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Empirical Characteristic Function Approach to Goodness of Fit Tests for the Logistic Distribution under SRS and RSS

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The integral of the squares modulus of the difference between the empirical characteristic function and the characteristic function of the hypothesized distribution is used by Wong and Sim (2000) to test for goodness of fit. A weighted version of Wong and Sim (2000) under ranked set sampling**,** a sampling technique introduced by McIntyre (1952), is examined. Simulations that show the ranked set sampling counterpart of Wong and Sim (2000) is more powerful.

Key words: Goodness of fit test, empirical distribution function, logistic distribution, ranked set sampling, simple random sampling.

Introduction

In any one-sample goodness of fit test problem where a random sample $X_1, X_2,...,X_r$ from an unknown distribution function $F(x)$ is given in order to test the hypothesis $H_o: F(x) = F_o(x)$ for all x against the hypothesis $H_1: F(x) \neq F_o(x)$, where $F_o(x)$ is a known distribution function. Stephens (1974) provided a practical guide to goodness of fit tests using statistics based on the empirical distribution function (EDF). Green and Hegazy (1976) studied modified forms of the Kolmogorov-Smirnov D, Cramer-von Mises W^2 and the Anderson-Darling A^2 goodness of fit tests. Stephens (1979) gave goodness of fit tests for the logistic distribution. A comprehensive survey for goodness of fit tests can be found in the book of D'Agostino and Stephens (1986).

Gürtler and Henze (2000) used another approach to test for goodness of fit for the Cauchy distribution. They built their test based on the weighted distance between the empirical characteristic function of the sample and the characteristic function of the null distribution, that is, they considered the test statistic of the form:

$$T = r\int_{-\infty}^{\infty}\left|\Phi_r(t) - e^{-|t|}\right|^2 w(t)dt, \quad w(t) = e^{-\kappa|t|}, \quad \kappa > 0,$$
$$= \frac{2}{r}\sum_{j,\kappa=1}^{r}\frac{\kappa}{\kappa^2 + (y_j - y_\kappa)^2} - 4\sum_{j=1}^{r}\frac{1+\kappa}{(1+\kappa)^2 + y_j^2} + \frac{2r}{2+\kappa}, \quad (1)$$

where $y_j = (x_j - \hat{\alpha})/\hat{\beta}$, and

$$\Phi_r(t) = \frac{1}{r}\sum_{j=1}^{r}\exp(Ity_j)$$

is the empirical characteristic function of the sample. The function $w(t)$ is a weight function and $\hat{\alpha}, \hat{\beta}$ are the Maximum Likelihood

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Estimators (MLE) of α and β, the location and the scale parameters of the Cauchy distribution. Wong and Sim (2000) studied the test statistic T when $w(t) \equiv 1$, for different distributions. Matsui and Takemura (2005) also considered the problem of Gürtler and Henze (2000) but used a different research design. For more information about the application of the empirical characteristic function to goodness of fit test see Feuerverger and Mureika (1977), Meintnis (2004), Epps (2005) and Towhidi & Salmanpour (2007).

In various situations, visual ordering of sample units (with respect to the variable of interest) is less expensive against its quantification. For statistical populations with such a property, McIntyre (1952) was the first to employ the visual ranking of sampling units in order to select a sample that is more informative than a simple random sample. Later, his sampling technique was known as Ranked Set Sampling (RSS). Without any theoretical developments, he showed that the RSS is more efficient and cost effective method than the Simple Random Sampling (SRS) technique. An RSS sample can be obtained as follows:

1. Select m random samples from the population of interest each of size m.
2. From the i^{th} sample detect, using a visual inspection, the i^{th} order statistic and choose it for actual quantification, say, Y_i, $i = 1,...,m$.
3. RSS is the set of the order statistics $Y_1,...,Y_m$.
4. The technique could be repeated r times to obtain additional observations.

Takahasi and Wakimoto (1968) developed the theoretical framework for RSS.

