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A Robust One-Sided Variability Control Chart

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A new control charting technique to monitor the variability of any distribution is proposed. The simulation study shows that the new method outperforms all the existing methods in controlling the Type I error rates and it also has good power performance for all distributions considered in the study.

Key words: Edgeworth expansion, Type I error rate, power performance.

Introduction

A major objective of statistical quality control is to quickly detect any sustained shift of central tendency and variability of a process. The control chart, proposed by Shewhart (1931), is an on-line process-monitoring technique widely used for this purpose. The Shewhart chart contains three lines: the center line, which represents the average value of the quality characteristic, and two control limit lines, the UCL (Upper Control Limit) and LCL (Lower Control Limit). These lines are chosen in such a way that, if the process is in control, nearly all the sample points will fall between the lines. If a sample point plots between the two control limits, the process is assumed to be in control. If any point plots above UCL or below LCL, then it is reasonable to suspect that the process is out of control. In this case, investigations and corrective actions are required to find and to eliminate the assignable cause responsible for this behavior.

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Much work has been done to develop and improve control charts that are able to detect small and large shifts in the process mean. However, less work has been done to control the process variability. One of the most widely used methods to control variability of a process is the Shewhart R-chart. The UCL and LCL for the standard three-sigma chart are as follows:

$$UCL = \overline{R} + 3\hat{\sigma}_R = \overline{R} + 3d_3\frac{\overline{R}}{d_2},$$

and

$$LCL = \overline{R} - 3\hat{\sigma}_R = \overline{R} - 3d_3\frac{\overline{R}}{d_2},$$

with a $\text{Center line} = \overline{R}$, where

$$\overline{R} = \frac{R_1 + R_2 + ... + R_m}{m},$$

and R_i is the range (difference between the largest and the smallest observation) of the i^{th} preliminary sample; d_2 and d_3 are the mean and the standard deviation of $W = \frac{R}{\sigma}$, respectively. Tables of d_2 and d_3 are available for various sample sizes (Montgomery, 1996). The Shewhart R-charts are constructed under the assumption that the underlying process is normally distributed.

An alternative to the R-chart is the S-chart. Rather than using range as a measure of variability, the S-chart uses the standard deviation. The UCL and the LCL of an S-chart are formulated as follows:

$$UCL = \bar{s} + 3\frac{\bar{s}}{c_4}\sqrt{1-c_4^2}\ ,$$

and

$$LCL = \bar{s} - 3\frac{\bar{s}}{c_4}\sqrt{1-c_4^2}\ ,$$

with a Center line $\bar{s}$ where

$$\bar{s} = \frac{1}{m}\sum_{i=1}^{m} s_i\ ,\ s_i$$

is the sample standard deviation of the i^{th} preliminary sample, c_4 is a constant such that the statistic s is an unbiased estimator of $c_4\sigma$. Tables of c_4 for various sample sizes are available in many statistical quality control books (Montgomery, 1996). Similar to the R-chart, all c_4 tables are constructed under the assumption of normal process.

Another alternative charting technique recommended by many practitioners, is the Shewhart S^2-chart. In the construction of a S^2-chart, the fact that $\frac{(n-1)S^2}{\sigma^2}$ has $\chi^2_{(n-1)}$ distribution under normality is used. The control limits for this chart are:

$$UCL = \frac{\bar{s}^2}{n-1}\chi^2_{\frac{\alpha}{2},n-1}\ ,$$

with $\text{Center line} = \bar{s}^2$ and

$$LCL = \frac{\bar{s}^2}{n-1}\chi^2_{1-(\frac{\alpha}{2}),n-1}$$

where $\chi^2_{\frac{\alpha}{2},n-1}$ and $\chi^2_{1-(\frac{\alpha}{2}),n-1}$ denote the upper and lower $\frac{\alpha}{2}$ percentage points of the Chi-square distribution with $n-1$ degrees of freedom, and $\bar{s}^2$ is the average sample variances of m preliminary samples.

In many situations the underlying distribution of the process might not be normal. For example, the distributions of measurements from chemical processes and cutting tool wear processes are often skewed. Burr (1967) and Chan, Hapuarachchi and Macpherson (1988) have examined the effect of non-normality on R-charts. They found that, for skewed populations, Type I risk probabilities grow larger as the skewness of the distribution increases. The problem is in the "discrepancy between the variability pattern of the asymmetric distribution and the normality assumed in placing control limits on Shewhart R-chart." (Bai & Choi, 1995, p. 120). The impact of non-normality on the S-chart and S^2-chart is also expected.

To remedy the non-normal problem, Bai and Choi (1995) proposed a heuristic method for controlling variability of the skewed distributions based on the Weighted Variance (WV) method. Their chart is an R chart with 3-sigma control limits:

$$UCL = \bar{R}\left[1 + 3\frac{d_3'}{d_2'}\sqrt{2\hat{P}_X}\right] = V_U\bar{R}\ ,$$

and

$$LCL = \bar{R}\left[1 - 3\frac{d_3'}{d_2'}\sqrt{2(1-\hat{P}_X)}\right] = V_L\bar{R}\ ,$$

where

$$\hat{P}_X = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\delta\left(\bar{\bar{X}} - X_{ij}\right)}{m \cdot n}$$

with

$$\delta(x) = \begin{cases} 1 & for\ \ x \ge 0 \\ 0 & for\ \ x < 0 \end{cases},$$

n is the sample size and m is the number of preliminary samples. Therefore, $\hat{P}_X$ is the proportion of observations from pre-run stage that are less than or equal to the estimated process mean. Bai and Choi (1995) used $\hat{P}_X$ as an estimator of the measure of the degree of skewness of the process. They claimed that, if the underlying distribution is symmetric, then $P_X = 0.5$. If the population is positively or negatively skewed then P_X becomes greater than 0.5 or less than 0.5, respectively. The constants d_2' and d_3' are the mean and the standard deviation of $W = \frac{R}{\sigma}$ for the given skewed population.

Bai and Choi investigated five different families of distributions: Weibull, lognormal and three different forms of distributions from Burr's family. For each of the five families, they selected the proper parameter values, such that P_X is equal to a fixed quantity. For each value of P_X considered, they computed d_2' and d_3' via numerical integration for each distribution. They found that the values of d_2' and d_3' are similar across the distributions for each given P_X, so they took the average of those d_2' and d_3' as constants to construct tables for V_L and V_U for selected values of n and P_X.

Although Bai and Choi only considered five families of distributions in the computation of d_2' and d_3', they used the results to apply to all skewed distributions. Due to the limited choices of the skewed distributions, one may suspect that any distribution other than those considered, even with the same skewness but different kurtosis, may produce different constants d_2' and d_3'. Furthermore, Chan and Cui (2003) raised the question of using P_X as a measure of the degree of skewness in the WV method. They indicated that many skewed distributions may have a P_X value of 0.5, which leads to an incorrect control charting procedure.

To correct the skewness problem produced by WV method, Chan and Cui (2003) proposed the Skewness Correction (SC) method to construct R-control charts for skewed process distributions. The two control limits for SC R-chart are:

$$UCL = \bar{R}\left[1 + (3 + d_4^*)\frac{d_3^*}{d_2^*}\right] = D_4^*\bar{R}$$

and

$$LCL = \bar{R}\left[1 + (-3 + d_4^*)\frac{d_3^*}{d_2^*}\right]^+ = D_3^*\bar{R}$$

where chart constants d_2^* and d_3^*, as d_2 and d_3 in Shewhart control charts for the normal distribution, are defined as the mean and standard deviation of the relative range $\frac{R}{\sigma}$,

$$a^+ = \begin{cases} a & for \quad a \geq 0 \\ 0 & for \quad a < 0 \end{cases},$$

$$d_4^* = \frac{\frac{4}{3}k_3(R)}{1 + 0.2k_3^2(R)},$$

where $k_3(R)$ is the coefficient of skewness of the sample range *R*. The values of d_2^*, d_3^* and d_4^* can be obtained directly through numerical integration if the process distribution and sample size are specified.

In Chan and Cui's (2003) research, a collection of Weibull, lognormal, and four forms of distributions from the Burr's family are considered as representatives of all skewed distributions. The values of d_2^*, d_3^* and d_4^* are computed for selected values of k_3, the skewness of the distribution, in each family of distributions. Due to the similar values of the six d_2^*, d_3^* and d_4^* across the distributions for each given k_3, Chan and Cui took the averages of these d_2^*, d_3^* and d_4^* as constants for all the skewed distribution with a given k_3 to construct tables of D_4^* and D_3^* for various sample sizes. Skewness of the distribution k_3 is estimated by the sample skewness

$$k_3^* = \frac{1}{nm-1}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{X_{ij}-\overline{\overline{X}}}{S_{nm}}\right)^3,$$

where

$$\overline{\overline{X}} = \frac{1}{nm}\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij},$$

and

$$S_{nm} = \sqrt{\frac{1}{nm-1}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(X_{ij}-\overline{\overline{X}}\right)^2}.$$

Although the authors introduced the skewness correction to resolve the problem with P_X, the other potential problem is still unsolved. The tables are constructed based on three families with coefficient of skewness ranging from -4 to 4. It would be problematic for the practitioner to determine the control limits if the estimated coefficient of skewness is outside of this range. For example, the Weibull distribution with unit scale parameter and 0.5 shape parameter has $k_3 = 6.62$.

In a real life situation, it is more important to detect upper sustained shift than the lower shift in the process variability because the goal of statistical process control is to reduce the variability in the process as much as possible, the upper limit becomes more critical. As noted, it is common that the data has a non-normal underlying distribution; hence, the goal of this study is to develop an upper control chart for controlling the variability of the process that will work for any non-normal distribution, including both skewed and symmetric distributions.

Methodology

Long and Sa (2005) proposed a method that uses Edgeworth expansions to perform a hypothesis test for the variance for non-normally distributed populations. Their test controls type I error rates well and has power performance comparable to tests that have been developed in the past. The proposed control chart is an adaptation of their test.

Edgeworth expansion is an approximation to the distribution of the estimate $\hat{\theta}$ of the unknown quantity θ_0. If $\sqrt{n}(\hat{\theta}-\theta_0)$ is asymptotically normally distributed with mean zero and variance σ^2, then the distribution function of $\sqrt{n}(\hat{\theta}-\theta_0)$ may be expanded as a power series in $\sqrt{n}$ (Hall, 1992):

$$P\left\{\frac{\sqrt{n}(\hat{\theta}-\theta_0)}{\sigma} \le x\right\} =$$

$$\Phi(x) + n^{-\frac{1}{2}}p_1(x)\varphi(x) + \ldots + n^{-\frac{j}{2}}p_j(x)\varphi(x) + \ldots,$$

where $\phi(x) = (2\pi)^{-\frac{1}{2}}e^{-\frac{x^2}{2}}$ is the standard normal density function and $\Phi(x) = \int_{-\infty}^{x}\phi(u)du$ is the standard normal distribution function. The functions $p_j(x)$ are polynomials with coefficients depending on cumulants of $\hat{\theta}$-θ_0.

Hall (1992) provided the Edgeworth expansion for the sample variance

$$P\left\{\frac{\sqrt{n}(S^2-\sigma^2)}{\tau} \le x\right\} =$$

$$\Phi(x) + n^{-\frac{1}{2}}p_1(x)\varphi(x) + \ldots + n^{-\frac{j}{2}}p_j(x)\varphi(x) + \ldots,$$

where

$$p_1 = -\left(B_1 + B_2\frac{x^2-1}{6}\right),$$

$$B_1 = -(\nu_4-1)^{-\frac{1}{2}},$$

$$B_2 = (\nu_4-1)^{-\frac{3}{2}}(\nu_6 - 3\nu_4 - 6\nu_3^2 + 2),$$

$$\nu_j = E\left[\frac{X-\mu}{\sigma}\right]^j,$$

and

$$\tau = \sqrt{E(X-\mu)^4 - \sigma^4},$$

where $\tau/\sqrt{n}$ is the standard deviation of the estimator S^2.

The variable S^2 admits the inversion of Edgeworth expansion as follows:

$$P\left\{\frac{\sqrt{n}(S^2-\sigma^2)}{\tau} \le x + n^{-\frac{1}{2}}\left(B_1 + B_2\frac{x^2-1}{6}\right)\right\} = \Phi(x) + o(n^{-\frac{1}{2}}).$$

Long and Sa (2005) adapted the inversion formula of the Edgeworth Expansion to test $H_0: \sigma^2 = \sigma_0^2$ versus $H_a: \sigma^2 > \sigma_0^2$. An intuitive decision rule is to reject H_0 if

$$Z > z_\alpha + n^{-\frac{1}{2}}\left(B_1 + B_2\frac{z_\alpha^2-1}{6}\right), \qquad (1)$$

where z_α is the upper α percentage point of the standard normal distribution and

$$Z = \frac{S^2-\sigma_0^2}{\tau/\sqrt{n}}.$$

They first estimated B_1 and B_2 by

$$\hat{B}_1 = -\left(\frac{S^4}{\kappa_4 + 2S^4}\right)^{1/2}$$

and

$$\hat{B}_2 = \frac{\kappa_6 + 12\kappa_4 S^2 + 4\kappa_3^2 + 8\left(S^2\right)^3}{\left(\kappa_4 + 2S^4\right)^{3/2}},$$

where κ_3, κ_4, and κ_6 are the third, fourth and sixth sample cumulants, respectively. They then investigated six different forms of Z and found that

$$Z6 = \frac{S^2-\sigma_0^2}{\sqrt{\frac{\kappa_4\sigma_0^2}{nS^2} + \frac{2\sigma_0^4}{n-1}}} \qquad (2)$$

yields the best results for controlling the type I error rates as well as satisfying power performance; their final decision rule is to reject H_0 if:

$$Z6 > z_\alpha + n^{-\frac{1}{2}}\left(\hat{B}_1 + \hat{B}_2\frac{z_\alpha^2-1}{6}\right). \qquad (3)$$

The decision rule (3) of Long and Sa (2005) can be used in the construction of the upper-sided control chart for variability with some modifications. Population variance σ_0^2 can be estimated in the preliminary stage by the sample variance $\widetilde{S}^2$. $\hat{B}_1$ and $\hat{B}_2$ can also be calculated based on the preliminary samples.

