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A Decision Modeling For Phasor Measurement Unit Location Selection In Smart Grid Systems

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**A DECISION MODELING FOR PHASOR MEASUREMENT UNIT LOCATION
SELECTION IN SMART GRID SYSTEMS**

by

SEUNG YUP LEE

THESIS

Submitted to the Graduate School

of Wayne State University,

Detroit, Michigan

in partial fulfillment of the requirements

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Approved by:

Advisor

Date

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2014

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DEDICATION

To my wife,

my son,

my parents,

my parents-in-law,

my friends,

and my church family members of Central Alliance Church of Detroit.

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I received a lot of support from many people during the course of this research. First of all, I would like to express my special appreciation and thanks to my advisors, Dr. Kyoung-Yun Kim and Dr. Evrim Dalkiran, for their advice, inspiration, and support in this work. You have been tremendous mentors for me. Your knowledge and logical way of thinking have been of great value for me and provided a good basis for this research. None of this research would have been possible without your supportive effort. My gratitude also goes to the entire faculty and staff of the Department of Industrial and Systems Engineering at Wayne State University, for the support and kindness they have shown me over the last two years. I especially thank Dr. Richard Darin Ellis for serving on my thesis committee, and for providing invaluable advice.

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CHAPTER 1

INTRODUCTION

The smart grid has been proposed as an alternative modern power grid system, which is an enhancement of the 20th century power grid [1, 2]. With various characteristics of the smart grid, the different perspectives of smart grid functions have been highlighted for extending the boundaries of the smart grid [3-5]. However, the realization of those functionalities causes complicated questions. Especially, as a prerequisite for the initiation of the smart grid, the allocation of smart grid components needs to be properly determined with the consideration of the actual functions of components in the system.

As an effort to lead healthy modern power systems, the utility industry across the world has tried to overcome the inherent insensibility of existing electricity system [6], which is resulted from unidirectional and non-time synchronized characteristics of traditional grid. With the need of robust modern electricity grid, the smart grid is expected to achieve the reliability and security in power grid system. Phasor Measurement Units (PMUs) are essential power system devices that provide time synchronized information about dynamic performance of power network [7]. The information derived from measurements are same-time sampled in voltage and current waveforms from Global Positioning System Satellites (GPS), which enable PMU data from different utilities to be time-synchronized and combined to create a comprehensive view of the broader electrical system [8]. Synchronization of sampling of phasor is achieved using a common timing signal available locally at the substation. Figure 1-1 shows a diagram which illustrates architecture of a general PMU measurement system. [9]

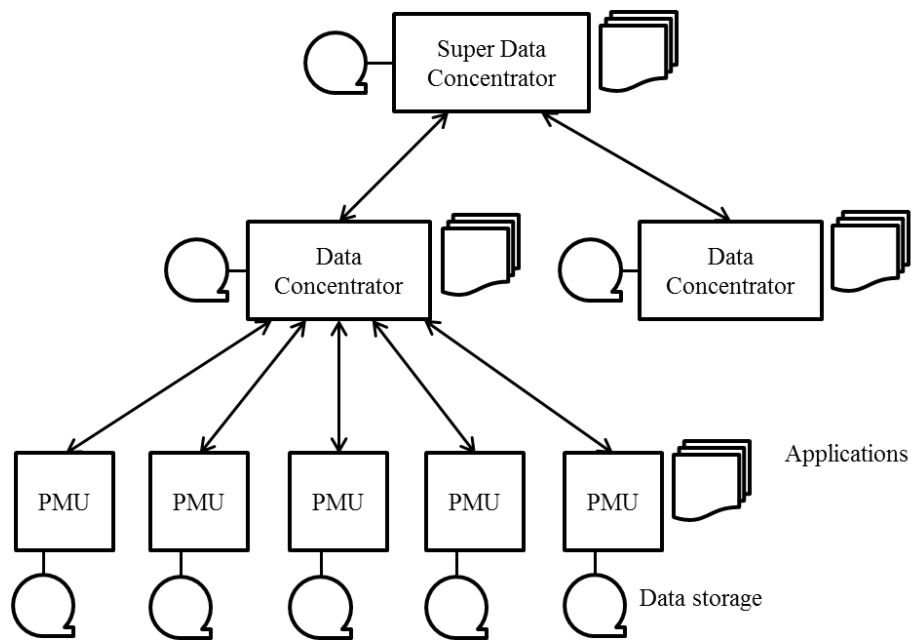


Figure 1-1. Hierarchy of the phasor measurement systems

In Figure 1-1 the PMUs are placed in power grid system substations, and the real-time data gathered at each PMU is used for analyzing the state of voltages and currents of buses and feeders monitored. Although actions based on the measurements are usually made by applications in higher level concentrators, some local application tasks are done by local PMUs, in which case necessary data is available locally for such tasks [9]. When these PMUs are installed on a power grid system, the both phasor of the bus voltage and of the line currents can be measured. Therefore, the voltage phasor of incident buses can be calculated using Kirchhoff's law and it is unnecessary to install PMUs on every single bus in system [10]. It is an essential characteristic of PMU allocation task since it is impossible to install the PMUs on all of the buses in power grid systems. So one of the imperative questions in the installation of PMUs is about the optimal number of PMUs and their location for covering a given power grid network according to decision makers' objectives. The main objective of optimal PMU placement

problem is to ensure the full measurement over a power grid system while minimizing the number of PMUs required [11]. Especially, when the measurement of a substation is possible, it is said that a substation, i.e., a bus, is observable by PMU.

The major purpose of this thesis is the development of models and modeling processes for supporting decisions on optimal PMU placement in smart grid context. The investigated topics are as follows:

- 1) The development of optimal PMU placement models based on the observability rules, which are already discovered. This task aims to successfully fulfill two fundamental objectives of PMU allocation task, i.e., minimization of the number of PMUs and maximization of level of redundancy. The result of this study is an effective model that ensures optimal placement of PMUs.
- 2) The development of modeling processes that considers the circumstantial factors around the phasor measurement systems. It extends the scope of PMU allocation to overall system requirement analysis, which considers not only system itself but also circumstances in which the functionality of system will be exhibited.

This thesis is composed of three main chapters including this introduction. The chapters are presented in such a way that each of two main objectives presented above is contained in each chapter. Then chapter 4 summarizes all the research work of this thesis and recommends direction for future study.

CHAPTER 2

PHASOR MEASUREMENT UNIT LOCATION SELECTION

2.1 Introduction

The primary purpose of PMU placement problem is to discover the minimum number of PMUs and their location, providing perfect observability over all buses, i.e. no bus unobservable, in a given electricity system. Through this process, the minimum cost of installation of PMUs is found. In addition to pursuing the efficient placement of PMUs, the goal of maximizing the observability in the given number of buses also needs to be taken into account, since a decision maker wants to maximize the effectiveness of installation of PMUs after finding minimum number of them. For representing the level of observability of a bus, the concept of redundancy is introduced. The redundancy R is defined as below.

R_i = The number of times bus i is observed by PMUs in a given system.

$R_T = \sum_i^{N_{bus}} R_i$ = total number of times all buses in a given system are observed by PMUs.

The objective of PMU placement problem is to determine the minimum number of PMUs to be installed and the set of optimal PMUs' location, which make a problem a combinatorial optimization solving process. Since the PMU placement problem finding minimum set of PMUs is NP-complete with a solution space of 2^N possible sets of combinations for given N -bus power system [12], various approaches have been implemented in order to achieve valuable solutions as close as possible to global optimal solutions of the objective function below.

$$\min_{N_{PMU}} \{ \max R_T(N_{PMU}, S(N_{PMU})) \}$$

This chapter is divided into five sections. The first section reviews the previous approaches and methods which were used based on various assumptions of researchers. Each research work has its own strengths and applications and this thesis has been significantly motivated by each work. The second section deals with the concept of observability in PMU allocation. Here we introduced how the buses in a given power grid system can be observed by PMUs according to three measurement modes which are explained and illustrated. Then the concept of overlap prevention rule in PMU allocation is proposed. This rule significantly contributed to reduction of number of PMUs required. A deterministic formulation of overlap prevention rule and indicator variables is derived in next section. By using an integer programming model described in this section, a decision making on optimal PMU placement can be supported effectively. A set of numeric formulas for IEEE 30 bus system is exemplified. The results and discussion is shown in last section.

2.2 Review of Previous PMU Allocation Strategy

The previously proposed research works are categorized into two, depending on the used methodology for solving the problem. First, heuristic approaches are used to exploit the benefit of meta-heuristic methodologies to overcome the inherent complexity of optimal PMU placement problem. Meta-heuristics is a process seeking a way to efficiently explore the search space so as to find near optimal solution [13] and incorporating its own mechanisms to avoid getting trapped in confined areas of the search space. In optimal PMU placement problem, it includes genetic algorithm, tabu search, simulated annealing, particle swarm optimization, ant colony optimization, differential evolution, and immune algorithm.

A genetic algorithms are search algorithms inspired by natural selection and natural genetics which insist that nature has capability to evolve living beings well adapted to their

environment. Marin et al. [14] used genetic algorithm to solve the optimal PMU placement problem. Through the algorithm, each bus and line in the power network was assigned to gene for forming the chromosome and a new generation was produced by three operator; selection, crossover, mutation, from the old generation. Then, a new generation started again with the fitness evaluation process. A tabu search uses the history of the search, both to escape from local minima and to implement an explorative strategy. The use of a tabu list prevents from returning to recently visited solutions therefor it prevents from endless cycling and forces the search to accept even uphill moves [13]. A tabu search algorithm for minimizing the number of PMUs and maximizing the redundancy is introduced by Peng et al [28]. with the augmented matrix. This research defined the redundancy measurement index as the sum of voltage redundancy and current redundancy. By applying heuristic node selection method at following iteration, the solution search speed and accuracy improved. A Simulated Annealing is an algorithm which has a fundamental idea of allowing moves resulting in worse quality solution than the current solution, i.e. uphill moves, in order to escape from local minima. It starts from an initial configuration and new ones are proposed through local changes, and accepted according to a given probability function. A simulated annealing algorithm was adopted by Nugui et al. [15] to solve the communication facility limited optimal PMU placement problem by implementing a metropolis algorithm. In this research, the definition of configuration, energy function, and penalty of configuration of optimal PMU placement problem is introduced in order to properly use simulated annealing. A particle swarm optimization provides a population-based search procedure in which individuals, called particles, change their position with time. During the movement of particles, each particle changes its own position based on previous position, velocity, private thinking, and cooperation with other particles. Each particle updates its best

solution by comparing its individual solutions, and the global best is replaced with best individual solution, if there is better individual solution than previous global solution. Hajian et al. [16] introduced the discrete binary version of particle swarm optimization in which the search space is discrete so variables can only take on values of 0 and 1. In this algorithm, each particle corresponded to a PMU placement configuration for a power network and the direction of these particles was determined by the set of particles neighboring and its history experience.

In addition to meta-heuristic methodologies, various approaches based on the deterministic techniques have also been proposed. Deterministic approaches make extensive use of integer programming and numerical based methods [17] by exploiting various computational solvers. Xu and Abur [18] considered PMU placement problem with consideration on the conventional power flows and injections as well as phasor measurements measured by PMUs. In this study, the nonlinear constraints were formed based on the network configuration and the knowledge about the locations and types of existing measurements. In [19], Chen and Abur argued that PMUs will provide increased bad data detection and identification capability, which may be useful during contingencies and existence of bad data in low redundancy pockets of the system. They utilized integer programming to solve both systems with conventional measurements and without them. Dua et al. [20] introduced two indices, bus observability index (BOI), which corresponds to the level of redundancy of a bus, and system observability redundancy index (SORI), which corresponds to total level of redundancy of a system, to calculate the observability redundancy over the system. Also a methodology finding optimal multistage scheduling of PMU placement was proposed, which uses the number of incident lines of buses as a parameter. Gou [21] proposed a generalized integer linear programming formulation for optimal PMU placement under different cases of redundancy PMU placement,

full observability and incomplete observability. The author found solutions for depth-of-one unobservability and depth-of-two unobservability cases with zero injection measurements. Sodhi and Srivastava [22] proposed a two level approach for solving optimal PMU placement problem for achieving complete observability of the power system. First, decomposition of the power system network was carried out with integer linear programming approach, where objective function utilizes the eigenvectors of the spanning tree adjacency matrix. Then, locations of PMUs are determined in the sub-networks in order to minimize their cost of installation. In [23], Chakrabarti *et al.* presented an integer quadratic programming approach that minimizes the total number of PMU required and maximizes the measurement redundancy at the power grid system. They considered the outage of a single transmission line or a single PMU. Aminifar *et al.* [24] assumed that the observability of power network and its outage possibility can be analyzed in a probabilistic manner. The authors define a distinct set of probabilistic indices for individual buses and the entire system. Based on this this idea, a mathematical model for the probabilistic observability of the PMU placement at the horizontal year was derived. The mixed-integer programming was used for the optimization and an efficient linearization technique was proposed to convert the nonlinear function representing the probability of observability into a set of linear expression. Mahaei and Tarafdar Hagh [25] took into account that the buses that have injection measurements may be connected to each other, and animated the consideration on suitable unequal constraints for these buses. This work stressed the modeling of zero injection buses to consider the topology conditions of power grid network. Enshaee *et al.* [26] pointed out the drawbacks from the previous research works. The main idea of them is that if a bus is connected to two or more zero-injection buses, there is no need to the corresponding observability variable to appear in all of the inequalities corresponding to those zero injection

buses. To overcome this, authors generated many different kinds of variables and formulas. The optimization problem was introduced in the form of a binary integer programming, and the optimal placement of PMUs was determined in the contingencies of a single PMU loss and a single line outage. Gómez and Ríos [27] proposed an integer linear programming approach for the optimal multistage placement of PMUs, which finds the number of PMUs and its placement in separate stages. It also incorporated the economic constraints at each stage considering the financial budget limitation of a decision maker. In addition, a methodology to identify specific buses to be observed for dynamic stability monitoring was introduced.

