A Re-Examination Of Money And Business Cycles

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A RE-EXAMINATION OF MONEY AND BUSINESS CYCLES

by

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DISSERTATION

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DEDICATION

To my wife Lindsey and my sons, Carson and Landon, for their love and support. And to my parents, Rob and Patty, who taught me the value of hard work.
ACKNOWLEDGEMENTS

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Chapter 1 Introduction

Over the last several years, dynamic stochastic general equilibrium (DSGE) models have become the predominant tool of modern macroeconomics and, in particular, monetary policy research. In general, the baseline New Keynesian framework has emerged as “the workhorse for the analysis of monetary policy, fluctuations, and welfare” (Gali [2008]: 41). This framework uses optimizing agents and firms to generate a dynamic, rational expectations model analogous to traditional IS-LM analysis in which the LM curve has been replaced with a monetary policy rule. As a result, the New Keynesian framework, even when extended to include capital accumulation, assumes that the traditional interest rate channel is the sole transmission mechanism of monetary policy.

The importance assigned to the interest rate in monetary policy transmission is consistent with neoclassical economic theory. For example, changes in the interest rate should result in intertemporal substitution of consumption as a higher real interest rate should result in lower consumption in the present period. In addition, the permanent income or life-cycle hypothesis suggests that higher real interest rates reduce the demand for assets thereby resulting in lower prices, a decline in wealth, and a corresponding decline in consumption. Finally, consistent with the neoclassical theory of investment, an increase in the real interest rate causes an increase in the user cost of capital and a corresponding reduction in investment.

Exclusive emphasis on the short term interest rate necessarily downplays the role for monetary aggregates. In fact, in most cases, money is excluded from the model altogether. This exclusion is the result of an emerging consensus in the literature that money aggregates can be altogether ignored without the loss of significant information. The justification for this consensus is based on four factors. First, the Federal Reserve and other central banks around the world use an interest rate as their monetary policy instrument. As such it is not

---

1For examples of monetary policy analysis using the baseline New Keynesian model, see Clarida et al. [1999], Clarida et al. [2000], and Rotemberg and Woodford [1997].

2For an overview of the New Keynesian framework, see Clarida et al. [1999], Ch. 3 in Woodford [2003], Walsh [2003], or Gali [2008].
clear a priori whether money aggregates provide additional information not communicated by movements in the interest rate. Second, there is a widespread belief that the demand for money is unstable (Friedman and Kuttner [1992]; Estrella and Mishkin [1997]; Woodford [1998]) and as a result money aggregates do not have a predictable influence on other economic variables. Third, empirical estimation of backward-looking IS equations do not find a statistically significant relationship between real money balances and the output gap (Rudebusch and Svensson [2002]). Finally, the dynamic New Keynesian model abstracts from money completely based on the claim that money is redundant in the model.\(^3\) In fact, McCallum [2001a] notes that the quantitative implications of this omission are quite small.

Nevertheless, there remains reason for skepticism about the strong assumptions regarding the role of money and the transmission of monetary policy. For example, the idea that the interest rate is sufficient for describing the monetary transmission process has long been questioned. Two predominant critiques, and those directly addressed in this paper, are those levied by monetarists and those who advocate the credit view.\(^4\) For example, monetarists often emphasized the nature of relative price adjustment for a multitude of assets, of which the interest rate is the price of only one such asset (Cf. Friedman and Schwartz [1963]; Brunner and Meltzer [1963]; Laidler [1982]). In fact, the transmission mechanism of monetary shocks was often the primary grounds for criticism of the traditional IS-LM model among monetarists (Brunner and Meltzer [1976], Brunner and Meltzer [1993]). In addition, advocates of the credit channel of monetary transmission argue that the interest rate alone is insufficient for describing the transmission process and emphasize the role of net worth (Gertler and Gilchrist [1993]; Bernanke and Gertler [1995]).

Also, while the empirical research cited above casts doubts on the role of money serving as either an information variable or an intermediate target for monetary policy, this evidence

---

\(^3\)For a textbook treatment, see Woodford [2003] or Gali [2008].

\(^4\)There are certainly other channels of policy transmission emphasized in the literature, most notably Tobin’s \(q\) and the exchange rate channel. These are not discussed in this paper as the baseline New Keynesian model assumes that the capital stock is fixed – as in the traditional IS-LM model – and, while the framework can be extended to the open economy, using a closed economy approach seems reasonable for the analysis of a large, open economy such as the United States.
is potentially flawed by the use of simple sum money aggregates. The use of simple sum aggregates is problematic because this aggregation procedure is only valid in the case in which all money assets in the particular aggregate are perfect substitutes. This limiting case is not supported empirically.

An alternative to the simple sum aggregates is the monetary services index, first derived by Barnett [1980] and available through the St. Louis Federal Reserve FRED database.\(^5\) The advantage of using the monetary services index is that the index is derived from microtheoretic foundations and is consistent with aggregation and index number theory.

The purpose of this dissertation is to re-examine the empirical results that justify the exclusion of money from DSGE models as well as assess alternate assumptions about the monetary transmission mechanism. This is accomplished as follows. First, the empirical evidence that supports the exclusion of money is re-examined using the monetary services indexes rather than simple sum aggregates as the measure of money. Second, the baseline New Keynesian model is extended to include asset prices, net worth, and a richer specification of the money demand function in order to compare and contrast the implications of alternative assumptions regarding the monetary transmission mechanism. The dissertation makes a significant contribution to the literature by demonstrating that the use of a more theoretically sound measure of money provides empirical support for stable money demand and the appearance of real money balances in the IS equation. In addition, the results from the extension of the New Keynesian model suggest that the interest rate is not sufficient to capture the monetary transmission mechanism.

\(^5\)There exist corresponding indexes for M1, M2, M3, and MZM. The database also includes the currency equivalent aggregates developed by Rotemberg et al. [1995]. These latter aggregates are not used in this paper.
Chapter 2 The New Keynesian Framework

As alluded to in the introduction, the basic New Keynesian model serves as the predominant framework for monetary analysis. The purpose of this chapter is to outline the New Keynesian model and the implications for the monetary transmission mechanism. Section 2.1 introduces the basic New Keynesian model. Section 2.2 discusses two predominant, alternative transmission mechanisms in the literature and section 2.3 discusses the empirical evidence used to justify the assumptions of the New Keynesian model.

2.1 Theory

The baseline New Keynesian model consists of a representative household that chooses consumption and labor to maximize utility, a sticky price firm, and a monetary authority that sets the interest rate according to a monetary policy rule. The model can be summarized by the following three equations:

\[
\begin{align*}
\bar{y}_t &= \beta E_t \bar{y}_{t+1} - (1/\sigma)(R_t - E_t \pi_{t+1}) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa \bar{y}_t \\
R_t &= \phi_\pi \pi_t + \phi_{\bar{y}} \bar{y}_t + \epsilon_t^R
\end{align*}
\]

where \(\bar{y}_t\) is the output gap, \(R_t\) is the nominal interest rate, \(\pi_t\) is inflation, and \(\epsilon_t^R\) is a monetary policy shock. Equation (1) is a dynamic IS equation, equation (2) is the New Keynesian Phillips curve, and equation (3) is the monetary policy rule. The framework therefore resemble IS-LM analysis where the LM curve has been replaced by a monetary policy rule that describes the path of the interest rate.\(^6\)

When solved forward, the IS equation implies that the demand for the output good is a

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\(^6\)The model can easily be extended to include capital accumulation, but this adjustment does not effect the monetary transmission mechanism.
function of the expectation of the future real interest rate. Alternatively, when interpreted in light of the expectations theory of the term structure, this implies that the output good is a function of the long term real interest rate. A positive monetary policy shock reflected in ϵ^R_t increases the nominal interest rate and, because prices are sticky, the real interest rate. In conjunction with the expectations theory of the term structure this implies that the long term interest rate rises as well. The size of the response of output to a monetary policy shock is then determined by the interest elasticity given in the IS equation.

This model therefore makes strong assumptions about the transmission of monetary shocks. Namely, it assumes that monetary shocks are transmitted solely through a single interest rate. Other asset prices are ignored. Given the implications of the model, it is important to consider whether this claim is consistent with empirical evidence and to investigate how well this model can explain the properties of macroeconomic variables relative to one in which the transmission mechanism is more richly specified. These topics make up the remainder of the dissertation.

2.2 Alternative Mechanisms

2.2.1 The Monetarist Transmission Channel

Notably absent from the New Keynesian model is an explicit representation of money. Whereas the traditional IS-LM model includes a money demand function, the New Keynesian framework replaces money demand with an interest rate rule. Money demand can be modeled explicitly, but movements in real balances simply reflect quantities necessary to clear the market given the nominal interest rate and the level of output. As a result, money is redundant and often excluded from the model.

The exclusion of money, or the cashless approach, is typically justified by the absence of a meaningful real balance, or wealth effect, in the IS equation. For example, Woodford [2003] shows that if real money balances are non-separable with consumption in the utility function, real balances enter the structural IS equation, shown as equation (1) above. However, for
reasonable parameterizations of the model the impact of real balances on demand is quite small. A somewhat similar analysis is conducted by Ireland [2004] who develops a model in which real money balances enter both the IS equation and the forward-looking Phillips curve (equation 2 above). Estimation of the model suggests that real balances should be absent from both equations. Similarly, McCallum [2001a] broadly concludes that the exclusion of money does not greatly alter the results of a cashless model.

The exclusion of money is at odds with the role that money plays in the monetarist transmission mechanism in which real money balances convey information about the transmission process not captured by the interest rate. Whereas both traditional Keynesian and New Keynesian IS-LM-type analysis emphasizes the effect of monetary policy on "the" interest rate as a sufficient description of the transmission process, the monetarist approach puts emphasis on the idea that monetary shocks affect a number of asset prices and the corresponding yields on that asset. For example, following an open market purchase, financial asset prices increase and yields on such assets correspondingly decline. As these prices increase, they become expensive relative to non-financial assets. Through attempts to reallocate portfolios, this provides an incentive to increase the demand for nonfinancial assets. This increase in the demand for nonfinancial assets, in turn, increases the price of existing assets relative to newly produced assets, which provides the incentive for the purchase of newly produced nonfinancial assets, such as capital. What’s more, the rising prices of nonfinancial assets increases wealth and therefore the demand for newly produced goods and services. If the money demand specification is such that real balances are a function of a number of asset prices, as in Friedman [1956], and not a single short term interest rate, the behavior of real balances will reflect the various portfolio reallocations and substitution effects induced by the open market operation.

Thus, while the exclusion of money is justified, at least in part, by the absence of a meaningful real balance effect, the monetarist transmission mechanism provides an alternative explanation for the role of money in the transmission process. Rather than describing a
direct wealth effect from a change in real money balances as emphasized by Patinkin [1965], the monetarist transmission mechanism emphasizes that changes in real balances are akin to an index that reflects the relative price adjustments and corresponding changes in explicit and implicit yields of a number of assets. Changes in real balances thus reflect substitution rather than wealth effects. This distinction is important because it implies that real balances can contain important information for explaining movements in aggregate demand without the existence of a real balance effect and without a real balance term in the IS equation. Finally, this channel can potentially explain the empirical significance of real money balances for a variety of definitions of money found in estimated IS equations by Nelson [2002], Hafer et al. [2007], and in Chapter 3.

2.2.2 The Credit Channel

Those who advocate the credit view similarly charge that the traditional interest rate channel is insufficient to explain the real effects generated by monetary disturbances. However, the literature on the credit channel emphasizes the role of informational asymmetries between borrowers and lenders. For example, the borrower often has better information about the prospects of a particular project. As a result, the presence of imperfect information drives a wedge between the cost of internal and external finance known as an external finance premium that serves to compensate the lender for monitoring and assessing the value of the project, or agency costs. What’s more, the existence of this premium implies that informational asymmetries increase the cost of borrowing and therefore real economic

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7This point should not be controversial. For example, the quintessential monetarist Milton Friedman (1976: 317) wrote: “I have never myself thought that wealth effects of changes in the quantity of money, or of prices changes which altered the real quantity of money, were of any empirical importance for short-run economic fluctuations. I have always believed that substitution effects were the important way in which changes in money exerted influence.” Friedman and other monetarists have very similar arguments elsewhere as well [Nelson, 2003].

8The credit channel can actually be divided into two sub-channels. The first is the bank lending channel, which emphasizes the role of bank balance sheets in amplifying the effects of monetary policy. The second subgroup, which is described in this section, emphasizes the role external and internal finance in economic decisions. The latter is what is explored in this paper and thus the discussion in this section neglects to discuss the bank lending channel.
decision-making.

As a result, the external finance premium is central to the monetary transmission mechanism in the credit channel literature. Specifically, the external finance premium represents an amplification mechanism following a monetary disturbance. For example, a change in monetary policy that increases the interest rate simultaneously lowers the discounted present value of assets. As a result, net worth and, correspondingly, collateral values decline. The decline in net worth serves to increase the external finance premium and propagate the monetary disturbance.9

The baseline New Keynesian model abstracts from information asymmetries and implicitly accepts the Modigliani and Miller [1958] Theorem under which the structure of the financial system is irrelevant for analysis. While this characteristic is useful in cases in which financial market frictions are small, empirical evidence suggests that net worth, cash flow, and firm-specific measures of finance are important in the decision-making of firms.10

2.3 Evidence

It is by now a well-accepted axiom that a stable money demand function is a necessary condition for money to exert a predictable influence on economic variables. What’s more, there exists an emerging consensus in the literature that money demand has been unstable since the beginning of the 1980s and that money is not useful as an information variable. Indeed, this is a primary justification for the cashless approach outlined above. Specifically, the work of Friedman and Kuttner [1992] and Estrella and Mishkin [1997] are often cited as providing comprehensive evidence of this view. These results are discussed in turn below.

Friedman and Kuttner [1992] conduct a comprehensive analysis of money demand stability and the role of money as an information variable by employing two broad approaches. First, they examine the role of money growth in influencing nominal income growth and

---

9Bernanke et al. [1996] refer to this propagation mechanism as the “financial accelerator.”
10Cf. Fazzari et al. [1988]; Cantor [1990]; Gertler and Gilchrist, 1993; Cummins et al. [1994]; Himmelberg and Peterson [1994]; Gilchrist and Himmelberg [1995]; Hubbard et al. [1995]; Bernanke et al. [1996]; Gilchrist and Zakrajsek [2007]).
inflation under the assumption that if money is useful as an information variable, it should be the primary predictor of each. The second approach is to measure the stability of money demand using a cointegrated vector autoregressive (VAR) approach. For each of their approaches, they estimate results for three samples, the first sample runs from 1960:2 - 1979:3, the second from 1960:2 - 1990:4, and the final from 1970:3 - 1990:4.

In the first stage of analysis, the authors begin by using a three variable system consisting of nominal income, a fiscal variable and a money variable to estimate a VAR. Using the results, they use Granger causality tests of the null hypothesis that all coefficients on the lagged growth rates of money are equal to zero in the nominal income equation. For the first sample period the null hypothesis is rejected for the monetary base, M1, and M2. When the sample is expanded to 1990, the null cannot be rejected for the monetary base. For the third sample, the null can only be rejected for M1. Removing the fiscal variable yields similar results. What’s more, when authors expand the data set to include the price index as well, the null hypothesis is rejected for M1 and M2 in the first two samples, but cannot be rejected for any money aggregate in the final sample. They argue that the experience of the 1980s seems to have altered previous empirical relationships between money and nominal income.

The second stage continues this analysis by examining the stability of money demand for the same three samples above with the variables expressed in levels rather than differences. For example, a typical long run money demand function is given by:

\[ m_t - p_t = \gamma_0 + \gamma_y y_t + \gamma_r r_t + e_t \]  \hspace{1cm} (4)

where \( m \) is the money supply, \( p \) is the price level, \( y \) is a scale variable of real economic activity, \( r \) is the interest rate, and the variables are expressed in logarithms. For money demand to be considered stable in the long run, any deviations in money demand must be temporary.

\[^{12}\text{It is important to consider the level specification because of the potential for lost information when the data is first-differenced. This point was recognized by Friedman and Kuttner.}\]
The problem in estimating equation (4) is that all variables follow non-stationary I(1) processes and as a result have no tendency to return to a long run level. Nevertheless, it remains possible to examine the stability of money demand. For example, if deviations from equation (4) are temporary, $e_t$ should be stationary. This will be the case if the I(1) variables are cointegrated, or share a common trend.

Friedman and Kuttner test the null hypothesis of no cointegration using the Johansen maximal eigenvalue likelihood ratio statistic using the unrestricted model above and by imposing two separate restrictions on equation (4); namely, a unitary income elasticity ($\beta = 1$) and an exclusion of the interest rate ($\gamma = 0$).\textsuperscript{13} For the unrestricted case, they reject the null of no cointegration for the monetary base, M1, and M2 in the first sample. The null hypothesis is rejected only for M2 in the second sample and cannot be rejected for any measure of money in the final sample. Similar results hold for the restricted models. As a result, Friedman and Kuttner (1992: 490) conclude that, “whatever the situation may have been before the 1980’s, it is no longer possible to discern from the data a stable long-run relationship between income and the monetary base, M1, or credit, either with or without allowance for the effect of interest rates, and the evidence of such stability in the case of M2 strictly depends on the inclusion of data from the 1960’s.”

More recently, Estrella and Mishkin [1997] have used an approach similar to that of Friedman and Kuttner to examine the role of money growth in determining inflation and nominal income growth. Specifically, they estimate a VAR model that includes nominal income growth, inflation, and either the monetary base of M2 as the preferred measure of money growth using monthly data.\textsuperscript{14} The data set covers the period 1960:3 - 1995:12. Also, the model is estimated for a subsample for the period beginning in October 1979 that coincides with the appointment of Paul Volcker as chairman of the Federal Reserve and is an important break point for the analysis of simple sum money aggregates.

\textsuperscript{13}A restriction of $\alpha = 0$ is imposed on all models as the authors do not make explicit use of a constant term in the paper.

\textsuperscript{14}Inflation is thus defined as the change in the consumer price index. Nominal income growth is composed of the Commerce Department’s index of coincident indicators and the consumer price index.
Using the estimates from the three variable VAR, the authors conduct Granger causality tests of the null hypothesis that the lags of a given variable are all equal to zero. In estimation over the entire sample using the monetary base, the null hypothesis is rejected for the influence of lagged money growth on both nominal income growth and inflation. What’s more, nominal income growth and inflation do not predict money growth. However, when the model is estimated in the subsample beginning in October 1979, the null hypothesis the coefficients on lagged money are equal to zero cannot be rejected in the nominal income or inflation equations. In fact, the null hypothesis is only rejected for the own lags of each variable in the subsample.

The results using M2 are not promising either. For the entire sample, lagged money does help to explain the growth in nominal income, but not inflation. Additionally, lagged values of inflation and nominal income growth are found to influence money growth. In the subsample, lagged values of money growth do not help to predict nominal income growth or inflation growth. Finally, lagged values of inflation help to predict the movements in money growth.

Overall, these results do not support the notion that money growth is a useful predictor of nominal income growth and inflation in the period since 1979. Nevertheless, it is possible that the poor performance of money as an information variable could be the result of countercyclical movements in money as a result of attempts to smooth fluctuations in inflation and nominal income growth. Estrella and Mishkin investigate this claim by measuring the size and significance of the sum of the coefficients on lagged nominal income growth and inflation in the money growth equation. The results show that the coefficient sum is often either not statistically significant or has the wrong sign. This is the case for both the monetary base and M2. As a result there is little reason to believe that the changes identified in the Granger causality tests are due to countercyclical movement in the money aggregates.

Whereas the literature discussed above focuses on the ability of money growth to predict nominal output growth and inflation, a second major empirical claim of the cashless approach
is that money does not provide any additional information to explain fluctuations in the output gap. Empirically, this hypothesis can be tested by estimating the IS equation outlined above. Such empirical analysis has been conducted by Rudebusch and Svensson [2002] using a backward-looking IS equation of the form:

\[ y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 (i_{t-1} - \pi_{t-1}) + \varepsilon_t \]  

(5)

where \( y \) is the output gap defined as the percentage deviation of real output from the Congressional Budget Office’s measure of potential, \( i \) is the federal funds rate, and \( \pi \) is the average rate of inflation rate as measured by the GDP deflator.

The parameter estimates are obtained using a sample of quarterly data from 1961 - 1996. They find that the output gap has a strong autoregressive component and is negatively related to the lagged real interest. All three parameters are statistically significant and they report that these estimates are stable over time.\(^\text{15}\) Notably missing from this analysis is money as the authors (ibid: 423) acknowledge that, “lags of nominal money (in levels or growth rates) were insignificant when added” to the IS equation above. These results are consistent with a complementary VAR approach used by Gerlach and Smets [1995] that suggests that money aggregates fail to provide additional information when added to an endogenous vector of output, inflation, and the interest rate.

