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# An integrated decision support framework for remanufacturing in the automotive industry

Akhilesh Kumar  
*Wayne State University,*

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**AN INTEGRATED DECISION SUPPORT FRAMEWORK FOR  
REMANUFACTURING IN THE AUTOMOTIVE INDUSTRY**

by

**AKHILESH KUMAR**

**DISSERTATION**

Submitted to the Graduate School

of Wayne State University,

Detroit, Michigan

in partial fulfillment of the requirements

for the degree of

**DOCTOR OF PHILOSOPHY**

**2011**

MAJOR: INDUSTRIAL ENGINEERING

Approved by:

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Advisor

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Date

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## **DEDICATION**

**To my parents, Mrs. Shanti Ray and Mr. Birda Ray**

## **ACKNOWLEDGEMENTS**

First and foremost I would like to thank my PhD advisor Prof. Ratna Babu Chinnam. Without his inspirational guidance, enthusiasm, and encouragements this feat would not have been possible. My special thanks go to my committee members Prof. Richard Darin Ellis, Prof. Alper E Murat and Prof. John C Taylor for their suggestion for improvement. I am very grateful to present chair Prof. Leslie Monplaisir and ex-chair Prof. Kenneth Chelst for their support and enthusiasm to accomplish this goal.

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## Chapter 1 : **INTRODUCTION**

In today's global economy, firms are seeking any and every possible opportunity to differentiate themselves from competitors, to reduce their costs, and to add value to their supply chains and end customers. One increasingly popular option, under growing consumer awareness and increasing legislation, is to reintegrate used or returned product into the supply chain to regain the materials for economic and sustainability purposes (Schultmann et al., 2006). An important class of such "reverse" goods flows has to do with remanufacturing, which refers to activities that restore used products or their major modules to operational condition for use in place of new product or for other channels (e.g., spare parts). U.S. Environmental Protection Agency (EPA) advocates the practice of remanufacturing as economical, energy-efficient and environmentally friendly approach to reduce industrial waste (US EPA, 1997). Another important reason for improving reverse logistics is to cope with returns that have become endemic in many industries. For example, according to a recent Consumer Electronics Industry survey by the Reverse Logistics Executive Council, the average return rate is 8.46% in the high-tech industry (Thrikutam and Kumar - infosys.com 2004), with return rates as high as 20% for certain product segments. The value of these returned consumer electronic goods in the U.S. is estimated at \$104B for 2004 with the cost of managing the returns running around \$8B. While there are several types of returns (commercial returns, repairable returns, end-of-use returns, end-of-life returns, recalls, and others ...), the 8.46% return rate mostly covers commercial returns (that occur in the sales phase or shortly after) with immediate demand at another market location or segment. While efficient management of

commercial returns is challenging and necessary, particularly given the growth in return rates, remanufacturing is often far more complex. It not only deals with other types of returns that bring about lot more uncertainties (e.g., timing/location of return, return volume, quality), but also have to address complexities associated with reman production planning and control. Remanufacturing has traditionally been prevalent in such industries as automotive, electrical equipment, furniture, machinery, tires, and toner cartridges.

In the automotive industry, production parts can be roughly divided into Original Equipment (OE) parts and Aftermarket parts. OE parts refer to parts used in producing new vehicles, whereas, aftermarket parts refers to parts traded after original equipment sale, which includes both OE service (for parts under OEM warranty) and independent aftermarket (IAM) services. The automotive aftermarket industry is estimated at \$198B annually in the US, with IAM sales estimated at \$142B, mostly from collision centers and independent mechanics<sup>1</sup>. While the remanufacturing business was traditionally dominated by IAM companies, hefty profit margins and growing pressures to improve corporate citizenship, are encouraging more and more OEM and tier-1 suppliers to pursue remanufacturing. According to a recent survey by Inmar (Inmar, Special Report 2009), in the automotive industry, return rates are known to vary between 5%-25%. Survey also identifies various factors leading to poor returns: 1) Poor information flow, 2) Multiple networks that poorly interface with one another, 3) Different part numbering schemes for the same replacement parts, 4) Data entry order errors, 5) Incorrect shipments, 6) Mis-diagnosis, 7) Over ordering , and 8) Defective parts. Given returns and the size of the

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<sup>1</sup> <http://www.oemalliance.com/industry.htm>

aftermarket business, there are tremendous opportunities for OEMs and suppliers to engage in remanufacturing business to improve profitability and sustainability.

While all these opportunities abound, key complications for OEMs and suppliers is the difficulty in making decisions related to launch of remanufacturing program and efficient management of remanufacturing operations and logistics. There is lack of a structured and holistic decision support framework, which can guide firms in decision making related to timing the launch of the remanufacturing program, capacity installation/management etc. Further, efficient production and inventory management of remanufacturing parts for the supplier heavily impinges on the ability to accurately forecast these core returns from customers (besides forecasting demand for remanufacturing parts and securing cores from the open market, as necessary). All these factors are motivation for the proposed research.

### **1.1 Research Setting**

For a typical automotive product targeted for reman, production during its life-cycle can be roughly divided into three phases. Phase I more or less deals with the production of OE parts to support demand for new OEM product and tends to be relatively high volume production. Phase II covers the period of transition from production of just OE parts to both OE and OE service (OES) parts production and eventually just OE service and the independent after-market (IAM). Phase III covers the production of parts for just the IAM. Phase 0, preceding all the production phases, encompasses the various phases of product development with considerations for remanufacturing.

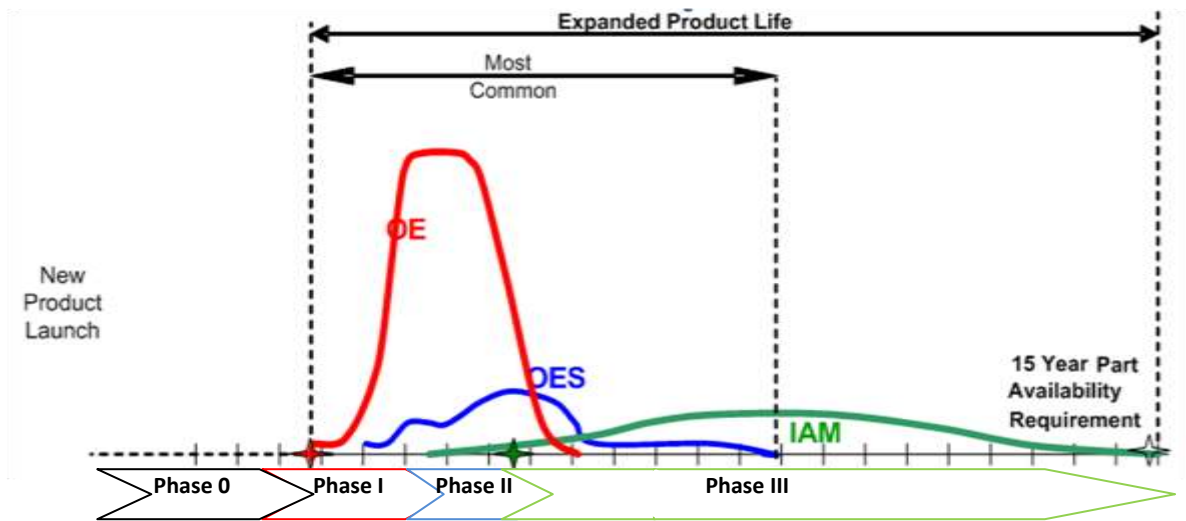


Figure 1.1 Typical production pattern of an automotive product considered for reman over its life-cycle

Figure 1.1 illustrates these phases along with representative production levels. For firms that do not engage in IAM or reman, in the automotive industry, the product from the end of the OE production cycle is often stocked to meet the 15 years spare-parts availability requirement.

For firms that engage in OE production as well as remanufacturing, the second and third phases impose new challenges apart from traditional forward supply chain management. In other words, presence of reverse logistic flows in a supply chain magnifies the variability and its effects. Following are the remanufacturing decision making needs during these different phases.

*Phase 0:* At this phase firms need to establish the business case for remanufacturing depending on the product attributes. This will trigger product development for remanufacturing.

*Phase I:* During OE production, firms need to establish contracts with dealers and third party collectors of “cores” or used product to establish return flow channels.

*Phase II:* At this stage, firms need to evaluate various decisions. Whether to launch remanufacturing program or keep producing only OE parts to meet new all demand? Decision to launch remanufacturing program depends among other things (e.g., potential margins) the product life cycle, demand pattern for new product, demand pattern for reman product, and availability/reliability of core returns. If firm decides to launch a reman program for the product under consideration, then decisions need to be taken on the timing of the program launch and reman capacity installation and management. In addition, since firm is in a hybrid production state (involving both manufacturing and remanufacturing), production planning and control becomes crucial because material flows from both the channels are dependent on each other. It should be noted here that core returns for OE service parts are often very reliable for they involve a fast trading cycle. The cycle is initiated with the receipt of a core or defective unit by the supplier from the dealer followed by an often overnight or same day delivery of a reman unit to the dealer from very limited finished goods inventory (FGI). The supplier then remans the core (often the same day) and stocks the unit for the next cycle. Given the cycle speed, the OE remanufacturing activity can be relatively efficient, at least from the perspective of core inventory and reman FGI.

*Phase III:* Decisions at this phase are similar to Phase II decisions. Here, high volume OE production is over. Firms need to make decision over launching remanufacturing program for IAM, if not done during Phase II. Depending on the returns from warranty claims,

firms either can launch the remanufacturing for IAM along with reman parts for OE service or wait for more core returns and establishment of core supply contracts with independent collectors. The major difference from Phase II is the significant uncertainty in core returns. Unlike the OE service setting, the trade in process is often not initiated with the receipt of a core but with an order. The reman product is shipped to the customer along with an RMA for the cores and a core charge (customer will not be reimbursed for the core charge until the cores are returned). However, our experience with a major Tier-1 supplier shows that customers can take months and even years to return cores. Hence, inventory management (of cores as well as FGI) becomes more critical as well as overall production planning and control.

## **1.2 Research Objectives**

The objective of this thesis is to develop an integrated framework, for industries supporting OE, OES and IAM business, to guide transition from OE production to hybrid settings. The specific objectives are as follows:

1. To develop models that can facilitate better timing of the launch of remanufacturing program for OE service and IAM and reman capacity planning. This is of particular concern to our collaborator Delphi Automotive LLP. While the literature offers no guidance/models, there are risks associated with both premature launch (reman OES parts are priced differently and the absence of reliable core supply due to premature launch can force the supplier to provide virgin parts in place of reman parts and poor utilization of reman capacity) or delayed launch (lost opportunity of provide reman product).

2. To develop a modelling framework for core-return forecasting to facilitate decision-making at different phases. The prerequisites for this objective are:

- Ability to forecast core returns for product as well as product families; A key requirement here is the ability of the modelling framework to support data sparsity (a lesson learnt from our work with Delphi Automotive LLP)
- Ability to forecast when there is long lag between product shipment/sale and core return
- Ability to support/exploit different levels/sets of information regarding historical sales, return rates, market inventory etc.
- Ability to provide feedback to timing the launch of a new product remanufacturing program and reman capacity planning

### **1.3 Research Scope**

In this dissertation, we assume that the business case for remanufacturing has already been established by the firm. Thus, our study will focus on developing an integrated framework for decision support during phases II of the production life-cycle (see Figure 1.1). In this dissertation, we have considered phase II and III jointly.

Scope of this work includes new models for core return, timing the launch of a remanufacturing program, and capacity planning. We have validated the overall framework and the associate models and methods through case studies with Delphi.



The rest of the dissertation is organized as follows. Chapter 2 presents strategic capacity management of remanufacturing. Chapter 3 offers models for core-returns forecasting. Finally, Section 4 presents conclusion and future research directions.

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## Chapter 2 : **STRATEGIC CAPACITY PLANNING AND MANAGEMENT OF REMANUFACTURED PRODUCTS**

Strategic capacity planning plays an important role in the effective management of product life-cycles and improving their profitability. In particular, decisions related to determining the sizes and timing of capacity investments. To effectively decide on '*timing the launch*', a firm must tradeoff the cost of capacity, supply, and inventories, with the revenues from the product demand over its life cycle. In addition, firm needs to make an important decision at the operations level on '*how much capacity to install*'. These decisions impose more challenges for firms that engage in original equipment (OE) production as well as remanufacturing. The presence of reverse logistic flows magnifies the variability in a supply chain due to uncertainty in timing/location of returns, return volume, quality etc. In other words, the timing and volume of used product returns are binding supply constraints for remanufacturing. Capacity management is, thus, even more complex and critical for supply chains that involve reverse logistics and remanufacturing.

The original motivation for our research came from the request of a leading global tier-1 automotive supplier, Delphi Automotive LLP, engaged in OE production as well as providing products to the aftermarket (both for OE service and the independent aftermarket). Key complications faced by the company were the difficulties in making decisions relating to proper timing of the launch of the reman product program, capacity installation, and efficient management of remanufacturing operations and logistics. Overall, there is recognition for the lack of a structured and holistic decision support

framework that can guide firms in decision making related to both, timing the launch of the remanufacturing program and capacity installation/management.

For a typical automotive product, production during its life cycle can be roughly divided into two phases. Phase I deals with the production of OE parts to support demand for new OEM product and tends to be relatively high volume production. Phase II covers the period of transition from OE parts to both OE parts and service parts including both OE service parts (OES) and eventually independent aftermarket (IAM) demand too. At the end of the regular production cycle, firms usually make a last run production to stock parts to meet the spare-parts availability requirement (in the US, the legal requirement is 15 years from the end of production

One increasingly popular option to support aftermarket demand (partially or fully) has to do with remanufacturing. For firms that engage in OE parts production as well as remanufacturing to support aftermarket services (as is the case with our collaborator, Delphi), it is seldom optimal to start the reman product program with the start of the earliest core returns. The reason being, in the absence of a reliable core supply for remanufacturing due to a premature reman product launch, the supplier is forced to provide new or virgin parts in place of reman parts to cover demand for reman product that exceeds reman production and inventory, a costly affair and in addition results in poor utilization of remanufacturing capacity. Therefore, it is more common for firms to delay the start of the reman program to the end of the OE production cycle. By delaying the launch to the end of the OE production cycle, firm can accumulate enough core returns to build up a large strategic recoverable inventory. This helps in better utilization

of remanufacturing capacity, avoids backorders, and also reduces the need for serviceable inventory of virgin parts. On the contrary, the delayed reman product launch may result in a lost opportunity to provide reman parts for OE service and there is also the possibility of not being able to take advantage of recoverable inventory due to insufficient orders for reman product post OE production. Our collaborator was already implementing the latter option to support demand for independent aftermarket services. Management was interested in knowing whether it is cost-effective to launch the remanufacturing program before the end of the OE production cycle and still be able to effectively utilize the remanufacturing capacity. Our research aims to build models that can effectively answer these types of questions.

Automotive products usually fall under the category of durable products, which means they remain with the customers for a considerable amount of time compared to the time horizon in which they were sold. For such products, demand may be subjected to a dynamic process due to product life cycle effects and models that treat demand to be stationary and address average cost/profit are often inappropriate; a dynamic discounted cash flow framework is more suitable. Further, in the presence of supply constraints, both in terms of availability and yield of returns, it becomes imperative to obtain dynamic optimal policies regarding production and remanufacturing decisions.

In light of the preceding discussion, this research proposes an approach to derive optimal remanufacturing policy and then simultaneously decide on the best time to launch a remanufacturing program and the overall capacity requirement. To the best of our knowledge, this research is a first attempt of its kind in the remanufacturing literature,

as prior research treated these interrelated decisions separately. The primary focus of this study is to develop intuition for drivers of cost-efficient remanufacturing program for aftermarket services while taking life-cycle dynamics into account. The insights are obtained by minimizing the discounted cash outflows caused by appropriate investment and return inventory building decisions. Though a simplistic deterministic sales and return dynamics are analyzed, our analysis of stochastic returns scenario revealed that proposed deterministic approach is sufficient enough to capture the important dynamics of cost-effective remanufacturing programs.

Remainder of this chapter is organized as follows: Section 2.1 outlines related literature. Proposed model is discussed in section 2.2. Numerical investigation is presented in Section 2.3. Finally, conclusion and future research in section 2.4.

## **2.1 Related Literature**

There is a vast body of literature dealing with operational issues of decision making in reverse logistics e.g. material resource planning (Ferrer and Whybark (2001), scheduling and shop floor management (Guide et al. 1997, 1998), inventory control (Van der Laan et al. 1999, Toktay et al. 2000), logistic network design (Fleischmann 2001), and routing (Beullens 2001). We encourage readers to refer a recent survey by Ilgin and Gupta (2010) for detailed overview of this literature. In contrast, today, the important problems of business are not tactical or operational but tend to be strategic and mostly unstructured (Guide, 2006). According to Valchos (2007), despite considerable emphasis over the last decade on long-term strategic management problems in reverse logistics, there are almost

no studies in the literature thus far. Further, one of the most influential aspects of investment decision, financial justification has widely been neglected in most of the studies (Kleber 2006).

Most of the research considering strategic issues in reverse logistics have been confined to network design in a single-period (see, e.g. Barros et al. 1998; Louwers et al. 1999) and less commonly a multi-period (Realf et al. 2004) setting with given product characteristics. Shih (2001) studied reverse logistics planning for electronic products in Taiwan. Using historical data, the author presented a model to determine the optimal capacity expansion plans of storage and disassembly facilities for different product take-back rates. Franke et al. (2005) developed a model for mobile phone remanufacturing to determine the required capacities for remanufacturing operations. They used information about uncertainties in the amount and conditions of returns as well as combinatorial optimization to determine the capacities of work stations. Francas (2009) developed a network configuration model for a multi-product supply chain in which a firm manufactures new products and remanufactures used products. Built on a stochastic programming approach that accounts for uncertainty in demand and returns, they studied capacity investment from a newsvendor network perspective and compare the performance of simultaneous and sequential design. Ryan (2010) developed a single-period model for capacity planning that determines the optimal amount of expansion for different lead times to obtain remanufacturing capacity. They stated that the difference between their research and past work is that they focus jointly on the forecasting and capacity management of returned products. Mutha (2010) presented a mathematical

model for handling product returns. The focus is on deciding the number of facilities, their locations and allocation of corresponding flow of used products and modules at an optimal cost for a given market demand and used product returned quantities. In all these approaches, the decision at which time to set up the respective facilities has already been made or facilities were already in place.