Visual ranking is accomplished based on an experimenter's experience. Hence, two factors affect the efficiency of an RSS: the set size and the ranking errors. The larger the set size, the larger the efficiency of the RSS; however, the larger the set size, visual ranking is more difficult and the ranking error is larger (Al-Saleh & Al-Omari, 2002). For this, several authors have modified MacIntyre's RSS scheme to reduce the error in ranking and to make visual ranking easier for an experimenter. Samawi, et al. (1996) investigated Extreme Ranked Set Sample (ERSS), i.e. they quantified the smallest and the largest order statistics. Muttlak (1997) introduced Median Ranked Set Sampling (MRSS) which consists of quantifying only the median in each set. Bhoj (1997) proposed a modification to the RSS and called it new ranked set sampling (NRSS). Al-Odat and Al-Saleh (2001) introduced the concept of varied set size RSS, which is called later by moving extremes ranked set sampling (MERSS). For more details about these developments see Chen (2000).

Stockes and Sager (1988) were the first who proposed a Kolmogorov-Smirnov goodness of fit test based on the empirical distribution function of an RSS. In addition, they derived the null distribution of their proposed test. Al-Subh, et al. (2008) studied the Chi-square test for goodness of fit test under the RSS technique and its modifications. Their simulation showed that the Chi-square test for the null logistic distribution is more powerful than its counterpart under SRS technique. This article examines the power of the test given in equation (1) when sample is selected using one of the modifications of the RSS, specifically, the modification that chooses only the i^{th} order statistic for quantification.

Problem Formalization

It can be noted that testing the hypotheses:

$$H_o : F(x) = F_o(x), \quad \forall x$$

vs.

$$H_1 : F(x) \neq F_o(x)$$

is equivalent to testing the hypothesis

$$H_o^* : G_i(y) = G_{io}(y), \quad \forall y$$

vs.

$$H_1^* : G_i(y) \neq G_{io}(y)$$

for some i, where $G_i(y)$, $G_{io}(y)$ are the cdf's of the i^{th} order statistics of random samples of size $2m-1$ chosen from $F(x)$ and $F_o(x)$, respectively. The rationale behind choosing an odd set size - rather than an even one - is to simplify the comparison with the median RSS, because an even set size produces two middle values. Moreover, quantifying the two middle sample units is more expensive than quantifying one sampling unit. If $f(y)$ and $f_o(y)$ are the corresponding pdf's of $F(x)$ and $F_o(x)$, respectively, then according to Arnold, et al. (1992), $G_i(y)$ and $G_{io}(y)$ have the following representations:

$$G_i(y) = \sum_{j=i}^{2m-1} \binom{2m-1}{j} [F(y)]^j [1-F(y)]^{(2m-1)-j}$$

and

$$G_{io}(y) = \sum_{j=i}^{2m-1} \binom{2m-1}{j} [F_o(y)]^j [1-F_o(y)]^{(2m-1)-j},$$

respectively. The corresponding pdf's are

$$g_i(y) = \frac{(2m-1)!}{(i-1)!(2m-1-i)!} F(y)^{i-1}(1-F(y))^{2m-1-i} f(y)$$

and

$$g_{io}(y) = \frac{(2m-1)!}{(i-1)!(2m-1-i)!} F_o(y)^{i-1}(1-F_o(y))^{2m-1-i} f_o(y),$$

respectively. It can be shown that $G_i(y)$=$G_{i0}(y)$ if and only if $F(x) = F_o(x)$, which means this statistical testing problem is invariant.

If ranked set sampling is employed to collect the data using the i^{th} order statistic, then the resulting data is used to build a test based on the empirical characteristic function of these data as described in equation (1). The empirical characteristic function and the population characteristic function that should be used, respectively, are:

$$\Phi_{ri}(t) = \frac{1}{r} \sum_{j=1}^{r} \exp(ItY_j),$$

and

$$\Phi_i(t) = \int_{-\infty}^{\infty} \exp(Ity) dG_{io}(y).$$

Hence, a ranked set sample counterpart of the test T is given by

$$T_i^* = r\int_{-\infty}^{\infty} \left|\Phi_{ri}(t) - \Phi_i(t)\right|^2 w(t)dt, \qquad (2)$$

where $w(t)$ is a suitable weight function. Using complex integration, it may be shown that:

$$\Phi_{2m-1}(t) = Beta(1-I\kappa t,\ 1-I\kappa t).$$

The test rejects H_o^* for large values of T_i^*. Attention is restricted to the case when $F_o(x) = (1+e^{-(x-\theta)/\sigma})^{-1}$, that is, for the logistic distribution. Even for logistic distribution, the test T_i^* has no closed form as in the Cauchy case; for this, a simulation study is conducted to study the power of the test T_i^* and its counterpart T. The two tests will be compared in terms of power based on samples of the same size. The power of the T_i^* test can be calculated according to the equation

$$\text{Power of } T_i^*(H) = P_H(T_i^* > d_\alpha), \qquad (3)$$

where H is a cdf under the alternative hypothesis H_1^*. Here d_α is the 100α percentage point of the distribution of T_i^* under H_o. The efficiency of the test statistic T_i^* relative to T is calculated as a ratio of powers:

$$eff\ (T_i^*,\ T) = \frac{\text{power of } T_i^*}{\text{power of } T},$$

thus, T_i^* is more powerful than T if $eff(T_i^*, T) > 1$.

