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Hazard rate models for early warranty issue detection using upstream supply chain information

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HAZARD RATE MODELS FOR EARLY WARRANTY ISSUE DETECTION USING UPSTREAM SUPPLY CHAIN INFORMATION

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DEDICATION

Dedicated to my wife and daughter —Wei and Qijia

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CHAPTER 1 INTRODUCTION

The automotive industry spends roughly $10-$13 billion per year in the U.S. on warranty claims (Arnum, 2011b) and up to $40 billion globally (MSX, 2010), consuming roughly 1-5.2% of original equipment manufacturers' (OEM) product revenue and roughly 0.5-1% of suppliers' product revenue (Arnum, 2011a). Warranty claims refer to customer claims for repair or replacement of, or compensation for non-performance or under-performance of an item, as provided for in its warranty document. Historically, the leading Japanese automotive OEMs, i.e. Honda and Toyota, had significantly lower warranty cost relative to product revenue than their U.S. counterparts. For example, between the years 2003 and 2011, the warranty costs for Toyota and Honda were around 1-1.7% of product revenue, whereas the costs for the U.S. OEMs (Ford, GM, and Chrysler) were between 2.2% to 5.2% (Arnum, 2011a). OEMs typically incur 70% of the warranty costs, including those associated with engineering, manufacturing, and suppliers (MSX, 2010). Early detection of reliability problems can help OEMs and suppliers take corrective actions in a timely fashion to minimize warranty costs and loss of reputation due to poor quality and reliability. A compelling example is the case of the recent product recalls from Toyota in the U.S. and around the world, attributed to pedal assembly and floor mat entrapment issues, involving 12 vehicle nameplates and 8.5 million vehicles produced between 1998 to 2010 (Takahashi, 2010; Toyota, 2010), costing the company over $2 billion (Carty, 2010) and caused its warranty costs to jump to around 2.5% of its product revenue (Arnum, 2011a).

1.1 Research Motivation

Improving reliability and reducing warranty costs is the joint objective and responsibility of both OEMs and suppliers. This is especially true when the recent trends show OEMs have increased pace of shifting warranty cost to their suppliers (Arnum, 2011a). A highly engineered product such as an automobile consists of many modular systems (e.g., electrical, powertrain, chassis, seating), subsystems (e.g., wiring harnesses, alternators, motors), and thousands of components that are supplied through an extensive supply network. Before a vehicle is produced, these systems, subsystems, and components have to undergo design, testing and build at supplier and OEM sites. Therefore, reliability problems don't just start from vehicles reaching customer's hands, but can start far early at suppliers' sites and are heavily influenced by operations at all tiers of suppliers. For example, a quality lapse in a supplier's plant may be the first indication of an unusually high warranty claim rate. There are rich sources of upstream production quality/testing information regarding components and sub-systems residing in the supplier network and accumulating long before the final vehicles are assembled. Figure 1 illustrates some of the major sources of information for developing early warranty detection models in the automotive industry. This echoes to Murthy's four notations of reliability: design, inherent, sale and field reliability (Murthy, 2010). If this prior upstream information can be utilized in a statistical framework to correlate to warranty claims, the detection power of an early warranty model might improve. Such an early warning system can also be used to monitor the effectiveness of corrective actions. While there is a growing body of literature on warranty modeling and detection, to the best of my knowledge, there is no model in the literature that explicitly links information from the supplier network to improve early warranty detection.

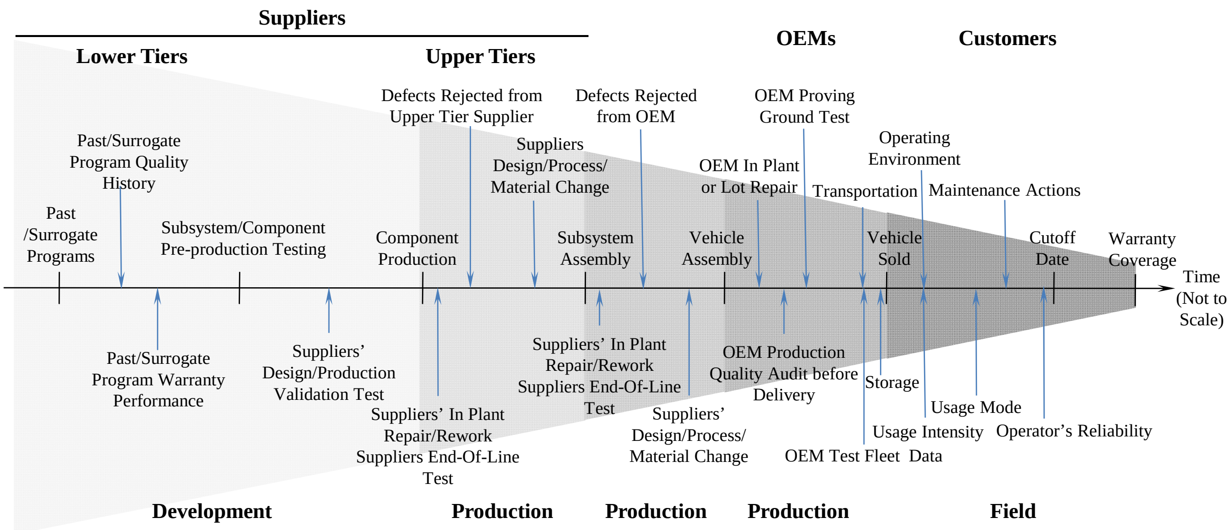

Figure 1: Data sources for modeling warranty issues from supply network upstream to customer downstream; Adopted from (Majeske, 2007; Murthy, 2010).

My research is motivated by the need for models to explicitly utilize upstream manufacturing process and quality/testing information from suppliers. With over 10 years of professional experience in the warranty and reliability area with automotive OEMs and suppliers, I can personally attest to these needs and progressive OEMs are demanding the same. In the current highly competitive environment, suppliers are being pushed to improve warranty performance for their responsible subsystems in the vehicles. When a warranty issue develops in the field, the issue is normally traced from the top to the bottom of the pyramid structure in Figure 1. In many cases, the precursors to the issue could be found at suppliers' sites months or even years earlier in terms of a quality spill, a design error etc. In addition, to address these warranty issues, suppliers often implement corrective actions without good knowledge for their effectiveness, leading to instances where the effectiveness is revealed through future claims to be less than expected. Facing such embarrassing situations, management is often raising the following sorts of questions:

- Can we act on warranty issues more proactively instead of reactively?
- Can we estimate our warranty risk early on?
- How to verify such a warranty risk quickly?
- Once a corrective action is implemented to address a warranty issue, how to confirm its effectiveness quickly?

In the context of warranty issues, to answer the above sorts of questions, we need to rethink the pyramid structure of Figure 1 in a different way: improving warranty performance should start from the bottom of the pyramid to the top whereas requirements (form, function, and fit) often flow the top to the bottom.

1.2 Research Objectives and Scope

The primary objective of this research is to introduce a statistical modeling framework that explicitly utilizes upstream supply chain information to: 1) allow early detection of warranty issues, 2) facilitate early validation of the effectiveness of corrective actions, and 3) to aid in predicting the warranty claim rates. By utilizing hazard rate models and further extending it to incorporate Bayesian analysis, upstream supply chain information is directly linked to expected warranty claims as explanatory covariates to achieve this goal.

While warranty claims can relate to reliability for the whole product life cycle at different stages: design reliability due to reliability specification at product development stage, inherent reliability due to assembly errors and component non-conformance, sale reliability due to damage or deterioration in transportation and storage, and field reliability due to customer usage mode/intensity and operating environment (Murthy, 2010); warranty claims can also related to human factors such as misuse, neglect, fraud or lack of training on product operation (Wu, 2011), this research is from a supplier's point of view, focuses on linking warranty claim rates to design and inherent reliability, to which the upstream supply chain information are available and can be extracted and on which a supplier has a control. However the statistical modeling framework from this research can easily extended to sale and field reliability by including the available relevant information as explanatory covariates.

While much of this research focuses on application of the proposed warranty issue detection models to the automotive industry, the models are also relevant to other industries that rely on a supply network to build parts of the product.

CHAPTER 2 HAZARD RATE MODELS FOR EARLY WARRANTY ISSUE DETECTION USING UPSTREAM SUPPLY CHAIN INFORMATION

2.1 Introduction

This chapter is organized as follows. Section 2.2 reviews relevant literature. Section 2.3 describes the structure of suppliers' manufacturing and quality/testing data sources that might be indicative of future warranty claims. Section 2.4 outlines the proposed methodology of utilizing hazard rate models to correlate upstream data sources to warranty claims. Section 2.5 develops an enhanced early warranty detection scheme by incorporating upstream suppliers' quality/testing data. Section 2.6 reviews the performance of the proposed method through a case study. Finally, Section 2.7 provides summary remarks and directions for further study.

2.2 Literature Review

Detection of a reliability problem often involves several steps: a) Statistical modeling of warranty claims so that those factors influencing product reliability can be selected and the parameters in the model can be estimated; b) Baselines for the parameters are obtained or predicted from historical warranty claims and/or from subject matter experts (SMEs) in the absence of any historical information, c) Critical values for the parameters are set to balance power of detection and false alarm probability, and d) Observed parameters for the current product model cycle are compared against the critical values to trigger out-of-control signals.

There is a growing body of literature discussing statistical modeling of warranty claims. In the automotive industry, as the number of expected warranty claims is often small under any given failure mode (claim rates are typically measured as claims per thousands of vehicles) compared to the large number of vehicles in field, from a reliability point of view, such warranty claims are often treated as rare and independent events, making the Poisson model an appealing statistical model for warranty claims. Since the seminal paper by (Kalbfleisch et al., 1991) that proposed a Poisson model to analyze warranty claims, many papers have been authored that focus on predicting future warranty claims for the remainder of warranty life based on existing/past warranty claims for the early portion of warranty life. Models have been developed to also deal with such issues as warranty report delay (Kalbfleisch and Lawless, 1991; Lawless, 1994); (Lawless, 1998), sales delay (Lawless, 1994; Majeske et al., 1997), two dimensional warranty policy such as 3 years/36,000 miles whichever comes first (Yang and Zaghati, 2002); (Krivtsov and Frankstein, 2004; Majeske, 2007), treatment of incomplete data (Hu and Lawless, 1996, 1997; Oh and Bai, 2001; Rai and Singh, 2003, 2004; Mohan et al., 2008), treatment of warranty claims related human factors such as non-failed but reported (NFBR), failed but not reported (FBNR) and claims from intermittent failures claims (Wu, 2011). While the vast majority of the literature assumes that the customer will file at most a single claim for a particular warranty issue/system and hence the focus on survival analysis methods that experience a single event, (Lawless, 1995; Lawless and Nadeau, 1995; Fredette and Lawless, 2007) also provided methods to forecast warranty claims based on a recurring event perspective that allows the customer to file multiple claims over time for the same system/issue. (Blischke and Murthy, 1996), and more recently (Karim and Suzuki, 2005; Wu and Akbarov, 2011), have reviewed the literature on mathematical and statistical techniques for analysis of warranty. The above studies typically serve the purpose of financial planning (warranty accruals) and taxation.

There is also literature focusing on detecting an emerging quality or reliability problem by predicting warranty claims for new vehicles (such as vehicles produced in current production month) based on warranty claims available for older vehicles (such as vehicles produced in past production months). These early/accurate warranty issue detection methods can actively reduce warranty cost by facilitating implementation of corrective actions in time, directly impacting company's bottom line. In this regard, some researchers have adopted the Poisson model discussed earlier to model warranty claims and establish the baseline, then utilizing the conventional statistical process control techniques to detect emerging quality or reliability problems month by month either by production or calendar months. (Wu and Meeker, 2002) stratified warranty claims by vehicle production month and age in terms of months in service (i.e., the difference between vehicle repair date and vehicle sold date). Assuming that warranty claims for vehicles from different production months and ages follow independent Poisson models with different claim rates, they proposed a sequential test procedure for early warranty detection. Such a scheme generalized the conventional process control chart by sequentially comparing predefined baseline claim rates from historically stable production periods to those from current production month for corresponding ages (available sequentially), so that an emerging quality or reliability problem can be detected with a predefined Type-1 error (i.e., false alarm error). (Oleinick, 2004) improved the conventional control chart (u chart) by applying standard reliability growth models to calibrate the variability in the reliability of vehicles among calendar months not accounted for by conventional reliability bathtub curves.

Another group of researchers adopted computational intelligence techniques such as artificial neural network methods to model warranty claims. They argue that the traditional distribution classes may not be flexible enough to capture the failure distributions observed in actual warranty claims and that qualitative factors are difficult to incorporate into traditional statistical models, compromising the accuracy required for early warranty detection. (Lindner and Klose, 1997) and (Lindner and Studer, 1999) observed that warranty claim rate curves along production months are rather similar for different ages, but different only in rate level, and they applied machine learning and neural network models to integrate warranty claim information about the interdependency between vehicle production month and age, and managed to provide trend prognoses several months in advance with good accuracy. (Grabert et al., 2004) estimated warranty claim rates using the multi-layer perceptron model, then, besides warranty claims, they further include OEM's quality data such as production audits before delivery into the analysis to establish the baseline. (Lee et al., 2007) included qualitative factors such as product type, warranty service area, part significance, seasons into their study on warning/detection of warranty issues. (Wu and Akbarov, 2011) introduced a weighted support vector regression (SVR) and weighted SVR-based time series model to forecast warranty claims.

As we look back on the warranty timeline starting from current time (cut-off date) in Figure 1, a data source pyramid forms along the timeline: towards the top of the pyramid is the field data from customer (warranty claims) at the vehicle level, available relatively late with scarcity. Towards the bottom of the pyramid is data from suppliers regarding lower-level components and subsystems, available earliest and abundant. Lot of this upstream information is available long before a vehicle is built and any warranty claims are filed. More importantly, since this upstream data is at the level of subsystems and components, it is more physics and failure mechanism relevant and might help identify, early on, root-causes of output warranty claims. Vast majority of the extant warranty detection literature focuses on warranty claims themselves, while few suggested the utilization of upstream OEM data. For example, (Grabert et al., 2004) utilize OEM's plant quality data for warranty detection, with an OEM perspective. To the best of our knowledge, there is not a single article in the literature that exploits further upstream data, in particular, the wealth of production quality/testing data from suppliers, for improved warranty detection. The primary objective of this paper is to address this short-coming in the literature and propose models that exploit upstream warranty relevant data sources from suppliers, so that any emerging quality/reliability problem can be detected earlier with more power.

2.3 Upstream Data Sources for Early Warranty Detection

As OEMs globalize their vehicle production and component sourcing, more and more suppliers are supplying components/subsystems to multiple OEMs, or to multiple vehicle platforms within one OEM (platform is a shared set of common design/engineering efforts and major components over a number of outwardly distinct models). Therefore, warranty detection has become more complex requiring increased active involvement from suppliers. To meet these requirements, OEMs cascade vehicle level warranty down to subsystem/component levels, which is often the responsibility of suppliers. These joint responsibilities and objectives are defined by warranty agreements and warranty sharing programs. To support this process, the Original Equipment Suppliers Association (OESA) drafted “Suppliers Practical Guide to Warranty Reduction” in 2005 (OESA, 2005) and later published in 2008 through the Automotive Industry Action Group (AIAG) as (AIAG, 2008). In recent years, the warranty reduction focus has shifted from warranty cost settlement or transfer to preventing or quickly and effectively eliminating reliability problems, which has greatest impact on warranty cost reduction in the long-term. To assist suppliers in meeting the warranty objective, OEMs typically allow suppliers to access their warranty claims database, warranty returned parts from end customer, test data from proving ground or test fleet vehicles, and plant audit and quality data. Some OEMs even allow suppliers to call dealer technicians within days of a claim to better link failure-modes to warranty claim data. All these initiatives provide suppliers with a great opportunity to correlate and exploit their internal quality/testing data (“Suppliers” in Figure 1), to OEM data (“OEMs” in Figure 1) to warranty claims (“Customer” in Figure 1).

2.3.1 OEM Warranty Data Sources/Structure

The structure of OEM warranty data has been explained in detail by (Wu and Meeker, 2002). It is worth noting that the key for this data structure is the vehicle identification number (VIN). From VIN, we can trace the vehicle built date, repair date, and all other vehicle production and warranty repair related information. More importantly, as will be explained later, using VIN, we can also trace back to production related information from suppliers responsible for components/subsystems. While the literature discusses warranty data complications associated with claims reporting delays, including (Wu and Meeker, 2002), fortunately, this is no longer an issue because of effective and near real-time IT integration of dealer network and repair shops to OEM warranty database systems. Given that suppliers have direct electronic access to this database, they can also obtain warranty claims related to their responsible subsystems/components without delay.

2.3.2 Supplier Network Warranty Data Sources/Structure

Modern production information technology and extended Enterprise Resource Planning (ERP) software is equipping OEMs and suppliers with enhanced warranty traceability, i.e., the ability to link upstream production quality/testing data to the warranty process timeline in Figure 1. We illustrate this using a typical Tier-1 supplier's production example. A typical Tier-1 supplier's production process starts first from receiving a VIN specific bill-of-material (BOM) from the OEM vehicle assembly plant. These BOMs are sent from OEM's production system to suppliers' production system electronically (typically, using some form of an electronic data interchange (EDI) system). The BOM defines the configuration/options for the supplier's subsystem for each VIN. For example, in the case of a seating supplier, the BOM will identify seat model type, material (leather/fabric), and optional content (e.g., active head-restraints, heated seats). Upon receiving the BOM, the supplier's production system typically generates a unique sequence number for the subsystem corresponding to each VIN. These sequence numbers are then sent to the first station of the supplier's assembly line for building the desired subsystem. At each station of the assembly line, certain components are added and then tested against production specifications by measuring functional parameters such as noise, current, voltage, resistance, speed, count etc. For key components, their unique identification numbers, typically part of bar codes, are also scanned into the production system before test; this provides the traceability from Tier-1's sub-system to lower tiers' components. If any measurement is out of specification, the in-process subsystem is rejected and the assembly line is stopped until the problem is fixed. This process will repeat for each station until the subsystem corresponding to the sequence number is completely built and passes all the function tests for all stations. Finally, the subsystem is put on a shipping rack ready for shipment to OEMs' vehicle assembly plant. As each sub-system is built, all its function test results and component scan results are stored in the production database, tied to sub-system sequence number and VIN, and are available for access. As each sub-system may consist of many components, an assembly line may consist of many stations and each station may conduct many test and scan activities, the amount of data stored is huge but rich: for annual production of 200K vehicles, a typical Tier-1 supplier's production database stores millions of records for its responsible subsystems. Likewise, the production data collection process can be cascaded down to lower tier suppliers. Therefore, suppliers' production database has a wealth of information that can support early warranty detection:

1. The core element of the production database is strong traceability. It uniquely maps each vehicle unit (VIN) to its corresponding subsystem (sequence number), then from the sequence number, it uniquely maps the subsystem to its components (through bar codes). From VIN, sequence number and bar code,

quality data from suppliers' production such as production date, function test results can be directly linked to vehicle warranty claims.

2. Since warranty detection methods can benefit from specific information regarding the failure mechanisms, including manufacturing, production/quality data from suppliers' production database can aid these detection methods.
3. Unlike OEMs' production quality audit, which typically samples 1% of the sub-systems that enter vehicle production (Grabert et al., 2004), suppliers' production databases often provide a complete history on 100% of the sub-systems (for all vehicles with and without warranty claims).
4. Despite its huge amount of information, it is well organized and structured, allowing us ready access to information critical for warranty detection.

Besides function test data, a typical Tier-1 supplier also stores information regarding units rejected by the OEM to its production database. After the subsystems are shipped to OEMs' vehicle assembly plant, some of them may be rejected by OEMs due to defects and shipped back to suppliers. Suppliers may repair them by rework or replace them. The sequence numbers and related events are then recorded in the production database.

In addition to function test data and customer quality audit/reject data, suppliers also have other quality/testing data that may be linked to warranty claims. Examples of such information include:

- Number of quality alerts generated each month due to defects found in OEM's plant
- Process capability information (e.g., Cpk) from all stations (by week/month)

- Internal scrap/rework rates (by date/batch)
- Component reject rates (by date/batch)
- Process and design change history (during production phase)
- Design and production validation test history (during development phase)
- Historical warranty claims and quality/testing data from similar subsystems on different vehicle lines or different OEMs

The extant literature is quite lacking in offering early warranty detection methods that can exploit the wealth of such upstream production quality/testing information. The primary objective of this manuscript is to propose methods that can begin to address this gap.

2.4 Correlating Upstream Quality/Testing Data to Warranty Claims

In the recent literature, a popular approach to modeling warranty claims is a nonparametric approach based on warranty claim counts modeled with a Poisson distribution with claim intensities that depend on production period and number of periods in service. The other standard assumption following the statistical model used by (Kalbfleisch and Lawless, 1991) is that the claims for vehicles from production period i and j periods in service (for the particular subsystem or labor code under consideration) can be described as independently distributed Poisson random variables. (Wu and Meeker, 2002) argue that this probability model is strongly supported by most warranty applications where there is a large number of units in the field, but the occurrence of any given failure mode, when reliability is as expected, should be rare and statistically independent from unit to unit. However, to reduce the need for estimation of a (potentially) large number of report intensity parameters for each production period and number of periods in service for each sub-system or labor code (which can run into thousands), we directly model the claim rate as a hazard function $h_i(t)$ over service age t (herein simply referred to as age) for each production period i. This not only reduces the need for independent estimation of a large number of report intensity parameters for each period in service but also allows us to avoid the need for the assumption of independently distributed Poisson random variables. Instead, we propose the more exact Binomial distribution to model warranty claims for each sub-system or labor code. In addition, while (Wu and Meeker, 2002) employ a nonparametric approach for modeling warranty claims over fitting a standard parametric distribution such as a Weibull or a lognormal distribution for each subsystem or labor code, given the challenges associated with identifying the right model for each of the hundreds to thousands of subsystems and labor codes of interest, our proposed method fully supports both a nonparametric as well as a parametric treatment of the hazard rate function.

We use the claim rate function $h_i(t)$ to estimate the probability $p_i(t)$ that any individual vehicle unit from production period i will generate a claim (for the particular labor code or sub-system) by age t:

$$p_i(t) = 1 - \exp\left(-\int_0^t h_i(\tau)d\tau\right) = 1 - \exp[-H_i(t)] \qquad (1)$$

$H_i(t) = \int_0^t h_i(\tau)d\tau$ is defined as the culmulative hazard rate. Assuming that n_i units are produced in production period i, the total number of claims expected from the vehicles of this production period by age t then follows a Binomial distribution

$B(n_i, p_i(t))$. In the sections that follow, we also outline a method that eliminates the need for separately estimating the hazard rate function for each production period through the use of production month covariates based on upstream supply chain information.

In order to correlate upstream supply chain quality/testing information with warranty claim rate, we propose the use of hazard rate models. By treating upstream supply chain quality/testing information as explanatory covariates of warranty claim rate, we directly link warranty claims with them. Even though one can use conventional models such as linear regression, log-linear regression, logit, probit and inverse polynomials analysis, the special properties of warranty claims make these models inappropriate due to their inefficiency, bias, inconsistency and insufficiency. Warranty claim data are heavily right censored (>90%); the conventional models can lead to biased estimates of the covariate effects by not incorporating this available censoring information (Hardin and Hilbe, 2007). In addition, if the explanatory covariates associated with warranty claims are time dependent (such as product usage rates/patterns), the conventional models have difficulty handling these situations. Note however that time dependent covariates are not considered in this manuscript and will be the focus of future work. Literature from other research areas such as marketing and political science (King, 1988; Helsen and Schmittlein, 1993; Soyer and Tarimcilar, 2008) confirm the above limitations of conventional models on certain datasets which share a lot of the same properties as warranty claims, and demonstrate that hazard rate models are able to overcome these limitations and outperform conventional models in terms of estimate stability and predictive accuracy.