The upper control limit can then be set as:

$$UCL = z_\alpha + n^{-\frac{1}{2}}\left(\hat{B}_1 + \hat{B}_2\frac{z_\alpha^2-1}{6}\right). \qquad (4)$$

In the control charting stage, a sample is selected and

$$Z6 = \frac{S_{(i)}^2 - \widetilde{S}^2}{\sqrt{\frac{\kappa_{4,(i)}\widetilde{S}^2}{nS_{(i)}^2} + \frac{2\widetilde{S}^4}{n-1}}}, \qquad (5)$$

where $S_{(i)}^2$ and $\kappa_{4,(i)}$ are variance and fourth cumulant of the i^{th} sample, is calculated. An out-of- control signal occurs when:

$$Z6 > UCL. \qquad (6)$$

The proposed Variability Control Chart can be constructed as follows:

1. Based on process requirements, select a significance level α and find the corresponding critical point z_α;

2. Assuming the process is under statistical control, select m preliminary samples of size n to calculate all the process quantities ($\widetilde{S}^2, \kappa_3, \kappa_4, \kappa_6, \hat{B}_1, \hat{B}_2$). Two methods are employed to calculate these quantities. The

first is called the combined sample method, which merges all m samples in the preliminary run as one sample with $m \cdot n$ observations to compute the process quantities. The second is the not combined sample method in which all the process quantities are equal to the averages of the m corresponding preliminary sample values.

3. Calculate UCL using (4);

4. Start the quality control stage. Select samples of size n periodically. After the i^{th} sample is selected, calculate the sample variance $S_{(i)}^2$ and sample cumulant $\kappa_{4,(i)}$;

5. Plug them into (5) to get the sample point Z6 for this sample;

6. Plot the sample point Z6 on the chart and draw the conclusion about this sample (in-control or out-of-control);

7. If the process is in-control, then go back to step 4 to select next sample; otherwise quality control team should investigate and possibly remove the assignable causes.

Simulation Study

In order to compare different control charts for variability of a process, a simulation study to investigate the type I error rates and power performance is performed. The methods compared include the Shewhart R-chart, S-chart, S^2-chart, WV R-chart, SC R-chart and the proposed control charts.

Distributions Examined

A large collection of distributions with a wide range of skewness and kurtosis are investigated via a simulation study. Distributions considered are separated into two groups: skewed and symmetric.

The skewed family includes eight Weibull distributions with scale parameter $\lambda = 1$ and shape parameter k from 0.5 to 3.5; exponential with $\lambda = 1$; Gamma with scale parameter $\beta = 1$ and shape parameters $\alpha = 0.15$, 1.2 and 4.0; Chi-square with ν degrees of freedom ($\nu = 1$ to 24); lognormal with $\mu = 0$ and $\sigma^2 = 1$; and the Barnes2 distribution which is a polynomial function of the standard normal distribution (Fleishman, 1978). For comparison purposes, the standard normal distribution is also studied and reported.

The symmetric distributions considered include: Student's T ($\nu = 5$, 6, 8, 16, 32, 40), JTB ($\mu = 0, \sigma = 1, \alpha, \tau$) with various α and τ (Johnson, Tietjen & Beckman, 1980) and special designed distributions Barnes1 and Barnes3 with respective kurtosis 6.89 and 1049 (Fleishman, 1978). All the distributions in this group are symmetric with the exception of Barnes3 with a coefficient of skewness of 3.00, which is negligible in comparison to its kurtosis of 1049.

Random number generators from the Fortran 90 IMSL library are used to generate normal, Weibull, exponential, lognormal, Chi-square, Gamma and Student's T variates. In addition, the Barnes1, Barnes2, Barnes3 and JTB random variates were created with Fortran 90 program subroutines using IMSL library's random number generators for standard normal, gamma and uniform distributions in various parts of the program.

Simulation Description

The simulation study includes two parts: (1). Process is in-control (type I error rate comparisons) and (2). Process is shifted (the power study). In both studies, the process parameters are assumed unknown and therefore need to be estimated. The number of samples used in the preliminary run is $m = 30$; a relatively small sample size of 10 and a moderate sample size of 25 are used in the study. The steps of the simulation take place in two parts: steps 1 – 4 are preliminary runs and steps 5 – 9 are the quality control stages.

Preliminary Runs (assumes the process is in-control):

1. Set up the nominal level $\alpha = 0.0027$ (which corresponds to the frequently used Average Run Length, ARL, = 370) and select the parent distribution;

2. Generate 30 samples of size n from the parent distribution;

3. Calculate the necessary quantities used in different methods: $\overline{R}, \overline{S}, \overline{S}^2$, $\hat{P}_X, k_3, \hat{B}_1, \hat{B}_2$ $\widetilde{S}^2, \kappa_3, \kappa_4, \kappa_6$; both the combined sample method and not combined sample method are used to calculate the process quantities for the proposed methods.

4. Calculate the appropriate upper control limit for each of the control methods; the control limits of the Shewhart R-chart, S-chart, S^2-chart, WV R-chart and SC R-chart are adjusted to meet the purpose of the comparisons. In order to achieve the desired nominal level of $\alpha = 0.0027$ for a one-sided control chart, $z_\alpha = 2.78215$ is used to construct the appropriate upper control limits for all the methods.

The Quality Control Stage: Steps (5) – (9)

5. Generate 1,000 samples of size n from the same parent distribution and calculate the statistic to be plotted for each of the control methods (sample range R for the Shewhart, WV and SC R-charts, sample standard deviation S for S-chart, sample variance for S^2-char, and Z6 for the proposed method);

6. Compare the statistic with the corresponding control limits and tabulate the number of out-of-control signals;

7. Calculate type I error rate for each method by finding the proportion of out-of-control signals in the 1000 samples;

8. Repeat steps (2) – (7) 4,000 times;

9. Calculate the average of 4,000 type I error rates.

In the power study each generated variate is multiplied by a pre-determined $\sqrt{k}$, where $k = 1,2,3,4,5,6$; thus, a new set of observations is created with variance k times larger than the variance of the original distribution. Steps (5) – (9) are then repeated for each value of k to investigate the power of each charting technique. The corresponding ARL can be calculated for an in-control or an out-of-control process by inverting the average type I error rate or power from step (9).

Results

The Study of Type I Error Rates

Tables 1 through 4 provide comparisons of type I error rates for skewed and symmetric distributions with sample sizes $n = 10$ and $n = 25$. The first and the second columns are the type I error rates of the proposed method Z6 using the combined sample and the not combined sample methods in the calculations of the process quantities. Three critical points z_α, $\frac{z_\alpha + t_{n-1,\alpha}}{2}$ and $t_{n-1,\alpha}$ are used in construction of the upper control limits for the proposed method with sample size $n = 10$; results shown are the first, second and third numbers in the respective column.

Skewed Distributions

Table 1 shows that all traditional control charts (the Shewhart R-chart, S-chart and S^2-chart) fail to maintain the type I error rates under nominal level $\alpha = 0.0027$ when the parent distribution is skewed. In general, the larger the degree of skewness, the bigger the type I error rate. For example, considering a $\chi^2_{(24)}$ distribution with skewness 0.58, the type I error rates of the three traditional charts are 0.0235, 0.0224 and 0.00566 with corresponding ARLs 42.55, 44.64 and 176.68 for the Shewhart R-chart, S-chart and S^2-chart respectively. Those rates change to 0.124, 0.132 and 0.0624 with respective ARLs 8.06, 7.58 and 16.03 when the parent distribution is $\chi^2_{(1)}$, with a more severe skewness of 2.83.

Among the three traditional charts, the S^2-chart tends to outperform the other two, however, it still consistently yields inflated type I error rates which result in very short ARLs. It usually performs reasonably well for distributions with low skewness. The best cases

Table 1: Skewed Distributions, Comparisons of Type I Error Rates when $n = 10$

Distribution (skewness) (kurtosis)	Combined Sample	Not Combined	R-chart	S-chart	S^2-chart	WV-chart	SC-chart
Normal (0,1)	3.97E-03	1.09E-02					
(0.00)	1.23E-03	9.36E-03	1.45E-02	1.24E-02	1.86E-03	1.24E-02	6.40E-03
(0.00)	3.58E-04	9.77E-03					
Exponential (1)	5.82E-03	4.71E-02					
(2.00)	2.73E-03	5.00E-02	8.71E-02	9.26E-02	4.10E-02	1.15E-02	7.58E-03
(6.00)	1.31E-03	5.39E-02					
Lognormal (0,1)	5.80E-03	4.19E-02					
(6.18)	2.85E-03	4.23E-02	1.37E-01	1.44E-01	7.61E-02	2.52E-02	2.33E-02
(110.93)	1.48E-03	4.38E-02					
Weibull (0.5)	5.83E-03	4.10E-02					
(6.62)	2.82E-03	4.28E-02	1.66E-01	1.75E-01	8.74E-02	3.11E-02	3.07E-02
(84.72)	1.48E-03	4.48E-02					
Weibull (0.75)	5.50E-03	4.64E-02					
(3.12)	2.69E-03	4.85E-02	1.27E-01	1.35E-01	6.62E-02	1.18E-02	1.30E-02
(16.0)	1.38E-03	5.10E-02					
Weibull (0.85)	5.70E-03	4.70E-02					
(2.56)	2.76E-03	4.95E-02	1.10E-01	1.17E-01	5.56E-02	1.14E-02	1.02E-02
(10.35)	1.41E-03	5.26E-02					
Weibull (1)	5.82E-03	4.71E-02					
(2.00)	2.73E-03	5.00E-02	8.71E-02	9.26E-02	4.10E-02	1.15E-02	7.58E-03
(6.00)	1.31E-03	5.39E-02					
Weibull (1.2)	5.70E-03	4.56E-02					
(1.52)	2.53E-03	4.89E-02	6.24E-02	6.61E-02	2.59E-02	1.06E-02	5.52E-03
(3.24)	1.14E-03	5.34E-02					
Weibull (1.5)	5.10E-03	4.13E-02					
(1.07)	2.05E-03	4.48E-02	3.73E-02	3.91E-02	1.22E-02	8.87E-03	3.90E-03
(1.39)	8.18E-04	4.97E-02					
Weibull (2.0)	4.08E-03	2.85E-02					
(0.63)	1.41E-03	3.12E-02	1.71E-02	1.77E-02	3.65E-03	7.41E-03	2.70E-03
(0.25)	4.66E-04	3.57E-02					
Weibull (3.5)	3.36E-03	1.07E-02					
(0.03)	9.14E-04	9.33E-03	6.53E-03	7.07E-03	6.93E-04	5.43E-03	2.10E-03
(-0.29)	2.13E-04	1.00E-02					
Barnes 2	7.25E-03	4.86E-02					
(1.75)	3.39E-03	5.19E-02	7.41E-02	7.82E-02	2.95E-02	4.77E-03	3.90E-03
(3.75)	1.63E-03	5.69E-02					

Table 1 (Continued): Skewed Distributions, Comparisons of Type I Error Rates when $n = 10$

Distribution (skewness) (kurtosis)	Combined Sample	Not Combined	R-chart	S-chart	S^2-chart	WV-chart	SC-chart
Chi (1)	6.31E-03	5.13E-02					
(2.83)	3.13E-03	5.48E-02	1.24E-01	1.32E-01	6.24E-02	7.89E-03	1.06E-02
(12.0)	1.64E-03	5.84E-02					
Chi (2)	5.82E-03	4.71E-02					
(2.00)	2.73E-03	5.00E-02	8.71E-02	9.26E-02	4.10E-02	1.15E-02	7.58E-03
(6.00)	1.31E-03	5.39E-02					
Chi (3)	5.72E-03	4.46E-02					
(1.63)	2.60E-03	4.75E-02	6.86E-02	7.28E-02	3.05E-02	1.27E-02	6.74E-03
(4.00)	1.19E-03	5.10E-02					
Chi (4)	5.44E-03	4.30E-02					
(1.41)	2.42E-03	4.56E-02	5.83E-02	6.14E-02	2.43E-02	1.32E-02	6.54E-03
(3.00)	1.07E-03	4.91E-02					
Chi (8)	4.87E-03	3.81E-02					
(1.00)	1.99E-03	4.12E-02	3.93E-02	4.03E-02	1.37E-02	1.30E-02	6.03E-03
(1.50)	8.23E-04	4.51E-02					
Chi (10)	4.67E-03	3.48E-02					
(0.89)	1.88E-03	3.76E-02	3.49E-02	3.53E-02	1.14E-02	1.32E-02	6.10E-03
(1.20)	7.45E-04	4.15E-02					
Chi (12)	4.52E-03	3.37E-02					
(0.82)	1.77E-03	3.64E-02	3.16E-02	3.16E-02	9.71E-03	1.29E-02	5.95E-03
(1.00)	6.77E-04	4.05E-02					
Chi (16)	4.45E-03	2.62E-02					
(0.71)	1.71E-03	2.76E-02	2.82E-02	2.75E-02	7.89E-03	1.35E-02	6.08E-03
(0.75)	6.39E-04	3.01E-02					
Chi (24)	4.22E-03	2.51E-02					
(0.58)	1.54E-03	2.64E-02	2.35E-02	2.24E-02	5.66E-03	1.30E-02	5.71E-03
(0.50)	5.42E-04	2.95E-02					
Gamma (0.15)	6.00E-03	4.54E-02					
(5.16)	2.99E-03	4.71E-02	1.85E-01	1.97E-01	9.43E-02	2.91E-02	2.82E-02
(40.0)	1.61E-03	4.90E-02					
Gamma (1.2)	6.07E-03	5.12E-02					
(1.83)	2.87E-03	5.46E-02	7.89E-02	8.38E-02	3.65E-02	1.23E-02	7.27E-03
(5.00)	1.34E-03	5.88E-02					
Gamma (4.0)	4.95E-03	3.97E-02					
(1.00)	2.07E-03	4.22E-02	3.91E-02	3.99E-02	1.36E-02	1.29E-02	6.02E-03
(1.50)	8.29E-04	4.58E-02					

