } N \qquad (4\text{-}18)$$

$$\sum_{\forall o \in OPTIONS} (package_{s,o,c} * option\_price_o) \leq upper\_price_{s,c}$$

$$\forall s \text{ in } SERIES, c \text{ in } PACKAGES \qquad (4\text{-}19)$$

$$lower\_price_{s,c} \leq \sum_{\forall o \in OPTIONS} (package_{s,o,c} * option\_price_o)$$

$$\forall s \text{ in } SERIES, c \text{ in } PACKAGES \qquad (4\text{-}20)$$

$$\sum_{\forall o \in OPTIONS} package_{s,o,c} \leq upper\_size_{s,c} \qquad \forall s \text{ in } SERIES, c \text{ in } PACKAGES \qquad (4\text{-}21)$$

$$lower\_size_{s,c} \leq \sum_{\forall o \in OPTIONS} package_{s,o,c} \qquad \forall s \text{ in } SERIES, c \text{ in } PACKAGES \qquad (4\text{-}22)$$

$$Package_{s,o,c} \leq P_o \qquad \forall o \text{ in } OPTIONS, s \text{ in } SERIES, c \text{ in } PACKAGES \qquad (4\text{-}23)$$

$$P_o \leq \sum_{s \text{ in } SERIES, c \text{ in } PACKAGES} Package_{s,o,c} \qquad \forall o \text{ in } OPTIONS \qquad (4\text{-}24)$$

$$X\_Series_{i,s} \leq \sum_{c \text{ in } PACKAGES} Package_{s,o,c} \qquad \forall s \text{ in } SERIES, o \text{ in } OPTIONS, i \text{ in } N{:}\ \beta_i^o = 1 \qquad (4\text{-}25)$$

$$X\_Series_{i,s} * \beta_i^o \leq excluded\_options_{s,o} \qquad \forall s \text{ in } SERIES, o \text{ in } OPTIONS, i \text{ in } N \qquad (4\text{-}26)$$

$$X\_Series_{i,s} \leq X_i \qquad \forall s \text{ in } SERIES, i \text{ in } N \qquad (4\text{-}27)$$

$$X_i \leq \sum_{s \text{ in } SERIES} X\_Series_{i,s} \qquad \forall i \text{ in } N \qquad (4\text{-}28)$$

$$\sum_{\forall i \in CONFIGS:\ SP_i \geq max\_price\_range_s} X\_Series_{i,s} = 0 \qquad \forall s \text{ in } SERIES \qquad (4\text{-}29)$$

$$\sum_{\forall i \in \mathrm{CONFIGS:\,SP}_i \le \mathrm{min\_price\_range}_s} X\_Series_{i,s} = 0 \qquad \forall \mathrm{s\ in\ SERIES} \tag{4-30}$$

$$X\_Series_{i,s} \le 2 - X\_Series\_prime_{i,s,c} - X\_Series\_second_{i,s,c}$$

$$\forall \mathrm{i\ in\ N, s\ in\ SERIES, c\ in\ PACKAGES} \tag{4-31}$$

$$X\_Series\_prime_{i,s,c} \le \sum_{o\ in\ OPTIONS} (Package_{s,o,c} * \mathrm{\beta_i^o})$$

$$\forall \mathrm{i\ in\ N, s\ in\ SERIES, c\ in\ PACKAGES} \tag{4-32}$$

$$X\_Series\_second_{i,s,c} \le \sum_{o\ in\ OPTIONS} Package_{s,o,c} - \sum_{o\ in\ OPTIONS} (Package_{s,o,c} * \mathrm{\beta_i^o})$$

$$\forall \mathrm{i\ in\ N, s\ in\ SERIES, c\ in\ PACKAGES} \tag{4-33}$$

$$\sum_{o\ in\ OPTIONS} (Package_{s,o,c} * \mathrm{\beta_i^o}) \le BigM * X\_Series\_prime_{i,s,c}$$