Algorithms for Power and Percentage Point

The following two algorithms approximate the power and the percentage of the tests T and T_i^*.

Percentage Point Algorithm:

1. Simulate $Y_1,\ldots,Y_r$ from $G_{io}(y)$.
2. Find T_i^* according to equation (2).
3. Repeat steps (1)-(2) to obtain $T_{i1}^*, \ldots, T_{i\,10{,}000}^*$.
4. Approximate d_α, the percentage point of T_i^*.

Power Algorithm:

1. Simulate $Y_1,\ldots,Y_r$ from H, a distribution under H_1^*.
2. Find T_i^* according to equation (2).
3. Repeat the steps (1)-(2) to obtain $T_{i1}^*, \ldots, T_{i\,10{,}000}^*$.
4. Approximate the power of T_i^* as:

$$\text{Power of } T_i^*(H) = \frac{1}{10{,}000}\sum_{t=1}^{10{,}000} I(T_{it}^* > d_\alpha),$$

where $I(.)$ stands for indicator function.

Results

To compare tests T and T_i^*, a Monte Carlo simulation study was conducted to approximate the power of each test based on 10,000 iterations according to the algorithms shown. Due to symmetry the first and the last order statistics produced the same power; therefore, simulation results for the largest order statistic are not presented. The powers of the two tests were compared for samples sizes $r = 10, 20, 30$, set sizes $m = 1, 2, 3, 4$ and alternative distributions *Normal* = *N*(0. 1), *Laplace* = *L*(0, 1), *Lognormal* = *LN*(0, 1), *Cauchy* = *C*(0, 1), *StudentT* = *S*(5), *Uniform* = *U*(0, 1), *Beta* (0, 1), *ChiSquare* (5) and *Gamma* (2, 1). In addition, the following weight functions were used in the simulation study:

$$w_1(t) = \text{Real Part of } \operatorname{Beta}(1-\kappa tI, 1-\kappa tI),$$

$$w_2(t) = \exp(-\kappa|t|),$$

$$w_3(t) = \exp(-\kappa t^2),$$

$$w_4(t) = |\cos(t)|\,e^{-\kappa t^2},$$

and

$$w_5(t) = (\kappa + t^2)^{-1}.$$

Simulation results are presented in Tables (1)-(5).

Simulation results for the uniform distribution show that the powers of all test statistics equal one, for this reason these powers are not reported in Tables (1)-(5). The simulation also shows that the efficiencies are equal to one for the non-symmetric alternatives: *Lognormal* = *LN*(0, 1), *ChiSquare* (5), *Gamma* (2, 1) and *Beta* (0, 1), thus, these are not presented in the tables.

Conclusion

Based on data in the tables, the following conclusions regarding T_i^* are put forth:

1. The efficiencies are greater than one for all alternatives, weight functions and all values of m, r and κ, thus indicating that the test T_1^* is more powerful than the test T.
2. It is noted that, for each alternative, the efficiency is increasing in m.
3. No clear pattern is observed in the efficiency values and the weight function, but for $\kappa = 1.5$ and $m = 4$, the efficiency has the highest values.
4. The worst value of the efficiency occurs when $H = N(0, 1)$ and $r = 50$.

This article considered a counterpart goodness of fit test based on the empirical characteristic function under ranked set sampling. The null distribution and the power of the new test have no closed forms; therefore they have been obtained using simulation. The simulation results show that the ranked set sampling counterpart is more powerful than the empirical characteristic function based on a simple random sample. In addition, it also possible to improve the power of the test statistic (1) (see introduction) under different ranked set sampling schemes, however, this discussion is avoided due to space limitations.