One major stream of capacity planning research in the reverse logistics domain is based on System Dynamics (SD) modeling. Georgiadis et al. (2003) introduce systematically the use of SD methodology in the analysis of closed-loop supply chains (CLSCs). They use a set of level of remanufacturing and collection capacities to study the effect of environmental issues on reverse channel's activities. Georgiadis and Vlachos (2004) further extend that SD model to account for environmental issues such as "green image" and effect of "take-back obligation" on product flows in the reverse channel, while considering the capacity levels exogenously. Vlachos et al. (2007) study capacity expansion policies in the reverse channel of a CLSC with remanufacturing activities assuming stationary demand, hence ignoring the concept of a limited product lifecycle and issues related to capacity contraction. Georgiadis et al. (2006) make a first attempt towards a more holistic approach, developing an SD model for a single product CLSC with remanufacturing activities in the reverse channel. They analyze the capacity planning policies both for collection and remanufacturing activities in the reverse channel, assuming that demand may follow different but standard lifecycle patterns consisting of the introduction, growth, maturity and decline stages. Specifically, they investigate how the lifecycle and return patterns of a product affect the near-optimal

capacity planning policies regarding expansion and contraction of collection and remanufacturing capacities. Georgiadis et al. (2010) further the earlier models by studying the capacity planning policies in the reverse channel for a portfolio of new and remanufactured versions of two sequential product-types (types 1 and 2). They investigated how different product lifecycles and different patterns of product returns affect the near-optimal expansion and contraction capacity planning policies for the collection and remanufacturing activities of two sequential product-types, under two alternative scenarios regarding the market preferences over them.

Debo et al. (2006) also captured life-cycle dynamics in the introduction and management of remanufactured products. They extended the Bass diffusion model in a way that maintains the two essential features of remanufacturing settings: (a) substitution between new and remanufactured products, and (b) a constraint on the diffusion of remanufactured products due to the limited supply of used products that can be remanufactured. They identified characteristics of the diffusion paths of new and remanufactured products and analyzed the impact of levers such as remanufacturability level, capacity profile and reverse channel speed on profitability.

To the best of our knowledge, the only research that has explicitly modeled reman product launch timing in reverse logistics is Kleber (2006). They focused on the timing of investment decisions, and concluded that by neglecting facility location and detailed capacity acquisition, for instance expenses for setting up facilities are set in such a way that a sufficient capacity is available, general insights can be obtained using an analytical approach.



To summarize this chapter makes several contributions to the literature. In this research we have focused on explicitly modeling both capacities as well timing of the launch of a remanufacturing program for a new product. Further, we also present the optimal remanufacturing policy and drivers of cost-effective remanufacturing program.

## **2.2 Capacity Planning Model**

The OEM's objective in reman capacity planning is to minimize the life-cycle cost of the reman program in supporting demand for service parts (both OE service for products under warranty and independent aftermarket demand for product out of warranty). To pursue this, we present a continuous time, finite-horizon, discounted cash outflow problem that attempts to satisfy all demand for service parts during the planning horizon at the lowest cost. This section first presents the necessary assumptions regarding product life-cycle and reverse channel flows and demands. We then provide a formulation for the OEM's aftermarket services optimization problem and characterize the optimal reman operations policy.

First, we will introduce the base case model with no remanufacturing option and then model the case with remanufacturing.

### **2.2.1 Base Case without Remanufacturing**

Consider firm introduces the product to the primary OE market at time  $t = 0$  and that OE sales evolve over the duration of the product life-cycle with rate  $q(t)$ . Our analysis assumes that  $q(t)$  is unimodal, deterministic, non-negative, and known. Given the

strategic nature of the capacity planning process, the assumption of deterministic sales rate is reasonable. The product resides for a finite period of time with the customer and can be referred to as residence time. We assume that residence time is a function of product durability characteristics and is randomly distributed with density function  $h(\Delta t)$ . Failure of the unit at the end of its residence time leads to service that triggers order for a replenishment service part. The demand for service parts  $d(t)$  can be described as a convolution of  $q(t)$  and  $h(\Delta t)$  (Geyer et al. 2007):

$$d(t) = \int_0^t q(s)h(t-s)ds \quad (1)$$

Initially, demand for service parts can be fulfilled by acquiring a “virgin” part from the OE production line. At the end of the OE production run,  $T_p$ , OEM makes a “last run” to support future demand for service parts and holds this inventory of virgin serviceable parts  $i_s(t)$  at a cost of  $h_n$  per unit per unit time. Thus, the net present value of the total discounted cash outflows to cover aftermarket services can be calculated as:

$$NPV_{base} = \int_{t_0}^{T_p} e^{-t\theta} (c_n d(t)) dt + e^{-T_p\theta} \int_{T_p}^T c_n d(t) dt + \int_{T_p}^T (h_n i_s(t)) dt \quad (2)$$

where,  $\theta$  is a discounting factor and  $T$  denotes the planning horizon.

### 2.2.2 Remanufacturing Case

Here firm tries to rely on remanufacturing to support demand for service parts. To pursue this, firm relies on dealers and repair shops (through contractual or other means) for used product or “core” returns to establish the remanufacturing program. This is essentially a trade-in process where the supplier provides a service part for a core return. We assume

here that the trade-in cycle is instantaneous or negligible as compared to product residence time or the life-cycle. This is a reasonable assumption for OE service parts under warranty, where dealers often return the cores to the OEM within days. There can however be significant delays in receiving cores from the independent aftermarket with even the possibility of permanent core loss to independent aftermarket companies. Future work will account for these losses and delays. Hence, we assume that a core is available to the firm for reman exactly at the end of its residence time. We also assume that all OE product generates demand for service parts at the end of their residence time. Thus, we can conclude that return rate  $v(t)$  is equal to demand for service parts  $d(t)$ . Henceforth, return rate  $v(t)$  will also be used to denote demand for service parts  $d(t)$ .

Firm, given the business case for aftermarket service remanufacturing, initiates core collection at time  $t_0$  to launch reman for services at time  $t_l(\geq t_0)$  with remanufacturing capacity level  $C$ . Let  $c_c$  be the variable cost of acquiring and maintaining one unit of capacity per unit time and  $c_a$  denotes the cost of core acquisition per unit including inspection and disassembly. Upon receipt of a core, and depending on whether remanufacturing program is already launched, the firm either processes the core to build up recoverable inventory  $i_r(t)$  of components/modules to be remanufactured at a future time or instantaneously remanufactures it with rate  $f_r(t)$  to fulfill immediate demand. Cost of holding one unit of recoverable inventory for unit of time is  $h_r$  and cost of remanufacturing per unit is  $c_r$ . By instantaneously, we mean here that there is no delay between pre-processing, order release and materials availability. Given that the firm

cannot remanufacture at a rate that exceeds the installed capacity level, constraint (3) limits the remanufacturing rate.

$$f_r(t) \leq C \quad (3)$$

To keep our analysis simpler, we assume that the firm never carries any remanned finished goods inventory (FGI), leading to constraint (4). This assumption is partially reasonable due to the fact that holding cost for serviceable inventory always exceeds the cost of holding recoverable inventory. In the presence of significant remanufacturing process lead times might warrant some FGI. Future work will address this case.

$$f_r(t) \leq v(t) \quad (4)$$

Yield issues are typical in most remanufacturing industries given that not all cores are viable candidates for remanufacturing (attributable to such factors as use or abuse of the product by the original customer and nature of the product). Let  $\gamma$  denote the remanufacturing yield percentage. Firm can to some extent control  $\gamma$  based on product design, materials/processes employed and so on. We assume here that  $\gamma$  is a product characteristic, deterministic, and known. Furthermore, we also assume here that a part can be remanufactured at most once during its life-cycle. Since,  $0 < \gamma \leq 1$ , remanufactured product from core returns in any period cannot exceed demand for service parts within that period. This combined with constrains (3) and (4), lead to the following upper and lower bounds on  $f_r(t)$ :

$$\min(\gamma * v(t), C) \leq f_r(t) \leq \min(v(t), C) \quad (5)$$

Now, we can add another constraint relating to rate of change of recoverable inventory:

$$\overline{i_r(t)} = \gamma * v(t) - f_r(t) \quad (6)$$

Constraint (6) implies that during pre-launch (before launch of the remanufacturing program), rate of change of recoverable inventory equals recovered inventory ( $f_r(t) = 0$ ). Whereas, post launch, it is the difference between rate of recoverable inventory and rate of remanufacturing.

Similar to the base case, during the phase of regular OE production, any excess demand for service parts beyond the rate of remanufacturing is met by acquiring virgin parts from manufacturing at a rate  $f_n(t)$ , at the cost of  $c_n$  per unit.

$$f_n(t) = v(t) - f_r(t) \quad \forall t \leq T_p \quad (7)$$

After the end of OE production cycle, any shortage is met by depleting inventory of virgin serviceable parts  $i_s(T_p)$ . We can then write an expression for the total service operations cost as:

$$TC_{reman}(t_l, C) = \int_{t_o}^T \left\{ c_n f_n(t) + c_r f_r(t) + c_a d(t) + h_s i_s(t) + h_r i_r(t) + c_c C \right\} dt \quad (8)$$

It should be noted here that for the remanufacturing case, we are not calculating the *NPV*, but the total cost of remanufacturing. This enables easier computation of the optimal remanufacturing policy. Once the optimal remanufacturing policy is obtained (i.e.,  $f_r^*(t)$ ), determination of the optimal capacity and launch timing parameters are deduced from minimizing the *NPV* of the total discounted cash flows within the planning horizon. Table 2.1 summarizes our key notations and figure 2.1 presents an illustrative example of the dynamics of sales, returns and yield during a typical product's life-cycle.

Table 2.1: Summary of Key Notations

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$t_l$ :	Time to launch of reman program
$C$ :	Remanufacturing capacity level
$c_n$ :	Cost of manufacturing one unit of virgin parts
$c_r$ :	Cost of remanufacturing one unit of reman parts
$c_c$ :	Variable cost acquiring and maintaining one unit of capacity per unit of time
$c_a$ :	Cost of acquisition of unit core
$h_r$ :	Cost of holding one unit recoverable inventory for per unit of time
$h_n$ :	Cost of holding one unit serviceable inventory for per unit of time
$\gamma$ :	Yield percentage
$\theta$ :	Discount factor
$\mu_{h(\Delta t)}$ :	Mean of residence time distribution
$\sigma_{h(\Delta t)}$ :	Standard deviation of residence time distribution
$T_p$ :	Time at end of production
$T$ :	Time horizon
$q(t)$ :	Sales rate
$h(\Delta t)$ :	Residence time distribution
$d(t)$ :	Demand for service parts
$v(t)$ :	Return rate
$f_r(t)$ :	Remanufacturing rate

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 $i_r(t)$ : Recoverable inventory rate

 $i_s(t)$ : Serviceable inventory rate

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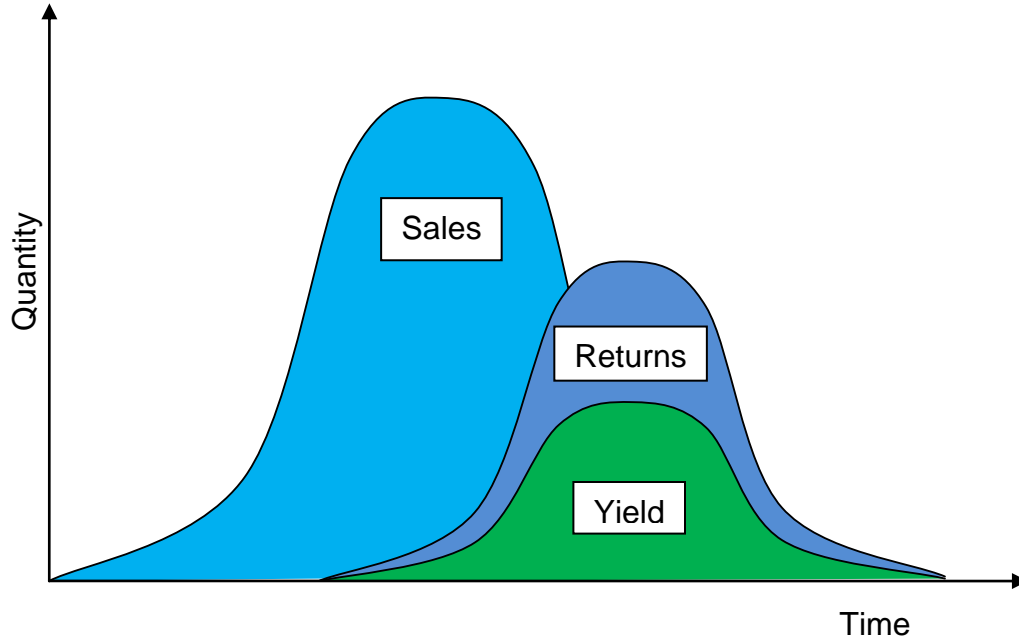


Figure 2.1: Illustrative example showing sales, returns and yield during a product's life-cycle

### 2.2.3 Optimization Problem

Even for the deterministic case, finding the optimal reman rate trajectory (i.e.,  $f_r(t)$ ) becomes intractable since many interacting effects determine the optimal path. We thus propose a simplification to the problem setting. We will assume that there is possibility of procuring virgin parts even after the end of the OE production run. This might be justified for two reasons. First, an external party might be willing to produce the parts (using maybe OE tooling). Secondly, if the OE facility has moved on to the next generation of

the product, it might still be able to support intermittent runs to build OE service parts on the same line. This is a common practice in many companies, such as Dana Holding Corporation (a leading global supplier of axles, drive-shafts among other systems to many automotive OEMs) and Continental AG (a supplier of chassis, safety, powertrain and interior systems among other things to automotive OEMs and other industries). In light of this assumption, we can rewrite the cost model equations to eliminate the serviceable inventory terms. Later in section 2.3, we discuss a solution algorithm to optimize the policy parameters, including estimation of serviceable inventory. Under the stated assumption, (8) can be re-written as:

$$TC_{reman} = \int_{t_0}^T \left\{ \begin{array}{l} c_n f_n(t) + c_r f_r(t) \\ + c_a v(t) + h_r i_r(t) + c_c C \end{array} \right\} dt \quad (9)$$

For a given  $t_l$  and  $C$ , we can choose  $f_r(t)$  to minimize the total cost:

$$TC_{reman}^*(t_l, C) = \min_{f_r(t) \geq 0} \int_{t_0}^T \left\{ \begin{array}{l} c_n(v(t) - f_r(t)) \\ + c_r f_r(t) + c_a v(t) + h_r i_r(t) + c_c C \end{array} \right\} dt \quad (10)$$

Once the optimal reman rate trajectory is derived (i.e.,  $f_r^*(t)$ ), we can derive the optimal policy parameters  $t_l$  and  $C$  by minimizing the total program cost as:

$$NPV_{reman}^* = \min_{t_l, C} \int_{t_0}^T e^{-t\theta} \left\{ \begin{array}{l} c_n(v(t) - f_r(t)) + c_r f_r(t) \\ + c_a v(t) + h_s i_s(t) + h_r i_r(t) + c_c C \end{array} \right\} dt \quad (11)$$

### 2.3. Optimal Policy

This section first presents the derivation of the optimal remanufacturing policy followed by optimization of launch timing and capacity parameters. We then discuss the structural properties of the optimal policy.



To obtain the optimal trajectory for  $f_r(t)$ , we partition the planning horizon into two regions based on the reman production launch timing: pre-launch and post-launch as illustrated in Figure 2.2. During the pre-launch phase, the optimal policy is obviously to meet all the demand for service parts using virgin parts. Thus, optimal remanufacturing is  $f_r^*(t) = 0$  and recovered cores will be stored into recoverable reman inventory and can be expressed as  $i_r(t) = \gamma v(t)$ .

Post-launch dynamics are far more involved. The decision is to choose optimal remanufacturing quantity  $f_r(t)$  that minimizes the  $TC_{reman}$  given fixed  $C$  and  $t_l$ . In the optimal control framework, this problem can be presented as minimization of the cost functional with state variable  $f_r(t)$  and can be solved using Pontryagin's minimum principle<sup>2</sup>.

$$Z = \min_{f_r(t) \geq 0} TC_{reman} \quad (12)$$

subject to control variable  $i_r(t)$ :

$$\overline{i_r(t)} = \gamma * v(t) - f_r(t) \quad (12.1)$$

$$\overline{V(t)} = v(t) \quad (12.2)$$

$$i_r(t_l) = V(t_l) \quad (12.3)$$

$$\min(\gamma * v(t), C) \leq f_r(t) \leq \min(v(t), C) \quad (12.4)$$

Equation (12.1) accounts for marginal increase/decrease in cumulative inventory at time  $t$  as the difference of recovered core rate and remanufacturing rate. Equation

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<sup>2</sup> Pontryagin's minimum principle is used in optimal control theory to find the best possible control for taking a dynamical system from one state to another, especially in the presence of constraints for state or input controls.

(12.2) is time derivative of return rate. Equations (12.3) and (12.4) form boundary conditions on control and state variables.

*Proposition 1. For any given  $C$  and  $t_l$ , the optimal remanufacturing rate is given by:*

$$f_r^*(t) = \begin{cases} \min(\gamma v(t), C) & i_r(t) = 0 \\ \min(v(t), C) & i_r(t) > 0 \end{cases} \quad (13)$$

*Proof: See Appendix*

Proposition 1 suggests that when there is no recoverable inventory, the only choice a firm has is to remanufacture returning cores, limited of course by reman capacity  $C$  (any excess in recovered enter recoverable inventory). However, when there is positive recoverable inventory, there are three possible scenarios: 1) if returns are less than capacity level, then remanufacture recovered cores and any surplus demand can be fulfilled by acquiring virgin units from manufacturing; 2) if capacity level is less than returns but more than recovered cores, remanufacture up to capacity level using recovered cores and recoverable remanufacturing inventory; 3) if capacity level is less than returns as well as recovered cores, remanufacture up to capacity level using recovered cores and any extra units enter recoverable reman inventory.