Table 2: Skewed Distributions, Comparisons of Type I Error Rates when $n = 25$

Distribution	(skewness) (kurtosis)	Combined Sample	Not Combined	R-chart	S-chart	S2-chart	WV-chart
Normal (0,1)	(0.00) (0.00)	3.41E-03	3.50E-03	1.83E-02	1.64E-02	1.82E-03	1.70E-02
Exponential (1)	(2.00) (6.00)	2.59E-03	3.03E-03	1.09E-01	1.19E-01	5.32E-02	8.16E-03
Lognormal (0,1)	(6.18) (110.93)	1.72E-03	2.51E-03	1.68E-01	1.85E-01	1.05E-01	2.49E-02
Weibull (0.5)	(6.62) (84.72)	1.97E-03	2.98E-03	1.97E-01	2.20E-01	1.24E-01	2.84E-02
Weibull (0.75)	(0.12) (1.23)	1.67E-03	2.22E-03	1.53E-01	1.71E-01	9.10E-02	9.19E-03
Weibull (0.85)	(2.56) (10.35)	2.02E-03	2.51E-03	1.35E-01	1.49E-01	7.49E-02	8.71E-03
Weibull (1)	(2.00) (6.00)	2.59E-03	3.03E-03	1.09E-01	1.19E-01	5.32E-02	8.16E-03
Weibull (1.2)	(1.52) (3.24)	3.18E-03	3.61E-03	8.06E-02	8.47E-02	3.16E-02	8.08E-03
Weibull (1.5)	(1.07) (1.39)	3.50E-03	3.88E-03	4.98E-02	5.03E-02	1.35E-02	8.91E-03
Weibull (2.0)	(0.63) (0.25)	3.20E-03	3.45E-03	2.27E-02	2.30E-02	3.54E-03	9.28E-03
Weibull (3.5)	(0.03) (-0.29)	2.72E-03	2.77E-03	6.67E-03	9.91E-03	7.08E-04	6.08E-03
Chi (1)	(2.83) (12.0)	2.10E-03	2.60E-03	1.46E-01	1.65E-01	8.38E-02	5.01E-03
Chi (2)	(2.00) (6.00)	2.59E-03	3.03E-03	1.09E-01	1.19E-01	5.32E-02	8.16E-03
Chi (3)	(1.63) (4.00)	2.78E-03	3.24E-03	8.80E-02	9.35E-02	3.77E-02	9.73E-03
Chi (4)	(1.41) (3.00)	2.84E-03	3.31E-03	7.58E-02	7.88E-02	2.92E-02	1.17E-02
Chi (8)	(1.00) (1.50)	2.90E-03	3.26E-03	5.17E-02	5.15E-02	1.51E-02	1.47E-02
Chi (10)	(0.89) (1.20)	2.92E-03	3.27E-03	4.61E-02	4.49E-02	1.22E-02	1.56E-02
Chi (12)	(0.82) (1.00)	3.03E-03	3.32E-03	4.24E-02	4.12E-02	1.06E-02	1.63E-02
Chi (16)	(0.71) (0.75)	3.03E-03	3.28E-03	3.71E-02	3.56E-02	8.25E-03	1.69E-02
Chi (24)	(0.58) (0.50)	3.16E-03	3.43E-03	3.14E-02	2.95E-02	5.79E-03	1.70E-02
Gamma (0.15)	(5.16) (40.0)	1.52E-03	2.26E-03	2.04E-01	2.36E-01	1.30E-01	1.77E-02
Gamma (1.2)	(1.83) (5.00)	2.56E-03	3.06E-03	9.83E-02	1.06E-01	4.51E-02	8.31E-03
Gamma (4.0)	(1.00) (1.50)	2.97E-03	3.38E-03	5.19E-02	5.15E-02	1.51E-02	1.46E-02
Barnes 2	(1.75) (3.75)	4.09E-03	4.73E-03	8.85E-02	9.66E-02	3.55E-02	2.08E-03

Table 3: Symmetric Distributions, Comparisons of Type I Error Rates when $n = 10$

Distribution (skewness) (kurtosis)	Combined Sample	Not Combined	R-chart	S-chart	S^2-chart	WV-chart	SC-chart
Normal (0,1)	3.97E-03	1.09E-02					
(0.00)	1.23E-03	9.36E-03	1.45E-02	1.24E-02	1.86E-03	1.24E-02	6.40E-03
(0.00)	3.58E-04	9.77E-03					
JTB (2.0, 1.0)	4.58E-03	1.50E-02					
(0.00)	1.89E-03	1.39E-02	6.60E-02	6.30E-02	2.39E-02	5.77E-02	4.23E-02
(3.00)	7.78E-04	1.46E-02					
JTB (0.75, 0.5)	5.69E-03	1.37E-02					
(0.00)	2.22E-03	1.10E-02	3.93E-02	3.62E-02	9.57E-03	3.33E-02	2.21E-02
(1.20)	8.52E-04	1.06E-02					
JTB (4.0, 1.0)	4.33E-03	1.43E-02					
(0.00)	1.59E-03	1.31E-02	3.31E-02	2.78E-02	7.42E-03	2.95E-02	1.88E-02
(0.78)	5.71E-04	1.35E-02					
JTB (1.0, 0.5)	5.08E-03	1.30E-02					
(0.00)	1.83E-03	1.07E-02	2.71E-02	2.41E-02	5.18E-03	2.34E-02	1.40E-02
(0.60)	6.38E-04	1.09E-02					
JTB (1.25, 0.5)	4.47E-03	1.30E-02					
(0.00)	1.52E-03	1.13E-02	1.94E-02	1.68E-02	3.03E-03	1.67E-02	9.29E-03
(0.24)	4.69E-04	1.17E-02					
JTB (1.35, 0.5)	4.19E-03	1.12E-02					
(0.00)	1.38E-03	9.34E-03	1.69E-02	1.45E-02	2.44E-03	1.46E-02	7.86E-03
(0.13)	4.11E-04	9.76E-03					
JTB (1.5, 0.5)	4.00E-03	1.18E-02					
(0.00)	1.26E-03	1.01E-02	1.45E-02	1.24E-02	1.88E-03	1.25E-02	6.44E-03
(0.00)	3.70E-04	1.05E-02					
JTB (2.0, 0.5)	3.22E-03	1.26E-02					
(0.00)	9.07E-04	1.18E-02	8.52E-03	7.47E-03	8.68E-04	7.08E-03	3.27E-03
(-0.30)	2.43E-04	1.29E-02					
JTB (4.0, 0.5)	1.65E-03	1.22E-02					
(0.00)	3.43E-04	1.28E-02	1.55E-03	2.00E-03	1.02E-04	1.20E-03	3.71E-04
(-0.75)	6.33E-05	1.52E-02					
JTB (9.0, 0.5)	8.09E-04	1.86E-02					
(0.00)	1.28E-04	2.27E-02	6.28E-05	4.81E-04	9.75E-06	4.28E-05	5.75E-06
(-1.00)	1.33E-05	2.92E-02					

Table 3 (continued): Symmetric Distributions, Comparisons of Type I Error Rates when $n = 10$

Distribution (skewness) (kurtosis)	Combined Sample	Not Combined	R-chart	S-chart	S^2-chart	WV-chart	SC-chart
Barnes 3	2.04E-03	1.73E-02	1.10E-01	9.09E-02	6.07E-02	1.06E-01	7.76E-02
(3.00)	8.02E-04	1.71E-02					
(1049)	3.41E-04	1.77E-02					
Barnes 1	3.97E-03	1.09E-02	1.45E-02	1.24E-02	1.86E-03	1.24E-02	6.40E-03
(0.00)	1.23E-03	9.36E-03					
(6.89)	3.58E-04	9.77E-03					
Student (5)	3.27E-03	1.66E-02	6.24E-02	5.61E-02	2.52E-02	5.76E-02	4.17E-02
(0.00)	1.27E-03	1.58E-02					
(6.00)	5.04E-04	1.66E-02					
Student (6)	3.41E-03	1.43E-02	5.31E-02	4.67E-02	1.90E-02	4.87E-02	3.44E-02
(0.00)	1.31E-03	1.35E-02					
(3.00)	5.15E-04	1.40E-02					
Student (8)	3.79E-03	1.44E-02	4.23E-02	3.63E-02	1.27E-02	3.84E-02	2.62E-02
(0.00)	1.41E-03	1.35E-02					
(1.50)	5.15E-04	1.39E-02					
Student (16)	4.09E-03	1.32E-02	2.68E-02	2.22E-02	5.48E-03	2.38E-02	1.46E-02
(0.00)	1.42E-03	1.22E-02					
(0.50)	4.80E-04	1.27E-02					
Student (25)	4.20E-03	1.15E-02	2.22E-02	1.83E-02	3.86E-03	1.94E-02	1.13E-02
(0.00)	1.41E-03	9.63E-03					
(0.29)	4.68E-04	9.64E-03					
Student (32)	4.01E-03	9.66E-03	1.97E-02	1.64E-02	3.21E-03	1.72E-02	9.79E-03
(0.00)	1.35E-03	7.51E-03					
(0.21)	4.24E-04	7.32E-03					
Student (40)	4.08E-03	1.26E-02	1.89E-02	1.57E-02	2.93E-03	1.65E-02	9.18E-03
(0.00)	1.34E-03	1.09E-02					
(0.17)	4.14E-04	1.11E-02					

Table 4: Symmetric Distributions, Comparisons of Type I Error Rates when $n = 25$

Distribution	(skewness) (kurtosis)	Combined Sample	Not Combined	R-chart	S-chart	S2-chart	WV-chart
Normal (0,1)	(0.00) (0.00)	3.41E-03	3.50E-03	1.83E-02	1.64E-02	1.82E-03	1.70E-02
JTB (2.0, 1.0)	(0.00) (3.00)	2.31E-03	2.51E-03	8.16E-02	7.97E-02	2.84E-02	7.63E-02
JTB (0.75, 0.5)	(0.00) (1.20)	4.42E-03	4.57E-03	4.41E-02	4.45E-02	9.86E-03	4.03E-02
JTB (4.0, 1.0)	(0.00) (0.78)	3.26E-03	3.38E-03	4.47E-02	3.55E-02	7.69E-03	4.24E-02
JTB (1.0, 0.5)	(0.00) (0.60)	4.36E-03	4.48E-03	3.18E-02	3.05E-02	5.21E-03	2.97E-02
JTB (1.25, 0.5)	(0.00) (0.24)	3.81E-03	3.96E-03	2.36E-02	2.16E-02	2.86E-03	2.20E-02
JTB (1.35, 0.5)	(0.00) (0.13)	3.72E-03	3.83E-03	2.15E-02	1.97E-02	2.39E-03	2.00E-02
JTB (1.5, 0.5)	(0.00) (0.00)	3.36E-03	3.46E-03	1.80E-02	1.63E-02	1.77E-03	1.68E-02
JTB (2.0, 0.5)	(0.00) (-0.30)	2.42E-03	2.50E-03	1.11E-02	9.94E-03	7.17E-04	1.02E-02
JTB (4.0, 0.5)	(0.00) (-0.75)	9.33E-04	9.70E-04	2.29E-03	3.03E-03	8.20E-05	2.02E-03
JTB (9.0, 0.5)	(0.00) (-1.00)	3.23E-04	3.33E-04	1.01E-04	8.21E-04	8.25E-06	8.53E-05
Barnes 3	(3.00) (1.49)	6.14E-04	9.31E-04	1.91E-01	1.47E-01	9.66E-02	1.88E-01
Barnes 1	(0.00) (6.89)	3.41E-03	3.50E-03	1.83E-02	1.64E-02	1.82E-03	1.70E-02
Student (5)	(0.00) (6.00)	1.30E-03	1.42E-03	9.00E-02	7.69E-02	3.36E-02	8.69E-02
Student (6)	(0.00) (3.00)	1.64E-03	1.75E-03	7.67E-02	6.35E-02	2.45E-02	7.39E-02
Student (8)	(0.00) (1.50)	2.32E-03	2.46E-03	6.11E-02	4.82E-02	1.50E-02	5.86E-02
Student (16)	(0.00) (0.50)	3.16E-03	3.29E-03	3.76E-02	2.88E-02	5.56E-03	3.56E-02
Student (25)	(0.00) (0.29)	3.28E-03	3.43E-03	2.95E-02	2.32E-02	3.64E-03	2.78E-02
Student (32)	(0.00) (0.21)	3.42E-03	3.50E-03	2.72E-02	2.17E-02	3.17E-03	2.56E-02
Student (40)	(0.00) (0.17)	3.35E-03	3.44E-03	2.50E-02	2.04E-02	2.75E-03	2.35E-02

other than the standard normal distribution produced by S^2-chart are for the Weibull(2.0) with skewness 0.63 and Weibull(3.5) with skewness 0.03 which result respective type I error rates 0.00365 and 0.000693 equivalent to ARLs 273.97 and 1443. However, when skewness increases, such as $\chi^2_{(8)}$ with skewness of 1.00, the performance goes down dramatically with type I error rate 0.0137 and ARL = 72.99.