Table 1: Values of $eff\ (T_i^*,\ T)$ Using $w_1(t)$ for $r = 10,\ 20,\ 30,\ 50$, $m = 1,\ 2,\ 3,\ 4$ and $\alpha = 0.05$

	r = 10, $w_1(t) =$ Real Part of $\text{Beta}(1-\kappa tI, 1-\kappa tI)$											
	$\kappa = 0.5$				$\kappa = 1$				$\kappa = 1.5$			
$H\backslash m$	1	2	3	4	1	2	3	4	1	2	3	4
$N(0,\ 1)$	1	1.31	2.13	3.25	1	1.77	4.2	6.94	1	4.25	12.26	23.52
$L(0,\ 1)$	1	1.16	1.38	1.67	1	1.60	2.82	3.76	1	3.33	6.89	10
$C(0,\ 1)$	1	1.52	2.89	4.14	1	1.85	3.4	5.12	1	1.51	2.71	3.729
$S(5)$	1	1.23	1.7	2.20	1	1.48	2.73	4.02	1	2.63	6.21	10.04
r = 20												
$N(0,\ 1)$	1	1.43	2.05	2.45	1	1.45	2.26	2.62	1	2.42	4.6	5.53
$L(0,\ 1)$	1	1.15	1.35	1.56	1	1.47	2.25	2.79	1	2.27	4.35	5.94
$C(0,\ 1)$	1	1.79	3.66	5.36	1	2.06	3.83	5.47	1	1.69	3.25	4.12
$S(5)$	1	1.22	1.69	2.22	1	1.45	2.31	3.13	1	1.92	4.45	6.14
r = 30												
$N(0,\ 1)$	1	1.25	1.51	1.57	1	1.26	1.49	1.53	1	1.52	1.91	1.97
$L(0,\ 1)$	1	1.12	1.29	1.42	1	2.32	3.01	3.75	1	2.3	3.93	5.25
$C(0,\ 1)$	1	2.07	3.77	5.48	1	1.35	2.56	3.42	1	1.77	2.98	3.67
$S(5)$	1	1.18	1.63	1.89	1	1.32	1.82	2.33	1	1.69	2.98	3.8
r = 50												
$N(0,\ 1)$	1	1.06	1.09	1.09	1	1.05	1.06	1.06	1	1.08	1.09	1.09
$L(0,\ 1)$	1	1.05	1.12	1.21	1	1.16	1.35	1.52	1	1.44	2.06	2.37
$C(0,\ 1)$	1	1.57	2.81	3.53	1	2.08	3.36	3.83	1	1.73	2.53	2.74
$S(5)$	1	1.18	1.38	1.5	1	1.13	1.39	1.51	1	1.26	1.73	1.9

Table 2: Values of $eff\ (T_i^*,\ T)$ Using $w_2(t)$ for $r = 10,\ 20,\ 30,\ 50$, $m = 1,\ 2,\ 3,\ 4$ and $\alpha = 0.05$

	r = 10, $w_2(t) = \exp(-\kappa\lvert t\rvert)$											
	$\kappa = 0.5$				$\kappa = 1$				$\kappa = 1.5$			
$H\backslash m$	1	2	3	4	1	2	3	4	1	2	3	4
$N(0,\ 1)$	1	1.31	1.91	2.82	1	1.35	2.4	3.88	1	1.57	3	4.82
$L(0,\ 1)$	1	1.13	1.22	1.38	1	1.15	1.61	1.87	1	1.49	2.11	2.69
$C(0,\ 1)$	1	1.67	2.44	3.56	1	1.76	3.34	4.85	1	1.53	3.13	4.43
$S(5)$	1	1.25	1.55	2.06	1	1.29	1.91	2.6	1	1.47	2.11	3.35
r = 20												
$N(0,\ 1)$	1	1.38	2.07	2.57	1	1.44	2.11	2.48	1	1.5	2.28	2.68
$L(0,\ 1)$	1	1.12	1.28	1.45	1	1.16	1.5	1.7	1	1.26	1.68	2.06
$C(0,\ 1)$	1	1.63	2.7	4.04	1	2.08	3.7	5.51	1	1.92	3.63	5.24
$S(5)$	1	1.3	1.68	2.2	1	1.3	1.79	2.37	1	1.39	2	2.64
r = 30												
$N(0,\ 1)$	1	1.28	1.64	1.78	1	1.26	1.49	1.55	1	1.28	1.51	1.55
$L(0,\ 1)$	1	1.11	1.15	1.28	1	1.21	1.38	1.55	1	1.19	1.55	1.8
$C(0,\ 1)$	1	1.54	2.73	3.99	1	1.86	3.58	4.77	1	1.78	3.37	4.41
$S(5)$	1	1.17	1.57	1.9	1	1.22	1.64	1.94	1	1.29	1.77	2.15
r = 50												
$N(0,\ 1)$	1	1.11	1.16	1.16	1	1.07	1.09	1.09	1	1.05	1.07	1.07
$L(0,\ 1)$	1	1.06	1.1	1.17	1	1.08	1.15	1.25	1	1.13	1.26	1.37
$C(0,\ 1)$	1	1.5	2.65	3.53	1	1.77	2.97	3.55	1	1.85	2.92	3.36
$S(5)$	1	1.21	1.43	1.6	1	1.13	1.34	1.47	1	1.18	1.37	1.5