Figure 2.2 presents an illustrative example of these dynamics. We can see from Figure 2.2 that during pre-launch all the demand is fulfilled using virgin parts;  $f_n(t) = v(t)$ . Pre-launch, all recovered cores enter recoverable reman inventory,  $i_r(t) = \gamma v(t)$ . Once remanufacturing is launched at time  $t_l$ , stored recoverable reman inventory is depleted to meet all the demand until recoverable inventory becomes zero at some time  $t_{i1}$ . During time period  $[t_{i1}, t_1)$ , both  $v(t), \gamma v(t) < C$ , firm remains available recovered cores to meet the partial demand and any excess demand is fulfilled with virgin

parts.  $v(t) > C$  and  $\gamma v(t) < C$  in time period  $[t_1, t_2)$ , so firm continues to reman available recovered cores while fulfilling surplus demand by acquiring virgin parts. Firm runs at full capacity when both  $v(t), \gamma v(t) > C$ , in time period  $[t_2, t_3)$  by just remanufacturing recovered cores. Here, recoverable inventory again starts building up with  $i_r(t) = \gamma v(t) - C$ . The recoverable inventory build up during this period is depleted until it becomes zero at some time  $t_{i2}$ . Remanufacturing during period  $[t_3, T]$  follows the same pattern as in time period  $[t_l, t_3]$ . In this particular case, it is optimal to reman all recovered cores. This example clearly shows that optimal remanufacturing rate profile depends on vector  $\mathbb{I}(t_l, t_{i1}, t_1, t_2, t_3, t_4, t_{i2})$ .

### 2.3.1 Optimization of Policy Parameters

In general, without imposing strict assumptions, it is not possible to estimate  $\mathbb{I}(t_l, t_{i1}, t_1, t_2, t_3, t_4, t_{i2})$  as a closed-form solution. Hence, we rely on numerical analysis to derive some additional structural properties. In this section, we present a solution algorithm in Table 2.2, incorporating serviceable inventory, to compute  $C^*$  and  $t_l^*$ .

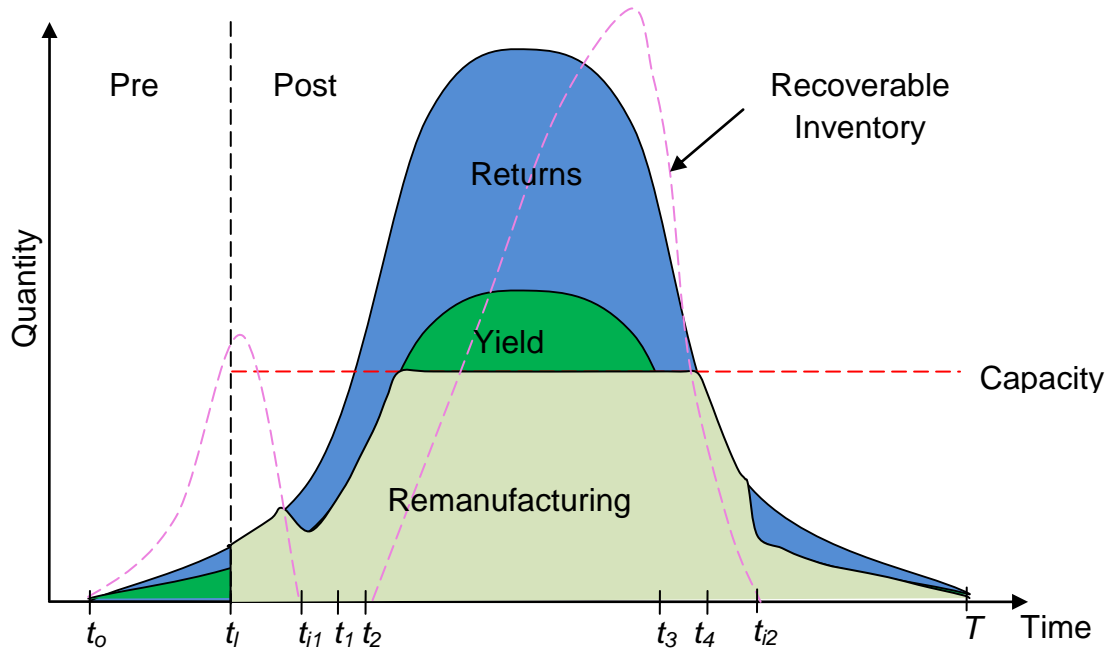


Figure 2.2: Illustrative example showing returns, yield, optimal remanufacturing, and recoverable service inventory profiles at different stages of the product life-cycle

Table 2.2: Heuristic algorithm to compute optimal policy parameters  $C^*$  and  $t_l^*$

---

Step 1. For a given  $C$  and  $t_l$ , compute optimal reman trajectory  $f_r^*(t)$  and the

corresponding  $f_n^*(t)$  by solving equation (12)

Step 2. Compute  $F_n(T_p) = \sum_{t=T_p}^T f_n(t)$

Step 3. Replace  $f_n(t) = 0 \forall t > T_p$

Step 4: Construct a new variable vector  $i_s(t) = \begin{cases} 0 & \forall t < T_p \\ F_n(T_p) - f_n(t) & \forall t \geq T_p \end{cases} \quad (14)$

Step 5. Solve equation (10) to compute  $NPV(C, t_l)$

Step 6. Repeat steps 1-5 for different  $(C, t_l)$

Step 7. Compute  $\arg \min(NPV(C, t_l))$  to obtain  $C^*$  and  $t_l^*$

---

### 2.3.2 Structural Properties of the Optimal Policy

We analytically derive structural properties of the optimal policy under few special cases of  $\mathbb{T}(t_l, t_{i1}, t_1, t_2, t_3, t_4, t_{i2})$ . Then, based on these results, we characterize the optimal policy for general settings.

*Proposition 2.*  $\mathbb{T}$  is a solution candidate to (12) if the following conditions are satisfied:

$$t_{i1} = t_2 \quad (15)$$

$$t_{i2} = T \quad (16)$$

*Proof:*

Given that  $t_{i1} = t_2$ , the recoverable inventory built up before pre-launch is completely exhausted before  $\gamma v(t) \geq C$ . Thus, the following equation holds:

$$\int_{t_0}^{t_l} (\gamma v(t)) dt = \int_{t_l}^{t_1} (v(t) - \gamma v(t)) dt + \int_{t_1}^{t_{i1}} (C - \gamma v(t)) dt \quad (17)$$

Whereas, the condition  $t_{i2} = T$  states that the entire recoverable inventory built up, when  $f_r^*(t) = C$  and  $\gamma v(t) \geq C$ , is used up to satisfy the surplus demand over instantaneous reman rate until the end of the planning horizon. Then,

$$\int_{t_{i1}}^{t_3} (\gamma v(t) - C) dt = \int_{t_3}^{t_4} (C - \gamma v(t)) dt + \int_{t_4}^{t_{i2}} (v(t) - \gamma v(t)) dt \quad (18)$$

Equation (18) discloses that the area formed by  $[C, \gamma v(t)]$  is equals to the area formed by  $[Cv(t), \gamma v(t)]$ . Henceforth, this will be referred to as equilibrium of  $\Delta_{C, \gamma v(t)}$  and  $\Delta_{Cv(t), \gamma v(t)}$ . In conclusion, maximum possible remanufacturing is possible in this scenario. Mathematically,

$$F_r = \gamma V(t) \quad (19)$$

Hence the proof. ♦

By solving equation (18), a threshold value for  $C$  can be obtained. Once we have  $C_{threshold}$ , then  $t_1, t_2$  can be computed by solving the following two equations, respectively:

$$C_{threshold} = v(t_1) \quad (20)$$

$$C_{threshold} = \gamma v(t_2) \quad (21)$$

Inserting values of  $t_1, t_2$  and  $C_{threshold}$  into equation (17), results in threshold value for  $t_l$ :

$$t_{l_{threshold}} = v^{-1} \left( \frac{\theta \cdot c_c \cdot C_{threshold}}{\gamma(c_n - c_r)} \right) \quad (22)$$

Based on  $C_{threshold}$  and  $t_{l_{threshold}}$ , we can now characterize the optimal policy for generic settings, which is a “threshold policy” in  $C$  and  $t_l$ .

*Proposition 3.* For  $C \geq C_{threshold}$  and  $t_l \leq t_{l_{threshold}}$ ,  $F_r = \gamma V(t)$

*Proof:*  $t_l \leq t_{l_{threshold}} \Rightarrow t_{i1} \leq t_2$  and  $C \geq C_{threshold} \Rightarrow t_{i2} \leq T$ . Thus,  $F_r = \gamma V(t)$ . ♦

*Proposition 4.* For  $C < C_{threshold}$  and  $t_l < t_{l_{threshold}}$ , or for  $C < C_{threshold}$  and  $t_l > t_{l_{threshold}}$ ;  $F_r < \gamma V(t)$

*Proof:*  $t_l < t_{l_{threshold}} \Rightarrow t_{i1} \leq t_2$  and  $C < C_{threshold} \Rightarrow t_{i2} > T$ . Thus,  $F_r < \gamma V(t)$ .

$t_l > t_{l_{threshold}} \Rightarrow t_{i1} > t_2$  and  $C < C_{threshold} \Rightarrow t_{i2} > T$ . Thus,  $F_r < \gamma V(t)$ . ♦

*Proposition 5.* For  $C > C_{threshold}$  and  $t_l > t_{l_{threshold}}$ ,  $F_r \leq \gamma V(t)$

*Proof:*  $t_l > t_{l_{threshold}} \Rightarrow t_{i1} > t_2$  and  $C > C_{threshold} \Rightarrow t_{i2} = T$ . Thus,  $F_r = \gamma V(t)$ .

$t_l > t_{l_{threshold}} \Rightarrow t_{i1} > t_2$  and  $C > C_{threshold} \Rightarrow t_{i2} > T$ . Thus,  $F_r < \gamma V(t)$ . ♦

## 2.4. Numerical Investigation

In this section, first we numerically examine the structural properties of the optimal reman policy given a fixed  $C$  and  $t_l$ . We then investigate the effect of cost and life-cycle parameters on  $C^*$ ,  $t_l^*$ , and the expected savings from launching the reman program for aftermarket services. It should be noted here that the primary focus in doing this is to develop a good intuition for drivers of cost-efficient remanufacturing program for aftermarket services. Since this study entertains the possibility of a reman program launch before the end-of the OE production run, the analysis is particularly relevant to parts for which aftermarket services start well before the last run or end-of the production of virgin parts.

For all our experiments, we employ a trapezoidal sales rate function with total sales of 30M units and a product life-cycle of 8 years. To better represent real-life operations, we allow for a faster growth phase, a long maturity phase, and a slow decline phase. A gamma distribution is used to represent the residence time distribution for its flexibility. The parameter settings and their ranges (partly including extreme values) are reported in Table 2.3. In selecting the parameter settings, it should be noted that we tried our best to capture some real-life scenarios from the automotive industry (e.g.,  $h_n$  and  $h_r$  are in the range of 12% of  $c_n$  and  $c_r$ , respectively). Since the effects of changes in the interacting parameters are manyfold, we decided to perform the study based on a large number of randomly generated examples.

Table 2.3: Parameter settings employed for numerical experiments

Parameter Ranges/Settings	
$c_n = [1]$	$\gamma = [0.2-0.8]$
$c_r = [0.6]$	$\theta = [0.01]$
$c_c = [0.01-0.3]$	$\mu_{h(\Delta t)} = [24, 48, 60]$
$c_a = [0.05]$	$\sigma_{h(\Delta t)} = [24]$
$h_r = [0.01-0.11]$	$T_p = [96]$
$h_n = [0.05-0.15]$	$T = [272]$

We caution here that while care has been exercised in conducting these numerical experiments to best extract and illustrate the dynamics at play, all the while coping with a large number of parameters, the patterns/effects reported can change somewhat as a function of the parameter levels. However, the essential dynamics/insights from these results are expected to hold strongly in most settings.

The section is organized as follows: section 2.4.1 illustrates the characterization of optimal decision surfaces and optimal costs; section 2.4.2 outlines the effect of different costs and life cycle parameters on  $t_l^*$  for a given  $C$ .

#### 2.4.1 Structural Properties of the Optimal Solution and Optimal Cost

We numerically investigate the structure of the optimal decision surface as presented in the propositions 3-5. Following parameter set is used to generate the plots:  $C =$



$[0, 2.2 \times 10^4], t_l = [10 - 96], c_n = 1, c_r = 0.6, c_c = 0.1, c_a = 0.05, h_r = 0.05, h_n = 0.09, \gamma = 0.5, \theta = 0.01,$   
 $\mu_{h(\Delta t)} = 60.$

Figure 2.3 presents the structure of optimal decision for combination of parameters listed above. Figure clearly shows that the optimal policy is a threshold policy in  $C$  and  $t_l$ . As we see from the figure for  $C=15,200$  and  $t_l=58$  are the respective values of  $C_{threshold}$  and  $t_{l_{threshold}}$ . To facilitate better understanding of the associated dynamics, we will now investigate the optimal decision at  $C_{threshold}=15,200$  and  $t_{l_{threshold}}=58$ . Figure 2.4 shows the trajectory of remanufacturing  $f_r(t)$ , along with demand  $v(t)$ , and recoverable inventory  $i_r(t)$  at the  $C = C_{threshold}$  and  $t_l = t_{l_{threshold}}$ . The recoverable inventory built up during pre-launch is consumed well before time  $t_2$ . Conversely, a careful assessment of the figure also discloses that equilibrium of areas  $\Delta_{C,\gamma v(t)}$  and  $\Delta_{C,v(t),\gamma v(t)}$  is achieved. This means, recoverable inventory built up during time period  $[t_2 - t_3]$  is depleted to support surplus demand over recovered cores in the time period  $[t_3 - T]$ . In conclusion, it is optimal to remanufacture all returns after yield, which is in accordance with structural property proposition 3.

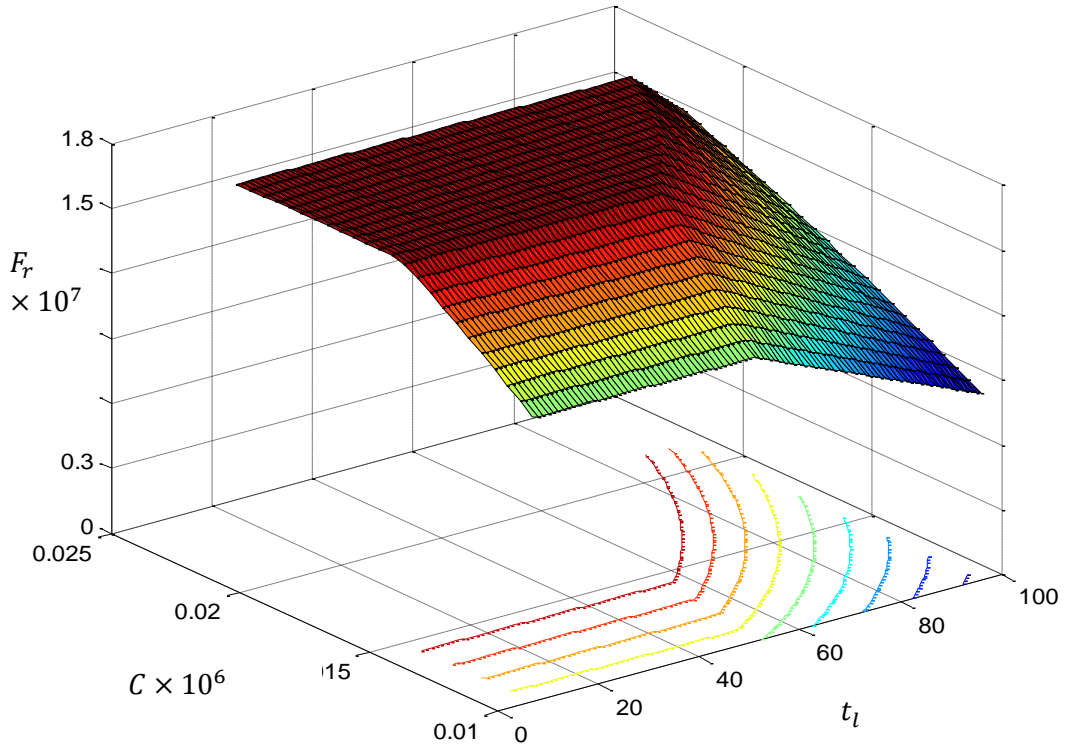


Figure 2.3: Optimal total reman volume as a function of  $C$  and  $t_l$

We can see from the plot that any deviation from the thresholds  $t_l \leq t_{l_{threshold}}$  and  $C \geq C_{threshold}$ , may either result in  $i_r(t_2) > 0$ , or lead to an imbalance of areas  $\Delta_{C,\gamma v(t)}$  and  $\Delta_{C,v(t),\gamma v(t)}$ . The result being the total volume of remanufactured parts comes down, leaving recoverable inventory at time  $T$ . To fix that, we present three scenarios discussed in propositions 4 and 5; i)  $t_l < t_{l_{threshold}}$  and  $C < C_{threshold}$ , ii)  $t_l > t_{l_{threshold}}$  and  $C < C_{threshold}$  and iii)  $t_l > t_{l_{threshold}}$  and  $C > C_{threshold}$ .

