Admixture and ancestry inference from ancient and modern samples through measures of population genetic drift

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Admixture and ancestry inference from ancient and modern samples through measures of population genetic drift

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Abstract: Methods that leverage the information about population history contained within the increasingly abundant genetic sequences of extant and extinct Hominid populations are diverse in form and versatile in application. Here, we review key methods recently developed to detect and quantify admixture and ancestry in modern human populations. We begin with an overview of the $f$- and $D$-statistics, covering their conceptual principles and important applications, as well as any extensions developed for them. We then cover a combination of more recent and more complex methods for admixture and ancestry inference, discussing tests for direct ancestry between two populations, quantification of admixture in large datasets, and determination of admixture dates. These methods have revolutionized our understanding of human population history and additionally highlighted its complexity. Therefore, we emphasize that current methods may not capture this population history in its entirety, but nonetheless provide a reasonable picture that is supported by data from multiple methods, and from the historical record.

Keywords: Introgression, Contamination, Ancient DNA, Genetic drift, Microsatellite

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The wealth of genome-wide polymorphism data from diverse human populations around the world (Cann et al., 2002; Gibbs et al., 2003; Altshuler et al., 2010; McVean et al., 2012) has allowed researchers to access a record of human history unprecedented in its breadth, spanning thousands of years (Patterson et al., 2012; Jones et al., 2015; Mendez et al., 2015; Lazaridis et al., 2016). With this data, new insights into human migration (Rasmussen et al., 2014; Lazaridis et al., 2014; Skoglund and Reich, 2016) and interbreeding (Hellenthal et al., 2014) during the peopling of the world have emerged. Specifically, the recent publication of
Archaeogenomes from ancient human remains (Rasmussen et al., 2010; Olalde et al., 2014; Fu et al., 2014; Lindo et al., 2016, 2017), Neanderthals (Green et al., 2010; Prüfer et al., 2014), and the Denisovan (Reich et al., 2010) have further complemented, corroborated, and enhanced our understanding of these events, and spurred the development of new tools and novel applications to existing ones. Here, we review some of the key methods developed to detect and quantify admixture through measurements of genetic drift that are currently in use, summarizing the conceptual and mathematical principles that underlie them, as well as the significant discoveries they have produced. We additionally show the applicability of methods across different data types where possible, including the $f$-statistics (Reich et al., 2009), $D_{FOH}$ (Pease and Hahn, 2015), and TreeMix (Pickrell and Pritchard, 2012).

These developments have thus opened the field to questions that were previously difficult or impossible to answer. These include inquiries about the date of an admixture event, the admixture proportion of one population in an interacting population, whether a population is truly admixed from two selected putative progenitor lineages, whether an ancient population is ancestral to a modern one, and how a population of interest is related by a graph to other studied populations. In addition to disentangling the genetic heritage of modern populations, the answers to these questions provide clues about the migration patterns that shaped the distribution of modern humans, including the occurrence of multiple migrations into certain geographic regions, the order in which these occurred, and the manner in which these contributed to the human genetic variation succeeding them.

**Admixture inference from an unrooted phylogeny using $f$-statistics**

We begin with the $f$-statistics, originally introduced by Reich et al. (2009) as tools to determine the ancestry of Indian populations, which were found to be heavily structured by caste and geographic location. Broadly, these statistics, $f_2$, $f_3$, and $f_4$, are interpretable as measures of genetic drift applied to unrooted population phylogenies. They require only allele frequency data from each of two, three, or four populations, respectively, for computation, and are therefore convenient to use in the absence of genome-wide sequence data. Additionally, these methods are robust to ascertainment bias outside of extreme cases (Patterson et al., 2012). Because the two-population statistic $f_2$ is similar in interpretation to $F_{ST}$ as a measure of differentiation between a pair of populations (but Patterson et al., 2012, note that $f_2$ values, unlike $F_{ST}$, are additive along the branch of a phylogenetic tree, and smaller for parts of the tree farther from the root), we will not go into greater detail about this statistic here. A recent review by Peter (2016) provides a myriad of additional interpretations of the $f$-statistics, including as coalescence times, pairwise
differences, tree topologies, and genealogies, and we briefly cover the major results from this work as well.