The results outlined above cast serious doubt on the ability of money aggregates to explain economic activity. The evidence suggests that money demand is unstable and monetary aggregates are unable to explain movements in prices and nominal income. What’s more, the lack of an identified role for money in explaining deviations in the output gap imply that movements in money aggregates are not useful as an information variable.

Nevertheless, there are reasons to be skeptical of this analysis. For example, recent estimates by Nelson [2002] show that lags of the real monetary base do have a positive and statistically significant effect on the output gap when added to Rudebusch and Svensson’s IS

\(^{15}\)The specific results are listed below in direct comparison to the empirical analysis in this paper.
equation. Similarly, Hafer et al. [2007] show real M2 has a positive and statistically significant impact on the output gap independent of the real interest rate even in the subsample that begins in the 1980s. What’s more, Hoffman et al. [1995] show that the demand for real M1 is stable when a unitary income elasticity is imposed on the data. Also, Anderson and Rasche [2001] find that the demand for the real monetary base is stable using annual data from 1919 - 1999.

This dissertation similarly argues that the results outlined above should be met with skepticism. However, contrary to others, the analysis that follows suggests that the failure to identify stability in the demand for money and a role for money in business cycles is the result of the mismeasurement of the money aggregates. The idea of mismeasurement and its implications are the subject of Chapter 3.
Chapter 3 Redundancy or Mismeasurement?

The empirical literature summarized in Chapter 2 is often cited as a justification for excluding monetary aggregates from business cycle and monetary policy analysis. These aggregates, however, are potentially flawed as they are not consistent with economic, index number, or aggregation theory. This chapter re-examines the empirical evidence summarized in Chapter 2 using monetary aggregates that are theoretically superior to their simple sum counterparts. The chapter proceeds as follows. Section 3.1 discusses an alternative to the simple sum monetary aggregates known as the monetary services index, highlighting both the qualitative and quantitative superiority of the latter. Sections 3.2, 3.3, and 3.4 re-examine the results of Friedman and Kuttner [1992], Estrella and Mishkin [1997], and Rudebusch and Svensson [2002], respectively, using the monetary services index to measure money rather than the simple sum aggregates to determine whether their results are driven by mismeasurement. Finally, section 3.5 concludes.

3.1 Alternative Measures of Money

The vast majority of the empirical literature that estimates money demand functions and the effects of money on real economic activity employs simple sum money aggregates in which different monetary components are added together with equal weights. This procedure has long been considered inadequate for measuring money.\footnote{The earliest critic of simple arithmetic index numbers is likely Fisher [1922].} For example, in assessing different measures of money included in their Monetary Statistics of the United States, Friedman and Schwartz (1970: 151) noted that it would be more appropriate for the components of money aggregates to be assigned a weight based on their degree of “moneyness.”

The reason that the weights of each asset are important is because the simple sum money aggregates imply that each asset is a perfect substitute for all other assets in the index. This is problematic because it is contrary to empirical evidence and as a result simple...
sum aggregates fail to capture pure substitution effects across assets. The failure of simple sum aggregates to capture these substitution effects is important as it necessarily implies that there has been some change in the subutility function pertaining to monetary services and thus, potentially, the observed instability of money demand discussed above.

An alternative to the simple sum aggregates is the monetary services index derived by Barnett [1980] in which monetary assets are weighted by their expenditure shares. Formally, this can be expressed as follows

$$d \log M_t = \sum_{i=1}^{n} \bar{w}_{it} d \log x_{it}$$

where $\bar{w}_{it}$ is the expenditure share averaged over the two periods and $x_{it}$ is the quantity of component $i$ at time $t$. The numerator of the expenditure share, $w_{it}$, is the product of the user cost of the particular asset and dollar quantity of that asset. The denominator is the summation of these products over all assets in the index. Here the user cost of an asset is derived from Barnett [1978] as

$$u_{it} = \frac{(R_t - r_{it})}{(1 + R_t)} P_t$$

where $u_{it}$ is the nominal user cost of asset $i$ at time $t$, $R$ is the benchmark rate of return, $r_i$ is the return on asset $i$, and $P$ is the price level.

The derivation of the monetary services index (henceforth MSI) is important for two reasons. First, these aggregates are derived from explicit microfoundations and are consistent with aggregation and index number theory. Second, the MSI aggregates are capable of adapting to financial innovation both through the introduction of new money assets or a change in the interest-bearing properties of a given asset. Simple sum indexes do not satisfy

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17Cf. Barnett et al. [1992]; Serletis [2001]

18These aggregates have often been called “Divisia” aggregates in the literature because they are constructed using the Divisia method of aggregation. The term monetary services index is the name chosen by the St. Louis Federal Reserve in the official publication of the data (Anderson et al. [1997a]; Anderson et al. [1997b]). This name is meant to reflect the fact that these aggregates measure a flow of services from a class of assets rather than a stock of assets.

19Donovan [1978] argues that the user cost concept is more appropriate for determining the price of money than the traditional form of assigning a price of unity.
either criteria.

Despite the clear theoretical superiority of the weighted money aggregates, it is not clear a priori that this necessarily implies a corresponding quantitative difference with simple sum aggregates. In order to facilitate such a comparison, the differences between MSI M1, MSI M2, MSI MZM, and the simple sum counterparts are plotted in Figures 1 - 3.\textsuperscript{20} It is important to note that differences in the growth rates are most notable in the 1980s, the decade in which money supposedly became less useful as both an intermediate target and an information variable.\textsuperscript{21} In addition, the growth of simple sum M2 is greater than the MSI counterpart for most of sample.

These differences in growth rates are, in fact, quantitatively important. For example, Belongia [2005] finds that during the 1960s and 1970s, the differences between the growth rates of simple sum M1 have a unit root. Thus, following the simple sum aggregate could potentially result in vastly divergent predictions than the MSI counterpart.

What’s more, these differences seem to be most important during the time in which money is thought to have lost its predictive ability. Barnett [1997] highlights the so-called monetarist experiment of 1979 - 1982, in which the Federal Reserve targeted the money supply, as well as the remainder of the early 1980s as two such instances. Specifically, for the period encapsulating the monetarist experiment, Barnett [1997] shows that simple sum M2 and M3 grew at an average rate of 9.3% and 10%, respectively, whereas the MSI counterparts grew at 4.5% and 4.8%. These average rates came on the heels of double-digit growth rates

\textsuperscript{20} Throughout the paper, MSI M1, MSI M2, and MSI MZM are used as money aggregates. These aggregates are the monetary services index counterparts to the simple sum aggregates M1, M2, and MZM. M1 consists of currency, demand deposits, traveler’s checks, as well as other checkable deposits such as negotiable order of withdraw (NOW) accounts. M2 includes all components of M1 as well as savings deposits, money market deposit accounts, small-denomination time deposits, and retail money market mutual funds. Finally, MZM (money with zero maturity) consists of the components of M2 (less time deposits) as well as institutional money market mutual funds. In choosing these aggregates, this paper explicitly ignores a second problem with monetary aggregation, which is the composition of assets within the aggregate. In other words, assigning weights to assets within an aggregate is not sufficient for designing a valid aggregate. Nonetheless, the emphasis in this paper is in providing analysis with aggregates in which the composition is well-known.

\textsuperscript{21} This point is potentially of importance. In the examination of simple sum aggregates, some such as Carlson et al. [2000] have argued that changes in the stability of money demand driven by structural shifts should be adjusted accordingly. Others, such as Woodford [1998], have argued that such shifts are what make money demand unstable and unusable.
Figure 1: Differences in Growth Rates – M1

Figure 2: Differences in Growth Rates – M2
for both the simple sum and MSI aggregates. Thus, using the MSI aggregates, it is much easier to understand why contractionary monetary policy resulted in a severe recession rather than a mild disinflation.

Figure 3: Differences in Growth Rates – MZM

The differences in growth rates between MSI and simple sum aggregates can also explain why dire monetarist predictions of rising inflation in the subsequent period were incorrect. For example, in a September 1983 *Newsweek* article, Milton Friedman predicted the accelerated money growth would lead to stagflation. Ironically, on the same day, William Barnett argued in *Forbes* that the concerns over rapid money growth were a statistical blip of simple sum aggregation. Specifically, Barnett argued that the sudden increase in money growth from 1982 - 1983 observed in simple sum aggregates was the result of the addition of new financial assets, such as NOW accounts and money market deposit accounts. As noted by Barnett [1997], the differences in the growth rates of the monetary aggregates can be explained by the way in which these new assets are introduced. New assets are simply added to simple sum aggregates. In contrast, new assets are introduced to MSI aggregates using the appropriate weight. Thus, given that the interest rates on these new assets were relatively
high, the weight was correspondingly low thereby making for a smooth introduction to the MSI aggregate. Note that this change is highlighted by the largest spike in the growth rate differences shown in Figures 1 - 3.\textsuperscript{22} It is also important to highlight the fact that among the aggregates the difference is largest for MZM, which includes both new assets.\textsuperscript{23}

In addition to the differences highlighted by casual inference, recent empirical evidence does suggest that the way in which money is measured has important implications for one’s results. For example, Belongia [1996] re-examines five puzzling results from the monetary literature by utilizing MSI aggregates rather than their simple sum counterparts. He finds that four of the five puzzling results exist only when simple sum aggregates are used. The results are mixed for the fifth result. In addition, international evidence collected in Belongia and Binner [2000] shows that MSI aggregates outperform their simple sum counterparts for most countries.

Given these results, it is important to re-examine the empirical evidence using the MSI measures of money rather than the simple sum indexes, which are at best theoretically flawed and at worst empirically misleading. This re-examination is the subject of the three subsequent sections.

### 3.2 Money Demand in a Time Series Framework

As discussed in Chapter 2, Friedman and Kuttner [1992] examine the stability of long-run money demand within the context of cointegration. This section outlines the analysis of money demand within a time series framework for both full sample and recursive estimation. This framework is then used to test for cointegration, estimate the parameters of the money demand function, and analyze the stability of each across samples. The results are then compared with those using simple sum aggregates.

\textsuperscript{22}Note that even if one recognized that this was the reason for the spike in the growth rate in simple sum aggregates, it would still be difficult to assess how much of the change was the result of the introduction of new assets and how much was the result of monetary policy.

\textsuperscript{23}M2 also includes both assets, but also includes time deposits that are not included in MZM.
3.2.1 Long-run Stability of Money Demand

Recall the long run money demand equation outlined in equation (4) in Chapter 2:

\[ m_t - p_t = \gamma_0 + \gamma_y y_t + \gamma_r r_t \]

where \( m \) is the money supply, \( p \) is the price level, \( y \) is a scale variable of real economic activity, \( r \) is a price variable usually measured by an interest rate, and the variables are expressed in logarithms.\(^{24}\)

As previously mentioned, given that each of these variables are non-stationary, there must exist a linear combination of the variables that is stationary for money demand to be considered stable in the long run.\(^{25}\) In other words, money demand stability requires that deviations from equilibrium are temporary. Formally, this can be shown by re-writing equation (4) as

\[ m_t - p_t - \gamma_0 - \gamma_y y_t - \gamma_r r_t = e_t \]

where \( e_t \) represents the deviation of the money demand from its long run equilibrium. If money demand is stable, \( e_t \) should be stationary with mean zero. As noted above, if \( e_t \) is stationary, the variables that comprise the money demand function are said to be cointegrated.

In order to determine whether there exists a stable money demand function, it is useful to employ an error correction VAR approach. The use of this approach is important because in addition to testing for cointegration, it provides an estimate of the money demand function parameters. Formally, the \( p \)-dimensional error correction VAR model is given by:

\[ \Delta x_t = \Gamma_1 \Delta x_{t-1} + \cdots + \Gamma_k \Delta x_{t-k} + \Pi x_{t-1} + \varepsilon_t \]

\(^{24}\)Whether or not \( r_t \) is measured in logarithms depends on how the variable is defined. Traditionally, if \( r_t \) is measured by an interest rate it is not expressed in logarithmic form. Below, \( r_t \) is measured as the price dual of the monetary aggregate and is expressed in logarithmic form.

\(^{25}\)Evidence of non-stationarity is found in Appendix 1.
where \( x_t \) is a vector of non-stationary I(1) endogenous variables, and \( \Gamma_i, i = 1, \ldots, k \), and \( \Pi \) are \((p \times p)\) parameter matrices.\(^{26}\) Within this context, the cointegration hypothesis is expressed as a reduced rank restriction on \( \Pi \), which can be written as the product of two matrices:

\[
\Pi = \alpha \beta'
\]

where \( \alpha \) and \( \beta \) are \((p \times r)\), \( r \leq p \), matrices of adjustment coefficients and long run equilibrium coefficients, respectively, and \( \text{rank}(\Pi) = r \). As a result, the cointegrated VAR can be re-written:

\[
\Delta x_t = \Gamma_1 \Delta x_{t-1} + \cdots + \Gamma_k \Delta x_{t-k} + \alpha \beta' x_{t-1} + \varepsilon_t
\]

where \( \beta' x_t \) is an \((r \times 1)\) vector of long-run cointegrating relationships. It follows that the rank of \( \Pi \) is equal to the number of cointegrating vectors. The existence of a stable long run money demand function is therefore consistent with a cointegrating vector:

\[
\beta' x_t = (m_t - p_t) - \gamma_0 - \gamma_0 y_t - \gamma_r r_t = 0
\]

One can determine the rank of \( \Pi \) and therefore the number of cointegrating vectors using Johansen’s trace test statistic, which is given by:

\[
\tau(p - r) = -T \sum_{i=r+1}^{p} \ln(1 - \hat{\lambda}_i)
\]

where \( \lambda_i \) are the eigenvalues from the estimated matrix \( \Pi \) and \( T \) is the number of observations. This test statistic can be used to test the null hypothesis that the number of cointegrating vectors is less than or equal to \( r \). Once the number of cointegrating vectors are identified, one can impose the corresponding rank of \( \Pi \) on the cointegrated VAR to estimate the parameters of the money demand function (should a single cointegrating vector exist).

\(^{26}\)In accordance with equation (4) above, a constant is included in the cointegrating vector.
3.2.2 Examining Structural Stability

While the existence of cointegration is a necessary condition for money demand stability, it is not sufficient. For example, the results in Friedman and Kuttner [1992] suggest that the ability to identify cointegration is potentially dependent on the sample. Intuitively, this can be understood by considering the conditions under which cointegration will exist. As outlined above, when the disturbance $e_t$ is stationary, the variables that comprise the money demand function are cointegrated. Nonetheless, as McCallum [1993] notes, the unique properties of money in facilitating transactions might lead to the non-stationarity of $e_t$. For example, innovations in transactions technology are unlikely to be captured by any measurable variable and will be reflected in $e_t$. Since such innovations are not likely to be reversible, it is possible that there will be a permanent component in the $e_t$ process that makes it non-stationary. It follows that the existence of money demand stability, examined within the context of cointegration, might differ over time. As a result, the present analysis seeks to consider the stability of the number of cointegrating relationships as well as the coefficient estimates across samples using recursive estimation.

In order to evaluate stability, the model is estimated for an initial sample period, $1, \ldots, T1$, and then recursively extending the endpoint of the subsample until the complete sample, $1, \ldots, T$, is estimated. Recursive tests outlined in Hansen and Johansen [1999] and Juselius [2006] can then be used to examine the constancy of the trace test statistics, the eigenvalues of $\Pi$, and the parameters of the cointegrating vector.

The first test of stability is to consider the hypothesis of cointegration across samples. As shown in Juselius [2006], the stability of cointegrating relationships can be examined using

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\[27\] McCallum [1993] argues that the failure to identify cointegration does not necessarily imply a rejection of money demand stability. Others, such as Hoffman et al. [1995] and Carlson et al. [2000], have argued this point as well and have therefore used dummy variables in the cointegrating vector to control for purported structural breaks associated with financial innovation or deregulation. However, as noted by Christ [1993], a good economic model should fit the data well and be useful for predicting future relationships. While incorporating dummies might improve the fit of the model, it is unlikely to be used successfully for prediction until structural shifts are identified. The monetary services indexes have the potential to solve this problem. More will be said on this final point below.
recursively calculated trace statistics:

\[
\tau(j) = \left\{ -t_1 \sum_{i=1}^{j} \ln(1 - \hat{\lambda}_i) \right\} \quad j = 1, \ldots, p; t_1 = T_1, \ldots, T
\]

where \(\lambda_i, i = 1, \ldots, p,\) are the eigenvalues of \(\Pi.\) This is identical to trace statistic outlined above except that it is estimated for each subsample in the recursive estimation. As a result, one can determine whether the existence of cointegration is dependent on the endpoint of the sample.

In addition to the existence of a single cointegrating vector across samples, a stable money demand function should also be associated with the constancy of parameters within the existing cointegrating vector across samples. Hansen and Johansen [1999] provide methods by which to examine parameter constancy. Two such methods are employed presently.

The first method of evaluating parameter stability is the eigenvalue fluctuations test. As shown in Juselius [2006], the eigenvalues of \(\Pi\) can be expressed as a quadratic function of \(\alpha_i\) and \(\beta_i, i = 1, \ldots, r.\) It follows that fluctuations in the parameters in the \(i^{th}\) column of \(\alpha\) or \(\beta\) will be reflected in the eigenvalue \(\lambda_i.\) The eigenvalue fluctuations test examines whether the eigenvalues are constant across samples. What’s more, given that the trace statistic is a function of the sample size and the eigenvalues, the fluctuations test enables one to determine whether a failure to identify a cointegrating vector is the result of a small sample or non-constant eigenvalues, which might reflect instability.

Formally, the test statistic for the fluctuations test is expressed as:

\[
\tau_i(t_1) = \frac{t_1}{T} \sqrt{T} \Sigma_{ii}^{-1/2} (\hat{\lambda}_{i,t_1} - \hat{\lambda}_{i,T}) \quad t_1 = T_1, \ldots, T
\]

where \(\Sigma_{ii}\) is the variance of \(\lambda_i\) defined by Hansen and Johansen [1999], \(\lambda_{i,t_1}\) is the eigenvalue \(i, i = 1, \ldots, p,\) of the subsample ending at \(t_1,\) and \(\lambda_{i,T}\) is the eigenvalue \(i\) of the complete sample. For the model estimated here, \(\text{rank}(\Pi) = 1\) and thus there is one eigenvalue to examine for each monetary aggregate.
The second test to examine parameter stability is the max test of a constant $\beta$ defined in Hansen and Johansen [1999], which tests the null hypothesis that $\hat{\beta}_t = \hat{\beta}_T$, where $t = T_1, \ldots, T$ is the endpoint of the sample. Formally, this test is based on the Nyblom [1989] statistic for examining parameter stability. The Nyblom statistic is defined as follows. Let $\theta$ be a vector of parameters. The score statistic, $S^{(T)}$, and the information statistic, $J^{(T)}$, are defined, respectively:

$$S^{(T)}(\theta) = \frac{\partial L^{(T)}(\theta)}{\partial \theta}$$

$$J^{(T)}(\theta) = -\frac{\partial^2 L^{(T)}(\theta)}{\partial \theta^2}$$

where $L$ is the likelihood function. The Nyblom test statistic can then be written

$$Q_T(t) \approx \text{tr}\{ (\hat{\theta}^{(t)} - \hat{\theta}^{(t)\prime})' J^{(t)}(\hat{\theta}^{(t)}) J^{(T)}(\hat{\theta}^{(T)})^{-1} J^{(t)}(\hat{\theta}^{(t)}) (\hat{\theta}^{(T)} - \hat{\theta}^{(t)}) \}$$

This statistic is adapted to the cointegrated VAR model as follows. First, the likelihood function can be written:

$$L^{(t)}(\theta_1, \theta_2, \theta_3) = \prod_{s=1}^{t} f(X_s | X_{s-1}, \ldots, X_{s-k+1}, \theta_1, \theta_2, \theta_3)$$

where $\theta_1 = \beta$, $\theta_2 = (\alpha, \Omega)$, and $\theta_3 = (\Gamma_1, \ldots, \Gamma_k)$.

Second, the coefficient estimates, $\hat{\beta}^{(t)}$, are normalized for all $t$. Defining

$$c = \begin{pmatrix} \hat{\beta}^{(T)} \\ 0 \end{pmatrix},$$

$$c_\perp = \begin{pmatrix} \hat{\beta}^{(T)}_\perp \\ 0 \end{pmatrix},$$

and

$$\bar{c} = c(c'c)^{-1}$$
the normalized coefficients can be written:

\[ \hat{\beta}_c^{(t)} = \hat{\beta}^{(t)} \{ \hat{\epsilon}' \hat{\beta}^{(t)} \}^{-1} \]

\[ \hat{\alpha}_c^{(t)} = \hat{\alpha}^{(t)} \hat{\beta}^{(t)'} \hat{c} \]

The test statistic for examining the constancy of the parameters can therefore be written

\[ Q_T^{(t)} = \left( \frac{t}{T} \right)^2 tr[V^{(T)}q^{(t)'}M^{(t)} \{ M^{(T)} \}^{-1} M^{(t)} q^{(t)}] \]

where

\[ q^{(t)} = T \hat{c}' \{ \hat{\beta}^{(t)} - \hat{\beta}^{(T)} \} \]

\[ V^{(T)} = \hat{\alpha}_c^{(T)'} \{ \hat{\Omega}^{(T)} \}^{-1} \hat{\alpha}_c^{(T)} \]

\[ M^{(t)} = T^{-1} c'_t S_11^{(t)} c_{\perp} \]

\[ M^{(T)} = T^{-1} c'_T S_11^{(T)} c_{\perp} \]

\[ S_11^{(t)} = \frac{1}{t} \sum_{s=1}^{t} R_{1t}^{(t)} R_{1t}' \]

\[ S_11^{(T)} = \frac{1}{t} \sum_{s=1}^{t} R_{1t}^{(T)} R_{1t}' \]

where \( R_{1t} \) is defined in the R-form of the model below and \( \hat{\Omega} = Var(\varepsilon_t) \). The distribution of the test statistic is determined by simulation.

