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Said Ali Al-Hadhrami
College of Applied Sciences, Nizwa, Oman, abur1972@yahoo.co.uk

Amer Ibrahim Al-Omari
Al al-Bayt University, Mafraq, Jordan, alomari_amer@yahoo.com

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Said Ali Al-Hadhrami
College of Applied Sciences, Nizwa, Oman

Amer Ibrahim Al-Omari
Al al-Bayt University, Mafraq, Jordan

Bayesian inference of the variance of the normal distribution is considered using moving extremes ranked set sampling (MERSS) and is compared with the simple random sampling (SRS) method. Generalized maximum likelihood estimators (GMLE), confidence intervals (CI), and different testing hypotheses are considered using simple hypothesis versus simple hypothesis, simple hypothesis versus composite alternative, and composite hypothesis versus composite alternative based on MERSS and compared with SRS. It is shown that modified inferences using MERSS are more efficient than their counterparts based on SRS.

Key words: Moving extremes ranked set sampling (MERSS), confidence interval, test hypothesis, Bayesian approach.

Introduction

Ranked set sampling (RSS) for estimating a population mean was suggested by McIntyre (1952) as a cost efficient alternative to simple random sampling (SRS) if the units of a sample can be easily ranked according to the variable of interest rather than actual measurements. The RSS involves randomly selecting m^2 units from the population and randomly allocating them into m sets, each of size m. The m units of each sample are ranked visually (or by any inexpensive method) with respect to the variable of interest. From the first set of m units, the smallest unit is measured. From the second set of m units, the second smallest unit is measured, the process continues until the largest unit is measured from the m^{th} set of m units. Repeating the process r times results in a set of size mr from initial m^2r units.

Takahasi and Wakimoto (1968) provided the mathematical theory for RSS. Muttlak (1996) proposed pair ranked set sampling instead of RSS, and Samawi, et al. (1996) suggested using extreme ranked set sampling to estimate the population mean. Muttlak (1997) also suggested using median ranked set sampling. Al-Saleh and Al-Kadird (2000) considered double ranked set sampling (DRSS). Al-Saleh and Al-Omari (2002) generalized DRSS to multistage RSS. Muttlak (2003) proposed quartile ranked set sampling. Weighted modified RSS was put forward by Muttlak and Abu-Dayyeh (2004).

Al-Odat and Al-Saleh (2001) introduced the concept of varied set size RSS. They investigated this modification non-parametrically and found that the procedure can be more efficient than the simple random sampling technique. Al-Saleh and Al-Hadhrami (2003a) considered the work of Al-Odat and Al-Saleh (2001) and investigated parametrically the mean of exponential distribution; they coined their method of moving extremes ranked set sampling (MERSS). Investigation of the mean of the normal distribution under MERSS was considered by Al-Saleh and Al-Hadhrami

Said Ali Al-Hadhrami is an Assistant Professor in the Department of Mathematics, College of Applied Sciences. Email him at: abur1972@yahoo.co.uk. Amer Ibrahim Al-Omari is an Assistant Professor in the Department of Mathematics. Email him at: alomari_amer@yahoo.com.

(2003b). They showed that the suggested estimators of the population mean are unbiased and more efficient than those based on SRS. Abu-Dayyeh and Al-Sawi (2007) studied the scale parameter of exponential distribution based on MERSS. (For more about RSS see Chen, et al., 2004; Al-Saleh & Al Ananbeh, 2007; Al-Omari & Jaber, 2008; Al-Nasser, 2007; Tseng & Wu, 2007; and Balakrishnan & Li, 2008.)

Methodology

The MERSS General Process

The MERSS can be described as follows:

Step 1: Select m random samples sized 1, 2, 3,…, m, respectively.

Step 2: Identify the maximum of each set by eye or by some other inexpensive method, without actually measuring the characteristic of interest.

Step 3: Accurately measure the selected judgment identified maxima.

Step 4: Repeat Steps 1, 2, 3 but for the minimum.

Step 5: Repeat the above steps r times until the desired sample size, $n = 2rm$ is obtained. The sample of these units is called moving extremes ranked set sample (MERSS).