A simple admixture test using the $f_3$-statistic

The $f_3$-statistic emerges from a test of three populations that explicitly asks whether a population of interest (say, $A$), is the result of admixture between two other populations (say, $B$ and $C$). It measures the covariance of the difference in allele frequencies between populations $A$ and $B$ and populations $A$ and $C$ across genomic loci. It is calculated across $J$ biallelic loci as

$$f_3(A; B, C) = \frac{1}{J} \sum_{j=1}^{J} (a_j - b_j)(a_j - c_j), \quad (1)$$

where $a_j$, $b_j$, and $c_j$ are the frequencies of derived allele at site $j$ in populations $A$, $B$, and $C$, respectively. Because the magnitude of $f_3$ (and the other $f$-statistics) depends highly on the distribution of allele frequencies within the three populations, smaller values of the derived allele frequency contribute less to the value of $f_3$. Patterson et al. (2012) address this issue by normalizing $f_3$ across $J$ loci such that

$$f_3(A; B, C) = \frac{\sum_{j=1}^{J} (a_j - b_j)(a_j - c_j)}{\sum_{j=1}^{J} 2a_j(1 - a_j)}, \quad (2)$$

where the denominator is the expected heterozygosity of population $A$ summed across $J$ loci.

If the test result is positive, i.e., $f_3(A; B, C) > 0$, then there is no evidence that $A$ is descended from an admixture event of $B$ and $C$. Interpreting this value as genetic drift, we can see that $f_3(A; B, C)$ is the length of the branch in the unrooted three-population phylogeny leading to $A$ from the internal node (Figure 1A). Meanwhile, if $f_3(A; B, C)$ is significantly negative, then $A$ may be admixed from $B$ and $C$. Significance of results against the null hypothesis of no admixture is evaluated by weighted block jackknife to obtain a mean Z-score, which is the weighted mean value of the statistic across all blocks of equal length, divided by the standard error of the statistic. The length of the blocks is the smallest value for which increasing the length does not increase the standard error (the point at which estimated standard errors converge; Schaefer et al., 2016). This method assumes a normal distribution of the statistic. For $M$ blocks and any test statistic $T$, this computation is


\[ Z = \frac{\bar{T}}{SE_T} = \frac{\sum_{i=1}^{M} W_i T_i}{\sqrt{\sum_{i=1}^{M} W_i [T_i - \bar{T}]^2 / M}} \]  

(3)

where the weight \( W_i = N_i / \sum_{j=1}^{M} N_j \) is the number of informative sites in block \( i \) \((N_i)\) divided by the total number of informative sites across all blocks. An \( f_3 \)-statistic for which \( Z < -3 \) is significantly negative. We can therefore represent the relationship of the three populations in this scenario as an admixture graph (Figure 1B). Here, we assign the value \( p \) to represent the proportion of \( A \)'s ancestry that is derived from an ancestor of \( B \), and the value \( 1 - p \) to represent the remaining proportion of \( A \)'s ancestry that is derived from an ancestor of \( C \). Quantities \( p \) and \( 1 - p \) are thus the probabilities that a randomly-chosen allele at the site under investigation has descended from the same lineage as a specific reference population.

To understand the manner in which genetic drift contributes to the expected value of \( f_3 \), we demonstrate the four ways in which the two alleles drawn at a site from an individual in admixed population \( A \) (descended from the mixture of the ancestors of populations \( B \) and \( C \)) trace their ancestry to \( B \) and \( C \), weighted by \( p \) or \( 1 - p \) (Figure 2A). Tracing ancestry to \( B \) with red arrows, and ancestry to \( C \) with blue arrows, we see that for two alleles descended from the \( B \) lineage, the red and blue paths overlap a branch length totaling \( i + ii \). Likewise, if both alleles are more closely related to the \( C \) lineage, then they will only overlap over a branch length totaling \( i + iii \). Two possibilities exist for a pair of alleles in which one allele is more closely related to the \( B \) lineage and one is more closely related to the \( C \) lineage. The two paths may only overlap in the same direction over branch length \( i \), or they may overlap in opposite directions over branch length \( v + vi \), in addition to overlapping in the same direction over \( i \). Overlap in the same direction is weighted positively because this is shared drift between the two drawn alleles, while overlap in opposite directions is weighted negatively, because this is not shared drift but divergence.

