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An Overview Of The Respondent-Generated Intervals (RGI) Approach To Sample Surveys

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This article brings together many years of research on the Respondent-Generated Intervals (RGI) approach to recall in factual sample surveys. Additionally presented is new research on the use of RGI in opinion surveys and the use of RGI with gamma-distributed data. The research combines Bayesian hierarchical modeling with various cognitive aspects of sample surveys.

Keywords: anchoring, Bayesian, confidence scale, recall, surveys

I. Introduction

This work provides an overview of the research to date on the Respondent-Generated Intervals, or RGI, protocol for asking questions in sample surveys. It brings together a body of research that started in 1996 with some theoretical ideas about how survey questionnaire design might be improved by asking respondents for more than just a basic answer to a question, but by also trying to elicit information about how certain the respondents might be about their answers. Over the years we developed various theoretical models for analyzing such RGI data from a survey, culminating in the current Bayesian hierarchical model detailed in Section II. With the development of a theoretical model came the need to explore how well the model might work in practice, with real people and real empirical data.

We examined pencil-and-paper classroom surveys, and a telephone survey using Census data. We have thought about possible internet surveys, but have not yet fielded this type of survey. Various surveys we carried out under the different survey protocols are described in Section III. Our conclusions so far can be found in Section IV.

The RGI protocol was originally developed to deal with survey questions requiring recall of numerical facts; it has since been extended to address questions of opinion as well. This extension will be discussed below. In its original form, the RGI protocol for asking questions in sample surveys involves asking each respondent not only for a basic answer to a recall-type question (an answer we call a “usage quantity”) but also, for a smallest value his/her true answer could be, and a largest value his/her true answer could be. We’ll refer to these values as the lower and upper bounds. The result of the RGI protocol is that the respondents themselves generate the intervals in which their true beliefs

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lie, instead of having their quantitative beliefs forced into intervals pre-assigned by the survey designer, as is often done in other survey protocols. (For a discussion of other survey protocols using intervals or brackets, see Press, 2004).

Interval-Response Surveys

Survey protocols that permit the respondent to give answers in intervals, self-determined, or pre-assigned by the survey designer, are often preferred by respondents for sensitive questions because the respondent need not be specific about the exact value being requested. Interval response protocols are also often preferred by respondents for questions for which the answers are not very well known. By responding in intervals for such questions, respondents need not be precise about the exact answer (see Lusinchi, 2003). Respondents prefer the RGI technique because it allows them to have control over their disclosures, and RGI allows respondents to feel confident about the accuracy of the information they provide. The intervals RGI respondents provide tend to be narrower than pre-defined intervals (see Schwartz and Paulin, 2000).

Genesis of RGI

The RGI protocol for questionnaire design has its origins in Bayesian assessment procedures. In that context, for a specific individual, we might assess an entire prior distribution about an unknown parameter. That prior distribution represents the individual's degrees of uncertainty about that unknown parameter. In certain contexts, we might assess many points on the individual's subjective probability distribution for that parameter by means of a sequence of elicitation questions, and then connect those points by a smooth curve that purports to represent the underlying distribution. In the RGI protocol, because of concern for respondent burden in surveys, we ask for only three points on the recall distribution.

For example, using some purely hypothetical numbers, suppose an individual has a normal subjective probability distribution representing "θ_0", the true (but unknown) change in the number of doctor visits he/she believes he/she made last year, compared with the previous year, so that $\theta_0 \sim N(4,1)$. (We use "change" in doctor visits as our illustrative variable in order to provide for both positive and negative values of the variable; thus we make the assumption of normality more plausible.) In such a case, the individual believes that it is most likely that he/she visited a doctor 4 more times last year than the previous year, with a standard deviation of 1.

So this individual equivalently believes that there is a 99.7% chance that he/she visited a doctor between 1 and 7 more times last year, or that there is really almost no chance that the true number of additional times was less than 1 or greater than 7. This probability distribution is subjective, in that it represents a specific individual's degrees of belief about his/her uncertainty about the underlying quantity, in the case of this example, the individual's uncertainty about how many more visits he/she believes he/she truly made to the doctor last year compared with the previous year.

We postulate that: in a factual survey each respondent has a distinctive recall distribution, and in an attitude or opinion survey he/she has an underlying probability distribution for his/her opinion or attitude about some issue. In the case of a recall-type question, we assume that the respondent knew the true value at some time in the past (or knew enough to construct an accurate answer) but because of imperfect recall, he/she is not now certain of the true value. He/she may feel confident that he/she knows the true value (but may be wrong in spite of high confidence), or he/she may be quite uncertain of the true value (and conceivably could be correct about the true value, but not realize it). We furthermore assume that the respondent is not purposely trying to deceive. In the case of opinion or attitude questions, the respondent may have a very fuzzy idea of his/her attitude about an issue, or he/she may feel quite strongly and specifically about it.

II. Theoretical Developments

A. Normal Data for Recall Questions

Suppose respondents answer independently and suppose respondent i gives a point response, y_i, and bounds (a_i, b_i), $a_i \le b_i$, $i = 1,\ldots, n$, as his/her answers to a factual recall question. We'll refer to y_i as respondent i's "usage quantity" (the term "usage quantity" was introduced originally to reflect estimated frequency of a behavior). The random quantities (y_i, a_i, b_i) are jointly distributed. Assume:

$$(y_i | \theta_i, \sigma_i^2) \sim N(\theta_i, \sigma_i^2). \tag{A1}$$

The normal distribution will often be appropriate in situations for which the usage quantity corresponds to a change in some quantity of interest. In other situations the gamma or another sampling distribution might be more appropriate. In such a case, we assume the y_i's (and the (a_i, b_i)) have been pre-transformed, so that after the transformation, the resulting variables are approximately normally distributed. Assume the means of the usage quantities are themselves exchangeable, and normally distributed about some unknown population mean of fundamental interest, θ_0:

$$(\theta_i | \theta_0, \tau^2) \sim N(\theta_0, \tau^2). \tag{A2}$$

Thus, respondent i has a recall distribution whose true mean value is θ_i (e.g., each respondent is attempting to recall his/her particular number of visits to the doctor last year). It is desired to estimate θ_0. Assume $(\sigma_1^2, \ldots, \sigma_n^2, \tau^2)$ are known; they will be assigned later. Denote the column vector of usage quantities by $\underset{\sim}{y} = (y_i)$, and the column vector of means by $\underset{\sim}{\theta} = (\theta_i)$. Let $\underset{\sim}{\sigma}^2 = (\sigma_i^2)$ denote the column vector of data variances. The joint density of the y_i's is given in summary form by:

$$p(\underset{\sim}{y} | \underset{\sim}{\theta}, \underset{\sim}{\sigma}^2) \propto \exp\left\{(-\frac{1}{2})\sum_1^n \left(\frac{y_i - \theta_i}{\sigma_i}\right)^2\right\}. \tag{A3}$$