### 3.2.3 Estimation Results

Before estimating the model it is important to determine how the variables are measured. Real money balances are measured by MSI M1, MSI M2, and MSI MZM adjusted by the GDP deflator. Typically, the scale variable is measured by some measure of real economic activity such as real Gross Domestic Product. However, given that money demand is derived
from consumer choice theory, real GDP is not likely to be the proper measure of income even for a representative agent. As a result, the scale variable used in this paper is the real final sales of domestic production.\textsuperscript{28}

The own price of money is generally proxied by the use of the opportunity cost of holding money, which is often measured by a short term interest rate.\textsuperscript{29} Ultimately, however, the use of the interest rate as the price of money is incorrect. As Belongia (2006: 240) notes, this “confuses the concepts of 'credit' and 'money'.” The appropriate measure of the own price of money when using a monetary services index is straightforward as it is given by the price dual of the monetary services index.\textsuperscript{30,31}

With the variables now properly defined, the cointegrated VAR model is estimated using a sample of quarterly data that spans 1960 - 2005 for each measure of money using lag lengths determined by Hanann-Quinn information criteria [Hannan and Quinn, 1979].\textsuperscript{32}

**Full Sample Results** The existence of cointegration is tested using the Johansen trace statistic. Estimates are shown in Table 1. For each definition of money, the null hypothesis of no cointegration is rejected. What’s more, the null hypothesis that \( r \leq 1 \) cannot be rejected for any measure of money. This is important because it provides evidence of the existence of a single cointegrating vector, which is consistent with the idea of a stable long run money demand function.

With the rank of \( \Pi \) identified, the model is now re-estimated by imposing the restriction, \( \text{rank}(\Pi) = 1 \). With the coefficient on real money balances normalized to unity, the corre-

\begin{itemize}
  \item \textsuperscript{28}The results are not sensitive to this specification.
  \item \textsuperscript{29}Poole [1988] suggests that a long term interest rate should be used. Hoffman et al. [1995] note that it is of little consequence in a cointegrating VAR model as such interest rates are typically cointegrated and thus adding an additional interest rate would simply result in an additional cointegrating vector.
  \item \textsuperscript{30}A price index number is the dual of the quantity index when the product of the quantity index and the price dual equal the total expenditure on the assets in the quantity aggregate. This can be thought of as the price of one unit of monetary services. For more, see Anderson et al. [1997b].
  \item \textsuperscript{31}It remains possible that an interest rate can serve as an opportunity cost variable that shifts the demand curve. However, using the yield on the 90-day Treasury bill, the hypothesis that the interest rate could be excluded from the money demand function as tested by a restriction on the cointegrating vector could not be rejected. As such, the interest rate is excluded from the results below.
  \item \textsuperscript{32}The end date of the sample is 2005 because that is the terminal date for which data on the monetary services indexes are available.
\end{itemize}
Table 1: Trace Statistics

<table>
<thead>
<tr>
<th>Monetary Variable</th>
<th>r</th>
<th>Trace</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSI M1</td>
<td>0</td>
<td>51.67</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>11.95</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.43</td>
<td>0.52</td>
</tr>
<tr>
<td>MSI M2</td>
<td>0</td>
<td>79.29</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>9.93</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.10</td>
<td>0.57</td>
</tr>
<tr>
<td>MSI MZM</td>
<td>0</td>
<td>86.39</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>12.61</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.37</td>
<td>0.71</td>
</tr>
</tbody>
</table>

Table 2: Cointegrated VAR Parameter Estimates

<table>
<thead>
<tr>
<th>Monetary Variable</th>
<th>$\gamma_y$</th>
<th>$\gamma_r$</th>
<th>$\gamma_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSI M1</td>
<td>0.39</td>
<td>-0.72</td>
<td>1.18</td>
</tr>
<tr>
<td>MSI M2</td>
<td>0.77</td>
<td>-0.61</td>
<td>1.45</td>
</tr>
<tr>
<td>MSI MZM</td>
<td>1.07</td>
<td>-1.14</td>
<td>1.88</td>
</tr>
</tbody>
</table>

Corresponding coefficient estimates of the cointegrating vector are shown in Table 2. In accordance with economic theory, one would expect that $\gamma_y > 0$ and $\gamma_r < 0$. As shown in Table 2, these conditions are satisfied for each measure of money. Overall, these results provide evidence of a stable long run money demand function for the estimation of the full sample.

**Recursive Estimation Results** Each of the test statistics from recursive estimation is calculated using the X-form and the R-form of the cointegrated VAR. The X-form of the cointegrated VAR is given by equation (6) above. The R-form of the model concentrates out the short-run dynamics, $\Gamma_i, i = 1, \ldots, k$, of equation (6). The derivation of the R-form is as follows. Define $Z_{0,t} = \Delta x_t, Z_{1,t} = x_{t-1},$ and $Z_{2,t} = [\Delta x_{t-1}', \ldots, \Delta x_{t-k}'],$ and $\Psi = [\Gamma_1, \ldots, \Gamma_k].$ The VAR can now be re-written as

$$Z_{0,t} = \alpha \beta' Z_{1,t} + \Psi Z_{2,t} + \varepsilon_t$$
Following the Frisch-Waugh theorem one can concentrate out the short-run dynamics of the model, $\Psi Z_{2,t}$, to obtain an estimate of $\alpha\beta'$ in three steps. First, regress $Z_{0,t}$ on $Z_{2,t}$ and obtain the residuals

$$R_{0,t} = Z_{0,t} - \hat{B}_1'Z_{2,t}$$

where $\hat{B}_1'$ are the OLS estimates.

Second, regress $Z_{1,t}$ on $Z_{2,t}$ and obtain the residuals

$$R_{1,t} = Z_{1,t} - \hat{B}_2'Z_{2,t}$$

where $\hat{B}_2'$ are the OLS estimates.

Finally, regress the residuals from the first regression, $R_{0,t}$, on the residuals from the second regression, $R_{1,t}$ to obtain the estimate of $\alpha\beta'$:

$$R_{0,t} = \alpha\beta'R_{1,t} + \epsilon_t$$

Equation (7) is known as the R-form of the model.\(^{33}\)

The purpose of estimating the test statistics from both the X-form and the R-form is that it enables one to determine whether the results are driven by the short-run dynamics of the model. For example, if a hypothesis can be rejected only in the R-form of the model, one can reasonably assert that the failure to reject the hypothesis in the X-form of the model is the result of short-run dynamics. This distinction is important because, in the case of money demand, the long run is of primary importance. Finally, for both the X- and R-form of the model, the test statistics have been divided by the 95% quantile of the corresponding distribution and plotted graphically. A rejection of the null hypothesis is therefore shown as a value greater than unity on the appropriate graph.

Figures 4, 5, and 6 plot the recursive trace statistics for MSI M1, MSI M2, and MSI

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\(^{33}\)The derivation of the R-form is based on Juselius [2006] and Hansen and Johansen [1999]. As with the X-form of the model, equation (7) is estimated using maximum likelihood estimation.
MZM, respectively. As shown in Figure 4, the null hypothesis of no cointegration can be rejected in all samples for the X-form of the model. For the R-form of the model, the null hypothesis of no cointegration cannot be rejected until the late 1980s. This latter result would seem to provide evidence against a stable money demand function. However, it is important to consider sample size in the context of the test. As shown above, the recursive trace test statistic is a function of the sample size and the eigenvalues of \( \Pi \). As a result, this test is not conclusive.\(^{34}\)

Figures 5 and 6 show that the null hypothesis of no cointegration is rejected for both the X-form and the R-form of the models for MSI M2 and MSI MZM. Curiously, the null hypothesis of \( r \leq 1 \) is also rejected in the X-form of the model for both of these measures of money in the earliest samples. This suggests that two cointegrating vectors exist. However, this is an instance where the distinction between the X-form and R-form of the model is important. The fact that the R-form of the model for both MSI M2 and MSI MZM suggests that there is one cointegrating relation for all samples implies that the results from the X-form of the model are driven by the model’s short-run dynamics. In other words, Figures 5 and 6 provide strong evidence for the existence of one cointegrating vector across all samples. This is contrary to the results of Friedman and Kuttner [1992].

\(^{34}\)As noted in Johansen [2002], the small sample properties of the trace test are different from the asymptotic properties. As a result, for the smaller samples in recursive estimation, it is possible that the failure to reject the null hypothesis of no cointegration is due to sample size. More will be said on this point below.
Figure 4: Recursive Trace Statistics – MSI M1
Figure 6: Recursive Trace Statistics – MSI MZM

The test statistics are scaled by the 5% critical values.
Having tested for the existence of cointegration across samples, it is now important to consider the constancy of the parameters within the cointegrating vector. Given that the eigenvalues of $\Pi$ can be shown to be a quadratic function of $\beta$, the existence of constant parameters in the cointegrating vector across samples implies that the eigenvalues should also be constant. What’s more, the recursively calculated trace statistics in Figure 4 provide mixed evidence for the existence of a cointegrating vector. Since the trace statistic is a function of the sample size and eigenvalues, the eigenvalue fluctuations test allows one to determine whether the failure to identify cointegration in the earliest samples is the result of non-constant eigenvalues or the small size of the sample.

The recursive eigenvalue fluctuation test statistics are plotted in Figures 7 - 9. As shown, one cannot reject the null hypothesis that the estimated eigenvalue in each subsample is equal to that of the entire sample for any measure of money in both the X- and R-form of the model. These results are important because they provide evidence, for all measures of money, that the parameters of the cointegrating vector are constant across samples. In addition, the fact that one cannot reject the null hypothesis of constant eigenvalues for MSI M1 can be taken as evidence that the failure to identify cointegration in the R-form of the model for the earliest samples is the result of the small size of the sample.

![Figure 7: Fluctuations Test – MSI M1](image)

Finally, Figures 10 - 12 plot the test statistic associated with the max test of a constant
Figure 8: Fluctuations Test – MSI M2

Figure 9: Fluctuations Test – MSI MZM
Again, this test is used to determine whether the parameters in $\beta$ for each subsample are equal those from the entire sample. For both MSI M1 and MSI MZM, one cannot reject the null hypothesis of constant parameters in the cointegrating vector. For MSI M2, however, the hypothesis can be rejected for one subsample. Nonetheless, the null hypothesis cannot be rejected for any other subsample. Broadly, these results suggest that the parameters of the cointegrating vector are constant for each measure of money.

\[\text{\textsuperscript{35}}\] It is interesting to note that this result is sensitive to lag length. Using Akaike information criteria rather than Hannan-Quinn, the choice of lag length is one period longer. Using that lag specification, one cannot reject the null hypothesis for any subsample. The change in lag length does not influence any of the other results.
Figure 10: Max Test of a Constant Beta – MSI M1
Figure 11: Max Test of a Constant Beta – MSI M2
Figure 12: Max Test of a Constant Beta – MSI MZM
A Comparison with Simple Sum Aggregates  As a method of comparison, the cointegrated VAR model is now estimated using the simple sum counterparts to MSI M1, MSI M2, and MSI MZM. In addition, following convention in traditional money demand estimation, \( r_t \) is defined as the yield on the 90-day Treasury bill. This alternative specification can be used to examine the hypothesis of mismeasurement.

Consistent with the analysis above, recursively estimated trace statistics are used to test for the existence of cointegration across samples. Economic theory implies that there should be evidence of a single cointegrating vector. The recursively estimated trace statistics are divided by the 95% quantile and plotted in Figures 13 - 15 for simple sum M1, M2, and MZM, respectively.
Figure 14: Recursive Trace Statistics – Simple Sum M2
Figure 15: Recursive Trace Statistics – Simple Sum MZM
The recursively calculated trace statistics plotted in Figure 13 show that the null hypothesis of no cointegration can be rejected for all samples in the X-form of the model using simple sum M1. These results are mostly consistent with the R-form of the model as well. As shown in Figure 14, however, there is evidence of multiple cointegrating relations when money is measured by simple sum M2 in the X-form of the model for most samples. Isolating the short-run dynamics of the model removes some evidence of multiple cointegrating relations. However, for samples ending in the 1970s, there remains evidence of multiple cointegrating relations. In addition, for samples ending in the late 1990s and early 2000s, one can marginally reject the null hypothesis that $\text{rank}(\Pi) \leq 1$ at the 5% level, which provides evidence of multiple cointegrating relations. Finally, in Figure 15 there is evidence of multiple cointegrating relations in the X-form of the model using simple sum MZM for samples ending in the 1970s. As shown in the lower panel of Figure 15, isolating the short-run dynamics of the model does not remove the evidence of multiple cointegrating relations as it does for MSI MZM. In fact, in the R-form of the model, one cannot reject the null hypothesis of no cointegration for most samples that end in the 1980s.

Overall, these results do not provide consistent evidence of a stable money demand function for simple sum M2 and simple sum MZM. In both the X-form and the R-form of the model, there is evidence of multiple cointegrating relations in the samples that end in the 1970s, which is not consistent with economic theory. While it is true that there was similar evidence of multiple cointegrating vectors in the X-form of the model for MSI M2 and MSI MZM, isolating the short-run dynamics was sufficient to identify one cointegrating relation across samples. The same cannot be said for the simple sum counterparts.

In addition, there is evidence of multiple cointegrating relation for samples ending in the late 1990s in both the X- and R-form of the model using simple sum M2. Finally, for many samples ending in the 1980s, one cannot reject the null of no cointegration in the R-form of the model using simple sum MZM as the monetary aggregate. These results are particularly important for the R-form of the model, which should provide the best evidence regarding the
long-run structure of the model since it isolates the short-run dynamics. Most importantly
for the hypothesis of mismeasurement, the results using simple sum aggregates are in stark
contrast to those in the R-form of the model using MSI M2 and MSI MZM, in which there
is consistent evidence of one cointegrating relation across all samples.

Summary  Overall, the results from the cointegrated VAR models using the monetary
services indexes are important because they contrast significantly with those of Friedman
and Kuttner [1992]. Whereas their work suggests that money demand is not stable since
the 1980s, the evidence presented above suggests that the stability of money demand is an
empirical reality when a monetary services index is used as the monetary aggregate. For
each measure of the monetary services index, there does exist a single cointegrating relation
with reasonable and stable parameter values for a money demand function across samples.
What’s more, the only instances in which the null of no cointegration cannot be rejected
are those in which the sample size is too small as evident in the eigenvalue fluctuations test
above. In contrast, when using either simple sum M2 or simple sum MZM as the monetary
aggregate, the number of cointegrating relations is dependent on the end date of sample.
This latter conclusion is not fundamentally altered by isolating the short-run dynamics of
the model. Taken together, these results therefore provide credence to the hypothesis that
empirical failures relating to money demand are an issue of mismeasurement.

3.3 Money, Income, and Prices

A central tenet of any quantity theoretic framework is that changes in the money supply
should help predict subsequent changes in nominal income and prices. As described in
Chapter 2, Estrella and Mishkin [1997] fail to find evidence of such a relationship using
Granger causality tests on a three variable system consisting of nominal income growth,
inflation, and money growth. This section re-examines their results using two approaches.
The first approach is to estimate a standard VAR model of the three variable system of
Table 3: Granger Causality Tests – Pre-1979

<table>
<thead>
<tr>
<th>Variable</th>
<th>Nom. GDP</th>
<th>Inflation</th>
<th>MSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSI M1</td>
<td>0.81</td>
<td>0.82</td>
<td>0.23</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.39</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>MSI M1</td>
<td>0.09</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>MSI M2</td>
<td>0.64</td>
<td>0.73</td>
<td>0.09</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.34</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>MSI M2</td>
<td>0.17</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>MSI MZM</td>
<td>0.60</td>
<td>0.78</td>
<td>0.07</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.36</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>MSI MZM</td>
<td>0.09</td>
<td>0.02</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Estrella and Mishkin [1997] and use Granger causality tests to determine whether money growth, as measured by the MSI data, can predict nominal income growth and inflation. The role of countercyclical monetary policy is also considered. Second, given the fact that the levels of nominal income, the price level, and the monetary services indexes each have a unit root and are cointegrated, a cointegrated VAR model is estimated and Granger causality tests conducted for the three variable system in levels. The results of each approach are then contrasted with those using simple sum aggregates.

3.3.1 Causality in a Standard VAR

This section adopts the three variable system of Estrella and Mishkin, estimates the corresponding VARs, and conducts Granger causality tests of the null hypothesis that lags of a given variable do not effect the particular variable in question. Nominal income growth is measured by nominal GDP growth, inflation by the change in the GDP deflator, and money growth by the monetary services indexes. To facilitate comparison with Estrella and Mishkin, the model is estimated over two samples, with the break point occurring in October 1979.  

Table 3 presents the p-values of the Granger causality tests for the three variable system for the pre-1979 sample. The results are printed such that the null hypothesis is that the

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36The lag length for the VARs are determined by lag length specification tests.
Table 4: Granger Causality Tests – Post-1979

<table>
<thead>
<tr>
<th>Variable</th>
<th>Nom. GDP</th>
<th>Inflation</th>
<th>MSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSI M1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nom. GDP</td>
<td>0.00</td>
<td>0.94</td>
<td>0.31</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.15</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>MSI M1</td>
<td>0.61</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>MSI M2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nom. GDP</td>
<td>0.00</td>
<td>0.45</td>
<td>0.31</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.48</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>MSI M2</td>
<td>0.06</td>
<td>0.25</td>
<td>0.00</td>
</tr>
<tr>
<td>MSI MZM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nom. GDP</td>
<td>0.00</td>
<td>0.25</td>
<td>0.19</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.65</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>MSI MZM</td>
<td>0.00</td>
<td>0.31</td>
<td>0.00</td>
</tr>
</tbody>
</table>

column variable does not predict the row variable. These results suggest that money, as measured by MSI M2 and MSI MZM, does help to predict nominal GDP growth. What’s more, the lags of all measures of money help to predict the inflation rate.

The p-values for the Granger causality tests for the post-1979 era are shown in Table 4. These results are different both from the pre-1979 results and those shown in Estrella and Mishkin for the same period. In contrast to the earlier sample and consistent with the work highlighted above, each measure of the monetary aggregate cannot predict fluctuations in nominal GDP growth. Contrary of Estrella and Mishkin, however, is that all of the money aggregates are useful for predicting inflation.

3.3.2 The Role of Countercyclical Monetary Policy

While the ability of the growth rates of the monetary services indexes to predict inflation is a notable improvement over the results shown in Estrella and Mishkin [1997], it remains somewhat puzzling that the same aggregates cannot predict nominal income growth in this latter period. One potential reason for this failure could be the result of the fact that movements in the money supply are reflecting an increased responsiveness of monetary policy to fluctuations in nominal income growth post-1979.\footnote{Hendrickson [2010] shows this to be the case, but uses the federal funds rate as the measure of monetary policy rather than a measure of the money supply.} For example, if the central bank is responding to both changes in inflation and the output gap, as is a generally accepted
proposition for the United States, this will likely be reflected in the bank’s responsiveness to nominal income growth. Upon casual inspection, this is potentially the case as nominal income growth does help predict the growth rates of two of the three money aggregates in the VAR model as shown in Table 4.

The idea that central bank policy might explain the failures of monetary aggregates to predict nominal income can be directly tested in two different ways. First, one can test this hypothesis by estimating a central bank reaction function in which the bank’s intermediate target is a money aggregate and its target is nominal income growth. Formally, this can be tested by estimating the following regression:

$$\Delta M_t = \alpha + \beta \Delta x_t + e_t$$

where $\Delta M_t$ is the growth rate of the money supply, $\Delta x_t$ is nominal income growth, and $\alpha$ is a constant. For this regression, nominal income growth is measured by the Federal Reserve’s Greenbook forecast of nominal income growth. The use of the forecast of nominal income growth is important because it eliminates the possibility of capturing reverse causation while also using data that was available to policymakers in real time.

The results of this regression are shown in Table 5. The results indicate that the Federal Reserve forecast of nominal income growth did have a negative and statistically significant effect on the growth rate of MSI M2 and MSI MZM in the post-1979 era. This therefore lends credence to the claim that the failure of monetary aggregates to predict nominal income is the result of the fact that changes in nominal income have feedback effects on money aggregates through monetary policy.