2.3 Concept of observability in PMU Allocation

Basically, wide spread installation of PMUs enhances the reliability and security of electricity grid, and PMU placement at all substations guarantees the thorough measurements over all buses in a given energy network. However, the placement of PMU at each bus is both cost-inefficient and structurally unnecessary because of the characteristics of measurement and calculation conducted with phasor information. For instance, when a PMU is placed at a bus, neighboring buses can be observable depending on the network configuration among buses. Hence, it is necessary to introduce some conditions how the electricity performances of buses become observable by PMUs and how the connection between buses affects the feasibility of measurement over buses. Although there have been various approaches to find the optimal location for PMU placement, selection of PMU location is conducted based on the following rules [28-30]. The feasibility of measurement is now referred to as ‘observability’.

- 1) Direct measurement: Installation of a PMU at a given bus makes a bus itself observable.

- 2) Pseudo measurement: Installation of a PMU at a given bus makes the buses incident to that bus observable.
- 3) Zero-injection bus: If all buses are observable but one among a zero-injection bus and its entire incident buses, one unobservable bus can be observable by Kirchhoff's Current Law (KCL) at the zero-injection bus.

where zero-injection bus means a bus where there is no current injection. So it can be regarded as a transshipment bus in the system. Especially direct measurement rule and pseudo measurement rule are straightforward rules as network expressions, and Fig. 2 describes those two rules concisely, which has a bus network consisting of 11 buses.

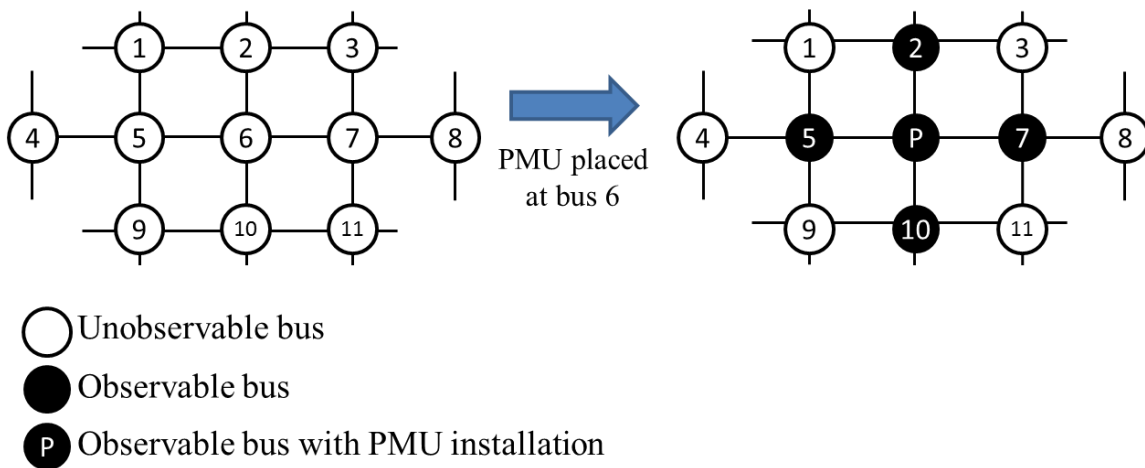


Figure 2-1. Observability decision in PMU allocation

In addition to rule 1 and 2, by utilizing the characteristics of zero-injection bus, the rule 3 is realized so that the number of PMUs in a given system can be reduced. Consider a zero-injection bus network as shown in Fig. 2. In this figure, it is assumed that the bus 1, 2, 3 and 4 are observable, i.e., their voltage phasors are known, but bus 5 is unobservable in terms of observability rule 1 and 2. Since the voltage phasors of bus 1, 2, 3 and 4 are known, current

between them (i.e., $I_{2,1}$, $I_{3,1}$, and $I_{4,1}$) can be known either from direct measurement by PMU or calculation from the equation (2.1).

$$I_{ij} = y_{ij}(V_i - V_j) \quad (2.1)$$

where y_{ij} is the line admittance between bus i and j . In addition, bus 5 is also observable by calculating the bus voltage by applying KCL at the zero-injection bus as follows:

$$\sum_{i=1}^{n_j} I_{ij} = 0, \quad \forall j \in J \quad (2.2)$$

$$V_5 - V_1 = -z_{15} \sum_{i=2}^4 I_{i1} \quad (2.3)$$

where n_j is the total number of branches with currents towards or away from the node j , and z_{ij} is the line impedance between bus i and j . Equation (2.2) is formulated according to the situation of Figure 2-2.

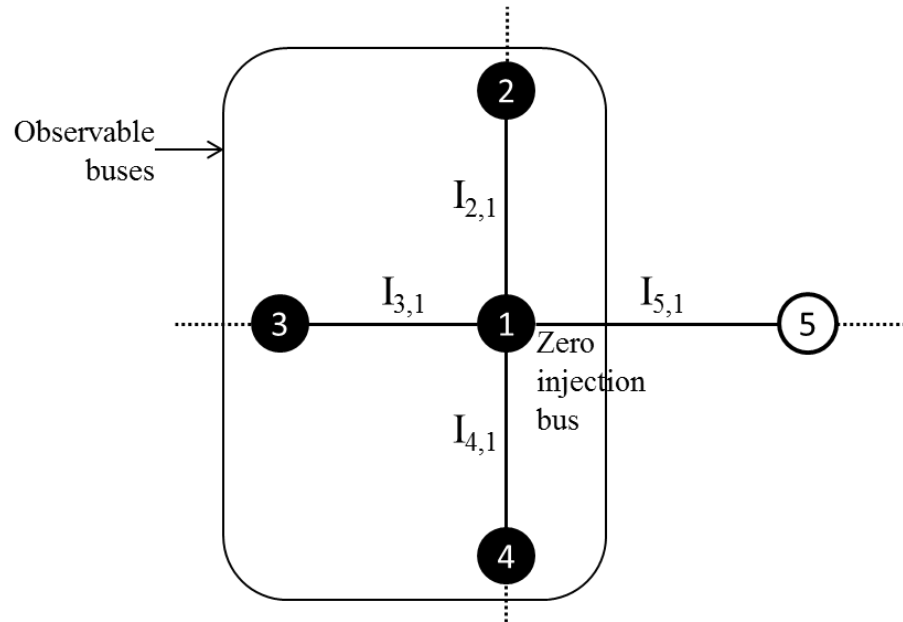


Figure 2-2. Zero-injection bus network modeling

Based on this relationship between zero-injection node and non-zero-injection nodes, a decision maker can have one more opportunity to reduce the minimum number of PMU required to fully observe a given energy network system.

2.4 Concept of overlap prevention rule in PMU allocation

Although the total number of PMU required making a full observability over a given network system is reduced by incorporating the concept of zero-injection node, there is an additional possibility (according to [18]) to decrease the number of PMU needed. This additional reduction comes from an idea that when a bus is connected to multiple zero-injection buses, it doesn't need to consider all connections between a bus and zero-injection buses connected to that bus. As Figure 2-2 shows, a zero-injection network (bus 1, 2, 3, 4, and 5) includes all buses connected with a zero-injection bus, as well as a zero-injection bus itself, and this zero-injection (bus 1) has a capability to accord an observability to a currently unobservable bus (bus 5) among its zero-injection network. It is plausible to think that there is another zero-injection bus, which connects to bus 5. If there is another zero-injection bus connected to bus 5, it means that there is another possibility to make bus 5 observable besides bus 1 centered zero-injection network. Figure 2-3 describes this occasion with an example. In figure 2-3, there are two zero-injection networks, which are formed by zero-injection bus 4 and 6, and Bus 5 is overlapped by two different zero-injection networks. According to the zero-injection bus rule, a decision maker can have two options to make bus 5 observable. Let the observability of a bus be f_i . If a bus i is observable, $f_i=1$, and otherwise, $f_i=0$, formulating the equations based on the zero-injection bus rule are as follow,

$$\begin{aligned}
 f_1 + f_3 + f_4 + f_5 + f_8 &\geq 4, \text{ at zero-injection network 4,} \\
 f_2 + f_5 + f_6 + f_7 + f_9 &\geq 4, \text{ at zero-injection network 6}
 \end{aligned}
 \tag{2.3}$$

For eliminating the redundant application of zero-injection bus rule from both zero-injection network 4 and 6, which may cause inefficient use of the rule, the overlap prevention rule is devised and applied. Table 2-1 indicates how overlap prevention rule works.

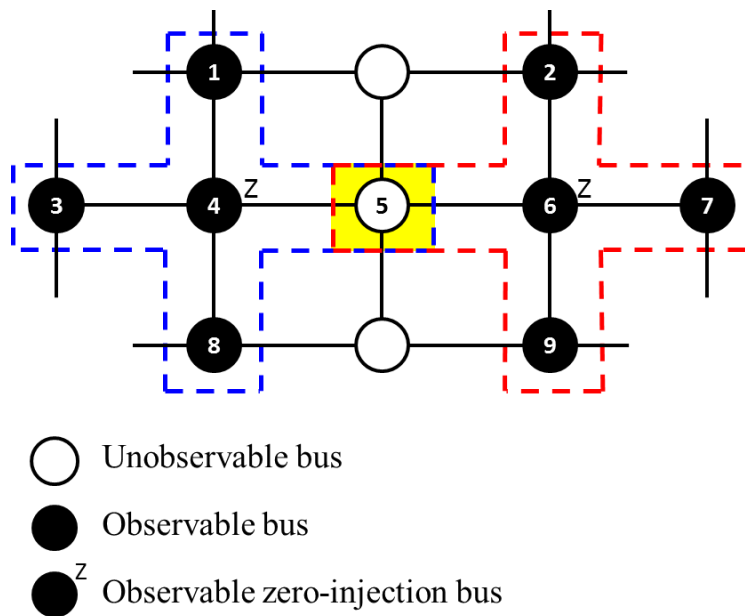


Figure 2-3. Zero-injection network coverage modeling

	1	2	3	4	5	6	7	8	9
Zero-injection network by bus 4					$g_{4,5}$				
Zero-injection network by bus 6					$g_{6,5}$				

Table 2-1. Zero-injection network coverage matrix

Table 2-1 organizes the coverage of zero-injection networks. A new variable g_{ij} is introduced for preventing the overlapped coverage by two different zero-injection networks, as well as guaranteeing the observability rule 3. Now a set of inequalities (2.3) is reformulated to (2.4).

$$\begin{aligned}
f_1 + f_3 + f_4 + f_{4,5} + f_8 &\geq 3 + g_{4,5}, \text{ at zero-injection network 4,} \\
f_2 + f_{6,5} + f_6 + f_7 + f_9 &\geq 3 + g_{6,5}, \text{ at zero-injection network 6,} \\
g_{4,5} + g_{6,5} &\geq 1, \\
g_{4,5} &\geq f_{4,5}, \text{ and} \\
g_{6,5} &\geq f_{6,5}.
\end{aligned} \tag{2.4}$$

Let g_{ij} variables be overlap indicator variables, because those variables indicate which zero-injection network's voltage phasor is used to observe the phasor of bus j . If $g_{4,5}$ is 1, which means that if bus 5 is observed by zero-injection network of zero-injection bus 4, then $g_{6,5}$ doesn't need to be 1, and the first two equations in (4) will be

$$\begin{aligned}
f_1 + f_3 + f_4 + f_5 + f_8 &\geq 4, \text{ at zero-injection network 4, and} \\
f_2 + f_6 + f_7 + f_9 &\geq 3, \text{ at zero-injection network 6,}
\end{aligned}$$

which could reduce the right-hand side value of the second equation from 4 to 3, allowing a decision maker a better feasible region in terms of a minimization of number of PMUs problem. On the other hand, if $g_{6,5}$ is 1, which means that if bus 5 is observed by zero-injection network of zero-injection bus 6, then $g_{4,5}$ doesn't need to be 1, and the first two equations in (4) will be

$$\begin{aligned}
f_1 + f_3 + f_4 + f_8 &\geq 3, \text{ at zero-injection network 4, and} \\
f_2 + f_5 + f_6 + f_7 + f_9 &\geq 4, \text{ at zero-injection network 6.}
\end{aligned}$$

Whichever zero-injection network is to be chosen, that can reduce the right-hand side value of constraints for the other zero-injection networks by 1. This relationship provides ‘lower’ lower bound to this minimization problem, which is not discovered by observation rule 1 and 2. Finally, observation rules used in this thesis are listed below.

