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Frequency Domain Modeling with Piecewise Constant Spectra

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Using piecewise constant functions as models for the spectral density of the differenced log real U.S. GDP it was found that these models have the capacity to compete with the spectral densities implied by ARMA models. According to AIC and BIC the piecewise constant spectral densities are superior to ARMA.

Key words: Spectral analysis, piecewise constant spectra, ARMA spectra, aggregate output.

Introduction

Univariate ARMA models are used in empirical economics as simple, purely statistical models for properly transformed macroeconomic time series (such as the first differences of the logs of the real GDP), and for the description of the serial correlation in the errors of more complex models such as linear or nonlinear multivariate regression models. A typical example of the first type is the study by Campbell & Mankiw (1987) who used ARMA(p,q) models with $p \leq 3$ and $q \leq 3$ to investigate the long-run behavior of aggregate output. The persistence of output shocks can be measured by the cumulative impulse response or, equivalently, by the value of the spectral density at frequency zero, however, two drawbacks exist. The first is that the model parameters must be estimated by numerical optimization routines, which depend heavily on the starting values and can easily get stuck at local optima (e.g., Hauser, et al., 1999). The second is the extreme sensitivity of inference to the order of the ARMA representation (e.g., Christiano & Eichenbaum, 1990).

Recently, interest has shifted from univariate to multivariate modeling (e.g., Blanchard & Quah, 1989; Pesaran, et al., 1993; Pesaran & Shin, . However, a multivariate approach based on economic theory and the information contained in a much larger data set is not necessarily better than a simple univariate time series model, because both the estimation and the identification of multivariate models is many orders of magnitude more difficult. But even in situations where multivariate models outperform univariate models, the latter are often used as benchmarks for the former (see, e.g., Schumacher & Dreger, 2004). Thus, univariate ARMA models still have an important role to play. This article proposes competitive alternatives to ARMA models for the purpose of estimating the spectral densities of macroeconomic time series.

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Methodology

The following piecewise constant functions are proposed:

$$g_r(\omega) = a(b_1 1_{[\alpha_0,\alpha_1)} + b_2 1_{[\alpha_1,\alpha_2)} + \ldots + b_{r-1} 1_{[\alpha_{r-2},\alpha_{r-1})} + 1_{[\alpha_{r-1},\alpha_r]}),\ \omega \in [0,\pi], \quad (1)$$

where $r \geq 2$ and $0=\alpha_0<\alpha_1<\ldots<\alpha_r=\pi$, for the approximation of the spectral densities of macroeconomic time series. There are $2(r-1)+1$ parameters that must be estimated, a, b_1, …, b_{r-1}, α_1, …, α_{r-1}. An obvious choice for the first parameter is:

$$a=\frac{I_1+...+I_m}{b_1 s_1+b_2(s_2-s_1)+...+b_{r-1}(s_{r-1}-s_{r-2})+(m-s_{r-1})}, \quad (2)$$

where s_j is the largest integer such that:

$$\frac{2\pi s_j}{n}<\alpha_j. \quad (3)$$

The parameters $b_1, \ldots, b_{r-1}, \ldots, s_1, \ldots, s_{r-1}$ can be found by maximizing the Whittle likelihood

$$\prod_{k=1}^{m}\frac{1}{g_r(\omega_k)}\exp\left(-\frac{I_k}{g_r(\omega_k)}\right), \quad (4)$$

or, equivalently,

$$-\sum_{k=1}^{n}\log(g_r(\omega_k))-\sum_{k=1}^{m}\frac{I_k}{g_r(\omega_k)}, \quad (5)$$

where

$$\omega_k=\frac{2\pi k}{n}, \; k=1,\ldots,m. \quad (6)$$

The parameters $\alpha_1, \ldots, \alpha_{r-1}$ can be obtained from $s_1, \ldots, s_{r-1}$ via

$$\alpha_j=\frac{2\pi s_j}{n}+\frac{\pi}{n}. \quad (7)$$

To demonstrate the usefulness of this approach, the seasonally adjusted quarterly real U.S. GDP from 1947.1 to 2007.1 was downloaded from FRED® (Federal Reserve Economic Data) and the spectral density of the first differences of the log GDP was approximated by the piecewise constant functions g_j, j=2,3,4.

Results

Figure 1 compares the three piecewise constant spectral densities with the best three ARMA spectral densities selected by BIC. One of these three ARMA models, namely the ARMA(3,2) model, is the best ARMA model according to AIC. Apart from the ARMA models of order (2,3) and (3,3), whose spectral densities are very similar to that of the ARMA(3,2) model, all other ARMA models ($p\leq 8$ & $q=0$, $p=0$ & $q\leq 8$, $1\leq p, q\leq 3$) have much higher AIC values than the ARMA(3,2) model. To facilitate the comparison between the piecewise constant spectral densities g_2, g_3, and g_4, and the ARMA spectral densities slightly modified AIC and BIC values (AIC* and BIC*) obtained from the Whittle likelihood were used. Among the top models both according to AIC* and BIC* (see Tables 1 and 2) are g_2, g_3, and g_4. Overall, g_2 has the smallest BIC* value and g_4 has the smallest AIC* value.

Table 1: AIC values (obtained from the Whittle likelihood) for piecewise constant spectral densities g(r) & ARMA(p,q) spectral densities, respectively, fitted to the differenced log real U.S. GDP

	1	2	3	4	5	6	7	8
g(r)		-2456.8	-2457.5	-2461.6				
AR(p)	-2447.3	-2446.9	-2447.7	-2448.6	-2448.1	-2446.2	-2444.2	-2442.4
MA(q)	-2440.5	-2448.2	-2447.8	-2445.9	-2447.7	-2445.7	-2443.8	-2445.2
ARMA(1,q)	-2446.2	-2447.2	-2445.8					
ARMA(2,q)	-2445.8	-2450.7	-2457.6					
ARMA(3,q)	-2449.1	-2459.2	-2457.1					

Table 2: BIC values (obtained from the Whittle likelihood) for piecewise constant spectral densities g(r) & ARMA(p,q) spectral densities, respectively, fitted to the differenced log real U.S. GDP

	1	2	3	4	5	6	7	8
g(r)		-2446.3	-2440.1	-2437.3				
AR(p)	-2440.3	-2436.5	-2433.7	-2431.2	-2427.3	-2421.9	-2416.4	-2411.1
MA(q)	-2433.5	-2437.8	-2433.9	-2428.5	-2426.8	-2421.3	-2416.0	-2413.9
ARMA(1,q)	-2435.8	-2433.3	-2428.4					
ARMA(2,q)	-2431.9	-2433.3	-2436.8					
ARMA(3,q)	-2431.7	-2438.3	-2432.8					

Conclusion

The results obtained show that piecewise constant spectral densities are extremely useful tools for the spectral analysis of macroeconomic time series and can outperform the more sophisticated ARMA spectral densities. This finding is striking given that twenty-five ARMA spectral densities were tried but only three piecewise constant spectral densities. It may also serve as a severe warning not to over-interpret certain characteristics of estimated ARMA spectral densities such as a decline or incline near frequency zero.

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Figure 1: Periodogram of differenced log GDP together with piecewise constant spectral densities (with two, three, and four pieces) & ARMA spectral densities AR(1), MA(2), ARMA(3,2)