The WV R-chart is also unable to maintain the type I error rate for skewed distributions; although it works well for a few distributions, in general it produces false alarms too often. The SC R-chart has better performances among the existing variability control charts. It shows a degree of robustness when the coefficient of skewness is small, but if the skewness becomes somewhat severe, it fails to keep the type I error rates close to the nominal level. For example, the SC R-chart produces type I error rates of 0.0233 and 0.0307 with corresponding ARL of 42.92 and 32.57 for the standard lognormal with skewness = 6.18 and Weibull (0.5) with skewness = 6.62.

For the proposed method, results show that the combined sample method, which merges all the samples in the preliminary runs as one large sample to compute the process quantities, consistently outperforms all the other methods with very few exceptions. The worst case is for the Barnes2 distribution with skewness 1.75. It produces the highest type I error rate of 0.0075 with a corresponding ARL 137.93 when z_α is used as the critical point. However, it drops to 0.00339 with ARL 294.99 when $\frac{z_\alpha + t_{n-1,\alpha}}{2}$ is used. When $t_{n-1,\alpha}$ is used as a critical point, the proposed method becomes too conservative, which is not recommended because it will become more difficult to detect shifts if present.

When a larger sample size $n = 25$ is used in the simulation study (see Table 2), the performances of the Shewhart R-chart, S-chart, S^2-chart and WV R-chart do not change much. Type I errors rates for these charts are still inflated for distributions with high degrees of skewness such as the standard lognormal and Weibull with shape = 0.5, etc. Conversely, the proposed method with combined sample produces type I error rates close to the nominal level even with z_α as the critical point. The highest type I error rate produced by the proposed method is 0.00409 (ARL = 244.5) for the Barnes2 distribution.

Table 1 shows that the proposed method with combined sample can also be used for the standard normal distribution. The type I error rates produced are smaller than those of all the charts except the S^2-chart, even though it is not designed for the normal distribution. This nice performance adds another desirable property to the proposed method.

Note that the SC R-chart is not used in the simulation study with sample size $n = 25$ because Chan and Cui (2003) do not provide constants for calculations of the control limits for any sample size larger than 10. It is extremely difficult for the practitioners to implement this control chart if the situation requires collecting a sample size larger than 10.

Symmetric Distributions

Table 3 provides type I error rate comparisons for the symmetric distributions with sample size 10. The proposed method is the only one that holds the type I error rates almost all the time. Although some of the type I error rates for the proposed method are a little higher than 0.0027, they are all within an acceptable range. The worst case found in the study is for the JTB distribution ($\alpha = 0.75$, $\tau = 0.5$) with kurtosis 1.2 using z_α as a critical point producing the lowest ARL = 175.75 with type I error rate 0.0057. However, once the critical point is changed to $\frac{z_\alpha + t_{n-1,\alpha}}{2}$, the ARL increases to 454.55 with type I error = 0.0022. Again when the critical point $t_{n-1,\alpha}$ is used, the proposed method becomes unnecessarily conservative.

The two traditional methods, Shewhart R-chart and S-chart, are not robust at all, but the S^2-chart performs surprisingly well when the kurtosis of the distribution is either very close to zero or negative. However, the good performance soon disappears once the distribution has a kurtosis larger than 0.5. It is expected that WV and SC methods will not perform very well, because they only try to correct the skewness of the distribution, not the kurtosis.

It can be observed that the type I error rates for all the existing charts are strongly affected by the kurtosis of the distributions. The type I error rate increases when the kurtosis increases. When Barnes3 with kurtosis 1049 is the parent distribution, all the other charting techniques fail. The type I error rates for Shewhart R-chart, S-chart, S^2-chart, WV R-chart and SC R-chart are 0.11, 0.091, 0.0607, 0.106 and 0.0776 with corresponding ARL 9.09, 11, 16.47, 9.43 and 13.04, respectively.

Table 4 provides type I error rate comparisons for the symmetric distributions with sample size 25. Similar results to those shown in Table 3 are observed in this table. The proposed method is the only one with robust performance. The highest type I error rate is 0.00442 with ARL = 226.24 for JTB ($\alpha = 0.75$, $\tau = 0.5$) with kurtosis = 1.2. All other methods are not able to maintain type I error rates for distributions with kurtosis greater than 0.78. When the coefficient of kurtosis is in negative values, the type I error rates are generally much lower than the desired nominal level; this is observed in all the methods studied except in R-chart which generally fails in nearly all cases.

Power Study

The primary goal of the power study is to find the control charts with improved type I error rates and power performance comparable to other charts. It is reasonable to expect that more conservative charts might produce lower power than other charts because it is more difficult to detect an out-of-control state with these charts.

The results of the power study for skewed distributions are presented in Table 5 for sample size 10 with z_α and $\frac{z_\alpha + t_{n-1,\alpha}}{2}$ as critical points; results for symmetric distributions are reported in Table 6 for sample size 10. A power study was also conducted for cases with sample size 25. For complete simulation results, please see Borysov and Sa (2010).

The following similarities in the power performances of all the control charting methods are observed: As sample size increases from 10 to 25, power increases; as k in $k\sigma^2$ increases, the power increases; as the skewness of the skewed distribution increases, the power tends to decrease; and as kurtosis of the symmetric distributions increases, the power also tends to decrease.

It can be observed that the power performance of the proposed method is relatively good and is similar to other charts. In the cases of highly skewed distributions with large kurtosis (e.g., standard lognormal with skewness 6.18 and kurtosis 110, Weibull (0.5) with skewness 6.62 and kurtosis 84.72, Gamma (0.15) with skewness 5.16 and kurtosis 40), the power of the proposed method tends to be lower than those of other charts. However, recall that the proposed chart is the only one able to control the type I error rates for those distributions. When the shift in process variability increases the proposed scheme with $\frac{z_\alpha + t_{n-1,\alpha}}{2}$ becomes compatible to the WV and SC control charts.

Although the three Shewhart charts generally have higher power than the proposed control chart, it must be restated that power performance of the control chart is useless if it cannot preserve an appropriate type I error rate. Frequent false alarms can create more damage than quick shift detections can benefit. If sample size 25 is used, the proposed method has better power performance than the WV R-chart for almost all the distributions considered, even for small shifts of the variability.

Simulation Study Summary

The proposed Variability Control Chart which plots Z6 against UCL with combined sample should be used with decision rule (6) in order to achieve controllable type I error rates as well as to detect shifts in variability. It can be implemented for a process with any form of the underlying distribution consisting of skewed and and/or symmetric distributions including normal.

Table 5: Power Comparison Study for Skewed Distributions ($n=10$)

Distribution	(skewness) (kurtosis)	k	Combined Sample	R-chart	S-chart	S2-chart	WV-chart	SC-chart
Normal (0,1)	(0.00) (0.00)	k=1	3.97E-03 1.23E-03	1.24E-02	1.86E-03	1.24E-02	6.40E-03	1.24E-02
Exponential (1)	(2.00) (6.00)		5.82E-03 2.73E-03	8.71E-02	9.26E-02	4.10E-02	1.15E-02	7.58E-03
Lognormal (0,1)	(6.18) (110)		5.80E-03 2.85E-03	1.37E-01	1.44E-01	7.61E-02	2.52E-02	2.33E-02
Weibull (0.5)	(6.62) (84.72)		5.83E-03 2.82E-03	1.66E-01	1.75E-01	8.74E-02	3.11E-02	3.07E-02
Weibull (0.85)	(0.12) (1.23)		5.50E-03 2.69E-03	1.27E-01	1.35E-01	6.62E-02	1.18E-02	1.30E-02
Weibull (0.75)	(2.56) (10.35)		5.70E-03 2.76E-03	1.10E-01	1.17E-01	5.56E-02	1.14E-02	1.02E-02
Weibull (1)	(2.00) (6.00)		5.82E-03 2.73E-03	8.71E-02	9.26E-02	4.10E-02	1.15E-02	7.58E-03
Weibull (1.2)	(1.52) (3.24)		5.70E-03 2.53E-03	6.24E-02	6.61E-02	2.59E-02	1.06E-02	5.52E-03
Weibull (1.5)	(1.07) (1.39)		5.10E-03 2.05E-03	3.73E-02	3.91E-02	1.22E-02	8.87E-03	3.90E-03
Weibull (2.0)	(0.63) (0.25)		4.08E-03 1.41E-03	1.71E-02	1.77E-02	3.65E-03	7.41E-03	2.70E-03
Weibull (3.5)	(0.03) (-0.29)		3.36E-03 9.14E-04	6.53E-03	7.07E-03	6.93E-04	5.43E-03	2.10E-03
Barnes2	(1.75) (3.75)		7.25E-03 3.39E-03	7.41E-02	7.82E-02	2.95E-02	4.77E-03	3.90E-03
Normal(0,1)	(0.00) (0.00)	k=2	1.66E-01 1.01E-01	2.54E-01	2.95E-01	1.46E-01	2.37E-01	1.78E-01
Exponential (1)	(2.00) (6.00)		6.02E-02 3.53E-02	2.94E-01	3.25E-01	1.97E-01	7.20E-02	5.44E-02
Lognormal (0,1)	(6.18) (110)		2.54E-02 1.41E-02	2.78E-01	2.98E-01	1.75E-01	6.53E-02	6.12E-02
Weibull (0.5)	(6.62) (84.72)		2.16E-02 1.18E-02	2.89E-01	3.07E-01	1.76E-01	7.48E-02	7.38E-02
Weibull (0.85)	(0.12) (1.23)		3.69E-02 2.10E-02	3.01E-01	3.26E-01	1.99E-01	5.08E-02	5.48E-02
Weibull (0.75)	(2.56) (10.35)		4.56E-02 2.64E-02	2.99E-01	3.27E-01	2.00E-01	5.74E-02	5.35E-02
Weibull (1)	(2.00) (6.00)		6.02E-02 3.53E-02	2.94E-01	3.25E-01	1.97E-01	7.20E-02	5.44E-02
Weibull (1.2)	(1.52) (3.24)		8.02E-02 4.74E-02	2.84E-01	3.20E-01	1.88E-01	8.99E-02	5.90E-02
Weibull (1.5)	(1.07) (1.39)		1.09E-01 6.45E-02	2.69E-01	3.09E-01	1.73E-01	1.14E-01	7.05E-02
Weibull (2.0)	(0.63) (0.25)		1.48E-01 8.89E-02	2.51E-01	2.96E-01	1.51E-01	1.58E-01	9.67E-02
Weibull (3.5)	(0.03) (-0.29)		1.97E-01 1.23E-01	2.45E-01	2.90E-01	1.35E-01	2.28E-01	1.62E-01
Barnes2	(1.75) (3.75)		8.32E-02 5.13E-02	3.01E-01	3.35E-01	2.00E-01	5.57E-02	5.13E-02

Table 5 (continued): Power Comparison Study for Skewed Distributions ($n = 10$)

Distribution	(skewness) (kurtosis)	k	Combined Sample	R-chart	S-chart	S2-chart	WV-chart	SC-chart
Chi (1)	(2.83)	k=1	6.31E-03	1.24E-01	1.32E-01	6.24E-02	7.89E-03	1.06E-02
	(12.0)		3.13E-03					
Chi (2)	(2.00)		5.82E-03	8.71E-02	9.26E-02	4.10E-02	1.15E-02	7.58E-03
	(6.00)		2.73E-03					
Chi (3)	(1.63)		5.72E-03	6.86E-02	7.28E-02	3.05E-02	1.27E-02	6.74E-03
	(4.00)		2.60E-03					
Chi (4)	(1.41)		5.44E-03	5.83E-02	6.14E-02	2.43E-02	1.32E-02	6.54E-03
	(3.00)		2.42E-03					
Chi (8)	(1.00)		4.87E-03	3.93E-02	4.03E-02	1.37E-02	1.30E-02	6.03E-03
	(1.50)		1.99E-03					
Chi (10)	(0.89)		4.67E-03	3.49E-02	3.53E-02	1.14E-02	1.32E-02	6.10E-03
	(1.20)		1.88E-03					
Chi (12)	(0.82)		4.52E-03	3.16E-02	3.16E-02	9.71E-03	1.29E-02	5.95E-03
	(1.00)		1.77E-03					
Chi (16)	(0.71)		4.45E-03	2.82E-02	2.75E-02	7.89E-03	1.35E-02	6.08E-03
	(0.75)		1.71E-03					
Chi (24)	(0.58)		4.22E-03	2.35E-02	2.24E-02	5.66E-03	1.30E-02	5.71E-03
	(0.50)		1.54E-03					
Gamma (0.15)	(5.16)		6.00E-03	1.85E-01	1.97E-01	9.43E-02	2.91E-02	2.82E-02
	(40.0)		2.99E-03					
Gamma (1.2)	(1.83)		6.07E-03	7.89E-02	8.38E-02	3.65E-02	1.23E-02	7.27E-03
	(5.00)		2.87E-03					
Gamma (4.0)	(1.00)		4.95E-03	3.91E-02	3.99E-02	1.36E-02	1.29E-02	6.02E-03
	(1.50)		2.07E-03					
Chi (1)	(2.83)	k=2	4.35E-02	3.09E-01	3.35E-01	2.05E-01	4.31E-02	5.37E-02
	(12.0)		2.55E-02					
Chi (2)	(2.00)		6.02E-02	2.94E-01	3.25E-01	1.97E-01	7.20E-02	5.44E-02
	(6.00)		3.53E-02					
Chi (3)	(1.63)		7.21E-02	2.84E-01	3.18E-01	1.89E-01	9.10E-02	5.90E-02
	(4.00)		4.24E-02					
Chi (4)	(1.41)		8.05E-02	2.79E-01	3.14E-01	1.84E-01	1.04E-01	6.44E-02
	(3.00)		4.72E-02					
Chi (8)	(1.00)		1.02E-01	2.67E-01	3.05E-01	1.70E-01	1.33E-01	8.13E-02
	(1.50)		6.01E-02					
Chi (10)	(0.89)		1.09E-01	2.64E-01	3.02E-01	1.66E-01	1.43E-01	8.88E-02
	(1.20)		6.73E-02					
Chi (12)	(0.82)		1.15E-01	2.61E-01	3.00E-01	1.63E-01	1.51E-01	9.40E-02
	(1.00)		7.28E-02					
Chi (16)	(0.71)		1.24E-01	2.60E-01	2.99E-01	1.60E-01	1.66E-01	1.04E-01
	(0.75)		6.42E-02					
Chi (24)	(0.58)		1.32E-01	2.56E-01	2.95E-01	1.54E-01	1.80E-01	1.15E-01
	(0.50)		7.81E-02					
Gamma (0.15)	(5.16)		2.42E-02	3.15E-01	3.34E-01	1.96E-01	8.04E-02	7.84E-02
	(40.0)		1.34E-02					
Gamma (1.2)	(1.83)		6.66E-02	2.90E-01	3.23E-01	1.95E-01	8.15E-02	5.72E-02
	(5.00)		3.94E-02					
Gamma (4.0)	(1.00)		1.03E-01	2.66E-01	3.04E-01	1.70E-01	1.32E-01	8.13E-02
	(1.50)		6.05E-02					