The joint density of the θ_i's is given by:

$$p(\underset{\sim}{\theta} | \theta_0, \tau^2) \propto \exp\left\{(-\frac{1}{2})\sum_1^n \left(\frac{\theta_i - \theta_0}{\tau}\right)^2\right\}. \tag{A4}$$

So the joint density of $(\underset{\sim}{y}, \underset{\sim}{\theta})$ is given by:

$$p(\underset{\sim}{y}, \underset{\sim}{\theta} | \theta_0, \tau^2, \underset{\sim}{\sigma}^2) = p(\underset{\sim}{y} | \underset{\sim}{\theta}, \underset{\sim}{\sigma}^2) p(\underset{\sim}{\theta} | \theta_0, \tau^2)$$

or, multiplying eqn. (A3) and eqn. (A4), gives:

$$\begin{aligned} &p(\underset{\sim}{y}, \underset{\sim}{\theta} | \theta_0, \tau^2, \underset{\sim}{\sigma}^2) \\ &\propto \exp(-\frac{1}{2})\left[\sum_1^n \left(\frac{y_i - \theta_i}{\sigma_i}\right)^2\right] \\ &\quad \bullet \exp(-\frac{1}{2})\left[\sum_1^n \left(\frac{\theta_i - \theta_0}{\tau}\right)^2\right] \\ &\propto \exp\left\{(-\frac{A(\underset{\sim}{\theta})}{2})\right\}, \end{aligned} \tag{A5}$$

$$\text{where: } A(\underset{\sim}{\theta}) \equiv \sum_1^n \left(\frac{y_i - \theta_i}{\sigma_i}\right)^2 + \sum_1^n \left(\frac{\theta_i - \theta_0}{\tau}\right)^2. \tag{A6}$$

Expand eqn. (A6) in terms of the θ_i's by completing the square. This takes some algebra. Then:

$$A(\underset{\sim}{\theta}) = \sum_1^n \left\{\alpha_i \left[\left(\theta_i - \frac{\beta_i}{\alpha_i}\right)^2 + \left(\frac{\gamma_i}{\alpha_i} - \frac{\beta_i^2}{\alpha_i^2}\right)\right]\right\}, \tag{A7}$$

$$\alpha_i = \frac{1}{\sigma_i^2} + \frac{1}{\tau^2}, \quad \beta_i = \frac{y_i}{\sigma_i^2} + \frac{\theta_0}{\tau^2}, \quad \gamma_i = \frac{\theta_0^2}{\tau^2} + \frac{y_i^2}{\sigma_i^2}. \tag{A8}$$

Now find the marginal density of $\underset{\sim}{y}$ by integrating eqn. (A5) with respect to $\underset{\sim}{\theta}$. Then:

$$p(\underset{\sim}{y}|\theta_0, \tau^2, \underset{\sim}{\sigma}^2) \propto J(\theta_0) \exp\left\{\left(-\frac{1}{2}\sum_1^n \alpha_i \delta_i\right)\right\},$$

where

$$J(\theta_0) \equiv \int \exp\left\{\left(-\frac{1}{2}\right)\sum_1^n \alpha_i \left(\theta_i - \frac{\beta_i}{\alpha_i}\right)^2\right\} d\underset{\sim}{\theta},$$

$$\delta_i = \left(\frac{\gamma_i}{\alpha_i} - \frac{\beta_i^2}{\alpha_i^2}\right). \qquad \text{A9)}$$

Rewriting eqn. (A9) in vector and matrix form, to simplify the integration, it is found that if

$$\underset{\sim}{f} \equiv \left(\frac{\beta_i}{\alpha_i}\right), \qquad \underset{\sim}{K}^{-1} \equiv diag(\alpha_1, ..., \alpha_n),$$

$$(\underset{\sim}{\theta} - \underset{\sim}{f})' \underset{\sim}{K}^{-1} (\underset{\sim}{\theta} - \underset{\sim}{f}) = \sum_1^n \alpha_i \left(\theta_i - \frac{\beta_i}{\alpha_i}\right)^2. \quad \text{(A10)}$$

Carrying out the (normal) integration gives:

$$p(\underset{\sim}{y}|\theta_0, \tau^2, \underset{\sim}{\sigma}^2) \propto \frac{1}{\left|\underset{\sim}{K}^{-1}\right|^{1/2}} \exp\left\{\left(-\frac{1}{2}\sum_1^n \alpha_i \delta_i\right)\right\}. \qquad \text{(A11)}$$

Now note that $\left|\underset{\sim}{K}^{-1}\right| = \prod_1^n \alpha_i =$ constant and the constant can be absorbed into the proportionality constant, but δ_i depends on θ_0. So:

$$p(\underset{\sim}{y}|\theta_0, \tau^2, \underset{\sim}{\sigma}^2) \propto \exp\left\{\left(-\frac{1}{2}\sum_1^n \alpha_i \delta_i\right)\right\}. \qquad \text{(A12)}$$

Note that the proportionality constant in eqn. (A12) does not depend upon θ_0. Now apply Bayes' theorem to θ_0 in eqn. (A12).