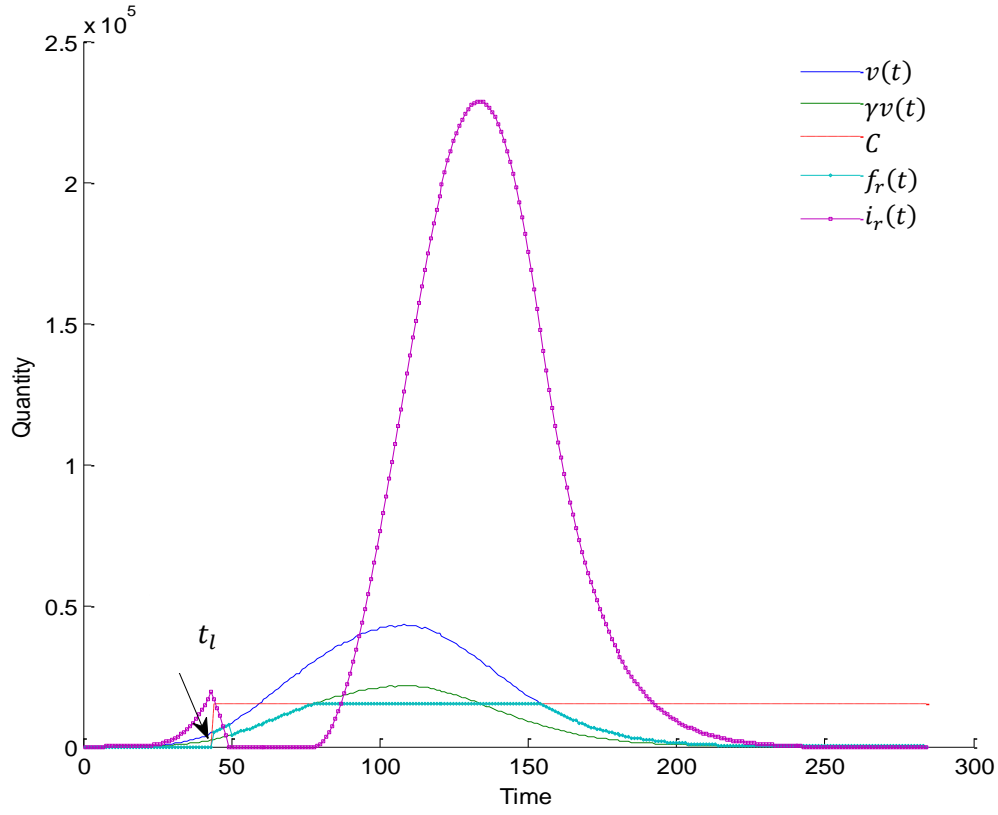


Figure 2.4: Trajectory of optimal states and control variables at  $C_{threshold}$  and  $t_{l_{threshold}}$

As can be seen from Figure 2.5, the serviceable inventory built up during the pre-launch phase is completely depleted well before  $t_2$ . Now,  $C < C_{threshold}$  means  $\Delta_{C,\gamma^*v(t)} > \Delta_{C,v(t),\gamma^*v(t)}$ , resulting in collection of more recoverable cores during the time period  $[t_2, t_3]$  than required during the time period  $[t_3, t_4]$ . Thus, it is not optimal to remanufacture all the recoverable service inventory, all in accordance with proposition 4.

Figure 2.6 shows that when  $t_l > t_{l_{threshold}}$ , due to delay in the reman program launch, a bigger recoverable inventory is built up and before it could be exhausted completely, firm starts to operate at full capacity level,  $f_r^*(t) = C$ . This imbalances the

desired equilibrium,  $i_r(t_2) + \Delta_{C,\gamma^*v(t)} > \Delta_{C,v(t),\gamma^*v(t)}$  and thus resulting in  $i_r(T) > 0$ . Therefore, again it is not optimal to remanufacture all recoverable cores and is in accordance with proposition 4 of the structural properties.

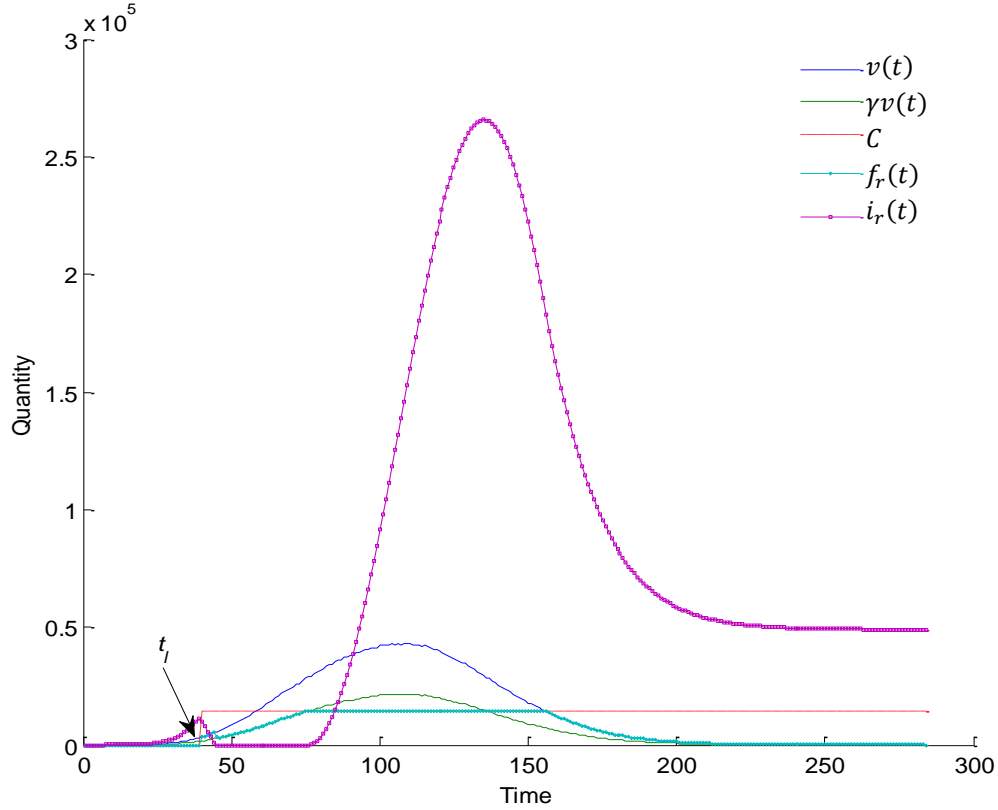


Figure 2.5: Trajectory of optimal states and control variables at  $C < C_{threshold}$  and  $t_l < t_{l_{threshold}}$

But, if both  $C$  and  $t_l$  is increased simultaneously, as in proposition 5 of the structural property, from the threshold value, it is still optimal to remanufacture all recoverable cores. In this case, recoverable inventory built up during pre-launch would be non-zero at  $t_2$ . However, this inventory is used to compensate for the difference in the area

$\Delta_{C,\gamma^*v(t)}$  and  $\Delta_{C,v(t),\gamma^*v(t)}$  in such a way that  $i_r(t_2) + \Delta_{C,\gamma^*v(t)} = \Delta_{C,v(t),\gamma^*v(t)}$ . Figure 2.7 shows this underlying scenario.

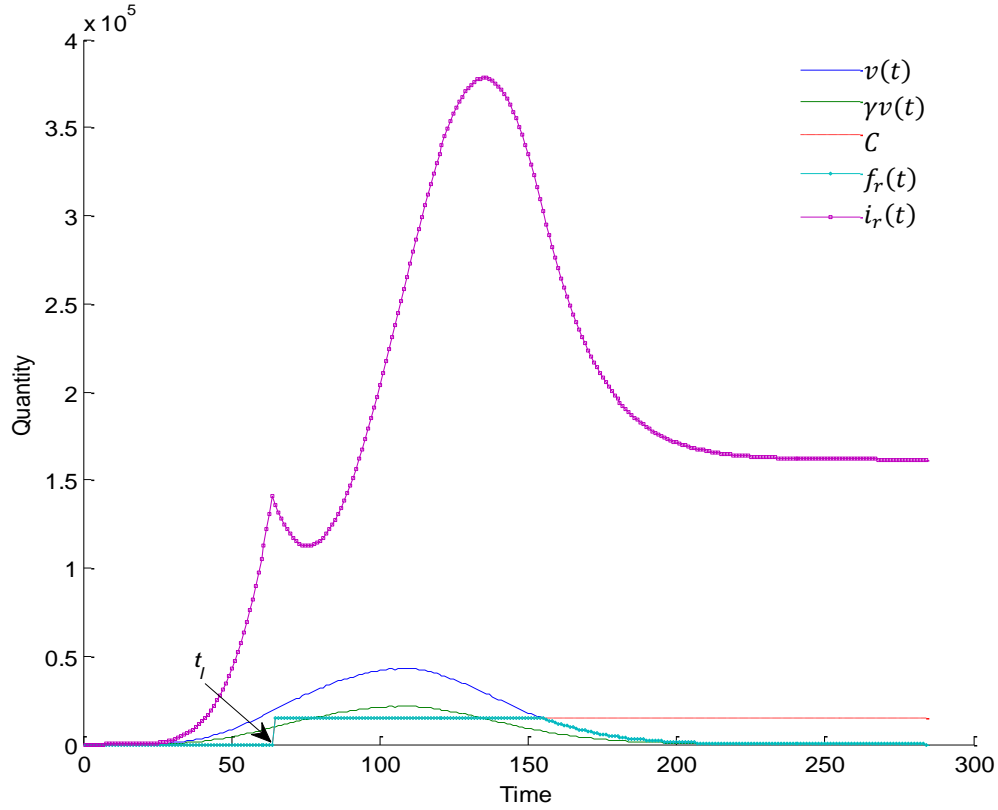


Figure 2.6: Trajectory of optimal states and control variables at  $C < C_{threshold}$  and  $t_l > t_{l_{threshold}}$

Next, we present the corresponding NPV associated with the optimal decision. Figure 2.8 shows the optimal NPV surface. The optimal total cost is  $2.17 \times 10^7$  for  $C = 15,200$  and  $t_l = 44$ . Please note here that this solution was found at a higher resolution in step of 10 in the interval  $C \in (14000, 16000)$ . As expected, the optimal minimum is found in the region satisfying proposition 3 of the structural property. The optimal minimum suggests that it

is judicious decision is to delay the launch of remanufactured parts while satisfying  $t_l \leq t_{l_{threshold}}$ . By delaying the launch, a strategic level of recoverable core inventory is built and thus needs of virgin parts are reduced after launch of remanufacturing program. Whereas, installing corresponding capacity level at  $C_{threshold}$  reduces otherwise high virgin serviceable parts inventory level needed after end-of the production cycle.

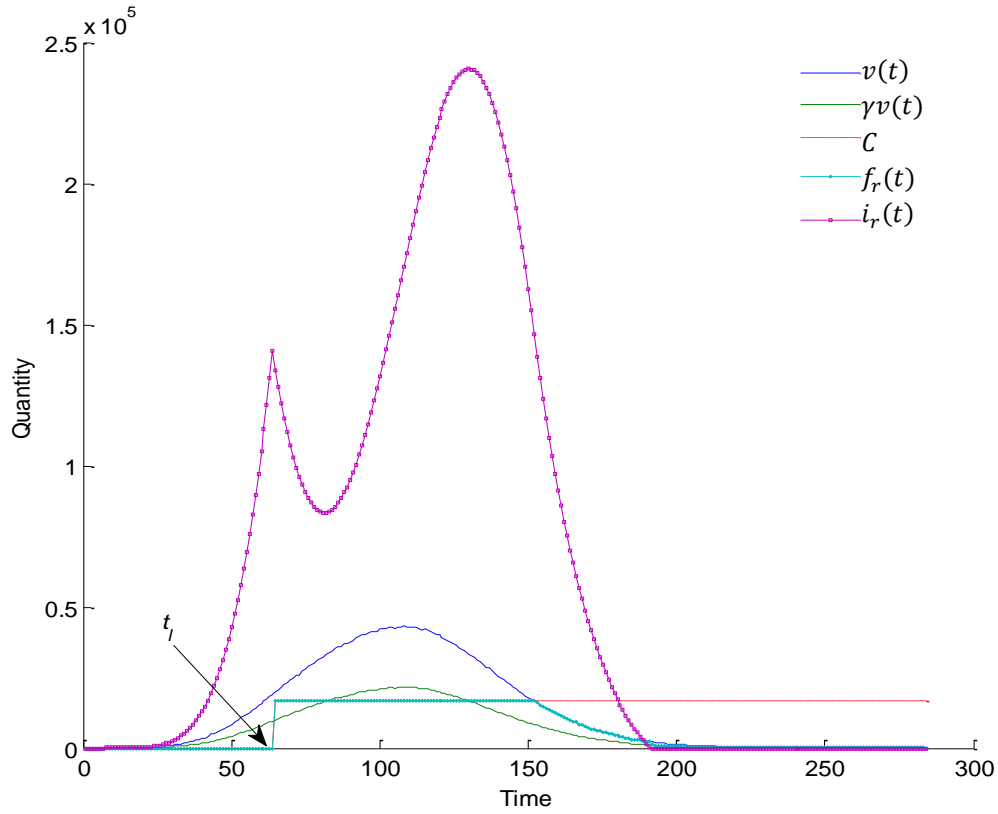


Figure 2.7: Trajectory of optimal states and control variables at  $C > C_{threshold}$  and  $t_l > t_{l_{threshold}}$

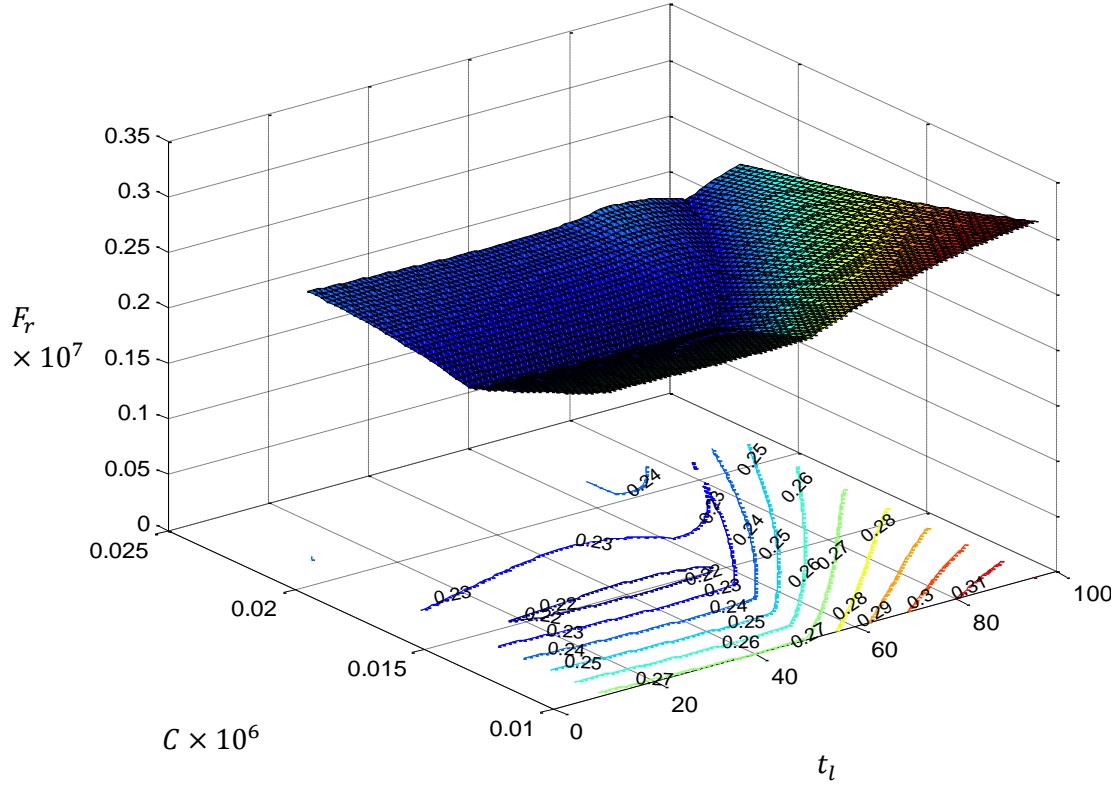


Figure 2.8: The structure of the optimal NPV

According to proposition 4, when  $t_l < t_{l_{threshold}}$ , since capacity is less than the  $C_{threshold}$  that means manufacturing of more virgin parts, non-zero recoverable core inventory at  $T$ , and also virgin parts inventory will be held for longer period because  $t_4$  is shifted to the right. From cost perspective, although cost is saved by installing less capacity, but it doesn't compensate for increase in cost due to total manufacturing cost, total recoverable inventory holding cost and total serviceable inventory holding cost as compared to case in proposition 3. For case, when  $t_l > t_{l_{threshold}}$ , all costs is increased as in case  $t_l < t_{l_{threshold}}$ , including more recoverable inventory at  $T$  because  $i_r(t_2) \neq 0$ .

For proposition 5, we have two options, one suggests remanufacturing all recoverable cores and other advocates it is optimal to produce less. In the first sub-case, in comparison to proposition 3, cost is incurred due to increase in total capacity cost and also pre-launch recoverable inventory holding cost. For second sub-case, cost is incurred as described for proposition 4. Arguably cost incurred due to surplus recoverable inventory could have been reduced if we had considered disposal activity in our model. On the contrary, costs still have increased for all the cases as compared to proposition 3 due to inclusion of disposal cost of surplus cores. It should be noted here that in the preceding analysis we kept all the parameters at nominal level otherwise it is trade-off between core holding cost vs. virgin part inventory cost vs. capacity installation cost. We will investigate this in next sub-section.