- 1) Rule 1: Installation of a PMU in a given bus makes itself and other buses incident to that bus observable. This implies that the voltage phasors of these buses are known.
- 2) Rule 2: If only one bus is unobservable among a zero-injection bus and its entire incident buses, it can be observable by using the Kirchhoff’s current law (KCL) at the zero-injection bus.
- 3) Rule 3: If a bus is connected to two or more zero-injection buses, there is no need to the bus to be observed by all of the connected zero-injection bus.

2.5 A Deterministic Approach Using Overlap Prevention Rule and Indicator Variables

In this section, a deterministic approach is proposed to solve the optimal PMU allocation by applying integer programming. Deterministic approach assumes that there is no randomness involved in the operations of systems. Although actual smart grid, especially situational awareness system including PMU measurement, can have uncertainties on many grounds of operations, starting with considerations on deterministic property of PMU measurement system operation gives the fundamental inspiration to the decision maker. For maintaining determinability in PMU allocation, basic assumptions of modeling are introduced as follows.

First, the network configuration in a given bus system is deterministic. In fact, since the cost of installation and repair of PMU system is expensive, e.g., EPRI estimated the cost for the installation of one unit of PMU as \$125,000 [8], the installation of multiple PMUs in a given system may take a substantial period of time. So it is plausible that during this period of time,

there would be certain changes in the network configuration, i.e., the addition or removal of bus or line. In this deterministic approach, it is assumed that a decision maker is only interested in the optimal allocation of PMUs for a current fixed network configuration. Second, there is no possibility to incorporate another measurement device besides PMUs. Basically, PMU is one of the most prominent alternatives for real-time wide-area situational awareness of given electricity grid system. However, PMU is not the only one option for measurement, and if different kinds of measurement systems would be installed within the period of PMUs installation, a solution, which is acquired at the beginning of planning for PMUs' installation will not be an optimal solution anymore. Based on these two assumptions, the PMU allocation problem is to be solved with deterministic approach.

Remarkable advantages of deterministic optimization are that the convergence to a solution is much faster and straightforward, compared to the stochastic approach, and the results of optimization process are unequivocal [31]. So the mathematical programming expression of deterministic optimization in network problem accords the sense of characteristics of a given network to a decision maker. Also, the constant results at a given bus system is appropriate to present a fundamental understanding on the overall network structure of the system to the decision maker in PMU allocation task.

The objective function of PMU allocation has two objectives, a minimization of total number of PMUs needed for ensuring full observability, and a maximization of total redundancy, which maximize the robustness of monitoring over a given power bus network system.

The first objective and its constraints can be readily formulated, considering the network configuration among the buses in a given system. For an N_{bus} bus system, the PMU allocation vector \mathbf{x} has elements x_i , which can be defined as (2.5), and the primitive first objective can be

formulated as (2.6). As a primary constraint, equation (2.7) is formulated, forcing each bus in the bus system to be observed at least once by PMUs.

$$x_i = \begin{cases} 1, & \text{if a PMU is placed at bus } i, \\ 0, & \text{otherwise} \end{cases} \quad (2.5)$$

$$\text{Min } Z = \sum_{i=1}^{N_{bus}} x_i. \quad (2.6)$$

subject to

$$\mathbf{Ax} \geq \mathbf{b}, \quad \forall i \in I. \quad (2.7)$$

where \mathbf{A} is a matrix, which has elements indicating connectivity between buses as shown at (2.8), and \mathbf{x} is a column vector having element of x_i . \mathbf{b} is a column vector, which has 1 as elements. (2.5) – (2.7) are a set of prototype formulation of optimal PMU placement. Those expressions are now modified.

$$a_{ij} = \begin{cases} 1, & \text{if } i = j, \\ 1, & \text{if bus } i \text{ and } j \text{ are connected,} \\ 0, & \text{otherwise.} \end{cases} \quad (2.8)$$

In addition, in this thesis an observability indicator variable f_i is proposed. f_i is a variable, which indicates whether a bus i is observable or not. As shown at (2.9), if a bus i is observable, f_i is 1, otherwise 0.

$$f_i = \begin{cases} 1, & \text{if bus } i \text{ is observable,} \\ 0, & \text{otherwise} \end{cases} \quad (2.9)$$

For making the relationship between x_i variable and f_i variable, Let us consider the implication that needs to be modeled. Logically, if there is a PMU either at bus i or at incident buses of bus i , the bus i will become observable according to the rule 1 or 2. This implication is described as (2.10).

$$\sum_{j \in C_i} a_{ij} x_j \geq 1 \Rightarrow f_i = 1 \quad (2.10)$$

where C_i is the set of buses which contains bus i itself and buses incident to bus i . The derivation for modeling the constraints based on the implication (2.11) is as follows.

$$\begin{aligned} \sum_{j \in C_i} a_{ij} x_j \geq 1 &\Rightarrow f_i = 1 \\ \rightarrow f_i = 0 &\Rightarrow \sum_{j \in C_i} a_{ij} x_j < 1 \\ \rightarrow f_i = 0 &\Rightarrow \sum_{j \in C_i} a_{ij} x_j \leq 0 \\ \rightarrow \sum_{j \in C_i} a_{ij} x_j - f_i &\leq 0 \end{aligned} \quad (2.11)$$

for making it always true, increase the coefficient of f_i by the upper bound of $\sum_{j \in C_i} a_{ij} x_j$.

$$\begin{aligned} \rightarrow \sum_{j \in C_i} a_{ij} x_j - U f_i &\leq 0 \\ \rightarrow \sum_{j \in C_i} a_{ij} x_j - \left(\sum_{j \in C_i} a_{ij} \right) f_i &\leq 0, \quad \forall i \in I \end{aligned} \quad (2.12)$$

This constraint successfully leads f_i to become 1 if bus i is observable. However, it is possible that f_i will have 1 as a value, even if bus is unobservable, i.e., $\sum_{j \in C_i} a_{ij} x_j = 0$. This dissatisfaction is removed by eliminating this case, by adding left-hand side value. (2.13) is only

required for the buses not in zero-injection network, since the buses, which are in zero-injection network, always satisfy $\sum_{j \in C_i} a_{ij} x_j \geq 1$.

$$\sum_{j \in C_i} a_{ij} x_j - f_i \geq 0, \quad \forall i \in I_{zn} \quad (2.13)$$

This constraint ensures that $f_i = 1$ if and only if $\sum_{j \in C_i} a_{ij} x_j \geq 1, \forall i \in I_{zn}$, where I_{zn} is a set of buses belonged in any zero-injection networks.

In addition to indicator variable for observability, there is another possibility to reformulate the constraints of $\mathbf{Ax} \geq \mathbf{b}$. Since \mathbf{b} is a column vector of which elements are 1, it implies that at least one PMU is necessary either at the bus i or at the buses incident to bus i in order to make bus i observable. But when the concept of zero-injection bus is considered, which is dealt with in section 2.3, the buses within zero-injection network don't need to be observable by direct or pseudo measurement. It means that one bus can be observable, even if there is no PMU around that bus. Consequentially, $\mathbf{Ax} \geq \mathbf{b}$ is necessary only for buses which are in non-zero-injection network. \mathbf{A} , a connectivity matrix, is divided into two parts, \mathbf{A}_{nn} for buses in non-zero-injection network, and \mathbf{A}_{zn} for buses in zero-injection network, and (2.7) is applied only to \mathbf{A}_{nn} .

$$\mathbf{A}_{nn} \mathbf{x}_{nn} \geq \mathbf{b}_{nn} \quad (2.14)$$

where \mathbf{b}_{nn} is a column vector, which only has element of buses not linked to zero-injection buses, and the values of elements are all 1.

On the other hand, regarding buses in zero-injection networks, the total number of observability within a zero-injection network has to be at least greater than equal to the number of buses in the corresponding zero-injection network minus 1, so that the observation rule 2 can

be applied. Unlike with previous constraints, this constraint is formulated for each zero-injection bus as follows.

$$\sum_{j \in C_i} a_{ij} f_j \geq \sum_{j \in C_i} a_{ij} - 1, \quad \forall i \in I_{zb} \quad (2.15)$$

Just as the definition of observation rule 2, this formulation clearly shows that if only one bus is unobservable among a zero-injection bus and its entire incident buses, it can be observable by using the Kirchhoff's current law (KCL) at the zero-injection bus.

Now consider observability rule 3, the overlap prevention rule. This rule is only targeting the buses which are overlapped by more than two different zero-injection networks. As shown in Figure 2-3, the buses, which are overlapped by multiple zero-injection networks, can bring additional opportunity to reduce the total number of PMUs needed in a given power grid system. So when a zero-injection network is considered, if there is no bus overlapped with another zero-injection network, observability rule 3 makes no claim. For modeling overlap prevention rule, another variable $g_{i,j}$ acts as an indicator, which shows whether bus j is observed by a zero-injection network of zero-injection bus i .

$$g_{i,j} = \begin{cases} 1, & \text{if bus } j \text{ is observed by zero-injection network of zero-injection bus } i, \\ 0, & \text{otherwise} \end{cases} \quad (2.16)$$

Since overlap prevention rule is based on the situation that a bus is overlapped by multiple zero-injection networks, g variables always exist as a pair, which has same j and different i s. In order to achieve the additional reduction of number of PMUs to be installed, (2.15) has to be modified as below.

$$\sum_{j \in Z_{NO}^i} a_{ij} f_j + \sum_{j \in Z_o^i} f_{i,j} \geq \sum_{j \in C_i} a_{ij} - 1 + \sum_{j \in Z_o^i} g_{i,j}, \quad \forall i \in I_{zb} \quad (2.17)$$

where Z^i is set of buses in zero-injection network, which is formed by zero-injection bus i . Z_o^i is set of buses that are overlapped by another zero-injection networks. On the other hand, Z_{NO}^i is set of buses that are not overlapped by another zero-injection networks, meaning that bus i is the only one zero-injection bus that makes bus j to be included in zero-injection network. So, $Z_o^i \cup Z_{NO}^i = Z^i$ and $Z_o^i \cap Z_{NO}^i = \emptyset$. In order to consider the buses overlapped by multiple zero-injection networks, a term of $\sum_{j \in Z_o^i} g_{i,j}$ is added. Moreover, observability variables f_j are also divided into two different types of variables, f_j and $f_{i,j}$, so that overlapped buses look obvious in the formulation. Each $f_{i,j}$ variable corresponds to each $g_{i,j}$ variable. If any $f_{i,j}$ variable for j is observable, i.e., $f_{i,j}=1$, f_j corresponding that j will become 1. This relationship can be written as (2.18). Moreover, g variables for one bus overlapped by multiple zero-injection networks don't need to be 1 for all g variables. This is a key concept of overlap prevention rule, and this condition is included in inequality (2.19).

$$\sum_{i \in O_{zn}^j} f_{i,j} - |O_{zn}^j| f_j \leq 0, \quad \forall j \in J_{zn} \quad (2.18)$$

$$\sum_{i \in O_{zn}^j} g_{i,j} \geq 1, \quad \forall j \in J_{zn} \quad (2.19)$$

where O_{zn}^j is a set of zero-injection buses at which bus j is overlapped multiple times. Lastly, if and only if g variable for certain j th bus is 1, f variable, an observability variable, can become 1. However, $g_{i,j}=1$ doesn't necessarily mean $f_j=1$. This relationship is formulated as (2.20)

$$g_{i,j} \geq f_{i,j}, \quad \forall i \in O_{zn}^j, \forall j \in Z_o^i \quad (2.20)$$

As a whole, the mathematical programming used in this thesis is the integer programming, and solves problem by categorizing the buses into two types of buses, non-zero-injection bus and zero-injection bus. The set of formulas of this integer programming designed for solving optimal PMU allocation is summarized as below table.

rules	Constraints
Rule 1 and 2	$\sum_{j \in C_i} a_{ij} x_j - f_i \geq 0, \quad \forall i \in I_{zn}$
	$\sum_{j \in C_i} a_{ij} x_j \geq 1, \quad \forall i \in I_{nzn}$
	$\sum_{j \in C_i} a_{ij} x_j - \left(\sum_{j \in C_i} a_{ij} \right) f_i \leq 0, \quad \forall i \in I$
Rule 3	$\sum_{j \in Z_{NO}^i} a_{ij} f_j + \sum_{j \in Z_O^i} f_{i,j} \geq \sum_{j \in C_i} a_{ij} - 1 + \sum_{j \in Z_O^i} g_{i,j}, \quad \forall i \in I_{zb}$
	$\sum_{i \in O_{zn}^j} f_{i,j} - O_{zn}^j f_j \leq 0, \quad \forall j \in J_{zn}$
	$\sum_{i \in O_{zn}^j} g_{i,j} \geq 1, \quad \forall j \in J_{zn}$
	$g_{i,j} \geq f_{i,j}, \quad \forall i \in O_{zn}^j, \forall j \in Z_O^i$

Table 2-2. Constraints categorization according to observation rules

The solving process applying devised integer programming is presented from this paragraph. After IEEE 30 bus system is exemplified, results for other IEEE power bus system is also to be introduced. Figure 2-4 shows the IEEE 30 bus system. A bus, which doesn't have any AC source and demand (arrow), is regarded as zero-injection bus, while a bus, which has either

AC source or injection, or both of them, is regarded as non-zero-injection bus. Therefore, it is clear that there are six zero-injection buses within IEEE 30 bus system, which are bus 6, 9, 22, 25, 27, and 28.