$$p(\theta_0 | \underset{\sim}{y}, \tau^2, \underset{\sim}{\sigma}^2) \propto p(\theta_0) \exp\left\{\left(-\frac{1}{2}\sum_1^n \alpha_i \delta_i\right)\right\}, \qquad \text{(A13)}$$

where $p(\theta_0)$ denotes a prior density for θ_0. Prior belief (prior to observing the point and bound estimates of the respondents) is that for the large sample sizes typically associated with sample surveys, the population mean, θ_0, might lie, with equal probability, anywhere in the interval (a_0, b_0), where a_0 denotes the smallest lower bound given by any respondent, and b_0 denotes the largest upper bound. So adopt a uniform prior distribution on (a_0, b_0). To be fully confident of covering all possibilities, however, adopt an (improper) prior density. Therefore adopt a prior density of the form:

$$p(\theta_0) \propto \text{ constant}, \qquad \text{(A14)}$$

for all θ_0 on the entire real line. (In some survey situations the same survey is carried out repeatedly so that there is strong prior information available for providing a realistic finite range for θ_0; in such cases we could improve on our estimator by using a proper prior distribution for θ_0 instead of the one given in eqn. (A14).) The development for a normal (rather than a vague) prior distribution on the population mean is simple and analogous.

Inserting (A14) into (A13), and noting that $p(\theta_0) \propto$ constant, gives:

$$p(\theta_0 | \underset{\sim}{y}, \tau^2, \underset{\sim}{\sigma}^2) \propto \exp\left\{\left(-\frac{1}{2}\sum_1^n \alpha_i \delta_i\right)\right\}. \qquad \text{(A15)}$$

Next substitute for δ_i and complete the square in θ_0 to get, after some algebra, the final result that if:

$$\lambda_i \equiv \frac{\left(\frac{1}{\sigma_i^2 + \tau^2}\right)}{\sum_1^n \left(\frac{1}{\sigma_i^2 + \tau^2}\right)}, \quad \sum_1^n \lambda_i = 1, \; 0 \le \lambda_i \le 1, \qquad \text{(A16)}$$

the conditional posterior density of θ_0 is seen to be expressible as:

$$(\theta_0 \big| \underset{\sim}{y}, \tau^2, \underset{\sim}{\sigma}^2) \sim N(\tilde{\theta}, \omega^2), \tag{A17}$$

where:

$$\tilde{\theta} = \sum_1^n \lambda_i y_i, \tag{A18}$$

$$\omega^2 = \frac{1}{\sum_1^n \left(\frac{1}{\sigma_i^2 + \tau^2} \right)}. \tag{A19}$$

Thus, the mean, $\tilde{\theta}$, of the conditional posterior density of the population mean, θ_0, is a convex combination of the respondents' point estimates, that is, their usage quantities. It is an unequally weighted average of the usage quantities, as compared with the sample mean estimator of the population mean, which is an equally weighted estimator, $\bar{y}$. Interpret $(\sigma_i^2 + \tau^2)^{-1}$ as the precision attributable to respondent *i*'s response, and $\sum_1^n (\sigma_i^2 + \tau^2)^{-1}$ as the total precision attributable to all respondents; then, λ_i is interpretable as the proportion of total precision attributable to respondent *i*. Thus, the greater his/her precision proportion, the greater the weight that is automatically assigned to respondent *i*'s usage response. We must still assess the variances $(\sigma_1^2, \sigma_2^2, ..., \sigma_n^2, \tau^2)$.

Assessing the Variances

Suppose that in addition to eqn. (A1):

$$a_i \big| a_{i0}, \psi_{ai}^2 \sim N(a_{i0}, \psi_{ai}^2); \tag{A1.1}$$

$$b_i \big| b_{i0}, \psi_{bi}^2 \sim N(b_{i0}, \psi_{bi}^2), \tag{A1.2}$$

where θ_i in eqn. (A.1) denotes the true population value for the mean usage for respondent *i*; a_{i0}, b_{i0} denote the true population values for respondent *i*'s lower and upper bounds, respectively; and $(\sigma_i^2, \psi_{ai}^2, \psi_{bi}^2)$ denote the corresponding population variances, respectively. Next, using the structure of the normal distribution, assume the approximate bounds for all subjects in the population are approximately 2 standard deviations on either side of the respective means. Accordingly, take approximately, $4\sigma_i \doteq b_i - a_i$, $i = 1, ..., n$, as our assessments for the σ_i's.

Then, define:

$$a^* = \frac{1}{N}\sum_1^N a_{i0}; \qquad b^* = \frac{1}{N}\sum_1^N b_{i0};$$

$$\bar{a} = \frac{1}{n}\sum_1^n a_i; \qquad \bar{b} = \frac{1}{n}\sum_1^n b_i,$$

where: a^*, b^* are averages of the *true* (unobserved) values of these bounds over the entire population; $\bar{a}, \bar{b}$ are the averages of the *observed* values of the bounds over the sample.

Assume approximately:

$$\psi_{a1}^2 = \psi_{a2}^2 = ... = \psi_a^2; \quad \psi_{b1}^2 = \psi_{b2}^2 = ... = \psi_b^2.$$

Then,

$$\bar{a} \sim N(a^*, \frac{\psi_a^2}{n}); \qquad \bar{b} \sim N(b^*, \frac{\psi_b^2}{n}). \tag{A20}$$

Next note that the true population mean value for respondent *i* must be between its bounds,

$$a^* \le \theta_0 \le b^*. \tag{A21}$$

Case 1—Extended Average Estimator

For 95% credibility on a^* with respect to a vague prior we have (approximating 1.96 by 2, here and throughout, for convenience):

$$\bar{a} - 2\frac{\psi_a}{\sqrt{n}} \le a^* \le \bar{a} + 2\frac{\psi_a}{\sqrt{n}}; \tag{A22}$$

for 95% credibility on b^* with respect to a vague prior we have:

$$\overline{b}-2\frac{\psi_b}{\sqrt{n}}\leq b^*\leq\overline{b}+2\frac{\psi_b}{\sqrt{n}}. \quad \text{(A23)}$$

From eqns. (A21), (A22) and (A23) we get:

$$\overline{a}-2\frac{\psi_a}{\sqrt{n}}\leq a^*\leq\theta_0\leq b^*\leq\overline{b}+2\frac{\psi_b}{\sqrt{n}},$$

or:

$$\overline{a}-2\frac{\psi_a}{\sqrt{n}}\leq\theta_0\leq\overline{b}+2\frac{\psi_b}{\sqrt{n}}. \quad \text{(A24)}$$