Table 3: Values of $eff\ (T_i^*,\ T)$ Using $w_3(t)$ for $r = 10,\ 20,\ 30,\ 50$, $m = 1,\ 2,\ 3,\ 4$ and $\alpha = 0.05$

	r = 10, $w_3 = e^{-\kappa t^2}$											
	$\kappa = 0.5$				$\kappa = 1$				$\kappa = 1.5$			
$H\backslash m$	1	2	3	4	1	2	3	4	1	2	3	4
$N(0,\ 1)$	1	1.35	2.33	3.66	1	1.46	3.43	5.6	1	1.59	5.21	9.18
$L(0,\ 1)$	1	1.48	2	2.46	1	1.88	2.98	3.85	1	2.45	4.48	6.17
$C(0,\ 1)$	1	1.78	3.37	5.53	1	1.97	3.91	5.88	1	1.82	3.29	4.79
$S(5)$	1	1.15	1.8	2.45	1	1.37	2.55	3.97	1	1.6	3.67	6.5
r = 20												
$N(0,\ 1)$	1	1.45	2	2.33	1	1.41	2.27	2.63	1	1.41	2.46	2.86
$L(0,\ 1)$	1	1.12	1.44	1.72	1	1.36	1.99	2.39	1	1.47	2.54	3.14
$C(0,\ 1)$	1	1.91	3.85	6.03	1	2.17	4.71	6.72	1	2.35	3.79	5.87
$S(5)$	1	1.29	1.73	2.3	1	1.34	2.13	2.9	1	1.51	2.68	3.7
r = 30												
$N(0,\ 1)$	1	1.19	1.4	1.45	1	1.24	1.45	1.48	1	1.34	1.6	1.64
$L(0,\ 1)$	1	1.12	1.33	1.52	1	1.19	1.73	2	1	1.56	2.27	2.81
$C(0,\ 1)$	1	1.99	3.95	5.35	1	2.26	4.26	5.78	1	2.84	4.48	5.76
$S(5)$	1	1.22	1.59	1.95	1	1.27	1.8	2.19	1	1.25	2	2.51
r = 50												
$N(0,\ 1)$	1	1.06	1.07	1.07	1	1.04	1.05	1.05	1	1.05	1.06	1.06
$L(0,\ 1)$	1	1.08	1.15	1.24	1	1.12	1.34	1.52	1	1.13	1.51	1.76
$C(0,\ 1)$	1	2.04	3.29	4.12	1	2.32	3.86	4.52	1	2.22	3.47	3.94
$S(5)$	1	1.15	1.35	1.45	1	1.12	1.37	1.48	1	1.17	1.49	1.62

Table 4: Values of $eff\ (T_i^*,\ T)$ Using $w_4(t)$ for $r = 10,\ 20,\ 30,\ 50$, $m = 1,\ 2,\ 3,\ 4$ and $\alpha = 0.05$