2.4.1 Hazard Rate Models

Hazard rate models can be approached a number of ways. Since the seminal work by Cox on the so called proportional hazard (PH) models (Cox, 1972; Cox, 1975), they have been extensively used in survival analysis to provide a statistically rigorous estimation and prediction of survival rates based on explanatory covariates (Klein and Moeschberger, 2003; Lawless, 2003; Li et al., 2007). There are also a number of non-proportional hazard rate models, with a popular option in survival analysis being the accelerated failure time (AFT) model. Whereas a PH model assumes that the effect of a covariate is to multiply the hazard by some constant, AFT model assumes that the effect of a covariate is to multiply the predicted event time by some constant. For a detailed discussion on the choices and tradeoffs for hazard models and their parameter estimation processes, see (Lawless, 2003; Hosmer et al., 2008). In what follows, we employ the PH model for linking warranty claim rates to upstream supply chain quality/testing information. However, the methodology is equally relevant if an alternate hazard rate model is employed.

2.4.2 Proportional Hazard (PH) Model

Let $h(t)$ denote the hazard rate extracted from warranty claims corresponding to a subsystem or labor code for which a supplier is responsible. The subsystem's $h(t)$ can be calculated from OEM's warranty database by selecting the first claim for each VIN under a chosen set of labor codes or defect codes defined by OEM's warranty database (given our interest here in early warranty detection, the focus here is on the first claim and not repeat claims). Let $h(t|\boldsymbol{x})$ denote the hazard rate for the subsystem of interest for a vehicle with age t under given known p fixed covariates $\boldsymbol{x} = [x_1, x_2, \dots, x_p]'$; $h(t|\boldsymbol{x})$ is assumed to have the following form by the proportional hazard (PH) model:

$$h(t|\boldsymbol{x}) = h_0(t)\exp(\boldsymbol{\beta}'\boldsymbol{x}) \quad (2)$$

Here $\boldsymbol{x}$ is the upstream supply chain quality/testing characteristic covariate vector extracted from functional tests and/or reject data of suppliers' production database; $h_0(t)$ is the baseline hazard function; i.e., the hazard function when $\boldsymbol{x} = 0\mathrm{x} = 0$; $\boldsymbol{\beta} = [\beta_1, \beta_2, \dots, \beta_p]'$ is the regression coefficient vector to quantify the relative failure rate impact from the corresponding covariates.

2.4.3 Parametric vs. Semi-Parametric PH Models

Depending on the assumed structure of the baseline hazard function, the PH model comes in one of two forms: parametric or semi-parametric. In the case of the parametric PH model, the baseline hazard function is assumed to follow a standard parametric distribution such as a Weibull or a lognormal distribution. In the case of the semi-parametric PH model, the baseline hazard function is allowed to be arbitrary or nonparametric. Selection between a parametric and a semi-parametric PH model in the end depends on the warranty claim data, mathematical convenience, and researcher's preference. Since warranty claim data may not fit well to the traditional parametric distribution classes due to mixed failures, subpopulations being under different operating conditions and so on, mixture models such as mixed-Weibull (Attardi et al., 2005), uniform-Weibull mixture (Majeske, 2003), piecewise Weibull-exponential mixture (Kleyner and Sandborn, 2005) type models can be adopted to fit the baseline hazard rate function. On the other hand, semi-parametric PH model where $h_0(t)$ is left arbitrary (non-parametric) offers considerable flexibility to support arbitrary failure modes/mechanisms and freedom from any shape/scale constraints (Helsen and Schmittlein, 1993). We do note that this added flexibility comes with some risk in that the hazard rate estimation is relatively more vulnerable to noise in the data (which might lead to 'artificial' fluctuations in hazard rate). However, given that warranty monitoring often involves very large datasets, this risk is bounded. The major assumption for all PH models is that the multiplicative or log-additive hazard structure from Eq.(2) is correct. Such an assumption needs to be validated formally and is discussed in later sections.

2.4.4 Estimating PH Model Parameters from Past/Current Warranty Datasets

Let n be the number of vehicles for which there exists a partial or full warranty claim history. The censored service age life times $(t_i, \delta_i), i = 1, \ldots, n$, and corresponding covariate vectors $\boldsymbol{x}_i$ are assumed to be known for each vehicle i. The indicator variable $\delta_i = 1$ if the ith vehicle experienced a warranty claim for the subsystem or labor code of interest at service age t_i, $\delta_i = 0$, if the ith vehicle has not produced any warranty claim until age t_i. Using the warranty dataset, one could estimate the baseline hazard function and covariate effects through maximum likelihood estimation (MLE) procedures. Depending on whether we employ a fully parametric or semi-parametric PH model, here is the process:

1. If the baseline hazard function $h_0(t)$ can be represented by one from some family of parametric models with parameter vector $\boldsymbol{\theta}$ with form $h_0(t; \boldsymbol{\theta})$, then the full log-likelihood function will apply (Lawless, 2003):

$$l(\boldsymbol{\theta}, \boldsymbol{\beta}) = \sum_{i}^{n} \delta_i \{\log h_0(t_i; \boldsymbol{\theta}) + \boldsymbol{\beta}' \boldsymbol{x}_i\} - \sum_{i}^{n} H_0(t_i; \boldsymbol{\theta}) \exp(\boldsymbol{\beta}' \boldsymbol{x}_i) \quad (3)$$

where $H_0(t_i; \boldsymbol{\theta})$ is the cumulative baseline hazard function:

$$H_0(t_i; \boldsymbol{\theta}) = \int_0^{t_i} h_0(\tau; \boldsymbol{\theta})\, d\tau \quad (4)$$

2. If the baseline hazard function $h_0(t)$ is left arbitrary, then the semi-parametric Cox's partial log-likelihood will apply (Lawless, 2003):

$$l(\boldsymbol{\beta}) = \sum_{i}^{n} \delta_i \left[\boldsymbol{\beta}' \boldsymbol{x}_i - \log \left(\sum_{l=1}^{n} Y_l(t_i) e^{\boldsymbol{\beta}' \boldsymbol{x}_l} \right) \right] \quad (5)$$

where the variable $Y_l(t_i)$, called the risk indicator, equals 1 if and only if the lth vehicle has no warranty claim and is still in service at time t_i, and hence at risk of generating a claim at time t_i; otherwise equals 0.

For both the parametric and semi-parametric PH models, $\boldsymbol{\theta}$ and $\boldsymbol{\beta}$ can be readily estimated by solving the so-called maximum likelihood equation via Newton-Raphson iteration or other methods:

$$U_j(\boldsymbol{\alpha}) = \frac{\partial l(\boldsymbol{\alpha})}{\partial \alpha_j} = 0, j = 1, \ldots, m \quad (6)$$

where $\boldsymbol{\alpha} = [\boldsymbol{\theta}, \boldsymbol{\beta}]$ for parametric model and $\boldsymbol{\alpha} = \boldsymbol{\beta}$ for PH model and m is the number of elements in $\boldsymbol{\alpha}$.

Under large-sample theory with mild “regularity” conditions (Cox and Hinkley, 1974), $\boldsymbol{\alpha}$ and its statistics estimates, standard errors and confidence intervals can be given by any one of the following three inference procedures:

1. Score procedure: $\boldsymbol{U}(\boldsymbol{\alpha}) \cong N_{(m)}[\mathbf{0}, I(\boldsymbol{\alpha})]$

2. MLE-based (Wald) procedure: $\hat{\boldsymbol{\alpha}} \cong N_{(m)}[\boldsymbol{\alpha}, I^{-1}(\boldsymbol{\alpha})]$

3. Likelihood ratio procedure: $\Lambda(\boldsymbol{\alpha}) = 2l(\hat{\boldsymbol{\alpha}}) - 2l(\boldsymbol{\alpha}) \cong \chi^2_{(m)}$

Here $N_{(m)}$ refers to m-dimensional normal distribution, and $\chi^2_{(m)}$ refers to Chi-squared distribution with m degrees of freedom.

Here $\hat{\boldsymbol{\alpha}}$ is maximum likelihood estimate of $\boldsymbol{\alpha}$ and $I(\boldsymbol{\alpha})$is the information matrix. Under the parametric PH model, it is $I(\boldsymbol{\theta}, \boldsymbol{\beta})$ with following components:

$$
\begin{aligned}
-\frac{\partial^2 l}{\partial\theta_j \partial\theta_k} &= \sum_{i=1}^{n} \left\{ \frac{\partial^2 H_0(t_i; \boldsymbol{\theta})}{\partial\theta_j \partial\theta_k} \exp(\boldsymbol{\beta}'\boldsymbol{x}_i) - \delta_i \frac{\partial^2 \log h_0(t_i; \boldsymbol{\theta})}{\partial\theta_j \partial\theta_k} \right\} \\
-\frac{\partial^2 l}{\partial\theta_j \partial\beta_k} &= \sum_{i=1}^{n} \frac{\partial H_0(t_i; \boldsymbol{\theta})}{\partial\theta_j} x_{ik} \exp(\boldsymbol{\beta}'\boldsymbol{x}_i) \\
-\frac{\partial^2 l}{\partial\beta_j \partial\beta_k} &= \sum_{i=1}^{n} x_{ij} x_{ik} H_0(t_i; \boldsymbol{\theta}) \exp(\boldsymbol{\beta}'\boldsymbol{x}_i)
\end{aligned}
\tag{7}
$$

Under the semi-parametric PH model, it is $I(\boldsymbol{\beta})$:

$$
\begin{aligned}
I(\boldsymbol{\beta}) &= \sum_{i=1}^{n} \delta_i \left\{ \frac{\sum_{l=1}^{n} Y_l(t_i) e^{\boldsymbol{\beta}'\boldsymbol{x}_l} [\boldsymbol{x}_l - \overline{\boldsymbol{x}}(t_i, \boldsymbol{\beta})][\boldsymbol{x}_l - \overline{\boldsymbol{x}}(t_i, \boldsymbol{\beta})]'}{\sum_{l=1}^{n} Y_l(t_i) e^{\boldsymbol{\beta}'\boldsymbol{x}_l}} \right\} \\
\overline{\boldsymbol{x}}(t, \boldsymbol{\beta}) &= \frac{\sum_{l=1}^{n} Y_l(t) \boldsymbol{x}_l e^{\boldsymbol{\beta}'\boldsymbol{x}_l}}{\sum_{l=1}^{n} Y_l(t) e^{\boldsymbol{\beta}'\boldsymbol{x}_l}}
\end{aligned}
\tag{8}
$$

The above estimation procedures are available in commercial statistical software.

2.4.5 Selection of Covariates

Selection of right covariates $\boldsymbol{x}$ from upstream supply chain information is the key to building an effective hazard rate model for modeling warranty claims. The explanatory covariates $\boldsymbol{x}$ can be selected as either quantitative or qualitative variables from the supply chain illustrated in Figure 1. Such covariates may be process, quality, design or product related. They may be from the production database that ties to VIN or from other heterogeneous sources that may only be available in aggregate form. The selection depends on the application and the kind of warranty issues that need to be detected:

- If a significant "process deterioration or improvement" is sensed, and its impact on warranty performance is desired to be detected, process related covariates such as quantitative variables noise (dB), current, voltage, resistance, speed, count etc. or qualitative variables such as pass/fail could be selected from functional test results extracted from a supplier's production database.
- If a significant "quality deterioration or improvement" is sensed, and its impact on warranty performance is desired to be detected, quality related covariates such as customer reject data in terms of reject rate or defective parts per million (PPM) could be selected from functional test results extracted from a supplier's production database.
- If there is a design or material change being implemented to address a previous reliability problem or reduce cost, and if we hope to evaluate the effectiveness of such a corrective action, we may apply qualitative coded covariate $x = 0(x = 1)$ $\mathrm{x} = 0(\mathrm{x} = 1)$to VINs before and after the corrective actions correspondingly. If we have validation test results such as a life-testing Weibull plot to demonstrate the reliability improvement, we may apply a quantitative covariate such as the Weibull location parameter $\alpha = \alpha_0(\alpha = \alpha_1)$ $\mathrm{\alpha} = \mathrm{\alpha}_0(\mathrm{\alpha} = \mathrm{\alpha}_1)$to VINs before and after the corrective actions correspondingly.

- If we want to monitor the overall warranty performance for the subsystem a supplier is responsible under known current process, quality and design conditions, we may include all of the above possible covariates.

2.4.6 PH Model Development

Evaluation of the regression coefficients of the covariates (i.e., $\boldsymbol{\beta}$) in Eq.(2) requires a reasonably large training dataset to achieve considerable accuracy due to large-sample theory (Cox and Hinkley, 1974). If current production vehicle model has significant history or is a carryover model from prior years, normally abundant historical warranty claims exist to form the PH model training dataset. If current production vehicle model is a newly launched vehicle model, the historical warranty claims from surrogate/similar vehicle models may be used to form the training dataset. If historically a supplier supplied similar subsystems for different vehicle models to either the same OEM or different OEMs, such historical warranty claims can be tailored or calibrated to form a surrogate training dataset by considering different applications, customer usage and operating conditions on the newly launched vehicle model.

Initially, we may include all covariates believed to impact $h(t)$, based on engineering experience and judgment, in developing the PH model. In reality, not all of the covariates might prove to be statistically significant in impacting $h(t)$ due to heterogeneity in customer usage and/or operating conditions. For example, certain features of the subsystem are seldom used by customers or the subsystem is seldom operated under certain conditions. Under such situations, it may take a long time for certain covariates to demonstrate their impact on $h(t)$. Also, not all candidate covariates are independent explanatory variables to $h(t)$. Forward and backward stepwise selection procedure (Klein and Moeschberger, 2003) can be applied to sequentially remove confounding and statistically insignificant covariates to arrive at a final candidate multivariate PH model. Such covariate screening process may take several iterations in association with good engineering experience and judgment.

For covariates without any history, such as a major design/process change to address a previous reliability problem, the corresponding model coefficient cannot be evaluated due to the lack of a training dataset representative of vehicles that incorporate the design change. In such cases, PH model has to be extended to incorporate Bayesian analysis which will be discussed in Chapter 3.

2.5 Early Warranty Detection Scheme

Once the PH model is established and validated, we are ready to set up the warranty issue detection rule and conduct formal hypothesis tests. Early warranty detection scheme monitors warranty claims, vehicle service age, and supply chain quality/testing covariates over the vehicle production life cycle. The vehicle production period is often stratified by date, week or month depending on the monitoring frequency, and so are the warranty claims, vehicle service ages, and covariates.

2.5.1 Notation and Assumptions

In our study, we define the beginning of life of the subsystem to be the time when its vehicle was produced (if appropriate, one can also use the time of production of the subsystem to be the starting point). Hence, for vehicles that produced a warranty claim on the subsystem or labor code of interest, the non-censored life of the subsystem (t in Eq.(2)) is the difference between the date of repair/diagnosis and the vehicle production date; for vehicles without a warranty claim it is the difference between the most current monitoring date and the vehicle production date. Although other definitions could be employed (e.g. vehicle sold date which coincides with the beginning of the warranty period), this definition provides convenience for suppliers: sales information (which provides vehicle sold dates) is only available to suppliers on vehicles with warranty claims from the OEM warranty claims database (this information is generally not available for vehicles without warranty claims). Also, the defined starting time can coincide with suppliers' subsystem production date, especially for Tier-1 suppliers that build their respective subsystems in "Just-In-Time" (JIT) plants nearby OEMs' vehicle assembly plant; individual vehicle units might be built within a day or two of when its subsystems are built. Moreover, before a vehicle is sold, dealers conduct routine pre-delivery inspections and any defect or failure noticed will be reported as a warranty claim to OEMs' warranty claims database, so that the warranty claims include sale reliability due to possible transportation damage or deterioration. For our study, since this definition assumes that vehicles are produced and enter service on the same date, it avoids the complications of sales delay analysis (which might be necessary in some cases).

Following the notation from (Wu and Meeker, 2002), let n_i denote the number of vehicles produced in period i and R_{ij} denote the number of first warranty claims during jth period in service for units that are manufactured in period i. Since there is no warranty claim report delay these days in OEM warranty databases (due to direct computer entry through OEM's dealer network), R_{ij} first becomes available in period $i + j$.

2.5.2 Binomial Distribution Model for Monitoring Warranty Claims

As stated in 2.4, we propose the Binomial distribution model to model warranty claims. We treat R_{ij} as an independently distributed Binomial $B(n_i, p_{ij})$ random variable, where p_{ij} represents the probability that the subsystem of interest manufactured in period i will produce the first warranty claim during the jth period in service. The reference value for p_{ij}, denoted by p_{ij}^0, can be obtained from (1) as:

$$p_{ij}^0 = \exp[-H(j-1|\boldsymbol{x}_i)] - \exp[-H(j|\boldsymbol{x}_i)] \tag{9}$$

where $\boldsymbol{x}_i$ is the fixed covariate vector for production period i and $H(j|\boldsymbol{x}_i)$ is the cumulative hazard rate until the jth period in service. Once p_{ij}^0 is known, the upper and lower confidence limits of R_{ij}, C_{ij}^U and C_{ij}^L, respectively, can be easily calculated from the Binomial distribution.

To evaluate $h(t|\boldsymbol{x}_i)$ for a supplier's subsystem, warranty claims for a chosen (set) of categorization codes are extracted from OEMs' warranty database. Each code represents causal component of a vehicle and the kind of repair taken, and all codes are structured in function groups. To have better statistical reliance and reduce the probability of the code being wrongly binned by dealers, we cluster a group of codes to represent a supplier's subsystem so that even if a component repaired is binned to a wrong code, the wrong code still falls in the chosen group of codes with high possibility due to its local or functional relation to the causal component.

Our study focuses on early detection of a warranty issue, normally within 12 months after a vehicle is produced. Hence, the issue of warranty "drop-out" due to two-dimensional warranty policy is not a problem here when compared to OEMs' 36 months/36,000 miles or even 60 months/60,000 miles warranty policies typical in North America. Moreover, as (Wu and Meeker, 2002) pointed out, the warranty “drop-out” due to accumulated mileage will be reflected in the PH model through the historical training dataset.

The vehicles in the field may be subject to heterogeneous environment/usage and our model captures the variability through the model training dataset. By assuming that the variability is stable over each production period, our PH model can focus on variability in the reliability of the manufactured subsystem from the upstream supplier chain over production periods.

2.5.3 Hypothesis Test

Along the lines of (Wu and Meeker, 2002), the formal problem of detection can be formulated as a test of the multiple-parameter hypothesis:

$$\begin{gathered} H_0: p_{i1} = p_{i1}^0, p_{i2} = p_{i2}^0, \ldots, p_{ij} = p_{ij}^0, \ldots, p_{iM} = p_{iM}^0 \\ versus \\ H_a: p_{i1} \neq p_{i1}^0 \ or \ p_{i2} \neq p_{i2}^0 \ or \ \ldots, or \ p_{iM} \neq p_{iM}^0 \end{gathered} \tag{10}$$

where M is the pre-specified number of future periods for which the Binomial distribution probabilities will be monitored for units manufactured in any given period. For a given overall false alarm rate, increasing M will require a reduction in power to spread protection over a larger number of monitoring periods.

Consider production period i. In this period, n_i units were manufactured and sold. At the end of production period i since all covariates $\boldsymbol{x}_i$ associated with production period i are available, the claim probabilities $p_{i1}^0, p_{i2}^0, \ldots, p_{iM}^0$ can be predicted from Eq.(9) for all periods in service. Among these, there were R_{i1} warranty reports during their first period of service, and these R_{i1} reports first became available in period $i+1$. Note that $R_{i1} \sim B(n_i, p_{i1})$, and in period $i+1$, we can test only $p_{i1} = p_{i1}^0$ versus $p_{i1} \neq p_{i1}^0$; no information is available on $p_{i2}, \dots, p_{iM}$. In general, in period $i+j$, j periods after the units in the ith production period were produced, we can test the joint hypothesis of whether:

$$\begin{pmatrix} p_{i1} \neq p_{i1}^0 & p_{i2} \neq p_{i2}^0 & \dots & p_{i,j-1} \neq p_{i,j-1}^0 & p_{ij} \neq p_{ij}^0 \\ p_{i+1,1} \neq p_{i+1,1}^0 & p_{i+1,2} \neq p_{i+1,2}^0 & \dots & p_{i+1,j-1} \neq p_{i+1,j-1}^0 & \\ \dots & \dots & \dots & \dots & \\ p_{i+j,1} \neq p_{i+j,1}^0 & & & & \end{pmatrix}$$

For testing p_{ij}, only the R_{ij}, $j = 1,2,\dots$ are relevant; the other $R_{ik}(j \neq k)$ contains no information about p_{ij}. Formally, the binomial variables R_{ij} and $R_{ik}(j \neq k)$ are not independent, but through the standard "Poissonization" in large samples, they are almost independent for any practical purpose. Therefore testing $H_0: p_{i1} = p_{i1}^0, p_{i2} = p_{i2}^0, \dots, p_{ij} = p_{ij}^0, \dots, p_{iM} = p_{iM}^0$ versus $H_a: p_{ij} \neq p_{ij}^0$ for some $j = 1,2,\dots,M$ can be done by testing, individually, $H_0^j: p_{ij} = p_{ij}^0$ versus $H_a^j: p_{ij} \neq p_{ij}^0$ for $j = 1,2,\dots,M$.

Consider first testing $H_0^1: p_{i1} = p_{i1}^0$ versus $H_a^1: p_{i1} \neq p_{i1}^0$, the warranty claim probability for a vehicle produced in period i for the first period in service. In period $i+1$, we conclude that $p_{i1} \neq p_{i1}^0$ if $R_{i1} \geq C_{i1}^U$ or $R_{i1} \leq C_{i1}^L$ for some critical values C_{i1}^L and C_{i1}^U (to be determined). Similarly, for testing $H_0^j: p_{ij} = p_{ij}^0$ versus $H_a^j: p_{ij} \neq p_{ij}^0$ (the warranty claim probability for a vehicle produced in period i for the jth period in service), in period $i+j$, we conclude that $p_{ij} \neq p_{ij}^0$ if $R_{ij} \geq C_{ij}^U$ or $R_{ij} \leq C_{ij}^L$ for some critical values C_{ij}^L and C_{ij}^U (to be determined).

The primary difference between our hypothesis tests and those from (Wu and Meeker, 2002) is that their null hypothesis p_{ij}^0 is “static” and is generally expected to be constant across production periods; our null hypothesis is “dynamic” and potentially varies across the production periods. For each production period i, $\lambda^{0}_{ij}\, p^0_{ij}$ in our null hypothesis varies and is estimated from a hazard rate model driven by different covariates x_i. This varying nature of the expected hazard rate as a function of the production period and its corresponding covariates is illustrated in Figure 3 (presented in full detail in the case study section). Under the proposed model, besides using historical warranty information, we are exploiting upstream supply chain information which constitutes a partial precursor signature for later warranty claims. Unlike (Wu and Meeker, 2002), we also propose a two-sided hypothesis test to detect both unforeseen process improvements as well as warranty issues.

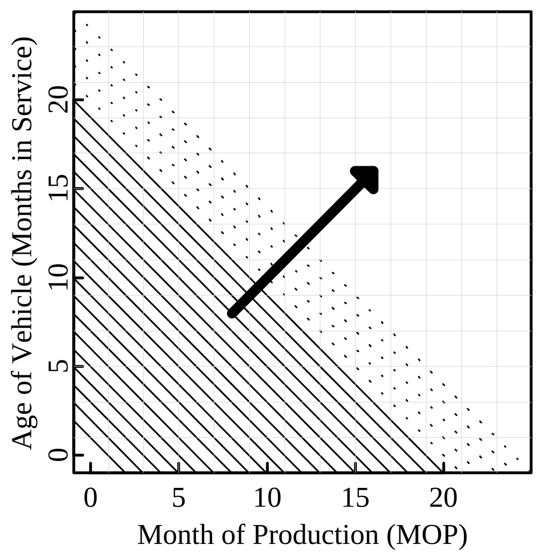

Figure 2: Warranty claims, covariates and vehicle volumes growth diagram stratified by production.