Table 5 (continued): Power Comparison Study for Skewed Distributions ($n=10$)

Distribution	(skewness) (kurtosis)	k	Combined Sample	R-chart	S-chart	S2-chart	WV-chart	SC-chart
Normal (0,1)	(0.00) (0.00)	k=3	4.36E-01 3.29E-01	5.49E-01	6.17E-01	4.38E-01	5.28E-01	4.54E-01
Exponential (1)	(2.00) (6.00)		1.53E-01 9.96E-02	4.72E-01	5.17E-01	3.63E-01	1.59E-01	1.29E-01
Lognormal (0,1)	(6.18) (110)		5.36E-02 3.19E-02	3.91E-01	4.19E-01	2.65E-01	1.08E-01	1.02E-01
Weibull (0.5)	(6.62) (84.72)		4.13E-02 2.42E-02	3.78E-01	4.00E-01	2.48E-01	1.16E-01	1.14E-01
Weibull (0.85)	(0.12) (1.23)		8.54E-02 5.28E-02	4.37E-01	4.72E-01	3.20E-01	1.01E-01	1.07E-01
Weibull (0.75)	(2.56) (10.35)		1.10E-01 7.00E-02	4.53E-01	4.92E-01	3.40E-01	1.20E-01	1.14E-01
Weibull (1)	(2.00) (6.00)		1.53E-01 9.96E-02	4.72E-01	5.17E-01	3.63E-01	1.59E-01	1.29E-01
Weibull (1.2)	(1.52) (3.24)		2.12E-01 1.43E-01	4.93E-01	5.44E-01	3.85E-01	2.14E-01	1.56E-01
Weibull (1.5)	(1.07) (1.39)		2.96E-01 2.08E-01	5.17E-01	5.74E-01	4.07E-01	2.91E-01	2.09E-01
Weibull (2.0)	(0.63) (0.25)		4.03E-01 2.99E-01	5.44E-01	6.07E-01	4.29E-01	4.12E-01	3.10E-01
Weibull (3.5)	(0.03) (-0.29)		4.93E-01 3.88E-01	5.65E-01	6.30E-01	4.47E-01	5.45E-01	4.63E-01
Barnes 2	(1.75) (3.75)		2.02E-01 1.40E-01	4.94E-01	5.40E-01	3.87E-01	1.46E-01	1.39E-01
Normal (0,1)	(0.00) (0.00)	k=4	6.35E-01 5.30E-01	7.38E-01	7.97E-01	6.61E-01	7.21E-01	6.60E-01
Exponential (1)	(2.00) (6.00)		2.52E-01 1.77E-01	6.03E-01	6.50E-01	5.00E-01	2.51E-01	2.11E-01
Lognormal (0,1)	(6.18) (110)		8.59E-02 5.37E-02	4.80E-01	5.13E-01	3.43E-01	1.49E-01	1.41E-01
Weibull (0.5)	(6.62) (84.72)		6.24E-02 3.82E-02	4.44E-01	4.70E-01	3.07E-01	1.53E-01	1.51E-01
Weibull (0.85)	(0.12) (1.23)		1.40E-01 9.13E-02	5.38E-01	5.76E-01	4.21E-01	1.53E-01	1.62E-01
Weibull (0.75)	(2.56) (10.35)		1.83E-01 1.23E-01	5.67E-01	6.09E-01	4.56E-01	1.86E-01	1.78E-01
Weibull (1)	(2.00) (6.00)		2.52E-01 1.77E-01	6.03E-01	6.50E-01	5.00E-01	2.51E-01	2.11E-01
Weibull (1.2)	(1.52) (3.24)		3.47E-01 2.54E-01	6.44E-01	6.94E-01	5.46E-01	3.41E-01	2.67E-01
Weibull (1.5)	(1.07) (1.39)		4.70E-01 3.63E-01	6.90E-01	7.42E-01	5.97E-01	4.62E-01	3.64E-01
Weibull (2.0)	(0.63) (0.25)		6.04E-01 4.96E-01	7.36E-01	7.87E-01	6.47E-01	6.20E-01	5.18E-01
Weibull (3.5)	(0.03) (-0.29)		6.91E-01 5.96E-01	7.60E-01	8.12E-01	6.80E-01	7.45E-01	6.80E-01
Barnes 2	(1.75) (3.75)		3.19E-01 2.37E-01	6.28E-01	6.74E-01	5.33E-01	2.46E-01	2.39E-01

Table 5 (continued): Power Comparison Study for Skewed Distributions ($n=10$)

Distribution	(skewness) (kurtosis)	k	Combined Sample	R-chart	S-chart	S2-chart	WV-chart	SC-chart
Chi (1)	(2.83)	k=3	9.98E-02	4.49E-01	4.85E-01	3.35E-01	9.30E-02	1.11E-01
	(12.0)		6.36E-02					
Chi (2)	(2.00)		1.53E-01	4.72E-01	5.17E-01	3.63E-01	1.59E-01	1.29E-01
	(6.00)		9.96E-02					
Chi (3)	(1.63)		1.90E-01	4.84E-01	5.33E-01	3.76E-01	2.08E-01	1.49E-01
	(4.00)		1.27E-01					
Chi (4)	(1.41)		2.18E-01	4.94E-01	5.47E-01	3.85E-01	2.42E-01	1.68E-01
	(3.00)		1.47E-01					
Chi (8)	(1.00)		2.86E-01	5.14E-01	5.73E-01	4.04E-01	3.20E-01	2.26E-01
	(1.50)		1.98E-01					
Chi (10)	(0.89)		3.07E-01	5.19E-01	5.79E-01	4.08E-01	3.46E-01	2.49E-01
	(1.20)		2.15E-01					
Chi (12)	(0.82)		3.21E-01	5.22E-01	5.83E-01	4.11E-01	3.65E-01	2.66E-01
	(1.00)		2.26E-01					
Chi (16)	(0.71)		3.44E-01	5.29E-01	5.91E-01	4.17E-01	3.97E-01	2.94E-01
	(0.75)		2.46E-01					
Chi (24)	(0.58)		3.66E-01	5.33E-01	5.96E-01	4.19E-01	4.29E-01	3.26E-01
	(0.50)		2.65E-01					
Gamma (0.15)	(5.16)		4.69E-02	4.00E-01	4.21E-01	2.73E-01	1.29E-01	1.26E-01
	(40.0)		2.77E-02					
Gamma (1.2)	(1.83)		1.72E-01	4.79E-01	5.26E-01	3.71E-01	1.82E-01	1.39E-01
	(5.00)		1.14E-01					
Gamma (4.0)	(1.00)		2.88E-01	5.13E-01	5.72E-01	4.03E-01	3.19E-01	2.27E-01
	(1.50)		2.00E-01					
Chi (1)	(2.83)	k=4	1.61E-01	5.52E-01	5.90E-01	4.40E-01	1.47E-01	1.71E-01
	(12.0)		1.08E-01					
Chi (2)	(2.00)		2.52E-01	6.03E-01	6.50E-01	5.00E-01	2.51E-01	2.11E-01
	(6.00)		1.77E-01					
Chi (3)	(1.63)		3.15E-01	6.31E-01	6.81E-01	5.30E-01	3.27E-01	2.50E-01
	(4.00)		2.27E-01					
Chi (4)	(1.41)		3.60E-01	6.51E-01	7.03E-01	5.53E-01	3.80E-01	2.86E-01
	(3.00)		2.64E-01					
Chi (8)	(1.00)		4.61E-01	6.89E-01	7.44E-01	5.96E-01	4.94E-01	3.84E-01
	(1.50)		3.52E-01					
Chi (10)	(0.89)		4.89E-01	6.98E-01	7.53E-01	6.06E-01	5.29E-01	4.19E-01
	(1.20)		3.78E-01					
Chi (12)	(0.82)		5.07E-01	7.03E-01	7.59E-01	6.13E-01	5.53E-01	4.43E-01
	(1.00)		3.96E-01					
Chi (16)	(0.71)		5.35E-01	7.12E-01	7.69E-01	6.24E-01	5.90E-01	4.82E-01
	(0.75)		4.23E-01					
Chi (24)	(0.58)		5.62E-01	7.19E-01	7.76E-01	6.33E-01	6.25E-01	5.24E-01
	(0.50)		4.50E-01					
Gamma (0.15)	(5.16)		7.02E-02	4.60E-01	4.83E-01	3.32E-01	1.71E-01	1.68E-01
	(40.0)		4.34E-02					
Gamma (1.2)	(1.83)		2.83E-01	6.17E-01	6.66E-01	5.16E-01	2.87E-01	2.30E-01
	(5.00)		2.02E-01					
Gamma (4.0)	(1.00)		4.63E-01	6.88E-01	7.43E-01	5.95E-01	4.94E-01	3.84E-01
	(1.50)		3.54E-01					

Table 5 (continued): Power Comparison Study for Skewed Distributions ($n = 10$)

Distribution	(skewness) (kurtosis)	k	Combined Sample	R-chart	S-chart	S2-chart	WV-chart	SC-chart
Normal (0,1)	(0.00)	k=5	7.61E-01	8.44E-01	8.88E-01	7.95E-01	8.33E-01	7.88E-01
	(0.00)		6.71E-01					
Exponential (1)	(2.00)		3.46E-01	6.97E-01	7.41E-01	6.05E-01	3.38E-01	2.92E-01
	(6.00)		2.54E-01					
Lognormal (0,1)	(6.18)		1.20E-01	5.52E-01	5.87E-01	4.10E-01	1.89E-01	1.79E-01
	(110)		7.75E-02					
Weibull (0.5)	(6.62)		8.36E-02	4.97E-01	5.25E-01	3.56E-01	1.87E-01	1.85E-01
	(84.72)		5.29E-02					
Weibull (0.85)	(0.12)		1.94E-01	6.15E-01	6.54E-01	5.02E-01	2.04E-01	2.15E-01
	(1.23)		1.32E-01					
Weibull (0.75)	(2.56)		2.53E-01	6.51E-01	6.93E-01	5.48E-01	2.50E-01	2.41E-01
	(10.35)		1.78E-01					
Weibull (1)	(2.00)		3.46E-01	6.97E-01	7.41E-01	6.05E-01	3.38E-01	2.92E-01
	(6.00)		2.54E-01					
Weibull (1.2)	(1.52)		4.64E-01	7.47E-01	7.90E-01	6.64E-01	4.55E-01	3.73E-01
	(3.24)		3.58E-01					
Weibull (1.5)	(1.07)		6.05E-01	7.99E-01	8.40E-01	7.27E-01	5.99E-01	5.02E-01
	(1.39)		4.94E-01					
Weibull (2.0)	(0.63)		7.36E-01	8.45E-01	8.81E-01	7.83E-01	7.56E-01	6.73E-01
	(0.25)		6.39E-01					
Weibull (3.5)	(0.03)		8.06E-01	8.63E-01	8.99E-01	8.14E-01	8.53E-01	8.09E-01
	(-0.29)		7.29E-01					
Barnes 2	(1.75)		4.21E-01	7.20E-01	7.62E-01	6.40E-01	3.41E-01	3.35E-01
	(3.75)		3.26E-01					
Normal (0,1)	(0.00)	k=6	8.38E-01	9.04E-01	9.35E-01	8.73E-01	8.96E-01	8.65E-01
	(0.00)		7.64E-01					
Exponential (1)	(2.00)		4.28E-01	7.65E-01	8.04E-01	6.84E-01	4.16E-01	3.67E-01
	(6.00)		3.26E-01					
Lognormal (0,1)	(6.18)		1.54E-01	6.10E-01	6.46E-01	4.67E-01	2.26E-01	2.16E-01
	(110)		1.02E-01					
Weibull (0.5)	(6.62)		1.04E-01	5.40E-01	5.69E-01	3.99E-01	2.18E-01	2.15E-01
	(84.72)		6.77E-02					
Weibull (0.85)	(0.12)		2.47E-01	6.74E-01	7.12E-01	5.68E-01	2.52E-01	2.64E-01
	(1.23)		1.73E-01					
Weibull (0.75)	(2.56)		3.19E-01	7.15E-01	7.54E-01	6.20E-01	3.10E-01	3.01E-01
	(10.35)		2.32E-01					
Weibull (1)	(2.00)		4.28E-01	7.65E-01	8.04E-01	6.84E-01	4.16E-01	3.67E-01
	(6.00)		3.26E-01					
Weibull (1.2)	(1.52)		5.60E-01	8.16E-01	8.53E-01	7.49E-01	5.50E-01	4.67E-01
	(3.24)		4.49E-01					
Weibull (1.5)	(1.07)		7.02E-01	8.67E-01	8.97E-01	8.12E-01	7.01E-01	6.13E-01
	(1.39)		5.98E-01					
Weibull (2.0)	(0.63)		8.19E-01	9.05E-01	9.30E-01	8.64E-01	8.41E-01	7.78E-01
	(0.25)		7.38E-01					
Weibull (3.5)	(0.03)		8.73E-01	9.19E-01	9.43E-01	8.88E-01	9.12E-01	8.82E-01
	(-0.29)		8.13E-01					
Barnes 2	(1.75)		5.05E-01	7.84E-01	8.21E-01	7.17E-01	4.25E-01	4.20E-01
	(3.75)		4.05E-01					