For one cycle, let

$$\left\{ \begin{matrix} X_{m:m}, X_{m-1:m-1}, X_{m-2:m-2}, \ldots, \\ X_{1:1}, Y_{1:m}, Y_{1:m-1}, Y_{1:m-2}, \ldots, Y_{1:1} \end{matrix} \right\}$$

be a MERSS from a normal distribution mean μ and variance σ^2. If judgment ranking is perfect, then for $i = 1, 2, \ldots, m,$ $X_{i:i}$ has the same density as the i^{th} order statistic of a SRS of size i from $f(x;\theta)$, i.e., $X_{i:i}$ has the density:

$$f_{i:i}(x) = if(x;\theta)\left[F(x;\theta)\right]^{i-1}. \qquad (2.1)$$

In addition, $Y_{1:i}$ has the same density as the first order statistic of a SRS of size i from $f(y;\theta)$, i.e., $Y_{1:i}$ has the density

$$f_{1:i}(y) = if(y;\theta)\left[1 - F(y;\theta)\right]^{i-1}, \quad (2.2)$$

and the likelihood function of θ is given by

$$L(\theta) = \prod_{i=1}^{m} if\left(x_{i:I},\theta\right)\left[F\left(x_{i:i},\theta\right)\right]^{i-1} if\left(y_{1:i},\theta\right)\left[1 - F\left(y_{1:i},\theta\right)\right]^{i-1}. \qquad (2.3)$$

Assuming the random variable X is normally distributed with mean μ and variance σ^2, then the probability density function (pdf) of X is given by

$$\begin{aligned} f_X\left(x,\theta\right) &= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\theta)^2}{2\sigma^2}} \qquad -\infty < x < \infty, \\ &= \frac{1}{\sigma}\phi\left(\frac{x-\theta}{\sigma}\right), \end{aligned} \qquad (2.4)$$

and the cumulative distribution function is

$$F_X\left(x;\theta\right) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(u-\theta)^2}{2\sigma^2}} du = \Phi\left(\frac{x-\theta}{\sigma}\right), \qquad (2.5)$$

where ϕ and Φ are the density and cumulative distribution of the standard normal distribution, respectively.

Generalized Maximum Likelihood Estimator (GMLE)

In the case of estimating the population variance, the information number is proportional to $1/\sigma^2$ (see Al-Hadhrami, et al., 2009), allowing the Jeffery prior for σ to be written as

$\pi(\sigma)\alpha 1/\sigma$. The posterior distribution for σ is then given by

$$h(\sigma \mid x, y) \propto \frac{1}{\sigma} \prod_{i=1}^{m} i^2 \left(\frac{1}{\sigma}\right) \phi(z_i) \left(\frac{1}{\sigma}\right) \phi(w_i) \left[\Phi(z_i)\right]^{i-1} \left[1-\Phi(w_i)\right]^{i-1}. \tag{3.1}$$

The log of both sides of (3.1) is

$$L^*(\sigma) = C - \log\sigma + \sum_{i=1}^{m} \left[\log\left(\frac{1}{\sigma}\right)\phi(z_i) + \log\left(\frac{1}{\sigma}\right)\phi(w_i)\right] + \sum_{i=1}^{m}(i-1)\left[\log\Phi(z_i) + \log\left(1-\Phi(w_i)\right)\right], \tag{3.2}$$

where C is a constant. The first derivative of (3.2) is given by

$$\frac{\partial L^*}{\partial \sigma} = -\frac{1}{\sigma} + \left[\frac{1}{\sigma}\sum_{i=1}^{m} z_i^2 - \frac{m}{\sigma}\right] + \left[\frac{1}{\sigma}\sum_{i=1}^{m} w_i^2 - \frac{m}{\sigma}\right] + \sum_{i=1}^{m}(i-1)\left[\frac{\left(\frac{w_i}{\sigma}\right)\phi(w_i)}{1-\Phi(w_i)} - \frac{\left(\frac{z_i}{\sigma}\right)\phi(z_i)}{\Phi(z_i)}\right].$$

Let $\frac{\partial L^*}{\partial \sigma} = 0$, then the likelihood equation is defined as

$$-\frac{2m+1}{\sigma} + \frac{1}{\sigma}\sum_{i=1}^{m}\left[z_i^2 + w_i^2\right] + \sum_{i=1}^{m}(i-1)\left[\frac{\frac{w_i}{\sigma}\phi(w_i)}{1-\Phi(w_i)} - \frac{\frac{z_i}{\sigma}\phi(z_i)}{\Phi(z_i)}\right] = 0,$$

which may be written as

$$1 - \frac{1}{2m+1}\sum_{i=1}^{m}\left(z_i^2 + w_i^2\right) - \frac{1}{2m+1}\sum_{i=1}^{m}(i-1)\left[\frac{w_i\phi(w_i)}{1-\Phi(w_i)} - \frac{z_i\phi(z_i)}{\Phi(z_i)}\right] = 0. \tag{3.3}$$