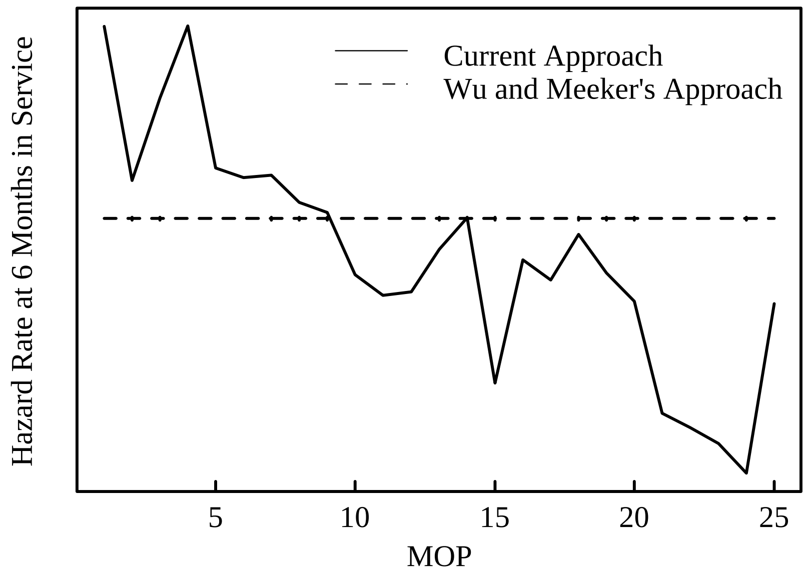

Figure 3: Illustration of null hypothesis difference between current and Hu and Meeker's approach.

2.5.4 Allocation of False Alarm Probability and Power for Detection

Let α_{ij} be the nominal false alarm probability for testing the sub hypothesis H_0^j versus H_a^j about p_{ij}, corresponding to the jth period in service for units from production period i. If we set the overall false alarm probability as α_i, from Boole's inequality, $\alpha_i \leq \sum_{j=1}^{M} \alpha_{ij}$, taking the conservative case, we have:

$$\alpha_i = \sum_{j=1}^{M} \alpha_{ij} \tag{11}$$

To balance between quick detection and the overall probability of detection (power) over potential reliability problems over the first M periods of a unit's life, we follow (Wu and Meeker, 2002) and choose α_{ij} to be proportional to the information available for testing H_0^j versus H_a^j (this information is proportional to the expected number of reports during the jth period in service). Since age is here defined as the difference between warranty repair date and vehicle production date and instead of the difference between warranty repair date and vehicle sold date, and we don't have the implication of sales delay problem:

$$\alpha_{ij} = \rho\, p_{ij}^{0} \tag{12}$$

From Eqs.(11) and (12), α_{ij} can be approximated by:

$$\alpha_{ij} = \frac{p_{ij}^{0}\alpha_i}{p_{i1}^{0} + p_{i2}^{0} + \ \dots + \ p_{iM}^{0}}, j = 1,2, \dots, M, \tag{13}$$

Note that unlike (Wu and Meeker, 2002), the nominal false alarm probability for testing is different here in ages but same for each $MOP(i)$; since λ_{ij}^{0} p_{ij}^{0} is different for each production period $MOP(i)$, α_{ij} is different for both production period $MOP(i)$ and age j. This is due to the "dynamic" nature of our null hypothesis $H_0^j \mathrm{H}_0 \mathrm{H}_0$. Once α_{ij} is determined, the critical values for carrying out the hypothesis tests, $\mathrm{C}_{\mathrm{ij}} C_{ij}^{L}$ and C_{ij}^{U}, can be easily calculated from the Binomial distribution.

2.6 Case Study

In order to illustrate and test our statistical framework for early warranty issue detection, we used a Tier-1 automotive seating supplier as an example. To illustrate the monitoring scheme from section 2.5, we follow the OEMs' typical practice of monthly monitoring frequency and define production period as a production month (MOP). Accordingly, vehicle ages, warranty claims and covariates $\boldsymbol{x}$ are also stratified by month.

2.6.1 Data

Two datasets are collected retrospectively:

1. Warranty claims dataset is collected from the supplier's OEM customer warranty database for supplier's seat related warranty claims
2. Production dataset is collected from the seat supplier's plant production database from which covariates $\boldsymbol{x}$ can be extracted.

The two datasets are linked through VIN and can be used to estimate $h(t|\boldsymbol{x})$. In our case, they cover 275,231 vehicles and 550,462 seats (one driver and one passenger seat for each vehicle) spanning over 25 production months, 3 model years with each vehicle having at least 9 months of age. The warranty claim dataset contains 11,915 (4.3%) non-censored data (warranty claims) and 263,316 (95.7%) censored data (vehicles that did not experience any seat related warranty claim). The large sample size facilitates us to effectively estimate the PH model. The non-parametric Fleming-Harrington (FH) estimation of the cumulative hazard plot (Figure 4) for all 275,231 vehicles shows a very smooth line with narrow 95% confidence bands. For confidential reason, the actual cumulative hazard rate is masked to protect proprietary information but kept as the same scale as Figure 5 and Figure 6 for relative comparison.

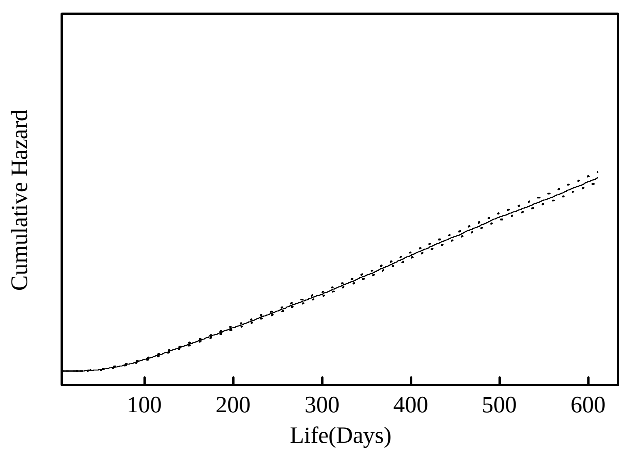

Figure 4: Cumulative hazard plot for all 275,231 vehicle seats

Since this vehicle model has 60 months/60,000 miles warranty policy, the maximum time in service for this dataset is 1,033 days, so the early warranty claim dataset will not be affected by warranty drop out due to accumulated mileage. Since the warranty claim dates are recorded by day, even though the monitoring frequency is defined as monthly, to maintain the claim date resolution, $h(t)$ estimations are based on the actual claim dates and not by monthly groupings.

2.6.2 Covariates

The production dataset contains in total 27 million function test results for the 550,462 seats. For each seat, there are about 60 function tests depending on the seat type. The 63 covariates $\boldsymbol{x}$ are stratified by production month (MOP) and extracted from supplier's production database:

- Monthly process capability indices (Cpk) for function tests - quantitative covariates: These are process state indicators for each of the 60 function tests.
- $SeatType\ (1, 2, 3 \text{ and } 4)$ - qualitative covariate: Is an indicator variable that identifies the type of seat going into the vehicle. The supplier's plant produced four different seat types from low end (#1) with fewer features and base material to high end (#4) with more features and premium material. Higher end seats with more features/content are expected to have a higher warranty claim rate.
- $\mathrm{NOKFstN}\,NOKFstN$ - quantitative covariate: Denotes the fraction of seats, by month, that did not pass at least one of the function tests in the first pass. This is the aggregate indicator for overall process state. Before each seat is shipped out from the supplier's plant, it has to pass all the 60 function tests either by repair or replacement. Even though function tests can catch all of the defects exhibited

during function testing, they may not catch certain defects such as intermittent defects which may not show up during functional testing but show up later in the field. Higher levels of $NOKFstN$ indicate higher risk of warranty claims.

- $RejectN$ - quantitative covariate: Denotes the fraction of seats rejected by the OEM vehicle assembly plant, by month, due to various seat defects. This is again an aggregate indicator for various seat defects either not covered by function tests or not caught by function tests. Higher levels of $RejectN$ indicate higher risk of warranty claims.

2.6.3 Model Estimation

The initial candidate covariates are $\boldsymbol{x} = [SeatType, RejectN, NOKFstN, Cpk1, Cpk2, \dots, Cpk60]$. $\mathrm{x} = [\mathrm{SeatType}, \mathrm{RejectN}, \mathrm{NOKFstN}, \mathrm{Cpk1}, \mathrm{Cpk2}, \dots, \mathrm{Cpk60}]$ These covariates are chosen due to their direct traceability from supplier system to end product (seats to vehicles). The particular seat system under consideration is a "carryover" design from a previous model year without any major design change. Hence, the covariates $\boldsymbol{x}$ reflect well the impact of the manufacturing process on warranty performance for this supplier's plant.

The cumulative hazard plot in Figure 5, stratified by $MOPs$, seem to clearly reveal that different $MOPs$ have distinctly different hazard rates. The purpose of the hazard rate covariate model is to explore the relationship between the above 63 covariates and $H(t)$ (or $h(t)$) so that any differences in the warranty claim rates across the different $MOPs$ can be explained by the corresponding covariate vectors ($\boldsymbol{x}_i$).

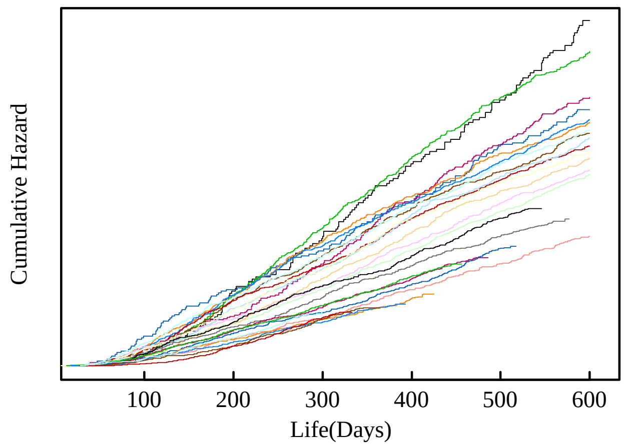

Figure 5: Cumulative hazard plot stratified by MOPs

We use warranty claim and production datasets from $MOP(1)$MOP(1) to $MOP(20)$ as training dataset to construct the PH model and estimate regression coefficients of covariates $\boldsymbol{\beta}$. This training dataset is represented by solid lines in Figure 2. The training set has a total of 206,412 vehicles with 4,093 non-censored data (warranty claims) and 202,319 censored data (vehicles never experience any seat related claims). The data from the remaining five production months, $MOP(21)$ to $MOP(25)$, are used to form the detection dataset to conduct sequential hypothesis tests. The detection dataset is represented by dotted lines in Figure 2.

To construct PH model for this case study, the baseline hazard function $h_0(t)$ $\mathrm{h}_0(\mathrm{t})$is left arbitrary, and we employed Cox's partial log-likelihood procedure for estimating the same (available from most statistical software).

It is possible that not all of the 63 covariates $(SeatType, RejectN, NOKFstN, Cpk1, Cpk2, \dots, Cpk60)$ are statistically significant in impacting the claim rate $h(t)$. The screening process is to find the significant covariates. Past experience from the plant tells us that $SeatType$ and

$RejectN$ are important covariates: normally when plant produced higher percentage of high end seats, the warranty claim rate was higher due to reasons explained above. The non-parametric FH estimation of cumulative hazard plot with 95% confidence band (Figure 6) stratified by $SeatType$ clearly shows the significantly different hazard functions for the four seat types. Also, whenever the plant received higher customer rejects ($RejectN$), later such defects showed up in warranty. The Wald test on the single covariate $RejectN$ and $SeatType$ confirms its significance with $p = 1.4 \times 10^{-8}$ $\mathrm{p} = 1.4 \times 10^{-8}$$\mathrm{p} = 1.4 \times 10^{-8}$and $p = 0$ correspondingly.

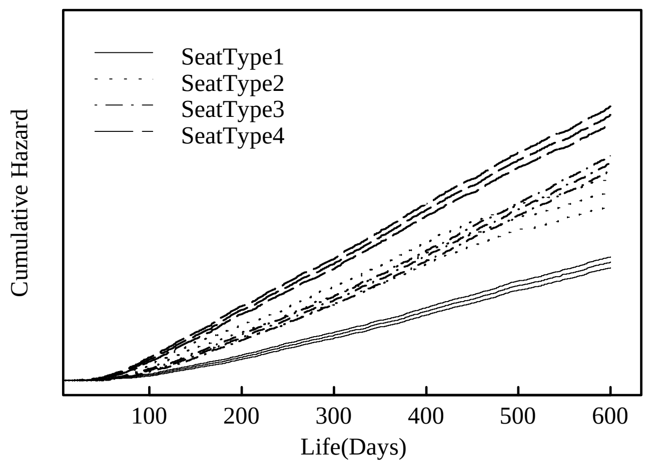

Figure 6: Cumulative hazard plot stratified by SeatType

We use $SeatType$ and $RejectN$ as the primary covariates. As for the remaining covariates, we only retained $NOKFstN$ and the 40 process capability covariates (Cpk) that exhibited a value of less than or equal to 2.0 in any month of the training dataset (by definition of process capability, the higher the Cpk, the lower the risk of a defect). These 41 covariates are candidates for the standard forward model construction procedure under the Akaike information criterion (AIC)(Akaike, 1974):

$$AIC = -2\log L + 2p \tag{14}$$

where p is the number of covariates in the PH model and L is the likelihood of the model. The forward procedure is conducted as follows:

1. Include only primary covariates $[SeatType, RejectN]$ to fit the PH model, compute its AIC and set it as the original model.
2. Add an additional covariate one by one from the 41 covariates to the original model to construct 41 1st iteration models and compute AICs. Compare the smallest AIC among them with the original model's, if this AIC is smaller than the original model's, update this model as the original model.
3. Repeat step 2 until no more covariates can be added.

After creating a multivariate PH model by the above procedure, a backward procedure is applied to remove any covariate with $p > 0.05$ and keep covariates with sound physical effect on the PH model. $NOKFstN$ did not prove to be a significant covariate due to its strong correlation with some of the 60 $Cpks$. Since $NOKFstN$ is an aggregate indicator for the 60 Cpks, it becomes redundant.

The model diagnosis revealed that there is a strong interaction between covariate $SeatType$ and time/age, which invalidates the PH model assumption. In order to account for this interaction in the PH model, we stratified the data by $SeatType$ and allowed a different baseline hazard function for each $SeatType$.

The final PH model can be expressed as:

$$h_{ST1}(t|\boldsymbol{x}) = h_{ST1,0}(t)e^{\begin{pmatrix}23.5RejectN-0.21Cpk56-0.40Cpk50-0.57Cpk27\\-1.47Cpk54-0.88Cpk26-0.24Cpk25\end{pmatrix}}$$
$$h_{ST2}(t|\boldsymbol{x}) = h_{ST2,0}(t)e^{\begin{pmatrix}23.5RejectN-0.21Cpk56-0.40Cpk50-0.57Cpk27\\-1.47Cpk54-0.88Cpk26-0.24Cpk25\end{pmatrix}}$$
$$h_{ST3}(t|\boldsymbol{x}) = h_{ST3,0}(t)e^{\begin{pmatrix}23.5RejectN-0.21Cpk56-0.40Cpk50-0.57Cpk27\\-1.47Cpk54-0.88Cpk26-0.24Cpk25\end{pmatrix}} \tag{15}$$
$$h_{ST4}(t|\boldsymbol{x}) = h_{ST4,0}(t)e^{\substack{(23.5RejectN-0.21Cpk56-0.40Cpk50-0.57Cpk27\\-1.47Cpk54-0.88Cpk26-0.24Cpk25)}}$$

where the subscript ST denotes $SeatType$. As expected, the PH model reveals some strong relationships between the covariates and the warranty claim rate:

- Reject rate does increase warranty claims (every thousandth of reject increase results in 2.3% increase of warranty claims).
- Improved process capability (Cpk) results in reduced warranty claims (every tenth increase of $Cpk56, Cpk50, Cpk27, Cpk54, Cpk26, Cpk25$ results in 2.1%, 3.9% , 5.5%, 13.7%, 8.4% and 2.4% of warranty claim reduction).

2.6.4 Model Validation

The above PH model is formally diagnosed from three aspects: violation of the assumption of proportional hazards, overly influential data, and nonlinearity in the relationship between the log hazard and the covariates.

Assessing Proportional Hazards:

The plot of the scaled Schoenfield residuals against transformed time (Figure 7) shows no systematic departures from a horizontal line, indicating no concern with the proportional hazards assumption.

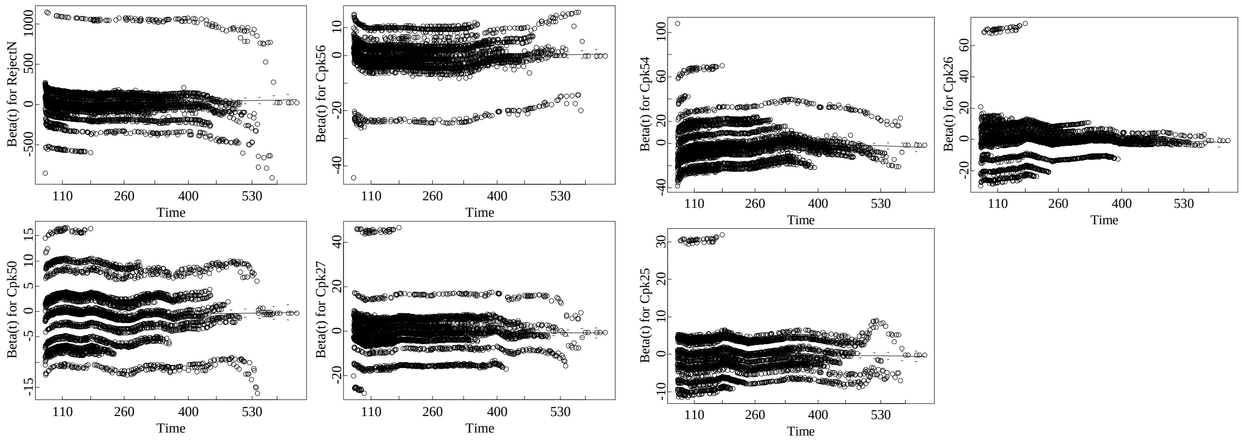

Figure 7: Plots of scaled Schoenfield residuals against transformed time for the different covariates.

Identifying Influence Points:

Using the changes in the estimated scaled coefficient due to dropping each observation from the fit as a measure of influence, a set of plots (Figure 8) are created and suggests that none of the observations are terribly influential individually.

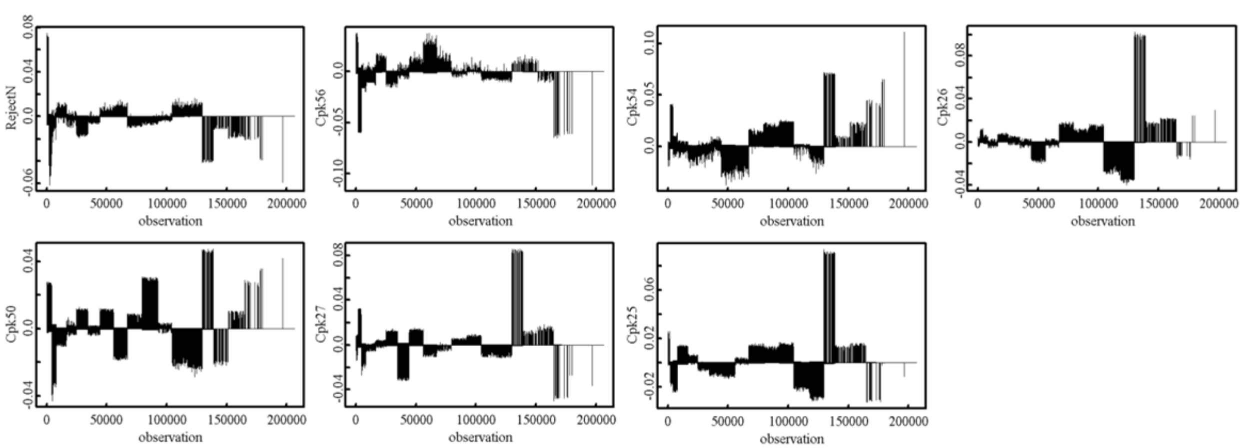

Figure 8: Plots influence by observation number for the different covariates.

Assessing Non-linearity:

Nonlinearity – that is, an incorrectly specified functional form in the parametric part of the Cox model – is a potential problem in Cox regression. The martingale residual may be plotted against covariates to detect nonlinearity. The martingale-residual plots (Figure 9) suggest that all the relationships are reasonably linear.

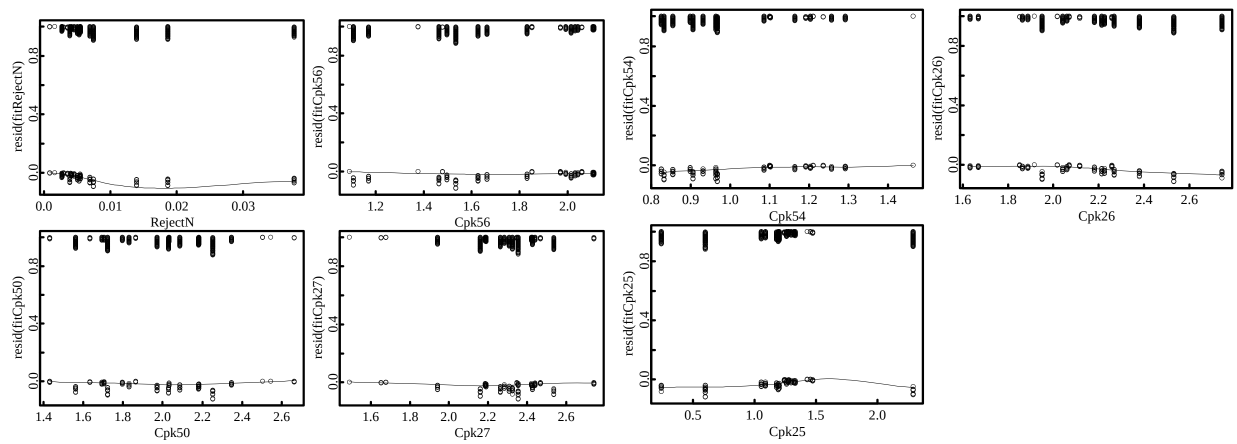

Figure 9: Martingale-residual plots for the different covariates.

Overall Results

As stated earlier, we used data from the first 20 months of production to build/calibrate the PH model and the data from the remaining 5 months of production for assessing the performance of the model in carrying out early warranty issue detection. We set overall false alarm probability at 0.1% (consistent with (Wu and Meeker, 2002) for reducing false alarms) and the monitoring period M as the first 9 months (270 days) in service. The false alarm probabilities are spread across the 9 months in service using Eq. (13). The test results are summarized in Figures (10~18). The actual claim rates are masked to protect proprietary and confidential information; however, all the figures are kept at the same scale for relative comparison. To compare the results with (Wu and Meeker, 2002), we created a constant baseline hazard rate h_j^0 for their model utilizing the training data as above (data from first 20 months of production); this constant baseline hazard rate is also revealed in Figure 3.