$$\forall \mathrm{i\ in\ N, s\ in\ SERIES, c\ in\ PACKAGES} \tag{4-34}$$

$$\sum_{o\ in\ OPTIONS} Package_{s,o,c} - \sum_{o\ in\ OPTIONS} (Package_{s,o,c} * \mathrm{\beta_i^o}) \le BigM * X\_Series\_second_{i,s,c}$$

$$\forall \mathrm{i\ in\ N, s\ in\ SERIES, c\ in\ PACKAGES} \tag{4-35}$$

$$\mathrm{\tilde{z}_l^p} \in \{0,1\},\ \mathrm{Z_{Binary}}_\mathrm{l}^\mathrm{p} \in \mathbb{R}^+\ \forall \mathrm{p, l} \tag{4-36}$$

$$\mathrm{X_i} \in \{0,1\}\ \ \forall \mathrm{i} \in \mathrm{N} \tag{4-37}$$

$$\mathrm{Y_i^r} \in \mathbb{R}^+\quad \forall \mathrm{i} \in \mathrm{N}, \forall \mathrm{r} \in \mathrm{R} \tag{4-38}$$

$$\mathrm{Z^p} \in \mathbb{N} \cup \{0\},\ \mathrm{P_p} \in \{0,1\}\ \ \forall \mathrm{p} \in \mathrm{P} \tag{4-39}$$

$$\mathrm{Y_i^r} \in \mathbb{R}^+\ \ \forall \mathrm{i} \in \mathrm{N}, \forall \mathrm{r} \in \mathrm{R} \tag{4-40}$$

$$\mathcal{L}_\mathrm{i}, \mathrm{E_i} \in \mathbb{R}^+\ \forall \mathrm{i} \in \mathrm{N} \tag{4-41}$$

$$X\_Series_{i,s} \in \{0,1\}\ \ \forall \mathrm{i} \in \mathrm{N, s\ in\ SERIES} \tag{4-42}$$

$$\mathrm{package}_{s,o,c} \in \{0,1\}\ \ \forall \mathrm{s\ in\ SERIES, o\ in\ OPTIONS, c\ in\ PACKAGES} \tag{4-43}$$

$$X\_Series\_prime_{i,s,c}, X\_Series\_second_{i,s,c} \in \{0,1\}\ \ \forall i \in \mathrm{N}, s\ \mathrm{in\ SERIES, c\ in\ PACKAGES} \tag{4-44}$$

Equation (4-4) is to calculate the observed (effective) demand while equation (4-5) determines the left-over inventory of configuration i. Equations (4-6 and 4-7) are to make sure that there is no production for a configuration that is not built. Equation (4-8) is to guarantee that total production does not exceed the maximum production limit. Equations (4-9 and 4-10) finds total units of option p required for the production of all configurations. Equations (4-11 to 4-16) are to choose a discount level (for economies-of-scales) for each option in the assortment and equation (4-17) calculates piece-wise linear manufacturing complexity cost. Equation (4-18) states that each configuration should be assigned to no more than one series. Equations (4-19 to 4-20) put a limit on total cost of all options in a package (or a standalone option) while equations (4-21 and 4-22) limit the number of options in package. Equation (4-23) states that when a part is assigned to any package, it is forced to be built and equation (4-24) determines when a part is not assigned to any package in no series, it is forced to be not built. Equation (4-25) requires that when a series doesn't have a particular option in any of the packages, that series shouldn't offer any configuration containing that option. Equation (4-26) prohibits option p to be offered on series s in case of exogenous restrictions and equations (4-27 and 4-28) enforce a one to one relation between X_Series and X decision variables. Equations (4-29 and 4-30) force the configurations in each series to take prices within a pre-determined range (with some possible overlaps across some series). Finally equations (4-31 to 4-35) are a set of linear rules to make sure that if series s includes package p with some options, all the configurations in series s have to contain either all options in package p or none of those options is allowed for configurations within series s.