#### 2.4.2 Effect of Parameters on Optimal Time to Launch and Optimal Capacity

In this section we will present the effect of each parameter-  $h_r, h_n, c_c, \gamma, \mu_h(\Delta t)$  on  $C, t_l$ , and %Rel.Savings. %Rel.Savings is calculated as follows:

$$\% \text{ Rel. Savings} = \frac{(\text{NPV}_{\text{base}} - \text{NPV}_{\text{reman}})}{\text{NPV}_{\text{base}}} * 100 \quad (23)$$

Figure 2.9 presents the effect of  $h_r$  on optimal capacity, optimal time to launch and %Rel.Savings. We observe that for lower values of  $h_r$ , firm tends to install more capacity and time to launch is also delayed. We believe this is because of two reasons. First, at lower values of  $h_r$ , it is cheaper to hold larger recoverable inventory for longer period and then to capitalize on the high level of held inventory a larger capacity is installed. Secondly, after the end of the production, recoverable inventory substitutes for



serviceable inventory. Therefore, at lower values firm tries to minimize the needs for serviceable inventory after end of the production. Whereas, in case of higher values of  $h_r$ , as expected, time to launch shortens but surprisingly capacity increases. This can be attributed to fact that at higher values firm doesn't differentiate much between recoverable and serviceable inventory because cost associated is almost similar. Thus, it tries to reduce cost that might be incurred due to holding large recoverable inventory which can be done by launching earlier and installing more capacity so as to increase differences in equilibrium of areas. Figures also reveal an interesting qualitative result. The %Rel.Savings, first increases and reaches a peak and then decreases. For a small value of  $h_r$ , when it is optimal to carry a large recoverable inventory and capacity is less likely to be constrained, the %Rel.Savings is low due to high capacity cost. Whereas, for higher values, when there is not much of difference between recoverable and serviceable inventory, still due to increase in capacity cost %Rel.Savings decrease. It is also important to note here that when  $h_r > h_n$ , it is not optimal to remanufacture as %Rel.Savings becomes non-positive. This certifies our initial assumption regarding remanufacturing that for remanufacturing to be profitable  $h_r$  should be less than  $h_n$ .

Effect of  $h_n$  on optimal capacity, optimal time to launch and %Rel.Savings is presented in Figure 2.10. It can be seen from the figure that  $h_n$  doesn't much affect the decision related to time to launch and capacity except at very high value as compared to  $h_r$ . We believe for lower values, firm tries to maintain the equilibrium of areas. By doing so, it reduces two important costs which are incurred due to manufacturing and carrying serviceable inventory for longer duration. Whereas, for higher values, it picks maximum

possible capacity, so that there is no need of virgin parts at all after reman is launched. As mentioned earlier, to maximize the utilization at this capacity level, time to launch is increased so that a large recoverable inventory could be built. Figure 2.10 also presents the corresponding effect of  $h_n$  on %Rel.Savings.

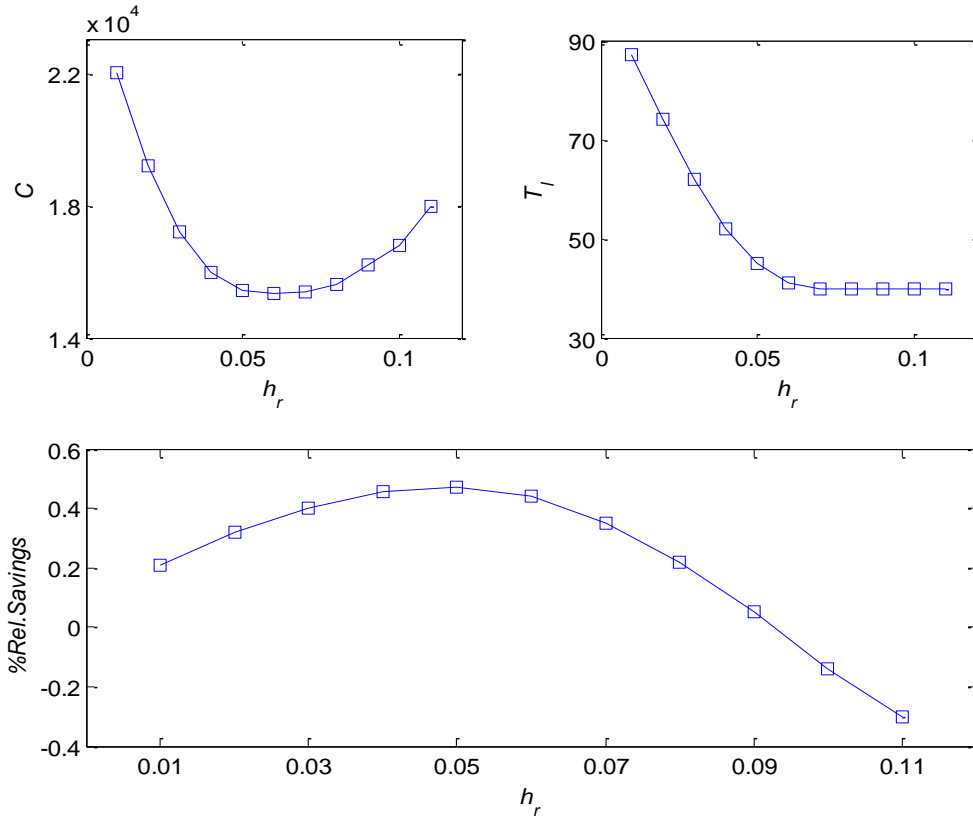


Figure 2.9: Effect of  $h_r$  on  $C$ ,  $t_l$  and %Rel. Savings ( $c_n = 1, c_r = 0.6, c_c = 0.1, c_a = 0.05, h_r = [0.01 - 0.11], h_n = 0.09, \gamma = 0.5, \theta = 0.01, \mu_{h(\Delta t)} = 60$ )

Results obtained are very intuitive in the sense that as  $h_n$  increases there is more and more value in doing the reman to reduce the costs that might incur due to carrying more

serviceable inventory. But, interesting dynamics for higher values of  $h_n$ , we can see from figure %Rel.Savings starts decreasing. In this case, firm tends to completely substitute serviceable inventory with recoverable inventory. However in doing so, capacity cost has increased resulting in less %Rel,Savings.

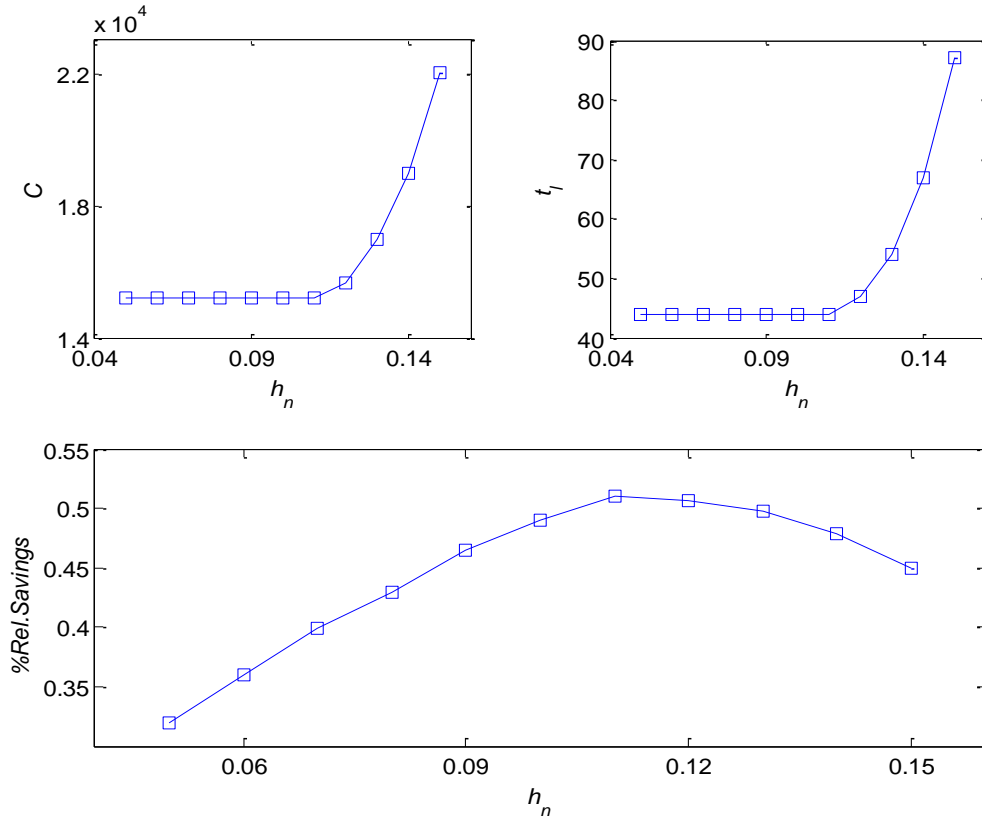


Figure 2.10: Effect of  $h_n$  on  $C$ ,  $t_l$  and %Rel. Savings ( $c_n = 1, c_r = 0.6, c_c = 0.1, c_a = 0.05, h_r = 0.05, h_n = [0.05 - 0.15], \gamma = 0.5, \theta = 0.01, \mu_{h(\Delta t)} = 60$ )

Figure 2.11 presents the effect of  $c_c$  on optimal capacity, optimal time to launch and %Rel.Savings. It is interesting to realize that  $c_c$  also doesn't affect the capacity decision

as long as remanufacturing is a viable option. On the other hand time to launch increases non linearly with increase in  $c_c$ . This can be understood in following manner. The cost associated with capacity is  $c_c * C * (T - t_l)$  and the objective is minimization. Thus, in order to offset the increase in  $c_c$ , length of interval  $(T - t_l)$  should be reduce and that can be only done by increasing  $t_l$ . In other words, firm tries to delay the launch as late as capacity cost increases. Though, we are not sure at this point of time that why capacity didn't decrease with increase in capacity cost. We are assuming that staying at equilibrium is more beneficial in terms of reducing the costs otherwise incurred due to recoverable and serviceable inventory holding cost. Given this argument, effect of  $c_c$  on %Rel.Savings is quiet straightforward as shown figure.

Figure 2.12 presents the effect yield percentage,  $\gamma$  on optimal capacity, optimal time to launch and %Rel.Savings. With increase in  $\gamma$ , it is obvious that  $C_{threshold}$  will increase and  $t_l$  will decrease accordingly. We also found out that for yield percentage less than a value of 0.4, remanufacturing is not a viable option since the combined effect of all the costs surpasses the benefits of reman. Though, an interesting observation is that even if yield percentage as high as 80%, it is not judicious to start reman earlier. This suggests that firm is better off carrying more than required recoverable inventory than installing capacity any earlier.

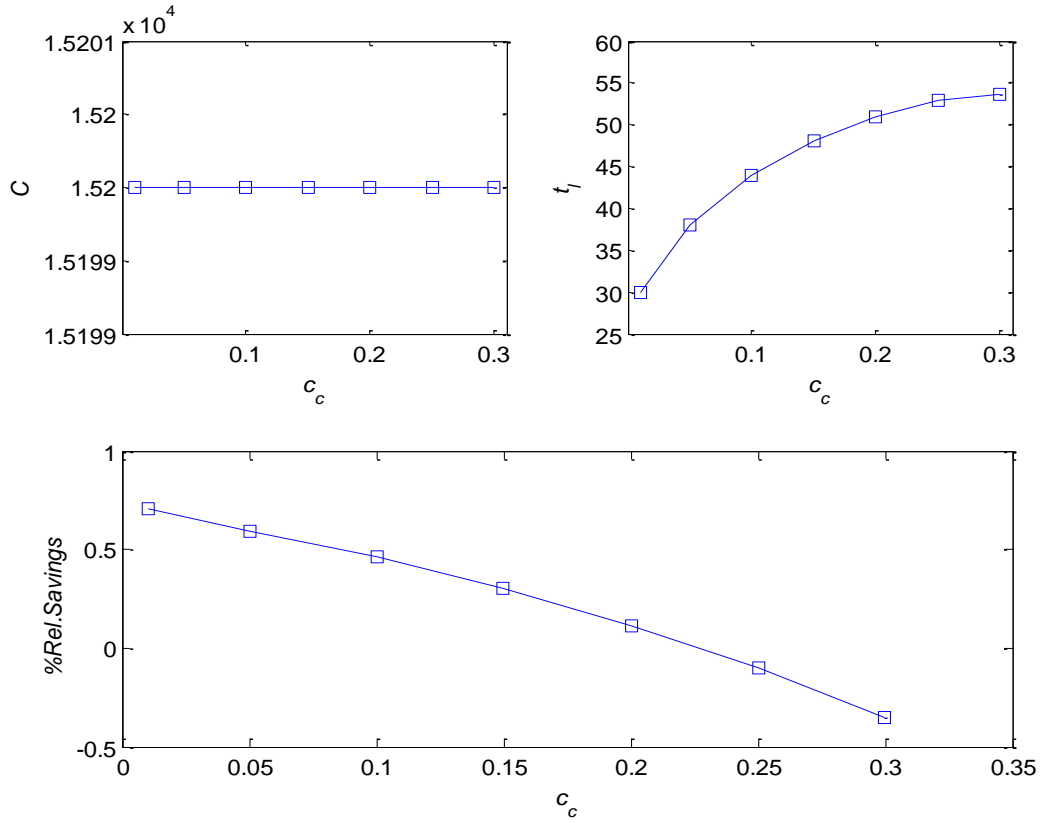


Figure 2.11: Effect of  $c_c$  on  $C$ ,  $t_l$  and %Rel. Savings ( $c_n = 1$ ,  $c_r = 0.6$ ,  $c_c = [0.01 - 0.3]$ ,  $c_a = 0.05$ ,  $h_r = 0.05$ ,  $h_n = 0.09$ ,  $\gamma = 0.5$ ,  $\theta = 0.01$ ,  $\mu_{h(\Delta t)} = 60$ )

Finally, we discuss the effect of durability, in terms of mean of residence time distribution,  $\mu_{h(\Delta t)}$ . Change in durability affects in terms of the position of the mean, e.g if durability increases, most of the demand falls near or after end-of-production, thus judicious decision is to delay the launch of remanufactured parts. By delaying the launch, high level of core inventory is built. To take advantage of high level of core inventory, firm installs a high remanufacturing capacity level. As mentioned earlier, high level of capacity is advantageous from the perspectives that it helps in reducing the needs of virgin products after the end-of-production. On the contrary, relative cost increases

because total core inventory holding cost as well as capacity installation cost has increased. Thus, %Rel.Savings decreases.

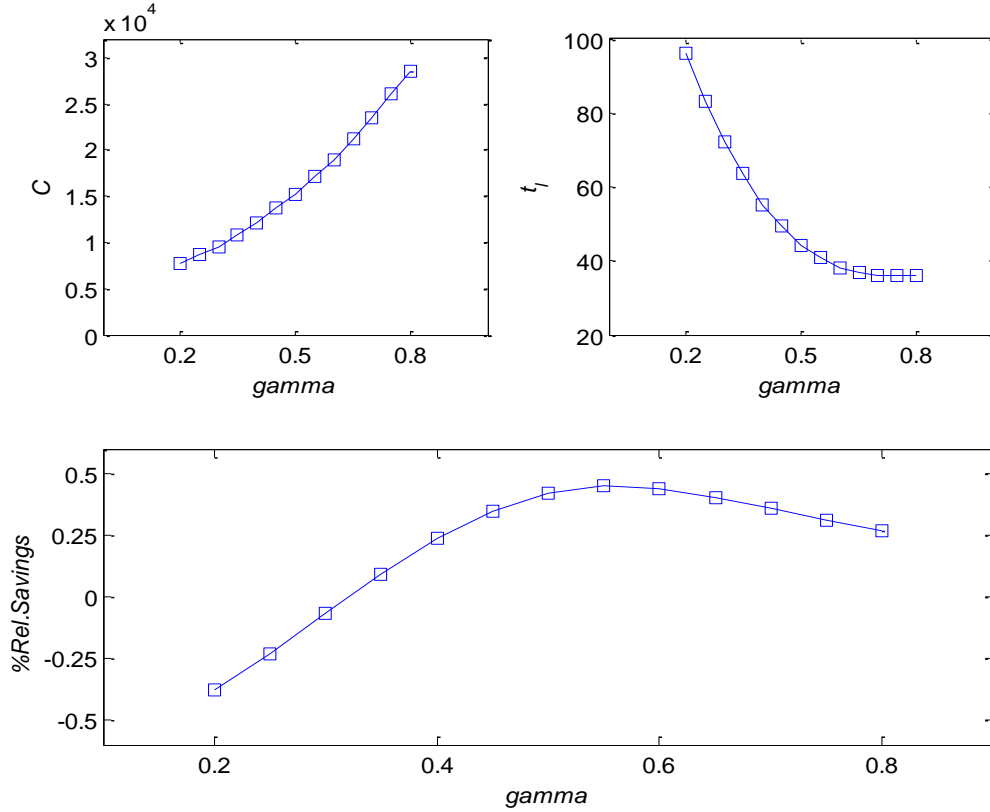


Figure 2.12: Effect of  $\gamma$  on  $C$ ,  $t_l$  and %Rel. Savings ( $c_n = 1, c_r = 0.6, c_c = 0.1, c_a = 0.05, h_r = 0.05, h_n = 0.09, \gamma = [0.2 - 0.8], \theta = 0.01, \mu_{h(\Delta t)} = 60$ )

One important aspect of this study is what happens when the demand of services parts are stochastic instead of deterministic as considered so far. To investigate this, we introduced randomness in  $\mu_{h(\Delta t)}$  and computed cumulative demand in each scenario. Figure 2.12 shows the cumulative demand profile obtained for 100 runs for randomness value 0.6. Figure clearly reveals the tightness of demand profiles. Given this behavior, despite

randomness, we suggest that deterministic study is very much valid and sufficient enough to understand the underlying dynamics of capacity management.