The plots from Figures (10~18) show that the PH model fit the training set quite well. All the figures report the claims and their prediction limits for the different months in production. The primary difference between the figures being the months in service (different months have different production volumes, and hence, contribute to estimation/variation in the prediction limits). It is readily apparent that the PH model produces rather “tight” prediction limits for the claims across the different production months (both for the training and testing datasets) as well as for the different months in service. This is in significant contrast to the limits produced by Wu and Meeker’s approach employing a constant baseline hazard rate function across all production months. While the vehicles from the 24th month in production produced claims slightly exceeding the PH model prediction limits during the seventh and ninth months in service, they are not alarmingly outside the limits. Overall, it is clear from these plots that the different covariates derived from the supply chain can greatly aid in improving the accuracy of prediction limits, and in turn, enhance the detection power for early detection of potential warranty issues. To be more conclusive, the proposed models have to be further tested using warranty data sources from other systems, products, and industries.

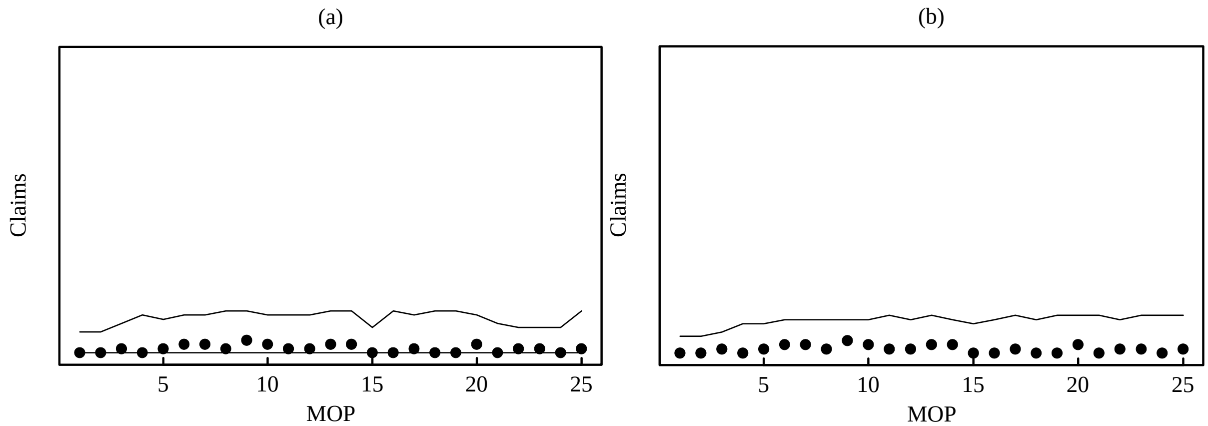

Figure 10: Hypothesis test results comparison between the proposed PH model (a) and Wu & Meeker (2002) (b) for warranty claims from vehicles during first month in service (prediction limits and actual claims).

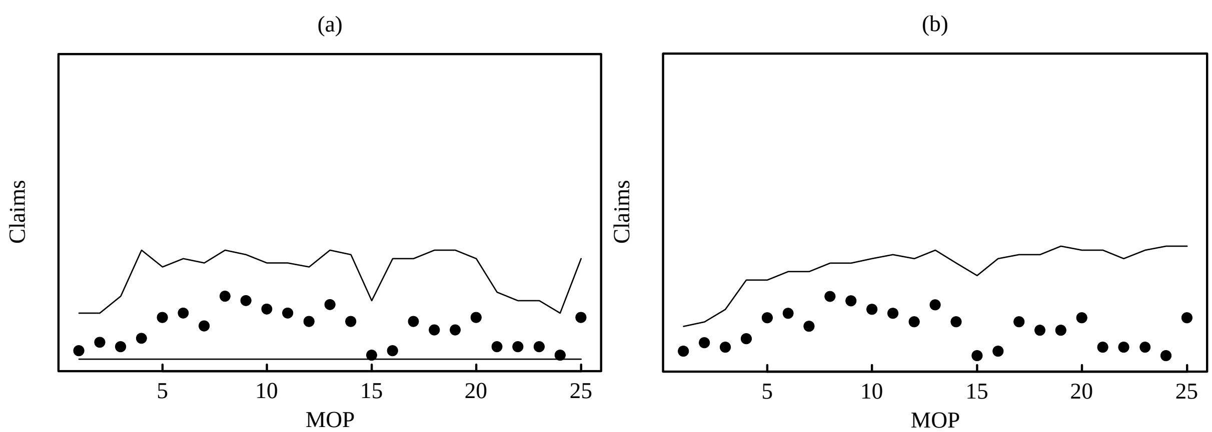

Figure 11: Hypothesis test results comparison between the proposed PH model (a) and Wu & Meeker (2002) (b) for warranty claims from vehicles during the second month in service.

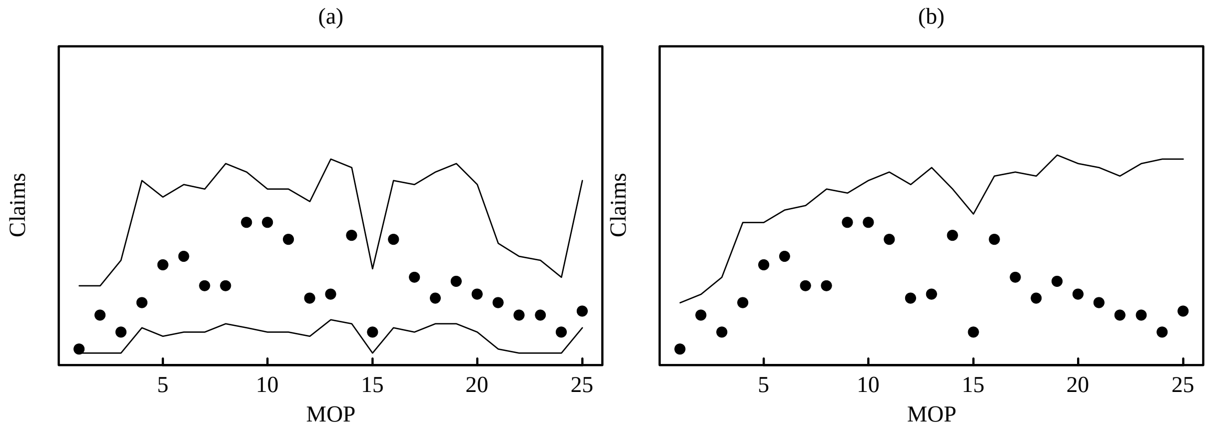

Figure 12: Hypothesis test results comparison between the proposed PH model (a) and Wu & Meeker (2002) (b) for warranty claims from vehicles during the third month in service.

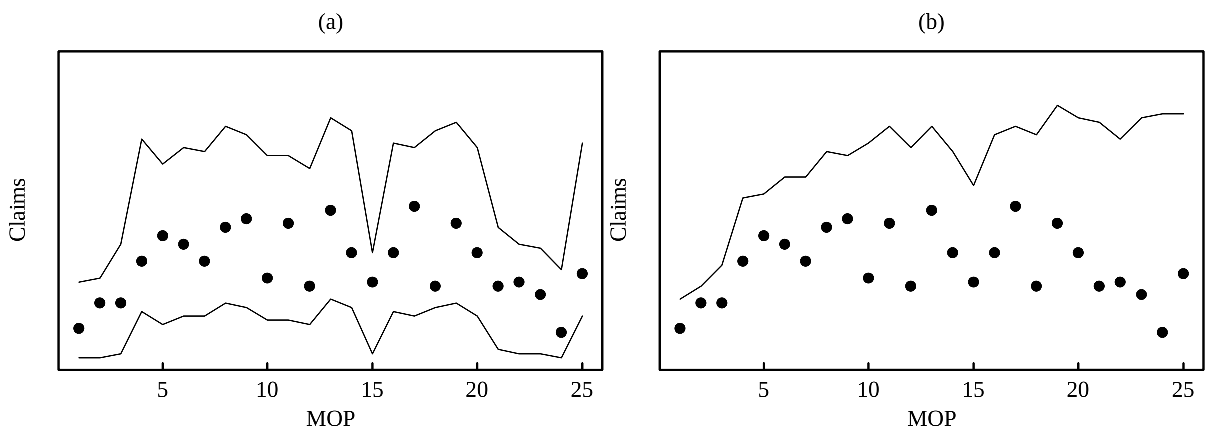

Figure 13: Hypothesis test results comparison between the proposed PH model (a) and Wu & Meeker (2002) (b) for warranty claims from vehicles during the fourth month in service.

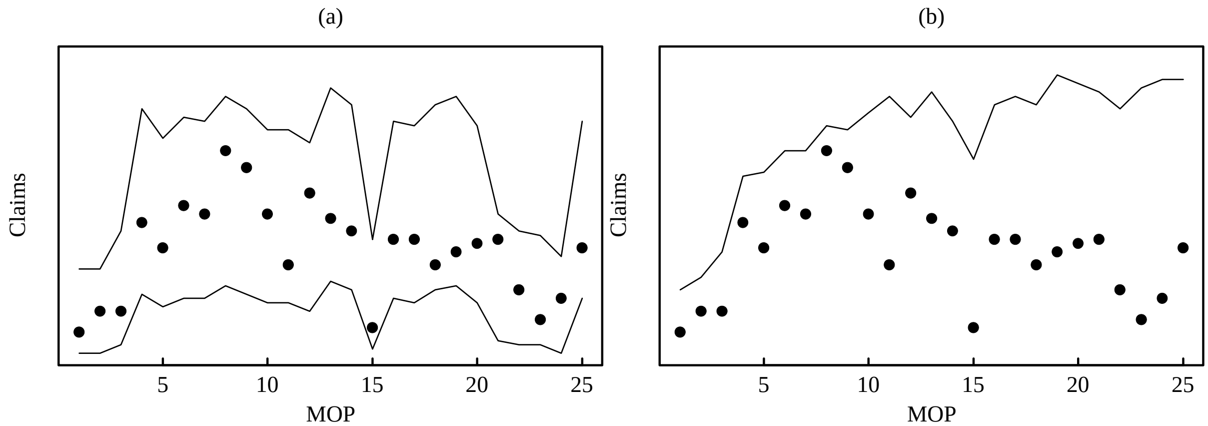

Figure 14: Hypothesis test results comparison between the proposed PH model (a) and Wu & Meeker (2002) (b) for warranty claims from vehicles during the fifth month in service.

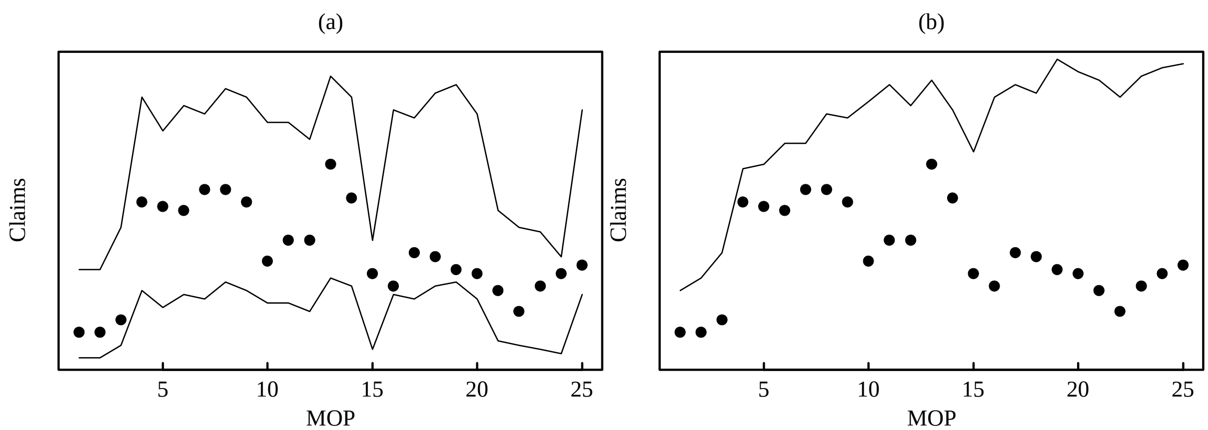

Figure 15: Hypothesis test results comparison between the proposed PH model (a) and Wu & Meeker (2002) (b) for warranty claims from vehicles during the sixth month in service.

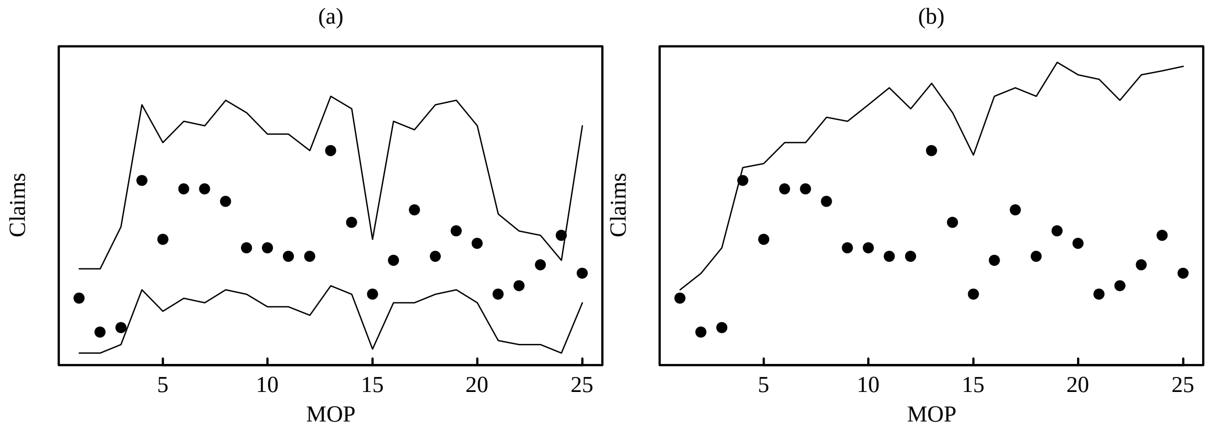

Figure 16: Hypothesis test results comparison between the proposed PH model (a) and Wu & Meeker (2002) (b) for warranty claims from vehicles during the seventh month in service.

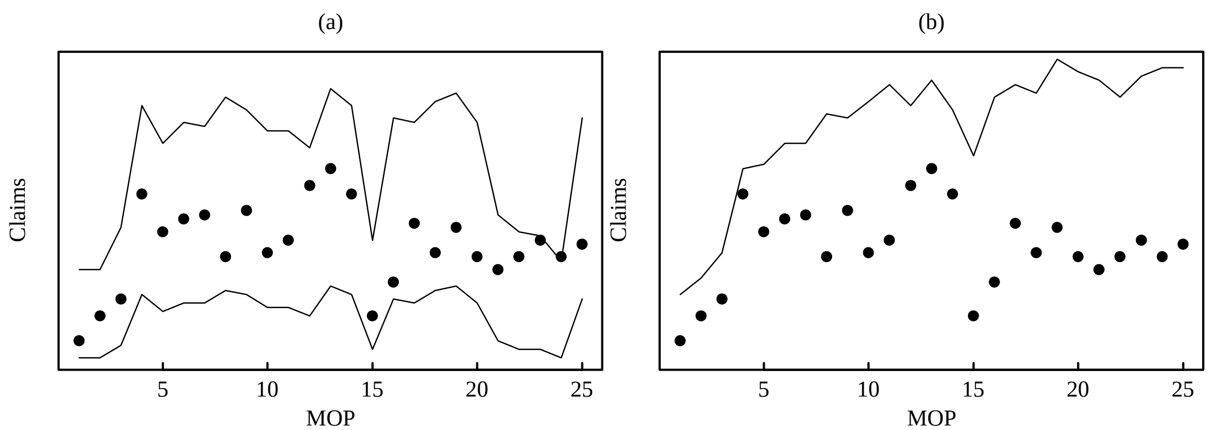

Figure 17: Hypothesis test results comparison between the proposed PH model (a) and Wu & Meeker (2002) (b) for warranty claims from vehicles during the eighth month in service.

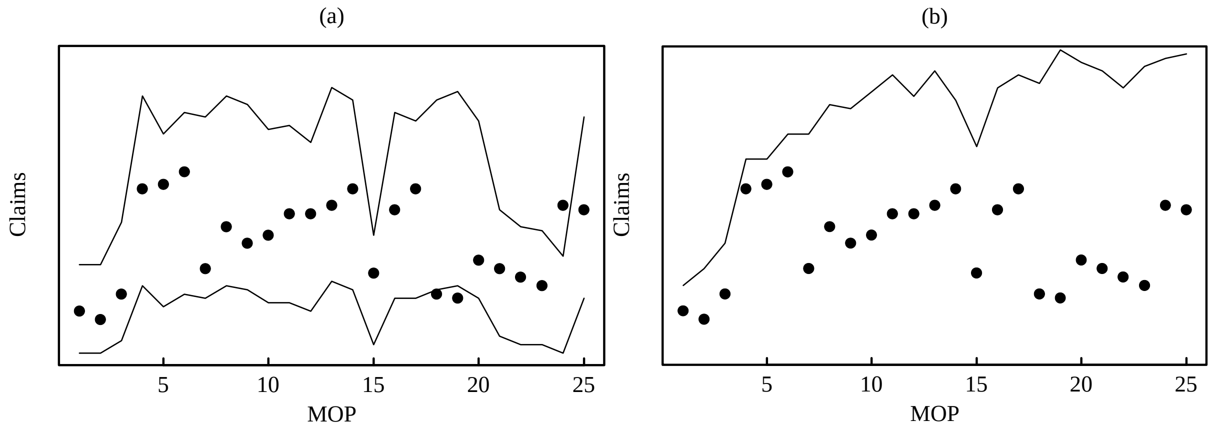

Figure 18: Hypothesis test results comparison between the proposed PH model (a) and Wu & Meeker (2002) (b) for warranty claims from vehicles during the ninth month in service.

2.7 Conclusion

Early detection of warranty issues can significantly aid companies reduce the associated warranty costs and improve customer satisfaction and brand image. Early warranty detection is a challenge when we deal with highly engineered products such as automobiles that involve complex and global supply chains and operations. Extant methods are mostly reactive and often rely only on data available from original equipment manufacturers (OEMs). Even these models do not try to explicitly link warranty claims to capability of manufacturing/assembly processes. This paper presents a statistical methodology to construct an early automotive warranty issue detection model based on upstream supply chain information. The paper proposes hazard rate models to link upstream supply chain quality/testing information as explanatory covariates for early detection of warranty issues. In doing so, it improves both the accuracy of warranty issue detection as well as the lead time for detection. The proposed methodology is illustrated and validated using real-world data from a leading Tier-1 automotive supplier.

There is other upstream supply chain information related major design/process change with which little to no warranty historical warranty claim data exists to associate, to link this type of information to warranty claims, the next chapter extend the proposed models to account for the warranty claim judgments of subject matter experts (e.g., opinions of design, process, quality and testing experts regarding design/process changes) and information from Tier-2 and further upstream suppliers. Suppliers that supply systems for multiple OEMs should also be able to exploit warranty claims information from multiple OEM customers.

CHAPTER 3 BAYESIAN APPROACH TO HAZARD RATE MODELS FOR EARLY WARRANTY ISSUES DETECTION

3.1 Introduction

In the previous chapter, we utilized upstream supply chain information such as product reject rate from end-of-line tests and manufacturing process capability in the form of covariates for hazard rate models to detect early on warranty issues and predict future warranty claims. The upstream supply chain information exploited is mostly from routine manufacturing process and historical data from the plant and observed warranty claims were used to build and calibrate the hazard rate models. However, as is evident from Figure 1, there are a number of other upstream supply chain data sources that can aid the development of effective warranty issue detection models. In particular, information from product development, major design change/upgrade efforts, manufacturing technology upgrades etc. This type of information might initially be available only in the form of results from prototype/bench tests and judgments from subject-matter-experts (SMEs) but there might be little to no warranty historical data to recalibrate the models to account for the changes. Here are some example scenarios:

- After product is launched, incoming warranty claims exhibit excessive design related fatigue failure due to certain customer usage patterns not being captured in verification tests during the product design phase. Once such warranty issue is realized, a design change is quickly implemented to address the issue.
- Due to process technology improvements, suppliers may make a major manufacturing process change (e.g., switch from gas metal arc welding to laser beam welding).

- An existing supplier's subsystem such as seat with warranty claim history will supplied to different vehicle models for the same or a new OEM.
- Suppliers add new features to their subsystem per OEM's request to meet consumers' rapidly evolving demand.

Under these types of scenarios, the management might hope to know the impact of such major changes on warranty performance early on so that any necessary counter measures can be quickly implemented to reduce risks. Unfortunately, we cannot blindly wait for the claims patterns/rates to be revealed from the field.

Suppliers often have some information for process/design and application changes. For examples, for a design related fatigue failure, suppliers may have run accelerated lab test for existing design and new design under newly realized customer usage patterns; for a major process change, such as a welding process change, suppliers may have information from production trials in the plant to evaluate scarp rate; for cases involving an existing subsystem being newly employed in other product models or with new features, suppliers may have extensive design verification results under the new applications. Also, suppliers often have good expert knowledge and opinion on the effects of the above changes from SMEs.

Unfortunately the above upstream supply chain information cannot be directly applied to hazard rate models for there is often no good field warranty claims data directly associated with it. We aim here to extend the hazard rate models proposed in Chapter 2 to exploit judgments of SMEs regarding the changes and information available from testing efforts (e.g., bench tests) to facilitate earlier and improved detection of warranty issues/improvements from the changes.

3.2 Bayesian Approach to Encapsulate Upstream Supply Chain Information

Let us revisit the proportional hazard (PH) model for warranty issue detection from the previous chapter:

$$h(t|\boldsymbol{x}) = h_0(t)\exp(\boldsymbol{\beta}'\boldsymbol{x}) \tag{16}$$

where $h_0(t)$ denotes the baseline (warranty claim) hazard rate and $\boldsymbol{x}$ the set of covariates (e.g., quality measured by end-of-line tests and process capability of different processes). We now adapt this model to account for the impact of design and/or process changes on warranty claims.

$\boldsymbol{x}$ now denotes the vector of binary covariates indicating the different design/process/application changes and $\boldsymbol{\beta}$ the regression coefficients for the same. Previously, we adopted the frequentist approach of the likelihood method and purely relied on historical warranty claims data to estimate the model coefficients. However, as stated earlier, in the presence of major design changes, we cannot afford to wait for the warranty claims patterns/rates to be revealed from the field. Instead, we adopt a Bayesian approach to exploit priors available based on the judgments of SMEs and bench tests/plant trials. We hypothesize that this extended Bayesian hazard rate model provides the potential to reduce warranty issue detection time with more power. It is however extremely important that the priors be reliable and accurate in terms of bias and precision, the construction of which can be quite involved. Guidelines for the same are provided in sections to follow.

More importantly, Bayesian method provides us a precise prescription to refine/update our prior distributions sequentially, as new warranty claims (associated with major design/process/application changes) get revealed over time from the field.

Based on the Bayes' theorem, the posterior distribution is proportional to the product of the conditional likelihood of the new warranty claims and the prior distribution for $\boldsymbol{\beta}$. The posteriors derived using data from the first monitoring period will form the updated priors for the second monitoring period and so on. After each sequential monitoring period, $\boldsymbol{\beta}$ will be closer to the true value with reduced variances, the estimated impact for the changes will be closer to true impact. Accurate prior distributions based on upstream supply chain information will make the subsequent posterior distributions converge faster to true $\boldsymbol{\beta}$, so that it can reduce warranty detection time with more power.

Since Bayesian inference is introduced (Lindley and Smith, 1972) it has been applied in reliability (Singpurwalla, 1988b), with an extensive review from a Bayesian perspective (Singpurwalla, 1988a, 2006). Bayesian analysis was also extended to proportional hazard model due to its popularity of being easy to interpret and well understood in engineering community.