If the second derivative of the likelihood with respect to σ is negative at the solution of $\frac{\partial L^*}{\partial \sigma} = 0$, then this solution is the GMLE of σ. The second derivative of the log likelihood with respect to σ is

$$\frac{\partial^2 L^*}{\partial \sigma^2} = T_1 + T_2 + T_3, \tag{3.4}$$

where

$$T_1 = \frac{2m+1}{\sigma^2} - \frac{3}{\sigma^2}\sum_{i=1}^{m}\left(z_i^2 + w_i^2\right) + \frac{3}{\sigma^2}\sum_{i=1}^{m}(i-1)\left[\frac{z_i\phi(z_i)}{\Phi(z_i)} - \frac{\phi(w_i)}{1-\Phi(w_i)}\right]$$
$$= \frac{2m+1}{\sigma^2}\left[1 - \frac{3}{2m+1}\sum_{i=1}^{m}\left(z_i^2 + w_i^2\right) + \frac{3}{2m+1}\sum_{i=1}^{m}(i-1)\left(\frac{z_i\phi(z_i)}{\Phi(z_i)} - \frac{\phi(w_i)}{1-\Phi(w_i)}\right)\right].$$

The value of T_1 at the solution of Equation (3.3) is

$$T_1 = -2\left(\frac{2m+1}{\sigma^2}\right),$$

and

$$T_2 = \sum_{i=1}^{m}(i-1)\left[\left(-\frac{z_i^2}{\sigma^2}\right)\frac{\phi(z_i)}{\Phi(z_i)}\left(z_i + \frac{\phi(z_i)}{\Phi(z_i)}\right)\right],$$

$$T_3 = \sum_{i=1}^{m}(i-1)\left[\frac{w_i^2}{\sigma^2}\left(\frac{\phi(w_i)}{1-\Phi(w_i)}\right)\left(w_i - \frac{\phi(w_i)}{1-\Phi(w_i)}\right)\right]$$

which are both negative. Therefore,

$\frac{\partial^2 L^*}{\partial \sigma^2} < 0$. Thus, the GMLE of σ is the solution of Equation (3.3) and the GMLE of variance is the square of this solution. Note that the GMLE of σ using SRS, when μ is known is given by:

$$\hat{\sigma}_{SRS} = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n+1}} . \qquad (3.5)$$

As shown in Table (1), the GMLE using MERSS is more efficient than its counterparts based on SRS, and the efficiency increases as the sample size increases.

Table 1: The Efficiency of $eff = MSE_{SRS} / MSE_{MERSS}$, $\sigma \sim \pi(\sigma) \propto 1/\sigma$, and $X \sim N(0,1)$

m	MSE_{SRS}	M_{MERSS}	eff
3	0.2721	0.2405	1.1313
5	0.1740	0.1230	1.4150
7	0.1304	0.0769	1.6951
11	0.0820	0.0375	2.1848
14	0.0661	0.0251	2.6261

Confidence Interval

From the sampling distribution of the variance, Table 2 shows the interval width (IW), lower bound (LB), upper bound (UB), and the approximated two-sided 95% confidence intervals (CI) for the variance of the normal distribution $N(3,1)$ using both MERSS and SRS methods. Table 3 shows the approximated two-sided 95% confidence intervals (CI) for the variance of $N(4,4)$ based on MERSS and SRS.

Based on Tables 2 and 3, it may be noted that the intervals using MERSS are shorter than that those based on SRS. Also, the width of the intervals becomes shorter as the set size increases. The width also depends on the population variance; the smaller the variance, the smaller the width.