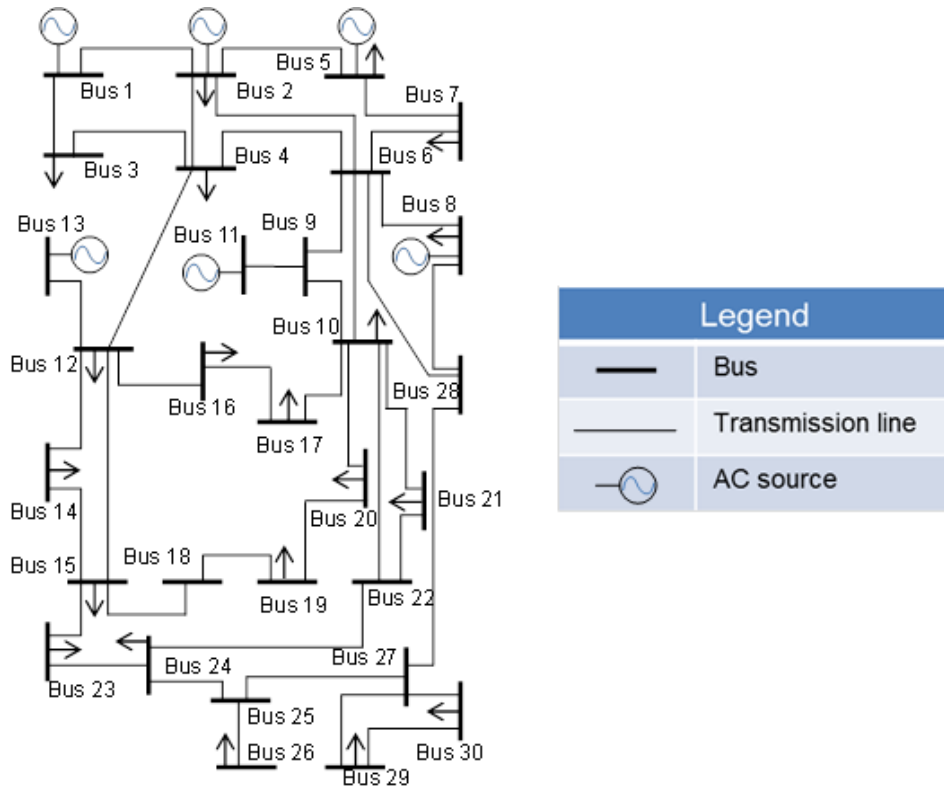


Figure 2-4. IEEE 30 bus system

Also, buses in IEEE 30 bus system can be categorized in accordance with the integer programming used in this thesis as a deterministic approach.

Category	Lists of buses
Zero-injection buses	6, 9, 22, 25, 27, 28
Non-zero-injection buses	1, 2, 3, 4, 5, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 26, 29, 30

Table 2-3. Non-zero-injection buses and zero-injection buses in IEEE 30 bus system

Based on the connections between buses in a given power grid diagram, a_{ij} is defined, and connectivity matrix \mathbf{A} can be created. After this, the matrix can be divided into two parts, which are \mathbf{A}_{nm} for buses not in zero-injection networks and \mathbf{A}_{zn} for buses in zero-injection networks.

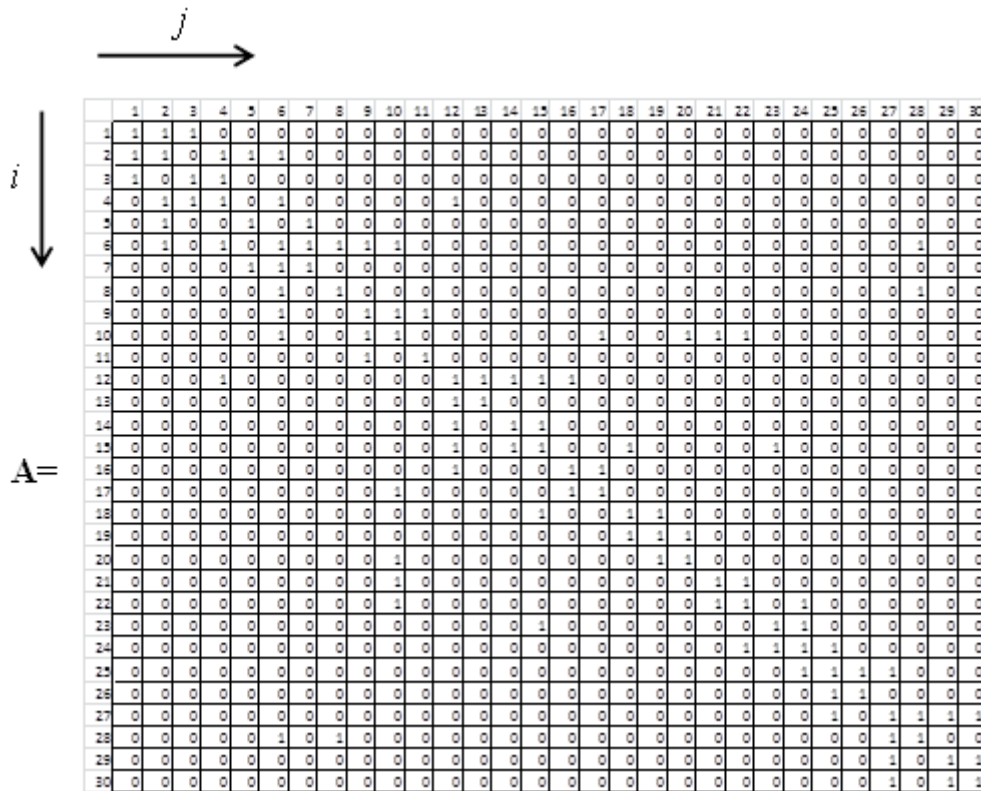


Figure 2-5. Connectivity matrix \mathbf{A} of IEEE 30 bus system

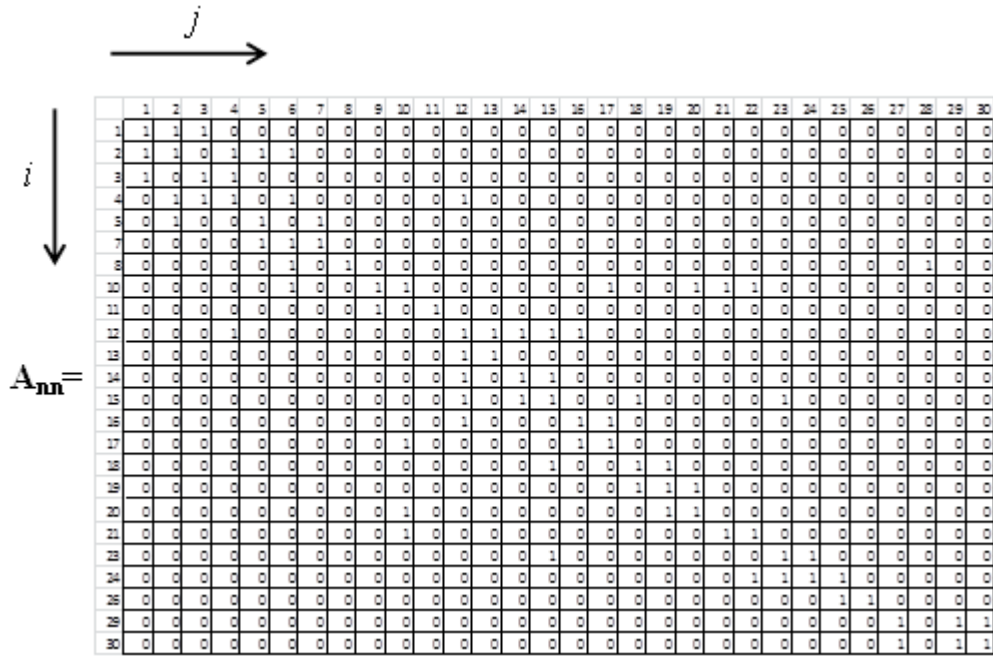


Figure 2-6. Non-zero-injection network connectivity matrix A_{nn} of IEEE 30 bus system

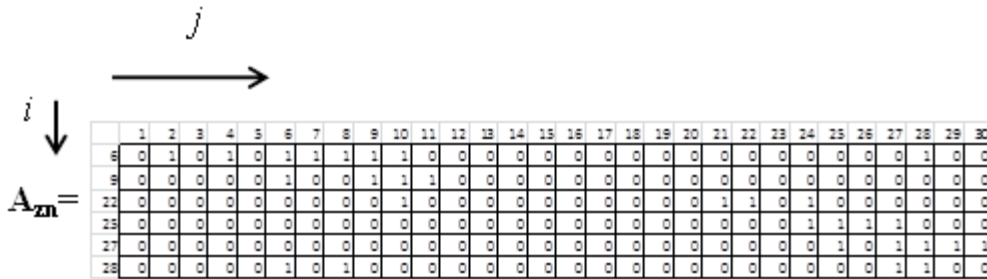


Figure 2-7. Zero-injection network connectivity matrix A_{zn} of IEEE 30 bus system

Category	Lists of buses
Buses in zero-injection networks	2, 4, 6, 7, 8, 9, 10, 11, 21, 22, 24, 25, 26, 27, 28, 29, 30
Buses not in zero-injection networks	1, 3, 5, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23

Table 2-4. Categorization of buses according to the networks buses belong to

Some elements in A_{zn} are required to be more stressed than the others, since before using A_{zn} for realizing observation rule 1, 2, and 3 for solving PMU allocation, buses overlapped by multiple zero-injection networks should be identified. Figure 2-8 explicitly represent which buses are overlapped by zero-injection networks, and integer programming model should reflect them in solving task.

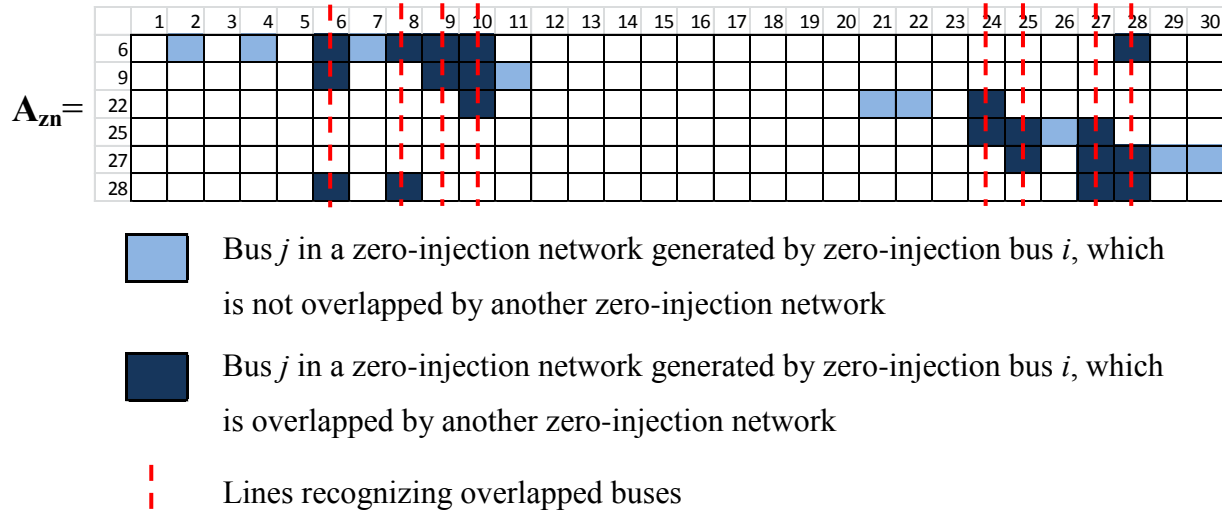


Table 2-5. Colored zero-injection network connectivity matrix A_{zn} of IEEE 30 bus system

As it was stated at the beginning of chapter 2, there are two objectives in this PMU allocation. The first objective is to minimize the number of PMUs placed and the second objective is to maximize the total redundancy over a given bus system. In this thesis, total redundancy is calculated as a sum of two types of redundancy. First type of redundancy comes from observation rule 1, which means that PMUs' direct or pseudo measurement can increase the level of redundancy of buses. The second redundancy comes from zero-injection measurement, and it's more complicated than the first redundancy due to the structural complexity of zero-injection measurement. However it can be simplified when the definition of observation from zero-injection network is modified. Assuming that this integer programming model guarantees

the perfect observability over a given system, it is plausible to redefine the second redundancy by zero-injection network that if redundancy value from observation rule 1 (redundancy 1) is 0 for bus i , redundancy value from observation rule 2 (redundancy 2) is 1. Observation rule 3 is not involved in redundancy calculation. For the redundancy 1, total number of redundancy 1 can be calculated by (2. 21).

$$R_1 = \sum_{i=1}^{N_{bus}} |A_i| x_i, \quad \forall i \in I \quad (2.21)$$

Redundancy 2 is then simply defined as (2.22). This simple expression is possible since (2.5) – (2.20) ensures complete observability over a power grid system.

$$R_2 = |I| - \sum_{i=1}^{N_{bus}} f_i, \quad \forall i \in I \quad (2.22)$$

where $|I|$ is total number of buses in a given system. If we have IEEE-30 bus system, $|I| = 30$.