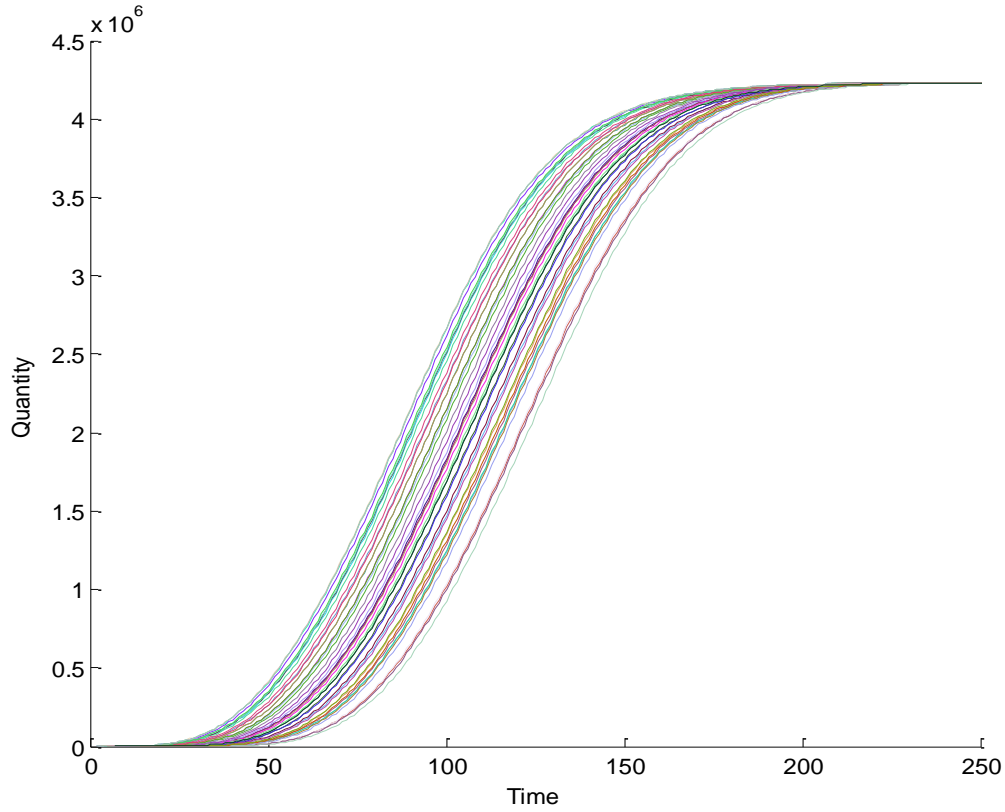


Figure 2.13: Cumulative stochastic returns with randomness in mean of 0.6

## 2.5 Conclusion and Future Research

In this chapter, we presented the drivers of optimal strategic capacity management for remanufactured products targeting aftermarket services. First, we analyzed properties of dynamic situation with regard to product life cycle and returns to establish optimal reman policy for aftermarket services. Then we presented an algorithm to compute optimal time to launch and overall capacity requirement given various costs and life cycle parameters.

We also presented the structural properties of the optimal reman policy and demonstrate how the optimal policy is a threshold policy in capacity and time to launch. Furthermore, we compared our solution with no remanufacturing scenario and established when it is optimal to reman.

Our analysis asserts that it is always optimal to delay the launch of remanufacturing program in order to build a strategic recoverable inventory. This helps in making the dynamics less supply constrained. But care should be taken in making such decision since it is a trade-off between recoverable inventory holding cost and potential relative savings. A high inventory holding cost decreases the profitability of remanufacturing, especially if it is stocked for future remanufacturing. We also found out that low cost of serviceable inventory of virgin parts doesn't affect decision regarding time to launch. This is because, at low cost, when there is smaller deviation from recoverable inventory holding cost, remanufactured units can imperfectly substitute the virgin parts. Thus, decision largely depends on the cost of holding recoverable inventory. But, at high cost of serviceable inventory, remanufactured units perfectly substitute virgin parts and remanufacturing becomes attractable. Though, firm also needs to take into account cost of capacity at high cost of serviceable inventory. For remanufacturing capacity level, it is not optimal to install maximum possible capacity. A capacity level should be selected such that it reduces the needs of serviceable inventory of virgin parts after end of the OE production run. In this study, we couldn't figure out why cost of capacity doesn't influence time to launch decision. Though, we presented a reasonable argument, but a better study needs to be carried out and thus subject of future research.



A number of possibilities exist for further research in this area. Though, we have shown that deterministic analysis is very powerful in realizing the important insights regarding effective reman program, yet a complete stochastic analysis could be very interesting and valuable. Further, we considered a single product environment; an extension of this work focusing in multi-product environment by analyzing joint distributions of the product returns are very much possible.

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## APPENDIX

### *Proof of proposition 1*

The problem can be modeled as fixed-time problem with free right hand conditions.

Introducing a new variable  $Z(t)$

$$Z(t) = \int_{t_l}^T (c_r * f_r(t) + c_n * (v(t) - f_r(t)) + c_a * v(t) + h_r * i_r(t) + c_c * C) dt$$

Shifting  $t_l$  to origin, for simplicity, new limits are  $[0, T']$

We obtain a new differential equation:

$$\overline{Z(t)} = (c_r * f_r(t) + c_n * (v(t) - f_r(t)) + c_a * v(t) + h_r * i_r(t) + c_c * C)$$

Rearranging terms,

$$\overline{Z(t)} = ((c_n - c_r) * f_r(t) + (c_n - c_a) * v(t) + h_r * i_r + c_c * C)$$

Let,  $(c_r - c_n) = c_o$ ,  $(c_n - c_a) = c_{na}$

Now, model can be re written as

$$\max_{f_r(t)} - Z(T)$$

Subject to

$$\begin{bmatrix} \overline{I_r} \\ \overline{V_r} \\ \overline{Z} \end{bmatrix} = \begin{bmatrix} 0 & \gamma & 0 \\ 0 & 1 & 0 \\ h_r & c_{na} & 0 \end{bmatrix} \begin{bmatrix} i_r \\ v \\ z \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ c_0 \end{bmatrix} f_r + \begin{bmatrix} 0 \\ 0 \\ c_c * C \end{bmatrix}$$

The auxiliary systems is given by

$$\dot{\bar{\psi}} = -A^t \bar{\psi} = \begin{bmatrix} 0 & \gamma & -h_r \\ -\gamma & -1 & -c_{na} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} \text{ and } \begin{bmatrix} \psi_1(t) \\ \psi_2(t) \\ \psi_3(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

Let,  $\bar{\psi}(t) = \phi(t)\bar{C}$ , where  $\phi(t)$  is a fundamental matrix and  $\bar{C}$  is a constant vector.

$$\phi(t) = e^{-A^T t} = \begin{bmatrix} 1 & 0 & -t h_r \\ -t\gamma + t^2\gamma & 1 - t + t^2 & -tc_{na} + t^2(\gamma h_r + c_{na}) \\ 0 & 0 & 1 \end{bmatrix}$$

From the boundary conditions  $\bar{C}$  can be determined as

$$\bar{C} = \begin{bmatrix} -t h_r \\ t(-c_{na} + tc_{na} + h_r t^2 \gamma)/(1 - t + t^2) \\ -1 \end{bmatrix}$$

Therefore, the solution of the auxiliary system is

$$\bar{\psi} = \begin{bmatrix} -T h_r + t h_r \\ -(-t\gamma + t^2\gamma)T h_r + \frac{(1 - t + t^2)T(-c_{na} + Tc_{na} + h_r T^2 \gamma)}{(1 - T + T^2)} + tc_{na} - t^2(\gamma h_r + c_{na}) \\ -1 \end{bmatrix}$$

And the Hamiltonian may then be written as:

$$H = [\psi_1 \psi_2 \psi_3] \begin{bmatrix} \overline{I_r} \\ \overline{V_r} \\ \overline{Z} \end{bmatrix}$$

After inserting  $\bar{\psi}$  in  $H$  and after rearranging the term we find that

$$f_r^*(t) = (T h_r + c_o - t h_r)f_r(t) \Rightarrow \text{maximum possible at time } t$$

maximizes the Hamiltonian  $H$ .

At  $t = 0, i_r(t) > 0$ , thus any surplus demand over instantaneous returns after yield  $\gamma v(t)$

can be fulfilled from recoverable core inventory  $i_r(t)$ . Thus,  $f_r^*(t) = \min(v(t), C)$ .

Once all the recoverable inventory is depleted then at some time  $t_{i1} > t, i_r(t) = 0$  only option is to process instantaneous returns  $\gamma v(t)$ . But as we know maximum can be produce is  $C$  so  $f_r^*(t) = \min(\gamma v(t), C)$ . In case if  $C \leq \max(\gamma v(t))$  recoverable inventory again start building up when  $f_r^*(t) = C$  and then depleted once demand falls below  $C$ . So summarizing results:

$$f_r^*(t) = \begin{cases} \min(\gamma v(t), C) & , i_r(t) = 0 \\ \min(v(t), C) & , i_r(t) > 0 \end{cases}$$

### Chapter 3 : **HAZARD RATE MODELS FOR CORE RETURNS FORECASTING IN REMANUFACTURING**

Efficient production planning is a very important lever of a profitable remanufacturing program at operational level. A production planning system for remanufacturing assists managers making decisions regarding disassembly, remanufacturing, manufacturing, and coordinating between disassembly and reassembly. Among various factors which affect the production planning, accurate estimation of core returns is an important input for an efficient planning. Nevertheless, the uncertainty in the timing and quantity of returns makes core returns forecasting a very challenging task in remanufacturing milieu.

As aforementioned in chapter 2, this research was also pursued on a request from a tier-1 automotive supplier engaged in OE production and also providing aftermarket services. Management was interested in improving the accuracy of their core forecasting method because the existing forecast method was too simplistic to capture the dynamics of core returns in the face of uncertainty in timing/location of return, return volume, quality etc. The particular interest was in understanding the dynamics of independent aftermarket (IAM) returns. IAM core returns is more challenging than original equipment services or warranty claims because of increased uncertainty in the returns. Unlike the OE service setting, the trade in process is often not initiated with the receipt of a core but with an order. The setting is as follows. Supplier receives orders for reman parts from a number of automotive aftermarket parts retailers/distributors (e.g., NAPA), OE service and parts operations organizations (e.g., GM SPO), and large dealers, referred to here upon as the “customer”. In shipping the order, the supplier imposes a “core charge” on



the customer, a debit that will be credited to the customer upon receiving the defective part or the “core”. The supplier issues a return material authorization (RMA) in shipping the order, to facilitate return shipment of cores. Efficient production and inventory management of reman parts for the supplier heavily impinges on the ability to accurately forecast these core returns from customers (besides forecasting demand for reman parts and securing cores from the open market, as necessary). There are several challenges to this, including, the volume and diversity of customers, differences among individual customer warehouses in returning cores, large reman product catalog, changing customer behaviors (often improving core return delays), and data sparsity.

In this chapter, we have reported the evidence for the effectiveness of hazard rate regression models to calculate return delay distribution in the context of remanufacturing. We extensively studied various types of hazard rate modelling technique (e.g., parametric, semi-parametric etc.) and its appropriateness. Further, we described various approaches when underlying proportionality assumptions is violated or when there is time-varying effect of covariates or there is randomness in one of the covariates. To the best of our knowledge, no existing literature has explored all these issues in context of returns modelling for remanufacturing.

Rest of the chapter is organized as follows: Related literature is presented in section 3.2. Proposed framework is discussed in section 3.3. A real world case study is presented in Section 3.4. Results and discussion have been presented in section 3.5. Finally, conclusion and future research in section 3.6.

### 3.2 Literature Review

Over the last two decades, there has been significant research in the area of remanufacturing and reverse logistics. Guide (2000) carried out an extensive survey of reman literature and identified future research needs. Based on existing literature, he divided reman research into five broad categories: forecasting, reverse logistics, production planning and control, inventory control and management, and general. Further, he identified seven complicating characteristics that complicate the production planning and control activities of reman industry: 1) the uncertain timing and quantity of returns, 2) the need to balance returns with demands, 3) the disassembly of returned products, 4) the uncertainty in materials recovered from returned items, 5) the requirement for a reverse logistics network, 6) the complication of material matching restrictions, and 7) the problems of stochastic routings for materials for remanufacturing operations and highly variable processing times. In recent years, the last four complication categories have been addressed extensively ( Aras (2008), Barba-Gutierrez (2008), Inderfurth (2004), Krikke (2008), Li (2009), Takahashi (2007), Tang (2005), Wang (2007)). Since, our research focus here will be on the first complicating characteristic; forecasting, we encourage readers to refer a recent survey by Ilgin and Gupta (2010) for research in the other categories.

Toktay et al. (2000) presented the role of forecasting in managing product returns and argued how predicting returns influences decision at strategic, tactical, and operational levels. They also quoted that there are only few documented business examples dealing with forecasting in reverse logistics. Ilgin and Gupta (2010), reiterate

this statement by citing only eight notable publications. However, most of these publications assume that the core return probability is known in advance (e.g., Goh & Varaprasad (1986), Kelle and Silver (1989))

Most of the extant literature exploited the fact that future returns are a function of past sales. Goh & Varaprasad (1986) are credited for being the first to develop such a model. They propose a transfer function model to estimate return quantities of Coca-Cola bottles in Malaysia and Singapore markets using Box-Jenkin's time-series techniques to compute life-cycle parameters. Kelle and Silver (1989) proposed four forecasting techniques based on available information sets to estimate the "net demand" during lead time of reusable containers. As noted earlier these models assumed that returns are Poisson with known rate. To overcome this limitation, Toktay et al. (2000) considered a queuing network based approach to achieve an optimal ordering policy for Kodak's single use-camera. The model utilized a Bayesian estimation and expectation optimization approach to forecast returns in a trackable as well as untrackable case. Although, their method doesn't require known return rate but makes assumption regarding the shape of lag distribution.

Aforementioned methods used past sales and return data to forecast returns. Hess and Mayhew (1997) employed split adjusted hazard model and time regression to model merchandise return in direct marketing. They incorporated explanatory variables in their regression. Marx-Gómez et al. (2002) develop a fuzzy inference system for the forecasting of returns. Their model included demand, life cycle parameters, and return incentives with the fuzzy rule-base developed from prior expert knowledge.

The extant literature offers a few returns forecasting models but these models are simply not practical for many suppliers, such as our collaborator Delphi Product & Service Solutions, a sub-division of Delphi Corporation that provides replacement parts and services to the automotive aftermarket, because:

1. Virtually all these models are applicable for forecasting returns of individual products/SKUs
2. The historical data is simply too sparse to facilitate modelling and calibration of models for individual SKUs.
3. Makes one or other assumption based on expert/prior knowledge (e.g., return rate is known or shape of lag model is known).

Thus, it becomes imperative to build effective and efficient models for forecasting core returns in the automotive IAM, from the perspective of a Tier-1 automotive parts supplier.

One of the most interesting characteristics of returns data is right-censoring, which means at any given time only a fraction of returns is observed whereas rest of them are outstanding, and thus requires analysis of duration time. Typically, an analyst tends to achieve three modeling objectives while investigating duration time data (Helsen and Schmittlen 1993): effects of covariate, dynamics of duration, and duration time forecasting. They also listed short-comings in conventional modeling approaches (duration time regression, logit, probit etc.) as follows:

1. Use of duration time regression in the face of censoring may lead to biased estimates of the covariate effects;

2. Time regression and logit/probit models are inappropriate when there are time varying covariates; and
3. In case of probit/logit models, for predictions, time intervals should be integer multiples of censoring times.

Literature suggests (Gupta 1991, Jain and Vilcassim 1991, Helsen and Schmittlen 1993) that hazard rate regression models can overcome the above listed shortcomings while achieving all three objectives within a single tractable class of duration time models. Further, Helsen and Schmittlen (1993) established that hazard rate regression models outperform conventional procedures (e.g. duration time regression, logit, probit etc.) in terms of stability of the estimates, face validity of parameter estimates, and predictive accuracy.

### 3.3 The Modelling Framework

As stated earlier, core returns can be modeled using duration time modeling within a single tractable class of hazard rate models. This section provides a brief overview of hazard rate models.

Let  $h(t|x_t)$  denote the hazard rate at time  $t$  for an individual having covariate values  $x_{1t}, x_{1t}, \dots, x_{kt} = x_t$  at time  $t$ . Thus, the covariate values may vary over time for any individual. This hazard rate is assumed to take the form

$$h(t|x_t) = h_o(t)\varphi(x_t, \beta) \quad (1)$$

Where  $\beta_j$  indicates the effect of covariate  $x_{jt}$  on the hazard rate, and  $h_o(t)$  is the baseline hazard function. Thus the model has two multiplicative components. The first,  $h_o(t)$ ,

captures the longitudinal regularities in duration time dynamics. The second,  $\varphi(x_t, \beta)$ , adjusts  $h_o(t)$  up or down proportionately to reflect the effect of the measured covariates. In light of this proportional adjustment of the baseline hazard rate, estimation of the  $\beta$  - vector in (1) is termed proportional hazards regression (PHR).

In most applications  $\varphi$  is formulated as an exponential function:

$$h(t|x_t) = h_o(t)e^{\beta x_t} \quad (2)$$

Which renders the estimation of  $\beta$  easier, given that no constraints need to be imposed to ensure non-negativity of  $\varphi$ .

### 3.3.1 Semi Parametric Modelling (Cox-proportional Hazard rate model)

Cox proportional hazard model is one of the most widely used tools in survival analysis. It gained a lot attention of researches since its development in 1972 due to its efficiency and flexibility. This could be attributed to semi-parametric nature of the model which doesn't make any special assumption regarding the distribution of failure occurrence also know as baseline hazard function. Cox's major contribution was to suggest an estimation technique- partial likelihood to purely estimate regression coefficients  $\beta$ , allowing for a general hazard function as nuisance parameter. He also suggested that this can result in slight loss of information about  $\beta$ . Efron (1977) and Oakes (1977) provide evidence indicating that maximizing the partial likelihood results in very efficient estimates of  $\beta$ . Tsiatis (1981) shows that under general conditions the partial MLE is consistent and asymptotically normal.