Table 2: 95% Confidence Intervals for σ^2 of the Normal Distribution, $N(3,1)$, Using MERSS and SRS

CI for σ^2 using SRS			
m	IW	LB	UB
3	2.3216	0.1676	2.4892
7	1.5719	0.3810	1.9530
10	1.2898	0.4701	1.7599
15	1.0330	0.5456	1.5787
CI for σ^2 using MERSS			
m	IW	LB	UB
3	1.8525	0.3952	2.2477
7	1.1120	0.4939	1.6059
10	0.8358	0.6188	1.4546
15	0.5628	0.7352	1.2981

Table 3: 90% Confidence Intervals for the Variance of the Normal Distribution, $N(4,4)$, Using MERSS and SRS with MLE

CI for σ^2 using SRS			
m	IW	LB	UB
3	8.0225	0.9010	8.9235
7	4.4965	2.0174	6.5139
10	3.9709	2.2040	6.1749
15	3.2805	2.4799	5.7605
CI for σ^2 using MERSS			
m	IW	LB	UB
3	6.3062	1.9864	8.2926
7	3.2390	2.5284	5.7675
10	2.3814	2.8574	5.2388
15	1.8909	3.1147	5.0056

Testing Hypothesis

Once a confidence interval about the parameter is obtained, a test hypothesis about this parameter can be constructed. For a two-sided hypothesis the two-sided confidence interval may be used and the upper or lower bound confidence interval is for one-sided hypotheses with same significance level.

Consider the test hypothesis about the variance σ^2 of the normal distribution with known mean based on Bayesian paradigm when the sample is drawn using MERSS. The decision is based on the Bayes factor which is of the form

$$B = \frac{p_0 / p_1}{\pi_0 / \pi_1} = \frac{p_0 \pi_1}{p_1 \pi_0}, \quad (4.1)$$

where

$\pi_0 = p(\theta \in \Theta_0)$: Prior probability for $\theta \in \Theta_0$.

$\pi_1 = p(\theta \in \Theta_1)$: Prior probability for $\theta \in \Theta_1$.

$P_0 = P(\theta \in \Theta_0 \mid x)$: Posterior probability for $\theta \in \Theta_0$.

$P_1 = P(\theta \in \Theta_1 \mid x)$: Posterior probability for $\theta \in \Theta_1$.

π_0 / π_1: Prior odds on H_0 versus H_1.

p_0 / p_1: Posterior odds on H_0 versus H_1.

Two Simple Hypotheses

Consider testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 = \sigma_1^2$, where σ^2 is the variance of a normal distribution with known mean, μ. The Bayes factor in this case is $B = p(x, y \mid \sigma_0^2) / p(x, y \mid \sigma_1^2)$ which can be written for a sample from a normal distribution using MERSS as

$$B = \frac{\prod_{i=1}^{m} i^2 \begin{pmatrix} f(x_i; \mu, \sigma_0^2) F^{i-1}(x_i; \mu, \sigma_0^2) \\ f(y_i; \mu, \sigma_0^2) \left[1 - F(y_i; \mu, \sigma_0^2)\right]^{i-1} \end{pmatrix}}{\prod_{i=1}^{m} i^2 \begin{pmatrix} f(x_i; \mu, \sigma_1^2) F^{i-1}(x_i; \mu, \sigma_1^2) \\ f(y_i; \mu, \sigma_1^2) \left[1 - F(y_i; \mu, \sigma_1^2)\right]^{i-1} \end{pmatrix}} \quad (4.2)$$

To test the null hypothesis, 1,000 numerical comparisons were made between MERSS and SRS. Results for tests of rejection of the true null hypothesis are summarized in Tables 4 and 5 for two normal distributions $N(4,1)$ and $N(-6,3)$, respectively, using SRS and MERSS methods.

Tables 4 and 5 show that the error in rejecting the null hypothesis using MERSS is less than the error when using SRS; the error in rejecting the true hypothesis also becomes smaller as the sample size increases. In addition, the error becomes smaller as the alternative moves farther from the value assumed for the null hypothesis.

Simple Null Hypothesis versus Composite Hypothesis

Next a simple hypothesis was tested against a composite hypothesis about the variance of normal distribution using MERSS. That is $H_0 : \sigma^2 = \sigma_0^2$ was tested against $H_1 : \sigma^2 \neq \sigma_1^2$ when the population mean was known. The following Bayes factor was used

$$B = \frac{p(x, y \mid \sigma_0^2)}{\int_{\sigma^2 \neq \sigma_0^2} p(x, y \mid \sigma^2) \pi(\sigma^2) d\sigma^2}, \quad (4.3)$$

where

$$p(x, y \mid \sigma_0^2) = \prod_{i=1}^{m} i^2 \begin{pmatrix} f(x_i; \sigma_0^2) F^{i-1}(x_i; \sigma_0^2) \\ f(y_i; \sigma_0^2) \left[1 - F(y_i; \sigma_0^2)\right]^{i-1} \end{pmatrix} = \sum_{k_1=0}^{0} \sum_{k_2}^{1} \cdots \sum_{k_m=0}^{m-1} \left\{ \prod_{i=1}^{m} \left[a(i, k_i) G(x, y, \sigma_0^2, i, k_i) \right] \right\},$$