4.5. Numerical Experiments

We generated an example with 192 potential configurations and made an artificial data set for cost of each part/option and based on that, derived the unit cost associated with each configuration. We also generate artificial data related to other parameters like substitution probabilities (α_{ij}), demand fractions ($\mathrm{d_i}$), fixed costs of offering configuration ($\mathrm{C_i^{fixed\_conf}}$), and fixed costs of offering option ($\mathrm{C_p^{fixed\_part}}$). All 192 potential configurations are made from 11 different options consisting of six powertrain technology choices (PT1: 2.5L i4 Manual FWD, PT2: 2.5L i4 Automatic FWD, PT3: 1.6L EcoBoost Automatic FWD, PT4: 1.6L EcoBoost Automatic AWD, PT5: 2.0L EcoBoost Automatic FWD, and PT6: 2.0L EcoBoost Automatic AWD), and 5 binary options: Ford Sync, Moon-roof, Rearview Camera, Navigation system, and Blind Spot Information System (BLIS). The generated data is presented in appendix B.

We would like to maximize total profit while meeting packaging requirements by identifying the supply amount (if any) of each configuration. We use ILog-Cplex 11.0 to run the mathematical model presented in this paper. A summary of results is presented in tables 4.1 and 4.2

Table 4.1: Average profit margin, price ranges and number of configurations in each series

	Series 1	Series 2	Series 3	Series 4
Total number of configurations	4	15	25	14
Minimum selling price	14,400	16,100	18,700	23,300
Maximum selling price	15,300	19,300	23,400	25,700
Average selling price	14,875	17,887	20,170	24,264

As seen in the table, different series are taking quiet different assortment sizes with Series 1 being the smallest and series 3 the largest one. This is mainly due to higher primary demand fractions of configurations that are allowed to be built on series 2 and 3. Also, the total size of assortment is 58 out of 192 potential configurations which is considerably small size.

Table 4.2: Options available over different series

	Powertrains (Baseline)	First Package		Second Package		First standalone	Second standalone
Series 1	PT1, PT2	Ford Sync	-	-		-	-
Series 2	PT1,PT2,PT3,PT4	Ford Sync	Moon roof	Rearview Camera	-	Navigation	-
Series 3	PT3,PT4	Rearview Camera	-	Ford Sync	Navigation	Moon roof	BLIS
Series 4	PT5,PT6	Moon roof	-	Ford Sync	Navigation	Rearview Camera	BLIS

Table 4.3: Optimal size of packages and standalones across all four series

	First package			Second package			First standalone			Second standalone		
Package Size	Optimal Size	Lower Limit	Upper Limit	Optimal Size	Lower Limit	Upper Limit	Optimal Size	Lower Limit	Upper Limit	Optimal Size	Lower Limit	Upper Limit
Series 1	1	0	1	-			-			-		
Series 2	2	1	2	1	1	1	1	0	1	-		
Series 3	1	1	2	2	1	3	1	0	1	1	0	1
Series 4	1	1	2	2	1	3	1	0	1	1	0	1

4.6. Conclusion

We present a framework for packaging product features for automotive assortments in order to maximize OEM's profit. Overall, it looks that feature packaging gives a very powerful tool to decision makers in auto industry to select the optimal assortment as well to represent it in an easy way to customers. It also helps automakers to considerably cut their assortment size to manage their manufacturing complexity costs.

4.7. Appendix

Appendix A: Notation

Table 4.4: Problem parameters, sets, and decision variables

Sets	
$i \in 1, \dots, N$	Set of all potential configurations,
$p \in 1, \dots, P$	Set of all parts/options that build configurations
$l \in 1, \dots, L$	Set of all discount levels for an option.
$k \in 1..\mathrm{NbComlevel}$	Set of all breakpoints of piecewise complexity cost
$s \in \mathrm{SERIES}$	Set of all vehicle series
$c \in \mathrm{PACKAGES}$	Set of all packages (including standalones) associated with all series
$o \in \mathrm{OPTIONS}$	Set of parts/options that can be offered to customers as a choice in any of the PACKAGES