By applying (2.5) – (2.20) to this network structure of IEEE 30 bus system, a set of integer programming is generated. Table 2-4 shows the constraints corresponding to observation rule 1.

	Bus in zero-injection network	Bus not in zero-injection network	Observability indication
Bus 1		$x_1 + x_2 + x_3 \geq 1$	$x_1 + x_2 + x_3 - 3f_1 \leq 0$
Bus 2	$x_1 + x_2 + x_4 + x_5 + x_6 - f_2 \geq 0$		$x_1 + x_2 + x_4 + x_5 + x_6 - 5f_2 \leq 0$
Bus 3		$x_1 + x_3 + x_4 \geq 1$	$x_1 + x_3 + x_4 - 3f_3 \leq 0$
Bus 4	$x_2 + x_3 + x_4 + x_6 + x_{12} - f_4 \geq 0$		$x_2 + x_3 + x_4 + x_6 + x_{12} - 5f_4 \leq 0$
Bus 5		$x_2 + x_5 + x_7 \geq 1$	$x_2 + x_5 + x_7 - 3f_5 \leq 0$
Bus 6	$x_2 + x_4 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{28} - f_6 \geq 0$		$x_2 + x_4 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{28} - 8f_6 \leq 0$
Bus 7	$x_5 + x_6 + x_7 - f_7 \geq 0$		$x_5 + x_6 + x_7 - 3f_7 \leq 0$

Bus 8	$x_6+x_8+x_{28}-f_8 \geq 0$		$x_6+x_8+x_{28}-3f_8 \leq 0$
Bus 9	$x_6+x_9+x_{10}+x_{11}-f_9 \geq 0$		$x_6+x_9+x_{10}+x_{11}-4f_9 \leq 0$
Bus 10	$x_6+x_9+x_{10}+x_{17}+x_{20}+x_{21}+x_{22}-f_{10} \geq 0$		$x_6+x_9+x_{10}+x_{17}+x_{20}+x_{21}+x_{22}-7f_{10} \leq 0$
Bus 11	$x_9+x_{11}-f_{11} \geq 0$		$x_9+x_{11}-2f_{11} \leq 0$
Bus 12		$x_4+x_{12}+x_{13}+x_{14}+x_{15}+x_{16} \geq 1$	$x_4+x_{12}+x_{13}+x_{14}+x_{15}+x_{16}-6f_{12} \leq 0$
Bus 13		$x_{12}+x_{13} \geq 1$	$x_{12}+x_{13}-2f_{13} \leq 0$
Bus 14		$x_{12}+x_{14}+x_{15} \geq 1$	$x_{12}+x_{14}+x_{15}-3f_{14} \leq 0$
Bus 15		$x_{12}+x_{14}+x_{15}+x_{18}+x_{23} \geq 1$	$x_{12}+x_{14}+x_{15}+x_{18}+x_{23}-5f_{15} \leq 0$
Bus 16		$x_{12}+x_{16}+x_{17} \geq 1$	$x_{12}+x_{16}+x_{17}-3f_{16} \leq 0$
Bus 17		$x_{10}+x_{16}+x_{17} \geq 1$	$x_{10}+x_{16}+x_{17}-3f_{17} \leq 0$
Bus 18		$x_{15}+x_{18}+x_{19} \geq 1$	$x_{15}+x_{18}+x_{19}-3f_{18} \leq 0$
Bus 19		$x_{18}+x_{19}+x_{20} \geq 1$	$x_{18}+x_{19}+x_{20}-3f_{19} \leq 0$
Bus 20		$x_{10}+x_{19}+x_{20} \geq 1$	$x_{10}+x_{19}+x_{20}-3f_{20} \leq 0$
Bus 21	$x_{10}+x_{21}+x_{22}-f_{21} \geq 0$		$x_{10}+x_{21}+x_{22}-3f_{21} \leq 0$
Bus 22	$x_{10}+x_{21}+x_{22}+x_{24}-f_{22} \geq 0$		$x_{10}+x_{21}+x_{22}+x_{24}-4f_{22} \leq 0$
Bus 23		$x_{15}+x_{23}+x_{24} \geq 1$	$x_{15}+x_{23}+x_{24}-3f_{23} \leq 0$
Bus 24	$x_{22}+x_{23}+x_{24}+x_{25}-f_{24} \geq 0$		$x_{22}+x_{23}+x_{24}+x_{25}-4f_{24} \leq 0$
Bus 25	$x_{24}+x_{25}+x_{26}+x_{27}-f_{25} \geq 0$		$x_{24}+x_{25}+x_{26}+x_{27}-4f_{25} \leq 0$
Bus 26	$x_{25}+x_{26}-f_{26} \geq 0$		$x_{25}+x_{26}-2f_{26} \leq 0$
Bus 27	$x_{25}+x_{27}+x_{28}+x_{29}+x_{30}-f_{27} \geq 0$		$x_{25}+x_{27}+x_{28}+x_{29}+x_{30}-5f_{27} \leq 0$
Bus 28	$x_6+x_8+x_{27}+x_{28}-f_{28} \geq 0$		$x_6+x_8+x_{27}+x_{28}-4f_{28} \leq 0$
Bus 29	$x_{27}+x_{29}+x_{30}-f_{29} \geq 0$		$x_{27}+x_{29}+x_{30}-3f_{29} \leq 0$
Bus 30	$x_{27}+x_{29}+x_{30}-f_{30} \geq 0$		$x_{27}+x_{29}+x_{30}-3f_{30} \leq 0$

Table 2-6. Constraints for observability of buses based on observation rule 1

In addition to those constraints, constraints, which express observation rule 2 and 3, can also be formulated as Table 2-5.

For zero-injection networks	Constraints based on (2-17)
Bus 6	$f_2+f_4+f_{6,6}+f_7+f_{6,8}+f_{6,9}+f_{6,10}+f_{6,28} \geq 2+g_{6,6}+g_{6,8}+g_{6,9}+g_{6,10}+g_{6,28}$
Bus 9	$f_{9,6}+f_{9,9}+f_{9,10}+f_{11} \geq g_{9,6}+g_{9,9}+g_{9,10}$
Bus 22	$f_{22,10}+f_{21}+f_{22}+f_{22,24} \geq 1+g_{22,10}+g_{22,24}$
Bus 25	$f_{25,24}+f_{25,25}+f_{26}+f_{25,27} \geq g_{25,24}+g_{25,25}+g_{25,27}$
Bus 27	$f_{27,25}+f_{27,27}+f_{27,28}+f_{29}+f_{30} \geq 1+g_{27,25}+g_{27,27}+g_{27,28}$
Bus 28	$f_{28,6}+f_{28,8}+f_{28,27}+f_{28,28} \geq g_{28,6}+g_{28,8}+g_{28,27}+g_{28,28}-1$

Table 2-7. Constraints based on observation rule 2 and 3

For zero injection networks	Constraints based on (2-18)	Constraints based on (2-19)
Bus 6	$f_{6,6}+f_{9,6}+f_{28,6}-3f_6 \leq 0$	$g_{66}+g_{96}+g_{286} \geq 1$
Bus 8	$f_{6,8}+f_{28,8}-2f_8 \leq 0$	$g_{68}+g_{288} \geq 1$
Bus 9	$f_{6,9}+f_{9,9}-2f_9 \leq 0$	$g_{69}+g_{99} \geq 1$
Bus 10	$f_{6,10}+f_{9,10}+f_{22,10}-3f_{10} \leq 0$	$g_{610}+g_{910}+g_{2210} \geq 1$
Bus 24	$f_{22,24}+f_{25,24}-2f_{24} \leq 0$	$g_{2224}+g_{2524} \geq 1$
Bus 25	$f_{25,25}+f_{27,25}-2f_{25} \leq 0$	$g_{2525}+g_{2725} \geq 1$
Bus 27	$f_{25,27}+f_{27,27}+f_{28,27}-3f_{27} \leq 0$	$g_{2527}+g_{2727}+g_{2827} \geq 1$
Bus 28	$f_{6,28}+f_{27,28}+f_{28,28}-3f_{28} \leq 0$	$g_{628}+g_{2728}+g_{2828} \geq 1$

Table 2-8. Constraints for indicator variable f_j and g_i

For (i, j) pair	Constraints based on (2-19)
(6, 6)	$g_{66}-f_{66} \geq 0$
(6, 8)	$g_{68}-f_{68} \geq 0$
(6, 9)	$g_{69}-f_{69} \geq 0$
(6, 10)	$g_{610}-f_{610} \geq 0$
(6, 28)	$g_{628}-f_{628} \geq 0$

(9, 6)	$g_{96}-f_{96} \geq 0$
(9, 9)	$g_{99}-f_{99} \geq 0$
(9, 10)	$g_{910}-f_{910} \geq 0$
(22, 10)	$g_{2210}-f_{2210} \geq 0$
(22, 24)	$g_{2224}-f_{2224} \geq 0$
(25, 24)	$g_{2524}-f_{2524} \geq 0$
(25, 25)	$g_{2525}-f_{2525} \geq 0$
(25, 27)	$g_{2527}-f_{2527} \geq 0$
(27, 25)	$g_{2725}-f_{2725} \geq 0$
(27, 27)	$g_{2727}-f_{2727} \geq 0$
(27, 28)	$g_{2728}-f_{2728} \geq 0$
(28, 6)	$g_{286}-f_{286} \geq 0$
(28, 8)	$g_{288}-f_{288} \geq 0$
(28, 27)	$g_{2827}-f_{2827} \geq 0$
(28, 28)	$g_{2828}-f_{2828} \geq 0$

Table 2-9. Constraints for relationship between f_i and g_i variables

Finally, formulation on the redundancy calculation for IEEE 30 bus system can be made as Table 2-8.

	Equations based on (2-21) and (2-22)
Redundancy 1	$r_1=3x_1+5x_2+3x_3+5x_4+3x_5+8x_6+3x_7+3x_8+4x_9+7x_{10}+2x_{11}+6x_{12}+2x_{13}+3x_{14}+5x_{15}+3x_{16}+3x_{17}+3x_{18}+3x_{19}+3x_{20}+3x_{21}+4x_{22}+3x_{23}+4x_{24}+4x_{25}+2x_{26}+5x_{27}+4x_{28}+3x_{29}+3x_{30}$
Redundancy 2	$r_2=30 - (f_1+f_2+f_3+f_4+f_5+f_6+f_7+f_8+f_9+f_{10}+f_{11}+f_{12}+f_{13}+f_{14}+f_{15}+f_{16}+f_{17}+f_{18}+f_{19}+f_{20}+f_{21}+f_{22}+f_{23}+f_{24}+f_{25}+f_{26}+f_{27}+f_{28}+f_{29}+f_{30})$

Table 2-10. Equations for redundancy calculation

All those constraints and equations are solved with an objective function,

$$\text{Min } Z = \sum_{i=1}^{N_{bus}} x_i - (w_1 r_1 + w_2 r_2), \quad i \in I \quad (2.23)$$

where w_1 and w_2 are weights, which are given to redundancy 1 and 2, respectively according to decision maker's intention for finding optimal answer. These weight factors will be emphasized in chapter 3.

2.6 Results and Discussion

In this section, the results based on an optimal PMU placement model introduced in section 2.5 are shown. In this study, GAMS (General Algebraic Modeling System) software and a solver, BARON (Branch-And-Reduced Optimization Navigator) are used to solve this optimization problem. Figure 2-8 indicates the optimal PMU allocation point when the designed model is solved.

