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Sunil Kumar
University of Jammu, (J & K), India, hpsujn@rediffmail.com

Housila P. Singh
Vikram University, Ujjain, India

Sandeep Bhougal
Shri Mata Vaishno Devi University, Kakryal, Jammu (J & K), India, sandeep.bhougal@smvdu.ac.in

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A General Class of Chain-Type Estimators in the Presence of Non-Response Under Double Sampling Scheme

Sunil Kumar
University of Jammu,
(J & K), India

Housila P. Singh
Vikram University,
Ujjain, India

Sandeep Bhougal
Shri Mata Vaishno Devi University,
Kakryal, Jammu (J & K), India

General class chain ratio type estimators for estimating the population mean of a study variable are examined in the presence of non-response under a double sampling scheme using a factor-type estimator (FTE). Properties of the suggested estimators are studied and compared to those of existing estimators. An empirical study is carried out to demonstrate the performance of the suggested estimators; empirical results support the theoretical study.

Key words: Double sampling, factor-type estimator, chain ratio estimator, non-response.

Introduction

Over the last five decades one of the major developments in sample surveys is the use of an auxiliary variable x, correlated with the study variable y in order to obtain estimates of the population total or mean of the study variable. Various estimation procedures in sample surveys require advance knowledge of some auxiliary variable x_i, which is then used to increase the precision of estimates. When the population mean $\overline{X}$ is not known, it can be estimated from a preliminary large sample on which only the auxiliary characteristic x is observed. The value of $\overline{X}$ in the estimator is then replaced by its estimate, and a smaller second-phase sample of the variable of interest (study variable) y is taken. This technique, known as double sampling or two-phase sampling, is particularly appropriate if the x_i values are easily accessible and are much less expensive to collect than the y_i values (Sitter, 1997; Hidiroglou & Sarndal, 1998). Neyman (1938) was the first to describe the concept of double sampling in connection with collecting information on strata sizes is a stratified sampling (Singh & Espejo, 2007).

In some practical situations it is observed that, when conducting a sample survey, complete information for all the units selected in the sample is not obtained due to the occurrence of non-response. Hansen and Hurwitz (1946) considered the problem of non-response while estimating the population mean by taking a sub-sample from the non-response group with the help of an unbiased estimator; they suggested combining the information available from response and non-response groups. Further, rectification in the estimation procedure for the population mean in the presence of non-response using auxiliary variable was proposed by Cochran(1977), Rao (1986, 1987), Khare and Srivastava (1993, 1995, 1997), Okator and Lee (2000), Tabasum and Khan (2004, 2006), and Singh and Kumar (2008a, 2008b, 2008c, 2009a, 2009b) using the Hansen and Hurwitz (1946) technique. This article develops a one parameter family of chain ratio type estimators with two auxiliary variables in the presence of non-response. The proposed family is based on factor type estimators (FTE) developed by Singh and Shukla (1987) and Singh, et al. (1994) and empirical studies support the results.

Sunil Kumar is in the Department of Statistics in University of Jammu College. Housila P. Singh is a Professor in the School of Studies in Statistics. Email: hpsujn@rediffmail.com. Sandeep Bhougal is in the School of Mathematics.
Email: sandeep.bhougal@smvdu.ac.in.

The Proposed Strategy

Consider a finite population $U=(U_1,U_2,...,U_N)$ of size N. Let y be the study variable, x_1 be the main auxiliary variable with an unknown mean that is highly correlated with main character y, and x_2 be an additional auxiliary variable with known mean that is less correlated with y than is x_1. A large first phase sample of size n' from the finite population U is selected by simple random sampling without replacement (SRSWOR). A smaller second phase sample of size n is selected from n' by SRSWOR. Non-response occurs in the second phase sample of size n in which n_1 units respond and n_2 units do not. From the n_2 non-respondents, by SRSWOR a sample of size $r=n_2/k;\ k>1$ units is selected where k is the inverse sampling rate at the second phase sample of size n with all r units responding. Thus, (n_1+r) are the responding units on the study variable y, consequently the estimator for the population mean $\bar{Y}$ of the study variable y using a sub-sampling scheme envisaged by Hansen and Hurwitz (1946) is defined as