Thus, the expected value of \( f_3 \) is

\[ f_3(A; B, C) = i + p^2(ii) + (1 - p)^2(iii) - p(1 - p)(v + vi). \]  

(4)

Therefore, the only term in Equation 4 that contributes negatively to \( f_3 \) is the final term, which is proportional to the length of \( v + vi \). Once again, we can see that if \( A \) is not admixed from lineages \( B \) and \( C \), the value of \( p \) is zero and the expected value of \( f_3 \) equals \( i + iii \), the length of the branch between population \( A \) and the common ancestor of \( A \), \( B \), and \( C \) on the unrooted tree. It is important to note, however, that a significantly negative value of \( f_3 \) may not arise even if admixture has occurred. Patterson et al. (2012) point out that high population-specific drift in
the admixed population, which increases the value of branch length $i$, may mask the signal of admixture. Additionally, a significantly negative value of $f_3$ can emerge in what Patterson et al. (2012) call the outgroup case. Here, a misleading signal of admixture emerges for the test $f_3(A; B, O)$, where $A$ is admixed from $B$ and $C$ as in Figure 1B, but population $O$, an outgroup that diverged basally to the split of $B$ and $C$, is used as the second reference population. A signal of admixture is still detected because the drift paths of the two alleles draw at that site still overlap in opposite directions. Thus, even if the $f_3$-statistic is improperly set up in this manner, the admixed population is still identified, though the proper population history is not represented. For this reason, a significantly negative $f_3$-statistic should be interpreted as evidence that the target population is admixed, but not necessarily admixed with the two reference populations.

This formulation is different from the outgroup $f_3$-statistic presented in Raghavan et al. (2014b) to quantify the Western Eurasian-Siberian ancestry of modern Native American populations. The outgroup $f_3$-statistic measures the shared genetic drift between two populations relative to an outgroup, and the specific measurement of only shared genetic drift is the proposed advantage of this method over the use of pairwise distance measures such as $F_{ST}$, which are sensitive to lineage-specific genetic drift. Because the underlying phylogeny is a three-population tree (Figure 1A), the outgroup $f_3$ once again represents the length of the branch between the outgroup and the internal node. This approach necessarily yields a value greater than zero if the outgroup is properly assigned, and this is to be expected because it measures a positive branch length (or, an overlap of drift paths in the same direction). The quantity $f_3(O; W, X)$ increases with increasing shared ancestry of populations $W$ and $X$ and can be used to provide evidence of recent and exclusive common ancestry provided that $W$ and $X$ are not related by admixture (Raghavan et al., 2015). Therefore, typical use of the outgroup $f_3$ involves multiple calculations wherein $W$ and $O$ are fixed and $X$ is changed such that inferences can be made about the affinity of $W$ to all tested populations $X$.

We demonstrate this principle in Figure 3 with microsatellite data rather than allele frequencies from biallelic sites. The $f_3$-statistic can accommodate microsatellite data by measuring the covariance in mean microsatellite lengths between populations across loci. This was first proposed by Pickrell and Pritchard (2012) for TreeMix, which also normally uses allele frequency data from biallelic sites. For this set of tests, we prepared biplots in which each axis represents the shared ancestry between fixed population $W$ and other global human populations $X$, compared with the sub-Saharan Yoruba (genotypes from the dataset assembled by Pemberton et al., 2013) as the outgroup: $f_3(Yoruba; W, X)$. Using the outgroup $f_3$ in this manner allows us to resolve clusters within population data and display ancestry intuitively, when the two axes are appropriately selected.
In Figure 3A, we see that all populations have shared ancestry to the Native American Pima and Huilliche populations at approximately equal levels, falling on the diagonal line indicating equal affinity. Middle Eastern populations (yellow) yielded the lowest levels of shared ancestry with the Pima and Huilliche, and other Native American (purple) populations yielded the highest levels. Outgroup $f_3$ biplots of shared ancestry with two populations from different geographic regions provide a greater ability to separate populations in two dimensions, highlighting differences in population affinities to one geographic region relative to another. Figure 3B compares affinity with the East Asian Han to affinity with the European Sardinian population. This test has a greater ability to resolve the Oceanian (green) populations from the Central/South Asian (red) populations than does the first because it exploits their differing levels of shared ancestry to the European lineage. The biplots in Figures 3C and D demonstrate the effect of changing a single axis. The East Asian (pink) and Native American populations overlap substantially in their affinity to the Han and admixed Australian populations (Figure 3C), but are noticeably different in their affinity to the Native American Karitiana population and unambiguously cluster separately for this comparison (Figure 3D).