\footnote{Formally, this model could be re-written as:

$$\Delta M_t = \delta_0 + \delta_1(\Delta \bar{x} - \Delta x_t) + e_t$$

where $\bar{x}$ is the nominal income growth target. Thus, in the regression above, $\alpha = \delta_0 + \delta_1 \Delta \bar{x}$ and $\beta = -\delta_1$.}

\footnote{This data is readily available through the Philadelphia Federal Reserve. The sample estimated is from 1979:4 - 2003:4 as that is the latest data available.}

\footnote{The t-statistics correspond to Newey-West standard errors due to the evidence of serial correlation in a standard, ordinary least squares regression.}
<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSI M1</td>
<td>Constant</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>∆NGDP Forecast</td>
<td>0.03</td>
</tr>
<tr>
<td>MSI M2</td>
<td>Constant</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>∆NGDP Forecast</td>
<td>-0.06</td>
</tr>
<tr>
<td>MSI MZM</td>
<td>Constant</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>∆NGDP Forecast</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

A second way to examine whether monetary policy can explain why money growth cannot predict nominal income growth is to consider a case in which money aggregates are truly exogenous. For example, Rowe and Rodriguez [2007] find that changes in the growth rate of U.S. simple sum monetary aggregates do not Granger cause fluctuations in the growth rate of U.S. real GDP. However, changes in the growth rate of U.S. simple sum monetary aggregates do explain fluctuations in the real GDP of Hong Kong, whose currency is pegged to the U.S. dollar. Intuitively, this is the case because U.S. monetary policy reacts to fluctuations in measures of real activity in the U.S., but not Hong Kong. As a result, changes in the money supply in the U.S. should be exogenous to Hong Kong.

Thus, as a further method of comparison, the three variable system outlined above is re-estimated using nominal income growth and inflation from Hong Kong and the MSI aggregates from the U.S. The sample period runs from 1983:1 - 1997:2. The results are shown in Table 6. Based on the results, the growth of monetary aggregates can be considered exogenous for MSI M1 and MSI M2 as these are not predicted by nominal income growth or inflation. In addition, the same monetary aggregates do help predict Hong Kong’s nominal income growth and inflation rate. Again, this tends to lend credence to the view that the failure to identify a role for monetary aggregates in predicting nominal income growth is the result of monetary policy.

Overall, the results from the three variable VAR using the monetary services index as

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41 The sample is chosen as representing the period from the first peg of the Hong Kong dollar to the U.S. dollar until the handing over of Hong Kong to the Chinese and the re-pegging that occurred shortly thereafter.
the method of aggregation for the money supply indicate that money growth is an important predictor of inflation for all measures of money and all sample periods. This is important because previous research that relies on simple sum monetary aggregates find no such relation in the post-1979 era. The results with regards to nominal income, however, are somewhat mixed. In the pre-1979 era, MSI M2 and MSI MZM do help to predict nominal income growth. However, consistent with earlier research, this relationship does not hold in the post-1979 era. Nevertheless, this failure in the latter period is likely the result of countercyclical monetary policy. As evidence in support of this claim MSI M2 and MSI MZM demonstrate a statistically significant response to the Federal Reserve’s forecast of nominal GDP growth. What’s more, the U.S. MSI aggregates do help to predict nominal GDP growth and inflation in Hong Kong, a country whose currency is pegged to the U.S. dollar. Ultimately, the results in this subsection cast doubt on earlier research that finds money curiously unable to predict inflation. In addition, and again contrary to earlier research, the inability of money aggregates to explain nominal income growth is shown to be the result of countercyclical monetary policy.

### 3.3.3 Causality in a Cointegrated VAR

Estrella and Mishkin [1997] use the three variable system of growth rates because nominal income, the price level, and each of the monetary services indexes are difference stationary.
Nonetheless, since the levels of these variables are cointegrated, it is necessary to estimate a cointegrated VAR.\footnote{The trace statistics are shown in Appendix 1.} As a method of comparison with the standard VAR above as well as the analysis of Estrella and Mishkin [1997], the cointegrated VAR is estimated for period 1960 - 1979:3 and 1979:4 - 2005 at quarterly frequencies to capture the policy change that occurred at the Federal Reserve under Paul Volcker. In addition, the cointegrated VAR is estimated imposing $\text{rank}(\Pi) = 1$ thereby implying one cointegrating vector as evident from the trace tests. Granger causality tests are then performed within this context.

The p-values from the Granger causality tests for the pre-1979 sample are shown in Table 7. The p-values are printed such that the null hypothesis is that the column variable does not predict the row variable. In the pre-Volcker era, one can reject the null hypothesis that the levels of the monetary services index do not predict nominal income for each measure of money. However, one cannot reject the null hypothesis that the monetary services indexes do not affect the price level. In fact, the evidence suggests that the price level is only predicted by its previous values. Nonetheless, there is evidence that the monetary services indexes are important for predicting nominal spending as implied by the quantity theory of money.

Table 8 reports the p-values from the Granger causality tests for the post-1979 era. Again, the p-values are printed such that the null hypothesis is that the column variable does not predict the row variable. The results show that the null hypothesis that money predicts

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**Table 7: Granger Causality Tests, cointegrated VAR – Pre-1979**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Nom. GDP</th>
<th>Price Level</th>
<th>MSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSI M1</td>
<td>0.83</td>
<td>0.96</td>
<td>0.05</td>
</tr>
<tr>
<td>Price Level</td>
<td>0.13</td>
<td>0.00</td>
<td>0.66</td>
</tr>
<tr>
<td>MSI M1</td>
<td>0.85</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>MSI M2</td>
<td>0.59</td>
<td>0.69</td>
<td>0.05</td>
</tr>
<tr>
<td>Price Level</td>
<td>0.20</td>
<td>0.00</td>
<td>0.36</td>
</tr>
<tr>
<td>MSI M2</td>
<td>0.31</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>MSI MZM</td>
<td>0.65</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>Price Level</td>
<td>0.26</td>
<td>0.00</td>
<td>0.86</td>
</tr>
<tr>
<td>MSI MZM</td>
<td>0.35</td>
<td>0.05</td>
<td>0.00</td>
</tr>
</tbody>
</table>
nominal income cannot be rejected when money is measured by MSI M1. However, this can be rejected at the 5%-level when money is measured by MSI M2 and at the 1%-level when measured by MSI MZM. In addition, while one cannot reject the null hypothesis that money predicts the price level when money is measured by MSI M1, this hypothesis can be rejected at the 5%-level when money is measured by MSI M2 or MSI MZM. These results are important because they are once again in contrast to conclusions of Estrella and Mishkin [1997].

Overall, the results from the Granger causality tests provide evidence that money is useful in predicting nominal income in the pre-1979 sample and both nominal income and the price level in the post-1979 sample if money is measured by MSI M2 or MSI MZM. These results are important because they are in contrast to the previous findings of Estrella and Mishkin [1997]. Curiously, and again in contrast to Estrella and Mishkin [1997], the results suggest that the predictive qualities of money might have improved in the post-1979 era at least in the case of the price level. It is interesting to consider why this might be the case. Nonetheless, more work is necessary to consider this anomaly.

3.3.4 A Comparison With Simple Sum Aggregates

The previous results can now be compared with those using simple sum aggregates. Following Estrella and Mishkin [1997], Tables 9 and 10 report the p-values from Granger

| Table 8: Granger Causality Tests, cointegrated VAR – Post-1979 |
|----------------|---------|---------|---------|
| Variable       | Nom. GDP | Price Level | MSI     |
| MSI M1         | 0.02    | 0.62    | 0.12    |
| MSI M2         | 0.00    | 0.80    | 0.04    |
| MSI MZM        | 0.01    | 0.59    | 0.01    |

Following Estrella and Mishkin [1997], Tables 9 and 10 report the p-values from Granger
Table 9: Granger Causality Tests – Simple sum, Pre-1979

<table>
<thead>
<tr>
<th>Variable</th>
<th>Nom. GDP</th>
<th>Inflation</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>Nom.GDP</td>
<td>0.15</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>Inflation</td>
<td>0.45</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>M1</td>
<td>0.55</td>
<td>0.01</td>
</tr>
<tr>
<td>M2</td>
<td>Nom.GDP</td>
<td>0.56</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Inflation</td>
<td>0.24</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>0.35</td>
<td>0.09</td>
</tr>
<tr>
<td>MZM</td>
<td>Nom.GDP</td>
<td>0.86</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Inflation</td>
<td>0.36</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MZM</td>
<td>0.03</td>
<td>0.10</td>
</tr>
</tbody>
</table>

causality tests on the three variable system of nominal income growth, inflation, and money growth for the pre- and post-1979 eras, respectively. Money growth is now defined by the simple sum counterparts to the monetary services indexes. Also, given evidence of cointegration, Tables 11 and 12 report the p-values of Granger causality tests on the three variable system of nominal income, the price level, and the money stock. The results in each table are printed such that the null hypothesis is that the column variable does not predict the row variable.

As shown in Table 9, there is evidence that simple sum M2 and simple sum MZM are useful in predicting both nominal income growth and inflation in the pre-1979 period. In addition, simple sum M1 is useful in predicting inflation. These results are largely consistent with those using the monetary services indexes for the same period. For the post-1979 period, the simple sum aggregates perform markedly worse. As shown in Table 10, one cannot reject the null hypothesis that money growth does not predict inflation for any of the simple sum monetary aggregates. This is in stark contrast to the monetary services indexes, which are all useful in predicting inflation in the post-1979 period. Only simple sum MZM is useful for predicting nominal income growth. These results are consistent with those of Estrella and Mishkin [1997]. However, the differences in performance of the monetary services indexes and their simple sum counterparts provide evidence for the hypothesis of mismeasurement.
Table 10: Granger Causality Tests – Simple sum, Post-1979

<table>
<thead>
<tr>
<th>Variable</th>
<th>Nom. GDP</th>
<th>Inflation</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>0.00</td>
<td>0.50</td>
<td>0.23</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.02</td>
<td>0.00</td>
<td>0.95</td>
</tr>
<tr>
<td>M1</td>
<td>0.11</td>
<td>0.08</td>
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</tr>
<tr>
<td>M2</td>
<td>0.02</td>
<td>0.37</td>
<td>0.47</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.01</td>
<td>0.00</td>
<td>0.91</td>
</tr>
<tr>
<td>M2</td>
<td>0.22</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>MZM</td>
<td>0.00</td>
<td>0.49</td>
<td>0.06</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.01</td>
<td>0.00</td>
<td>0.34</td>
</tr>
<tr>
<td>MZM</td>
<td>0.05</td>
<td>0.17</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 11: Granger Causality Tests, Cointegrated VAR – Simple sum, Pre-1979

<table>
<thead>
<tr>
<th>Variable</th>
<th>Nom. GDP</th>
<th>Price Level</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>0.21</td>
<td>0.72</td>
<td>0.09</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.54</td>
<td>0.00</td>
<td>0.72</td>
</tr>
<tr>
<td>M1</td>
<td>0.62</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>M2</td>
<td>0.12</td>
<td>0.50</td>
<td>0.44</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.81</td>
<td>0.02</td>
<td>0.10</td>
</tr>
<tr>
<td>M2</td>
<td>0.36</td>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td>MZM</td>
<td>0.14</td>
<td>0.74</td>
<td>0.09</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.65</td>
<td>0.00</td>
<td>0.22</td>
</tr>
<tr>
<td>MZM</td>
<td>0.23</td>
<td>0.22</td>
<td>0.00</td>
</tr>
</tbody>
</table>

When the three variable system is expressed in levels rather than growth rates, the results imply similar conclusions. As shown in Table 11, and consistent with evidence for the monetary services indexes, simple sum M1 and simple sum MZM are both useful in predicting nominal income in the pre-1979 period. In contrast to the analysis of the monetary services indexes, simple sum M2 is no longer useful for predicting nominal income, but is useful in predicting the price level. As shown in Table 12, simple sum M2 is useful for predicting the price level, but not nominal income whereas simple sum MZM is useful for predicting nominal income, but not the price level. Again, this is in contrast to the results outlined above, in which MSI M2 and MSI MZM were useful for predicting both the price level and nominal income for the same period.
Overall, these results are consistent with Estrella and Mishkin [1997] as they demonstrate that the simple sum monetary aggregates have limited predictive ability. Nonetheless, these results differ from those using the monetary services indexes as the appropriate measure of the monetary aggregate. Whereas Estrella and Mishkin [1997] interpret the failures of the simple sum aggregates as evidence that monetary aggregates are not useful for policy analysis, the empirical evidence using monetary services indexes coupled with the results in this section provide evidence that such conclusions are driven by mismeasurement.

### 3.4 Money and the Output Gap

The final method of empirical analysis is an examination of the role of real money balances in predicting movements in the output gap. As noted in Chapter 2, money is absent from contemporary analysis due to both the lack of a dynamic LM equation as well as the fact that real balances do not appear in the IS equation. The former is absent because it is replaced in typical analysis by an interest rate rule that governs monetary policy. For inclusion in a structural IS equation, it is necessary for real balances to be non-separable with consumption in the utility function. The exclusion of real balances from the IS equation is justified by the fact that, for reasonable parameterizations of the model, the effect of real balances on
the output gap would be small in magnitude.\footnote{See Woodford [2003].}

While there exists a theoretical basis for excluding real money balances from a structural IS equation, it remains possible that real balances contain information that is important for explaining the output gap. As discussed in Chapter 2, a key argument among monetarists is that the demand for real money balances is a function of a number of different asset prices and not simply the short-term interest rate. As a result, if monetary policy affects relative prices of financial assets other than the short-term interest rate, real balances are likely to capture the variety of substitution effects and channels through which monetary policy works. In this case, real money balances, despite their absence from a structural IS equation, might still convey information not captured by the short-term interest rate and therefore might help to predict movements in the output gap.

Empirical analysis of the effect of money on the output gap has been carried out by estimating a backward-looking IS equation. As previously mentioned, Rudebusch and Svensson [2002] estimate the particular specification given in equation (5) using quarterly data from 1961 - 1996. They find little evidence of a role for money. This paper extends their analysis in two ways. First, the sample is extended through 2005:4. Second, the model is expanded to include a one period lag of the quarterly growth rate of real money balances as measured by the monetary services indexes.

In addition to the estimates over the entire sample, the IS equations are also estimated for the subsample, 1979:4 - 2005:4. This date is chosen because it is consistent with the break point identified by Friedman and Kuttner [1992] and Estrella and Mishkin [1997] in their work described above. What's more, this date marks the change in monetary policy beginning with the appointment of Paul Volcker that has been documented by Taylor [1999], Clarida et al. [2000], and Hendrickson [2010] as well as the beginning of financial deregulation and innovation. Others, such as Bernanke and Mihov [1998], Leeper and Roush [2003] and Hafer et al. [2007] use 1983 as the break point. This is justified by the fact that this date
marked the shift from monetary targeting to interest rate targeting as well as shifts in the velocity of certain simple sum aggregates. Practically, this is not a particularly useful date, especially for the monetary services indexes, which do not experience structural shifts in velocity around 1983.

The model is estimated using OLS. The results are shown in Table 13. The first column are the results reported by Rudebusch and Svensson. The second and third columns re-estimate the model for the extended sample and the subsample, respectively. In each case, the output gap is shown to be strongly autoregressive and the real interest rate is shown to have a negative and significant impact on the output gap. Although there is a slight reduction in the parameter on the real interest in the subsample beginning in October 1979, the parameters are relatively constant across samples.

Columns 4 - 9 extend the model to include lagged quarterly growth of real money balances as measured by MSI M1, MSI M2, and MSI MZM adjusted by the GDP deflator. For the entire sample real money balances exhibit a positive and significant effect on the output gap. In the subsample, real MSI M1 does not have a statistically significant effect on the output gap. However, real balances do have a positive and statistically significant effect on the output gap when measured by MSI M2 or MSI MZM. In addition, the coefficient on the real interest rate declines in the subsample when real money balances are included and is not statistically significant when MSI M2 or MSI MZM is included in the regression. Overall, these results not only do not support omitting real money balances from IS equations, but also suggests that the exclusion of money leaves the estimated IS equations misspecified.
Table 13: IS equations

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output gap (-1)</td>
<td>1.16</td>
<td>1.22***</td>
<td>1.20***</td>
<td>1.25***</td>
<td>1.15***</td>
<td>1.24***</td>
<td>1.18***</td>
<td>1.25***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(14.50)</td>
<td>(17.46)</td>
<td>(13.29)</td>
<td>(17.00)</td>
<td>(13.24)</td>
<td>(16.67)</td>
<td>(13.60)</td>
<td>(17.28)</td>
<td></td>
</tr>
<tr>
<td>Output gap (-2)</td>
<td>-0.26</td>
<td>-0.29***</td>
<td>-0.31***</td>
<td>-0.26***</td>
<td>-0.30***</td>
<td>-0.22***</td>
<td>-0.31***</td>
<td>-0.23***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-3.25)</td>
<td>(-4.12)</td>
<td>(-3.31)</td>
<td>(-3.61)</td>
<td>(-3.25)</td>
<td>(-3.19)</td>
<td>(-3.44)</td>
<td>(-3.41)</td>
<td></td>
</tr>
<tr>
<td>Real Rate (-1)</td>
<td>-0.09</td>
<td>-0.07***</td>
<td>-0.05**</td>
<td>-0.07***</td>
<td>-0.04*</td>
<td>-0.05**</td>
<td>-0.03</td>
<td>-0.05**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.75)</td>
<td>(-2.85)</td>
<td>(-2.06)</td>
<td>(-2.73)</td>
<td>(-1.83)</td>
<td>(-2.01)</td>
<td>(-1.44)</td>
<td>(-2.22)</td>
<td></td>
</tr>
<tr>
<td>Money (-1)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.10*</td>
<td>0.02</td>
<td>0.23***</td>
<td>0.19***</td>
<td>0.12***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>(1.67)</td>
<td>(0.36)</td>
<td>(3.80)</td>
<td>(2.71)</td>
<td>(3.43)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.90</td>
<td>0.91</td>
<td>0.93</td>
<td>0.91</td>
<td>0.93</td>
<td>0.91</td>
<td>0.94</td>
<td>0.91</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson</td>
<td>2.08</td>
<td>2.15</td>
<td>2.12</td>
<td>2.16</td>
<td>2.12</td>
<td>2.19</td>
<td>2.15</td>
<td>2.19</td>
<td></td>
</tr>
</tbody>
</table>

***1% crit. value, **5%, *10%
3.5 Conclusion

The emerging consensus is that monetary aggregates are not useful in monetary policy and business cycle analysis. This view has largely been justified by empirical work that shows that the demand for money is unstable and that money does not help to explain fluctuations in the output gap. Modern business cycle theorists have used these results to develop models that completely abstract from money. At the core of these models is the dynamic New Keynesian IS-LM-type model where the LM curve has been replaced by an interest rate rule followed by the central bank. Money is inconsequential to the model as it merely reflects movements in the interest rate. In other words, money is redundant.

One potential problem with the empirical results that justify these cashless models is that they rely on the use of simple sum monetary aggregates. Such aggregates are theoretically flawed in that they treat all components of a particular aggregate as perfect substitutes; a result inconsistent with empirical evidence. Thus, previous results that employ simple sum aggregates are potentially flawed by mismeasurement.

As a result, this chapter re-examines the empirical findings of previous authors by using monetary services indexes rather than the simple sum counterparts. The advantage of using the monetary services index is that it is derived from microtheoretic foundations and is consistent with aggregation and index number theory. Using this alternative measure of money, this paper identifies a stable money demand function for each component class of monetary assets across samples. In addition, it demonstrates that real money balances not only have a positive and significant impact on the output gap, but that this effect is often larger in magnitude than that of the real interest rate. Overall, the results suggest that previous findings are likely the result of mismeasurement with regards to monetary aggregates.
Chapter 4 Monetary Transmission in the New Keynesian Framework

A key implication of the New Keynesian model is that the monetary transmission mechanism is solely captured by the path of the short term interest rate. Both monetarists and advocates of the credit channel argue that the interest rate is insufficient to capture the transmission process. This chapter extends the baseline New Keynesian model to incorporate these other mechanisms and compares the results of the model to empirical characteristics observed in the data.

4.1 Extending the Model

This chapter extends the baseline New Keynesian model in two directions. First, following Cuthbertson and Taylor [1987], Christiano et al. [1998], and Nelson [2002], it is assumed that there is disutility associated with adjusting real money balances. Under this assumption, the money demand function has desirable properties, specifically dependence on permanent income, the long term interest rate, and lagged real balances.

Second, following Carlstrom et al. [2010], the model introduces agency costs such that the model includes endogenously determined asset prices and net worth.\textsuperscript{44} The addition of agency costs is important because it creates an additional channel for the monetary transmission mechanism as emphasized by the credit channel literature. What’s more, it represents an important extension to the work of Nelson [2002] by incorporating additional financial assets to the model; a characteristic often emphasized in the work of Brunner and Meltzer [1989, 1993].