From the normality and 95% credibility,

$$\begin{aligned}4\tau&=\left(\overline{b}+2\frac{\psi_b}{\sqrt{n}}\right)-\left(\overline{a}-2\frac{\psi_a}{\sqrt{n}}\right)\\&=\left(\overline{b}-\overline{a}\right)+\frac{2}{\sqrt{n}}\left(\psi_a+\psi_b\right).\end{aligned} \quad \text{(A25)}$$

But ψ_a and ψ_b are unknown. Estimate them by their sample quantities:

$$s_a^2\equiv\hat{\psi}_a^2\equiv\frac{1}{n}\sum_1^n(a_i-\overline{a})^2;$$

$$s_b^2\equiv\hat{\psi}_b^2\equiv\frac{1}{n}\sum_1^n(b_i-\overline{b})^2.$$

(A26)

Then, the assessment procedure for τ becomes:

$$4\tau\doteq\left(\overline{b}-\overline{a}\right)+\frac{2}{\sqrt{n}}\left(s_a+s_b\right). \quad \text{(A27)}$$

There is a Minitab 13 macro for computing the Bayesian RGI extended average estimator (See Miller, 2003).

Case 2—Extended Range Estimator

From eqn. (A24), since $a_0<\overline{a}$, and $\overline{b}<b_0$, we can consider for an alternative assessment procedure,

$$a_0-2\frac{\psi_a}{\sqrt{n}}\leq\theta_0\leq b_0+2\frac{\psi_b}{\sqrt{n}}. \quad \text{(A28)}$$

Then, (A25) becomes:

$$\begin{aligned}4\tau&=\left(b_0+2\frac{\psi_b}{\sqrt{n}}\right)-\left(a_0-2\frac{\psi_a}{\sqrt{n}}\right)\\&=\left(b_0-a_0\right)+\frac{2}{\sqrt{n}}\left(\psi_a+\psi_b\right).\end{aligned} \quad \text{(A29)}$$

Using eqn. (A26) gives:

$$4\tau\doteq\left(b_0-a_0\right)+\frac{2}{\sqrt{n}}\left(s_a+s_b\right). \quad \text{(A30)}$$

Note that the second term in (A27), and in (A30) disappear for large sample sizes, leaving us with just the average or range of the bounds, but for smaller sample sizes, the second term can have a substantial effect.

B. Non-Normal Data for Recall Questions

Suppose the usage quantity data, the y_i's, follow a 2-parameter gamma distribution instead of the normal distribution assumed in Section IIA. Adopt the probability density structure:

$$f(y|\alpha,\beta)=\frac{1}{\Gamma(\alpha)\beta^\alpha}y^{\alpha-1}e^{-\frac{y}{\beta}}\quad y>0,$$
$$\alpha>0,\ \beta>0, \quad \text{(B1)}$$

with:

$$E(Y)=\alpha\beta\equiv\mu,\ mode(Y)=(\alpha-1)\beta,$$
$$var(Y)=\alpha\beta^2. \quad \text{(B2)}$$

Define a new transformation parameter μ by:

$$\beta=\frac{\mu}{\alpha}$$

We can rewrite the gamma distribution in terms of μ as:

$$f(y|\alpha,\mu)=\frac{1}{\Gamma(\alpha)\left(\frac{\mu}{\alpha}\right)^\alpha}y^{\alpha-1}e^{-\frac{\alpha y}{\mu}}, \quad \text{(B3)}$$

with: mean $E(Y)=\mu$,

$$mode(Y)=\mu-\frac{\mu}{\alpha},$$

$$var(Y) = \frac{\mu^2}{\alpha}.$$

Now make the normalizing transformation (see McCullagh and Nelder, 1983, Chapter 7.2):

$$Z = 3\left[\left(\frac{y}{\mu}\right)^{1/3} - 1\right], \tag{B4}$$

so that now, the transformed variable is approximately a standard normal variable; i.e., $Z \sim N(0,1)$. Under this transformation the precision parameter $\alpha = \sigma^{-2}$ is assumed constant for all observations. Applying this transformation to all the variables creates a new set of standard normal variables. Modifying their locations and scales, as shown below, reduces the problem, approximately, to the one discussed in IIA.

Applying the transformation in (B4) to all the usage quantities gives:

$$Z_i = 3\left[\left(\frac{y_i}{\theta_i}\right)^{1/3} - 1\right]. \tag{B5}$$

Now, the Z_i's are independent, and approximately,

$$Z_i \sim N(0,1). \tag{B6}$$

Next define the new variables, Z_i^* by:

$$Z_i^* \equiv \theta_i + \sigma_i Z_i. \tag{B7}$$

Now we have the Z_i^*'s mutually conditionally independent, and

$$(Z_i^* \mid \theta_i, \sigma_i^2) \sim N(\theta_i, \sigma_i^2),\ i = 1,\ldots,n. \tag{B8}$$

Suppose the θ_i's are exchangeable, with

$$\theta_i \sim N(\theta_0, \tau^2). \tag{B9}$$

Assume

$$p(\theta_0) \propto \text{constant}. \tag{B10}$$

We would like to find a Bayesian estimator of the population mean, θ_0.

We already know that for given $(\sigma_1^2,\ldots,\sigma_n^2,\tau^2)$, by Bayes' theorem,

$$(\theta_0 \mid Z_1^*,\ldots,Z_n^*,\sigma_1^2,\ldots,\sigma_n^2,\tau^2) \sim N(\tilde{\theta},\omega^2), \tag{B11}$$

where the posterior mean of θ_0 is given by:

$$\tilde{\theta} = \sum_1^n \lambda_i Z_i^*, \tag{B12}$$

$$\lambda_i \equiv \frac{\left(\frac{1}{\sigma_i^2 + \tau^2}\right)}{\sum_1^n \left(\frac{1}{\sigma_i^2 + \tau^2}\right)}, \qquad \sum_1^n \lambda_i = 1,$$

$$0 \le \lambda_i \le 1,$$

and

$$\omega^2 = \frac{1}{\sum_1^n \left(\frac{1}{\sigma_i^2 + \tau^2}\right)}. \tag{B13}$$

Now we substitute approximations for the unknown parameters.