We conclude our overview of $f_3$ with a proposed redefinition of $f_3$ from Peter (2016). Throughout this work, the author emphasizes the usefulness of defining the $f$-statistics using coalescent theory. This redefinition is to alleviate the computational difficulty of tracing all allele paths in admixture plots, especially as the number of admixture events increases, and to avoid the restriction that admixing subpopulations cannot be structured themselves. Thus, the coalescent theory perspective does not require a defined admixture graph. The $f_3$-statistic can be written in terms of $f_2$ (the measure of drift between two populations; Patterson et al., 2012) and $f_2$ can be written as the difference of expected coalescence times (Peter, 2016). We can therefore write $f_3$ in terms of the expected coalescence times of lineages drawn from populations $A$, $B$, and $C$ as

$$f_3(A; B, C) = \frac{\theta}{2} (T_{AB} + T_{AC} - T_{BC} - T_{AA}), \quad (5)$$

where $\theta$ is the population-scaled mutation rate and $T_{AB}$ is the expected time to coalescence of lineages from populations $A$ and $B$.

Interestingly, Peter (2016) demonstrated that the use of the mean pairwise sequence difference $\pi_{BC}$ between populations $B$ and $C$ has a stronger correlation with the divergence time of $B$ and $C$ than does $f_3(A; B, C)$. To illustrate this result, the author considers the outgroup $f_3$-statistic in terms of expected coalescence times. In determining the affinity of a test population, $B$, to a series of known populations, each taken as $C$ in a separate test, the only term in Equation 5 that changes across tests is $T_{BC}$. Therefore, measurement of $\pi_{BC}$ alone is sufficient to compare the
difference in affinity between population $B$ and all populations $C$. Because the measurement of $\pi_{BC}$ has a smaller variance than the measurement of $f_3(A; B, C)$, we can see that the correlation of the former with the time of divergence between $B$ and $C$ is greater than that of the latter. For this reason, the author suggests that $\pi_{BC}$ should supplant $f_3(A; B, C)$ as a measure of affinity between populations.

### A model-based test of treeness with the $f_4$-statistic

Reich et al. (2009) additionally used measurements of shared drift as a method of validating a proposed tree topology, and we therefore refer to the $f_4$-statistic as a test of treeness. That is, it tests whether a particular unrooted, four-population phylogeny (of which there are three; see Felsenstein, 2004) accurately describes the relationship between the tested populations. Similarly to $f_3$, the formula for $f_4$ is based on the difference in allele frequencies at biallelic loci (but the difference in mean microsatellite lengths is also compatible here, as with $f_3$; see Pickrell and Pritchard, 2012). Here, the $f_4$-statistic represents the covariance of allele frequency differences between populations $A$ and $B$ and populations $C$ and $D$, and is calculated across all $J$ loci as

$$f_4(A, B; C, D) = \frac{1}{J} \sum_{j=1}^{J} (a_j - b_j)(c_j - d_j),$$

where $a_j$, $b_j$, $c_j$, and $d_j$ are the frequencies of a reference allele at site $j$ in populations $A$, $B$, $C$, and $D$, respectively. The particular test in Equation 6 is a test of whether an unrooted tree wherein populations $A$ and $B$ form a cluster, and $C$ and $D$ form a cluster, is correct (Figure 1C). Because the $f_4$-statistic is based on the difference of allele frequencies, normalizing the statistic may be required, as with $f_3$. The authors suggest a normalized $f_4$-statistic of the form

$$f_4^*(A, B; C, D) = \frac{\sum_{j=1}^{J} (a_j - b_j)(c_j - d_j)}{\sum_{j=1}^{J} x_j(1 - x_j)}.$$

Here, the choice of denominator is flexible, and so population $X$, whose derived allele frequency at site $j$ is denoted by $x_j$, can be any of the four populations ($A$, $B$, $C$, or $D$) incorporated into the test. The authors explain that in principle, normalizing by the most diverged population (e.g., an African population such as Yoruba or San, whose diversity encompasses most of the diversity other human populations; see Rosenberg, 2011) is a reasonable choice. However, if one is interested in measuring the drift specific to a branch of the tree highly diverged
from an African outgroup, then normalizing \( f_4 \) using a population more closely related to the branch of interest may be more appropriate. The authors suggest, for example, normalizing using Han allele frequencies for a set of East Asian ingroup populations. In this way, the value of the denominator is reduced and misleadingly small \( f_4 \) values are avoided (Reich et al., 2009).