$$G(x, y, \sigma_0^2, i, k_i) = F^{i-1}(x_i; \sigma_0^2) F^{k_i}(y_i; \sigma_0^2) f(x_i; \sigma_0^2) f(y_i; \sigma_0^2),$$

and

$$a(i, k_i) = i^2 (-1)^{k_i} \binom{i-1}{k_i}, \quad k_i = 0, 1, 2, ..., i-1.$$

Table 4: Comparison Between MERSS and SRS When a Simple Hypothesis about the Variance of the Normal Distribution, $N(4,1)$ was Tested 1,000 Times

Number of Rejections of the Null Hypothesis						
H_1	$m=3$		$m=6$		$m=15$	
	MERSS	SRS	MERSS	SRS	MERSS	SRS
1.2	287	302	252	284	111	218
1.4	223	237	123	163	14	85
1.6	151	154	62	92	0	40
1.8	88	131	28	75	0	7
2	67	79	16	40	0	6
2.2	43	69	6	18	0	2
2.4	37	47	5	15	0	0

Table 5: Comparison Between MERSS and SRS When a Simple Hypothesis about the Variance of the Normal Distribution, $N(-6,3)$ was Tested 1,000 Times

Number of Rejections of the Null Hypothesis						
H_1	$m=6$		$m=10$		$m=15$	
	MERSS	SRS	MERSS	SRS	MERSS	SRS
3.2	378	396	363	380	327	346
3.4	317	321	243	307	167	290
3.6	245	271	172	231	107	228
3.8	213	251	122	186	54	159
4	173	205	83	172	23	109
4.2	119	141	59	129	8	79
4.4	99	127	32	95	2	52
4.6	82	120	22	71	1	32
4.8	53	91	6	63	0	31

Also,

$$\int_{\sigma^2\neq\sigma_0^2} p\left(x,y\,|\,\sigma^2\right)\pi\left(\sigma^2\right)d\sigma^2 = \left(\begin{array}{c}\sum_{k_1=0}^{0}\sum_{k_2}^{1}\cdots\sum_{k_m=0}^{m-1}\prod_{i=1}^{m}a\left(i,k_i\right)\\ \int_{\sigma^2\neq\sigma_0^2}\prod_{i=1}^{m}G\left(x,y,\sigma^2,i,k_i\right)\pi\left(\sigma^2\right)d\sigma^2\end{array}\right),$$

therefore, the Bayes factor can be written as:

$$B=\frac{\sum_{k_1=0}^{0}\sum_{k_2}^{1}\cdots\sum_{k_m=0}^{m-1}\left\{\prod_{i=1}^{m}\left[\frac{a\left(i,k_i\right)}{G\left(x,y,\sigma_0^2,i,k_i\right)}\right]\right\}}{\left(\begin{array}{c}\sum_{k_1=0}^{0}\sum_{k_2}^{1}\cdots\sum_{k_m=0}^{m-1}\prod_{i=1}^{m}a\left(i,k_i\right)\\ \int_{\sigma^2\neq\sigma_0^2}\prod_{i=1}^{m}G\left(x,y,\sigma^2,i,k_i\right)\pi\left(\sigma^2\right)d\sigma^2\end{array}\right)} \tag{4.4}$$

Using Monte Carlo methods, an approximation for the denominator of the Bayes factor is given by

$$\int_{\sigma^2 \neq \sigma_0^2} p\left(x, y \mid \sigma^2\right) \pi\left(\sigma^2\right) d\sigma^2 =$$

$$\frac{1}{r} \sum_{k_1=0}^{0} \sum_{k_2=0}^{1} \cdots \sum_{k_m=0}^{m-1} \sum_{t=1}^{r} \left[\prod_{i=1}^{m} a\left(i, k_i\right)\right] \prod_{i=1}^{m} G\left(x, y, \sigma_t^2, i, k_i\right) \tag{4.5}$$

If the underlying distribution is $N(2,1)$, assuming that $\pi_0 = \pi_1 = 0.5$, the test is executed 1,000 times using computer simulation using SRS and MERSS for m = 5, 10, 15; results are presented in Table 6 based on the constant prior.

