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Journal of Modern Applied Statistical Methods

Volume 3 | Issue 1 Article 21

5-1-2004

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Journal of Modern Applied Statistical Methods
May, 2004, Vol. 3, No. 1, 213-214

1538 – 9472/04/$95.00

BRIEF REPORTS
A Test-Retest Transition Matrix: A Modification of McNemar's Test

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McNemar introduced what is known today as a test for symmetry in a two by two contingency tables. The logic of the test is based on a sample of matched pairs with a dichotomous response. In our example, the sample consists of the scores before and after an education program and the responses before and after the program. Each pair of scores is from only one person. The pretest divides the group of responders according to their answers to a dichotomous question. The posttest divides the two groups into two groups of like labels. The result is a two by two table. We construct a test of homogeneity, where the proportion of initially partitioned subjects will be equally distributed over the same partition after the program is completed, conditioned on the initial distribution.

Key words: McNemar's test, test of homogeneity, contingency table

Introduction

McNemar (1947) introduced what is known today as a test for symmetry in contingency tables, although his table was only a two by two. What is more, his table is often illustrated via matched pairs and the joint classification of a dichotomy applied to each of the pair. Let zero (0) represent the absence and 1 the presence of the characteristic thus dichotomized.

Table 1 illustrates such a dichotomy. Since they were matched by some criterion, a zero response from a case ought to be matched with a zero from its control, but that does not always happen. The numbers N (0,1) and N (1,0) measure any departure from perfect correlation. McNemar's hypothesis was that these two numbers ought to be equal, or P(0,1) = P(1,0). In our illustration this hypothesis needs to be changed.

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Consider a pre-test and a post-test or a pre-program and post-program situation. A simple random sample of subjects is asked a question about a certain characteristic such as "Do you smoke?" There are N(0) who do not smoke and N(1) who do smoke prior to the application of a program on smoking cessation. Six months after the program is completed they are again asked the same question. A table such as Table 1 results, except that "Case" is now replaced by "Pre-Program" and "Controls" is replaced with "Post-Program".

Unless N(0) = N(1), N(0,1) cannot be expected to equal N(1,0). N(0,1) is the number of people who did not smoke, but six months after the program they were observed to be smoking. N(1,0) people were smoking before the program, and six months later they were not smoking. The correct null hypothesis is $P(1|0) = P(0|1)$. That is, the proportion of prior non-smokers who changed to smokers is equal to the proportion of prior smokers who changed to non-smokers.

The application of the question prior to the program stratifies the sample into two strata that cannot be expected to be the same size. If the program is expected à priori to work, the one-sided alternative should be used, namely $P(1|0) < P(0|1)$. That is, the proportion of non-smokers who changed to smokers should be significantly and clinically smaller than the proportion of smokers who changed to non-smokers. Table 2 is now rearranged and the people are relabeled as “Stayers” and “Movers.” Stayers are non-smokers who remain non-smokers, and similarly for smokers. Quitting and Beginning after the program label those who change and are called “Movers” in Table 3.

With this rearrangement, the hypothesis of homogeneity can be tested with the usual chi-squares, Pearson or Likelihood Ratio, and also with Fisher’s exact test. A significant chi-square at 0.5α coupled with $N(1,0)/N(1) > N(0,1)/N(0)$ signals a working program because a smaller fraction of non-smokers became smokers than the fraction of smokers who became non-smokers. The Fisher’s exact test would be one-tailed in the direction supporting the alternative hypothesis.

The first questionnaire revealed 142 non-smokers and 58 smokers in a teen smoking cessation project. Of the 142, after six months 11 had begun smoking, while 25 of the 58 smokers had quit smoking. Filling in Table 2 yields Table 3. Analyzing this table gives rise to X2 = 34.9, G2 = 31.9, DF = 1, $p < 0.0001$. Fisher’s exact test gives p = 2.4x 10-8 with proportions moving 11/142 = 0.077 < 25/58 = 0.431. Therefore there is statistical significance. Because only 7.7% moved from non-smoker to smoker while the reverse was true for 43.1% of the smokers, this is apparently clinically significant. Therefore the program works. If Odds Ratio is the measure of choice, the Odds of Quitting given the person was a smoker is 9.02 times the Odds of Beginning given that the person was a non-smoker with a 95% CI = (4.03, 20.2).

Table 1: A dichotomy applied to cases and their respective matched controls.

	Control: N(0)	Control: N(1)
Case: N(0)	N(0,0)	N(0,1)
Case: N (1)	N (1,0)	N(1,1)

Table 2: Stayers and Movers.

Stayers	Movers
N(0,0)	N(0,1)
N(1,1)	N(1,0)

Table 3: Stayers and Movers.

Stayers	Movers	Total
N(0,0) = 131	N(0,1) = 11	N(0) = 142
N(1,1) = 33	N(1,0) = 25	N(1) = 58
N-Stayers = 164	N-Movers = 36	N = 200

X2 = 34.9, G2 = 31.9, DF = 1, $p < 0.0001$

References

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