Table 5 (continued): Power Comparison Study for Skewed Distributions ($n = 10$)

Distribution	(skewness) (kurtosis)	k	Combined Sample	R-chart	S-chart	S2-chart	WV-chart	SC-chart
Chi (1)	(2.83)	k=5	2.21E-01	6.28E-01	6.66E-01	5.23E-01	2.00E-01	2.29E-01
	(12.0)		1.54E-01					
Chi (2)	(2.00)		3.46E-01	6.97E-01	7.41E-01	6.05E-01	3.38E-01	2.92E-01
	(6.00)		2.54E-01					
Chi (3)	(1.63)		4.27E-01	7.33E-01	7.78E-01	6.47E-01	4.35E-01	3.48E-01
	(4.00)		3.24E-01					
Chi (4)	(1.41)		4.83E-01	7.58E-01	8.02E-01	6.76E-01	5.00E-01	3.97E-01
	(3.00)		3.74E-01					
Chi (8)	(1.00)		5.96E-01	8.00E-01	8.44E-01	7.28E-01	6.31E-01	5.21E-01
	(1.50)		4.84E-01					
Chi (10)	(0.89)		6.26E-01	8.09E-01	8.52E-01	7.40E-01	6.66E-01	5.62E-01
	(1.20)		5.15E-01					
Chi (12)	(0.82)		6.45E-01	8.14E-01	8.58E-01	7.48E-01	6.90E-01	5.89E-01
	(1.00)		5.35E-01					
Chi (16)	(0.71)		6.71E-01	8.22E-01	8.65E-01	7.60E-01	7.23E-01	6.29E-01
	(0.75)		5.64E-01					
Chi (24)	(0.58)		6.96E-01	8.29E-01	8.72E-01	7.69E-01	7.55E-01	6.71E-01
	(0.50)		5.93E-01					
Gamma (0.15)	(5.16)		9.29E-02	5.07E-01	5.30E-01	3.80E-01	2.09E-01	2.05E-01
	(40.0)		5.93E-02					
Gamma (1.2)	(1.83)		3.86E-01	7.15E-01	7.59E-01	6.26E-01	3.84E-01	3.19E-01
	(5.00)		2.89E-01					
Gamma (4.0)	(1.00)		5.99E-01	7.99E-01	8.43E-01	7.28E-01	6.30E-01	5.22E-01
	(1.50)		4.87E-01					
Chi (1)	(2.83)	k=6	2.77E-01	6.86E-01	7.23E-01	5.90E-01	2.51E-01	2.83E-01
	(12.0)		1.99E-01					
Chi (2)	(2.00)		4.28E-01	7.65E-01	8.04E-01	6.84E-01	4.16E-01	3.67E-01
	(6.00)		3.26E-01					
Chi (3)	(1.63)		5.22E-01	8.04E-01	8.42E-01	7.32E-01	5.27E-01	4.37E-01
	(4.00)		4.11E-01					
Chi (4)	(1.41)		5.82E-01	8.29E-01	8.65E-01	7.63E-01	5.98E-01	4.95E-01
	(3.00)		4.69E-01					
Chi (8)	(1.00)		6.96E-01	8.68E-01	9.01E-01	8.16E-01	7.30E-01	6.31E-01
	(1.50)		5.89E-01					
Chi (10)	(0.89)		7.24E-01	8.76E-01	9.08E-01	8.27E-01	7.62E-01	6.71E-01
	(1.20)		6.20E-01					
Chi (12)	(0.82)		7.41E-01	8.80E-01	9.13E-01	8.34E-01	7.83E-01	6.98E-01
	(1.00)		6.40E-01					
Chi (16)	(0.71)		7.63E-01	8.87E-01	9.19E-01	8.44E-01	8.11E-01	7.35E-01
	(0.75)		6.68E-01					
Chi (24)	(0.58)		7.85E-01	8.92E-01	9.24E-01	8.53E-01	8.38E-01	7.72E-01
	(0.50)		6.95E-01					
Gamma (0.15)	(5.16)		1.15E-01	5.44E-01	5.67E-01	4.19E-01	2.42E-01	2.37E-01
	(40.0)		7.52E-02					
Gamma (1.2)	(1.83)		4.75E-01	7.85E-01	8.23E-01	7.08E-01	4.69E-01	4.00E-01
	(5.00)		3.68E-01					
Gamma (4.0)	(1.00)		6.99E-01	8.67E-01	9.01E-01	8.15E-01	7.30E-01	6.32E-01
	(1.50)		5.92E-01					

Table 6: Power Comparison Study for Symmetric Distributions ($n = 10$)

Distribution	(skewness) (kurtosis)	k	Combined Sample	R-chart	S-chart	S2-chart	WV-chart	SC-chart
Barnes 3	(3.00)	k=1	2.04E-03	1.10E-01	9.09E-02	6.07E-02	1.06E-01	7.76E-02
	(1049)		8.02E-04					
Barnes 1	(0.00)		3.97E-03	1.45E-02	1.24E-02	1.86E-03	1.24E-02	6.40E-03
	(6.89)		1.23E-03					
JTB (2.0, 1.0)	(0.00)		4.58E-03	6.60E-02	6.30E-02	2.39E-02	5.77E-02	4.23E-02
	(3.00)		1.89E-03					
JTB (0.75, 0.5)	(0.00)		5.69E-03	3.93E-02	3.62E-02	9.57E-03	3.33E-02	2.21E-02
	(1.20)		2.22E-03					
JTB (4.0, 1.0)	(0.00)		4.33E-03	3.31E-02	2.78E-02	7.42E-03	2.95E-02	1.88E-02
	(0.78)		1.59E-03					
JTB (1.0, 0.5)	(0.00)		5.08E-03	2.71E-02	2.41E-02	5.18E-03	2.34E-02	1.40E-02
	(0.60)		1.83E-03					
JTB (1.25, 0.5)	(0.00)		4.47E-03	1.94E-02	1.68E-02	3.03E-03	1.67E-02	9.29E-03
	(0.24)		1.52E-03					
JTB (1.35, 0.5)	(0.00)		4.19E-03	1.69E-02	1.45E-02	2.44E-03	1.46E-02	7.86E-03
	(0.13)		1.38E-03					
JTB (1.5, 0.5)	(0.00)		4.00E-03	1.45E-02	1.24E-02	1.88E-03	1.25E-02	6.44E-03
	(0.00)		1.26E-03					
JTB (2.0, 0.5)	(0.00)		3.22E-03	8.52E-03	7.47E-03	8.68E-04	7.08E-03	3.27E-03
	(-0.30)		9.07E-04					
JTB (4.0, 0.5)	(0.00)		1.65E-03	1.55E-03	2.00E-03	1.02E-04	1.20E-03	3.71E-04
	(-0.75)		3.43E-04					
JTB (9.0, 0.5)	(0.00)		8.09E-04	6.28E-05	4.81E-04	9.75E-06	4.28E-05	5.75E-06
	(-1.00)		1.28E-04					
Barnes 3	(3.00)	k=2	9.49E-03	1.80E-01	1.76E-01	1.19E-01	1.76E-01	1.35E-01
	(1049)		4.20E-03					
Barnes 1	(0.00)		1.66E-01	2.54E-01	2.95E-01	1.46E-01	2.37E-01	1.78E-01
	(6.89)		1.01E-01					
JTB (2.0, 1.0)	(0.00)		7.00E-02	2.90E-01	3.20E-01	1.87E-01	2.68E-01	2.22E-01
	(3.00)		3.93E-02					
JTB (0.75, 0.5)	(0.00)		1.14E-01	2.85E-01	3.20E-01	1.78E-01	2.62E-01	2.15E-01
	(1.20)		6.86E-02					
JTB (4.0, 1.0)	(0.00)		1.18E-01	2.68E-01	3.03E-01	1.63E-01	2.52E-01	1.98E-01
	(0.78)		6.85E-02					
JTB (1.0, 0.5)	(0.00)		1.36E-01	2.73E-01	3.10E-01	1.64E-01	2.54E-01	2.00E-01
	(0.60)		8.27E-02					
JTB (1.25, 0.5)	(0.00)		1.52E-01	2.61E-01	3.00E-01	1.53E-01	2.44E-01	1.87E-01
	(0.24)		9.25E-02					
JTB (1.35, 0.5)	(0.00)		1.57E-01	2.57E-01	2.96E-01	1.49E-01	2.40E-01	1.82E-01
	(0.13)		9.54E-02					
JTB (1.5, 0.5)	(0.00)		1.66E-01	2.54E-01	2.95E-01	1.45E-01	2.38E-01	1.78E-01
	(0.00)		1.01E-01					
JTB (2.0, 0.5)	(0.00)		1.87E-01	2.40E-01	2.85E-01	1.33E-01	2.22E-01	1.62E-01
	(-0.30)		1.14E-01					
JTB (4.0, 0.5)	(0.00)		2.34E-01	2.09E-01	2.67E-01	1.09E-01	1.89E-01	1.24E-01
	(-0.75)		1.44E-01					
JTB (9.0, 0.5)	(0.00)		2.80E-01	1.72E-01	2.54E-01	9.03E-02	1.50E-01	8.22E-02
	(-1.00)		1.76E-01					

Table 6 (continued): Power Comparison Study for Symmetric Distributions ($n = 10$)

Distribution	(skewness) (kurtosis)	k	Combined Sample	R-chart	S-chart	S2-chart	WV-chart	SC-chart
Student (5)	(0.00)	k=1	3.27E-03	6.24E-02	5.61E-02	2.52E-02	5.76E-02	4.17E-02
	(6.00)		1.27E-03					
Student (6)	(0.00)		3.41E-03	5.31E-02	4.67E-02	1.90E-02	4.87E-02	3.44E-02
	(3.00)		1.31E-03					
Student (8)	(0.00)		3.79E-03	4.23E-02	3.63E-02	1.27E-02	3.84E-02	2.62E-02
	(1.50)		1.41E-03					
Student (16)	(0.00)		4.09E-03	2.68E-02	2.22E-02	5.48E-03	2.38E-02	1.46E-02
	(0.50)		1.42E-03					
Student (25)	(0.00)		4.20E-03	2.22E-02	1.83E-02	3.86E-03	1.94E-02	1.13E-02
	(0.29)		1.41E-03					
Student (32)	(0.00)		4.01E-03	1.97E-02	1.64E-02	3.21E-03	1.72E-02	9.79E-03
	(0.21)		1.35E-03					
Student (40)	(0.00)		4.08E-03	1.89E-02	1.57E-02	2.93E-03	1.65E-02	9.18E-03
	(0.17)		1.34E-03					
Student (5)	(0.00)	k=2	6.15E-02	2.69E-01	2.98E-01	1.69E-01	2.54E-01	2.00E-01
	(6.00)		3.24E-02					
Student (6)	(0.00)		7.43E-02	2.67E-01	2.97E-01	1.66E-01	2.51E-01	1.97E-01
	(3.00)		4.02E-02					
Student (8)	(0.00)		9.47E-02	2.66E-01	3.00E-01	1.64E-01	2.50E-01	1.96E-01
	(1.50)		5.30E-02					
Student (16)	(0.00)		1.30E-01	2.61E-01	2.99E-01	1.56E-01	2.45E-01	1.89E-01
	(0.50)		7.59E-02					
Student (25)	(0.00)		1.44E-01	2.60E-01	2.98E-01	1.53E-01	2.43E-01	1.86E-01
	(0.29)		8.55E-02					
Student (32)	(0.00)		1.47E-01	2.56E-01	2.95E-01	1.50E-01	2.39E-01	1.82E-01
	(0.21)		8.77E-02					
Student (40)	(0.00)		1.52E-01	2.57E-01	2.96E-01	1.50E-01	2.41E-01	1.83E-01
	(0.17)		9.15E-02					
Student (5)	(0.00)	k=3	1.88E-01	4.81E-01	5.45E-01	3.73E-01	4.62E-01	3.86E-01
	(6.00)		1.17E-01					
Student (6)	(0.00)		2.24E-01	4.91E-01	5.57E-01	3.86E-01	4.72E-01	3.97E-01
	(3.00)		1.45E-01					
Student (8)	(0.00)		2.77E-01	5.08E-01	5.77E-01	4.03E-01	4.88E-01	4.16E-01
	(1.50)		1.87E-01					
Student (16)	(0.00)		3.60E-01	5.29E-01	5.98E-01	4.23E-01	5.10E-01	4.38E-01
	(0.50)		2.58E-01					
Student (25)	(0.00)		3.90E-01	5.37E-01	6.06E-01	4.29E-01	5.18E-01	4.45E-01
	(0.29)		2.86E-01					
Student (32)	(0.00)		3.97E-01	5.37E-01	6.06E-01	4.29E-01	5.17E-01	4.44E-01
	(0.21)		2.93E-01					
Student (40)	(0.00)		4.07E-01	5.41E-01	6.10E-01	4.32E-01	5.22E-01	4.48E-01
	(0.17)		3.03E-01					