Bayesian analysis was applied to proportional hazard model in two aspects: non-parametric and parametric. On the one hand, regression coefficient vector of covariates $\boldsymbol{\beta}$ is always assumed to have prior distributions with possibly unknown hyperparameters, on the other hand, the baseline hazard function $h_0(t)$ can be treated non-parametrically as in semi-parametric proportional hazard model (Cox, 1972), or parametrically as Weibull or extreme value distribution. Therefore its prior distributions have to be specified non-parametrically or parametrically accordingly.

The idea to handle the prior distribution of $h_0(t)$ non-parametrically is first to discretize $h_0(t)$ in the form of piecewise constant, non-decreasing jump etc, then apply stochastic processes which have a convenient property of conjugacy. The prior distribution can be treated as the realization of such stochastic processes. Several such stochastic processes have been proposed starting from Dirichlet processes on survival function (Ferguson, 1973), then neutral to the right (NTR) processes on survival function which overcomes limitation of Dirichlet process of being losing its property of conjugacy under right censoring (Doksum, 1974) and left truncation (Kim and Lee, 2003), then followed by gamma process (Kalbfleisch, 1978), beta process (Hjort, 1990) and random finite-mixture process (Gelfand and Mallick, 1995) on cumulative hazard function. The pros and cons for above methods have been extensively reviewed (Sinha and Dey, 1997; Singpurwalla, 2006).

The prior distribution of $h_0(t)$ can also be handled parametrically by fitting it to known parametric proportional hazards model such as Weibull and the extreme value model (Kim and Ibrahim, 2000; Zuashkiani et al., 2006)

But obtaining the posterior distribution is computationally challenging. Conjugate priors are convenient ways to obtain the closed form of posterior without computation burden. However conjugate priors are exception instead of rule in Bayesian analysis. Even for the simple and popular Weibull distribution without covariate, its posterior is not analytical tractable due to involving the integral of a non-linear function of the parameters; numerical integration and Monte Carlo simulation has to be resorted (Tsokos and Canavos, 1972; Canavos and Tsokos, 1973). Generally obtaining Bayesian posterior for hazard rate model is daunting and becomes a road block for implementation of Bayesian approach to hazard rate model. It triggers researchers to resort computation intensive numerical approach by taking advantage of modern computer power. This approach ranges from simple Monte Carlo sampling (Smith and

Gelfand, 1992) to more advanced Markov chain Monte Carlo (MCMC) algorithms such as data augmentation, Gibbs sampling, and sampling-importance-resampling (Gelfand and Smith, 1990; Gelfand and Mallick, 1995). Since MCMC is easily to be coded into computer program, it emerges as a standard procedure in statistical software to attack common computation problems encountered in Bayesian analysis.

In the following sections, we will tailor above Bayesian analysis methods to our early warranty detection scheme, so that base on the unique property of warranty claims with upstream supply chain information, we can implemented Bayesian analysis to hazard rate model in a practical way with computation efficiency.

3.3 Statistical Framework to Obtain Posterior Distribution of Hazard Rate Model

To fully examine the hazard rate model of Eq. (16) from a Bayesian prospective, we need priors for both the baseline hazard rate (i.e., $h_0(t)$) as well as the proportional model regression coefficient vector (i.e., $\boldsymbol{\beta}$). This is rather demanding for constructing priors for $\boldsymbol{\beta}$ in the absence of good historical data is rather challenging. One could partially overcome this difficulty by adopting a parametric PH model with the baseline hazard following a standard distribution (e.g., Weibull or Gamma). However, given that design and process/technology changes are made to existing products and processes, respectively, assuming that the impact of the change is proportional to the earlier hazard rate, we can utilize the baseline hazard rate function established from historical data for the earlier product. We adopt this approach throughout this chapter (i.e., $h_0(t)$ is known). Future work will account for $h_0(t)$ to be an unknown random variable or know with some uncertainty.

Overall, we divide the covariates into two groups:

- covariates employed by the PH model calibrated using historical data for the previous product (i.e., before the product/process underwent major changes), denoted by $\boldsymbol{x}^{(k)}$ with coefficients denoted by $\boldsymbol{\beta}^{(k)}$
- second group constitutes the binary covariates representative of the major changes, denoted by $\boldsymbol{x}^{(u)}$ with coefficients denoted by $\boldsymbol{\beta}^{(u)}$

As stated earlier, the Bayesian analysis will only be applied to $\boldsymbol{\beta}^{(u)}$. The proposed Bayesian approach to extend the hazard rate model is formally defined as:

$$h\left(t\middle|\boldsymbol{\beta}^{(u)}\right) = h_0(t)e^{\left(\left(\boldsymbol{\beta}^{(k)}\right)'\boldsymbol{x}^{(k)}+\left(\boldsymbol{\beta}^{(u)}\right)'\boldsymbol{x}^{(u)}\right)} \tag{17}$$

$$\boldsymbol{\beta}^{(u)} \sim \pi(\boldsymbol{\beta}^{(u)}) \tag{18}$$

Suppose $\boldsymbol{x}^{(u)}$, $\boldsymbol{\beta}^{(u)}$ are $q \times 1$ vectors $[x_1^{(u)}, x_2^{(u)}, \dots, x_q^{(u)}]'$ and $[\beta_1^{(u)}, \beta_2^{(u)}, \dots, \beta_q^{(u)}]'$. $x_j^{(u)}$ $(j = 1,2, \dots, q)$ is coded as a binary covariate with $x_j^{(u)} = 0$ if it is an existing design/process/application and with $x_j^{(u)} = 1$ if it is new design/process/application. $\beta_j^{(u)}$ $(j = 1,2, \dots, q)$ denotes the associated regression coefficient to quantify the relative hazard rate impact. Each $\beta_i^{(u)}$ is assigned a prior distribution of $\pi_i(\beta_i^{(u)})$.

We assume that $\boldsymbol{\beta}$ is independent of $h_0(t)$ and will not change the baseline hazard function. Also, we assume that $\boldsymbol{\beta}^{(u)}$ is independent of $\boldsymbol{\beta}^{(k)}$.

Based on the Bayes' law (Singpurwalla, 2006), the posterior distributions after n independent warranty claims $[(t_1, \delta_1, \boldsymbol{x}_1), (t_2, \delta_2, \boldsymbol{x}_2), \dots, (t_n, \delta_n, \boldsymbol{x}_n)]$ are observed during the 1st monitoring period are:

$$\pi\left(\boldsymbol{\beta}^{(u)}\middle|[(t_1,\delta_1,\boldsymbol{x}_1),(t_2,\delta_2,\boldsymbol{x}_2),\dots,(t_n,\delta_n,\boldsymbol{x}_n)]\right)$$
$$=\frac{L\left([(t_1,\delta_1,\boldsymbol{x}_1),(t_2,\delta_2,\boldsymbol{x}_2),\dots,(t_n,\delta_n,\boldsymbol{x}_n)]\middle|\boldsymbol{\beta}^{(u)}\right)\pi\left(\boldsymbol{\beta}^{(u)}\right)}{\int L([(t_1,\delta_1,\boldsymbol{x}_1),(t_2,\delta_2,\boldsymbol{x}_2),\dots,(t_n,\delta_n,\boldsymbol{x}_n)]|\boldsymbol{\beta}^{(u)})\pi(\boldsymbol{\beta}^{(u)})d\boldsymbol{\beta}^{(u)}} \tag{19}$$

where t_i is the censored ith unit service age with corresponding changes as covariate vectors $\boldsymbol{x}_i^{(u)}$. The indicator variable $\delta_i = 1$ if the ith unit experienced a warranty claim for the subsystem of interest at service age t_i; $\delta_i = 0$ if the ith unit has not produced any warranty claim until age t_i.

$L\left([(t_1,\delta_1,\boldsymbol{x}_1),(t_2,\delta_2,\boldsymbol{x}_2),\dots,(t_n,\delta_n,\boldsymbol{x}_n)]\middle|\boldsymbol{\beta}^{(u)}\right)\pi\left(\boldsymbol{\beta}^{(u)}\right)$ is the likelihood of $\boldsymbol{\beta}^{(u)}$ given the warranty claims $[(t_1,\delta_1,\boldsymbol{x}_1),(t_2,\delta_2,\boldsymbol{x}_2),\dots,(t_n,\delta_n,\boldsymbol{x}_n)]$. This likelihood can be estimated as:

$$\begin{aligned}
&L\left((t_1,\delta_1,\boldsymbol{x}_1),(t_2,\delta_2,\boldsymbol{x}_2),\dots,(t_n,\delta_n,\boldsymbol{x}_n)\middle|\boldsymbol{\beta}^{(u)}\right)\\
&=\prod_{i=1}^{n} f(t_i)^{\delta_i}S(t_i)^{1-\delta_i}=\prod_{i=1}^{n}[h(t_i)S(t_i)]^{\delta_i}[S(t_i)]^{1-\delta_i}=\prod_{i=1}^{n}h(t_i)^{\delta_i}S(t_i)\\
&=\prod_{i=1}^{n}\left[h_0(t_i)e^{\left(\left(\boldsymbol{\beta}^{(k)}\right)'\boldsymbol{x}_i{}^{(k)}+\left(\boldsymbol{\beta}^{(u)}\right)'\boldsymbol{x}_i{}^{(u)}\right)}\right]^{\delta_i} e^{-H_0(t_i)\exp\left[\left(\boldsymbol{\beta}^{(k)}\right)'\boldsymbol{x}_i{}^{(k)}+\left(\boldsymbol{\beta}^{(u)}\right)'\boldsymbol{x}_i{}^{(u)}\right]}\\
&=\prod_{i=1}^{n}\left[h_0(t_i)e^{\left(\boldsymbol{\beta}^{(k)}\right)'\boldsymbol{x}_i{}^{(k)}}\right]^{\delta_i} e^{-H_0(t_i)\exp\left[\left(\boldsymbol{\beta}^{(k)}\right)'\boldsymbol{x}^{(k)}\right]}\\
&\times\prod_{i=1}^{n}\left[h_0(t_i)e^{\left(\boldsymbol{\beta}^{(u)}\right)'\boldsymbol{x}_i{}^{(u)}}\right]^{\delta_i} e^{-H_0(t_i)\exp\left[\left(\boldsymbol{\beta}^{(u)}\right)'\boldsymbol{x}^{(u)}\right]}
\end{aligned} \tag{20}$$

Since $\boldsymbol{\beta}^{(k)}$,$\boldsymbol{x}_i{}^{(k)}$,$h_0(t_i)$ are all known, $\prod_{i=1}^{n}\left[h_0(t_i)e^{\left(\boldsymbol{\beta}^{(k)}\right)'\boldsymbol{x}_i{}^{(k)}}\right]^{\delta_i} e^{-H_0(t_i)\exp\left[\left(\boldsymbol{\beta}^{(k)}\right)'\boldsymbol{x}^{(k)}\right]}$ is a constant, set it as c_0, then:

$$L\left([(t_1,\delta_1,\boldsymbol{x}_1),(t_2,\delta_2,\boldsymbol{x}_2),\dots,(t_n,\delta_n,\boldsymbol{x}_n)]\middle|\boldsymbol{\beta}^{(u)}\right) \tag{21}$$

$$= c_0 \prod_{i=1}^{n} \left(\left[h_0(t_i) e^{\left(\boldsymbol{\beta}^{(u)}\right)' \boldsymbol{x}_i{}^{(u)}} \right]^{\delta_i} e^{-H_0(t_i) \exp\left[\left(\boldsymbol{\beta}^{(u)}\right)' \boldsymbol{x}_i{}^{(u)}\right]} \right)$$

$$= \left(c_0 \prod_{i=1}^{n} [h_0(t_i)]^{\delta_i} \right) \left(\prod_{i=1}^{n} \left[e^{\sum_{j=1}^{q} \beta_j^{(u)} x_{ij}^{(u)}} \right]^{\delta_i} \right) \left(\prod_{i=1}^{n} e^{-H_0(t_i) \exp\left[\sum_{j=1}^{q} \beta_j^{(u)} x_{ij}^{(u)}\right]} \right)$$

Set $\theta_j^{(u)} = e^{\beta_j^{(u)}} (j = 1,2, \dots, q)$. Also, given that $\prod_{i=1}^{n} [h_0(t_i)]^{\delta_i}$ is known and is a constant, set $c_1 = \prod_{i=1}^{n} [h_0(t_i)]^{\delta_i}$. Then:

$$L\left(\left(t_1, \delta_1, \boldsymbol{x}_1^{(u)}\right), \left(t_2, \delta_2, \boldsymbol{x}_2^{(u)}\right), \dots, \left(t_n, \delta_n, \boldsymbol{x}_n^{(u)}\right) \middle| \boldsymbol{\theta}^{(u)} \right)$$
$$= c_0 c_1 \left(\prod_{j=1}^{q} \left(\theta_j^{(u)}\right)^{\sum_i^n \delta_i x_{ij}^{(u)}} \right) \left(e^{-\sum_{i=1}^{n} \left[H_0(t_i) \prod_{j=1}^{q} \left(\theta_j^{(u)}\right)^{x_{ij}^{(u)}} \right]} \right) \tag{22}$$

Here, $\boldsymbol{\theta}^{(u)} = \left[\theta_1^{(u)}, \theta_2^{(u)}, \dots, \theta_q^{(u)}\right]'$. The posterior distribution of $\boldsymbol{\theta}^{(u)}$ can then be expressed as:

$$\pi\left(\boldsymbol{\theta}^{(u)} \middle| [(t_1, \delta_1, \boldsymbol{x}_1), (t_2, \delta_2, \boldsymbol{x}_2), \dots, (t_n, \delta_n, \boldsymbol{x}_n)]\right)$$
$$= \frac{c_0 c_1 \left(\prod_{j=1}^{q} \left(\theta_j^{(u)}\right)^{\sum_i^n \delta_i x_{ij}^{(u)}} \right) \left(e^{-\sum_{i=1}^{n} \left[H_0(t_i) \prod_{j=1}^{q} \left(\theta_j^{(u)}\right)^{x_{ij}^{(u)}} \right]} \right) \pi\left(\boldsymbol{\theta}^{(u)}\right)}{c_0 c_1 \int \left(\prod_{j=1}^{q} \left(\theta_j^{(u)}\right)^{\sum_i^n \delta_i x_{ij}^{(u)}} \right) \left(e^{-\sum_{i=1}^{n} \left[H_0(t_i) \prod_{j=1}^{q} \left(\theta_j^{(u)}\right)^{x_{ij}^{(u)}} \right]} \right) \pi(\boldsymbol{\theta}^{(u)}) d\boldsymbol{\theta}^{(u)}}$$
$$= \frac{\left(\prod_{j=1}^{q} \left(\theta_j^{(u)}\right)^{\sum_i^n \delta_i x_{ij}^{(u)}} \right) \left(e^{-\sum_{i=1}^{n} \left[H_0(t_i) \prod_{j=1}^{q} \left(\theta_j^{(u)}\right)^{x_{ij}^{(u)}} \right]} \right) \pi\left(\boldsymbol{\theta}^{(u)}\right)}{\int \left(\prod_{j=1}^{q} \left(\theta_j^{(u)}\right)^{\sum_i^n \delta_i x_{ij}^{(u)}} \right) \left(e^{-\sum_{i=1}^{n} \left[H_0(t_i) \prod_{j=1}^{q} \left(\theta_j^{(u)}\right)^{x_{ij}^{(u)}} \right]} \right) \pi(\boldsymbol{\theta}^{(u)}) d\boldsymbol{\theta}^{(u)}} \tag{23}$$

When there is only one design/process/application change (i.e., $q = 1$), $\boldsymbol{\theta}^{(u)} = \theta$ and $\boldsymbol{x}^{(u)} = x$. The likelihood function can then be simplified as:

$$\begin{aligned}
&L([(t_1, \delta_1, x_1), (t_2, \delta_2, x_2), \dots, (t_n, \delta_n, x_n)] | \theta) \\
&= c_0 c_1 \theta^{\sum_i^n \delta_i x_i} \left(e^{-\sum_{i=1}^n H_0(t_i) \theta^{x_i}} \right) \\
&= c_0 c_1 \theta^{\sum_i^n \delta_i x_i} e^{-\left(\sum_{i=1}^{n^{(0)}} H_0(t_i) + \theta \sum_{i=1}^{n^{(1)}} H_0(t_i) \right)} \\
&= c_0 c_1 e^{-\sum_{i=1}^{n^{(0)}} H_0(t_i)} \theta^{\sum_i^n \delta_i x_i} e^{-\theta \sum_{i=1}^{n^{(1)}} H_0(t_i)}
\end{aligned} \tag{24}$$

Here, $n^{(0)}$ denotes the number of units fielded prior to the change (i.e., data corresponding to $x_i = 0$). $n^{(1)}$ denotes the number of units fielded post change. We denote $n = n^{(0)} + n^{(1)}$. Setting $c_2 = e^{-\sum_{i=1}^{n^{(0)}} H_0(t_i)}, c_3 = \sum_{i=1}^n \delta_i x_i, c_4 = \sum_{i=1}^{n^{(1)}} H_0(t_i)$, which are all constants, we have:

$$\begin{aligned}
&L([(t_1, \delta_1, x_1), (t_2, \delta_2, x_2), \dots, (t_n, \delta_n, x_n)] | \theta) \\
&= c_0 c_1 c_2 \theta^{c_3} e^{-c_4 \theta}
\end{aligned} \tag{25}$$

The posterior distribution can then be simplified as:

$$\begin{aligned}
&\pi(\theta | [(t_1, \delta_1, x_1), (t_2, \delta_2, x_2), \dots, (t_n, \delta_n, x_n)]) \\
&= \frac{c_0 c_1 c_2 \theta^{c_3} e^{-c_4 \theta} \pi(\theta)}{\int c_0 c_1 c_2 \theta^{c_3} e^{-c_4 \theta} \pi(\theta) d\theta} \\
&= \frac{\theta^{c_3} e^{-c_4 \theta} \pi(\theta)}{\int \theta^{c_3} e^{-c_4 \theta} \pi(\theta) d\theta}
\end{aligned} \tag{26}$$

Obviously, if the prior distribution of θ is a gamma distribution:

$$\pi(\theta) = \frac{\lambda^k \theta^{k-1} e^{-\lambda \theta}}{\Gamma(k)} \sim Gamma(k, \lambda) \tag{27}$$

Its posterior distribution is also a gamma distribution:

$$\begin{aligned}
&\pi(\theta | [(t_1, \delta_1, x_1), (t_2, \delta_2, x_2), \dots, (t_n, \delta_n, x_n)]) \\
&= \frac{(\lambda + c_4)^{k + c_3}}{\Gamma(k + c_3)} \theta^{k + c_3 - 1} e^{-(\lambda + c_4)\theta} \sim Gamma(k + c_3, \lambda + c_4)
\end{aligned} \tag{28}$$

where $c_3 = \sum_{i=1}^{n} \delta_i x_i$ and $c_4 = \sum_{i=1}^{n^{(1)}} H_0(t_i)$. It is interesting to note that c_3 is the warranty claims associated with the change (i.e., the new design/process/application), c_4 is the sum of the cumulative hazard rates for units under the new design/process/application. The additional new warranty claims for units with existing design/process/application will not shed any light on the posterior distribution of θ.

When there are two or more design/process/application changes, multiple covariates will be necessary. Then, $\theta_j s$ in the exponent of the last item of Eq.(22) are compounded and cannot be separated; the closed-form of $\boldsymbol{\theta}$'s posterior does not exist, numerical Bayesian methods have to be pursued (see (Gelfand and Smith, 1990) for further information on numerical Bayesian methods).

We propose two numerical Bayesian methods to obtain $\boldsymbol{\theta}$'s posteriors when more than one change is involved. The first method is a non-parametric method whereas the second is a parametric method.

3.3.1 Large Sample Size Bayesian Analysis

Here, we adopt the large sample Bayesian analysis from (Faraggi and Simon, 1997). In the PH model shown in Eq.(16), since their interest focus on estimation of regression coefficients $\boldsymbol{\beta}$ rather than on prediction of the survival function, they avoided placing a prior distribution on $h_0(t)$ to derive the posterior distribution. Instead, they only assigned a multivariate Gaussian prior to $\boldsymbol{\beta}$, then obtained the large sample approximation of posterior $\boldsymbol{\beta}|\widehat{\boldsymbol{\beta}}$ via MLE using the standard Cox procedure (Cox, 1972). This method was adopted in our application as the following procedure:

1. *Estimate Baseline PH Model*: Use historical warranty claims with upstream supply chain information to estimate $h_0(t)$ and $\boldsymbol{\beta}^{(k)}$ in Eq.(17) by maximizing Cox's partial log-likelihood:

$$l\left(\boldsymbol{\beta}^{(\boldsymbol{k})}\right) = \sum_{i}^{n_0} \delta_i \left[\boldsymbol{\beta}^{(\boldsymbol{k})\prime}\boldsymbol{x}_i - log\left(\sum_{l=1}^{n_0} Y_l(t_i) e^{\boldsymbol{\beta}^{(\boldsymbol{k})\prime}\boldsymbol{x}_l}\right)\right] \quad (29)$$

where n_0 is the number of units from the existing design/process/application for which there exists a partial or full warranty claim history, variable $Y_l(t_i)$, called the risk indicator, equals 1 if and only if the lth unit has no warranty claim and is still in service at time t_i, and hence, at risk of generating a claim at time t_i. Hence, $h_0(t)$ and $\boldsymbol{\beta}^{(k)}$ become known variables.

2. *Assign Prior*: When the new design/process/application changes have been implemented into suppliers' subsystems, assign $\boldsymbol{\beta}^{(u)}$ with a Gaussian prior $\pi\left(\boldsymbol{\beta}^{(u)}\right) = N_q(\boldsymbol{\mu}^{(0)}, \Sigma^{(0)})$ from upstream supplier chain information where $\boldsymbol{\mu}^{(0)}$ is $q \times 1$ mean vector, $\Sigma^{(0)}$ is $q \times q$ variance matrix. Start monitoring warranty claims from units associated with these new design/process/application changes.