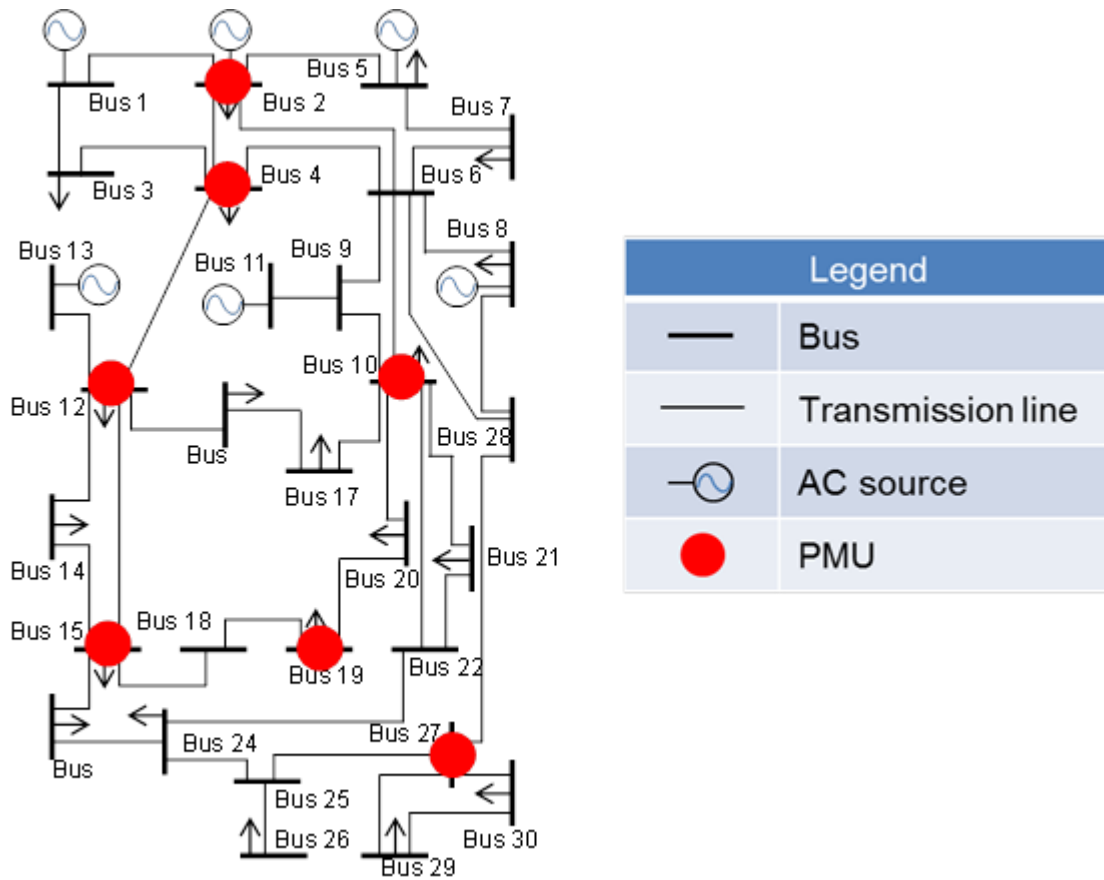


Figure 2-8. Optimal PMU location at IEEE 30 bus system

This result comes with a set of information regarding observability and redundancy for this power grid system. First, the 7 buses are chosen as a place that PMUs have to be located, which are bus 2, 4, 10, 12, 15, 19, and 27. In addition to this, total redundancy value is found as 41, 36 of them from redundancy 1 and 5 from redundancy 2. The outcome from IEEE 30 bus system is summarized in Table 2-9.

IEEE 30 Buses	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	Sum	
PMU placement	0	1	0	1	0	0	0	0	0	1	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	7
Redundancy 1	1	2	1	3	1	3	0	0	1	1	0	3	1	2	2	1	1	2	1	1	1	1	1	0	1	0	1	1	1	1	36	
Redundancy 2	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	5	
Total Redundancy	1	2	1	3	1	3	1	1	1	1	1	3	1	2	2	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	41	

Table 2-11. Optimal solution of IEEE 30 bus system

Since the number of PMUs is 7 and total number of buses is 30, percentage of buses occupied by PMUs is 23.3%, which is successfully fall into the range of 1/5 to 1/4 that is stated in [56]. This solving procedure can be applied to another power grid system. In this study, the optimal PMU location selection for IEEE 14, 30, 39, 57, 108 bus systems is executed. By applying proposed model, results of Table 2-10 are achieved.

IEEE system	# of PMU	Location of PMU	Percentage of buses occupied by PMUs	Redundancy
14 bus	3	2, 6, 9	21.4%	16
30 bus	7	2, 4, 10, 12, 15, 18, 27	23.3%	41
39 bus	8	8, 10, 16, 18, 20, 23, 25, 29	20.5%	43
57 bus	11	1, 4, 13, 19, 25, 29, 32, 38, 41, 51, 54	19.3%	61
118 bus	28	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110	23.7%	156

Table 2-12. Optimal solutions for IEEE bus systems

There have been many different kinds of approaches to solve optimal PMU placement problem. As stated in section 2.4, the authors utilized a broad spectrum of the fashion of the

moment algorithms and methods. At the same time the variety in those approaches has also brought out the variety in optimal values for problems. So, comparing the solutions generated by proposed methodology from this study with other approaches represents a good sense in terms of effectiveness of the algorithm. Table 2-11 and 2-12 clearly show the comparison between results of PMU allocation strategies from unique approaches.

Ref. #	Methods	14bus	30bus	39bus	57bus	118bus
	Proposed method (ILP)	3	7	8	11	28
[32]	Simulated annealing	3	-	8	-	29
[28]	Tabu search	3	-	10	13	-
[33]	Genetic algorithm	3	7	-	12	29
[16]	Particle swarm optimization	3	7	-	11	28
[21]	Generalized integer programming	3	7	-	11	-
[29]	Immunity genetic algorithm	3	7	-	11	28
[34]	Binary search algorithm	3	7	8	-	-
[18]	Integer non-linear programming	3	-	-	12	29
[20]	ILP by Dua et al.	3	-	-	14	29
[30]	ILP by Aminifar et al.	3	7	8	11	28
[26]	ILP by Enshae et al.	3	7	8	11	28
[35]	Imperialistic competition algorithm	3	7	-	11	28
[36]	Chemical reaction optimization	3	7	-	14	29
[37]	Three stage heuristic method	3	7	8	11	28

Table 2-13. Comparison in terms of minimizing the number of PMUs

Since each strategy has used its unique algorithms and methods, it is difficult to judge the superiority between those approaches. Although each approach has its own originality, the effectiveness of them, in this study, is measured exclusively based on the number of PMUs found and level of redundancy which is achieved from that set of PMUs. Table 2-11 represents the level of redundancy of some strategies, which are comparatively better than others in terms of effectiveness, i.e., minimum number of PMUs. Especially [16] using particle swarm optimization, [29] using immunity genetic algorithm, [34] using binary search algorithm, [30] and [26] using integer linear programming, [35] using imperialistic competition algorithm, [37] using three stages of heuristic method, and the integer programming approach proposed in this study are selected as most effective strategies in terms of PMU allocation efficiency. Table 2-22 shows the total redundancy levels of those selected effective strategies.

Ref. #	Methods	14bus	30bus	39bus	57bus	118bus
	Proposed method (ILP)	16	41	43	61	156
[16]	Particle swarm optimization	16	37	-	60	147
[29]	Immunity genetic algorithm	16	33	-	60	148
[34]	Binary search algorithm	16	39	43	-	-
[30]	ILP by Aminifar et al.	16	34	43	59	148
[26]	ILP by Enshae et al.	16	41	43	59	156
[35]	Imperialistic competition algorithm	16	41	-	59	156
[37]	Three stage heuristic method	16	36	43	60	148

Table 2-14. Comparison in terms of maximizing the level of redundancy

Table 2-12 clearly indicates that a strategy proposed in this study finds best answers with regard to effectiveness of solutions, i.e., minimization of number of PMUs and maximization of level of redundancy at each IEEE system.

Based on the fact that the proposed strategy is effective in finding both minimum number of PMUs and maximum level of redundancy, a practical assumption can be incorporated in this problem that can give a decision maker more choices on determining the location of PMUs. Previous results are only focused on optimum answers for overall power grid systems. However, it is possible that a decision maker wants to put more weight on a particular bus than other buses [38]. This can happen when a decision maker thinks that a particular bus is more important than other buses and that bus should have higher level of redundancy than other buses. A new set of experiments can be conducted, which reflects this idea and in this study, a constraint that a level of redundancy at a particular bus should be greater than or equal to 3 in IEEE 30 bus setting. Table 2-13 shows the results of a set of experiments for this consideration.

Bus	PMU placement	No.	R	Efficiency (R/PMU)
Normal case	2, 4, 10, 12, 15, 19, 27	7	41	5.86
$R \geq 3$ at bus 1	1, 2, 3, 10, 12, 15, 18, 27	8	42	5.25
2	2, 4, 6, 10, 12, 15, 18, 29	8	46	5.75
3	1, 2, 3, 4, 10, 12, 15, 18, 26	9	47	5.22
4	2, 4, 10, 12, 15, 19, 27	7	41	5.86
5	2, 4, 10, 12, 15, 18, 27	7	41	5.86
6	2, 4, 5, 7, 10, 12, 15, 18, 27	9	46	5.11
7	2, 4, 10, 12, 15, 20, 27	7	40	5.71
8	1, 5, 6, 7, 10, 12, 15, 18, 27	9	46	5.11

9	2, 4, 6, 8, 10, 12, 15, 18, 27, 28	10	54	5.40
10	2, 4, 6, 9, 10, 12, 15, 18, 27	9	50	5.56
11 (2)	2, 4, 6, 10, 12, 15, 20, 27	8	46	5.75
12	2, 4, 9, 10, 11, 12, 18, 24, 27	9	44	4.89
13 (2)	2, 4, 10, 12, 15, 20, 27	7	40	5.71
14	3, 7, 10, 12, 13, 15, 18, 27	8	39	4.88
15	2, 4, 10, 12, 14, 15, 20, 27	8	43	5.38
16	2, 4, 10, 12, 15, 18, 27	7	41	5.86
17	1, 7, 12, 16, 17, 19, 24, 30	8	34	4.25
18	3, 5, 10, 13, 15, 16, 17, 20, 29	9	37	4.11
19	2, 4, 10, 12, 15, 18, 19, 27	8	44	5.50
20	1, 5, 10, 12, 18, 19, 20, 24, 27	9	40	4.44
21	1, 5, 10, 12, 18, 19, 20, 24, 27	9	40	4.44
22	2, 4, 10, 12, 15, 18, 21, 22, 27	9	47	5.22
23	2, 4, 10, 12, 19, 22, 24, 27	8	42	5.25
24	2, 4, 10, 12, 15, 19, 23, 24, 27	9	46	5.11
25	1, 7, 10, 12, 18, 22, 24, 25, 27	9	41	4.56
26 (2)	2, 4, 10, 12, 19, 24, 25, 27	8	41	5.13
27	2, 4, 10, 12, 15, 18, 25, 26, 27	9	51	5.67
28	2, 4, 10, 12, 18, 24, 25, 27, 28	9	45	5.00
29	2, 4, 6, 10, 12, 15, 18, 27, 28	9	51	5.67
30	2, 4, 10, 12, 15, 18, 27, 29, 30	9	47	5.22

Table 2-15. Additional experiments considering emphasis on a particular bus

CHAPTER 3

HARMONIZED DECISION MODEL FOR PMU ALLOCATION IN SMART GRID

CONTEXT

3.1 Introduction

A primary role of the decision models for smart grid systems should be able to maximize the effectiveness of investment, by minimizing the cost for the optimal resource allocation in a given system. Based on the importance of the economic feasibility, there have been various topics of decision making for the optimal component allocation in the smart grid industry; however, there is a limited effort to realize the decision making framework, which can harmonize the physical and operational aspects of smart grid components. Due to the ruinous complexity of an exhaustive approach, each model has been designed separately based on its own assumptions without enough reflection of their functions. Although the functions of the smart grid significantly vary based on the definition of the smart grid systems and the scope of the investigation, several key functions that have higher priority and importance in the deployment of smart grid technologies are introduced – refereeing the reports from National Institute of Standard and Technology (NIST) [39] and Electrical Power Research Institute (EPRI) [8, 40].

This chapter extends the scope of PMU allocation task to overall system requirement analysis task and presents a harmonized decision modeling process that can be employed to realize a decision support system for the smart grid system analysis. This work is based on an idea that the component allocation strategy in smart grid systems should reflect the operational circumstances and should maintain the model hospitable for achieving a practical decision considering the functionality of smart grid systems. In this research, a new PMU allocation

modeling process is used to describe the proposed modeling framework and the IEEE bus systems are used to validate the work. In the next section, the existing literature related to the decision making for the smart grid resource is presented. In Section 3, the harmonized decision modeling process is described. In Section 4, a component allocation is modeled and solved by using the harmonized decision model.

3.2 Review of Decision Making in Smart Grid

For this literature review, four key functional areas (i.e., demand response, real-time wide-area situational awareness, distributed electric units, and distribution grid management) are selected based on the discussion in [8, 39-41].