For duration time processes, the usual ("total") likelihood has as the event of interest the fact that individuals  $i$ 's duration time (i.e., the random variable  $T_i$  took on the observed value  $T_i = t$  for individuals  $i = 1, \dots, N$ . The partial likelihood also focuses on the observed durations  $t_1, t_2, \dots$ , but considers them in a different way. Imagine that individual  $i$  has an uncensored duration  $T_i = t$ . At this duration time  $t$ , a number of other individuals were "at risk," i.e., had not yet experienced the duration event (the "risk set"). Of all those at risk, individual  $i$ , is the one who actually experienced the duration at  $t$ , and it is this selection event that the partial likelihood considers. Thus, the partial likelihood is the likelihood that individual  $i$  is the one, of those at risk, who has the duration of  $t$ , given that someone is known to have a duration of  $t$ .

Since the hazard rate  $h(t)$  measures the likelihood of the duration event happening at  $t$  for those who have made it up to time  $t$  without experiencing an event, this rate determines the odds of selection in the partial likelihood for each individual at risk. Thus, for an observed time  $t$  at which individual  $i$  experiences a duration ( $T_i = t$ ), the partial likelihood that this duration indeed happened to individual  $i$  (and not to one of the other individuals at risk) is

$$L(i|j_{1..j_{n(t)}}) = \frac{h_i(t)}{\sum_{k=1}^{n(t)} h_{jk}(t)} \quad (3)$$

Where  $n(t)$  is the number of individuals at risk at  $t$ , and these individuals are denoted  $j_{1..j_{n(t)}}$ . Substituting the proportional hazards model (2) in (3) yields

$$L(i|j_{1..j_{n(t)}}) = \frac{h_o(t)e^{\beta x_{it}}}{\sum_{k=1}^{n(t)} h_o(t)e^{\beta x_{jk}t}} \quad (4)$$

for which the longitudinal effect  $h_o(t)$  cancels, leaving

$$L(i|j_1..j_{n(t)}) = \frac{e^{\beta x_{it}}}{\sum_{k=1}^{n(t)} e^{\beta x_{jk}t}} \quad (5)$$

The partial likelihood estimate of is obtained by maximizing the product of expression (4) over all observed duration times. Note that, unlike the usual duration time regression models, the right-censored observations do enter the partial likelihood (5), i.e., these individuals, each having some covariate vector  $X_{jt}$  were at risk at  $t$  but did not experience the duration. The information in this event relevant for the response coefficient is appropriately taken into account in (4). To summarize, the only thing "partial" about the partial likelihood is in the event it chooses to model. The total likelihood is concerned with the total duration event, i.e., "When will the duration occur for each individual?" The partial likelihood considers only part of the total duration event, namely, "Given that a duration occurred to someone at a specific time, which individual, of those still at risk, experienced it?" Since the answer to this latter question hinges on the relative riskiness of various individuals all measured at the same duration time, it comes as no surprise that the longitudinal effects  $h_o(t)$  drop out in (3), leaving (4) dependent only on the desired response coefficients 1.

Cox-model has gained popularity because it works well in practice. Practitioner believes that in process of considering possible models, Cox model should always considered as an option. This is attributable to the flexibility of the model which only requires proportional hazard assumption. It has also been established that Cox model is reasonably robust to modest departure from proportional hazard. Further, in many cases variables can be transformed to show approximate proportional hazard (discusses in section 5.3).



### 3.3.2 Parametric Modeling

Parametric models assume that base-line hazard function;  $h_0(t)$  follows known functional form, e.g.: Exponential, exponential, logistics etc. Computationally, biggest advantage of parametric model is, one can use full maximum likelihood to estimate the parameters. This in turn provides meaningful estimate of effects. Parametric models are better choice if modeller has better knowledge of the aging process. In literature, researchers always caution the use of parametric model since most of the time prior knowledge is not always available. But, this does not rule out the option of comparing parametric models against semi-parametric models.

### 3.4 Case Study

To establish the empirical performance of the proposed framework, we tested it on IAM return data of an engine control module (ECM). Electronic Control Modules are subsystems consisting of CPUs and assorted signal inputs and outputs dedicated to controlling a component within the vehicle. They range in complexity from an Engine Control Unit which handles the logic for managing the power-train system efficiency, to an Anti-lock Braking (ABS) Control unit that monitors vehicle speed and brake fluid, to a simple body module that controls the automatic door locks or power windows (National Instruments).

### 3.4.1 Data

Data was collected over span of few years for 100's of parts and some 30 customers. Dataset consisted of customers, parts, shipping dates and return dates. Preliminary data cleaning reveals that there were many customers who never returned any parts back. Also, in some of the cases customers only did business for very small time period. Thus, for further analysis we only considered customers who returned at least 10 products. Figure 3.1 shows history of core return delays for a Delphi Product Family. The time-axes have been modified throughout this document for reasons of confidentiality<sup>3</sup>.

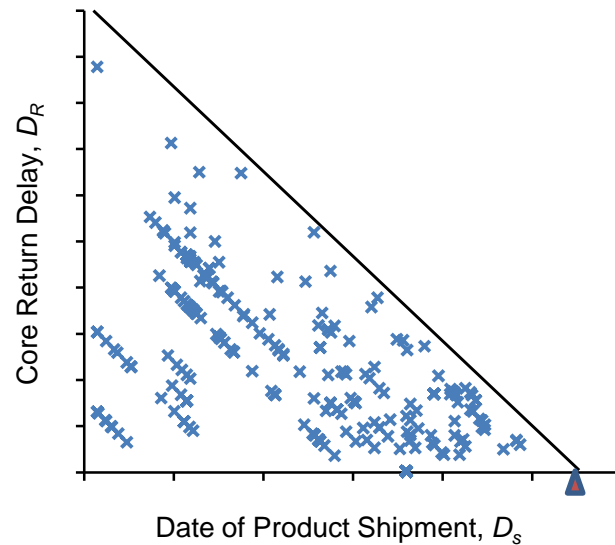


Figure 3.1 History of Core Return Delays for a Delphi Product Family (Source: Delphi).

Note: Red line at  $-45^\circ$  slope denotes censoring time (i.e., date for termination of data collection).

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<sup>3</sup> <http://www.ni.com/>

### 3.4.2 Nomenclature

This section provides necessary nomenclature to facilitate duration modelling in an IAM setting. In this case, we have used right censoring for truncation.

$D_A$ : Census Date

$D_S$ : Shipment Date

$D_R$ : Day product was returned (if it is returned)

$R$ : If product is returned it is 1 else 0

$T$ : It is defined as time since  $D_S$  until  $D_R$  or  $D_A$ , depending on returned or not

$P_i$ : Products,  $i=1, 2, 3 \dots$

$C_i$ : Customers,  $i=1, 2, 3 \dots$

Now, we can construct a hazard rate model with set of covariates  $X = [P, C]$ , and dependent variable  $T$  with censoring  $R$ . Mathematically,

$$h(T, R) = h_0(t)e^{(\beta_1 * P + \beta_2 * C)} \quad (6)$$

## 3. 5. Results and Discussion<sup>4</sup>

In this section we present results of the numerical case study. First we discuss the estimates of the covariates for parametric and semi-parametric hazard rate models. Then, we show the validity of the underlying models in the face of stability, efficiency and

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<sup>4</sup> All these experimentation were done using Survival package in Software R (<http://cran.r-project.org/>). For time-by-covariate interactions, we used COXPHF package (<http://www.meduniwien.ac.at/msi/biometrie/programme/fc/>)

predictive performance. Finally, we discuss how to pursue modelling using extended Cox ph models when proportionality assumption is violated.

### **3.5.1 Parameter Estimates**

To assess the suitability of parametric modelling, we chose widely known parametric proportional hazard model-Weibull; whereas, Cox proportional hazard rate model for semi-parametric modelling. First requirement was to ensure the effect of covariates.

Our initial analysis revealed that none of the products were statistically significant from each other. Thus, we assumed that all products are identical and chose customers as only covariate for the modelling. Table 3.1 summarizes the results obtained. Results indicated that both model have monotonically increasing hazard rate. Monotonically increasing hazard rates seems highly intuitive since we are modelling return- likelihood of return of a product increases with elapsed time. Statistically, both models are very significant with p-values at 0. Also, individual estimates for each customers obtained are highly significant. Further, the sign of coefficients explains the returning behaviour of a customer- positive value depicts that customer makes faster returns and vice-versa. Expected return behaviour of customers is shown in figure 3.2. It was surprising to realize, in case of most of the customer, that instead of returning core back after receiving a shipment from supplier immediately; customer tends to delay it infinitely. A possible reason seems to be that rather than trade-in, customer wants to stock the parts to handle stock-out situation. Other reasons could be a core-collector is buying it at high price than

supplier. Owing to such explanations monotonically increasing hazard rates seems highly plausible.

Table 3.1 Covariate Estimates

	Weibull			Cox		
	Coef	se	p	Coef	se	p
Intercept	-6.121	0.0492	0.0000	NA	NA	NA
Customer 2	-0.497	0.2245	0.0270	-0.73717	0.29604	0.0128
Customer 3	-0.401	0.1514	0.0082	-0.53514	0.20002	0.0075
Customer 4	-0.1	0.0671	0.1340	-0.17991	0.08874	0.0426
Customer 5	-0.537	0.1349	0.0001	-0.72724	0.17633	0.0000
Customer 6	0.228	0.0709	0.0013	0.29824	0.0936	0.0014
Customer 7	0.309	0.1036	0.0029	-0.31279	0.13717	0.0226
Log(Scale)	-0.278	0.0277	0.0000	NA	NA	NA
	Loglik	-6298.9		R <sup>2</sup>	0.049	

### 3.5.2 Validation

In this section, we present various performance measures to establish validity of these models. Readers should note here that our intention is to present validity of these models within single tractable class of duration time models in modelling returns -proportional hazard rate models. We are not promoting the use of one model over other. This is because direct comparisons of these models are not fair, since: 1) parametric model is

based on event times whereas Cox's model is based on rank of event times; 2) scales of the parameters may differ. Thus, all the comparative study presented in this section is to show suitability and relative performance of the models. We divided our validation process in two parts- 1) Stability of the estimates, and 2) Predictive performance of the models.

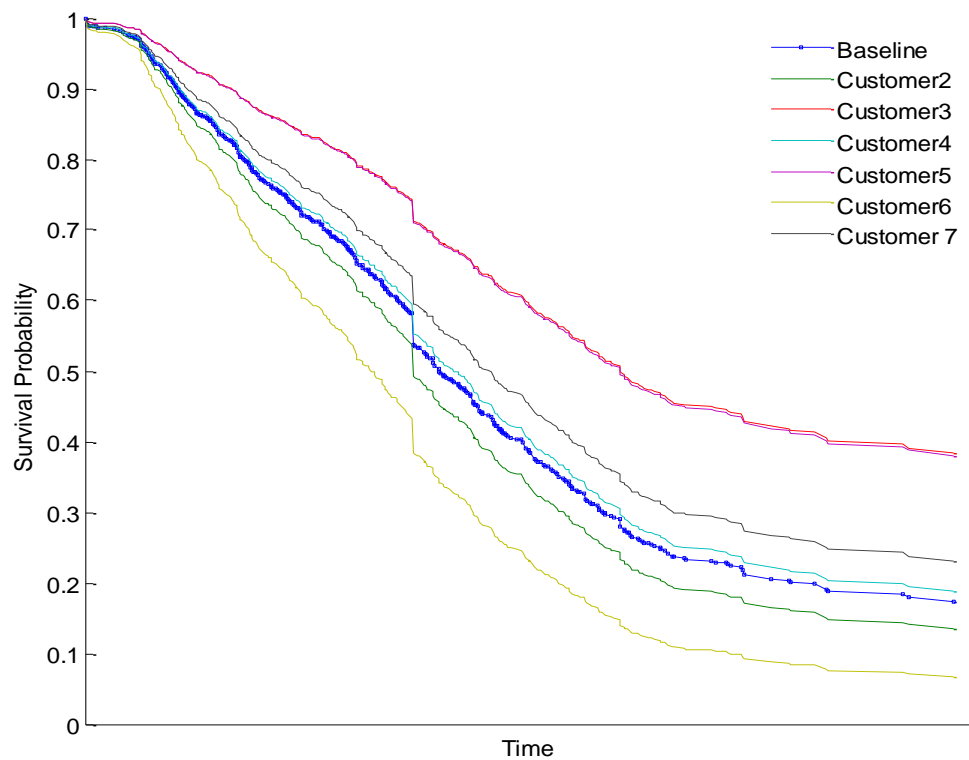


Figure 3.2 Expected return behavior of customers using Cox' model

### 3.5.2.1 Stability and Efficiency of the Estimates

To check the stability of the estimates, we referred to methods proposed by Krsitiaan and Helsen (1993). In their work, they considered two samples from different market as calibration and validation dataset. Since, in our case we do not have two different dataset thus, we considered these samples: 1) *Complete-dataset (I)*, 2) 50% of complete dataset as *Calibration-dataset (II)*, and 3) rest of the 50% as *Validation-dataset (III)*. One should note here that taking a totally random sample may compromise uniformity, since we have returns as well as non-returns. So, in order to retain uniformity across samples, we considered 50% of returns and 50% of non-returns for each calibration and validation samples. We re-estimated all the models for calibration and validation dataset. To evaluate the relative efficiency of the estimates, we evaluated standardized measures of variability,  $SV (= \sigma_{\beta}/|\beta|)$ , for all the models on all the samples.  $SV$  is analogous to the coefficient of variations, where cases with parameter estimates close to zero are emphasized (Nardi and Schemper, 2003).

Table 3.2 presents the estimated coefficients and  $SV$ 's (in parentheses) for each sample for both models. Table 3.2 shows incredible performance of the models in term of stability of the parameter estimates. There is remarkable consistency (ignoring minor discrepancies) between calibration and validation dataset (Only notable discrepancy in case of customer 2 where departure is as high as 10% for both the models). More interestingly, there is very small departure from estimates obtained from complete dataset versus calibration and validation dataset (mostly less than 5%). Also, there is no change in the sign of coefficients. This dictates the suitability of proportional hazard rate models for

modelling returns from the standpoint of stability of the parameter estimate, even when sample size was reduced to 50% of original sample.

Table 3.2 Estimated Coefficients and standardized measures of variability (*I: Complete dataset, II: Calibration-dataset, III: Validation-dataset*)

	Weibull			Cox		
	I	II	III	I	II	III
Customer 2	-0.497	-0.5199	0.471	-0.73717	-0.7735	-0.7115
	(0.45)	(0.62)	(0.67)	(0.40)	(0.54)	(0.59)
Customer 3	-0.401	-0.3448	0.455	-0.53514	-0.49	-0.6129
	(0.38)	(0.63)	(0.47)	(0.37)	(0.58)	(0.46)
Customer 4	-0.1	-0.0619	0.136	-0.17991	-0.1363	-0.2281
	(0.67)	(1.55)	(0.69)	(0.49)	(0.92)	(0.55)
Customer 5	-0.537	-0.5286	0.545	-0.72724	-0.6815	-0.7807
	(0.25)	(0.36)	(0.35)	(0.24)	(0.37)	(0.32)
Customer 6	0.228	0.2262	-0.231	0.29824	0.3171	0.2813
	(0.31)	(0.45)	(0.43)	(0.31)	(0.42)	(0.47)
Customer 7	-0.309	-0.2946	0.323	-0.31279	-0.2914	-0.3487
	(0.34)	(0.50)	(0.45)	(0.44)	(0.67)	(0.55)

To compare relative efficiency of the models, we compared  $SV$ 's of parameter estimates.

One can easily see in most of the cases (4 out of 6 for every sample) Cox's model  $SV$ 's



were closer to zero as contrast to Weibull model. Thus, in relative sense we can conclude that Cox's model performed better than Weibull based on standard measures of variability.

### **3.5.2.2 Predictive Performance of the Models**

We compared different model's prediction with observed returns. In order to facilitate these comparisons we considered two performance measures: hit rates (Krsitiaan and Helsen, 1993) and mean square errors (MSE) in forecast.

Hit rates can be defined as percentage of returns correctly classified. To calculate hit rates, we require hazard rate model forecasts for median duration and observed return for median duration. Hazard rate model forecasts for median duration implies, computation of time point at which the survival function drops below 0.5 and then interpolating linearly to produce forecast for median duration. Table 3.3 presents hit rates for both models for all the samples. For Cox's model hit rates are as high as 90%. Cox's model performed remarkably well as compared to Weibull model. This difference can be better understood by analyzing survival plot for base-line for both the models (Figure 3.3). One can easily see that Cox's model tries to fit to the data better due its flexibility as compared to Weibull with rigid structure.

Next, we considered mean square error in forecast. To achieve this objective, Data set is divided into 5 time period. The same procedure, as used to predict forecast for mean duration, can be used to generate forecasts for each time-periods. In this scenario, we will

compare number of estimated returns with observed returns for each time periods. Table 3.4 presents the overall MSE of forecast for each model for each customer. Overall, Cox's model performed better than Weibull.

In conclusion, for this particular case study, we can conclude that Cox model performed better than Weibull.