3. *MLE*: During the first monitoring period, suppose there are n_1 units with both existing design/process/application and new design/process/application for which there exists a partial or full warranty claim history, we estimate $\boldsymbol{\beta}^{(u)}$ as $\widehat{\boldsymbol{\beta}}^{(u)}$ in (17) by maximizing Cox's partial log-likelihood as follows:

$$l\left(\boldsymbol{\beta}^{(\boldsymbol{u})}\right) = \sum_{i}^{n_1} \delta_i \left[\boldsymbol{\beta}^{(\boldsymbol{u})\prime}\boldsymbol{x}_i - \log\left(\sum_{l=1}^{n_1} Y_l(t_i) e^{\boldsymbol{\beta}^{(\boldsymbol{u})\prime}\boldsymbol{x}_l}\right)\right] \quad (30)$$

Under large sample size, the probability density function of $\widehat{\boldsymbol{\beta}}^{(u)}$ can be approximated by a Gaussian distribution:

$$f\left(\widehat{\boldsymbol{\beta}}^{(u)}\middle|\boldsymbol{\beta}^{(u)}\right) \sim N_q(\boldsymbol{\beta}^{(u)}, I^{-1}(\widehat{\boldsymbol{\beta}}^{(u)})) \tag{31}$$

where $I(\widehat{\boldsymbol{\beta}}^{(u)})$ is the observed information matrix, which can be estimated as:

$$I(\widehat{\boldsymbol{\beta}}^{(u)}) = -\frac{\partial^2 l(\boldsymbol{\beta}^{(\boldsymbol{u})})}{\partial \boldsymbol{\beta}^{(\boldsymbol{u})} \partial \boldsymbol{\beta}^{(\boldsymbol{u})\prime}} \tag{32}$$

4. *Derive Posterior*: During the first monitoring period, the posterior distribution of $\boldsymbol{\beta}^{(u)}$ can be evaluated based on $\widehat{\boldsymbol{\beta}}^{(u)}$:

$$\begin{aligned} \pi(\boldsymbol{\beta}^{(u)}|\widehat{\boldsymbol{\beta}}^{(u)}) &= \frac{f\left(\widehat{\boldsymbol{\beta}}^{(u)}\middle|\boldsymbol{\beta}^{(u)}\right)\pi(\boldsymbol{\beta}^{(u)})}{\int f\left(\widehat{\boldsymbol{\beta}}^{(u)}\middle|\boldsymbol{\beta}^{(u)}\right)\pi(\boldsymbol{\beta}^{(u)})d\boldsymbol{\beta}^{(u)}} \\ &\propto N_q\left(\boldsymbol{\beta}^{(u)}, I^{-1}(\widehat{\boldsymbol{\beta}}^{(u)})\right) N_q(\boldsymbol{\mu}^{(0)}, \Sigma^{(0)}) \end{aligned} \tag{33}$$

The above posterior is proved to be Gaussian (Lindley and Smith, 1972):

$$\begin{aligned} &\pi(\boldsymbol{\beta}^{(u)}|\widehat{\boldsymbol{\beta}}^{(u)}) = N_q(\boldsymbol{\mu}^{(1)}, \Sigma^{(1)}) \\ &\boldsymbol{\mu}^{(1)} = \left[I(\widehat{\boldsymbol{\beta}}^{(u)}) + {\Sigma^{(0)}}^{-1}\right]^{-1}\left[I(\widehat{\boldsymbol{\beta}}^{(u)})\widehat{\boldsymbol{\beta}}^{(u)} + {\Sigma^{(0)}}^{-1}\boldsymbol{\mu}^{(0)}\right] \\ &\Sigma^{(1)} = \left[I(\widehat{\boldsymbol{\beta}}^{(u)}) + {\Sigma^{(0)}}^{-1}\right]^{-1} \end{aligned} \tag{34}$$

It is easy to see that the Bayes estimator for $\boldsymbol{\beta}^{(u)}$ is a weighted average of the MLE $\widehat{\boldsymbol{\beta}}^{(u)}$ and the mean of the prior $\boldsymbol{\mu}^{(0)}$. When $\boldsymbol{\beta}^{(u)}$ is a scalar variable with $\beta^{(u)} \sim N_q(\mu^{(0)}, {\sigma^{(0)}}^2)$, then the Bayes estimator for $\beta^{(u)}$ is the familiar form of posterior for Gaussian conjugate prior with known variance.

$$\begin{aligned} \mu^{(1)} &= \frac{{\sigma^{(0)}}^2}{\sigma^2_{\widehat{\beta}^{(u)}} + {\sigma^{(0)}}^2}\widehat{\beta}^{(u)} + \frac{\sigma^2_{\widehat{\beta}^{(u)}}}{\sigma^2_{\widehat{\beta}^{(u)}} + {\sigma^{(0)}}^2}\mu^{(0)} \\ {\sigma^{(1)}}^2 &= \frac{\sigma^2_{\widehat{\beta}^{(u)}}{\sigma^{(0)}}^2}{\sigma^2_{\widehat{\beta}^{(u)}} + {\sigma^{(0)}}^2} \end{aligned} \tag{35}$$

5. *Update* $h_0(t)$ and $\boldsymbol{\beta}^{(k)}$: update $h_0(t)$ and $\boldsymbol{\beta}^{(k)}$ per step 1 based on new incoming warranty claims from existing design/process/application during the first monitoring period. When the warranty claims for existing design/process/application are fully mature, $h_0(t)$and $\boldsymbol{\beta}^{(k)}$ will change little; when the warranty claims for existing design/process/application are not fully mature especially in high time in service, this updating can improve the accuracy of the model by iterative calibration per monitoring period.
6. *Use Old Posterior as New Prior*: During the second monitoring period, suppose there are n_2 units with both existing design/process/application and new design/process/application for which there exists a partial or full warranty claim history, replace the prior of $\boldsymbol{\beta}^{(u)}$ with its posterior $N_q(\boldsymbol{\mu}^{(1)}, \Sigma^{(1)})$ from the first monitoring period; estimate MLE $\widehat{\boldsymbol{\beta}}^{(u)}$ from Eq.(30) based on n_2; and then obtain the $\boldsymbol{\beta}^{(u)}$'s posterior $N_q(\boldsymbol{\mu}^{(2)}, \Sigma^{(2)})$.
7. *Repeat*: Repeat steps 2-6 for each monitoring period.

For M monitoring periods, $M+1$ sequence of posteriors of $\boldsymbol{\beta}^{(u)}$ will be obtained as $N_q\left(\boldsymbol{\mu}^{(0)}, \Sigma^{(0)}\right), N_q\left(\boldsymbol{\mu}^{(1)}, \Sigma^{(1)}\right), \dots, N_q\left(\boldsymbol{\mu}^{(M)}, \Sigma^{(\mathrm{M})}\right)$. It was shown (Faraggi and Simon, 1997) that such a Bayesian estimation process has superior performance over using Cox model directly without Bayesian treatment.

The advantage of the large sample size method is computational efficiency; eases burden of computing posterior of $h_0(t)$. The difficulty with the method proposed by (Faraggi and Simon, 1997) is that they lack prior knowledge of $h_0(t)$. Hence, the survival function cannot be predicted. In our case, we overcome that difficulty because of availability of historical claims data from the existing design/process/application to estimate $h_0(t)$ accurately (as discussed in Step-1 above). The only assumption made is that MLE $\widehat{\boldsymbol{\beta}}^{(u)}$ is approximately normal, valid under large sample sizes. (Faraggi and Simon, 1997) showed that when the ratio of the number of events (warranty claims in our case) to the number of covariates is larger than 15, the approximation is accurate. In our case, large sample size is the unique property of warranty claims due to large population of units in the field (in particular, for automotive industry). The implementation of this procedure is easy as standard modules are available in most statistical software to estimate MLE $\widehat{\boldsymbol{\beta}}^{(u)}$, and the posterior of $\boldsymbol{\beta}^{(u)}$ can be computed without special software due to the convenient properties of the Gaussian distribution.

3.3.2 Monte Carlo Markov Chain (MCMC) Bayesian Analysis

In the parametric method, we assume warranty claims follow a Weibull distribution with shape parameter $r > 0$ and scale parameter $\lambda > 0$:

$$h(t) = \lambda r t^{r-1} \tag{36}$$

We assume that only the scale parameter λ depends on covariates $\boldsymbol{x}$:

$$\lambda(\boldsymbol{x}) = \boldsymbol{e}^{(\beta_0 + \boldsymbol{\beta}'\boldsymbol{x})} \tag{37}$$

From Eq.(37), Eq.(36) takes the parametric form of Eq.(16):

$$h(t) = r t^{r-1} \boldsymbol{e}^{\beta_0} \boldsymbol{e}^{(\boldsymbol{\beta}'\boldsymbol{x})} \tag{38}$$

with $h_0(t) = rt^{r-1}\boldsymbol{e}^{\beta_0}$ as the baseline hazard function. Dividing $\boldsymbol{\beta}$ into $\boldsymbol{\beta}^{(k)}$ and $\boldsymbol{\beta}^{(u)}$ as before, Eq.(39) takes the parametric form of Eq.(17):

$$h\left(t \middle| \boldsymbol{\beta}^{(u)}\right) = r t^{r-1} e^{\beta_0} e^{\left(\left(\boldsymbol{\beta}^{(k)}\right)' \boldsymbol{x}^{(k)} + \left(\boldsymbol{\beta}^{(u)}\right)' \boldsymbol{x}^{(u)}\right)} \tag{39}$$

The Weibull family is used here because it is widely used and well understood in the engineering community to model failure modes. Its shape parameter r can represent decreasing ($r < 1$), constant ($r = 1$), and increasing ($r > 1$) failure rate, which makes it very flexible and attractive. Also, Weibull family is the only parametric family that yields both a proportional hazard (PH) model and an accelerated failure time (AFT) model.

The procedure here is similar to that of the non-parametric method discussed in Section 3.3.1. The major difference is that a closed-form posterior for $\boldsymbol{\beta}^{(u)}$ is not possible, hence, simulation techniques such as Markov Chain Monte Carlo methods will be applied:

1. *Estimate Baseline PH Model:* Use historical warranty claims with upstream supply chain information to estimate r, β_0 and $\boldsymbol{\beta}^{(k)}$ in Eq.(39) by maximizing the full log-likelihood function:

$$\begin{aligned} &l\left(r, \beta_0, \boldsymbol{\beta}^{(k)}\right) \\ &= \sum_{i}^{n_0} \delta_i \left[\log h_0(t_i; r, \beta_0) + \boldsymbol{\beta}^{(k)\prime} \boldsymbol{x}_i\right] - \sum_{i}^{n_0} H_0(t_i; r, \beta_0) \exp\left(\boldsymbol{\beta}^{(k)\prime} \boldsymbol{x}_i\right) \\ &= \sum_{i}^{n_0} \delta_i \left[\log r + (r-1)\log t_i + \beta_0 + \boldsymbol{\beta}^{(k)\prime} \boldsymbol{x}_i\right] - \sum_{i}^{n_0} t_i^r \exp\left(\beta_0 + \boldsymbol{\beta}^{(k)\prime} \boldsymbol{x}_i\right) \end{aligned} \tag{40}$$

where n_0 is the number of units from the existing design/process/application for which there exist a partial or full warranty claim history. Now r, β_0 and $\boldsymbol{\beta}^{(k)}$ become fixed and known variables.

Alternately, since a Weibull model is both PH and AFT model, we can re-parameterize the scale parameter λ:

$$\lambda(\boldsymbol{x})^{-1/r} = \exp(\beta_{AFT,0} + \boldsymbol{\beta}_{AFT}{}' \boldsymbol{x}) \tag{41}$$

Here $\beta_{AFT,0}, \boldsymbol{\beta}_{AFT}$ and $\beta_0, \boldsymbol{\beta}$ have the following relationship:

$$\beta_0 = -r\beta_{AFT,0}, \boldsymbol{\beta} = -r\boldsymbol{\beta}_{AFT} \tag{42}$$

Since t (of Eq.(40)) follows a Weibull distribution, $\log t$ follows extreme value distribution with the typical location-scale form. With some basic algebraic manipulation, its log-likelihood function can be shown take the form:

$$l\left(r, \beta_{AFT,0}, \boldsymbol{\beta}_{AFT}^{(k)}\right) = \log r \sum_{i}^{n_0} \delta_i + \sum_{i}^{n_0} (\delta_i z_i - e^{z_i}) \tag{43}$$

with $z_i = r(\log t_i - \beta_{AFT,0} - \boldsymbol{\beta}_{AFT}^{(k)'} \boldsymbol{x}_i)$. $r, \beta_{AFT,0}, \boldsymbol{\beta}_{AFT}^{(k)}$ can be obtained by maximizing the likelihood of Eq.(43), then, β_0 and $\boldsymbol{\beta}^{(k)}$ can be derived from Eq.(42), and also become known variables. It can be shown that Eq.(43) and Eq.(40) are identical with a constant difference. Therefore, in the later sections, we only mention Eq.(40) for brevity.

2. *Assign Prior*: When the new design/process/application changes have been implemented into suppliers' subsystems, assign $\boldsymbol{\beta}^{(u)}$ with a Gaussian prior $\pi\left(\boldsymbol{\beta}^{(u)}\right) = N_q(\boldsymbol{\mu}^{(0)}, \Sigma^{(0)})$ from upstream supplier chain information where $\boldsymbol{\mu}^{(0)}$ is $q \times 1$ mean vector, $\Sigma^{(0)}$ is $q \times q$ variance matrix. Start monitoring warranty claims from units associated with these new design/process/application changes.
3. *MLE*: During the first monitoring period, suppose there are n_1 units with both existing design/process/application and new design/process/application for which there exists a partial or full warranty claim history. Since r, β_0 and $\boldsymbol{\beta}^{(k)}$ are known variables, they can be combined to form constant terms $\mathbb{C}1$ and $\mathbb{C}2$. After some algebra, the log-likelihood function of $\boldsymbol{\beta}^{(u)}$ for n_1 units is:

$$l\left(n_1\ data|\boldsymbol{\beta}^{(u)}\right)$$
$$= \mathbb{C}1 + \sum_{i}^{n_1} \delta_i \boldsymbol{\beta}^{(u)\prime} \boldsymbol{x}_i - \mathbb{C}2 \sum_{i}^{n_1} t_i^r \exp\left(\boldsymbol{\beta}^{(u)\prime} \boldsymbol{x}_i\right) \tag{44}$$

4. *Derive Posterior*: During the first monitoring period, the posterior distribution of $\boldsymbol{\beta}^{(u)}$ can be evaluated as:

$$\pi\left(\boldsymbol{\beta}^{(u)}|n_1\ data\right) = \frac{e^{l\left(n_1\ data|\boldsymbol{\beta}^{(u)}\right)}\pi\left(\boldsymbol{\beta}^{(u)}\right)}{\int e^{l\left(n_1\ data|\boldsymbol{\beta}^{(u)}\right)}\pi(\boldsymbol{\beta}^{(u)})d\boldsymbol{\beta}^{(u)}} \tag{45}$$

As explained in Section 3.3, when $\boldsymbol{\beta}^{(u)}$ has two or more elements, this posterior is intractable due to nonlinearity of $\boldsymbol{\beta}^{(u)}$ in the likelihood function. But since the likelihood function is fully parametric, we can use MCMC to obtain the approximate marginal distributions of $\pi\left(\boldsymbol{\beta}^{(u)}|n_1\ data\right)$.

The main idea behind MCMC is to approximate the posterior by sampling. For any random variable Y, if we can independently sample the posterior through Monte Carlo, by law of large numbers, the mean of Y and its function g can be estimated by:

$$E_\pi[g(Y|data)] \approx \frac{1}{N}\sum_{i=1}^{N} g\left(Y^{(i)}\right) \tag{46}$$

Unfortunately, independent sampling from the posterior such as $\pi\left(\boldsymbol{\beta}^{(u)}|n_1\ data\right)$ in our case is difficult. This is why Markov chains are beneficial: if we can generate a Markov chain by independently sampling from a known distribution, and if the Markov chain converges to our target posterior after enough iterations $\mathbb{t}$, then the samples generated in iteration $\mathbb{t}$ can be used to estimate any function of the posterior random variables. There are many MCMC algorithms available, some of the popular methods being the Metropolis-Hastings, Gibbs sampling etc. See (Gelfand and Smith, 1990; Smith and Gelfand, 1992; Gelfand and Mallick, 1995) for more information on MCMC methods. Lot of commercial software provide modules for implementing these algorithms. After applying MCMC, we can obtain the marginal distributions for each element of $\boldsymbol{\beta}^{(u)}$ in terms of mean and standard deviation:

$$\pi_j\left(\beta_j^{(u)}|n_1\ data\right)\sim\pi_j\left(\mu_j^{(1)},\sigma_j^{(1)}\right); j = 1,2,\ldots,q \tag{47}$$

5. *Update* r, β_0 and $\boldsymbol{\beta}^{(k)}$: update r, β_0 and $\boldsymbol{\beta}^{(k)}$ per step 1 based on new incoming warranty claims from existing design/process/application during the first monitoring period. When the warranty claims for existing design/process/application are fully mature, r, β_0 and $\boldsymbol{\beta}^{(k)}$ will change little; when the warranty claims for existing design/process/application are not fully mature especially in high time in service, this updating can improve the accuracy of the model by iterative calibration per monitoring period.
6. *Use Old Posterior as New Prior*: During the second monitoring period, suppose there are n_2 untis with both existing design/process/application and new design/process/application for which there exists a partial or full warranty claim history, replace the prior of $\boldsymbol{\beta}^{(u)}$ with its posterior $\pi_j\left(\mu_j^{(1)},\sigma_j^{(1)}\right)$ from the first monitoring period; estimate posterior of $\boldsymbol{\beta}^{(u)}$ based on n_2 by MCMC $\pi_j\left(\mu_j^{(2)},\sigma_j^{(2)}\right)$.
7. *Repeat*: Repeat steps 2-6 for each monitoring period.

3.4 Constructing Prior for $\boldsymbol{\beta}^{(u)}$

The performance of all Bayesian methods rely on the quality of the priors. Accurate construction of the prior $\pi\left(\boldsymbol{\beta}^{(u)}\right)$ plays a key role for early warranty detection; a strong but correct prior can lead us toward the right direction earlier with more power. Constructing a prior is essentially the effort to abstract upstream supply chain information in a probability form with a few hyper-parameters.

Priors can come from two approaches: objective approach by exploring empirical data and subjective approach by subject matter expert (SME) judgment. Unlike some situations such as early development phase, objective data is scarce, and subjective approach is the only source. When it comes down to warranty issue detection, it is already in the late development phase and lab tests and production trial results can be available. So for warranty detection, our recommended strategy is to use expert judgment as a complement of empirical data: first get the initial uncertainty quantification of $\beta^{(u)}$ by exploring empirical data from lab tests and production trials, and fine tune the uncertainty using SME judgment.

Since not all SMEs are trained and think in terms of probability, $\beta^{(u)}$ prior has to be transformed into SME's natural language. A nice feature of the hazard rate model is the relatively easy interpretation of $\beta^{(u)}$ in terms of hazard rate, which is a familiar concept in reliability engineering and well understood by SMEs. As the covariates $\boldsymbol{x}^{(u)}$ are binary indicators ($\boldsymbol{x}^{(u)} = \mathbf{0}$ represents existing design/process/application and $\boldsymbol{x}^{(u)} = \mathbf{1}$ represents the major design/process/application change), the impact of such changes can be easily interpreted by relative hazard rate increase or decrease and readily quantified through regression coefficient vector $\boldsymbol{\beta}^{(u)}$. Suppose one such change $x_j^{(u)} = 1$ has the corresponding regression coefficient of $\beta_j^{(u)}$, from Eq.(16), the relative hazard rate change, fixing the other covariates, is:

$$\frac{h(t|x_j^{(u)} = 1)}{h(t|x_j^{(u)} = 0)} = \frac{h_0(t)e^{\beta_j^{(u)}\times 1}}{h_0(t)e^{\beta_j^{(u)}\times 0}} = e^{\beta_j^{(u)}} > 0 \tag{48}$$

For a major design or process change to improve warranty performance, we may anticipate relative hazard rate reduction with $e^{\beta_j^{(u)}} < 1$, where $(1 - e^{\beta_j^{(u)}})$ quantifies the potential effectiveness of such a change. For changes that involve adding features to an existing subsystem or subjecting an existing subsystem to a more complex application, we may anticipate the relative hazard rate to increase $(e^{\beta_j^{(u)}} > 1)$; $(e^{\beta_j^{(u)}} - 1)$ quantifies the potentially increased risk from such a change.

As $e^{\beta_j^{(u)}}$ is a bit easier for SMEs to understand than $\beta_j^{(u)}$, we recommend starting with the quantification of the uncertainties of $e^{\beta_j^{(u)}}$. Assuming that one has reached agreement with SMEs on its exact meaning; it is common to ask SMEs in term of its interval or ranges instead of mean and variance (Booker and McNamara, 2004; Cook, 2010), as the mis-concept of mean and variance from common people's intuitive understanding may mislead the SMEs toward symmetrical distribution, therefore creating bias. The next step is to refine the uncertainties by further asking SMEs how much confidence they have on the interval or ranges. The answers to the above questions can be summarized as two quantiles in term of the confidence level. For any of the common distributions with two parameters chosen for $e^{\beta_j^{(u)}}$, these parameters can be fully defined (Cook, 2010). The prior distribution for $\beta_j^{(u)}$ can then be obtained by transforming from $e^{\beta_j^{(u)}}$ to $\beta_j^{(u)}$.

In reality, the elicitation process can be complicated due to many human factors such as cognition, psychology involved during the process of interpreting diverse data, knowledge, and experience. Fortunately there are many techniques (Cooke, 1994) and tools (Booker et al., 2003) available. For example, if SMEs can only quantify the effectiveness of a design/process improvement in terms of scales 1 to 10, or in a natural language of excellent, good, average, poor, unacceptable, the method to link fuzzy set theory and probability can be used (Booker and Singpurwalla, 2003; Yadav et al., 2003).

The choice of prior distribution form for $\beta_j^{(u)}$ is more commonly based on mathematical convenience instead of physical justification. The common choice for $\beta_j^{(u)}$ is Gaussian due to its well known properties, so $e^{\beta_j^{(u)}}$ is a lognormal distribution. Sometimes, a gamma distribution is chosen for $e^{\beta_j^{(u)}}$ for its conjugate properties. (Clemen et al., 1996) show that a Gaussian prior offers the same level of performance as other complex models. The choice of prior distribution does not pose a serious limitation as there is rarely sufficient prior information to differentiate the difference.

To minimize the bias generated from expert judgments, we select multiple SMEs from diverse backgrounds. SMEs can be design/process engineers who create the design/process changes, quality and production engineers who work on the production lines, test engineers who test the design change, etc. From elicitation process, one prior probability distribution is generated from each expert. The multiple prior probability distributions should be combined into a single prior distribution. There are many combination procedures available such as: weighted arithmetic average, weighted geometric average, and Bayesian and Bayesian hierarchical models. An extensive review for these procedures (Clemen and Winkler, 1999) found that there is no clear-cut performance differences between simple and complex models. But a simple model has advantages of ease of use and interpretation. Therefore, we recommend and employ the simplest equal weighted average combination method for our case. For each $\beta_j^{(u)}$, this method can be mathematically presented as:

$$\pi\left(\beta_j^{(u)}\right) = \frac{1}{N}\sum_{i=1}^{N}\pi_i\left(\beta_j^{(u)}\right) \tag{49}$$

where N is the number of SMEs, $\pi_i\left(\beta_j^{(u)}\right)$ represents expert i th prior probability distribution for $\beta_j^{(u)}$, and $\pi\left(\beta_j^{(u)}\right)$ represents the combined probability prior distribution. For Gaussian priors, the combined prior also remains Gaussian.

3.5 Case Study

We illustrate and test our statistical framework to encapsulate upstream supply chain information via the Bayesian approach using a case study from a Tier-1 automotive seating supplier. For reasons of confidentiality, we are unable to reveal all the details. The company had relatively high warranty claims from an existing seating product supplied to a global automotive OEM. In depth analysis revealed that some of the structural components were failing due to fatigue failure. Hence, it was decided to both upgrade the material and dimensions of a critical component within the seat frame. We retrospectively investigate this case to illustrate how the proposed Bayesian hazard rate model can help assess the impact of the design change and improve the lead-time for the assessment.

3.5.1 Data

The retrospective warranty data covers 65 production months (MOP) with over 1.8 million vehicles; the design change was implemented at the beginning for the 49th production month ($MOP49$). The warranty claims for these vehicles were monitored for 87 months starting from 1st production month ($MOP1$), so all vehicles have at least 22 months of age.