Table 6: Numerical Comparison Between MERSS and SRS when Testing Hypothesis about the Variance of the Normal Distribution

Method	Number of rejections the null hypothesis while it is true		
	$m=5$	$m=10$	$m=15$
MERSS	300	168	119
SRSS	384	278	206

From Table 6, it is observed that the error in testing the hypothesis using MERSS is less than the error when using SRS, also the error becomes smaller as sample size increases.

Composite Null Hypothesis versus Composite Alternative Hypothesis

If the null and alternative hypotheses are composite, the Bayes factor

$$B = \frac{\int_{\sigma^2 \in \Theta_0} p(x \mid \sigma_0^2) \pi_0\left(\sigma^2\right) d\sigma^2}{\int_{\sigma^2 \in \Theta_1} p\left(x \mid \sigma_1^2\right) \pi_1\left(\sigma^2\right) d\sigma^2}, \tag{4.6}$$

may be used, where

$$\int_{\sigma^2 \in \Theta_0} p\left(x, y \mid \sigma_0^2\right) \pi_0\left(\sigma^2\right) d\sigma^2$$

$$= \left(\begin{array}{c} \sum_{k_1=0}^{0} \sum_{k_2=0}^{1} \cdots \sum_{k_m=0}^{m-1} \prod_{i=1}^{m} a\left(i, k_i\right) \\ \int_{\sigma^2 \in \Theta_0} \prod_{i=1}^{m} G\left(x, y, \sigma_0^2, i, k_i\right) \pi_0\left(\sigma^2\right) d\sigma^2 \end{array} \right)$$

and

$$\int_{\sigma^2 \in \Theta_1} p\left(x, y \mid \sigma_1^2\right) \pi_1\left(\sigma^2\right) d\sigma^2$$

$$= \left(\begin{array}{c} \sum_{k_1=0}^{0} \sum_{k_2=0}^{1} \cdots \sum_{k_m=0}^{m-1} \prod_{i=1}^{m} a\left(i, k_i\right) \\ \int_{\sigma^2 \in \Theta_1} \prod_{i=1}^{m} G\left(x, y, \sigma_1^2, i, k_i\right) \pi_1\left(\sigma^2\right) d\sigma^2 \end{array} \right)$$

with

$$G\left(x, y, \sigma^2, i, k_i\right) =$$
$$F^{i-1}\left(x_i; \sigma^2\right) F^{k_i}\left(y_i; \sigma^2\right) f\left(x_i; \sigma^2\right) f\left(y_i; \sigma^2\right)$$

where

$$a\left(i, k_i\right) = i^2 (-1)^{k_i} \binom{i-1}{k_i}, \; k_i = 0, 1, 2, \ldots, i-1 .$$

Suppose that the hypothesis to be tested is a one-sided hypothesis $H_0 : \sigma^2 \leq \sigma_0^2$ versus $H_1 : \sigma^2 > \sigma_0^2$. For example, let $\pi_0 = \pi_1 = 0.5$, $m = 5, 10, 15$, $H_0 : \sigma^2 \leq 9$ versus $H_1 : \sigma^2 > 9$, and assume that the hypothesis is tested 1,000 times. Table 7 shows the simulation comparison between MERSS and SRS based on Bayes factors.

Table 7 indicates that the error in rejecting the null hypothesis using MERSS is less than using SRS based on the same sample size. Also, the error decreases as the sample size increases. Furthermore, because $H_0 : \sigma^2 \leq 9$, the error decreases as the true value moves farther from 9.

Table 7: Results for Testing $H_0: \sigma^2 \le 9$ versus $H_1: \sigma^2 > 9$ Using SRS and MERSS

The Population Variance	Number of Rejections of the Null Hypothesis					
	$m = 5$		$m = 10$		$m = 15$	
	MERSS	SRS	MERSS	SRS	MERSS	SRS
$\sigma^2 = 7.84$	379	418	253	339	150	289
$\sigma^2 = 6.76$	246	317	112	185	41	177
$\sigma^2 = 5.76$	159	194	31	107	4	56
$\sigma^2 = 4.84$	16	65	5	43	0	18
$\sigma^2 = 4$	19	72	0	13	0	0

Conclusion

Bayesian inferences regarding the population variance of the normal distribution were considered based on the MERSS method. Results indicate that the confidence intervals based on MERSS are shorter than those from SRS. These intervals will be shorter as the set size and the width increases, and they depend on the population variance. For the hypothesis testing considered in this study, it was shown that the error in rejecting the null hypothesis using MERSS is less than the error observed when using SRS.

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