Parameters	
d_i	Primary (first-choice) demand fraction for configuration i when facing a full assortment
$\mathrm{option\_price}_o$	Price of standalone part/option o when offered to customers
$\mathrm{lower\_price}_{s,c}$	Minimum acceptable price of package (or standalone) c when offered at series s
$\mathrm{upper\_price}_{s,c}$	Maximum acceptable price of package (or standalone) c when offered at series s
$\mathrm{lower\_size}_{s,c}$	Minimum acceptable size of package (or standalone) c when offered

	at series s
$\mathrm{upper\_size}_{s,c}$	Maximum acceptable size of package (or standalone) c when offered at series s
$\mathrm{excluded\_options}_{s,o}$	Equal to one if option o is allowed to be offered in series s , otherwise zero
α_{ij}	Substitution probability of switching to configuration i after not finding the favorite choice, configuration j
SP_i	Selling price of configuration i,
$c_i^{variable}$	Variable cost of every unit of configuration i
C_i^{fixed}	Fixed cost of configuration i , if carried in the market.
$C_i^{overage}$	Overage cost of configuration i if not sold.
v_l^p	Amount of discount for each unit of option p if purchased at quantity level l
β_i^p	Bill of material parameter equal to one if configuration i requires option p and zero otherwise.
LP_l^p	The lower-point of quantity for level l of option p.
BigM	A sufficiently large number
Y_{max}	Maximum production capacity of the OEM
$ComY_k$	Total manufacturing complexity cost for k^{th} level of complexity
$ComX_k$	Assortment size at k^{th} level of complexity

Decision Variables	
Y_i^r	Number of vehicles of configuration i supplied to the region r of the market.
X_i	A binary decision variable equal to one if configuration i is built, and zero otherwise.
P_p	A binary decision variable equal to one if option p is required, and zero otherwise.
Z^p	Total required units of option p for the production of all configurations
$\tilde{z}_l^p$	Total required units of option p purchased at discount level l
$Z\_Binary_l^p$	A binary decision variable equal to one if option p is purchased at discount level l, and zero otherwise.
$L_{i\|Y}$	Left-over inventory of configuration i given supply vector is *Y*.
E_i	Effective demand for configuration i
Complexity_Cost	Total manufacturing complexity cost
$X\_Series_{i,s}$	A binary decision variable equal to one if configuration i is offered at series s
$package_{s,o,c}$	A binary decision variable equal to one if option o is offered at package c on series s
$X\_Series\_prime_{i,s,c}$	Intermediate binary variables
$X\_Series\_second_{i,s,c}$	Intermediate binary variables

Chapter 5. Conclusion & Future Research

In this dissertation, we study product assortment planning for a manufacturer of configurable products. In the second chapter, we study assortment planning of configurable products under stock-out based substitution. We use an exogenous demand model that accounts for product substitution, manufacturing complexity cost and economies-of-scale. We develop a mixed-integer-nonlinear program and propose an alternate (linear) Modified Branch and Bound procedure for solving the original problem. The result shows the importance of considering substitution effects for product assortment planning. We also observe that M-BnB method surpasses Bonmin in terms of solution time for all the selected experiments while in terms of objective function it returns a better solution in 11 out of 14 experiments with two other experiments returning the same objective value. In the third chapter, we present a framework for assortment planning of automotive products using an exogenous demand modeling. Our objective is maximizing OEM's profit through considering a set of environmental constraints. The numerical experiment presented in this chapter suggests that production rates have a different trend w.r.t primary demand fractions due to product substitution effects. Also, we show that hybrid technologies are not being selected in the optimal solution because of low profit margins while 2.0 L manual Diesel technologies have been selected as a reliable alternative to help meet CAFE requirements. Finally and in the fourth chapter, we present a framework for packaging product features for automotive assortments in order to maximize OEM's profit. We show that feature packaging is a very powerful tool for decision makers in auto industry to select the optimal assortment as well to represent it in an easy way to customers. It also helps automakers to considerably cut their assortment size to manage their manufacturing complexity costs.

5.1. Future Research

During different parts of this dissertation, we are using an exogenous demand model for the primary demand fraction and substitution parameters. However, it could be quiet difficult to estimate these parameters when dealing with real world problems. We see it very worthy to develop methodologies for estimating those parameters.