$$\bar{y}^* = w_1\bar{y}_1 + w_2\bar{y}_{2r}, \tag{1}$$

where

$$w_1 = n_1/n,\ w_2 = n_2/n,\ \bar{y}_1 = \sum_{i=1}^{n_1} y_i/n_1$$

and

$$\bar{y}_{2r} = \sum_{i=1}^{r} y_i/r.$$

It is known that the estimator $\bar{y}^*$ is an unbiased estimator of the population mean $\bar{Y}$ of the study variable y and has a variance as given by

$$Var(\bar{y}^*) = \theta_1 S_y^2 + \theta^* S_{y(2)}^2, \tag{2}$$

where

$$\theta_1 = \left(\frac{1}{n}-\frac{1}{N}\right),\quad \theta^* = \frac{W_2(k-1)}{n},\quad W_2 = N_2/N,$$

and S_y^2 and $S_{y(2)}^2$ are the population mean square of the variable y for the entire population and for the non-responding group of the population. Similarly, for estimating the population mean $\bar{X}_i$ of the auxiliary variable $x_i;\ (i=1,2)$, the unbiased estimator $\bar{x}_i^*$ is given by

$$\bar{x}_i^* = w_1\bar{x}_{i(1)} + w_2\bar{x}_{i(2r)}, \tag{3}$$

where $\bar{x}_{i(1)}$ and $\bar{x}_{i(2r)}$ are the sample means of the auxiliary variable $x_i;\ (i=1,2)$ based on n_1 and r units respectively.

The variance of $\bar{x}_i^*$ is given by

$$Var(\bar{x}_i^*) = \theta_1 S_{x_i}^2 + \theta^* S_{x_i(2)}^2, \tag{4}$$

where $S_{x_i}^2$ and $S_{x_i(2)}^2$ are the population mean square of $x_i;\ (i=1,2)$ for the entire population and the non-responding group of the population.

The Proposed Class of Strategy

Using an unknown constant $t>0$ and two auxiliary variables x_1 and x_2, a general class of chain ratio type of strategy $\left[D, \bar{y}_F^*(t)\right]$ is defined for estimating the population mean $\bar{Y}$ of the study variable y in the presence of non-response as follows:

$$\bar{y}_F^*(t) = \bar{y}^* g_3(1,0)\left[\frac{\varphi\{\lambda_1(t)\}}{\varphi\{\lambda_2(t)\}}\right], \tag{5}$$

where

$$\varphi\{\lambda_i(t)\} = \lambda_i(t) + \{1-\lambda_i(t)\}g_2(0,1);\ i=1,2,$$

$$\lambda_1(t) = \frac{\theta B}{A+\theta B+C},\ \lambda_2(t) = \frac{C}{A+\theta B+C},$$

$$A=(t-1)(t-2),\ B=(t-1)(t-4),$$

$$C=(t-2)(t-3)(t-4),\ \theta = n/N,$$

$$\theta_2 = \left(\frac{1}{n'}-\frac{1}{N}\right),\ \theta_3 = \left(\frac{1}{n}-\frac{1}{n'}\right),$$

$$g_1(\alpha,\beta)=\left(\frac{\bar{X}_1}{\bar{x}_1^*}\right)^{\alpha}\left(\frac{\bar{X}_2}{\bar{x}_2^*}\right)^{\beta},$$

$$g_2(\alpha,\beta)=\left(\frac{\bar{X}_1}{\bar{x}_1'}\right)^{\alpha}\left(\frac{\bar{X}_2}{\bar{x}_2'}\right)^{\beta},$$

$$g_3(\alpha,\beta)=\left(\frac{\bar{x}_1'}{\bar{x}_1^*}\right)^{\alpha}\left(\frac{\bar{x}_2'}{\bar{x}_2^*}\right)^{\beta},\ \bar{X}_1=\sum_{i=1}^{N_1}x_i/N_1,$$

$$\bar{X}_2=\sum_{i=1}^{N_2}x_i/N_2,\ \bar{x}_1'=\sum_{i=1}^{n_1'}x_i/n_1'$$

and

$$\bar{x}_2'=\sum_{i=1}^{n_2'}x_i/n_2'.$$

In order to identify some of the members of the proposed strategy and compare their efficiencies, certain classical strategies are put forth:

(i) $\left[D,\bar{y}_R^*\right];\ \bar{y}_R^*=\bar{y}^*g_3(1,0)$ (6)

by Khare and Srivastava (1993), Okafor and Lee (2000) and Tabasum and Khan (2004)

(ii) $\left[D,\bar{y}_P^*\right];\ \bar{y}_P^*=\bar{y}^*g_3(-1,0)$ (7)

by Khare and Srivastava (1993)

(iii) $\left[D,\bar{y}_C^*\right];\ \bar{y}_C^*=\bar{y}^*g_3(1,0)g_2(0,1)$. (8)

Some Strategies of the Class

For $t=1$ and 4 respectively,

(i) $\left[D,\bar{y}_F^*(1)\right]=\left[D,\bar{y}_C^*\right]$, (9)

(ii) $\left[D,\bar{y}_F^*(4)\right]=\left[D,\bar{y}_R^*\right]$. (10)

Further, for $t=2$ and 3,

$$\left[D,\bar{y}_F^*(2)\right];\ \bar{y}_F^*(2)=\bar{y}^*g_3(1,0)g_2(0,-1), \quad (11)$$

$$\left[D,\bar{y}_F^*(3)\right];\ \bar{y}_F^*(3)=\bar{y}^*g_3(1,0)\begin{Bmatrix}(1+h)\\ -hg_2(0,-1)\end{Bmatrix}, \quad (12)$$

where $h=n(N-n)^{-1}$, and $\bar{y}_F^*(2)$ is a chain type estimator in D in which $\bar{X}_1$ is estimated through the product estimator utilizing $\bar{X}_2$ where non-response on auxiliary variable x_1 and $\bar{y}_F^*(3)$ is a chain type estimator in D in which $\bar{X}_1$ is estimated utilizing a dual to ratio estimator with non-response on auxiliary variable x_1.

Properties of the Proposed Strategy

To obtain the bias and mean square error (MSE) of the proposed general class of strategy $\left[D,\bar{y}_F^*(t)\right]$, under the large sample approximation,

$$\bar{y}^*=\bar{Y}(1+\varepsilon_0),\ \bar{x}_1^*=\bar{X}_1(1+\varepsilon_1),$$
$$\bar{x}_2^*=\bar{X}_2(1+\varepsilon_2),\ \bar{x}_1'=\bar{X}_1(1+\varepsilon_1'),$$
$$\text{and } \bar{x}_2'=\bar{X}_2(1+\varepsilon_2'),$$

such that

$$E(\varepsilon_0)=E(\varepsilon_1)=E(\varepsilon_2)=E(\varepsilon_1')=E(\varepsilon_2')=0$$

and

$$E(\varepsilon_0^2)=\theta_1S_y^2+\theta^*S_{y(2)}^2,$$

$$E(\varepsilon_1^2)=\theta_1S_{x_1}^2+\theta^*S_{x_1(2)}^2,$$

$$E(\varepsilon_2^2)=\theta_1S_{x_2}^2+\theta^*S_{x_2(2)}^2,\ E(\varepsilon_1'^2)=\theta_2S_{x_1}^2,$$

$$E(\varepsilon_2'^2)=\theta_2S_{x_2}^2,\ E(\varepsilon_0\varepsilon_1)=\theta_1S_{yx_1}+\theta^*S_{yx_1(2)},$$

$$E(\varepsilon_0\varepsilon_1')=\theta_2S_{yx_1},\ E(\varepsilon_1\varepsilon_1')=\theta_2S_{x_1}^2,$$