The scope of the investigation here is limited to claims impacted by the design change (fatigue failure of a particular structural member of the seat frame; all other claim codes are filtered out from the dataset). Unlike the case study from Chapter 2, for this particular seating system, we also do not have any plant level quality or manufacturing process information to build the baseline model with covariates. Hence, $h_0(t)$ will completely account for the baseline hazard rate under the older seat-frame design. Also, as stated earlier, only one major design change was involved (change the material and a particular dimension of the structural component) to address the fatigue failure mode. So the covariate vector $\boldsymbol{x}^{(u)}$ and its associate regression coefficient vector $\boldsymbol{\beta}^{(u)}$ degrade to scalar $x^{(u)}$ and $\beta^{(u)}$. The Bayesian hazard rate model of Eqs.(17) and (18) is reduced to:

$$h\left(t\middle|\beta^{(u)}\right) = h_0(t)e^{\beta^{(u)}x^{(u)}} \tag{50}$$

$$\beta^{(u)} \sim \pi(\beta^{(u)}) \tag{51}$$

As discussed in Section 3.3, if the prior of $e^{\beta^{(u)}}$ is assumed to be a gamma probability distribution, the posterior of $e^{\beta^{(u)}}$ is also a gamma probability distribution. Hence, we can employ the closed-form early warranty detection method outlined early on in Section 3.3. The availability of a closed-form posterior facilitates us to also evaluate the precisions of the two numerical Bayesian approximation methods discussed in Sections 3.3.1 and 3.3.2.

The claims filtering was based on the facts and knowledge gathered from upstream supply chain information: 1) based on root-cause analysis of parts returned from the field, the design related fatigue failures only occurred in seats of SeatType1 rather than SeatType2. SeatType1 shares the same structural design with SeatType2, but with an added “feature”; the fatigue failure occurred within a “component” of this added feature. Hence, we only work with data from vehicles equipped with seats of SeatType1. 2) Further investigation has also revealed that any seats that have undergone this fatigue failure needed replacement during service. Hence, we excluded any warranty claims that did not involve seat replacement.

After filtering, there are total 4,531 warranty claims from 629,832 vehicles, among which 4,097 warranty claims are from 438,877 vehicles that were produced before the design change was put in place (labeled "Old Design" and assigned $x^{(u)} = 0$); 434 warranty claims were observed from 190,955 vehicles produced after the design change was implemented (labeled "New Design" and assigned $x^{(u)} = 1$).

3.5.2 Retrospective Estimation of Impact of New Design on Hazard Rate

The non-parametric Fleming-Harrington (FH) fit for the retrospective data (Figure 19) reveals that the New Design significantly improved the warranty performance by reducing the hazard rate. Also, both the Old and New Designs yield cumulative hazard plots with increasing hazard rate, typical for fatigue related failures. Note however that the difference in cumulative hazard rates between the Old and New Designs is not distinguishable until the seats in vehicles reach some 200 days in age.

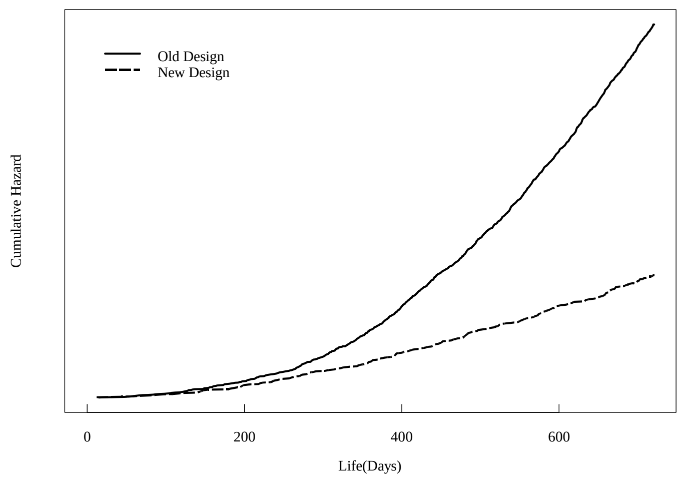

Figure 19: Cumulative hazard comparison between Old and New Design from retrospective data.

Plotting the above cumulative hazard rates on a log-log scale plot (Figure 20) shows that the cumulative hazard plot for the New Design is roughly parallel to the plot for the Old Design, supporting the proportional hazard assumption. In addition, both the cumulative hazard plots are roughly straight, supporting the accelerated failure time assumption and imply a Weibull model is an appropriate model for this case.

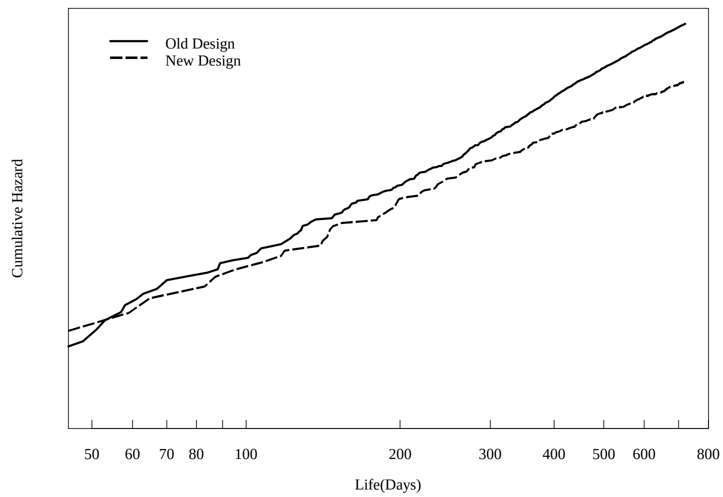

Figure 20: Cumulative hazard log-log scale plot comparing Old and New Design from retrospective data.

To estimate the impact of New Design in terms of $\beta^{(u)}$ or $e^{\beta^{(u)}}$, we fitted the retrospective data with a semi-parametric Cox PH model as well as a parametric Weibull PH model. The results (Figure 21) show that both the Cox and Weibull models fit the retrospective data reasonably well.

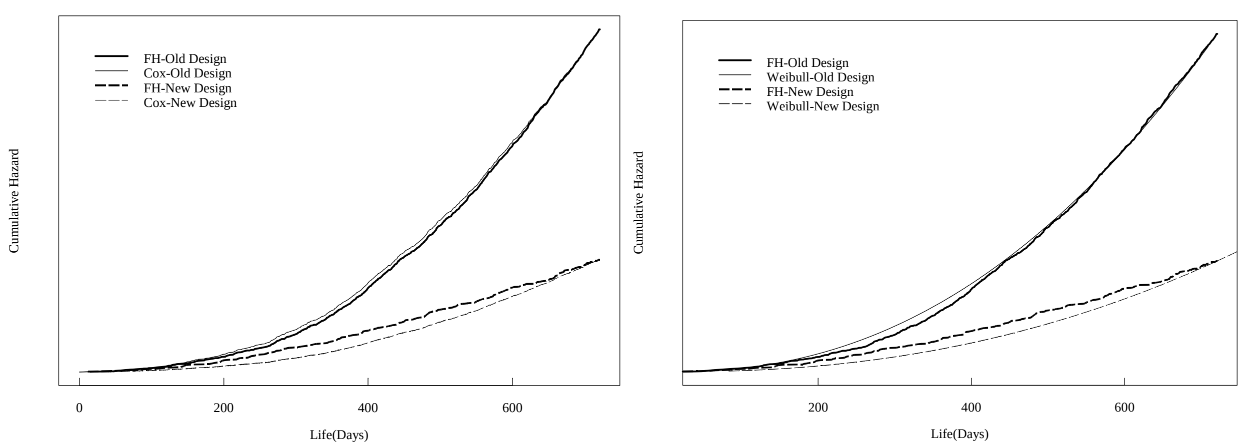

Figure 21: Cumulative hazard comparison among FH, Cox PH and Weibull PH models.

The estimations for the impact of the New Design in terms of $\beta^{(u)}$ or $e^{\beta^{(u)}}$ are almost identical between Cox and Weibull PH models to the third decimal place, yielding a $\beta^{(u)}$

mean of -1.115 with 95% confidence interval of $(-0.985, -1.245)$; $e^{\beta^{(u)}}$ mean of 0.328 with 95% confidence interval of $(0.288, 0.373)$. The model suggests that the New Design did reduce hazard rate proportionally by $1 - e^{\beta^{(u)}} = 1 - 0.328 = 67.2\%$, which represents the effectiveness of the New Design.

3.5.3 Estimation of Baseline Hazard

To validate the Bayesian hazard rate models proposed in Section 3.3, we artificially reset the present time as end of $MOP48$, the time when the New Design was implemented in production. We defined $MOP48$ as the observation period 0 ($OBS0$). At $OBS0$, no warranty claim data was available yet for the New Design seat, but there are 48 months of warranty claims history available for vehicles with the Old Design. We label this data as the training dataset and use it to establish the baseline cumulative hazard function $H_0(t)$ non-parametrically as FH fit for the model from Section 3.3.1 and parametrically as Weibull model for the model from Section 3.3.2. The shape parameter of the Weibull is estimated from Section 3.5.3 to be $r = 2.688$ and the scale parameter is estimated as $\beta_0 = -23.080$, which corresponds to a characteristic life of $5{,}356$ days. We assume that the historical warranty claims for Old Design are enough to define the baseline cumulative hazard function. To verify our assumption, we compare the above two training dataset fits to the full retrospective dataset, and they are very close to each other except beyond 630 days. Therefore, the baseline hazard remains reasonably stable over time.

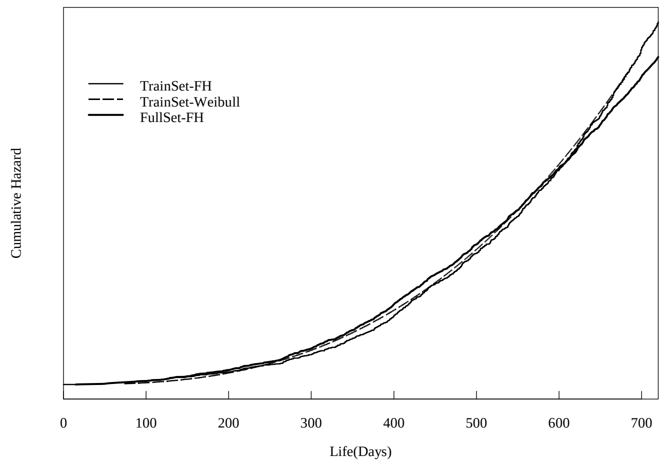

Figure 22: Cumulative hazard comparison between FH, Weibull fits for training dataset and FH fit for full retrospective dataset.

3.5.4 Eliciting Prior Distribution of $\beta^{(u)}$ For the New Design

To obtain the prior distribution of New Design $\beta^{(u)}$, we start with the objective testing data. Before the New Design was released to production, three prototype from the New Design as well as three Old Design production parts were each tested in the lab under the same accelerated test cycle plan per design specification from the OEM. The test results showed that all three Old Design samples failed and only one New Design sample failed with the other two samples not failing at the end of the testing cycle (censored). Given the limited testing data, we assume that the lab test follows the typical Weibull AFT model with the same shape parameter as originally estimated in Section 3.5.3 to be $r = 2.688$.

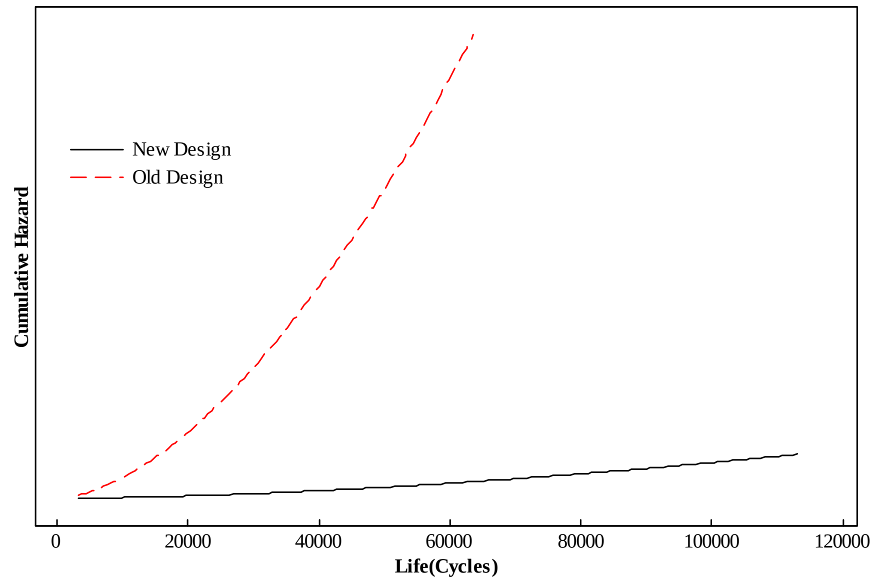

Figure 23: Cumulative hazard comparison between Old and New Design under the lab test.

The cumulative hazard plots for the Old and the New Design are shown in Figure 23 with $\beta^{(u)} \approx -2.343$ or $e^{\beta^{(u)}} \approx 0.096$, meaning that the New Design has reduced the proportional hazard by 90%. Realize that the test sample size is very small, test employed prototype parts of New Design and does not fully represent the production version of New Design, and the lab test may not fully capture all the real-world customer usage patterns of the seat, this value is a reference value for further elicitation from SMEs. We also compare historical cumulative hazards between SeatType1 and SeatType2, as SeatType1 is same as SeatType2 except for an added feature; the hazard rate for SeatType1 cannot be lower than that of SeatType2. Applying Cox PH model using SeatType as a covariate, it is found that $\beta^{(u)} > -3.56$ or $e^{\beta^{(u)}} > 0.03$, which indicates that New Design reduced the proportional hazard by less than 97%.

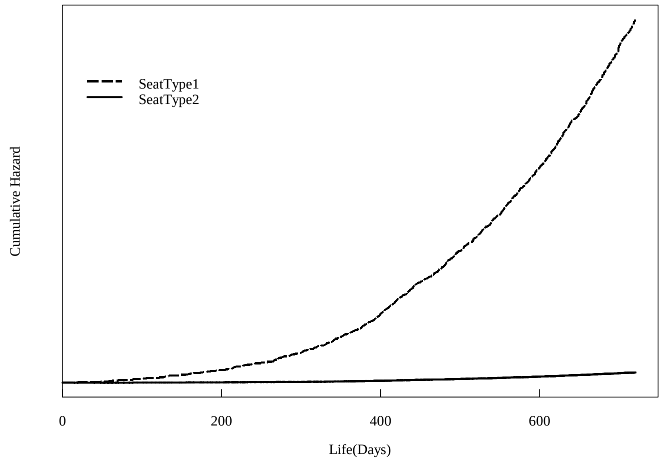

Figure 24: Cumulative hazard comparison between SeatType1 and SeatType2 under historical warranty claims.

Then, we consulted SMEs independently with the above objective information for their reference. To minimize the bias, we consulted four SMEs each from one of four areas: design, process, quality and testing by asking each the effectiveness of New Design in term of percentage of hazard rate reduction $(1 - e^{\beta^{(u)}})$ with 95% confidence level. The results are shown in column 2 of Table 1.

	Effectiveness $(1 - e^{\beta^{(u)}})$ 95% Interval	$e^{\beta^{(u)}}$ 95% Interval	$\beta^{(u)}$ (μ, σ)
Design Expert	(70%, 95%)	(0.05, 0.30)	(-2.100, 0.457)
Process Expert	(45%, 75%)	(0.25, 0.55)	(-0.992, 0.201)
Quality Expert	(40%, 70%)	(0.30, 0.60)	(-0.857, 0.177)
Testing Expert	(50%, 90%)	(0.10, 0.50)	(-1.498, 0.411)
Combined	(64.4%, 71.6%)	(0.184, 0.356)	(-1.362, 0.168)

Table 1: Elicitation of SME judgments, knowledge and opinions regarding the impact of the design change

Design Expert is more optimistic about the effectiveness of the New Design. The expert believes the upgrading material to be of higher strength and can greatly increase the seat structure resistance to fatigue; while Process and Quality Experts are more pessimistic as they are more concerned about the increased production variation due to new manufacturing processes induced by the material upgrading. Process and Quality Experts also observed that certain new processes can degrade the material strength and increase reject rate in the plant. All these are not reflected in prototype samples in the lab test. Testing Expert is more concerned about the small sample size.

To combine the SME judgments, knowledge and opinions, we first assume that $\pi(\beta^{(u)})$ is Gaussian distributed, therefore, $\pi(e^{\beta^{(u)}})$ is a log-normal distribution. Based on the 95% interval of $e^{\beta^{(u)}}$ in column 3 of Table 1, we can derive the parameters of $\pi(\beta^{(u)})$, i.e., the mean μ and standard deviation σ. Then, we apply the simplest equal weighted average combination method shown in Eq.(52) to derive the parameters for the prior:

$$\pi\left(\beta^{(u)}\right) = \frac{1}{4}\sum_{i=1}^{4}\pi_i\left(\beta^{(u)}\right) = \frac{1}{4}\sum_{i=1}^{4}N(\mu_i,\sigma_i^2) \sim N(-1.362, 0.168^2) \tag{52}$$

It follows that $\pi(e^{\beta^{(u)}})$ is log-normal with the same parameters, in which case, the combined 95% confidence interval for $e^{\beta^{(u)}}$ can be derived as (0.184, 0.356).

To obtain the conjugate closed-form posterior, we need to evaluate $\pi(e^{\beta^{(u)}})$ in the gamma distribution form. This can be done by fitting gamma to 95% confidence interval (0.184, 0.356), which yields a shape parameter $k = 35.764$ and scale parameter $\lambda = 135.960$.

To inspect the distribution-wise difference between log-normal and gamma distributions that fit the same 95% confidence interval, their probability density functions are overlaid in Figure 25. It appears that they are very close to each other.

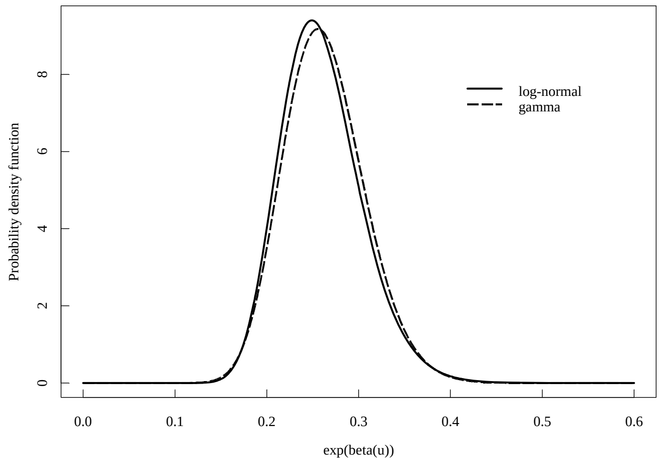

Figure 25: Probability density function comparison between estimated log-normal and gamma distributions given the same 95% confidence interval limits.

3.5.5 Estimating Posterior Distribution of $\beta^{(u)}$ or $e^{\beta^{(u)}}$ *for New Design*

Once the prior $\pi(e^{\beta^{(u)}})$ is defined, the posterior $\pi\left(\beta^{(u)}\right)$ can be computed according to procedures explained in Section 3.3. For brevity and illustration purposes, the observation period is set as every 3 months starting from $OBS0$, so $OBS1$ is 3 months after $OBS0$, $OBS2$ is 6 months after $OBS0$, and so on. The posterior mean and 95% confidence interval for $e^{\beta^{(u)}}$ in each observation period are listed on columns 2~4 of Table 2. and Table 3. In order to compare results between the Bayesian approach and the frequentist approach, we apply Cox and Weibull models purely on incoming warranty claims without considering the prior for each observation period and obtain mean and 95% confidence intervals for $e^{\beta^{(u)}}$. These results are listed in columns 5~6 of

Table 2 and Table 3. The frequentist approach is basically equal to a Bayesian approach with non-information prior.

Observation Period	$e^{\beta^{(u)}}$ Posterior Mean from Bayesian Approach			$e^{\beta^{(u)}}$ Mean from Frequentist Approach	
	Conjugate Gamma	Large Sample Approximation	MCMC Approximation	Cox	Weibull
$Prior$	0.263 Gamma	0.260 log-Gaussian	0.260 log-Gaussian	NA	NA
$OBS1$	0.269	0.271	0.271	2.198	6.004
$OBS2$	0.288	0.297	0.285	0.928	1.589
$OBS3$	0.329	0.349	0.354	0.787	1.009
$OBS4$	0.430	0.463	0.481	0.845	0.980
$OBS5$	0.440	0.435	0.487	0.606	0.705
$OBS6$	0.439	0.425	0.447	0.521	0.589
$OBS7$	0.404	0.398	0.406	0.450	0.498
$OBS8$	0.343	0.345	0.349	0.369	0.399

Table 2: Posterior mean of $e^{\beta^{(u)}}$ for each observation period from the elicited prior.

Observation Period	$e^{\beta^{(u)}}$ posterior 95% Confidence Interval from Bayesian Approach			$e^{\beta^{(u)}}$ 95% Confidence Interval from Frequentist Approach	
	Conjugate Gamma	Large Sample Approximation	MCMC Approximation	Cox	Weibull
$Prior$	(0.184, 0.356) Gamma	(0.184, 0.356) log-Gaussian	(0.184, 0.356) log-Gaussian	NA	NA
$OBS1$	(0.189, 0.362)	(0.193, 0.369)	(0.196, 0.365)	(0.172, 9.735)	(0.518, 25.773)
$OBS2$	(0.206, 0.382)	(0.215, 0.399)	(0.209, 0.378)	(0.333, 2.078)	(0.598, 3.458)
$OBS3$	(0.245, 0.426)	(0.261, 0.457)	(0.268, 0.443)	(0.445, 1.292)	(0.587, 1.623)
$OBS4$	(0.339, 0.531)	(0.364, 0.580)	(0.390, 0.572)	(0.598, 1.160)	(0.710, 1.320)
$OBS5$	(0.358, 0.530)	(0.352, 0.530)	(0.418, 0.607)	(0.463, 0.780)	(0.546, 0.896)
$OBS6$	(0.369, 0.515)	(0.355, 0.505)	(0.376, 0.527)	(0.421, 0.637)	(0.480, 0.715)
$OBS7$	(0.347, 0.466)	(0.340, 0.462)	(0.356, 0.473)	(0.377, 0.534)	(0.418, 0.588)
$OBS8$	(0.298, 0.391)	(0.299, 0.396)	(0.300, 0.398)	(0.314, 0.430)	(0.340, 0.465)

Table 3: Posterior 95% confidence interval of $e^{\beta^{(u)}}$ for each observation period from the elicited prior.

Without prior from upstream supply chain information, both Cox and Weibull models will declare the New Design to be ineffective for the first four observation periods (12 months), as their 95% confidence intervals of $e^{\beta^{(u)}}$ include one. Starting from the fifth observation period ($OBS5$), as more claims are observed, Cox and Weibull models begin to sense the effectiveness of New Design with 95% confidence intervals of $e^{\beta^{(u)}}$ excluding one. In the subsequent periods, effectiveness of the New Design increases more and more with narrower 95% confidence intervals due to more positive evidences from the warranty claims and finally converge towards the true limits. Both Cox and Weibull models perform quite similar except for the first observation period due to very few claims being observed. If we were to judge the effectiveness of the New Design solely based on the warranty claims for the first four observation periods, we would have generated a false alarm.

On the other hand, by extending hazard rate model to include prior from upstream supply chain information, the effectiveness of the New Design is statistically significant for all observation periods as none of 95% confidence interval of $e^{\beta^{(u)}}$ include one. The effectiveness of the New Design decreases somewhat at the beginning due to limited number of warranty claims to concur with the prior, but starting from the fifth observation period, the effectiveness of the New Design becomes more apparent due to more positive evidence from the warranty claims and finally converges towards the true effectiveness. Our model effectively avoids false alarm on the effectiveness of New Design in reducing warranty hazard rate.

As our hazard rate model from the Bayesian approach involves approximating using the asymptotical normal distribution on large sample size in Section 3.3.1 and sampling in Section 3.3.2, to evaluate their performance, their posterior means and the 95% confidence interval of $e^{\beta^{(u)}}$ are compared with the "exact" posteriors from the conjugate gamma prior; the results are reasonably close. For large sample size Bayesian approach, the posterior means of $e^{\beta^{(u)}}$ are very close to the “exact” means with a maximum 8% difference; the posterior 95% confidence intervals of $e^{\beta^{(u)}}$ are a little wider (2~13%) than the “exact” ones. This is partially due to approximation and partially due to different prior distributions.