As mentioned earlier, our scope is to provide strategic level guidance on the optimal assortment at different marketing regions followed by the production levels. A more tactical level decision would be achieved by extending this model to determine the optimal stock levels of configurations at dealer level. These types of models could be more attractive due to very high stock-out rates at dealers as well as a relatively high rate of transshipment among dealers in a region.

Another possible extension to our current work could be achieved in terms of solution strategies. Even though our Modified Branch and Bound method is able to solve larger models much faster compared to current MINLP solvers, there are still real world problems that our model is not capable of solving and returning an optimal solution in a reasonable amount of time. For this reason, we suggest enhancing our current procedure in order to deal with more real world problems.

We also assumed single period models with no replenishment of supply and exogenous prices; however, as an extension, we suggest building models with on-going replenishment process and/or models that account for pricing and price-demand elasticity.

In our proposed models we assume that individual market regions are served by a single dealer; however, in reality, there a number of dealers/region stocking limited product and making stock-out rates even higher and substitution options are more limited and come at expense if they involve transshipments. So, we suggest building models that explicitly account for distribution channel constraints. Lastly, we suggest considering the effect of assortment complexity not only on production and manufacturing costs (which we considered in this study) but also on the inventory holding costs as well as dealer capacities. This will provide more guidance to managers on the optimal level of assortment complexity for OEMs.

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ABSTRACT

AN INTEGRATED FRAMEWORK FOR CONFIGURABLE PRODUCT ASSORTMENT PLANNING

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May 2013

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Major: Industrial and Systems Engineering

Degree: Doctor of Philosophy

A manufacturer's assortment is the set of products or product configurations that the company builds and offers to its customers. While the literature on assortment planning is growing in recent years, it is primarily aimed at non-durable retail and grocery products. In this study, we develop an integrated framework for strategic assortment planning of configurable products, with a focus on the highly complex automotive industry. The facts that automobiles are highly configurable (with the number of buildable configurations running into thousands, tens of thousands, and even millions) with relatively low sales volumes and the stock-out rates at individual dealerships (even with transshipments) are extremely high, pose significant challenges to traditional assortment planning models. This is particularly the case for markets such as the U.S. that mostly operate in a make-to-stock (MTS) environment. First, we study assortment planning models that account for exogenous demand models and stock-out based substitution while considering production and manufacturing complexity costs and economies-of-scale. We build a mathematical model that maximizes the expected profit for an Original Equipment Manufacturer (OEM) and is a mixed-integer nonlinear problem. We suggest using linear

lower/upper bounds that will be solved through a Modified-Branch and Bound procedure and compare the results with commercial mixed-integer nonlinear solvers and show superiority of the proposed method in terms of solution quality as well as computational speed.

We then build a modeling framework that identifies the optimal assortment for a manufacturer of automotive products under environmental considerations, in particular, Corporate Average Fuel Economy (CAFE) requirements as well as life-cycle Greenhouse Gas (GHG) emission constraints. We present a numerical experiment consisting of different vehicle propulsion technologies (conventional, Diesel, and hybrid) and study the optimal shares of different technologies for maximizing profitability under different target levels of CAFE requirements. Finally, we develop assortment planning formulations that can jointly identify optimal packages and stand-alone options over different series of the product model. Our numerical experiment reveals that product option packaging has a considerable effect on managing product complexity and profitability.

AUTOBIOGRAPHICAL STATEMENT

Seyed Ali Taghavi received his B.S. and M.S. degrees in Industrial Engineering in 2005 and 2008, respectively, from Sharif University of Technology, Tehran, Iran. He received his PhD degree in Industrial Engineering from Wayne State University (Detroit, Michigan) in 2012. He has also experienced working as an operations research scientist for a supply chain design company.

His research interests are in planning and modeling configurable product assortments. He studied application of advanced Operations Research methodologies for assortment planning models under very high stock-out rates. He also studied the impact of environmental considerations as well as feature packaging requirements on the optimal assortment of an automotive industry.

Mr. Taghavi has published a number of peer reviewed journal articles and conference proceedings and made several technical presentations at INFORMS annual meetings and POMS conference.