The following subsections discuss each modification in more detail.

\textsuperscript{44}The present model modifies the framework of the previous authors by introducing an alternative assumption about the characteristics of the entrepreneurs. In the absence of this alternative assumption, the addition of agency costs does not affect the response of the output gap to a monetary shock. This is because the previous authors assume that entrepreneurs do not consume and, as a result, agency costs only affect net worth and asset prices. This is discussed in more detail below.
4.1.1 Money Demand

The first extension to the New Keynesian model is to assume that there are small costs in the form of disutility associated with adjusting real money balances. For example, Brunner et al. [1980], Brunner et al. [1983], and Meltzer [1998] suggest that agents will only want to adjust real balances if exogenous shocks are expected to persist. Formally, this is consistent with habit persistence in the level of real money balances. In this case, the representative agent would seek to smooth movements in real balances over time to prevent the problem of sudden movements to real balances caused by exogenous shocks.\(^4^5\) Given the preference to smooth money balances, it is assumed that there is disutility associated with changing money balances.

Ultimately, the cost of adjustment is assumed to be small, but existent.\(^4^6\) This approach also has desirable features as it implies that there is a dynamic adjustment in the demand for real money balances and that real balances are dependent on long run expectations of the interest rate and income. Each of these characteristics is justified below.

The existence of a dynamic adjustment is an implicit, and often neglected, characteristic of typical empirical approach to money demand. To illustrate this consider a cointegrated vector autoregressive (VAR) model typically used in money demand analysis:

\[
\Delta x_t = \Gamma_1 \Delta x_{t-1} + \cdots + \Gamma_k \Delta x_{t-k} + \alpha \beta' x_{t-1} + \varepsilon_t
\]

where \(\alpha\) is the adjustment matrix and \(\beta'\) is the cointegrating matrix. The former contains short run adjustment parameters whereas the latter explains long run relations. Thus, the

\(^{4^5}\)This would seem to be consistent with the role of money as a "temporary abode of purchasing power" as in Friedman and Schwartz [1982] in which money is valued beyond its role for transactions purposes and is held as part of a portfolio of assets. In this regard, partial adjustment in real money balances is not necessarily a quintessential monetarist point. For example, as Niehans [1978] explains, “The difference between desired and actual stocks [is] called excess demand . . . Excess demand refers . . . to the fact that more rapid adjustment of assets may be more expensive than slow adjustment, so that the individual finds it efficient to spread the adjustment over time. This approach to asset accumulation is familiar from investment and portfolio analysis.” This concept remains relevant to empirical analysis of money demand as outlined below.

\(^{4^6}\)The precise size of the cost is discussed in the calibration section below.
impetus behind the use of the cointegrated VAR model in the analysis of money demand is that it allows for a direct examination of the long run relationship between real money balances, a scale variable of economic activity, and a price variable such as an interest rate. Specifically, the nature of this relationship can be examined by testing for cointegration of the variables as well as testing restrictions on the cointegrating vector, $\beta$, associated with a cointegrating relation. For example, if a stable money demand function exists, this would imply the following:

$$\beta'x_t = m_t - \beta_0 - \beta_y y_t - \beta_r R_t = 0$$

where $m_t$ is real balances, $y_t$ is the scale variable, and $R_t$ is the interest rate. Even when such a relationship holds, however, at any point in time there might exist a vector $x_t = [m_t, y_t, R_t]'$ such that $\beta'x_t \neq 0$. As such, one might alternatively express $\beta'x_t$ as:

$$m_t - m_t^*$$

where $m_t$ is actual real balances and $m_t^* = \beta_0 + \beta_y y_t + \beta_r R_t$ represents desired money balances.

Returning to the cointegrated VAR, maintaining the definition that $\beta'x_t = m_t - m_t^*$, and suppressing lagged differences, the model can be re-written:

$$\begin{bmatrix}
\Delta m_t \\
\Delta y_t \\
\Delta R_t
\end{bmatrix} = \begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{bmatrix} \begin{bmatrix}
(m_t - m_t^*)
\end{bmatrix} + \cdots + \begin{bmatrix}
\varepsilon_{1,t} \\
\varepsilon_{2,t} \\
\varepsilon_{3,t}
\end{bmatrix}$$

Through simple algebraic manipulation the real money balances equation can then be expressed as:

$$m_t = (1 + \alpha_1)m_{t-1} - \alpha_1 m_t^* + \cdots + \varepsilon_{1,t}$$

where $\alpha_1 \leq 0$. This relation thus bears a strong resemblance to the dynamic adjustment approach of Goldfeld [1973].
Using this framework, the speed of adjustment of actual real money balances toward the desired level of real balances can be estimated directly. This characteristic is important because if individuals are able to accurately perceive the shocks and the costs of adjustment are sufficiently small, one would expect the adjustment parameter, $\alpha_1$ to be sufficiently close to unity in absolute value such that actual real money balances would immediately adjust to their desired level; a result not documented by empirical evidence.

As a result of the implications illustrated above, it seems justified to assume that there are small costs associated with changes in real money balances. Returning to the theoretical proposition, suppose that the following utility function is maximized subject to a typical budget constraint:

$$U = \beta t \left[ \left( \frac{C_t}{1 - \sigma} \right)^{1-\sigma} + \left( \frac{m_t}{1 - \epsilon_m} \right)^{1-\epsilon_m} + \ldots - \frac{\phi_m}{2} \left( \frac{m_t}{m_{t-1}} - 1 \right)^2 \right]$$

where $C_t$ is consumption, $m_t$ is real money balances, and $\sigma$, $\epsilon_m$, and $\phi_m$ are parameters.

Maximization yields the following first order condition expressed in log-deviations:

$$\lambda_t = -\sigma C_t$$

$$m_t + \alpha \Delta m_t - \beta \alpha \Delta m_{t+1} + \frac{1}{\epsilon_m} (\lambda_t + R_t) = 0$$

where $\lambda_t$ is the Lagrangian multiplier, $R_t$ is the nominal interest rate, and $\alpha$ is a parameter. Combing first order conditions and solving forward to eliminate the unstable root, as in Sargent (1987), the money demand equation can then be expressed as:

$$m_t = \mu_1 m_{t-1} + (\beta a_m)^{-1} \left[ \sum_{i=0}^{\infty} (\mu_2)^{-i+1} \left( \frac{\sigma}{\epsilon_m} y_{t+i} - \frac{1}{\epsilon_m} R_{t+i} \right) \right]$$

(8)

where $\mu_1$ is the dynamic adjustment parameter and is a nonlinear function of $\alpha$ and $\beta$. This money demand function has the desirable properties that it is both a function of the
long term interest rate as well as permanent income as implied by Friedman (1956). The appearance of the long term interest rate in the money demand function is consistent with propositions of monetarists such as Friedman [1956] and Meltzer [1998]. In addition, if the long term interest rate is present in the money demand function, it is possible that real money balances might convey additional information not explained by the short term interest rate adjusted by monetary policy as emphasized by in the monetarist transmission mechanism discussed above.

### 4.1.2 Agency Costs

The second addition to the model is the introduction of agency costs. Following Carlstrom et al. [2010], the framework is extended in two important directions. First, the representative household provides two types of labor. Second, the representative entrepreneur hires each type of labor to produce an intermediate good. However, the choice of one type of labor is subject to a collateral constraint due to information asymmetries in the labor market. Constrained optimization therefore implies that one input is financed in a matter consistent to an intratemporal loan with a credit distortion.

These two modifications introduce important aspects to the model. First, it can be shown that the credit distortion is positively related to the output-net worth ratio consistent with a financial accelerator model. Second, the model is isomorphic to a costly state verification framework used in previous agency cost models. Finally, and most importantly, the existence of agency costs generates endogenously determined asset prices and net worth. As a result, the model incorporates elements of the asset price channel and the credit channel emphasized in the literature discussed above.

The addition of agency costs in this framework is similar to the models used by Curdia and Woodford [2008] and DeFiore and Tristani [2009], who also extend the baseline New Keynesian model to include some type of credit friction in order to determine optimal mone-

\footnote{Note that in the baseline New Keynesian model \( y_t = c_t \).}

\footnote{See the appendix to Carlstrom et al. [2010].}
tary policy. Curdía and Woodford [2008] extend the baseline model to include a time-varying interest rate spread between borrowing rates and saving rates by considering the case of two types households, which differ according to their ”relative impatience to consume” (Curdía and Woodford [2008]: 8). These households are free to adjust their type. As a result, there is a reason for financial intermediation. The financial friction arises because households are only able to engage in financial contracting through the intermediary sector, which incurs a real resource cost through intermediation. This resource cost generates an equilibrium spread between borrowing and deposit rates. What’s more, it is assumed that this spread is also subject to an exogenous mark-up shocks analogous to a financial shock.

DeFiore and Tristani [2009] introduce credit frictions through the use of a costly state verification framework. In their model, firms have asymmetric information and must raise external funds in order to pay their labor force in advance of production. If the firms generate sufficient revenue from production, they pay back their debt and use any remaining profits for consumption. As implied by the costly state verification framework, a firm that fails to generate sufficient revenue defaults on the debt and the output that was produced is taken by the financial intermediary. Credit frictions arise as a result of the monitoring cost associated with the borrowing of external funds.

These models differ from the present analysis in a number of important respects. For example, both models neglect the endogenous role of net worth and asset prices. Curdía and Woodford [2008] offer only one endogenous mechanism through which the credit spread can be effected. What’s more, these authors explicitly assume that the monetary transmission mechanism is much the same as in the baseline New Keynesian model. Despite the fact that the interest rate spread affects both the IS equation and the Phillips curve relation, the spread effectively only affects the dynamics of the model through a new additive term in each equation akin to a new exogenous disturbance. As a result, the model relies on the

49 Households are also able to agree to state-contingent insurance contracts against aggregate and idiosyncratic risk, but do not have access to the insurance market every period. This enables aggregation in their model while maintaining a meaningful financial friction.
traditional interest rate channel of monetary transmission in which the future path of the policy interest rate is all that matters.

DeFiore and Tristani [2009] assume that entrepreneurial net worth is exogenous and given as an endowment each period. This similarly has important implications for the monetary transmission mechanism. Most notably, the existence of exogenous net worth implies that the model is not consistent with the typical literature on the credit channel, in which endogenous movements in net worth propagate monetary disturbances. In contrast, monetary policy affects financial decisions because the contract is priced in nominal terms.

4.2 The Model

4.2.1 Household

The representative household supplies two types of labor, \( L_t \) and \( u_t \), in exchange for real wages, \( w_t \) and \( r_t \), respectively. In addition, the household holds interest yielding bonds, \( B_{t-1} \), money balances, \( M_{t-1} \), and shares of the intermediate goods producing firm, \( e_{t-1} \), at the beginning of the period. The household uses the income generated from supplying labor and the value of its asset holdings to finance consumption and re-allocate its portfolio in the present period. The household budget constraint can be expressed in real terms as:

\[
w_t L_t + r_t u_t + (1 + R_{t-1}) b_{t-1} + e_{t-1} (q_t + d_t) = c_t + b_t + m_t + q_t e_t \quad (9)
\]

where \( R_t \) is the nominal interest rate, \( q_t \) is the real price of a share of the intermediate goods producing firm, and \( d_t \) is the real dividend paid by the intermediate goods producing firm.

The household’s utility function is given by:

\[
U(C, L, u, m) = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1 - \sigma} - \frac{L_t^{1+\theta}}{1 + \theta} - \frac{u_t^{1+\theta}}{1 + \theta} + \frac{m_t^{1-\epsilon_m}}{1 - \epsilon_m} - \frac{\varphi_m}{2} \left( \frac{m_t}{m_{t-1}} - 1 \right)^2 \right\}
\]

where \( \varphi_m \) is a parameter that measures the cost of portfolio adjustment associated with
changing real money balances and $\beta$ is the discount factor. This utility function and the existence of two types of labor can be justified as an aggregation procedure in which there are heterogeneous households that differ only in the sense that they provide two, distinct types of labor so long as the households insure one another against risk in terms of consumption.

The household maximizes utility subject to (9). Constrained maximization yields:

$$\lambda_t = C_t^{-\sigma}$$  \hspace{1cm} (10)

$$L^\theta_t = \lambda_t w_t$$  \hspace{1cm} (11)

$$u^\theta_t = \lambda_t r_t$$  \hspace{1cm} (12)

$$m_t^{-\epsilon_m} - \varphi_m \left( \frac{m_t}{m_{t-1}} - 1 \right) \frac{1}{m_{t-1}} - \varphi_m \left( \frac{m_t}{m_{t-1}} - 1 \right) \frac{m_{t+1}}{m_t^2} + \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t$$  \hspace{1cm} (13)

$$\beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} (1 + R_t) = \lambda_t$$  \hspace{1cm} (14)

$$\beta E_t \lambda_{t+1}(q_{t+1} + d_{t+1}) = \lambda_t q_t$$  \hspace{1cm} (15)

where $\lambda_t$ is the Lagrangian multiplier. Solving to eliminate the Langrangian multiplier yields two labor supply curves, a money demand function, a dynamic IS equation, and an asset demand function.

### 4.2.2 Entrepreneur

The representative entrepreneur uses two types of labor, $L$ and $u$, to produce good $x$ and sells the good for price $p$. The entrepreneur earns profits from production to purchase shares of the intermediate goods producing firm and for consumption. The entrepreneur thus solves two problems – a profit-maximization problem and a utility maximization problem. This differs from the model used by Carlstrom et al. [2010] as that model imposes the assumption that a fraction of entrepreneurs die each period and, as a result, entrepreneurial consumption is zero in equilibrium. This assumption assures that the collateral constraint is binding. Un-
fortunately, this limits the effects of agency costs to fluctuations in asset prices and net worth and fails to differentiate the output dynamics of the model from a baseline New Keynesian model. This model adopts the assumption that entrepreneurs have a higher discount rate than the representative household in order to ensure that the collateral constraint is binding and that entrepreneurial consumption is non-zero in equilibrium. Without this assumption, the entrepreneur would simply forego hiring workers of type L until accumulating enough net worth to fully cover the wage bill.

The entrepreneur produces output using the following production function:

$$x_t = L_t^\alpha u_t^{1-\alpha}$$ (16)

and purchases quantities of each type of labor from the household in competitive markets. A useful interpretation of this production function is as follows. Suppose that two distinct entrepreneur types exist. One of the entrepreneurs uses type-L labor in production and is subject to a collateral constraint. The other entrepreneur type uses type-u labor in production and faces no constraint. The production function can therefore be rationalized as a Cobb-Douglas aggregator of production in which the fraction of entrepreneurs using the L-type of labor is given by $\alpha$.51

The entrepreneur thus chooses $L_t$ and $u_t$ to maximize profits given by:

$$Profits = p_t x_t - r_t u_t - w_t L_t$$ (17)

50 It does generate important differences in welfare. Since the previous authors were conducting welfare analysis for alternate monetary policy rules, this is not a critique of their methods. Nonetheless, their assumption is not reasonable for the present analysis.

51 This interpretation is useful in the calibration below.
where the choice of labor is made subject to the collateral constraint:  

\[ w_t L_t \leq g(nw_t, p_t x_t - r_t u_t) \equiv nw_t^b(p_t x_t - r_t u_t)^{1-b} \]  

(18)

Where \( nw_t = e_{t-1}(q_t + d_t) \) is entrepreneurial net worth and \((p_t x_t - r_t u_t)\) are profits obtained without the use of the L-type of labor. The collateral constraint implies that the entrepreneur backs up the wage agreement with L-type labor suppliers with his net worth and the profits from using the u-type of labor.

Maximizing (17) in which equation (18) is a binding constraint for the choice of \( L \) yields the following optimization conditions:

\[ r_t u_t = (1 - \alpha)p_t x_t \]  

(19)

\[ w_t L_t (1 + b\phi_t) = \alpha p_t x_t \]  

(20)

where \( \phi_t \) is the Lagrangian multiplier from the constrained optimization problem.

Equations (19) and (20) reveal the nature of the credit market friction employed in this model. In comparison to the unconstrained choice, the optimization condition given by equation (20) is analogous to the case in which the entrepreneur must borrow funds at a real interest rate, in this case given by \( b\phi_t \), in order to pay L-type workers at the beginning of the period. The loan is subsequently paid off at the end of the period. In this case, \( b\phi_t \) represents an endogenous credit distortion, akin to an external finance premium, that is a function of the ratio of output to net worth.  

\(^{52}\) This constraint is desirable because, as Carlstrom, Fuerst, and Paustian show, it is isomorphic to the costly state verification framework used in Carlstrom and Fuerst [1997], Carlstrom and Fuerst [1998] and other agency cost literature.

\(^{53}\) Assuming that the collateral constraint is binding, one can combine the collateral constraint with the optimization conditions to yield:

\[ b\phi_t = \left( \frac{\alpha p_t x_t}{nw_t} \right)^b - 1 \]

The parameter \( b \) measures the sensitivity of the finance premium to the output-net worth ratio. For the case in which \( b = 0 \), the external finance premium collapse to zero. Details are shown in Appendix 2.
channel, a decline in net worth induced by a change in monetary policy results in a larger credit distortion and a corresponding increase in the cost of hiring workers of type $L$.

Finally, by substituting the optimization conditions given in equations (19) and (20) into the profit equation (17), one can derive the equilibrium level of profits earned by the entrepreneur:

$$\text{Profits}_t = \alpha p_t x_t \left( \frac{b\phi_t}{1 + b\phi_t} \right)$$

Thus, the existence of a collateral constraint for L-type labor implies that the entrepreneur yields a positive level of profits.

Like the household, the entrepreneur also buys and sells shares of the intermediate goods producing firm. The entrepreneur uses the value of the shares carried over from the previous period, the income generated through the collection of the corresponding dividends, and the profits generated by production to finance consumption and the purchase of shares in the present period. The entrepreneur’s budget constraint is therefore given by:

$$c^e_t + e_t q_t = e_{t-1}(q_t + d_t) + \text{profits}$$  \hspace{1cm} (21)

Using the definition of net worth combined with the optimization conditions, the collateral constraint, and the equilibrium level of profits, this budget constraint can be re-written:

$$c^e_t + e_t q_t = \alpha p_t x_t F(\phi_t)$$  \hspace{1cm} (22)

where:

$$F(\phi_t) = \left[ \left( \frac{b\phi_t}{1 + b\phi_t} \right) + \left( \frac{1}{1 + b\phi_t} \right)^{\frac{1}{\gamma}} \right]$$

The entrepreneur then chooses consumption, $c^e$, and the fraction of shares, $e$, to maximize utility:

$$U(c^e) = E_t \sum_{t=0}^{\infty} (\beta \gamma)^t c^e_t$$
subject to the budget constraint above. Here, $\gamma$ is an additional discount factor that ensures that the collateral constraint is binding. Constrained maximization yields the entrepreneur’s intertemporal Euler equation:

$$q_t = \beta \gamma E_t(q_{t+1} + d_{t+1})(1 + b\phi_{t+1})$$ (23)

### 4.2.3 Final Goods Producing Firm

The final good producing firm is perfectly competitive and produces, $y_t$. The firm purchases $y_t(j)$ units from firm $j \in [0,1]$ at price $P_t(j)$. The final good is a Dixit-Stiglitz aggregate of intermediate goods:

$$y_t = \left[ \int_0^1 y_t(j)(\varepsilon-1)/\varepsilon dj \right]^{\varepsilon/(\varepsilon-1)}$$ (24)

where $-\varepsilon$ is the price elasticity of demand for $y_t(j)$.

The final goods producing firm maximizes profits:

$$P_t y_t - \int_0^1 P_t(j)y_t(j)dj$$ (25)

subject to (24). This gives the following demand function for the each intermediate good:

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} y_t$$ (26)

Since the final goods producing firm is perfectly competitive it earns zero profits. Thus, combining (26) and (25), yields the price index:

$$P_t = \left[ \int_0^1 P_t(j)^{1-\varepsilon}dj \right]^{1/(1-\varepsilon)}$$
4.2.4 Intermediate Goods Producing Firm

The intermediate good producing firm is monopolistically competitive and purchases a quantity of the entrepreneurial good \( x_t(j) \) in a perfectly competitive market at price \( p_t \) from the entrepreneur and combine technology, \( a_t \) to produce \( y_t(j) \).

Under cost minimization, the firm would thus choose \( x_t(j) \) to minimize:

\[
p_t x_t(j)
\]

subject to:

\[
y_t(j) = a_t x_t(j)
\]

The first order condition is given by:

\[
p_t = z_t a_t
\]

where \( z_t \) is the Lagrangian multiplier. Substituting this into (27) yields:

\[
z_t y_t(j)
\]

where the Lagrangian multiplier can now be interpreted as the real marginal cost.