Table 3.3 Hit Rates

	Hit Rate	
	Weibull	Cox
I: Complete dataset	67.63	91.29
II: Calibration-dataset	66.28	91.09
III: Validation-dataset	68.31	91.46

Table 3.4 Mean Square Error (MSE) in Forecast

	Weibull	Cox
Customer 2	2.52	2.48
Customer 3	7.10	3.35
Customer 4	10.21	9.41
Customer 5	12.57	13.40
Customer 6	6.22	4.51
Customer 7	1.44	2.24

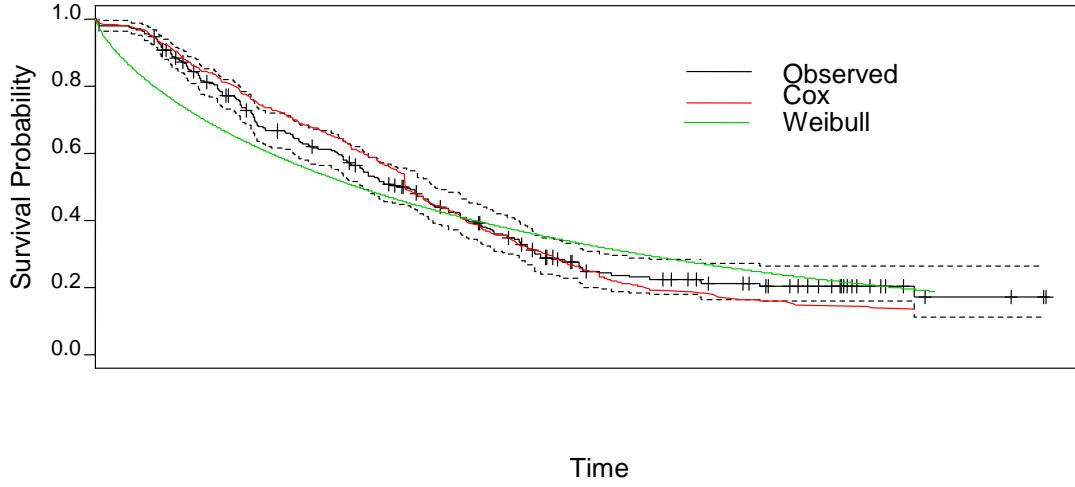


Figure 3.3 Baseline Survival Plot

### 3.5.3 An Important Note

One valid question arises, why for some of the customers Cox' model is better and for some Weibull? According to Cox and Oakes (1984), if there is strong time trend in covariates, a parametric model yields more efficient parameter estimates than Cox' model. To check time trend in the covariates, we performed Schonfeld residual test. Test revealed that there is evidence against proportional hazard for some of the customers, though there was modest departure from proportionality. Since, proportional hazard assumption is unclear; we performed Cox's model with time-by-covariate interaction fit for the data. Time-by-covariate interactions can be captured by simple monotonic function of time (Lehr and Schemper 2007). Mathematically,

$$h(T, X) = h_0(t)e^{\beta X + \gamma X f(t)}; \quad (7)$$

Where,  $f(t): t, \log(t)$

Table 3.5 presents the estimate obtained by time-by-covariate interactions. Although all the statistical tests are significant but parameter estimates are not easily interpretable. To better understand the dynamics let's consider customer 5 and different time-periods described earlier. Figure 3.4 presents effective  $\beta(= -49.80+7.36*\log(D_s))$  for each time-period. One can easily see for the first three time periods likelihood of return is almost zero but in last time period it became almost comparable to customer 6 estimates from regular Cox's model. This is because customer 5 started business with supplier in 5<sup>th</sup> time period. Regular Cox or Weibull model can only estimate average effect of baseline when there is no other information available. While, time-by-covariate interaction can capture the time dependent effects of covariate along with average effect of baseline.

Table 3.5 Estimates- Cox with Time-by-Covariate Interaction

	Coef	se	p
Customer 2	10.29	4.9628	0.0000
Customer 3	1.44	1.0278	0.0023
Customer 4	0.96	0.9792	0.00
Customer 5	-49.80	9.3076	0.0000
Customer 6	-0.26	0.6486	0.0000
Customer 7	-0.20	0.5363	0.6890
$\log(D_s)$	-0.10	0.0885	0.2802

Customer 2:log( $D_s$ )	-1.86	0.8484	0.0000
Customer 3:log( $D_s$ )	-0.36	0.2016	0.0290
Customer 4:log( $D_s$ )	-0.186	0.1643	0.0000
Customer 5:log( $D_s$ )	7.36	1.3952	0.0000
Customer 6:log( $D_s$ )	0.10	0.1090	0.0000
Customer 7:log( $D_s$ )	-0.07	0.0976	0.4974

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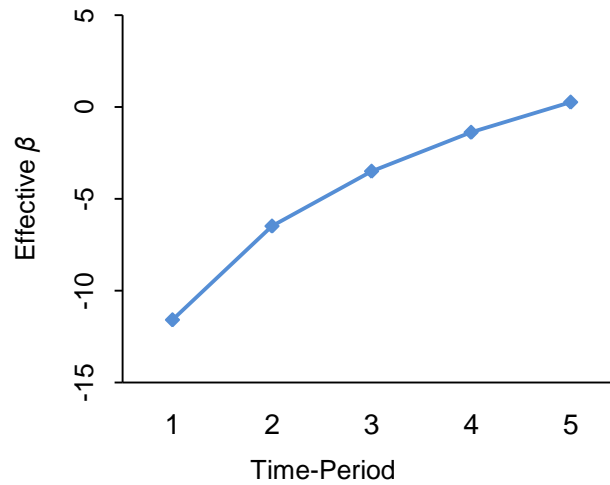


Figure 3.4 Effective  $\beta$  for different time periods

Other possible explanation, for deviation from proportional hazard, could be some random effect due to other covariates. To understand this phenomenon, we performed Cox's regression with customer and 'frailty due to products' as covariates. Results obtained clearly reveal that 'frailty due to products' is highly significant with variance

0.198. Although our initial experiments suggested that products (by itself) were statistically insignificant, but its random effect is highly significant. Further,  $R^2$  value increases from 0.049 to 0.151. Given the significant random effect attributable to products and the insignificance of products as fixed covariates within the hazard rate model, we should investigate the possibility to incorporate product attributes, such as product size, weight, core deposit, and demand etc., to improve model fidelity and explanation power. Table 2.6 presents the estimates obtained using frailty models.

Table 3.6 Cox with Random Effect (Frailty Model) Estimates

	Coef	se	p
Customer 2	-0.832	0.3012	0.01
Customer 3	-0.676	0.2049	0.00
Customer 4	-0.152	0.0936	0.10
Customer 5	-0.523	0.1829	0.00
Customer 6	0.275	0.1018	0.01
Customer 7	-0.213	0.1489	0.15
Frailty(Product)			0.00
Variance of random effect	0.198		
$R^2$	0.151		

Indeed, time-by-covariate interaction model was able to explain the dynamics better than other models (considering customer as only covariates) but, from

computational complexity point of view other models were far more superior. This is because, with time-by-covariate interactions, there will be a baseline hazard rate for every time stamp. Thus, choosing one of these models is trade-off between computational complexity and degree of accuracy one is intend to achieve. In our case, Cox's model performed satisfactory (ignoring modest departure from proportionality) to meet the requirements.

### **3.6. Conclusions and Future Work**

This research presents a unified approach for modeling returns in an automotive independent aftermarket setting. It helped in understanding the customer behaviour, which tends to be attracted by the open-market deals or try to stock the products instead of trading. Results are also beneficial when suppliers are planning to kick-off new reman product in the market. Further, based on our insights, products attribute can bring more robust and promising results than just considering products by itself.

A range of hazard rate models has been presented to facilitate returns modelling. This research does not try to advocate one type of models over other since it depends on experts/analysts discretion what he is trying to achieve. For our analysis, we found Cox's model sufficient enough to meet our requirements. We reiterate the flexibility and ease of use of Cox' model were outstanding. Although, we modeled a particular setting but presented model is capable of achieving higher level of scalability and can easily be replicated in any industry.

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## Chapter 4 : CONCLUSION AND FUTURE RESEARCH

Over the last few decades, OEMs and suppliers have realized that there are tremendous opportunities to engage in remanufacturing business to improve profitability and sustainability. However, efficient management of remanufacturing program is known for its complexity. This is mostly attributed to limited visibility in reverse logistics systems. Our collaboration with one of the tier 1 supplier indicated that there is lack of a structured and holistic decision support framework, which can guide firms in decision making related to timing the launch of the remanufacturing program, capacity installation/management etc. Further, efficient production and inventory management of remanufacturing parts for the supplier heavily impinges on the ability to accurately forecast these core returns from customers (besides forecasting demand for remanufacturing parts and securing cores from the open market, as necessary). Based on request from our collaborator and existing gaps in related literature survey, this research has proposed an integrated decision support framework for remanufacturing in aftermarket services. Though, focus of this thesis is mainly on automotive aftermarket services but models introduced are robust enough to fit in most of the remanufacturing environment.

In this research, we have tackled two interrelated problems of reman program at strategic and operational level. At strategic level, we have studied the capacity management in launch of reman program for aftermarket services. This objective requires making decision on optimal time to launch of the reman program and overall capacity

requirement. The pre-requisite of this objective is to first compute an optimal reman policy given a time to launch and capacity level. To pursue this, we have analytically derived optimal reman policy by minimizing total cost associated with reman. Our analysis revealed that in the presence of supply and capacity constraint, the optimal reman policy is a threshold policy in time to launch and capacity. This suggests that if time to launch and capacity is not in the range of their respective threshold values, it is not possible to reman all returns. Thus, total cost associated with reman program will be higher since firm couldn't exploit the option of remanufacturing all collected returns. Given optimal policy, it becomes evident that there exist an optimal time to launch and optimal capacity level for a reman program. To compute optimal time to launch and capacity level, we proposed a heuristics solution method to minimize the discounted cash outflow given an optimal reman policy. We found out that it is always in the best interest of the firm to delay the launch of a reman program to build a strategic recoverable inventory. Regarding capacity, most of the existing literature assumes that there is enough capacity level available for remanufacturing. On the contrary, our analysis suggests that it is not always optimal to install maximum capacity level. Working at maximum capacity level is only beneficial if reman program commences after end of the regular production. In that case, a high level of strategic inventory is built thus reman is not supply constrained. Further, we extensively studied the drivers of cost-effective remanufacturing in terms of various cost, product and life cycle parameters. Following are the specific contributions of our study at strategic level:

- (i) This is the first study that systematically accounted and explicitly modeled reman policy, time to launch and capacity level within a single modeling framework. Most of the prior research focused on evaluating these decisions disjointedly.
- (ii) We exploited the fact that each return generates demand for an aftermarket service parts due to trade-in process. Thus, demand for aftermarket service parts is same as core returns.
- (iii) An optimal reman policy is obtained analytically using pontygrain maximum principle.
- (iv) A heuristics solution algorithm is formulated to obtain the time to launch and capacity. Previous study in computation of time to launch did not account for optimal capacity level (Kleber, 2006). They assumed that there is sufficient capacity to reman most of the returns.
- (v) We have analytically derived the structural properties of optimal reman policy in presence of both supply and capacity constraints.
- (vi) A closed-form expression for threshold value of time to launch and capacity is accomplished in this study.
- (vii) Sensitivity analysis revealed many managerial insights important in achieving cost-effective reman program.
- (viii) Finally, our analysis of stochastic returns revealed that underlying deterministic analysis is very robust and efficient in capturing most important drivers of remanufacturing for aftermarket services.

At operational level, we studied the core-returns forecasting in remanufacturing. Most of the extant literature dealing with returns forecasting typically assumed that probability distribution of returns is already known. Furthermore, models were simply not practical for many suppliers, such as our collaborator, because the historical return data was simply too sparse to facilitate modelling and calibration of models for individual SKUs. Additionally, there were several challenges to this, including, the volume and diversity of customers, differences among individual customer warehouses, large remanufacturing product catalogue, and changing customer behaviours (often improving core return delays with time). To overcome these complications, we proposed an integrated modelling framework that relies on products and customers among others as covariates for forecasting returns among product families within a single tractable class of duration modelling.

In this thesis, we have reported the evidence for the effectiveness of hazard rate regression models to calculate return delay distribution in the context of remanufacturing. We extensively studied various types of hazard rate modelling technique (e.g., parametric, semi-parametric etc.) and its appropriateness. Further, we described various approaches when underlying proportionality assumptions is violated or when there is time-varying effect of covariates or there is randomness in one of the covariates. To the best of our knowledge, no existing literature has explored all these issues in context of returns modelling for remanufacturing. Furthermore, we also provided valuable insights based on our analysis regarding customer behaviour and made necessary

recommendation for firms in aftermarket remanufacturing business. Following are the specific contributions of our study at operational level:

- (i) We studied the effectiveness of hazard rate models in context of automotive remanufacturing targeted for independent aftermarket.
- (ii) Parametric, semi-parametric and extended Cox proportional models have been exploited in modeling core returns within a single tractable class of duration time modeling.
- (iii) For our analysis, we realized semi-parametric, Cox proportional hazard rate model, is powerful enough to understand the dynamics of IAM.
- (iv) Results obtained from extended Cox proportional hazard rate model revealed two important characteristics:
  - a. There is a time-varying effect of covariate and thus a time-by-covariate interaction is more appropriate approach to model IAM data. However, we showed that time-by-covariate interaction is very complicated modeling technique, thus selection models should be based on the trade-off between accuracy vs. complexity.
  - b. There is randomness due to covariate which is captured by implementation of covariates model.
- (v) Based on our analysis, we made following recommendations:

- a. Instead of trading in core, customer tends to stock the product to handle any stock-out situation.
- b. Customer is attracted the open-market deals on cores. Thus, firm needs to build a better incentive mechanism which can encourage customer to return the cores.
- c. Frailty model suggested that there is randomness in the process because of the product. Thus, there is opportunity of incorporating product attributes, such as product size, weight, core deposit, and demand etc., to improve model fidelity and explanation power.

#### **4.1 Future Research**

The undertaken research is a very first step in building integrated decision support framework for remanufacturing while catering needs to real world problem. Here, we briefly discuss a few potential areas that are worth exploring:

- (i) Our research study was focus on development of new facilities for a single remanufactured product. Since, product development and production and introduction are continuous process, it is important to incorporate the product portfolio instead of a single product analysis. Our research can be used as a starting point for such studies.
- (ii) We focused on making one time decision regarding capacity and assumed with time elapsed the investment can be considered as a sunk cost. But, typically, capacity management considers capacity expansion and contraction based on



market response. Thus, it is worth exploring dynamic capacity management in context of remanufacturing program.

- (iii) We considered the OES and IAM jointly with 100% service level. Generally, 100% service level constraint is not very valid assumption. Thus, explicit modeling of IAM could be more insightful in considering remanufacturing program for IAM. More explicit model should be able to answer questions such as; *should firm launch remanufacturing program for independent aftermarket, should firm operate at full capacity or capacity contraction is more attractive etc.*
- (iv) One important cost which undertaken research completely ignored is the cost of disposal. We believe that there is some value in incorporating disposal cost in the model. However, to account disposal, more sophisticated modeling is required.
- (v) In order to increase the explaining power and fidelity of the models for core-forecasting, product attributes can play a significant role.
- (vi) A new avenue for research in aftermarket services can be development of better incentive mechanism to encourage customer for quick returns.

**ABSTRACT****AN INTEGRATED DECISION SUPPORT FRAMEWORK FOR  
REMANUFACTURING IN THE AUTOMOTIVE INDUSTRY**

by

**AKHILESH KUMAR**

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**Advisor:** Dr. Ratna Babu Chinnam**Major:** Industrial Engineering**Degree:** Doctor of Philosophy

In today's global economy, firms are seeking any and every opportunity to differentiate from competitors by reducing supply chain costs and adding value to end customers. One increasingly popular option, under growing consumer awareness and increasing legislation, is to reintegrate returned products into the supply chain to achieve economic benefits as well as improve sustainability. An important class of such "reverse" goods flows has to do with remanufacturing (reman), which refers to activities that restore returned products ("cores") or their major modules to operational condition for using in place of new product or distributing through other channels (e.g., spare parts). While opportunities abound, some key complications reported in the literature include: 1) difficulty in timing the launch of reman product (while accounting for uncertainties associated with product life-cycle demand and core supply), 2) difficulty with capacity planning for remanufacturing (while accounting for the fact that volumes can be low and that facilities/lines should target multiple product families for economies of scale), and 3)

operational difficulties in maintaining efficiencies in production planning and control of remanufacturing activities. These difficulties are mostly attributable to limited visibility and higher levels of uncertainty in reverse logistics (in comparison with forward logistics). Despite advances in the remanufacturing literature in the last two decades (both in the academic literature and practitioner community), there is no integrated decision support framework that can guide companies to successful launch and execution of remanufacturing operations. This is particularly true for companies that engage in both original equipment (OE) service as well as the independent after-market (IAM) in the automotive industry. This research aims to address these limitations by developing a decision support framework and necessary models for effective remanufacturing in the automotive industry.

At the strategic level, we propose a unified approach to explicitly model and address issues of capacities as well timing the launch of remanufacturing programs for new product. We derive the optimal remanufacturing policy and extensively studied the drivers of cost-effective remanufacturing program for aftermarket services. Our policies exploit the ability to leverage OE production to support both the OE service operations as well as demand from the IAM. To the best of our knowledge, this research is the first attempt of its kind in the remanufacturing literature, as prior research treated these interrelated decisions separately. Valuable managerial insights are obtained by minimizing the discounted cash outflows caused by appropriate investment and core return inventory building decisions. We show that, under certain conditions, it may be optimal to delay the launch of the remanufacturing program to build up an adequate

initial core return inventory. This may help in perfectly substituting virgin parts with remanufactured parts after end of the OE production run.

At operational level, efficient production planning and control of reman parts for the supplier heavily impinges on the ability to accurately forecast core returns from customers (e.g., dealers, distributors). There are several challenges to this, including, the volume and diversity of customers served by the supplier, differences among individual customer warehouses in returning cores, large reman product catalogs, changing customer behaviors (often improving core return delays), and data sparsity. In this research we report the evidence for the effectiveness of hazard rate regression models to estimate core return delays in the context of remanufacturing. We investigate a number of hazard rate modelling techniques (e.g., parametric, semi-parametric etc.) using real-world datasets from a leading Tier-1 automotive supplier. Results indicate the effectiveness of the proposed framework in terms of stability and face validity of the estimates and in predictive accuracy.

## AUTOBIOGRAPHICAL STATEMENT

**NAME: AKHILESH KUMAR**

### EDUCATION

Ph. D: Industrial Engineering, Wayne State University, USA, 2011

B.Tech: Manufacturing Engineering, National Institute of Foundry and Forge Technology, India, 2005

### PUBLICATIONS

- “Timing the Launch of Reman Program and Optimal Capacity to Install in An Automotive Aftermarket Setting” to be submitted to *Production and Operations Management* (Ready for Submission)
- “Framework for Core Returns Forecasting of Auto Parts Remanufacturing” to be submitted to *International Journal of Production Economics* (Ready for Submission)
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