$$E(\varepsilon_0\varepsilon_2')=\theta_2S_{yx_2},\ E(\varepsilon_1\varepsilon_2')=\theta_2S_{x_1x_2},$$
$$E(\varepsilon_1'\varepsilon_2')=\theta_2S_{x_1x_2},$$

where

$$S_{yx_1} = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_{1i} - \bar{X}_1\right),$$

$$S_{yx_2} = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_{2i} - \bar{X}_2\right),$$

$$S_{yx_1(2)} = \frac{1}{(N_2-1)}\sum_{i=1}^{N_2}\left(y_i - \bar{Y}_2\right)\left(x_{1i} - \bar{X}_{1(2)}\right),$$

$$S_{x_1x_2} = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(x_{1i} - \bar{X}_1\right)\left(x_{2i} - \bar{X}_2\right),$$

$$\bar{X}_{1(2)} = \sum_{i=1}^{N_2} x_{1i}/N_2 ,$$

$$\bar{Y} = \sum_{i=1}^{N} y_i/N ,$$

$$\bar{Y}_2 = \sum_{i=1}^{N_2} y_i/N_2 ,$$

N_1 and $N_2\left(= N - N_1\right)$ are the sizes of the responding and non-responding units from the finite population N.

Expressing the proposed estimator $\bar{y}_F^*(t)$ in terms of $\varepsilon' s$,

$$\bar{y}_F^*(t) = \bar{Y}\left(1+\varepsilon_0\right)\frac{\left(1+\varepsilon_1'\right)}{\left(1+\varepsilon_1\right)}\left[\frac{\lambda_1(t)+\left\{1-\lambda_1(t)\right\}\left(1-\varepsilon_2'\right)^{-1}}{\lambda_2(t)+\left\{1-\lambda_2(t)\right\}\left(1-\varepsilon_2'\right)^{-1}}\right]. \tag{13}$$

It is assumed that $\left|\lambda_2(t)\varepsilon_2'\right| < 1$, because $\lambda_2(t) = \dfrac{C}{A+\theta B+C}$, for any choice of t, $\left|\lambda_2(t)\right| < 1$. Thus, if $\left|\varepsilon_2'\right| < 1$, $\left|\lambda_2(t)\varepsilon_2'\right| < 1$ is a valid assumption, expanding the right hand side of (13) and neglecting the terms involving powers of $\varepsilon' s$ greater than two results in

$$\bar{y}_F^*(t) = \bar{Y}\begin{Bmatrix} 1+\varepsilon_0-\varepsilon_1+\varepsilon_1'+\varepsilon_1^2-\varepsilon_1\varepsilon_1'-\varepsilon_0\varepsilon_1+\varepsilon_0\varepsilon_1' \\ +\lambda(t)\left(\varepsilon_2'-\varepsilon_1\varepsilon_2'+\varepsilon_1'\varepsilon_2'+\varepsilon_0\varepsilon_2'-\lambda_2\varepsilon_2'^2\right)\end{Bmatrix},$$

$$\left\{\bar{y}_F^*(t)-\bar{Y}\right\} = \bar{Y}\begin{Bmatrix} \varepsilon_0-\varepsilon_1+\varepsilon_1'+\varepsilon_1^2-\varepsilon_1\varepsilon_1'-\varepsilon_0\varepsilon_1+\varepsilon_0\varepsilon_1' \\ +\lambda(t)\left(\varepsilon_2'-\varepsilon_1\varepsilon_2'+\varepsilon_1'\varepsilon_2'+\varepsilon_0\varepsilon_2'-\lambda_2\varepsilon_2'^2\right)\end{Bmatrix}, \tag{14}$$

where $\lambda(t) = \lambda_1(t) - \lambda_2(t)$.