Table 6 (continued): Power Comparison Study for Symmetric Distributions ($n = 10$)

Distribution	(skewness) (kurtosis)	k	Combined Sample	R-chart	S-chart	S2-chart	WV-chart	SC-chart
Barnes 3	(3.00) (1049)	k=3	3.00E-02 1.32E-02	2.41E-01	3.62E-01	1.91E-01	2.33E-01	1.78E-01
Barnes 1	(0.00) (6.89)	k=3	4.36E-01 3.29E-01	5.49E-01	6.17E-01	4.38E-01	5.28E-01	4.54E-01
JTB (2.0, 1.0)	(0.00) (3.00)	k=3	1.91E-01 1.24E-01	4.93E-01	5.47E-01	3.88E-01	4.66E-01	4.10E-01
JTB (0.75, 0.5)	(0.00) (1.20)	k=3	2.95E-01 2.09E-01	5.23E-01	5.82E-01	4.18E-01	4.97E-01	4.40E-01
JTB (4.0, 1.0)	(0.00) (0.78)	k=3	3.28E-01 2.31E-01	5.22E-01	5.90E-01	4.17E-01	5.02E-01	4.34E-01
JTB (1.0, 0.5)	(0.00) (0.60)	k=3	3.55E-01 2.59E-01	5.34E-01	5.97E-01	4.27E-01	5.13E-01	4.47E-01
JTB (1.25, 0.5)	(0.00) (0.24)	k=3	3.99E-01 2.97E-01	5.41E-01	6.06E-01	4.31E-01	5.21E-01	4.49E-01
JTB (1.35, 0.5)	(0.00) (0.13)	k=3	4.13E-01 3.09E-01	5.42E-01	6.09E-01	4.32E-01	5.22E-01	4.50E-01
JTB (1.5, 0.5)	(0.00) (0.00)	k=3	4.36E-01 3.30E-01	5.49E-01	6.17E-01	4.37E-01	5.30E-01	4.54E-01
JTB (2.0, 0.5)	(0.00) (-0.30)	k=3	4.88E-01 3.78E-01	5.60E-01	6.29E-01	4.44E-01	5.39E-01	4.59E-01
JTB (4.0, 0.5)	(0.00) (-0.75)	k=3	5.92E-01 4.84E-01	5.90E-01	6.57E-01	4.59E-01	5.66E-01	4.72E-01
JTB (9.0, 0.5)	(0.00) (-1.00)	k=3	6.69E-01 5.70E-01	6.24E-01	6.78E-01	4.72E-01	5.96E-01	4.89E-01
Barnes 3	(3.00) (1049)	k=4	7.68E-02 3.47E-02	3.92E-01	6.06E-01	3.16E-01	3.67E-01	2.52E-01
Barnes 1	(0.00) (6.89)	k=4	6.35E-01 5.30E-01	7.38E-01	7.97E-01	6.61E-01	7.21E-01	6.60E-01
JTB (2.0, 1.0)	(0.00) (3.00)	k=4	3.20E-01 2.25E-01	6.37E-01	6.97E-01	5.51E-01	6.12E-01	5.57E-01
JTB (0.75, 0.5)	(0.00) (1.20)	k=4	4.59E-01 3.54E-01	6.83E-01	7.42E-01	6.03E-01	6.60E-01	6.09E-01
JTB (4.0, 1.0)	(0.00) (0.78)	k=4	5.15E-01 4.02E-01	6.97E-01	7.64E-01	6.21E-01	6.80E-01	6.18E-01
JTB (1.0, 0.5)	(0.00) (0.60)	k=4	5.38E-01 4.30E-01	7.06E-01	7.67E-01	6.29E-01	6.88E-01	6.31E-01
JTB (1.25, 0.5)	(0.00) (0.24)	k=4	5.92E-01 4.86E-01	7.23E-01	7.83E-01	6.45E-01	7.06E-01	6.46E-01
JTB (1.35, 0.5)	(0.00) (0.13)	k=4	6.09E-01 5.03E-01	7.28E-01	7.88E-01	6.50E-01	7.12E-01	6.51E-01
JTB (1.5, 0.5)	(0.00) (0.00)	k=4	6.35E-01 5.30E-01	7.38E-01	7.97E-01	6.61E-01	7.23E-01	6.60E-01
JTB (2.0, 0.5)	(0.00) (-0.30)	k=4	6.91E-01 5.93E-01	7.60E-01	8.15E-01	6.80E-01	7.43E-01	6.80E-01
JTB (4.0, 0.5)	(0.00) (-0.75)	k=4	7.86E-01 7.07E-01	8.07E-01	8.47E-01	7.19E-01	7.92E-01	7.27E-01
JTB (9.0, 0.5)	(0.00) (-1.00)	k=4	8.41E-01 7.80E-01	8.46E-01	8.67E-01	7.46E-01	8.31E-01	7.70E-01

Table 6 (continued): Power Comparison Study for Symmetric Distributions ($n = 10$)

Distribution	(skewness) (kurtosis)	k	Combined Sample	R-chart	S-chart	S2-chart	WV-chart	SC-chart
Student (5)	(0.00)	k=4	3.29E-01	6.40E-01	7.13E-01	5.51E-01	6.22E-01	5.44E-01
	(6.00)		2.28E-01					
Student (6)	(0.00)		3.82E-01	6.57E-01	7.29E-01	5.73E-01	6.39E-01	5.63E-01
	(3.00)		2.74E-01					
Student (8)	(0.00)		4.53E-01	6.81E-01	7.51E-01	6.02E-01	6.63E-01	5.95E-01
	(1.50)		3.40E-01					
Student (16)	(0.00)		5.54E-01	7.11E-01	7.76E-01	6.35E-01	6.94E-01	6.31E-01
	(0.50)		4.41E-01					
Student (25)	(0.00)		5.87E-01	7.22E-01	7.85E-01	6.45E-01	7.06E-01	6.43E-01
	(0.29)		4.76E-01					
Student (32)	(0.00)		5.95E-01	7.23E-01	7.86E-01	6.47E-01	7.07E-01	6.44E-01
	(0.21)		4.86E-01					
Student (40)	(0.00)		6.06E-01	7.27E-01	7.89E-01	6.51E-01	7.12E-01	6.49E-01
	(0.17)		4.98E-01					
Student (5)	(0.00)	k=5	4.54E-01	7.50E-01	8.17E-01	6.83E-01	7.34E-01	6.62E-01
	(6.00)		3.37E-01					
Student (6)	(0.00)		5.15E-01	7.68E-01	8.31E-01	7.07E-01	7.53E-01	6.85E-01
	(3.00)		3.96E-01					
Student (8)	(0.00)		5.90E-01	7.91E-01	8.50E-01	7.37E-01	7.77E-01	7.19E-01
	(1.50)		4.74E-01					
Student (16)	(0.00)		6.89E-01	8.20E-01	8.71E-01	7.70E-01	8.08E-01	7.58E-01
	(0.50)		5.83E-01					
Student (25)	(0.00)		7.18E-01	8.30E-01	8.78E-01	7.80E-01	8.18E-01	7.70E-01
	(0.29)		6.19E-01					
Student (32)	(0.00)		7.26E-01	8.32E-01	8.79E-01	7.82E-01	8.20E-01	7.73E-01
	(0.21)		6.28E-01					
Student (40)	(0.00)		7.36E-01	8.35E-01	8.81E-01	7.86E-01	8.24E-01	7.77E-01
	(0.17)		6.40E-01					
Student (5)	(0.00)	k=6	5.57E-01	8.24E-01	8.80E-01	7.75E-01	8.11E-01	7.48E-01
	(6.00)		4.35E-01					
Student (6)	(0.00)		6.19E-01	8.40E-01	8.92E-01	7.97E-01	8.28E-01	7.71E-01
	(3.00)		5.00E-01					
Student (8)	(0.00)		6.90E-01	8.60E-01	9.07E-01	8.23E-01	8.50E-01	8.04E-01
	(1.50)		5.80E-01					
Student (16)	(0.00)		7.79E-01	8.85E-01	9.23E-01	8.52E-01	8.76E-01	8.39E-01
	(0.50)		6.86E-01					
Student (25)	(0.00)		8.04E-01	8.93E-01	9.27E-01	8.61E-01	8.84E-01	8.49E-01
	(0.29)		7.18E-01					
Student (32)	(0.00)		8.11E-01	8.94E-01	9.28E-01	8.63E-01	8.86E-01	8.52E-01
	(0.21)		7.28E-01					
Student (40)	(0.00)		8.18E-01	8.97E-01	9.30E-01	8.65E-01	8.89E-01	8.55E-01
	(0.17)		7.38E-01					

Table 6 (continued): Power Comparison Study for Symmetric Distributions ($n=10$)

Distribution	(skewness) (kurtosis)	k	Combined Sample	R-chart	S-chart	S2-chart	WV-chart	SC-chart
Barnes 3	(3.00)	k=5	1.53E-01	5.89E-01	7.79E-01	4.59E-01	5.53E-01	3.69E-01
	(1049)		7.54E-02					
Barnes 1	(0.00)		7.61E-01	8.44E-01	8.88E-01	7.95E-01	8.33E-01	7.88E-01
	(6.89)		6.71E-01					
JTB (2.0, 1.0)	(0.00)		4.34E-01	7.36E-01	7.93E-01	6.69E-01	7.14E-01	6.65E-01
	(3.00)		3.24E-01					
JTB (0.75, 0.5)	(0.00)		5.85E-01	7.84E-01	8.34E-01	7.27E-01	7.65E-01	7.24E-01
	(1.20)		4.76E-01					
JTB (4.0, 1.0)	(0.00)		6.52E-01	8.06E-01	8.60E-01	7.55E-01	7.93E-01	7.43E-01
	(0.78)		5.41E-01					
JTB (1.0, 0.5)	(0.00)		6.68E-01	8.11E-01	8.60E-01	7.58E-01	7.97E-01	7.52E-01
	(0.60)		5.65E-01					
JTB (1.25, 0.5)	(0.00)		7.21E-01	8.29E-01	8.75E-01	7.79E-01	8.17E-01	7.71E-01
	(0.24)		6.25E-01					
JTB (1.35, 0.5)	(0.00)		7.37E-01	8.35E-01	8.80E-01	7.84E-01	8.23E-01	7.78E-01
	(0.13)		6.44E-01					
JTB (1.5, 0.5)	(0.00)		7.60E-01	8.45E-01	8.88E-01	7.95E-01	8.34E-01	7.88E-01
	(0.00)		6.71E-01					
JTB (2.0, 0.5)	(0.00)		8.09E-01	8.66E-01	9.03E-01	8.16E-01	8.55E-01	8.11E-01
	(-0.30)		7.30E-01					
JTB (4.0, 0.5)	(0.00)		8.80E-01	9.05E-01	9.27E-01	8.53E-01	8.96E-01	8.58E-01
	(-0.75)		8.25E-01					
JTB (9.0, 0.5)	(0.00)		9.15E-01	9.31E-01	9.40E-01	8.76E-01	9.23E-01	8.92E-01
	(-1.00)		8.76E-01					
Barnes 3	(3.00)	k=6	2.46E-01	7.49E-01	8.76E-01	5.85E-01	7.16E-01	4.87E-01
	(1049)		1.35E-01					
Barnes 1	(0.00)		8.38E-01	9.04E-01	9.35E-01	8.73E-01	8.96E-01	8.65E-01
	(6.89)		7.64E-01					
JTB (2.0, 1.0)	(0.00)		5.29E-01	8.04E-01	8.54E-01	7.53E-01	7.85E-01	7.42E-01
	(3.00)		4.12E-01					
JTB (0.75, 0.5)	(0.00)		6.79E-01	8.49E-01	8.90E-01	8.08E-01	8.34E-01	8.01E-01
	(1.20)		5.74E-01					
JTB (4.0, 1.0)	(0.00)		7.47E-01	8.72E-01	9.14E-01	8.38E-01	8.62E-01	8.24E-01
	(0.78)		6.46E-01					
JTB (1.0, 0.5)	(0.00)		7.58E-01	8.74E-01	9.12E-01	8.39E-01	8.64E-01	8.30E-01
	(0.60)		6.64E-01					
JTB (1.25, 0.5)	(0.00)		8.05E-01	8.91E-01	9.25E-01	8.58E-01	8.82E-01	8.49E-01
	(0.24)		7.23E-01					
JTB (1.35, 0.5)	(0.00)		8.19E-01	8.96E-01	9.29E-01	8.64E-01	8.88E-01	8.55E-01
	(0.13)		7.40E-01					
JTB (1.5, 0.5)	(0.00)		8.38E-01	9.04E-01	9.34E-01	8.73E-01	8.97E-01	8.65E-01
	(0.00)		7.64E-01					
JTB (2.0, 0.5)	(0.00)		8.76E-01	9.22E-01	9.46E-01	8.91E-01	9.15E-01	8.86E-01
	(-0.30)		8.16E-01					
JTB (4.0, 0.5)	(0.00)		9.28E-01	9.50E-01	9.62E-01	9.20E-01	9.45E-01	9.23E-01
	(-0.75)		8.89E-01					
JTB (9.0, 0.5)	(0.00)		9.51E-01	9.65E-01	9.70E-01	9.35E-01	9.61E-01	9.45E-01
	(-1.00)		9.24E-01					

Because the method involves higher sample cumulants κ_6 it is recommended to use a sample size of at least 10; the simulation study shows that critical point $\frac{z_\alpha + t_{n-1,\alpha}}{2}$ is preferable when sample size 10 is used.