Demand response is a management strategy, which encourages energy consumption to control energy use in response to supply condition. This function also enables less expensive management to intelligently influence a load than the establishment of a new utility facility [42]. Bakker et al. [43] try to design the optimization methodology, which can incorporate communication between different technologies to reshape the energy demand profile. Due to much computational power required, their planning and control methodology is organized in a tree structure applying three steps of optimization levels. Mohsenian-Rad and Leon-Garcia [44] point out problems in utilization of the potential benefits of real-time pricing tariffs. They propose an optimal and automatic residential energy consumption scheduling framework for achieving a desired trade-off between minimizing the electricity payment and minimizing the waiting time for the operation of each appliance in household.

Real-time wide-area situational awareness plays a crucial role in smart grid as a measure for grid protection and control by providing time-synchronized data of power system operating states [45]. The information that system operators have influences on how effective a grid

system's reaction will be against the contingencies. Zhu and Abur [46] describe the need for phasor measurements to overcome the limitation of conventional measurements. Authors show that by including redundant phasor information, errors in the parameters can be correctly identified. Aminifar et al. [30] present a model for the optimal placement of phasor measurement units (PMUs) considering contingency conditions (i.e., line outages and loss of measurements). Their work shows that integer programming can find the global optimality of PMU allocation problem with reasonable computational complexity.

The emergence of smart grid has stimulated the electric units to be distributed from one centralized spot [6]. This involves distributed generation unit, electricity storage, electric vehicles, and the qualitative improvement in demand side management. Bu et al. [47] present a distributed stochastic power generation unit commitment scheme by using hidden Markov models and a Markov-modulated Poisson process for modeling renewable energy resources and the power demand load, respectively. The effectiveness of their scheme is evaluated in terms of the cost of energy and pollutant emission through the simulation. Jia et al. [48] introduce the optimization process of the sizing and siting of electric vehicle charging stations. Their approach defines variables to represent the charging demand, and formulates the problem with a mixed integer quadratic programming with a graph theory.

Distribution grid management focuses on maximizing performance of electrical components of networked distribution systems and integrating them with transmission systems and customer operations [39]. Oshiro et al. [49] aims to perform voltage control in distribution system by the cooperative control between the interfaced inverter with distributed generation and the existing voltage control devices. In their work, a one-day schedule of voltage references for the control devices is determined by the optimization calculation. In [50], Soma et al. develop a

model of Information and Communication Technology (ICT) system that considers the position of ICT infrastructure, and then propose a decision making process for finding the optimal allocation of WiMAX antennas with an active distribution network planning algorithm. In addition, Galli et al. [51] point that there was not enough efforts to give quantitative guidelines on how to choose one communication technology over the other in the design of smart grid. They analyzed the role of power-line communications, and conducted electrical and topological analysis of the power distribution network.

3.3 Harmonized Decision Modeling Process

Since the purpose of the traditional decision making has been the minimization of the amount of financial investment while ensuring the normal and stable operations of a given system, the traditional processes have mainly stressed the aspect of economic feasibility rather than the considerations on the substantive operational aspects. However, the more suitable decision model process has to animate the model by incorporating the operational aspect of system. Specifically, the decision model should include the considerations on the functionality of component for enhancing the utility of solution, as well as the economic feasibility by minimizing cost. Feasibility of the model needs to be reinforced and confirmed by a decision maker for embracing the variability in operation of system.

Due to the complexity of smart grid system, it is neither an extemporary nor a simple task to find a generalized methodology that can define the model structure applied in smart grid context. In this article, we propose a general decision modeling process for smart grid component allocation as shown in Figure 3-1.

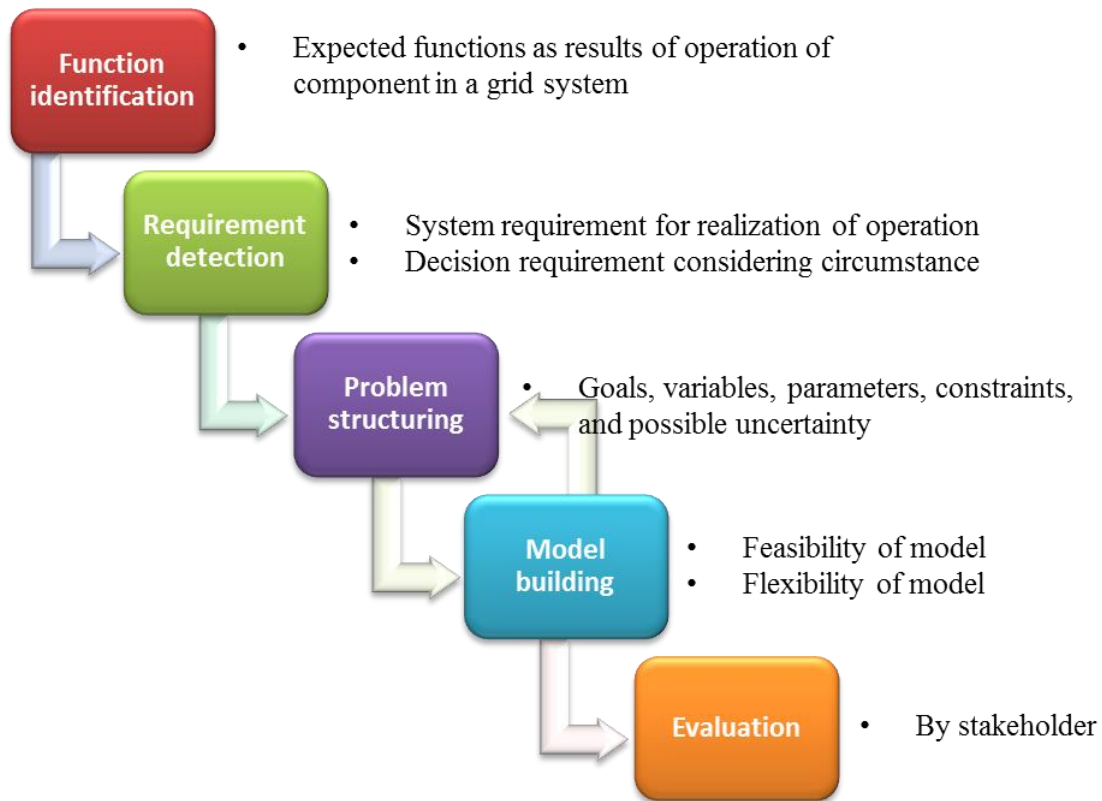


Figure 3-1. Harmonized decision modeling process

When applying this decision modeling process in the smart grid context, a decision maker needs to identify the functions, which are expected as results of the installation and operation of the component in a given grid system. Since the complexity in function identification (e.g., an entanglement between functionalities over multiple domains) is frequently arisen, this step encourages a decision maker to conduct the exhaustive review on the functional effects of the component.

While the step of function identification is for sketching a rough outline of decision to be made, the requirement detection process requires the decision maker to study the problem with various angles and depths for defining important points to be handled through the model. The requirements discovered in this process are the requirements of system, which is directly related

to the realization of elemental operation, and also the decision requirements, which should involve the circumstantial consideration.

The problem structuring is the next step, and a focused way of thinking [52] for solving the problem given by the function and the system requirement. Problem structuring can be conducted with identification of several parts of a problem, such as goals, variables, parameters, constraints, and possible uncertainties [53]. The model building is very dynamic process interacting with the problem structuring [54]. Particularly, the feasibility of model must be considered in this process. In contrast with the prior processes that specialize the decision model based on the functions and requirements discovered, the model building process must accord flexibility to the model, so that it can tolerate the inherent complexity of the problem and the variability in the operational application. After the solving process according to the harmonized decision modeling, the results need to be evaluated by the stakeholder.

3.4 Harmonized Decision Model Structure and Formulation for PMU Allocation

Although there has been a noticeable research works dealing with the PMU allocation [55] as stated in chapter 2, those research works have mainly focuses on the minimization of number of PMUs to be placed in a given system. As a result, PMU allocation has been apt to simply reduce the number of PMU, rather than to consider the harmonization of model with the environment of the region where PMUs will function and with the variability of system operation. Based on the proposed sequence of decision modeling methodology, PMU placement problem can be restructured.

The primary function that a decision maker or stakeholder in a business of PMU operation could anticipate is the electrical state measurement for determining the health of the electricity grid system. Based on this primary function, several derived functions can also be

discovered (e.g., prevention of power outage, load control including the load shedding, increase in power quality, system interconnection, generator and line modeling, renewable source integration, congested area control requiring online monitoring and so on). The examination considering the subsidiary functionalities of the component encourages a decision maker to expand the boundary of idea on requirement detection. As stated above, three concrete functions can be taken into account, that are prevention of power outage, load control including load shedding, and the increase in power quality. First, real-time monitoring can detect the fault in the energy grid system, and suppress the wide spread of power outage. Since the impact of power outage varies depending on the situation where it occurs, it is important to consider the factor that could affect the significance of impact. The power outage impact can be determined by considering the population that will be affected by a fault of a certain substation or lines linked to the substation, the significance of electrical facilities operated by substations, and the presence of interregional area in each region. For instance, the region that has more population would have greater importance than other regions in terms of the importance of prevention of power outage. And the region that has a governmental agency highly relies on the computer systems utilizing critical data would have to receive more significant attention than other regions. If a region is acting as an interregional gate where connects two different regions, more considerations need to be located on that region. Also, a load control is a noticeable function that would be performed by the utilization of PMU. When the load control function is considered, the amount of electricity consumed in a specific region will come into the spotlight due to the high possibility of the high demand region to be in need of the load control. As the last additional function, the increase in power quality is expected to be dealt with in the PMU allocation. This function attracts the entity that is sensitive to the quality of electricity. For example, to manufacturers

producing subminiature devices (e.g., semiconductor chips), even a minimal change in electrical performance can seriously affect their productivity and the quality of products. The requirements listed here are particularly meaningful in the demand side aspect, while other aspects also exist: that are system interconnection, generator and line modeling, renewable integration, and congested area requiring online monitoring. However, this paper focuses on the five selected requirements preferentially. The other requirements will be considered in future research.

In the problem structuring, the requirements are entered in the model as objectives. To earn the technical margin of modeling for further applicable operations, the design of model focuses on efficient solving process. Although the determination of the significance of each factor through the systematic calculation is required, this calculation is beyond the scope of this research. Thus, in this paper it is assumed that the valid calculation for each factor of each region is done by a statistical decision support tool.

As a whole, there are six objectives in this PMU allocation considering smart grid system context: 1) minimization of the number of PMUs to be installed; 2) maximization of population, which is supplied by substations observed by PMUs; 3) maximization of significance of facilities in regions, which are supplied by substations observed by PMUs; 4) maximization of level of observation for interregional area; 5) maximization of amount of electricity demand of regions, which are supplied by substations observed by PMUs; and 6) maximization of the number of facility sensitive to the quality of electricity in regions observed by PMUs. Based on them, a multi-objective problem having six objectives can be:

$$\begin{aligned} & \min F_1(x_i), \max F_2(x_i), \max F_3(x_i), \max F_4(x_i), \max F_5(x_i), \max F_6(x_i), \\ & \text{subject to } x_i \in S \end{aligned} \tag{3.1}$$

where S is the set of feasible solutions in which $x_i = 1$, if a PMU is placed at bus i , otherwise $x_i = 0$, for all $i \in \{1, 2, \dots, n\}$, and n is the number of buses in a given system.

Apparently, this is a complex problem, which involves six different objectives, and it would be very hard for these objectives to harmonize each other. In other words, these multi-objectives would be excessively competitive each other, which could lead to the invalid solution. It means that the best PMU allocation for the one objective may not be the best for the other objectives. Also, when it is recalled that the original PMU allocation has been a large-scale combinatorial optimization problem [28], to solve a hexa-objective combinatorial problem having two factors, number of PMU (N_{PMU}) and placement set $S(N_{PMU})$, becomes a formidable task.