Taking expectations of both sides of (14), results in the bias of $\bar{y}_F^*(t)$ to the first degree of approximation, as

$$B\left(\bar{y}_F^*(t)\right) = \begin{Bmatrix} \theta_3\left(1-K_{yx_1}\right)S_{x_1}^2+\theta^*\left(1-K_{yx_1(2)}\right)S_{x_1(2)}^2 \\ -\lambda(t)\theta_2\left(\lambda_2(t)-K_{yx_2}\right)S_{x_2}^2\end{Bmatrix}, \tag{15}$$

where

$$K_{yx_1} = \frac{S_{yx_1}}{S_{x_1}^2} = \rho_{yx_1}\frac{S_y}{S_{x_1}},\ K_{yx_2} = \frac{S_{yx_2}}{S_{x_2}^2} = \rho_{yx_2}\frac{S_y}{S_{x_2}},$$

$$\rho_{yx_1} = \frac{S_{yx_1}}{S_yS_{x_1}} \text{ and } \rho_{yx_2} = \frac{S_{yx_2}}{S_yS_{x_2}}.$$

Squaring both sides of (14) and neglecting terms of $\varepsilon' s$ involving power greater than two,

$$\left(\bar{y}_F^*(t)-\bar{Y}\right)^2 = \bar{Y}^2\left\{\varepsilon_0-\varepsilon_1+\varepsilon_1'+\lambda(t)\varepsilon_2'\right\}^2$$

$$\left(\bar{y}_F^*(t)-\bar{Y}\right)^2 = \bar{Y}^2\begin{Bmatrix} \varepsilon_0^2+\varepsilon_1^2+\varepsilon_1'^2+\lambda^2(t)\varepsilon_2'^2 \\ -2\varepsilon_0\varepsilon_1+2\varepsilon_0\varepsilon_1'+2\lambda(t)\varepsilon_0\varepsilon_2' \\ -2\varepsilon_1\varepsilon_1'-2\lambda(t)\varepsilon_1\varepsilon_2'+2\lambda(t)\varepsilon_1'\varepsilon_2'\end{Bmatrix}. \tag{16}$$

Taking expectations of both sides of (16), gives the mean square error of $\bar{y}_F^*(t)$ to the first degree of approximation as

$$MSE\left(\bar{y}_F^*(t)\right)= \begin{bmatrix} \theta_1 S_y^2+\theta_3\left(1-2K_{yx_1}\right)S_{x_1}^2 \\ +\lambda(t)\theta_2\left(\lambda(t)+2K_{yx_2}\right)S_{x_2}^2 \\ +\theta^*\left\{S_{y(2)}^2+\left(1-2K_{yx_1(2)}\right)S_{x_1(2)}^2\right\} \end{bmatrix}, \quad (17)$$

where

$$K_{yx_1(2)}=\frac{S_{yx_1(2)}}{S_{x_1(2)}^2}=\rho_{yx_1(2)}\frac{S_{yx_1(2)}}{S_{x_1(2)}}, \quad \rho_{yx_1(2)}=\frac{S_{yx_1(2)}}{S_{y(2)}S_{x_1(2)}}.$$

Corollary

Letting $\lambda(t)=-1, \lambda_2(t)=1$ for $t=1$ in (15) and (17), the bias and MSE of $\bar{y}_C^*$, respectively given by

$$B\left(\bar{y}_F^*(1)=\bar{y}_C^*\right)=\begin{bmatrix} \theta_3\left(1-K_{yx_1}\right)S_{x_1}^2 \\ +\theta^*\left(1-K_{yx_1(2)}\right)S_{x_1(2)}^2 \\ +\theta_2\left(1-K_{yx_2}\right)S_{x_2}^2 \end{bmatrix} \quad (18)$$

and

$$MSE\left(\bar{y}_F^*(1)=\bar{y}_C^*\right) =\begin{bmatrix} \theta_1 S_y^2+\theta_3\left(1-2K_{yx_1}\right)S_{x_1}^2 \\ +\theta_2\left(1-K_{yx_2}\right)S_{x_2}^2 \\ +\theta^*\left\{S_{y(2)}^2+\left(1-2K_{yx_1(2)}\right)S_{x_1(2)}^2\right\} \end{bmatrix}. \quad (19)$$