An Example

Suppose a chemical manufacturer wants to monitor the viscosity of a particular chemical from the production line and that it is important to detect disturbances which could result in increasing the variability of the process. The random measurements of the viscosity are selected until subgroups are obtained, and corresponding sample variances $S^2_{(i)}$, $\kappa_{4,(i)}$ and Z6 are calculated and presented in Table 7.

Necessary process parameters are estimated from the preliminary run stage which contains 30 samples sized 10 each. The estimated process variance $\tilde{S}^2 = 7.398$, process skewness k_3^*= 1.74 (positively skewed distribution), and process cumulants are $\kappa_3 = 33.654$, $\kappa_4 = 232.667$ and $\kappa_6 = 9598.75$. Because the proposed method recommends using a combined sample, all quantities are obtained from one large sample, *n* = 300, by merging the 30 size 10 samples together. Equation (4) is then used to obtain the upper limit of the control chart with critical point z_α

$$\mathrm{UCL} = \mathrm{z}_\alpha + \mathrm{n}^{-\frac{1}{2}}\left(\hat{\mathrm{B}}_1 + \hat{\mathrm{B}}_2 \frac{\mathrm{z}_\alpha^2 - 1}{6}\right) = 6.049.$$

The new method is used to construct the control chart for the variability of this positively skewed distribution. Each sample point is the test statistic Z6 of the sample where

$$Z6 = \frac{S^2_{(i)} - \tilde{S}^2}{\sqrt{\frac{\kappa_{4,(i)}\tilde{S}^2}{nS^2_{(i)}} + \frac{2\tilde{S}^4}{n-1}}};$$

and $S^2_{(i)}$ and $\kappa_{4,(i)}$ are variance and the fourth cumulant of the i^{th} sample; $\tilde{S}^2$ is estimated process variance calculated from the preliminary stage process. (See Figure 1). It can be observed that the process is under statistical control during the period of time when the 40 samples were collected, that is, all points are under the Upper Control Limit of 6.049.

If the traditional one-sided R-chart, S-chart, S^2-chart as well as WV and SC charts are also constructed, then one can observe that - in all charts except the SC-chart - at least one sample point, point 18, is above the Upper Control Limit, which gives a false out-of-control signal.

Conclusion

This study proposed a new charting scheme for the variability of a process. This technique is an adaptation of Long and Sa's (2005) testing procedure and is designed to control the variability of a process without any assumption regarding the form of the underlying distribution.

The Monte Carlo simulation study of type I error rates indicates that the proposed method is robust for all distributions studied. It can achieve significant improvement over the Shewhart R-chart, the S-chart and the S^2-chart, as well as the WV R-chart and the SC R-chart when the distribution is highly skewed and/or has large kurtosis. It can maintain the type I error rates close to the nominal level $\alpha = 0.0027$ and shows reasonably good power.

In a real life situation, control charts are constructed even when there is no information about the form of the distribution of the quality characteristic. The method presented herein works well for all distributions studied, which includes the normal distribution.

If sample size is small, then the average of z_α and $t_{\alpha,n-1}$ as the critical point is recommended to produce a small number of false alarms and detect shifts reasonably well. Because the proposed method involves higher moments, a sample size of at least 10 is recommended.

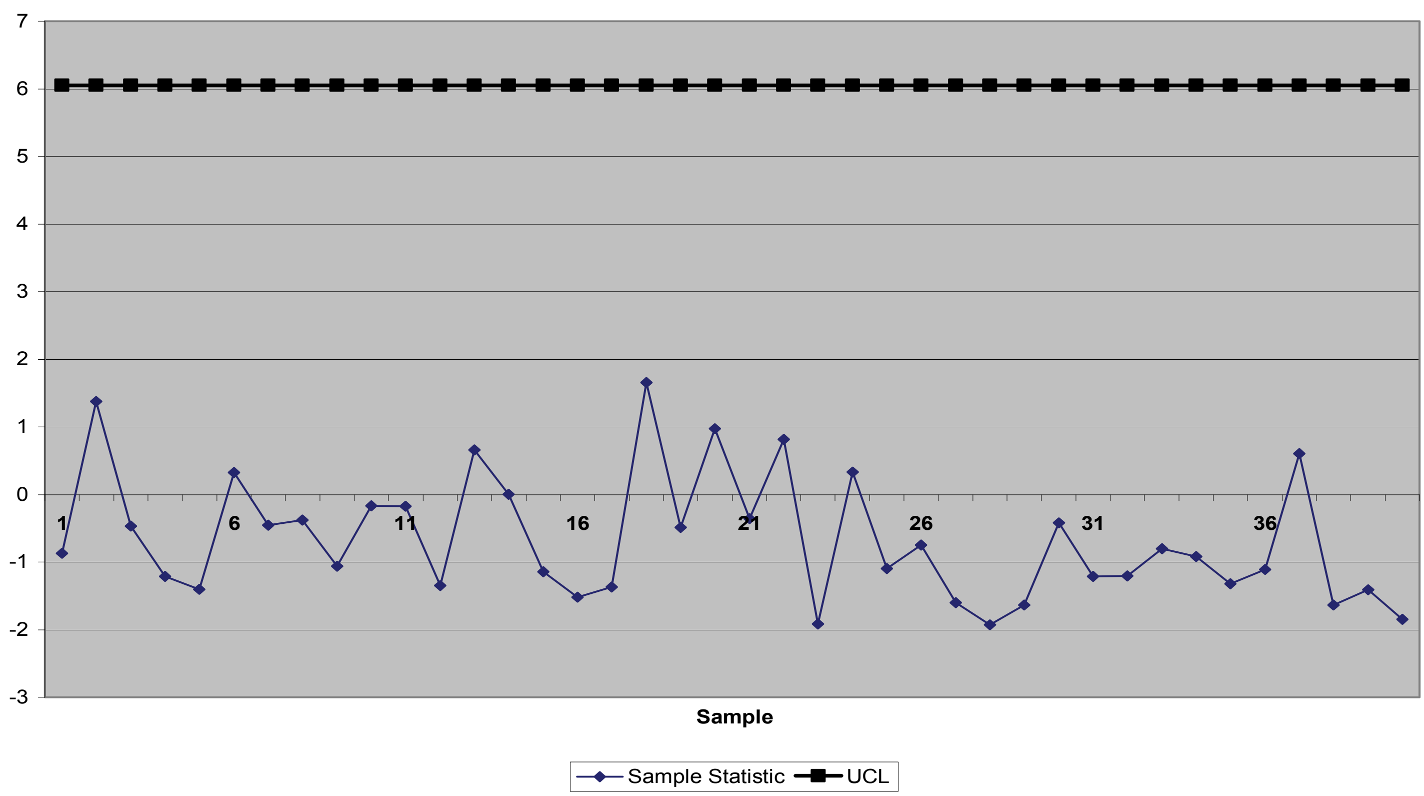
Table 7: Example Data ($n = 10$)

Sample	Data										S^2	κ_4	Z6
1	0.1753	0.1993	2.8990	2.6626	2.4803	3.9687	6.6066	1.7451	5.7174	2.3713	4.3754	0.0000	-0.8679
2	1.8987	9.5414	0.4717	3.7043	2.3586	9.9265	5.6399	0.8266	2.2084	1.0139	12.1981	0.0000	1.3730
3	6.7533	0.2629	4.2325	4.3115	3.1053	1.9042	4.7606	1.2661	8.1081	4.5336	5.7830	0.0000	-0.4647
4	1.3454	2.3209	2.8454	0.7297	2.0158	1.9803	1.0899	5.5295	0.2362	1.0541	2.2310	18.2422	-1.2115
5	1.2494	3.7607	0.7006	5.5135	1.6933	1.4810	3.3468	1.9125	2.0569	1.8984	2.0655	6.2677	-1.4055
6	4.9890	0.9675	11.7036	2.4818	4.8241	1.2914	1.6978	3.6436	4.6315	4.3729	9.5299	392.0086	0.3254
7	6.1313	6.3210	1.1931	1.6164	1.2826	6.4058	2.6711	0.4085	5.4223	4.6664	5.8097	0.0000	-0.4570
8	0.2906	1.7878	0.7905	1.1382	7.7531	0.2521	0.3012	2.1627	2.2058	0.6501	5.0435	179.6856	-0.3803
9	6.2531	6.0388	1.6128	2.8829	3.6935	0.7531	2.0489	4.8375	1.7980	2.1992	3.6892	0.0000	-1.0645
10	3.6747	2.2271	2.6493	4.3548	9.9896	2.9346	0.6846	2.2333	4.4169	2.1832	6.4037	217.7216	-0.1638
11	0.0888	1.1911	5.3541	2.9182	1.2550	8.7974	2.1795	2.0307	1.4277	2.0003	6.4750	138.9135	-0.1755
12	3.4825	2.3265	5.2447	0.7256	5.1806	1.6558	3.5688	2.8752	2.0482	0.5028	2.7078	0.0000	-1.3456
13	5.4607	0.7457	3.3457	9.8583	3.3429	2.9802	2.7684	0.1488	8.5222	5.5548	9.7128	0.0000	0.6611
14	0.6061	1.9028	4.7777	1.7799	0.6848	9.3894	1.4367	3.1679	5.5664	4.1000	7.4303	87.0290	0.0055
15	0.6069	0.2926	1.5002	0.9540	2.3739	3.5945	1.8038	1.7982	5.6449	0.2206	2.8074	15.3712	-1.1409
16	4.9356	2.4603	3.6291	4.4440	1.2951	2.3350	1.7887	1.5324	0.7752	1.0778	2.0949	0.0000	-1.5212
17	1.6166	0.6310	4.9091	3.8633	0.5929	2.9032	0.6788	0.8536	3.9993	1.5527	2.6392	0.0000	-1.3653
18	1.8073	4.3716	0.7548	4.0216	1.9322	17.5866	7.5443	1.2864	1.6462	1.7298	26.0279	4037.2395	1.6522
19	0.7649	3.4767	0.3363	3.7036	8.2392	3.9891	0.8094	4.0438	3.6515	2.9723	5.2400	51.3835	-0.4910
20	0.2054	14.2214	5.3427	2.4460	4.6481	5.7322	2.2885	4.3794	4.2059	0.8869	15.2693	1103.0417	0.9704
21	7.6503	1.8991	1.9254	0.8912	1.6838	2.3336	0.6366	6.2804	2.7257	0.2412	5.9172	38.7200	-0.3605
22	9.8933	7.5309	3.2807	1.5821	0.4120	3.3600	0.6842	0.2074	0.1167	1.4909	11.1675	135.4547	0.8177
23	0.9872	2.0582	0.3003	1.1105	1.9264	1.9017	0.1944	0.3366	1.5401	2.6845	0.7320	0.0000	-1.9116
24	2.8916	11.2486	0.7761	0.6566	1.2512	3.0792	4.1080	1.8494	0.6488	2.3294	9.9941	652.4974	0.3328
25	0.5455	2.9088	2.0852	2.5598	6.5651	0.8456	3.5727	2.9086	5.2798	2.2881	3.3818	5.6749	-1.0979
26	1.9177	2.7938	1.3091	7.1956	4.6303	1.6167	1.2484	0.5885	1.9009	1.5335	3.9685	48.8875	-0.7445
27	0.3927	2.2825	1.5281	1.3541	1.9966	0.3233	1.1659	4.0624	0.6790	0.5122	1.3077	4.0065	-1.6038
28	2.1452	2.2046	2.3803	3.4120	1.8255	3.1891	3.2583	1.5925	2.4079	0.7436	0.6824	0.0000	-1.9258
29	0.3452	1.1206	1.5241	2.0653	1.5242	1.7115	0.8049	3.7092	4.2871	1.6163	1.5126	1.5460	-1.6379
30	0.4881	7.1682	4.6524	0.4335	2.2388	0.8668	0.8453	4.9536	0.7459	0.3920	5.9185	1.4050	-0.4228

Table 7 (continued): Example Data ($n = 10$)

Sample	Data										S^2	κ_4	Z6
31	1.1352	3.7820	6.0729	2.0385	0.6038	4.9635	3.7010	2.3934	2.7716	1.0346	3.1780	0.0000	-1.2109
32	5.3424	1.4407	2.3453	1.5825	5.1191	3.9414	5.9906	2.5694	1.4657	1.6535	3.2021	0.0000	-1.2040
33	5.5480	2.2404	2.3234	3.5577	0.1689	6.1042	1.2915	3.8142	6.3927	1.6449	4.6073	0.0000	-0.8015
34	3.0710	0.6454	1.1600	2.5251	0.7999	3.8018	2.1754	0.6235	6.6516	2.0064	3.4462	29.9413	-0.9175
35	5.7266	2.5197	0.5746	4.0094	1.8757	3.1076	1.5696	2.2175	1.1754	1.2167	2.3779	7.1928	-1.3236
36	0.4678	3.0494	5.3371	2.9850	0.5926	0.8862	0.7395	4.7288	0.4933	0.7294	3.5218	0.0000	-1.1124
37	1.6734	12.9557	0.7857	0.8905	1.7642	0.7138	0.9902	2.1479	2.7973	2.4034	13.4796	1625.8027	0.6029
38	1.8289	2.9504	3.4233	0.5749	1.2637	4.2572	0.2638	1.0693	2.8841	1.8386	1.7003	0.0000	-1.6343
39	1.9636	4.6574	0.7779	1.2897	1.1516	2.1388	1.4726	4.9557	2.1783	4.4767	2.4909	0.0000	-1.4078
40	3.4110	2.8170	1.3172	3.2577	3.4979	1.7006	1.5000	1.0208	1.3976	1.4541	0.9690	0.0000	-1.8437

Figure 1 Example of the Proposed One-Sided Control Chart Method



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