Interpreting the value of the \( f_4 \)-statistic requires visualizing the shared drift of the two paths defined in the test. For \( f_4(A,B;C,D) \), the two defined paths are from \( A \) to \( B \) and from \( C \) to \( D \). It is evident from the first tree of Figure 2B that \( f_4(A,B;C,D) = 0 \) for a phylogeny in which \( A \) and \( B \) form a cluster, and in which \( C \) and \( D \) form a cluster. This is because there is no overlap (no correlation) in drift between members of the two clusters, indicating that they do not share a recent or significant population history. If, however, the true relationship between these four populations at the site under investigation is ((A,C), (B,D)), then \( f_4(A,B;C,D) \) is equal to the length of the internal branch of the tree, and positive because the drift paths overlap in the same direction (Figure 2B, second tree). Conversely, if the correct relationship is ((A,D), (B,C)), then the drift paths overlap in the opposite direction and \( f_4(A,B;C,D) \) is again equal in magnitude to the length of the internal branch, but negative. (Figure 2B, third tree). The \( f_4 \)-statistic can therefore be used to calculate the length of the internal branch for a phylogeny concordant with the test. This value is simply \( f_4(A,C;B,D) \) for cases in which the true tree is ((A,B), (C,D)) (Peter, 2016). As with the \( f_3 \)-statistic, the significance of the \( f_4 \)-statistic is based on a \( Z \)-score calculated by block jackknife (Equation 3), with significantly positive (\( Z > 3 \)) and significantly negative (\( Z < -3 \)) values rejecting the null hypothesis of correct tree topology ((A,B),(C,D)).

The properties of the \( f_4 \)-statistic make it a powerful tool for inferring admixture, especially in conjunction with an \( f_3 \) test. The result of a discordant tree topology (a significantly nonzero value for \( f_4(A,B;C,D) \) across all sites), is alone enough to suggest significant common ancestry between clusters, which traverses their divergence on the tree. The \( f_4 \)-statistic does not, though, indicate the direction of admixture. If the goal of the tests is to determine whether population \( D \) is admixed from populations \( B \) and \( C \), with \( A \) as a verified outgroup, an appropriate subsequent test is \( f_3(D;B,C) \). A significantly negative result here would represent strong evidence for admixture between \( B \) and \( C \) to produce \( D \). A caveat, however unlikely, to the result of the \( f_4 \)-statistic is that it may yield a false result of no admixture if \( D \) is admixed from \( B \) and \( C \) in equal proportions across the genome. This phenomenon occurs because the signal of discordance from each source of ancestry is equal in magnitude, opposite in direction, and weighed by admixture proportion (represented as \( p \) and \( 1 - p \) in Figures 1B and 2A). This does not affect the value of the \( f_3 \)-statistic, which would remain significantly negative in this scenario (Reich et al., 2009).
Two other important applications of the $f_4$-statistic exist, the $f_4$-ratio, and the $f_4$-rank test (Reich et al., 2009, 2012; Patterson et al., 2012; Peter, 2016). The $f_4$-ratio, introduced by Reich et al. (2009) as $f_4$ ancestry estimation, quantifies the proportion of ancestry that an admixed population derives from its progenitor lineages. The $f_4$-ratio is calculated from the quotient of two $f_4$-statistics generated from five populations wherein one is the result of admixture between two others, neither of which is the outgroup (Figure 1D). If the proportion of admixture of lineage $C$ into $A$ is defined as $p$, and the proportion of admixture of lineage $D$ into $A$ is $1 - p$, then $f_4(B,O;A,D) = pf_4(B,O;C,D)$. Therefore, the proportion of ancestry deriving from $C$ in admixed population $A$ is

$$\frac{f_4(B,O;A,D)}{f_4(B,O;C,D)}.$$ (8)