The intermediate goods producing firm is monopolistically competitive and chooses its price. However, there is a cost of adjusting the price when the change differs from the steady state inflation rate. Following Rotemberg [1982], the quadratic cost of nominal price adjustment and expressed in terms of final output:

\[
\frac{\varphi_p}{2} \left( \frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right)^2 y_t
\]

where \( \varphi_p \) measures the size of the price adjustment cost. Higher values of \( \varphi_p \) indicate greater price stickiness.

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54 Where \( x_t = \int_0^1 x_t(j) \, dj \)
The intermediate goods firm seeks to maximize the real present discounted marginal utility value of the dividend that it pays out from its profits:

$$E_t \left[ \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \frac{D_{t+i}(j)}{P_{t+j}} \right]$$

where the real value of the dividend is given by:

$$\frac{D_t(j)}{P_t} = \frac{P_t(j)}{P_t} y_t(j) - z_t y_t(j) - \frac{\varphi_p}{2} \left( \frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right)^2 y_t$$

Using this definition of the value of the dividend as well as the demand for $y_t$ given by (26), the intermediate goods producing firm chooses its price $P_t(j)$ to solve the following unconstrained maximization model:

$$E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \left[ \left( \frac{P_{t+i}(j)}{P_{t+i}} \right)^{1-\varepsilon} y_{t+i} - z_{t+i} \left( \frac{P_{t+i}(j)}{P_{t+i}} \right)^{-\varepsilon} y_{t+i} - \frac{\varphi_p}{2} \left( \frac{P_{t+i}(j)}{\pi P_{t+i-1}(j)} - 1 \right)^2 y_{t+i} \right]$$

Using the fact that, in equilibrium, $z_t$ and $y_t$ are the same for all intermediate goods firms, $P_t(j) = P_t$. Defining $\pi_t = P_t/P_{t-1}$, and multiplying the first-order condition from the unconstrained maximization problem by $p_t/y_t$, one can derive the marginal cost version of the New Keynesian Phillips curve:

$$(1 - \varepsilon)\lambda_t + \varepsilon \lambda_t z_t - \varphi_p \left( \frac{\pi_t}{\pi} - 1 \right) \left( \frac{\pi_t}{\pi} \right) + \beta \varphi_p E_t \left[ \lambda_{t+1} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \left( \frac{\pi_{t+1}}{\pi} \right) \left( \frac{y_{t+1}}{y_t} \right) \right] = 0 \quad (31)$$

The firm is monopolistically competitive and, as such, earns a profit. It uses the profit to pay a dividend to shareholders given, in real terms, by:

$$d_t = a_t x_t (1 - z_t) \quad (32)$$

Technology, $a_t$, used by the intermediate goods firm is exogenous and is expressed in
log-deviations by:

\[ \hat{a}_t = \rho_a \hat{a}_{t-1} + \hat{\nu}_t^a \]

where \( E(\nu^a) = 0 \) and \( SD(\nu_a) = \sigma_{\nu_a} \).

### 4.2.5 Monetary Policy

The central bank conducts monetary policy according to a Taylor rule. This is expressed in log-deviations as:

\[ \hat{R}_t = \phi_\pi \hat{\pi}_t + \phi_{\tilde{y}} \hat{\tilde{y}} + \hat{\epsilon}_t^R \]  \hspace{1cm} (33)

where \( R \) is the nominal interest rate, \( \pi \) is the inflation rate, \( \tilde{y} \) is the output gap, and \( \epsilon_t^R \) is the monetary shock.

### 4.2.6 Closing the Model

In equilibrium, \( y_t(j) = y_t, \ P_t(j) = P_t, \ x_t(j) = x_t \).

The goods market equilibrium is given by:

\[ y_t = c_t + c_t^e \]

The change in real balances is defined as:

\[ \Delta m_t = \frac{m_t}{m_{t-1}} \]

Finally, the output gap (expressed in log-deviations) is defined as:

\[ \hat{\tilde{y}}_t = \hat{y}_t - \hat{y}_t^n \]

where

\[ \hat{y}_t^n = \left( \frac{1 + \theta}{\sigma + \theta} \right) \hat{a}_t \]
Using the fact that \( p_t = z_t a_t \), \( y_t = a_t x_t \), and substituting equation (10) into the remaining household first-order conditions to eliminate the Lagrangian multiplier, the four equation above and equations (11) - (15), (16), (18) - (20), (22), (23), (31) - (33) are sufficient to solve for \( c, L, w, u, r, \pi, q, d, \overline{c}, e, z, \phi, m, \Delta m, \tilde{y}, \) and \( y^n \).

### 4.2.7 A Note on Simplifying the Model

The present model nests the baseline New Keynesian model as a special case and facilitates a direct comparison with that framework. Under the assumption that \( \alpha = 0 \), the L-type of labor is not used in production and the collateral constraint is of no significance. It can be shown that the model reduces to the baseline New Keynesian framework with the notable exception that the money demand function remains forward-looking. However, setting the parameter \( a_m = 0 \) reduces the money demand function to the standard, static model posited in the baseline New Keynesian model.\(^{55}\) By imposing one of these assumptions at a time, one can contrast the dynamics of each extension to the model.

### 4.3 Simulation and Results

#### 4.3.1 Calibration

The model is calibrated as follows. The parameters related to the household problem are either chosen to be in line with the literature or are implied by the estimation of a money demand function consistent with equation (8) above. The discount factor \( \beta \) is set to 0.99, which is consistent throughout the business cycle literature. Quarterly estimates for the demand for the real MSI M2 using a cointegrated VAR suggest that the income/consumption elasticity is equal to 0.64 and the interest elasticity is -0.38.\(^{56}\) As a result, \( \epsilon_m \) is set to 2.5 and \( \sigma \) is set to equal 1.6 in order to remain roughly consistent with equation (8). In addition,

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\(^{55}\)For example, as in Galí (2008: 43).

\(^{56}\)MSI M2 is the monetary services index counterpart to simple sum M2. This aggregate is used because it is consistent with economic, index number, and aggregation theory and is shown in Chapter 3, along with MSI MZM, to have a statistically significant impact on the output gap. The estimates are taken from that paper. Similar results can be obtained for the real monetary base.
\( \mu_1 \) is estimated to be around 0.95, which is consistent with the quarterly estimates of the reduced form models in Taylor [1993].\(^{57}\) Consistent with Nelson [2002], \( a_m \) is set equal to 10, which implies that \( \mu_1 = .7 \), which is a slightly more conservative estimate. This value combined with \( \epsilon_m \) implies that the cost of portfolio adjustment, \( \varphi_m = 22 \). Finally, the inverse Frisch labor supply elasticity is set to unity. This is consistent with the estimates in Fiorito and Zanella [2008] and the calibration of Gali [2008].

The parameters for the sticky price firm are chosen to be consistent with the literature. The sticky price literature assumes that the steady state mark up is between 10% and 40%. Since the markup can be expressed as the inverse of marginal cost, \( \epsilon \) is set equal to 10, which implies a steady state markup of 11%. In addition, the cost of adjustment parameter for price changes, \( \varphi_p \), is set to 173.08, which implies that the coefficient on marginal cost in the Phillips curve equals 0.05. Also, as shown in Keen and Wang [2005], the price adjustment parameter and the steady state markup are equivalent to a Calvo-type [1983] specification in which prices are adjusted approximately every 5 quarters.

The calibrated parameters for the entrepreneur are \( \alpha \) and \( b \) as well as the steady state values of \( \phi \) and \( e \). Recalling that the entrepreneur’s production function is analogous to a Cobb-Douglas aggregation of collateral constrained entrepreneurs and their unconstrained counterparts, \( \alpha \) is chosen to reflect the fraction of small firms likely to be collateral constrained. Following Carlstrom et al. [2010], \( \alpha \) is set to 0.5, which is the fraction of employment in firms of 500 employees or less. Also, the isomorphism of the collateral constraint to a costly state verification model implies that \( b = 0.2 \) and that \( b\phi_{ss} = 0.026 \).\(^{58}\) Finally, since the supply of shares is normalized to unity in equilibrium, \( e_{ss} \) is the fraction of shares held by the entrepreneur in the steady state. This value is set to 0.04 such that the steady state entrepreneurial consumption share of output is 0.01.\(^{59}\) In the present model, agency costs

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\(^{57}\)Taylor’s estimates are for real M1. The estimates of Anderson and Rasche [2001] for the demand for the real monetary base suggest \( \mu_1 = 0.82 \). However, that result is for annual data. A more conservative estimate is chosen because the cointegrated VAR is estimated with contemporaneous data rather than the expected values implied by the theory.

\(^{58}\)For details, see the appendix to Carlstrom et al. [2010].

\(^{59}\)For details, see Appendix 2.
affect output through entrepreneurial consumption. This value of $e_{ss}$ is therefore chosen to ensure that the entrepreneurial consumption share of output is small and, as a result, that additional output effects are not simply assumed to be large.\footnote{The results presented below are not sensitive to the calibration. Sensitivity analysis is contained in Appendix 3.}

The parameters of the monetary policy rule are set to be consistent with the Taylor rule ($\phi_{\pi} = 1.5$ and $\phi_{\gamma} = 0.5$). The monetary policy shock is assumed to follow an AR(1) process with a coefficient of 0.4 and an innovation standard deviation of 0.002, which is consistent with McCallum [2008] and McCallum and Nelson [1999]. The technology shock is also assumed to be an AR(1) process with a coefficient of 0.95 and an innovation standard deviation of 0.007, which is consistent with the real business cycle literature.

### 4.3.2 Evaluation

Perhaps as important as the articulation of the model is the method of evaluation. Traditionally, monetary models are evaluated by the quantitative and qualitative features of impulse response functions. In contrast, this paper takes a different approach.\footnote{Impulse response functions can be found in Appendix 3.} First, following McCallum [2001b], this paper evaluates the model by the comparison of the second moments and autocorrelations of the model with those found in the data. This type of analysis is especially important for examining the monetary transmission mechanism as it captures the effects of systematic monetary policy and not simply the effects of unanticipated shocks. This is important, as McCallum [2001b] notes, because systematic policy would seem to be more relevant for analyzing the monetary transmission mechanism as the unsystematic component explains only a very small fraction of the movement in the monetary policy instrument.\footnote{This evaluation procedure is by no means new to the literature. For example, the use of second moments can be found in Rotemberg and Woodford [1997] and has long been prevalent in real business cycle research as well. In addition, Fuhrer and Moore [1995], Fuhrer [2000], and Estrella and Fuhrer [2002] employ vector autocorrelations to evaluate the model.}

As a second method of analysis, IS-type equations are estimated from simulated data sets.
Table 14: Standard Deviations (in %)

<table>
<thead>
<tr>
<th></th>
<th>Output Gap</th>
<th>Inflation</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1954:3 - 2009:4</td>
<td>2.46</td>
<td>2.22</td>
<td>3.36</td>
</tr>
<tr>
<td>1979:4 - 2009:4</td>
<td>2.30</td>
<td>1.71</td>
<td>3.81</td>
</tr>
<tr>
<td>Agency Cost Model</td>
<td>2.52</td>
<td>2.29</td>
<td>2.18</td>
</tr>
<tr>
<td>New Keynesian Model</td>
<td>0.14</td>
<td>0.18</td>
<td>0.32</td>
</tr>
</tbody>
</table>

generated by the model. These estimates are carried out because the traditional monetarist view of the transmission mechanism predicts that real money balances are an important information variable for predicting movements in real output independent of the policy interest rate or some real balance effect. These estimates therefore provide a direct examination of whether real balances are important for the monetary transmission mechanism without assuming that they enter the structural IS equation in the model.

The first method of evaluating the systematic effects of monetary policy adjustment in the model is to compare the second moments in the data to those predicted by the model. Table 14 shows the standard deviations of the output gap, inflation, and the federal funds rate for the period 1954:3 - 2009:4 and the sub-period 1979:4 - 2009:4. The latter period is chosen to coincide with the beginning of Paul Volcker’s chairmanship at the Federal Reserve. The output gap is measured by the percentage deviation of real GDP from the Congressional Budget Office’s estimate of potential GDP, inflation is measured as the annual percentage change in the GDP deflator, and the interest rate is measured by the quarterly average of the federal funds rate at an annual percent.

The third and fourth rows in Table 14 list the corresponding standard deviations for the agency cost model employed in this paper and the baseline New Keynesian model, respectively.\textsuperscript{63} As shown, the agency cost model is able to reproduce the standard deviations observed for the whole sample quite well. For the sub-period, the model’s simulation is less favorable, but only moderately so. In contrast, the standard deviations in the New Keynesian model.

\textsuperscript{63}It is important to note that the existence of adjustment costs for real money balances does not have a bearing on these results because money is not a state variable in the analysis.
Table 15: First- and Second-order Autocorrelations

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Output Gap</th>
<th>Inflation</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1954:3 - 2009:4</td>
<td>0.92</td>
<td>0.97</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>0.79</td>
<td>0.93</td>
<td>0.88</td>
</tr>
<tr>
<td>1979:4 - 2009:4</td>
<td>0.93</td>
<td>0.84</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>0.81</td>
<td>0.66</td>
<td>0.87</td>
</tr>
<tr>
<td>Agency Cost Model</td>
<td>0.93</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>0.87</td>
<td>0.90</td>
<td>0.91</td>
</tr>
<tr>
<td>New Keynesian Model</td>
<td>0.52</td>
<td>0.94</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>0.32</td>
<td>0.88</td>
<td>0.81</td>
</tr>
</tbody>
</table>

The agency cost model therefore performs unequivocally better than the New Keynesian model in this regard.

The second method of evaluating the systematic component of monetary policy is to consider the behavior of the autocorrelation functions for each of the same three variables above. Table 15 lists the first- and second-order autocorrelations and Figure 16 plots the autocorrelation functions of the output gap, inflation, and the interest rate, respectively, from the entire sample as well as for each model.

As shown in Table 15, each model is able to replicate the autocorrelations of inflation and the short term interest rate quite well. The major difference between the two models is in regards to the behavior of the output gap. The agency cost model is able to generate first- and second-order autocorrelations consistent with the data. However, the autocorrelations generated from the New Keynesian model are substantially smaller than those observed in

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64 McCallum adds a preference shock to the IS equation. This increases the variability of the variables, but they are still much lower than the data for a variety of monetary policy rules. Specifically, the standard deviations of output and inflation are less than unity in that model for the same monetary policy rule.

65 The ability of the baseline New Keynesian model to replicate the persistence of inflation is contrary to the findings of Nelson [1998] and Estrella and Fuhrer [2002]. Nelson examines a variety of model specifications. Estrella and Fuhrer examine the baseline New Keynesian model as outlined and estimated in McCallum and Nelson [1999]. That model only differs from the model above in that it includes an IS shock of the form: \( ε_t^I = 0.3ε_t^I + v_t \) where \( E_tv_t = 0 \) and \( SD(v_t) = 0.01 \). Modifying the model above to include this specification results in a significant decline in the persistence of inflation, the details of which can be found in Appendix 3.
the data.

Figure 16: Autocorrelation Functions

Figure 16 plots the autocorrelation functions for the New Keynesian model, the agency cost model, and the data. Standard error bands for the autocorrelation functions in the data are shown by the dotted lines. As the results shown in Table 15 suggest, each model is able to replicate the autocorrelations of inflation and the interest rate quite well. Again, the difference in the performance of the models is illustrated in the behavior of the output gap. The New Keynesian model performs poorly in this regard as the entire autocorrelation function is outside the standard error band of that observed in the data. In contrast, the agency cost model performs better along this dimension as the autocorrelation function remains within the standard error bands through the third-order autocorrelation and is
more persistent than the data thereafter. Thus, the baseline New Keynesian model is unable to capture the persistence of the output gap evident in the data. When the model is amended to include agency costs, however, it is capable of capturing the persistence of the output gap reasonably well. Again, this suggests that the agency cost model is better able to replicate the empirical properties observed in the data.

The final method of analysis is to estimate a backward-looking IS-type equation using data generated from simulations of the model to determine the effects of monetary policy on aggregate demand. This equation is of the form:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 R_{t-1} + \beta_3 \Delta m_{t-1} + \epsilon_t$$  \hspace{1cm} (34)

where $y$ is output, $R$ is the nominal interest rate, and $\Delta m$ is the change in real money balances. Given the fact that the data are generated by a model in which real money balances are absent from the structural IS equation, these estimates represent a direct test of the monetarist transmission mechanism in which movements in real money balances reflect the substitution effects as a result of the relative price adjustments that follow a monetary shock. In addition, since the nominal interest rate is solely determined by monetary policy, this model is able to determine whether real balances contain any information not communicated by movements in the central bank’s policy instrument. It may seem strange to use the nominal interest rate rather than the real rate as it is the latter that enters the structural IS equation. However, the point of estimation is to determine to role of real balances in the monetary transmission mechanism. In a model with sticky prices and in which monetary policy satisfies the Taylor principle, inflation expectations will be anchored and movements in the nominal interest rate will cause corresponding changes in the real rate of interest.$^{66,67}$

$^{66}$In fact, the coefficient estimates obtained for the U.S. data below are quite similar in magnitude to the estimates in Chapter 3, in which the real interest rate specification is used.

$^{67}$For the same reason, the nominal federal funds rate is frequently used in the VAR literature. In addition, given the fact that the baseline model contains two exogenous shocks – technology and monetary – and the standard deviation of the technology shock is two and one-half times that of the monetary shock, movements...
To estimate equation (34), each model is simulated 100 times to generate a time series that spans 200 quarters. The coefficient estimates and t-statistics that are reported are the averages across simulations. The results for each model and the corresponding money demand specification are shown in Table 16 alongside results estimated from U.S. data.

The results of estimating equation (34) using U.S. data are obtained using linearly detrended real gross domestic product as the measure of output, the interest rate is measured by the federal funds rate, and the change in real balances is measured by the quarterly change in MSI M2 for the period 1979:4 - 2005:4. The time period is chosen because it represents a time in which monetary policy has satisfied the Taylor principle. These estimates are calculated using ordinary least squares with Newey-West standard errors as initial estimation indicated serial correlation.

in the real interest rate are likely to be dominated by shocks to technology. As such, a broader specification of the exogenous structure would be necessary to consider changes in the real rate. This, however, would represent a departure from the baseline New Keynesian model and is therefore not carried out presently. An example of this type of analysis in a broader model can be found in Nelson [2002], who identifies a positive and significant effect of changes in real balances on aggregate demand controlling for the real rate.

68 The data was obtained through the Federal Reserve Bank of St. Louis’s FRED database.

69 See for example Clarida et al. [2000] and Taylor [1999]. The end period of 2005 is chosen because it is the most recent year in which monetary services index data is available.
Table 16: IS Equation Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>New Keynesian Model</th>
<th>Agency Cost Model</th>
<th>Data – MSI M2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y(-1)</td>
<td>1.06***</td>
<td>1.06***</td>
<td>1.07***</td>
</tr>
<tr>
<td></td>
<td>(12.01)</td>
<td>(12.09)</td>
<td>(12.14)</td>
</tr>
<tr>
<td>R(-1)</td>
<td>0.19*</td>
<td>0.19*</td>
<td>0.21*</td>
</tr>
<tr>
<td></td>
<td>(1.72)</td>
<td>(1.71)</td>
<td>(1.85)</td>
</tr>
<tr>
<td>Δm(-1)</td>
<td>0.03</td>
<td>–</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>–</td>
<td>(0.26)</td>
</tr>
</tbody>
</table>

**t-stats** are in parentheses. Sig.: *** 1% ** 5% * 10%
The coefficient estimates from U.S. data show that the federal funds rate has a negative and significant impact on de-trended output. When the change in real MSI M2 is included in the analysis, real balances are found to have a positive and statistically significant impact on de-trended output. In this latter specification, the coefficient on the federal funds rate remains negative, but is no longer statistically significant. The existence of a positive and significant impact of real balances on de-trended output provides evidence for the monetarist transmission mechanism.

The agency cost model does a reasonable job of reproducing these results. For example, for the specification that excludes real money balances, the coefficient on the policy interest rate is statistically significant and close in magnitude to that estimated in U.S. data. What’s more, the results show that real money balances exert a positive and statistically significant impact on output when the model employs a forward-looking money demand specification. Although the magnitude of the effect is larger in the model than in U.S. data, this latter result is important because it provides evidence for the monetarist transmission mechanism similarly identified in the data, but also suggests that the specification of money demand is important to the conclusions generated from the model. Outright exclusions of money based on the static money demand specification thus appear misplaced.

In contrast to the agency cost model, the New Keynesian model predicts that increases in the nominal interest rate have a positive and statistically significant impact on de-trended output regardless of whether adjustment costs are present. What’s more, the coefficient on the change in real money balances is not statistically significant for either money demand specification. Also, it is important to note that the estimation results from the New Keynesian model are not dependent on inclusion of real balances in estimation. The coefficient on the policy interest rate remains positive and significant when real balances are excluded. This is contrary to both the predictions of the model and the estimation results from U.S. data.