To obtain the bias and *MSE* of $\bar{y}_R^*$, assume that $\lambda(t)=\lambda_2(t)=0$ for $t=4$, in (15) and (17),

$$B\left(\bar{y}_F^*(4)=\bar{y}_R^*\right)=\begin{bmatrix} \theta_3\left(1-K_{yx_1}\right)S_{x_1}^2 \\ +\theta^*\left(1-K_{yx_1(2)}\right)S_{x_1(2)}^2 \end{bmatrix} \quad (20)$$

and

$$MSE\left(\bar{y}_F^*(4)=\bar{y}_R^*\right) =\begin{bmatrix} \theta_1 S_y^2+\theta_3\left(1-2K_{yx_1}\right)S_{x_1}^2 \\ +\theta^*\left\{S_{y(2)}^2+\left(1-2K_{yx_1(2)}\right)S_{x_1(2)}^2\right\} \end{bmatrix}. \quad (21)$$

The $MSE\left(\bar{y}_F^*(t)\right)$ is minimized, when

$$\lambda(t)=-K_{yx_2}. \quad (22)$$

Thus, substituting (22) in (17), results in the optimum mean square error of $\bar{y}_F^*(t)$, as

$$MSE\left(\bar{y}_F^*(t)\right)_{opt}= \begin{bmatrix} \theta_1 S_y^2-\theta_2 K_{yx_2}^2 S_{x_2}^2+\theta_3\left(1-2K_{yx_1}\right)S_{x_1}^2 \\ +\theta^*\left\{S_{y(2)}^2+\left(1-2K_{yx_1(2)}\right)S_{x_1(2)}^2\right\} \end{bmatrix}. \quad (23)$$

Efficiency Comparisons

From (2), (19), (21) and (23),

$$Var\left(\bar{y}^*\right)-MSE\left(\bar{y}_F^*(t)\right)_{opt} =\begin{bmatrix} \theta_3\left(1-2K_{yx_1}\right)S_{x_1}^2-\theta_2 K_{yx_2}^2 S_{x_2}^2 \\ +\theta^*\left(1-2K_{yx_1(2)}\right)S_{x_1(2)}^2 \end{bmatrix}, \quad (24)$$

$$MSE\left(\bar{y}_C^*\right)-MSE\left(\bar{y}_F^*(t)\right)_{opt}=\theta_2\left(1-K_{yx_2}^2\right)^2 S_{x_2}^2>0 \text{ when } K_{yx_2}<1, \quad (25)$$

$$MSE\left(\bar{y}_k^*\right)-MSE\left(\bar{y}_F^*(t)\right)_{opt}=\theta_2 K_{yx_2}^2 S_{x_2}^2 \quad (26)$$

It is explicit from the equations (24)-(26) that the proposed class of estimator $\bar{y}_F^*(t)$ is more efficient than:

(i) The usual unbiased estimator $\bar{y}^*$;

(ii) The estimator $\bar{y}_C^*$ when $K_{yx_2}<1$; and

(iii) The estimator $\bar{y}_R^*$, the ratio type estimator proposed by Khare and Srivastava (1993), Tabasum and Khan (2004) and Okafor and Lee (2000).

Thus, it may be concluded that the general chain ratio type class of proposed strategy $\left[D, \bar{y}_F^*(t)\right]$ is more efficient than the usual unbiased estimator $\bar{y}^*$, the estimator $\bar{y}_C^*$ and the ratio type estimator $\bar{y}_R^*$.

Empirical Study

To examine the effectiveness of the suggested class of chain ratio types, data sets studied by Khare and Sinha (2007) are considered. The data, from the Department of Paediatrics, Banaras Hindu University during 1983-1984, is the physical growth of an upper socio economic group of 95 school age children of Varanasi under ICMR study. The first 25% (i.e., 24 children) have been considered as non-responding units. The descriptions of the variates are given below:

Population I:

- y: Height (in cm.) of the children,
- x_1: Skull circumference (cm) of the children,
- x_2: Chest circumference (cm) of the children.