C. RGI And Opinion Questions

Suppose there is a population of opinions about some issue, say, "Issue A". Perhaps the analyst would like to establish the mean of the opinions of all people living in the City of New York about Issue A. There is no "correct" answer for an opinion or for an attitude for a given respondent, as there would be for a person answering a recall-type of question. Similarly, response bias does not have the same meaning as in recall. (With a recall-type of question, one of the reasons for response bias arises out of faulty memory.)

When using RGI for attitudes or opinions we can find both point and interval estimators. The RGI point estimator provides some information about the intensity of opinions of New Yorkers about "Issue A", more so than would a mere traditional sample mean that includes some people with very fuzzy opinions, and some people who have very firm opinions. RGI can provide various measures of strength-of-opinion. One such is the average range of the bounds supplied by all respondents , $(\overline{b} - \overline{a})$. It can also supply a credibility interval measure of belief. Of course, a confidence interval can also supply an interval measure of belief, but the confidence interval only reflects sampling uncertainty, whereas the RGI credibility interval also reflects individual fuzziness of opinion. The range-of-belief also available with RGI, $(b_0 - a_0)$, is somewhat different in that it measures the distance between the extremes of opinion.

Another measure of strength-of-opinion is one we call "fuzziness." There is certainly no unique way to define such a quantity. One way might be to measure it using the following scale. Recall that the *i*th respondent's bounds are given by (a_i, b_i), and the usage quantity for respondent i is given by y_i. Now define the fuzziness of respondent *i*'s opinion as:

$$f_i = (b_i - a_i)\left[1 - \exp\left\{-\frac{(b_i - a_i)}{|y_i|}\right\}\right] . \tag{C1}$$

As y_i varies, this measure varies between 0 and $(b_i - a_i)$. It is a monotone increasing function of the range, ($b_i - a_i$). So the greater the range, the greater the degree of fuzziness, and conversely. Moreover, when $y_i = 0$, $f_i = (b_i - a_i)$. This definition is driven by the need to avoid mathematical difficulties using $(b_i - a_i) / y_i$ when y_i is near the origin.

III. Empirical Studies of RGI

During the time that we have worked on RGI, our thinking has evolved in several directions. We have improved our modeling, the way we assess parameters (the population variance and the prior mean), and the form of our questioning. These changes are reflected in the design, analyses, and findings of our empirical work.

In our very first empirical effort we ran parallel record-check surveys on our campuses, asking students questions about their life on campus. If the student-respondents gave their consent, their answers were verified through the appropriate campus offices. On both campuses we asked about the number of credits the student had earned (CREDITS), about his/her SAT math and verbal scores (SATM, SATV), his/her GPA, the number of grades of C or below s/he had received (Cs), and the number of parking tickets s/he had been given (TICKETS). At the University of California at Riverside (UCR) we also asked about the registration fee (REGFEE) and the recreation center fee (RECFEE) the student had paid at the start of the quarter. At the State University of New York at Stony Brook (SUNY-SB) we also asked about the student activities fee (SAFEE) and the health fee (HEALTH) the student had paid at the start of the semester, as well as the amount s/he had spent on food via the food plan (FOOD) and the number of library fines (FINES) s/he had been assessed.

In the campus surveys there were two versions of the questionnaire, both asking about the same usage quantities in the same order. In one version the first half of the items also asked the respondent to provide an answer for the item (such as credits earned) in the following form:

a) Please fill in the blank – "I would be surprised if I had earned more than _____ credits by the beginning of the quarter".

We refer to this question form as the "surprise form." The second half of the items on this ballot asked the respondent to answer a question of the form:

b) Please fill in the blanks – “There is almost no chance that the number of credits that I had earned by beginning of this quarter was less than ______ and almost no chance that it was more than ______.”

We refer to this form of the bounds question as the “interval form.” In the other version of the questionnaire the interval form was used for the first half of the items and the surprise form was used for the second half of the items, hence counterbalancing to control for any order effects.

By assuming normality of the responses, and by defining what we mean by “surprise” (which fractile of the recall distribution corresponds to “surprise”?), a complete recall distribution would therefore be defined for each respondent from the surprise form. Again assuming normality of the responses, and defining what is meant by “almost no chance” (which fractile corresponds to “almost no chance”?), we could also generate a complete recall distribution for each respondent from the interval form. But which of these two approaches, “surprise” or “interval,” was a better way to elicit the desired recall distribution information?

At the time we designed the survey instrument we knew that both methods would give us the respondent’s recall distribution (as described in Section II), and we wanted to compare the efficacies of the two forms. When it came time to analyze the data, however, we realized that the interval form, was preferable, a priori. It offered us a direct measure of the location of the respondent’s usage quantity, in case the respondent had not given an answer to the usage question, either as a midpoint of the interval given by the bounds, or as some weighted average of the bounds. This information was not available from the surprise form of the question. Also, lack of symmetry of the responses to the two questions required for the interval form immediately would signal the non-normality of the recall distribution. Hence we only analyzed the data from the interval form questions in this experiment, and only used that form in later experiments.

In an attempt to estimate the population mean, our initial estimation procedure for these experiments compared:

(1) the usual sample mean;

(2) the average of the midpoints of the intervals given by the respondents, designated the midpoint estimator; as well as

(3) a Bayesian point estimator.

That Bayesian estimator was the mean of the posterior distribution of the true population mean value obtained from a two-stage hierarchical model using an assumed normal likelihood, exchangeable normal priors for the means of each respondent’s data distribution, and an exponential distribution for the common precision parameter of the respondents’ exchangeable normal priors. In addition, we adopted a normal prior for the population mean, centered at the mean of the averages of the bounds provided by the respondents (this was called the midpoint estimator, $(\overline{a}+\overline{b})/2$).

The posterior distribution for the population mean was complicated (the ratio of multiple integrals), but was evaluated numerically by Gibbs sampling Markov Chain Monte Carlo (MCMC). (See Press, 1997 for a derivation of the estimator, and Press and Tanur, 2000, for further details about the campus experiments.) The results given by these estimators were compared in terms of their closeness to the true means found in record checks.