This is consistent with the findings of Chapter 3 where the real interest rate specification is used.
These results are important because a key prediction of the New Keynesian model is that the short term interest rate exclusively provides information about monetary policy. Under the assumption of sticky prices, changes in the nominal interest rate should lead to corresponding changes in the real interest rate and therefore have an impact on aggregate demand as implied by the structural, dynamic IS equation. Thus, one would expect that the short term interest rate controlled by the monetary authority to be negatively related to output. However, using data generated from the baseline New Keynesian model, estimation of the backward-looking IS-type equation not only does not replicate the results evident in the data, but also fails to generate predictions consistent with the model itself. Meanwhile, by extending the model to include agency costs and a richer specification of the money demand function, one can replicate the results evident in the data reasonably well and generate predictions consistent with economic theory.

4.4 Conclusion

Over the last several years, the baseline New Keynesian model has been widely used to examine monetary policy either in and of itself or at the core of a larger model specification. This model has gained popularity largely as a result of the fact that it represents a micro-founded, optimization-based, dynamic representation of the familiar IS-LM analysis. As such the model has desirable and familiar properties.

Given the frequent use of the baseline New Keynesian framework for monetary policy analysis, it is important to examine the properties of the model in order to determine the model’s usefulness for completing such a task. Specifically, the New Keynesian framework makes important assumptions about the transmission of monetary shocks. In particular, this framework implies that the short-term interest rate set by the central bank sufficiently captures the monetary transmission process. This view is at odds with the literature on the credit channel as well as that of the monetarist literature.

In an effort to examine the assumptions about the monetary transmission mechanism
embedded in the New Keynesian framework, this chapter extended the model to include agency costs and a richer specification of the money demand function to compare the empirical properties of each model to those observed in the data. The results show that the New Keynesian model performs very poorly in capturing the second moments of the output gap, inflation, and the interest rate. In addition, while the model does a good job replicating the first- and second-order autocorrelations for inflation and the interest rate, the same cannot be said for the autocorrelation properties of the output gap. In contrast, when extended to include agency costs, the model is able to capture the second moments and the first- and second-order autocorrelations observed in the data quite well.

What’s more, the analysis of aggregate demand suggests that the New Keynesian model is poorly specified. Estimates of a backward-looking IS-type equation using U.S. data show that detrended output has an autoregressive component, is negatively related to the central bank’s short term interest rate target (albeit insignificantly when real balances are included), and is positively related to real money balances. In contrast, estimates of the same equation using data generated by the New Keynesian framework suggest that output is positively related to the central bank’s interest rate. Estimates for the agency cost specification are sensitive to the characteristics of the money demand functions. For example, when money demand is static, as is standard in most New Keynesian analysis, the model is able to replicate the autoregressive component of detrended output and the negative effect of movements in the interest rate instrument. However, real money balances are not found to be statistically significant. Nonetheless, when money demand is forward-looking, the agency cost model is able to capture the positive and statistically significant relationship between output and real money balances.

Overall, the New Keynesian model performs quite poorly in replicating the empirical properties observed in the data. By extending the model to include agency costs, as emphasized by the credit channel literature, and a richer specification of money demand, long emphasized by monetarists, the model employed in this chapter is much better able to repli-
cate these empirical properties. These results would seem to suggest that the failures of the New Keynesian framework are, at least in part, the result of strong assumptions regarding the monetary transmission process. Given the wide dissemination of this framework for monetary policy analysis, it would seem prudent to reconsider these assumptions in future research.
Chapter 5 Summary and Extensions

The basic New Keynesian model is the predominant framework for monetary policy analysis in the literature. The model is a dynamic, optimization-based framework analogous to the traditional IS-LM model. The model consists of three equations: (1) a dynamic IS equation, (2) the New Keynesian Phillips curve, and (3) an interest rate rule for monetary policy. As a result, the model makes a strong assumption about the monetary transmission mechanism. Specifically, the model assumes that the transmission of monetary policy is solely captured by the behavior of the short term interest rate.

In order to assess the validity of the assumption about the monetary transmission process, this dissertation proposes two methods of analysis. First, the failure to identify a meaningful role for monetary aggregates in the New Keynesian framework is built upon empirical results that suggest the money demand is unstable as well as results which suggest that monetary aggregates do not enter the IS equation. However, it is possible that these results are incorrect due to the fact that they are based on the use flawed, simple sum measures of money. As a result, the empirical analysis in Chapter 3 consists of a re-examination of these previous results using the monetary services indexes available through the Federal Reserve Bank of St. Louis, which are consistent with economic, aggregation, and index number theory. The results suggest that the conclusions of earlier studies were biased due to mismeasurement.

The second method of analysis, presented in Chapter 4, extends the New Keynesian model to include a richer specification of the money demand function and the inclusion of agency costs to examine the implications for the monetary transmission mechanism. The New Keynesian model remains nested within the new framework. The implications of each addition to the basic model are contrasted with the New Keynesian model by computing autocorrelation functions and standard deviations of the three core variables of the basic framework as well as by estimating IS-type equations. The results suggest that the New Keynesian model does a poor job of replicating empirical evidence.

Overall, this dissertation presents evidence that raises doubts about the assumptions
of the basic New Keynesian model. Nonetheless, there remain important areas for future research. First, in the model presented above, the demand for money is characterized by a demand for the monetary aggregate itself. However, it should be noted that a key insight of the derivation of the monetary services indexes is that there is a unique demand function for each component of the monetary aggregate. An alternative and important extension of the New Keynesian model would be to incorporate the demand for each asset within the monetary aggregate in order to compare the results with those presented in Chapter 3. Research using such extensions have only recently begun (Belongia and Ireland [2010]) and focus on different metrics of performance than those presented in this dissertation.

A second area for future research would be to identify a solid theoretical foundation to explain why the monetary services indexes perform better than the simple sum counterparts beyond the simple explanation of mismeasurement. As noted by McCallum [1993], the failure to identify a cointegrating relationship does not necessarily imply the absence of a stable, long-run money demand function. For example, since the demand for money is based on the idea that money is necessary to facilitate transactions, it is likely that the disturbance in the money demand function follows a random walk due to changes in transactions technology. The fact that one is able to identify a stable cointegrating relationship for the monetary services indexes across samples begs the question as to why McCallum’s criticism seemingly does not apply to these indexes. One possible reason is the transactions technology is likely to alter the demand for certain assets within a given aggregate. Since the monetary services index captures the changes in the yield and the price of each asset, it is possible that these indexes are able to circumvent the criticism levied by McCallum by implicitly capturing of these substitution effects. Nonetheless, a more solid theoretical foundation is necessary.
APPENDIX 1: UNIT ROOT AND COINTEGRATION TESTS

Table A1: Unit Root Tests

<table>
<thead>
<tr>
<th>Variable</th>
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<tbody>
<tr>
<td>Real MSI M1</td>
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<tr>
<td>Real MSI M2</td>
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</tr>
<tr>
<td>Real MSI MZM</td>
<td>-0.16</td>
</tr>
<tr>
<td>Price Dual M1</td>
<td>-1.88</td>
</tr>
<tr>
<td>Price Dual M2</td>
<td>-1.79</td>
</tr>
<tr>
<td>Price Dual MZM</td>
<td>-1.79</td>
</tr>
<tr>
<td>Real Final Sales</td>
<td>-1.54</td>
</tr>
</tbody>
</table>

Sig. level – 1%: -3.467, 5%: -2.876, 10% -2.575

Table A2: Trace Statistics

<table>
<thead>
<tr>
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<th>1979:4 - 2005</th>
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<td>13.99</td>
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<td></td>
<td>3.30</td>
<td>3.55</td>
</tr>
<tr>
<td>MSI M2</td>
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<tr>
<td></td>
<td>10.77</td>
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<td></td>
<td>1.31</td>
<td>5.46</td>
</tr>
<tr>
<td>MSI MZM</td>
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<td></td>
<td>2.53</td>
<td>3.30</td>
</tr>
</tbody>
</table>
Appendix 2: The Complete DSGE Model

Household

The household chooses consumption, $C_t$, nominal money balances, $M_t$, and two types of labor, $L_t$ and $U_t$, nominal bond holdings, $B_t$, and the number of shares of the intermediate goods producing firm, $e_t$, to maximize utility:  

$$U(C, L, u, m) = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\theta}}{1+\theta} - \frac{u_t^{1+\theta}}{1+\theta} + \frac{m_t^{1-\epsilon_m}}{1-\epsilon_m} - \frac{\varphi_m}{2} \left( \frac{m_t}{m_{t-1}} - 1 \right)^2 \right\}$$

subject to

$$w_t L_t + r_t u_t + (1 + R_{t-1}) b_{t-1} + m_{t-1} + e_{t-1}(q_t + d_t) = c_t + b_t + m_t + q_t e_t$$

where $\varphi_m$ is a parameter that measures the cost of portfolio adjustment, $\beta$ is the discount factor, $P_t$ is the price level, $m_t = M_t / P_t$ is real money balances, $w_t$ is the real wage, $r_t$ is the real cost of $u_t$, $q_t$ is the real price of a share of the intermediate goods firm, $d_t$ is the real dividend earned from holding a share, $b_t = B_t / P_t$, and $R$ is the nominal interest rate on bond holdings.

Constrained maximization yields the following:

$$\lambda_t = C_t^{1-\sigma} \quad \text{(A1)}$$

$$L_t^\theta = \lambda_t w_t \quad \text{(A2)}$$

$$u_t^\theta = \lambda_t r_t \quad \text{(A3)}$$

$$m_t^{1-\epsilon_m} - \varphi_m \left( \frac{m_t}{m_{t-1}} - 1 \right) \frac{1}{m_{t-1}} - \varphi_m \left( \frac{m_t}{m_{t-1}} - 1 \right) \frac{m_{t+1}}{m_t^2} + \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t \quad \text{(A4)}$$

---

71 The supply of shares is normalized to unity in equilibrium.
\[ \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} (1 + R_t) = \lambda_t \] (A5)

\[ \beta E_t \lambda_{t+1} (q_{t+1} + d_{t+1}) = \lambda_t q_t \] (A6)

Solving to eliminate the Langrangian multiplier yields two labor supply curves, a money demand function, a dynamic IS equation, and an asset demand function.

**Entrepreneurs**

The representative entrepreneur uses two types of labor, \( L \) and \( u \), to produce good \( x \) and sells the good for price \( p \). The entrepreneur earns profits from production to purchase shares of the intermediate goods producing firm and for consumption. The entrepreneur thus solves two problems – a profit-maximization problem and a utility maximization problem. This differs from the model used by Carlstrom, Fuerst, and Paustian (2010) as that model imposes the assumption that a fraction of entrepreneurs die each period and, as a result, entrepreneurial consumption is zero in equilibrium. This assumption assures that the collateral constraint is binding. Unfortunately, this limits the effects of agency costs to fluctuations in asset prices and net worth and thus fails to differentiate the dynamics of the model from a baseline New Keynesian model.\(^{72}\) This model adopts the assumption that entrepreneurs have a higher discount rate than the representative household. This ensures that the collateral constraint is binding and that entrepreneurial consumption is non-zero in equilibrium.

**Entrepreneurial Production**

The entrepreneur produces output using the following production function:

\[ x_t = L_t^\alpha u_t^{1-\alpha} \]

\(^{72}\)It does generate important differences in welfare. Since the previous authors were conducting welfare analysis for alternate monetary policy rules, the assumption in this paper would have unnecessarily complicated their analysis.
and purchases quantities of each type of labor from the household in competitive markets.

The entrepreneur thus chooses \( L_t \) and \( u_t \) to maximize profits given by:

\[
Profits = p_t x_t - r_t u_t - w_t L_t \tag{A7}
\]

where the choice of labor is made subject to the collateral constraint:

\[
w_t L_t \leq g(n w_t, p_t x_t - r_t u_t) \equiv n w_t^b (p_t x_t - r_t u_t)^{1-b} \tag{A8}
\]

Where \( n w_t = e_{t-1} (q_t + d_t) \) is entrepreneurial net worth and \((p_t x_t - r_t u_t)\) are profits obtained without the use of the L-type of labor. The collateral constraint implies that the entrepreneur backs up the wage agreement with L-type labor suppliers with his net worth and the profits from using the u-type of labor.

Maximizing \((A7)\) in which equation \((A8)\) is a binding constraint for the choice of \( L \) yields the following optimization conditions:

\[
r_t u_t = (1 - \alpha) p_t x_t \tag{A9}
\]

\[
w_t L_t = \alpha p_t x_t \left( \frac{1 + \phi_t g_2}{1 + \phi_t} \right) \tag{A10}
\]

where \( \phi_t \) is the Lagrangian multiplier and \( g_2 \) is the partial derivative of \( g \) with respect to the second argument.

Re-arranging equation \((A9)\) yields:

\[
p_t x_t - r_t u_t = \alpha p_t x_t
\]

\(^{73}\)Carlstrom, Fuerst, and Paustian show that this collateral constraint is isomorphic to the costly state verification framework used in Carlstrom and Fuerst (1997, 1998).
Plugging this and equation (A10) into the profits equation gives:

\[
\text{Profits} = \alpha p_t x_t - \alpha p_t x_t \left( \frac{1 + \phi_t g_2}{1 + \phi_t} \right) = \alpha p_t x_t \left( 1 - \frac{1 + \phi_t g_2}{1 + \phi_t} \right) = \alpha p_t x_t \left( \frac{\phi_t - \phi_t g_2}{1 + \phi_t} \right) \quad (A11)
\]

Equivalently, if the collateral constraint is binding, one can write:

\[
\text{Profits} = p_t x_t - r_t u_t - w_t L_t = \alpha p_t x_t - g
\]

Given that \( g_2 = (1 - b)nw_t^b(p_t x_t - r_t u_t)^{-b} \) and \( p_t x_t - r_t u_t = \alpha p_t x_t \), \( g \) can be written:

\[
g = \alpha p_t x_t \frac{g_2}{1 - b}
\]

Thus,

\[
\text{Profits} = \alpha p_t x_t \left( \frac{1 - b - g_2}{1 - b} \right) \quad (A12)
\]

Equations (A11) and (A12) imply:

\[
\left( \frac{\phi_t - \phi_t g_2}{1 + \phi_t} \right) = \left( \frac{1 - b - g_2}{1 - b} \right)
\]

Solving for \( g_2 \):

\[
g_2 = \left( \frac{1 - b}{1 + b\phi_t} \right)
\]

Plugging this in to equation (A10) gives:

\[
w_t L_t (1 + b\phi_t) = \alpha p_t x_t \quad (A13)
\]

The optimization conditions reveal the credit distortion. Formally, equation (A13) is isomorphic to a model in which the wage, \( w_t \), must be paid in advance of production in which \( b\phi_t \) is the real interest cost of an intratemporal loan. Thus, in comparison to a model without the credit constraint, \( b\phi_t \) represents a credit distortion on the loan. To understand this further,
re-write equation (A13):

\[(1 + b\phi_t) = \frac{\alpha p_t x_t}{w_t L_t}\]

If the collateral constraint binds, one can substitute for \(w_t L_t\):

\[(1 + b\phi_t) = \frac{\alpha p_t x_t}{n w_t^t (p_t x_t - r_t u_t)^{1-b}}\]

Equation (A9) implies that \(p_t x_t - r_t u_t = \alpha p_t x_t\). Thus,

\[b\phi_t = \left(\frac{\alpha p_t x_t}{n w_t^t}\right)^b - 1\]

where the credit distortion is now expressed in terms of the output-net worth ratio where the elasticity of the credit distortion to this ratio is given by \(b\).

Finally, the existence of the collateral constraint implies that the entrepreneur will earn a profit. Substituting the above optimization conditions into equation (A7) yields:

\[\text{Profits} = p_t x_t - r_t u_t - w_t L_t = \alpha p_t x_t - \left(\frac{\alpha p_t x_t}{1 + b\phi_t}\right) = \alpha p_t x_t \left(\frac{b\phi_t}{1 + b\phi_t}\right)\]

Entrepreneurial Consumption

The entrepreneur’s budget constraint is given by:

\[c_t^e + e_t q_t = e_{t-1}(q_t + d_t) + \text{profits} \quad (A14)\]

Using the expression for entrepreneurial profits and the fact that \(n w_t = e_{t-1}(q_t + d_t)\), we can re-write the budget constraint as:

\[c_t^e + e_t q_t = n w_t + \alpha p_t x_t \left(\frac{b\phi_t}{1 + b\phi_t}\right) \quad (A15)\]
From the optimization conditions and the collateral constraint:

$$\alpha p_t x_t (1 + b \phi_t)^{-1} = w_t L_t = n w_t^b (p_t x_t - r_t u_t)^{1-b}$$

Using the optimality condition for labor type $u$ in which $p_t x_t - r_t u_t = \alpha p_t x_t$, this can be re-written:

$$\frac{\alpha p_t x_t}{(1 + b \phi_t)} = n w_t^b (\alpha p_t x_t)^{1-b}$$

Solving for $nw_t$ yields:

$$nw_t = \alpha p_t x_t \left( \frac{1}{1 + b \phi_t} \right)^{\frac{1}{b}}$$

Substituting this into (A15) gives:

$$c_t^e + e_t q_t = \alpha p_t x_t F(\phi_t)$$  \hspace{1cm} (A16)

where:

$$F(\phi_t) = \left[ \left( \frac{b \phi_t}{1 + b \phi_t} \right) + \left( \frac{1}{1 + b \phi_t} \right)^{\frac{1}{b}} \right]$$

The entrepreneur then chooses consumption, $c^e$, and the fraction of shares, $e$, to maximize utility:\footnote{The additional discount factor $\gamma$ ensures that the collateral constraint is binding.}

$$U(c^e) = E_t \sum_{t=0}^{\infty} (\beta \gamma)^t c_t^e$$

subject to the budget constraint above. Here, $\gamma$ is an additional discount factor that ensures that the collateral constraint is binding. Constrained maximization yields the entrepreneur’s intertemporal Euler equation:

$$q_t = \beta \gamma E_t (q_{t+1} + d_{t+1})(1 + b \phi_{t+1})$$  \hspace{1cm} (A17)
Firms

Final Goods Producing Firm

The final good producing firm is perfectly competitive and produces, $y_t$. The firm purchases $y_t(j)$ units from firm $j \in [0, 1]$ at price $P_t(j)$. The final good is a Dixit-Stiglitz aggregate of intermediate goods:

$$y_t = \left[ \int_0^1 y_t(j)^{(\varepsilon-1)/\varepsilon} dj \right]^\varepsilon/(\varepsilon-1)$$  \hspace{1cm} (A18)

where $-\varepsilon$ is the price elasticity of demand for $y_t(j)$.

The final goods producing firm maximizes profits:

$$P_t y_t - \int_0^1 P_t(j)y_t(j) dj$$  \hspace{1cm} (A19)

subject to (A18). This gives the following demand function for the each intermediate good:

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} y_t$$  \hspace{1cm} (A20)

Since the final goods producing firm is perfectly competitive it earns zero profits. Thus, combining (A20) and (A19), yields the price index:

$$P_t = \left[ \int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)}$$

Intermediate Goods Producing Firm

The intermediate good producing firm is monopolistically competitive and purchases a quantity of the entrepreneurial good $x_t(j)$ in a perfectly competitive market at price $p_t$ from the entrepreneur and combine technology, $a_t$ to produce $y_t(j)$.\textsuperscript{75} Under cost minimization,

\textsuperscript{75}Where $x_t = \int_0^1 x_t(j) dj$
the firm would thus choose $x_t(j)$ to minimize:

$$p_t x_t(j) \quad (A21)$$

subject to:

$$y_t(j) = a_t x_t(j) \quad (A22)$$

The first order condition is given by:

$$p_t = z_t a_t \quad (A23)$$

where $z_t$ is the Lagrangian multiplier. Substituting this into $(A21)$ yields:

$$z_t y_t(j)$$

where the Lagrangian multiplier can now be interpreted as the real marginal cost.

The intermediate goods producing firm is monopolistically competitive and chooses its price. However, there is a cost of adjusting the price when the change differs from the steady state inflation rate. Formally, this is based on Rotemberg’s (1982) quadratic cost of nominal price adjustment and expressed in terms of final output:

$$\frac{\varp}{2} \left( \frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right)^2 y_t$$

where $\varp$ measures the size of the price adjustment cost. Higher values of $\varp$ indicate greater price stickiness.