The estimation of prior plays a key role in our hazard rate model from a Bayesian approach. A better prior with mean closer to true mean and with a tighter confidence band will make our model perform better. To illustrate this point, when we use $e^{\beta^{(u)}}$ estimated from the retrospective full dataset as a prior, the posterior means and 95% confidence interval of $e^{\beta^{(u)}}$ show the same pattern of behavior as before, but are very stable with much less fluctuation per observation period. For large sample size Bayesian approach, the posterior means of $e^{\beta^{(u)}}$ is at most 2% different from the “exact”, the posterior 95% confidence intervals of $e^{\beta^{(u)}}$ are at most 4% wider than the “exact” ones.

Observation Period	$e^{\beta^{(u)}}$ Posterior Mean from Bayesian Approach			$e^{\beta^{(u)}}$ Mean from Frequentist Approach	
	Conjugate Gamma	Large Sample Approximation	MCMC Approximation	Cox	Weibull
$Prior$	0.329 Gamma	0.329 log-Gaussian	0.329 log-Gaussian	NA	NA
$OBS1$	0.330	0.330	0.330	2.198	6.004
$OBS2$	0.334	0.335	0.334	0.928	1.589
$OBS3$	0.342	0.344	0.339	0.787	1.009
$OBS4$	0.365	0.372	0.366	0.845	0.980
$OBS5$	0.373	0.371	0.374	0.606	0.705
$OBS6$	0.379	0.373	0.371	0.521	0.589
$OBS7$	0.371	0.367	0.373	0.450	0.498
$OBS8$	0.344	0.344	0.342	0.369	0.399

Table 4: Posterior mean of $e^{\beta^{(u)}}$ for each observation period from "exact" prior.

Observation Period	$e^{\beta^{(u)}}$ Posterior 95% Confidence Interval from Bayesian Approach			$e^{\beta^{(u)}}$ 95% Confidence Interval from Frequentist Approach	
	Conjugate Gamma	Large Sample Approximation	MCMC Approximation	Cox	Weibull
$Prior$	(0.288, 0.373) Gamma	(0.288, 0.373) log-Gaussian	(0.288, 0.373) log-Gaussian	NA	NA
$OBS1$	(0.289, 0.374)	(0.290, 0.375)	(0.291, 0.374)	(0.172, 9.735)	(0.518, 25.773)
$OBS2$	(0.292, 0.377)	(0.294, 0.380)	(0.299, 0.377)	(0.333, 2.078)	(0.598, 3.458)
$OBS3$	(0.300, 0.386)	(0.303, 0.390)	(0.294, 0.387)	(0.445, 1.292)	(0.587, 1.623)
$OBS4$	(0.323, 0.410)	(0.329, 0.419)	(0.328, 0.423)	(0.598, 1.160)	(0.710, 1.320)
$OBS5$	(0.331, 0.416)	(0.329, 0.416)	(0.339, 0.411)	(0.463, 0.780)	(0.546, 0.896)
$OBS6$	(0.339, 0.420)	(0.334, 0.416)	(0.333, 0.421)	(0.421, 0.637)	(0.480, 0.715)
$OBS7$	(0.334, 0.410)	(0.330, 0.407)	(0.341, 0.406)	(0.377, 0.534)	(0.418, 0.588)
$OBS8$	(0.311, 0.378)	(0.311, 0.380)	(0.312, 0.378)	(0.314, 0.430)	(0.340, 0.465)

Table 5: Posterior 95% confidence interval of $e^{\beta^{(u)}}$ for each observation period from "exact" prior.

3.5.6 Hypothesis Testing of $\beta^{(u)}$ or $e^{\beta^{(u)}}$

Once we obtain the posterior $\beta^{(u)}$ or $e^{\beta^{(u)}}$, we can conduct formal hypothesis testing according to early warranty detection scheme established in Section 2.5. As $\beta^{(u)}$ or $e^{\beta^{(u)}}$ is not observable, the warranty detection scheme first transfers $\beta^{(u)}$ or $e^{\beta^{(u)}}$ to probability by $P(t) = 1 - e^{-H_0(t)e^{\beta^{(u)}}}$, then stratify $P(t)$ per period in service by $p(j) = P(j) - P(j-1)$ where j represents each period in service and $P(0) = 0$. $p(j)$ represents the probability that a vehicle employing a New Design seat may have a warranty claim associated with failure during the jth period. In our case study, since the monitoring period is every three months, j represents every three months in service. Therefore, $p(1)$ represents the probability of failure within the first three months in service, $p(2)$

represents the probability between three and six months in service, and so on. We set our total monitoring periods as eight ($M = 8$), covering a total of twenty-four months in service.

As illustrated in Chapter 2, the warranty detection scheme determines upper and lower warranty claim limits for each period in service through the Binomial distribution. The observed warranty claims will be compared with the upper and lower limits for hypothesis testing.

As each stratified $p(j)$ is assumed to be independent, the warranty detection scheme can conduct multiple hypothesis tests independently, for each period in service. We set overall false alarm probability at $\alpha = 0.1\%$ consistent with (Wu and Meeker, 2002), then the overall false alarm probability is allocated to each period in service α_j based on Eq.(13). Since in our case study detection of New Design effectiveness is critical, and we believe that the effectiveness will demonstrate itself more in later periods in service due to increasing hazard rate exhibited in Figure 19, larger values will be assigned to α_j from later periods in service.

The hypothesis test results are summarized in Figure 26 to Figure 33. The actual claim rates are masked to protect proprietary and confidential information, however all figures are kept at the same scale for relative comparison. The figures show that the effectiveness of the New Design is predicted well up to twenty-four months in service.

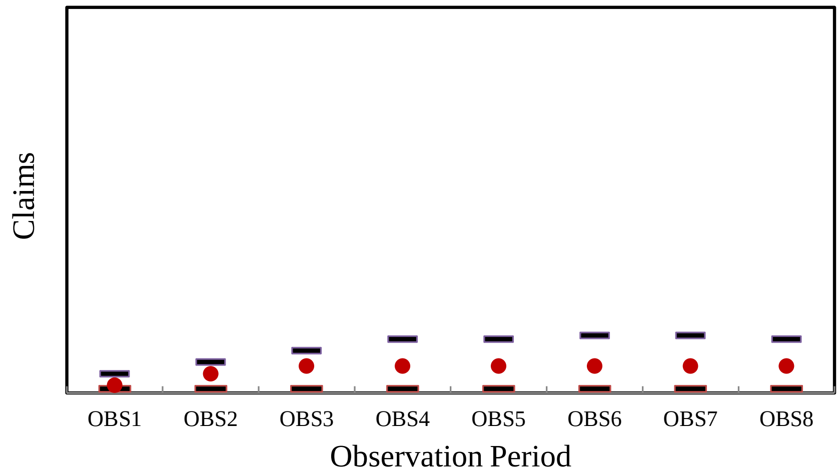

Figure 26: Hypothesis test on New Design results for warranty claims from vehicles between one to three months in service.

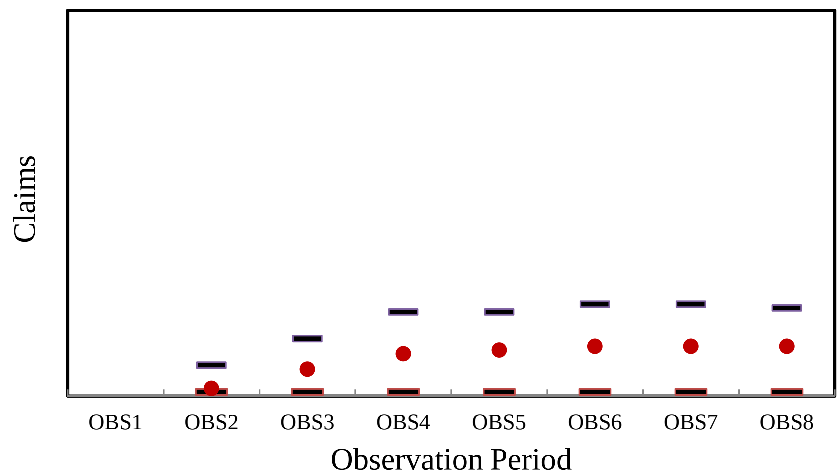

Figure 27: Hypothesis test on New Design results for warranty claims from vehicles between four to six months in service.

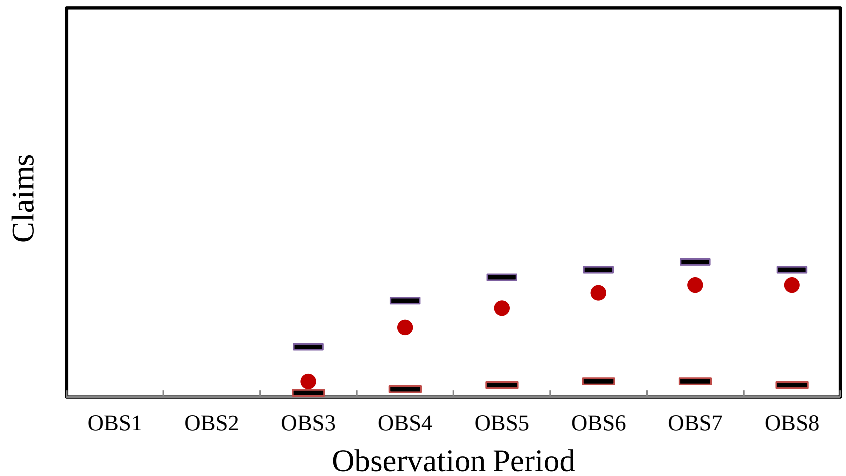

Figure 28: Hypothesis test on New Design results for warranty claims from vehicles between seven to nine months in service.

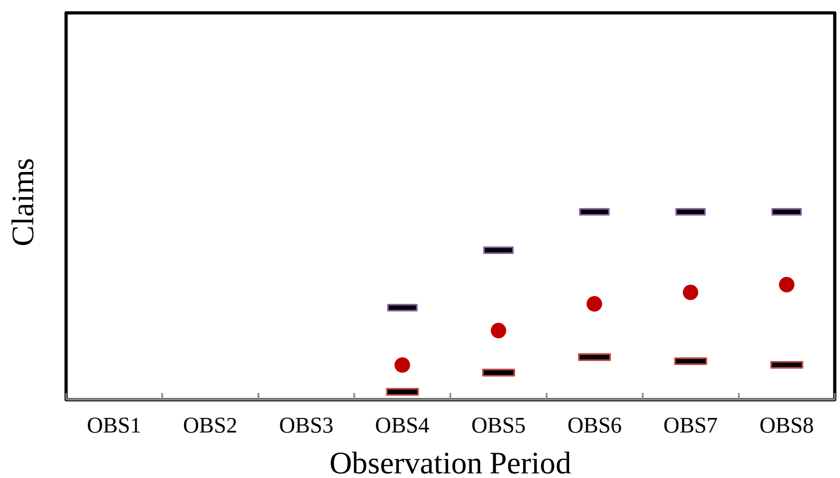

Figure 29: Hypothesis test on New Design results for warranty claims from vehicles between ten to twelve months in service.

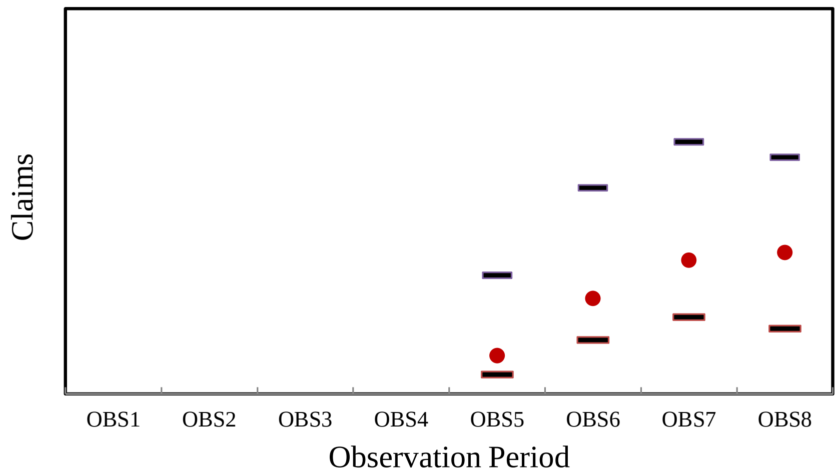

Figure 30: Hypothesis test on New Design results for warranty claims from vehicles between thirteen to fifteen months in service.

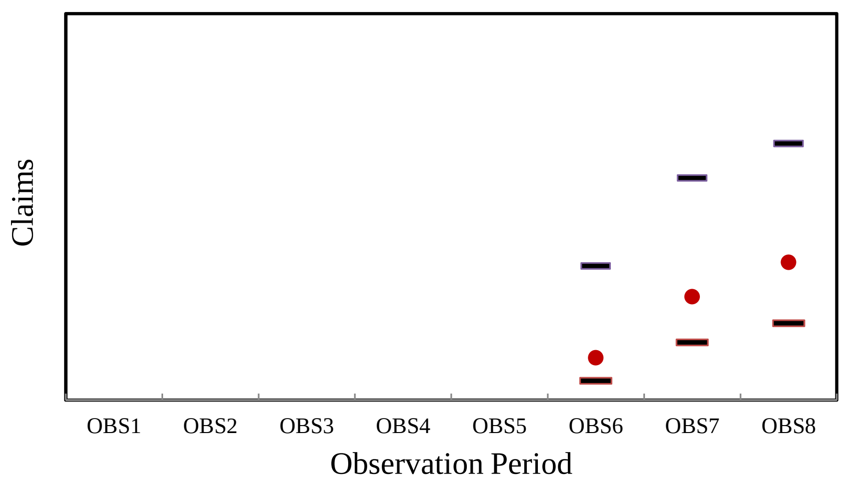

Figure 31: Hypothesis test on New Design results for warranty claims from vehicles between sixteen to eighteen months in service.

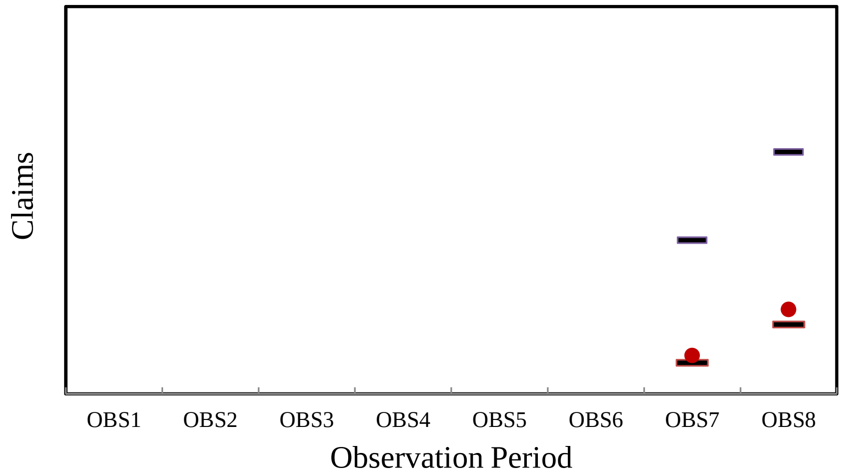

Figure 32: Hypothesis test on New Design results for warranty claims from vehicles between nineteen to twenty-one months in service.

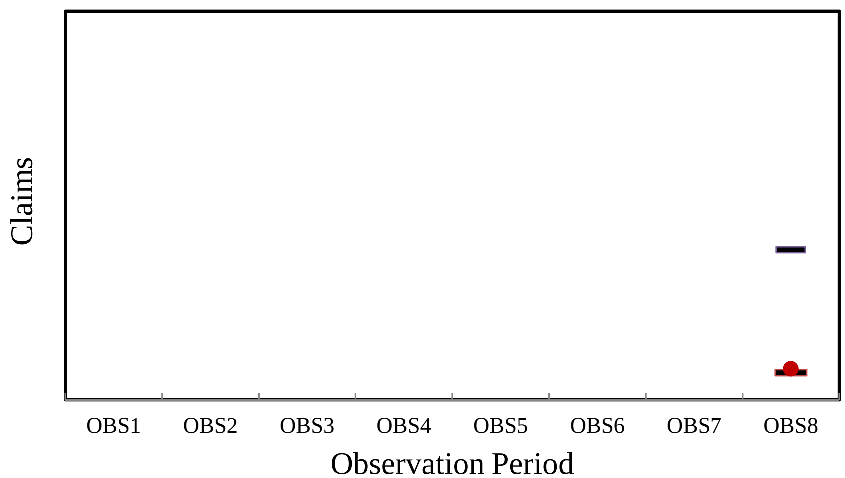

Figure 33: Hypothesis test on New Design results for warranty claims from vehicles between twenty-two to twenty-four months in service.

3.5.7 Performance Review: Large Sample Size vs. MCMC Bayesian Analysis

Even though the output results from the two Bayesian analysis are very close (Table 2 to Table 5), large sample size method definitely outperforms MCMC method on computation efficiency. Except PH model estimation from SPlus, large sample size Bayesian analysis does not require specialized software to obtain the posterior, so the posterior is obtained instantaneously, but MCMC Bayesian analysis requires both SPlus for PH model estimation and a specialized software (e.g. WinBUGS) for posterior estimation. For the large dataset in our case study, the run-times for convergent posteriors can be very long: it easily requires more than 10,000 iterations for 4 CPU hours under AMD quad-core 2.8GHz processor with 16G ram. Also the convergence cannot be guaranteed and sensitive to the choice of initial value.

3.6 Conclusion

In this chapter, we have extended our earlier hazard rate models by incorporating upstream supply chain information as priors through the Bayesian approach. This allows us to evaluate the impacts from brand new design/process changes on warranty performance even though there is no associated historical warranty claims available. By properly eliciting priors via objective data and SME knowledge, judgments and opinions available from suppliers, such impacts can be detected earlier with more power. Through proper priors, our model can avoid false alarms effectively during early warranty detection.

Also, by utilizing historical warranty claims associated with upstream supply chain information, our model relieves us from the heavy computational burden by avoiding estimating the posterior baseline hazard function. This will make our model more practical and computationally efficient. The case study shows that our models perform rather well.

CHAPTER 4 SUMMARY AND CONCLUSIONS

4.1 Summary and Research Contribution

The automotive industry spends \$10-\$13 billion per year on warranty claim (Arnum, 2011b) which consumes roughly 1- 5.2% of OEMs' product revenue and roughly 0.5-1% of suppliers' product revenue (Arnum, 2011a). Early detection of reliability problem can help OEMs and suppliers to take corrective actions as quick as possible to minimize warranty cost and reputation damage due to poor reliability.

Reducing warranty costs and improving product reliability is the joint objective and responsibility of OEMs and suppliers. As we know, a vehicle consists of many modularized subsystems and thousands of components which are supplied through suppliers at different tier levels. Before a vehicle is produced, these subsystems, components have to go through design, testing and manufacturing process on suppliers' sites. Therefore reliability problems don't just start from vehicles reaching customer's hands, but can start far early at suppliers' sites and are heavily influenced by operations at all tiers of suppliers as shown in Figure 1.There is a wealth of upstream supply chain information that exists long before vehicles are built but not being exploited. If this prior upstream information can be utilized in a statistical framework to correlate to warranty claims, the warranty issues and the effectiveness of corrective actions to address these issues may be predicted and detected earlier with more power

This research provides effective methods for suppliers to link their upstream supply chain information to warranty claims. The proposed models adopts hazard rate concept which is a well understood concept in the reliability engineering community, organizes the abundant but diversified upstream supply chain information into explanatory covariates and associates them with each vehicle and assuming their impact to hazard rate as constant multiples of each other.

When the covariates have direct historical warranty claims, the proposed model uses frequentist approach to establish their impacts as regression coefficients of the covariates directly from historical warranty claims. When the current covariates' values are available and known to suppliers, and before the vehicles associated with these covariates are serviced in the field, the established regression coefficients can be used to predict warranty claims rates from the field. Such predictions can be tested sequentially through hypothesis tests during each monitoring period in term of number of claims through the Binomial model.

When the covariates do not have direct historical warranty claims, the proposed model recommends a Bayesian approach to establish their impacts as regression coefficients of the covariates through priors elicited from SMEs, judgments and opinions. The priors are updated sequentially as posteriors every monitoring period as warranty claims become available. The posteriors can be used to predict warranty claim rates from the field. Again, these claim rates can be tested sequentially through hypothesis tests during every monitoring period through the Binomial model. To avoid heavy computation burden to estimate the posterior, the proposed model further utilizes any existing upstream supply chain information to establish the baseline hazard function, so that it is known and fixed during the Bayesian modeling.

The proposed model is practical to suppliers as the concept is more engineering oriented, moreover it is easy to apply as most statistical software have standard modules to implement this model without writing advanced codes.

The proposed model can help to address the following sorts of industry based problems especially from a supplier’s perspective:

- If we know status quo in suppliers' product, can we know the future warranty performance?
- How to verify what we claim to know?
- How to detect and detect earlier if what we claim to know is wrong?

4.2 Limitations and Recommendations for Future Research

As the proposed models are from a supplier’s point of view, focuses on linking warranty claim rates to design and inherent reliability, to which the upstream supply chain information are available and can be extracted and on which a supplier has a control, it assumes sale reliability due to transportation and storage, field reliability due to operating environment, usage mode/intensity and customer behavior are homogeneous over production periods. This reduces the need for collecting information regarding these factors. However these factors can be important factors impacting warranty claims; for example, the percentage of warranty claims due to user behavior can be as large as 10%+ (Wu, 2011). If the levels of heterogeneity for these factors are high over production periods, the models can be extended to account for those heterogeneities. If these factors can be transformed to explanatory covariates, for example, vehicles can be segmented as qualitative covariates by sold geographic region such as Northeast, Midwest, South and West in US to account for heterogeneity of operating environment; by sold country to account for heterogeneity of customer behavior; by sold type such as retail and fleet to account for heterogeneity of usage mode/intensity, by transportation route to account for heterogeneity of transportation/storage, they can be easily incorporated into the proposed models as additional covariates which can further improve the detection power of the proposed models. Frailty models are also candidates to model hazard rates in the presence of over-dispersion or group-specific random effects (Glidden and Vittinghoff, 2004). The latter are distinguished from the former by the term “shared” frailty models. Unfortunately, such data is not currently available to suppliers from OEM warranty databases.

The proposed model also assumes proportional hazards with respect to all covariates, as it provides a simple and easy way to estimate the effects of covariates. When the proportionality assumption of the hazard rate model does not hold, the time periods can be divided into several sub-time periods such that proportionality holds within each sub-time period, then, one can fit separate hazard rate models for each sub-time period.

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ABSTRACT

HAZARD RATE MODELS FOR EARLY WARRANTY ISSUE DETECTION USING UPSTREAM SUPPLY CHAIN INFORMATION

by

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This research presents a statistical methodology to construct an early automotive warranty issue detection model based on upstream supply chain information. This is contrary to extant methods that are mostly reactive and only rely on data available from the OEMs (original equipment manufacturers). For any upstream supply chain information with direct history from warranty claims, the research proposes hazard rate models to link upstream supply chain information as explanatory covariates for early detection of warranty issues. For any upstream supply chain information without direct warranty claims history, we introduce Bayesian hazard rate models to account for uncertainties of the explanatory covariates. In doing so, it improves both the accuracy of warranty issue detection as well as the lead time for detection. The proposed methodology is illustrated and validated using real-world data from a leading global Tier-one automotive supplier.

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