For the 18 items tested in the two campus experiments, this initial analysis found that the posterior mean was always very close to the midpoint estimator. This similarity was not surprising as we chose deliberately to use a sharp (non-vague) prior. The Bayesian estimator looked relatively good; but it was difficult to compute. Of the three estimates, the Bayesian estimate was least accurate for just one item, the midpoint estimate least accurate 7 times, and the sample mean of the usage quantities least accurate 10 times.

Our next empirical study was carried out during a fellowship at the Bureau of the Census held by S. J. Press. Census Bureau interviewers carried out telephone interviews with respondents from 500 households, asking questions about the household's economic situation. Respondents were asked questions about their income from salary and wages for the most recent calendar year and the year previous to that and about the change in their income from these sources over the previous 5 years. They were also asked similar questions about their income from interest and dividends. This study involved extensive cognitive testing of the question form (see Marquis and Press, 1999), and finally settled on asking 25% of the respondents the usage quantity first, followed by questions about the bounds (e.g.):

a) What is your best estimate of your household's income from salary and wages in 1997?

b) What is the lowest the correct value could be?

c) What is the highest the correct value could be?

Thus the bounds question was broken into two separate questions. In addition, for the remaining 75% of the respondents, the form of the bounds questions shown above was asked before the usage quantity question, rather than in the reverse order, to see whether the order of the questions would make any difference. Bayesian estimation was again carried out using normal priors, and MCMC, as described above. In this work, however, two versions of the estimation were carried out. One used the sample median of the usage quantities as the mean of the prior distribution and the other used the midpoint estimator as was done in the campus experiments.

Because of the split ballot nature of the experiment, there were 12 comparisons possible between the sample mean and the two Bayesian estimators. Of these comparisons with the sample mean, the sample mean was closest to the truth 4 times, the Bayesian estimator using the median closest to the truth 4 times, the Bayesian estimator using the midpoint estimator closest to the truth 3 times, and there was one tie between the Bayesian estimators. In a "head to head contest" between the two Bayesian estimators, the one using the median as the prior mean was closer to truth 5 times, the one using the midpoint estimator as the prior mean closer to the truth 6 times, and there was one tie. The order in which the usage and bounds questions were asked did not seem to make any difference in the accuracy of estimation. See Press and Marquis (2001) for more details on the Census experiment.

Meanwhile, other progress was being made. Schwartz and Paulin (2000) did a study at the Bureau of Labor Statistics comparing several techniques using bounds/interval questions. They found that respondents liked the RGI technique because they felt it gave them some control over their disclosures of income. They also found that the intervals offered by respondents tended to be smaller than those generated by the investigators themselves in another condition of the experiment. And intervals generated by the respondents had been used in several other contexts. Earlier rounds of the Survey of Consumer Finances used interval estimates to elicit answers from reluctant respondents (Kennickell, 1997) and the 2004 round was planning to put more emphasis on letting respondents who can't or won't give exact amounts determine their own ranges--rather than falling back on a range card or a decision tree (Kennickell, 2004).

Further, Lusinchi (2003) had encouraged respondents on a web survey to use such intervals when they were not sure of their answers. We ourselves (Press and Tanur, 2001) showed that in the early campus experiments up to 41% of respondents who did not choose to give a point estimate of a usage quantity did give a set of bounds. If we use the midpoint of the bounds as an approximation of what the respondent might have answered for the usage quantity, we see that the RGI protocol has the potential to reduce item nonresponse considerably. Clearly, RGI was useful, but we needed to work on the estimation strategy and the question format.

As our thinking evolved, we went on to develop a new model that allowed a closed form solution rather than the MCMC computer intensive numerical evaluation. That new model was presented in Section II above. The new modeling develops results for a vague prior for the population mean, but results for a proper (normal) prior for the population mean are analogous. We tried this model out on the data from the campus experiments described above. In order to assess the hyperparameters for a proper prior distribution we needed demographic information about respondents. (For example, we needed to know the composition of the sample in terms of year in school in order to derive a prior mean of the number of credits students would have earned. For a description of the how the prior means were derived see Press and Tanur, 2004, p. 272.)

Unfortunately, over the years some of those demographic data for the campus experiments became separated from respondents' reports on the items using the RGI questioning protocol. Hence our reanalysis of the campus experiments could use only 6 variables at SUNY-SB and only 4 at UCR. These results appear in Table 1. We see that the posterior mean, using a proper prior and the range of the bounds to estimate the population variance was closer to truth than the sample mean for 8 of the 10 items. Moreover, the Bayesian credibility interval covered truth for all 10 items, while the traditional confidence interval covered truth only for 6 of the 10.

Table 1 – Comparing Sample and RGI Posterior Means for Estimating Population Means in Campus Experiments Using Normal Priors and Range Estimator

Boldface point estimates denote "winners;" boldface interval estimates denote intervals that cover truth.

SUSB

	Truth	**x-bar**	**Conf. Int.**	**Post-Mean**	**Cred. Int.**
CREDITS	67.53	63.13	**(56.12, 70.14)**	**63.69**	**(55.54, 71.84)**
GPA	2.91	2.99	**(2.89, 3.09)**	**2.97**	**(2.85, 3.09)**
SATM	570.80	593.72	(572.40, 615.00)	**591.97**	**(553.15, 630.79)**
SATV	503.20	526.00	(503.80, 548.20)	**519.01**	**(478.52, 559.50)**
TICKETS	0.53	**0.92**	(.56, 1.28)	0.95	**(.32, 1.58)**
FINES	1.52	2.25	**(0, 5.41)**	**1.00**	**(.03, 1.96)**

UCR

	Truth	**x-bar**	**Conf. Int.**	**Post-Mean**	**Cred. Int.**
GPA	3.05	3.10	**(3.00, 3.20)**	**3.04**	**(2.88, 3.21)**
SATM	574.08	572.60	**(549.50, 595.60)**	**574.05**	**(537.54, 610.56)**
SATV	485.40	503.00	**(481.20, 524.80)**	**500.38**	**(463.74, 537.02)**
TICKETS	0.21	**0.51**	(.27, .75)	0.63	**(.09, 1.16)**

Clearly, the closed form estimation procedure was doing better than the MCMC procedure, but there was still room for improvement. We turned to issues of assessing the hyperparameters and to the questioning format to attempt further improvement.