The intermediate goods firm seeks to maximize the real present discounted marginal utility value of the dividend that it pays out from its profits:

$$E_t \left[ \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \frac{D_{t+i}(j)}{P_{t+j}} \right]$$
where the real value of the dividend is given by:

\[ \frac{D_t(j)}{P_t} = \frac{P_t(j)}{P_t} y_t(j) - z_t y_t(j) - \frac{\varphi_p}{2} \left( \frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right)^2 y_t \]

Using this definition of the value of the dividend as well as the demand for \( y_t \) given by (A20), the intermediate goods producing firm chooses its price \( P_t(j) \) to solve the following unconstrained maximization model:

\[ E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \left[ \left( \frac{P_{t+i}(j)}{P_{t+i}} \right)^{1-\varepsilon} y_{t+i} - z_{t+i} \left( \frac{P_{t+i}(j)}{P_{t+i}} \right)^{-\varepsilon} y_{t+i} - \frac{\varphi_p}{2} \left( \frac{P_{t+i}(j)}{\pi P_{t+i-1}(j)} - 1 \right)^2 y_{t+i} \right] = 0 \] (A24)

Maximization yields the following first-order condition:

\[ (1 - \varepsilon) \lambda_t \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} \left( \frac{y_t}{P_t} \right) + \varepsilon \lambda_t z_t \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon-1} \left( \frac{y_t}{P_t} \right) - \varphi_p \lambda_t \left( \frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right) \left( \frac{y_t}{\pi P_{t-1}(j)} \right) + \beta \varphi_p E_t \left[ \lambda_{t+1} \left( \frac{P_{t+1}(j)}{\pi P_{t+1}(j)} - 1 \right) \left( \frac{P_{t+1}(j)}{\pi P_{t+1}(j)} \right) \left( \frac{y_{t+1}}{y_t} \right) \right] = 0 \]

In equilibrium, \( z_t \) and \( y_t \) are the same for all intermediate goods firms and \( P_t(j) = P_t \). Thus, defining \( \pi_t = P_t / P_{t-1} \), and multiplying the first-order condition by \( p_t / y_t \), it can be re-written as:

\[ (1 - \varepsilon) \lambda_t + \varepsilon \lambda_t z_t - \varphi_p \left( \frac{\pi_t}{\pi} - 1 \right) \left( \frac{\pi_t}{\pi} \right) + \beta \varphi_p E_t \left[ \lambda_{t+1} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \left( \frac{\pi_{t+1}}{\pi} \right) \left( \frac{y_{t+1}}{y_t} \right) \right] = 0 \] (A25)

Equation (A25) is the marginal cost version of the New Keynesian Phillips curve.

The firm is monopolistically competitive and, as such, earns a profit. It uses the profit to pay a dividend to shareholders given, in real terms, by:

\[ d_t = a_t x_t (1 - z_t) \] (A26)

Technology, \( a_t \), used by the intermediate goods firm is exogenous and is expressed in log-deviations by:

\[ \hat{a}_t = \rho_a \hat{a}_{t-1} + \hat{\nu}_t^a \]
where $\nu^a$ is the technology shock.

**Natural Rate of Output**

The natural rate of output is defined to be the level of output that would exist in the absence of price and credit frictions. As a result, in log deviations this is defined as:

$$\hat{y}^n_t = \left(\frac{1 + \theta}{\sigma + \theta}\right) \hat{a}_t$$

**Monetary Policy**

The central bank conducts monetary policy according to a Taylor rule. This is expressed in log-deviations as:

$$\hat{R}_t = \phi_\pi \hat{\pi}_t + \phi_{\bar{y}} \hat{\bar{y}} + \epsilon^R_t$$

(A27)

where $R$ is the nominal interest rate, $\pi$ is the inflation rate, $\bar{y}$ is the output gap, and $\epsilon^R_t$ is the monetary shock with an AR(1) coefficient of 0.4.

**Closing the Model**

In equilibrium, $y_t(j) = y_t$, $P_t(j) = P_t$, $x_t(j) = x_t$.

The model is then closed by the goods market equilibrium:

$$y_t = c_t + c_t^e$$

---

76The derivation is shown in section 4.4 below.
Steady State

Output Shares

From the entrepreneur’s budget constraint,

\[ c^e = \alpha px F - eq \]

using the steady state condition given by equation (A23), this can be re-written:

\[ \frac{c^e}{y} = \alpha z F - e \frac{q}{y} = \alpha z F - e \frac{q}{x} \]

From (A6),

\[ q = \frac{\beta}{1 - \beta} d \]

Thus,

\[ \frac{c^e}{y} = \alpha z F - e \frac{\beta}{1 - \beta} \frac{d}{x} \]

From (A26),

\[ \frac{d}{x} = 1 - z \]

and from (A25),

\[ z = \frac{\varepsilon - 1}{\varepsilon} \]

Thus,

\[ \frac{c^e}{y} = \alpha \frac{\varepsilon - 1}{\varepsilon} F - e \frac{\beta}{1 - \beta} \frac{1}{\varepsilon} \]

Finally, from the goods market clearing equation:

\[ \frac{c}{y} = 1 - \frac{c^e}{y} \]
The Entrepreneur’s Discount Factor

The additional discount factor can be found by using equation (A6) and equation (A17):

\[ q = \beta (q + d) \]

\[ q = \beta \gamma (q + d)(1 + b\phi_{ss}) \]

Setting these equal and solving for \( \gamma \):

\[ \frac{1}{(1 + b\phi_{ss})} = \gamma \]

The parameterization of the model suggests that \( \gamma = .974 \). This is consistent with that of Carlstrom and Fuerst (1997).

Log-linearized System of Equations

The Household

Log-linearization of equation (A1) yields:

\[ \hat{\lambda} = -\sigma \hat{c}_t \]

Log-linearizing (A2), (A3), (A4), (A5), and (A6) and substituting the above condition yields:

\[ \sigma \hat{c}_t + \theta \hat{L}_t - \hat{w}_t = 0 \quad (A28) \]

\[ \sigma \hat{c}_t + \theta \hat{u}_t - \hat{r}_t = 0 \quad (A29) \]

\[ \hat{m}_t - \frac{\sigma}{\epsilon_m} \hat{c}_t + \frac{1}{\epsilon_m} R_t + a_m \Delta \hat{m}_t = \beta a_m E_t \Delta \hat{m}_{t+1} \quad (A30) \]

\[ \hat{R}_t + \sigma \hat{c}_t = \sigma E_t \hat{c}_{t+1} + E_t \hat{n}_{t+1} \quad (A31) \]
\[ \hat{q}_t - \sigma \hat{c}_t = \beta E_t \hat{q}_{t+1} + (1 - \beta) E_t \hat{d}_{t+1} - \sigma E_t \hat{c}_{t+1} \] (A32)

**The Entrepreneur**

Using the fact that \( p_t = z_t a_t \), log-linearization of the input demand curves given by equations (A13) and (A9) yields:

\[ \hat{z}_t + \hat{y}_t - \hat{w}_t - \hat{L}_t - b \hat{\phi}_t = 0 \] (A33)

\[ \hat{z}_t + \hat{r}_t - \hat{u}_t = 0 \] (A34)

Log-linearizing the collateral constraint given by equation (A8) yields:

\[ b \hat{c}_{t-1} + b \beta \hat{q}_t + b(1 - \beta) \hat{a}_t + (1 - b) \hat{z}_t + (1 - b) \hat{y}_t - \hat{w}_t - \hat{L}_t = 0 \] (A35)

Log-linearization of the entrepreneurial budget constraint [equation (A16)]:

\[ \frac{c^e}{\alpha z y F} \hat{c}_e + \frac{eq}{\alpha z y F} \hat{e}_t + \frac{eq}{\alpha z y F} \hat{q}_t - \hat{z}_t - \hat{y}_t - (b - 1) \hat{\phi}_t = 0 \] (A36)

Log-linearization of the entrepreneurial consumption Euler equation [equation (A17)]:

\[ \hat{q}_t = \beta E_t \hat{q}_{t+1} + (1 - \beta) E_t \hat{d}_{t+1} + b \phi_{ss} E_t \hat{\phi}_{t+1} \] (A37)

**The Intermediate Goods Firm**

Log-linearization of the production function:

\[ \hat{y}_t - \hat{a}_t - \alpha \hat{L}_t - (1 - \alpha) \hat{u}_t = 0 \] (A38)
Log-linearization of the Phillips curve given in equation (A25):

\[ \hat{\pi}_t - \left( \frac{\varepsilon - 1}{\varphi_p} \right) \hat{z}_t = \beta E_t \hat{\pi}_{t+1} \]  \hspace{1cm} (A39)

Finally, the dividend in equation (A26) is given by:

\[ \hat{d}_t - \hat{y}_t + (\varepsilon - 1)\hat{z}_t = 0 \]  \hspace{1cm} (A40)

Closing the Model

An identity defining the change in real balances gives:

\[ \Delta \hat{m}_t - \hat{m}_t + \hat{m}_{t+1} = 0 \]  \hspace{1cm} (A41)

The goods market is closed by:

\[ \hat{y}_t - \left( \frac{c}{y} \right) \hat{c}_t - \left( \frac{c^e}{y} \right) \hat{c}^e_t = 0 \]  \hspace{1cm} (A42)

The output gap is given by:

\[ \hat{y}_t - \hat{y}_t + \left( \frac{1 + \theta}{\sigma + \theta} \right) \hat{a}_t = 0 \]  \hspace{1cm} (A43)

Together with the monetary policy rule given in equation (A27), equations (A28) - (A43) are sufficient to solve for \( \hat{c}_t, \hat{L}_t, \hat{w}_t, \hat{u}_t, \hat{r}_t, \hat{R}_t, \hat{\pi}_t, \hat{q}_t, \hat{\phi}_t, \hat{e}_t, \hat{\sigma}_t, \hat{\pi}_t, \hat{\phi}_t, \hat{y}_t, \hat{\phi}_t, \hat{m}_t, \) and \( \Delta \hat{m}_t. \)

The Model Solution

The model can be expressed as a singular linear difference model:

\[ AE_t Y_{t+1} = BY_t + CX_t \]

\[ X_t = \rho X_{t-1} + G_t e_t \]
where $Y_t$ is the vector of endogenous variables, $X_t$ is a vector of exogenous variables, $A$, $B$, and $C$ are matrices that contain the parameter values that correspond to the relations between the variables described in the model above, $\rho$ is the matrix that details the dynamic behavior of the exogenous variables, $e_t$ is a vector of innovations.

Using King and Watson’s (2002) system reduction algorithm, the solution to the model can be expressed in state space form as:

$$Z_t = \Pi S_t$$

where

$$S_{t+1} = MS_t + G\epsilon_t$$

where $Z_t = [Y_t, X_t]'$ and $S$ is a vector of state variables.

This framework is useful as it is straightforward for simulation, impulse response functions, and population moment calculations (as shown in King, Plosser, and Rebelo, 2001).

**The Money Demand Function**

Recall the money demand function in the model with portfolio adjustment costs:

$$\hat{m}_t - \beta a_m E_t \Delta \hat{m}_{t+1} + a_m \Delta \hat{m}_t = \frac{\sigma}{\epsilon_m} \hat{c}_t - \frac{1}{\epsilon_m} R_t$$

First, it is important to note that when portfolio costs are zero, $a_m = 0$, and the money demand curve reduces to

$$\hat{m}_t = \frac{\sigma}{\epsilon_m} \hat{c}_t - \frac{1}{\epsilon_m} R_t$$

which is the standard money demand function found in the literature.

Second, the money demand function derived from the portfolio adjustment cost specification can be re-written (henceforth suppressing the expectations operator and the hat-
symbol):

\[ m_t - \beta a_m m_{t+1} + \beta a_m m_t + a_m m_t - a_m m_{t-1} = A_t \]

where \( A \) is given by the right-hand side of the original equation. Re-arranging:

\[ m_{t+1} - \frac{(1 + \beta a_m + a_m)}{\beta a_m} m_t + \frac{1}{\beta} m_{t-1} = -(\beta a_m)^{-1} A_t \]

or

\[ \left[ 1 - \frac{(1 + \beta a_m + a_m)}{\beta a_m} L + \frac{1}{\beta} L^2 \right] m_{t+1} = -(\beta a_m)^{-1} A_t \]

where \( L \) is the lag operator.

Factoring and multiplying both sides by the lag operator \( L \) yields:

\[ (1 - \mu_1 L)(1 - \mu_2 L)m_t = -(\beta a_m)^{-1} A_{t-1} \]

where

\[ \mu_1 + \mu_2 = \frac{(1 + \beta a_m + a_m)}{\beta a_m} \]

and

\[ \mu_1 \mu_2 = \frac{1}{\beta} \]

where \( \mu_1 \) is a stable root and \( \mu_2 \) is an unstable root.

Following Sargent (1987), one can solve the model forward to eliminate the unstable root:

\[ (1 - \mu_1 L)m_t = \frac{-(\beta a_m)^{-1} A_{t-1}}{(1 - \mu_2 L)} \]

where \( (1 - \mu_2 L) \) can be re-written:

\[ (1 - \mu_2 L)^{-1} = \frac{-(\mu_2 L)^{-1}}{1 - (\mu_2)^{-1}} = -\sum_{i=0}^{\infty} (\mu_2 L)^{-(i+1)} \]
Thus, 

\[(1 - \mu_1 L) m_t = \sum_{i=0}^{\infty} (\mu_2 L)^{-(i+1)}[(\beta a_m)^{-1} A_{t-1}] \]

Substituting for A and re-arranging:

\[m_t = \mu_1 m_{t-1} + (\beta a_m)^{-1}\left[ \sum_{i=0}^{\infty} (\mu_2)^{-(i+1)}\left( \sigma c_{t+i} - \frac{1}{\epsilon_m} R_{t+i} \right) \right] \]

Thus, the money demand function utilized in this paper can be viewed as one in which the demand for real balances is a function of permanent income and the long term interest rate.

**Simplifying the Model**

The model can be simplified to the baseline New Keynesian model by setting \(\alpha = 0\). Doing so eliminates the choice of \(L\) and thus the collateral constraint. The analysis can therefore be confined to the household and the firms and exclude the entrepreneur. The elimination of the entrepreneur implies that:

\[y_t = c_t\]

Thus, substituting the above condition and subtracting \(y_t^n\) from both sides, equation (A31) can be re-written:

\[\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma}(\hat{R}_t - E_t \hat{\pi}_{t+1}) \quad (A44)\]

which is the standard IS equation used in the New Keynesian framework.

Also, the real marginal cost facing the intermediate goods producing firm can now be derived by minimizing:

\[r_t u_t(j)\]
subject to:
\[ y_t(j) = a_t u_t \]

Constrained minimization yields:
\[ z_t = \frac{r_t}{a_t} \]
or in log-deviations:
\[ \hat{z}_t = \hat{r}_t - \hat{a}_t \]

Substituting \( \hat{r}_t \) from the household problem yields:
\[ \hat{z}_t = \sigma \hat{y}_t + \theta \hat{u}_t - \hat{a}_t \]

Substituting the \( u_t \) from the production function yields:
\[ \hat{z}_t = \sigma \hat{y}_t + \theta (\hat{y}_t - \hat{a}_t) - \hat{a}_t \]  \hspace{1cm} (A45)

In the absence of price adjustment costs and agency costs, the real marginal cost, \( z_t \) is constant. Thus, in log-deviations, \( \hat{z}_t = 0 \) and \( \hat{y}_t = \hat{y}_n^t \). Re-arranging yields:
\[ \hat{y}_n^t = \left( \frac{\theta + 1}{\sigma + \theta} \right) \hat{a}_t \]

which is the definition stated above.

To further simplify the matter, however, one can reduce the model further. Specifically, in the sticky price model \( \hat{z}_t \) in equation (A45) can be re-written as:
\[ \hat{z}_t = (\sigma + \theta) \hat{y}_n^t - (\theta + 1)\hat{a}_t \]  \hspace{1cm} (A46)
Subtraction \((A46)\) from \((A45)\) yields the real marginal cost in terms of the output gap:

\[
\hat{z}_t = (\sigma + \theta)(\hat{y} - \hat{y}_t^n)
\]

where \(\hat{y}_t - \hat{y}_t^n = \hat{z}_t\)

This latter expression of real marginal cost can then be substituted in the New Keynesian Phillips curve given in equation \((A39)\) to express the NK Phillips curve in terms of the output gap:

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \tilde{\eta}
\]

where \(\kappa = \left(\frac{\varepsilon - 1}{\varphi_p}\right)(\sigma + \theta)\).

The Phillips curve above together with equation \((A44)\) and the monetary rule specified in equation \((A27)\) make up the baseline New Keynesian framework.

Finally, the New Keynesian model used in this paper employs two versions of the money demand function. The first is that given in equation \((A30)\). The second sets the parameter \(a_m = 0\) in equation \((A30)\) such that:

\[
\hat{m}_t - \frac{\sigma}{\epsilon_m} \hat{y}_t + \frac{1}{\epsilon_m} R_t = 0
\]
APPENDIX 3: SUPPLEMENTAL RESULTS

As shown in Figure 16 in Chapter 4, the baseline New Keynesian model is able to capture the persistence of inflation quite well when the model is confined to two exogenous shocks: monetary and technology. These results are contrary to those presented in Estrella and Fuhrer (2002) for what is essentially the same structural model. The primary difference between that model and the baseline New Keynesian model employed in the present analysis is that the former includes an IS shock of the form:

\[ \varepsilon_t^{IS} = 0.3\varepsilon_{t-1}^{IS} + v_t \]

where \( E_t v_t = 0 \) and \( SD(v_t) = 0.01 \). (Estrella and Fuhrer assume that the IS shock is i.i.d. However, the model that it is based on – McCallum and Nelson [1999] – uses the specification above). To examine whether this difference is enough to explain the conflicting results, the baseline New Keynesian model is extended to include the IS shock as articulated above. The autocorrelation function for inflation generated by the model with the additional exogenous variable is plotted below along with that of the standard model employed in the paper and the autocorrelation function from the data. As in the paper, standard error bands are shown by the dotted line.

As seen in the figure, the extension of the model to include the IS shock significantly weakens the persistence of inflation. In fact, the entire autocorrelation function is outside of the standard error bands. Thus, it seems that the absence of the IS shock is sufficient to explain the differing results.

Figures A2 and A3 plot impulse response functions. Specifically, Figure A2 plots the impulse response function for a monetary shock for the inflation and the output gap, respectively, in each model. As noted in the footnote in the text, the agency cost model exhibits a larger response to a change in monetary policy. As with the results for the autocorrelation functions, the increased responsiveness and persistence is due to the effects of monetary
policy not only on the nominal interest rate, but also on net worth. Figure A3 plots the impulse response functions for the agency cost model.

Finally, Tables A3 and A4 examine the sensitivity of the results presented in Chapter 4 to the calibration of the model. The numbers in bold in the left-hand column denote the actual calibration of the model. As shown, the results are not sensitive to the particular calibration used in Chapter 4.
Figure A1: Autocorrelation Functions

Figure A2: Impulse Response Function Comparison
Figure A3: Monetary Shock – Agency Cost Model

- Response of $y$
- Response of $\phi$
- Response of $m$
- Response of $\pi$
- Response of $c$
- Response of $q$
- Response of $d$
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<th>Output Gap</th>
<th>Inflation</th>
<th>Interest Rate</th>
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<tbody>
<tr>
<td>1954:3 - 2009:4</td>
<td>2.46</td>
<td>2.22</td>
<td>3.36</td>
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<td>1979:4 - 2009:4</td>
<td>2.30</td>
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<tr>
<td><strong>b = 0.2</strong></td>
<td>2.52</td>
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<td>2.18</td>
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<td>2.21</td>
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<td><strong>e = 0.04</strong></td>
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**Monetary Policy**

**Standard Taylor rule (1.5, .5)**

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<tr>
<td>NK</td>
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**Clarida, Gali, Gertler (2.15, 0.93)**

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**Judd-Rudebusch (1.54, 0.99)**

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Table A4: First- and Second-order Autocorrelations

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<td>0.88</td>
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<td>Agency Cost Model</td>
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REFERENCES


ABSTRACT

A RE-EXAMINATION OF MONEY AND BUSINESS CYCLES

by

JOSHUA R. HENDRICKSON

December 2010

Advisor:  Dr. Robert J. Rossana
Major:   Economics
Degree:  Doctor of Philosophy

In the last several years, dynamic stochastic general equilibrium (DSGE) models have become the predominant tool of modern macroeconomics and, in particular, monetary policy research. Generally speaking, the basic New Keynesian model is the standard framework for analysis. This framework uses optimizing agents and firms to generate a dynamic, rational expectations model analogous to traditional IS-LM analysis in which the LM curve has been replaced with a monetary policy rule. As a result, the model makes a strong assumption about monetary policy. Namely, the New Keynesian model assumes that the short term interest rate is sufficient to capture the monetary transmission mechanism. Evidence that suggests that monetary aggregates are not useful for analysis of monetary shocks is viewed as prima facia evidence in favor of this view. Nonetheless, there remains cause for skepticism. This dissertation examines the implications of New Keynesian model in two important directions. First, the empirical evidence that examines the role of money relies on the use of simple sum aggregates, which are not consistent with aggregation theory. As a result, the dissertation re-examines previous results using monetary services indexes that are consistent with economic, aggregation, and index number theory. Second, the New Keynesian framework is nested in a model with agency costs and a richer specification of money demand than traditionally used to examine the implications of alternative transmission mechanisms.
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