For this population:

$$\bar{Y} = 115.9526,$$
$$\bar{X}_1 = 51.1726,$$
$$\bar{X}_2 = 55.8611,$$
$$S_y^2 = 35.6041,$$
$$S_{x_1}^2 = 2.3662,$$
$$S_{x_2}^2 = 10.7155,$$
$$S_{y(2)}^2 = 26.0532,$$
$$S_{x_1(2)}^2 = 1.6079,$$
$$S_{x_2(2)}^2 = 9.1060,$$
$$\rho_{yx_1} = 0.3740,$$
$$\rho_{yx_2} = 0.620,$$
$$\rho_{yx_1(2)} = 0.571,$$
$$\rho_{yx_2(2)} = 0.401,$$
$$\rho_{x_1x_2} = 0.2970,$$
$$\rho_{x_1x_2(2)} = 0.570,$$
$$N = 95,$$
$$n = 35,$$
$$n' = 45$$

Population II:

- y: Weight (kg) of the children,
- x_1: Chest circumference (cm) of the children,
- x_2: Mid-arm circumference (cm) of the children.

For this population,

$$\bar{Y} = 19.4968,$$
$$\bar{X}_1 = 55.8611,$$
$$\bar{X}_2 = 16.7968,$$
$$S_y^2 = 9.2662,$$
$$S_{x_1}^2 = 10.7155,$$
$$S_{x_2}^2 = 2.1115,$$
$$S_{y(2)}^2 = 5.5424,$$
$$S_{x_1(2)}^2 = 9.1060,$$
$$S_{x_2(2)}^2 = 1.4323,$$
$$\rho_{yx_1} = 0.846,$$
$$\rho_{yx_2} = 0.797,$$
$$\rho_{yx_1(2)} = 0.729,$$

$\rho_{yx_2(2)} = 0.757,$

$\rho_{x_1x_2} = 0.725,$

$\rho_{x_1x_2(2)} = 0.641,$

$N = 95,$

$n = 35,$

$n' = 45$

The percent relative efficiencies (PREs) of the estimators $\bar{y}_R^*$ and $\bar{y}_C^*$ have been computed along with the proposed estimator $\bar{y}_F^*(t)$ at its optimum with respect to the usual unbiased estimator $\bar{y}^*$ for two data sets for different values of k ; results are displayed in Table 1.

Results and Conclusion

Table 1 shows that the percent relative efficiency (*PRE*) of the proposed estimator $\bar{y}_F^*(t)$ at its optimum with respect to $\bar{y}^*$ is at its maximum over the ratio estimator $\bar{y}_R^*$ and the estimator $\bar{y}_C^*$ in both the populations. In population I, the *PREs* of all the estimators decreases with the increase in the value of k while in population II, the *PREs* of all the estimators increases with the increase in the value of k. Further, it is envisaged that the estimator $\{\bar{y}_F^*(t)\}_{opt}$ is the best estimator among $\bar{y}^*$, $\bar{y}_R^*$ and $\bar{y}_C^*$ in both the populations.

Table 1: Percent Relative Efficiencies (*PREs*) of the Different Estimators with Respect to $\bar{y}^*$ for Different Values of k

Estimator	Population - I				Population - II			
	$(1/k)$							
	$(1/5)$	$(1/4)$	$(1/3)$	$(1/2)$	$(1/5)$	$(1/4)$	$(1/3)$	$(1/2)$
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
$\bar{y}_R^*$	124.27	121.21	117.27	112.01	129.65	129.71	129.79	129.92
$\bar{y}_C^*$	144.83	144.27	143.52	142.45	168.52	175.91	186.72	204.05
$\{\bar{y}_F^*(t)\}_{opt}$	145.16	144.66	143.97	142.99	178.81	188.82	203.89	229.16

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