	r = 10, $w_4(t) = \|\cos(t)\| e^{-\kappa t^2}$											
	$\kappa = 0.5$				$\kappa = 1$				$\kappa = 1.5$			
*H**m*	1	2	3	4	1	2	3	4	1	2	3	4
$N(0,\ 1)$	1	2.15	4.99	8.55	1	2.92	8.08	15.10	1	5.61	15.6	31.60
$L(0,\ 1)$	1	1.95	3.54	4.56	1	3.35	6.40	9.25	1	3.41	7	11
$C(0,\ 1)$	1	1.95	3.51	5.02	1	1.84	3.26	4.82	1	1.61	3.01	4.09
$S(5)$	1	1.63	3.21	4.96	1	2.14	5.52	8.86	1	2.17	5.38	8.76
r = 20												
$N(0,\ 1)$	1	1.45	2.36	2.78	1	1.75	3.06	3.6	1	1.97	3.95	4.83
$L(0,\ 1)$	1	1.39	2.08	2.61	1	1.64	2.71	3.73	1	2.04	3.87	5.37
$C(0,\ 1)$	1	2.37	4.51	6.53	1	2.28	4.22	5.93	1	1.95	3.51	4.76
$S(5)$	1	1.54	2.37	3.18	1	1.57	3.02	4.35	1	2.13	3.90	5.60
r = 30												
$N(0,\ 1)$	1	1.27	1.49	1.53	1	1.24	1.55	1.59	1	1.36	1.77	1.81
$L(0,\ 1)$	1	1.33	1.76	2.17	1	1.39	2.23	2.71	1	1.87	3.23	3.98
$C(0,\ 1)$	1	2.59	4.83	6.2	1	2.1	4.17	5.43	1	1.89	3.41	4.33
$S(5)$	1	1.33	1.9	2.28	1	1.45	2.36	3.03	1	1.71	2.69	3.48
r = 50												
$N(0,\ 1)$	1	1.04	1.05	1.05	1	1.06	1.07	1.07	1	1.05	1.06	1.06
$L(0,\ 1)$	1	1.19	1.41	1.6	1	1.27	1.7	1.94	1	1.38	2.04	2.38
$C(0,\ 1)$	1	2.33	3.98	4.54	1	2.03	3.3	3.7	1	2.03	3.14	3.44
$S(5)$	1	1.2	1.46	1.59	1	1.24	1.57	1.71	1	1.24	1.62	1.83

Table 5: Values of $eff\ (T_i^*,\ T)$ Using $w_5(t)$ for $r = 10,\ 20,\ 30,\ 50$, $m = 1,\ 2,\ 3,\ 4$ and $\alpha = 0.05$

	$\alpha = 0.05$, r = 10, $w_5 = (k + t^2)^{-1}$											
	$\kappa = 0.5$				$\kappa = 1$				$\kappa = 1.5$			
$H\backslash m$	1	2	3	4	1	2	3	4	1	2	3	4
$N(0,\ 1)$	1	1.56	2.76	4.26	1	1.35	2.39	3.6	1	1.37	2.24	3.26
$L(0,\ 1)$	1	1.48	1.85	2.27	1	1.23	1.54	1.74	1	1.08	1.36	1.45
$C(0,\ 1)$	1	2.13	3.56	5.6	1	1.86	3.06	4.41	1	1.64	2.75	4.39
$S(5)$	1	1.2	2.09	3	1	1.18	1.7	2.44	1	1.15	1.69	2.21
r = 20												
$N(0,\ 1)$	1	1.35	2.17	2.59	1	1.31	1.92	2.31	1	1.38	2	2.43
$L(0,\ 1)$	1	1.3	1.53	1.85	1	1.17	1.4	1.66	1	1.11	1.25	1.45
$C(0,\ 1)$	1	1.95	3.47	4.98	1	1.84	3.51	5.21	1	1.77	3.33	5.13
$S(5)$	1	1.23	1.88	2.52	1	1.2	1.77	2.33	1	1.23	1.64	2.14
r = 30												
$N(0,\ 1)$	1	1.3	1.58	1.63	1	1.27	1.53	1.61	1	1.25	1.56	1.64
$L(0,\ 1)$	1	1.16	1.33	1.51	1	1.18	1.34	1.46	1	1.13	1.24	1.34
$C(0,\ 1)$	1	1.78	3.38	4.73	1	1.92	3.58	5.1	1	1.94	3.67	5.17
$S(5)$	1	1.22	1.69	2.05	1	1.21	1.64	1.98	1	1.25	1.58	1.94
r = 50												
$N(0,\ 1)$	1	1.05	1.07	1.07	1	1.07	1.09	1.09	1	1.08	1.11	1.11
$L(0,\ 1)$	1	1.08	1.2	1.28	1	1.07	1.16	1.24	1	1.07	1.12	1.17
$C(0,\ 1)$	1	1.74	2.77	3.22	1	1.63	2.82	3.44	1	1.73	2.9	3.69
$S(5)$	1	1.2	1.41	1.54	1	1.2	1.4	1.51	1	1.19	1.37	1.51

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