As a way to allow the model to keep computational tolerance for solving the problem, the objectives of (3.1) need to be restructured. In this study, the minimization of number of PMUs to be installed (i.e., $\min F_1(x_i)$) is regarded as a primary objective of PMU allocation and the other five objectives in (3.1), which are related to the requirement of harmonized modeling, are expressed as a function of redundancy. Equation (3.2) describes how six different objectives are standardized as a function of number of PMUs and level of redundancy. There are two distinctive features in this formulation. It uses the weighted sum method, which utilizes a priori articulation of preferences, one of the main methods solving multi-objective optimization, and it integrates all different parameters into the model as a function of redundancy, so that the model can be used in various applicable circumstances of operation and also can retain the computational margin in solving process.

$$\begin{aligned}
& \min \left[\sum_{i=1}^N x_i - \{ F_2(r_i) + F_3(r_i) + F_4(r_i) + F_5(r_i) + F_6(r_i) \} \right] \\
& = \min \left\{ \sum_{i=1}^N x_i - \sum_{i=1}^N \left(\frac{w_1 p_i r_i}{\sum_{i=1}^N p_i} + \frac{w_2 s_i r_i}{\sum_{i=1}^N s_i} + \frac{w_3 t_i r_i}{\sum_{i=1}^N t_i} + \frac{w_4 d_i r_i}{\sum_{i=1}^N d_i} + \frac{w_5 e_i r_i}{\sum_{i=1}^N e_i} \right) \right\} \quad (3.2) \\
& = \min \left[\sum_{i=1}^N x_i - \sum_{i=1}^N \left\{ \left(\frac{w_1 p_i}{\sum_{i=1}^N p_i} + \frac{w_2 s_i}{\sum_{i=1}^N s_i} + \frac{w_3 t_i}{\sum_{i=1}^N t_i} + \frac{w_4 d_i}{\sum_{i=1}^N d_i} + \frac{w_5 e_i}{\sum_{i=1}^N e_i} \right) r_i \right\} \right]
\end{aligned}$$

where, p_i = population of regions where bus i supplies electricity, s_i = significance of facilities in regions where bus i supplies electricity, t_i = index of interregional area, d_i = electrical demand of regions where bus i supplies electricity, e_i = level of sensitivity of facilities in regions, where bus i supplies electricity, and w_1, w_2, w_3, w_4 and w_5 = weights for $p_i, s_i, t_i, d_i,$ and e_i , respectively. Each parameter in the objective function of redundancy is normalized by dividing it by sum of parameters of all buses. Weight function w_i implies the level of importance which a decision maker attributes.

Equations (2.5) to (2.20) in Chapter 2 are used without any change, which formulate the relationship between buses in a power grid system. Since the model generated from this harmonized decision modeling process for PMU allocation should calculate the level of redundancy of each bus, (2.11) and (2.12) cannot be used as it is, and they should be modified. For calculating level of redundancy of each individual bus, new equations (3.3) – (3.6) are designed.

$$r_i^1 = \sum_{j=1}^N a_{ij} x_j, \quad \forall i \in I \quad (3.3)$$

$$r_i^2 + \sum_{j=1}^N a_{ij} x_j \geq 1, \quad \forall i \in I \quad (3.4)$$

$$\sum_{j=1}^N a_{ij} x_j - \left(\sum_{j=1}^N a_{ij} \right) (r_i^2 - 1) \leq 0 \quad (3.5)$$

$$r_i = r_i^1 + r_i^2 \quad (3.6)$$

3.5 Results and Discussion

As case studies, IEEE 30 bus system and 50 bus system are chosen and solved by using a mathematical model devised from harmonized decision modeling process. Artificially made data sets are also utilized in this problem. The population, and electrical demand are randomly generated integral values within $p_i(\text{people}) \in [5,000, 500,000]$, and $d_i(\text{kWh}) = p_i u$ where $u(\text{kWh}) \in [25, 50]$. The ranges of three integral indices are presupposed as $s_i \in [0, 5]$, $t_i \in [0, 2]$, and $e_i \in [0, 2]$, respectively. Figure 3-2 shows the different optimal PMU allocations based on the different modeling approaches.

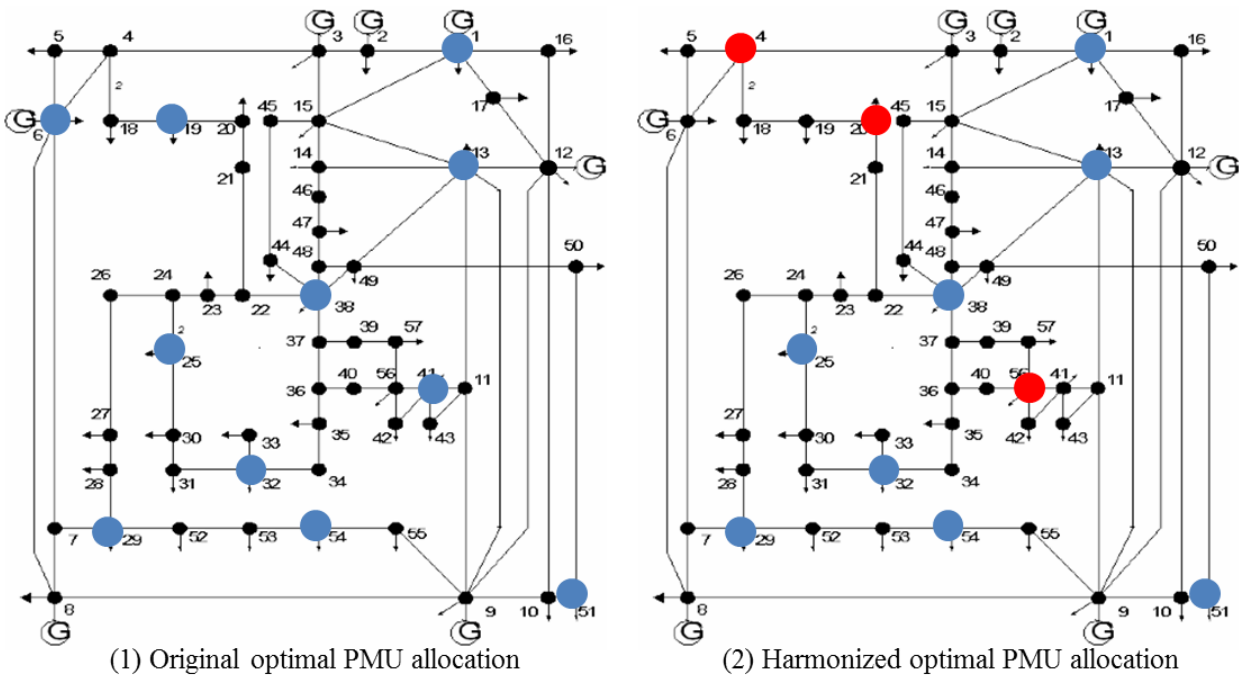


Figure 3-2. Changes in PMU location according to modeling approach

First diagram indicates the optimal PMU allocation point when this problem is dealt with as a mere location selection problem based on the network configuration, and bus 2, 4, 10, 12, 15, 18, and 27 are chosen to have PMUs. Second diagram show the optimal PMU allocation, which are solved by the harmonized decision model. The different circumstantial factors affect the component allocation layout with 27% disparity in PMUs allocation. This result explicitly describes that real world component allocation problem should incorporate the considerations on the operation condition of component according to the functionality in smart grid context. The following three tables validate that harmonized modeling process can make difference in optimal PMU placement plan. Two indices are used to show the disparity between the solutions of original model and harmonized model. Changes in PMU location of PMUs are calculated by equation (3.6) and improvement in redundancy is calculated by equation (3.7)

$$\text{Changes in PMU location} = \frac{\sum_{i=1}^{N_{bus}} |x_i^H - x_i|}{2} \quad (3.6)$$

$$\text{Improvement in redundancy} = \frac{R_H - R_T}{R_T} \quad (3.7)$$

where x_i^H is PMU placement variable of harmonized decision model and its meaning is same with (2.5). R_H is the total redundancy achieved by a solution found from the harmonized model and R_T is the total redundancy achieved by a solution found from the original model. To reflect decision maker's intention, weight values are applied to a model. Those weights indicate how much of importance a decision maker put on each factor. In other words, the harmonized decision model solves the problem based on decision maker's subjective intention as well as the objective parameters quantifying circumstantial factors.

IEEE-30	Weights (population/significance of facility/interregional area/electrical demand/sensitivity index)				
	2/2/2/2/2	5/2/1/1/1	0/5/2/1/2	1/1/1/6/1	0/0/2/2/6
Changes in PMU location	14.3%	14.3%	0%	14.3%	0%
Improvement in redundancy	1.2%	2.2%	0%	2.8%	0%

Table 3-1. Comparison of decisions in IEEE 30 bus system

IEEE-57	Weights (population/significance of facility/interregional area/electrical demand/sensitivity index)				
	2/2/2/2/2	5/2/1/1/1	0/5/2/1/2	1/1/1/6/1	0/0/2/2/6
Changes in PMU location	18.2%	18.2%	27.3%	18.2%	27.3%
Improvement in redundancy	2.5%	1.8%	1.8%	2.0%	5.7%

Table 3-2. Comparison of decisions in IEEE 57 bus system

IEEE-118	Weights (population/significance of facility/interregional area/electrical demand/sensitivity index)				
	2/2/2/2/2	5/2/1/1/1	0/5/2/1/2	1/1/1/6/1	0/0/2/2/6
Changes in PMU location	3.6%	25.0%	3.6%	28.6%	7.1%
Improvement in redundancy	0.2%	1.2%	1.0%	1.7%	0.6%

Table 3-3. Comparison of decisions in IEEE 118 bus system

The results show that harmonized decision model makes differences in the location of PMUs to be installed and the total redundancy in given power grid systems. The variation is also observed in comparison between different weighting values within a single parameter set. For example, even though same parameter values are generated and used for IEEE 118 bus system, there are significant changes in PMU location according to weighting values. When (5/2/1/1/1) is applied as weights for each factor, the 7 PMUs' locations are changed, while (2/2/2/2/2) changes only 1 PMU's location. Improvement in redundancy is expressed as percentiles to avoid the biased interpretation. The results show that harmonized model enhances the observability over a power grid system by changing the location of PMUs based on the values of both parameters and weights.

CHAPTER 4

DISCUSSION AND CONCLUSION

This thesis has presented an effective modeling strategy for optimal PMU placement associated with efficient allocation of resources and harmonized decision making process. The main objectives of the research are as follows:

1. The development of optimal PMU placement models based on the observability rules. The model tries to find the optimal allocation of PMUs by minimizing the number of PMUs required and maximizing the overall level of redundancy.
2. The development of modeling processes that incorporates the circumstantial factors around the operation of phasor measurement systems. This approach extends the boundary of PMU allocation from a network optimization problem to the system requirement analysis.

The introduction of redundancy prevention rule and indicator variable formulating redundancy prevention rule is one of the fundamental contributions of this thesis. The objective of this approach is to reduce the required number of PMUs by rigorously manipulating the network characteristics of PMU measurement. By using indicator variables, the number of variables and formulas was reduced and it enabled model to solve the problem more efficiently, finding better answers than other approaches. The model was tested on IEEE test systems. The results showed that PMU placement with proposed integer programming yields minimized number of PMUs among research works. Also, the model succeeded in maximizing the level of total redundancy compared to other research works. It is expected that this results can convince a decision maker of the reliability of this model and can be used as a basic structure of optimal PMU placement model. One of the future research topics that can be derived from this research

is a multi-stage scheduling of PMU placement. In real world situation of PMU installation, it is hard to install all of the PMUs at once due to the limitation in budget, and they are usually installed step by step according to a plan made by a utility or government. So the decision support system coming up with a solution for multi-stage installation scheduling is promising.

The development of harmonized modeling process was introduced and demonstrated. As a main contribution of this thesis, this modeling process contains the consideration on operational circumstance of systems and reflects the outcomes of system analysis in modeling process. The system analysis in this study has placed emphasis on function identification and requirement detection of PMU system, a component of smart grid systems. Since each smart grid component has its own functionalities and requirements, the modeling process of the resource allocation should deal with different factors in its decision making process. The harmonized modeling process tried to standardize this circumstantial aspect of resource allocation in smart grid context. It incorporated circumstantial factors as coefficients of redundancy variables into decision model to reduce computational burden. Also, the addition of weights was introduced. In conclusion, it turned out that the harmonized decision model can solve the optimal PMU placement problem from a different point of view, and the model can suggest a decision considering not only component network characteristics but also operational circumstance around the system and decision makers' own intention. The results made in IEEE bus systems indicated that harmonized decision model suggests an even better solution than original solution according to factors included in modeling process. In the future research, a decision model for different types of components or requirements can be discovered depending on component's own functionalities. Although that will require a decision modeler to build a decision model with a

different structure, this harmonized decision modeling process can serve as a guideline of establishment of a decision support system.

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ABSTRACT**A DECISION MODELING FOR PHASOR MEASUREMENT UNIT LOCATION
SELECTION IN SMART GRID SYSTEMS**

by

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As a key technology for enhancing the smart grid system, Phasor Measurement Unit (PMU) provides synchronized phasor measurements of voltages and currents of wide-area electric power grid. With various benefits from its application, one of the critical issues in utilizing PMUs is the optimal site selection of units.

The main aim of this research is to develop a decision support system, which can be used in resource allocation task for smart grid system analysis. As an effort to suggest a robust decision model and standardize the decision modeling process, a harmonized modeling framework, which considers operational circumstances of component, is proposed in connection with a deterministic approach utilizing integer programming. With the results obtained from the optimal PMU placement problem, the advantages and potential that the harmonized modeling process possesses are assessed and discussed.

AUTOBIOGRAPHICAL STATEMENT

Seung Yup Lee received his Bachelor's degree in Material Engineering at Sungkyunkwan University in South Korea and worked at Semiconductor R&D Center, Samsung Electronics as a researcher for four years. He has been studying towards his Master's degree in Industrial Engineering at Wayne State University since 2012. His research interests include healthcare operations, smart grid system analysis, and decision support system modeling.