We moved to expressing the hyperparameter τ according to Eqns. A27 (for the extended average estimator) and A30 (for the extended range estimator). (Earlier we had taken 4τ to be equal to the difference between the sample means of the bounds for the average estimator or equal to the difference between the highest sample upper bound and the lowest sample lower bound for the range estimator.)

From Equation. A16 it is clear that sample usage quantities that are coupled with narrow intervals receive greater weight in the Bayesian estimation than do sample usage quantities that are coupled with wide intervals. Hence it would improve estimation if respondents who give accurate usage quantities also gave narrow intervals and respondents who give inaccurate usage quantities gave wide intervals. We had found earlier that there is indeed a correlation between interval length and accuracy (see Press and Tanur, 2003); we set out to improve that correlation via our questioning.

To test these hypotheses we designed a new UCR classroom survey which was administered to a large undergraduate statistics class in spring, 2003. We worked through respondents' confidence, having earlier found a correlation between confidence and accuracy (see Press and Tanur, 2002).

Figure 1. Confidence Scale for RGI Protocol

1) What is your best guess as to what your score was on your first exam in this class? (Please don't answer if you've missed the first exam).____________________.

2) How confident are you about your answer to Question 1? Please answer on the following confidence scale. (Place a check in the first column next to the answer you prefer.)

Confidence Scale

Place a check somewhere in this column	**Numerical Score**	**Interpretation of confidence rating**	**Which question should I answer next?**
	0	**I have absolutely no idea what my exam score was**	**Go to Question 3b**
	2.5	**I am uncertain what my exam score was**	**Go to Question 3b**
	5.0	**I might be right and I might be wrong about what my exam score was**	**Go to Question 3b**
	7.5	**I think that I know what my exam score was**	**Go to Question 3a**
	10.0	**I am absolutely certain what my exam score was**	**Go to Question 3a**

In the questionnaires, we encouraged/prompted confident respondents to give narrow intervals and less confident respondents to give wide intervals. As in our earlier campus experiments, we asked students about everyday facts of their life on campus that we could verify – we asked for the score the respondent had earned in the midterm for that class, the score on the second homework, and again we asked about the registration fee paid at the beginning of the quarter (for details about this experiment, see Chu, Press, and Tanur, 2004). But before the respondent answered each question, s/he responded to a confidence scale we devised, as shown in Figure 1. The questions the respondent was directed to varied somewhat in format, but essentially they resembled the form shown in Figure 2.

Because we varied the amount of guidance we gave the respondent on how wide or narrow the intervals should be, we had 3 conditions for each of the 3 items we inquired about. Thus we had 9 chances to measure the accuracy of the extended range and extended average estimators against the accuracy of the sample mean. Using a vague prior, we found that in 6 of these cases the extended average estimate was closest to truth (and in all these cases, the extended range estimate was in second place), in one case the extended range estimate was closest to truth, and in the remaining two cases the sample mean "won." Using a normal prior (see Chu, in progress) the results are even more encouraging. For the question about the midterm grade the extended average estimate was closest to truth in all 3 cases, and for the other 2 questions the extended range estimate was closest to truth in all 6 cases.

In both this classroom experiment and in another that followed some months later (and is described just below), we varied the amount of guidance we gave the respondents about how wide their intervals should be if they were not confident about the accuracy of their recall. This manipulation worked in that those instructed to give a wider interval did indeed give a wider one on average than those who were instructed to give a less wide interval. Thus, the results given

Figure 2. Classroom Experiment – Form of RGI Question

3a) If your answer to Question 2 is 7.5 or more, please give the **smallest possible interval** in which you believe that the exam score is included. Please fill in:

The smallest my exam score could have been is %________,

The largest my exam score could have been is %__________.

NOW GO TO QUESTION 4.

3b) If your answer to Question 2 is 5 or less, give a **sufficiently wide interval** so that the interval will most likely include the actual exam score
Please fill in:
The smallest my exam score could have been is %________,

The largest my exam score could have been is %_________.

above and those to be presented below use only "obedient" respondents – those who followed our guidelines on how wide their intervals should be. For details on these guidelines and results for all respondents, see Chu, Press, and Tanur (2004).

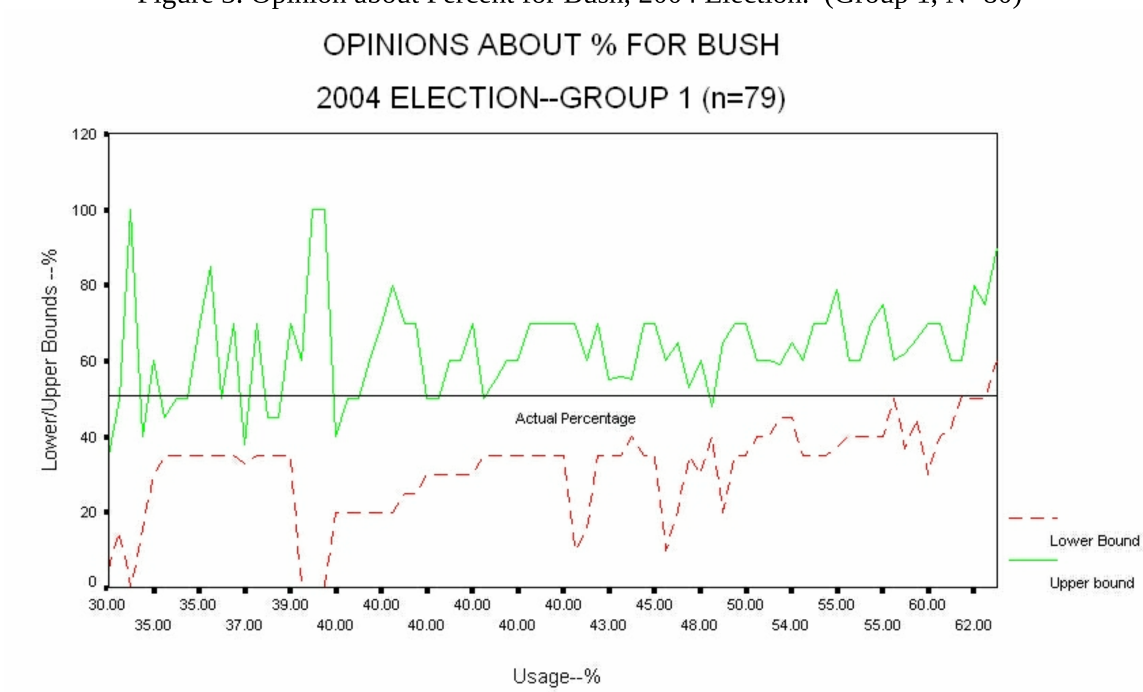
Because the sample sizes in the classroom experiment of spring, 2003 were small, we ran a similar experiment later (Nov. 5, 2003; see Chu, in progress). The questions were asked in the same form as in the spring, 2003 experiment (including the confidence scale), with the exception that instead of asking about scores on homework the student-respondents were asked for the number of movie videos they owned. (Verification data consisted of an earlier report these students had given to the professor in a questionnaire designed to acquaint the professor with the students' interests and given as part of regular classroom routine.) In this case, we again used both a vague prior and a normal prior and the extended range and extended average estimators. Again we had 9 cases for which we could compare the estimators. Using a vague prior we found that the extended average estimate was closest to truth in 3 cases, the extended range estimate closest once, and the sample mean closest for 5 cases. When we used a normal prior, the results were somewhat more encouraging, with the extended average estimate, the extended range estimate, and the sample mean each being closest to truth in 3 cases.

In the November, 2003 survey, almost exactly one year before the 2004 US presidential election we also asked our student respondents an opinion question: "In your opinion, what percentage of the total vote will Mr. George W. Bush receive in the 2004 presidential election (0-100%)?"

We found that the modal response was 40%, in contrast to the actual percent of the popular vote achieved by President Bush on November 2, 2004 of 51%. A graph of the respondents' bounds plotted against their usage quantities is shown in Figure 3, in which respondents have been ordered first by their usage quantity, then by their lower bounds within values of the usage quantity, and then by their upper bounds within values of the usage quantity and of the lower bounds to smooth the graph as much as possible.

Nevertheless, the many spikes in the graphs, and the wide variations in bounds from one respondent to another, in Figure 3, shows that about a year before the actual presidential election of 2004, these respondents were very uncertain (fuzzy) about how strong or weak the

Figure 3. Opinion about Percent for Bush, 2004 Election. (Group 1, N=80)



support for President Bush would be. It is also interesting to note from Figure 3 that as usage increase beyond about 40%, the spikiness of the graphs tends to decrease, and the lower and upper bounds tend to get closer.

For the opinion data in this example, we have calculated f_i (see Eqn. C1) for all respondents, and present a histogram of the distribution of the f_i's in Figure 4.

The mean fuzziness for this group of respondents on this question = 18.37; the corresponding standard deviation is 18.31 Note that these data are not available in a traditional survey of opinion where bounds information is not available. So there is an additional "intensity of belief " (or degree of fuzziness of belief) that is being provided by an RGI survey.

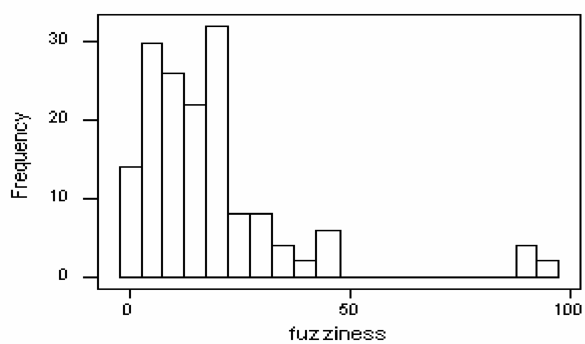
The data from this more recent classroom experiment presented an opportunity to refine our modeling. Note that the derivation in Section II assumes that the recall distribution for each respondent is normal. Of course this assumption is untestable, but evidence of possible violations of the normality assumption for the recall distributions might be reflected in a lack of normality in the sample distribution of recall quantities. Chu (in progress) studies the sample distributions for each of the items in the questionnaire. In particular, she finds for one treatment group the distribution of usage quantities for the question about the midterm examination seems to follow a gamma distribution. Applying the Wilson-Hillferty transformation (Eqn. B4, see McCullagh and Nelder, 1983), should transform the distribution of these data to approximate normality. Work is continuing on applying the gamma transformation to our data sets and on exploring the usefulness of other transformations of the data that will improve the normality of the sample distributions, and we are very hopeful that improving the conformity of the data to the assumptions of the model will improve our estimation results.

IV. Conclusions

The RGI approach to sample surveys has several advantages over more conventional methods of fielding and analyzing surveys.

1) It provides a method for getting respondents to give an answer to sensitive questions which they might not otherwise answer. Respondents generally feel that providing merely bounds to a question that has a numerical answer is less revealing to the interviewer than is answering a question that requires a specific point of estimate for an answer. Hence, RGI can be useful in reducing item nonresponse.

Figure 4. Fuzziness Histogram



2) Many respondents feel more comfortable giving their own point estimate and range that their true value could possibly be than merely giving a point estimate, because they feel it is more accurate.
3) Respondents to questions that use the RGI protocol are able to provide bounds for their responses as long as the bounds questions are carefully worded, and respondents are prompted with examples.
4) It is helpful to have respondents provide confidence scores for how sure they are of their answers.
5) Providing respondents with guidance in the width of intervals to use is an approach that can be used for the analyst to focus attention on the answers of those respondents who are most confident of their responses.
6) To improve accuracy it is helpful to study a measure of the distribution of the sample data. If the data are non-normal it is likely that a transformation of the data to approximate normality followed by an RGI estimation of the transformed data will generate accurate point and interval estimates of the population parameter.
7) When the RGI protocol is used with opinion questions it can provide various measures of intensity-of-belief in the opinions of a group.

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