The theory underlying the $f_4$-rank test, implemented in qpWave (see Reich et al., 2012, and Skoglund et al., 2015) is founded in linear algebra, and we will not discuss the mathematical details further here. The principle of the $f_4$-rank test is that by measuring the rank of a matrix of $f_4$-statistics, we can infer the number of admixture events in the history of a population of interest. Briefly, the matrix of $f_4$-statistics is of dimension $m \times n$ where $m$ is the number of putatively admixed sampled populations to test (drawn from set $A$, containing admixed and unadmixed populations), and $n$ is the number of sampled outgroup populations (e.g., more recently diverged than African lineages for analyses not involving African individuals, drawn from set $B$ of unadmixed populations). Each entry in the matrix is an $f_4$-statistic tested for a particular combination of admixed population $j$ (denoted $A_j$) drawn from the set of $m$ populations in $A$ and unadmixed outgroup population $k$ (denoted $B_k$) drawn from the set of $n$ populations in $B$. The $j$th row and $k$th column of the $f_4$ matrix is the $f_4$-statistic of the form $f_4(Y,A_j;Z,B_k)$, where $Y$ is a non-admixed sister population to the test population $A_j$ and is drawn from set $A$, and $Z$ is a fixed other outgroup (unadmixed) that is a sister population to $B_k$ and is drawn from set $B$. The rank of the matrix increases by one for each additional admixture event that occurred in the shared ancestry of the sampled populations, and is zero for population histories with no admixture. From this result, Reich et al. (2012) determined that Native American population history was consistent with at least three migrations from East Asia.

We finally note that $f_4$, similarly to $f_3$, has interpretations in terms of both $f_2$ and expected coalescence times and that this simplifies the estimation of ancestry proportion (Peter, 2016). First, $f_4$ can be written as the combination of the four possible $f_2$ drift values for all pairs of populations $A$, $B$, $C$, and $D$ (Patterson et al., 2012). Because $f_2$ can be written in terms of expected coalescence times, the $f_4$-statistic for populations $A$, $B$, $C$, and $D$ can be formulated as
\[ f_4(A, B; C, D) = \theta(T_{AD} + T_{BC} - T_{AC} - T_{BD})/2. \] From this notation, it is possible now to redefine the formula for the \( f_4 \)-ratio estimator of the admixture proportion. Because the expected coalescence times for any ingroup with the outgroup will be equal, \( f_4 \)-ratio simplifies to \( p = (T_{BD} - T_{BA})/(T_{BD} - T_{BC}) \). Lastly, Peter (2016) points out that substituting expected coalescence times for pairwise differences yields the admixture proportion \( p = (\pi_{BD} - \pi_{BA})/(\pi_{BD} - \pi_{BC}) \), and obviates the need for an outgroup in the \( f_4 \)-ratio test.

To conclude our discussion of four-population tests, we highlight another powerful tool, known as the \( h_4 \)-statistic, which uses differences in linkage disequilibrium (LD) rather than in allele frequencies to measure treeness (Skoglund et al., 2015). Specifically, the statistic is of the form \( h_4(A, B; C, D) \) and tests the null hypothesis that the unrooted topology is of the form \( ((A,B),(C,D)) \), with populations \( A \) and \( B \) forming a cluster and \( C \) and \( D \) forming a cluster. Under no admixture, \( h_4 \) is zero, and the interpretation of significant deviations from zero (computed with weighted block jackknife) is analogous to \( f_4 \). Based on haplotype frequencies, the \( h_4 \)-statistic can be employed to provide evidence of shared ancestry independent of the \( f_4 \)-statistic, which only uses allele frequencies. However, Skoglund et al. (2015) indicated that \( h_4 \) may be biased by demographic history such that the length of the region to consider for calculation of LD needs to be determined in advance in order to incorporate sufficient polymorphism. Further, the need for haplotype data may limit the application of \( h_4 \) to well-studied organisms such as humans, but may be more difficult to apply to other, less-studied primates.

**Testing for introgression using \( D \)-statistics**

First formulated for a three-taxon case by Huson et al. (2005) and reapplied to four-taxon *Drosophila* data by Kulathinal et al. (2009), then proposed in its most well-known form by Green et al. (2010) and since expanded by Eaton and Ree (2013) and Pease and Hahn (2015), the \( D \)-statistics represent a model-based approach for detecting gene flow between candidate populations using sequence data. As with the \( f \)-statistics, the \( D \)-statistics are robust to ascertainment bias (Patterson et al., 2012). For each type of test, an outgroup is selected (for applications to human data, this is typically a chimpanzee sequence), as well as three to four ingroup taxa of which two may have hybridized. Two of the ingroup taxa must be from sister lineages, only one of which has admixed with another (non-sister) ingroup lineage. The \( D \)-statistics therefore examine whether the frequency of apparent incomplete lineage sorting (ILS) between each sister lineage and the other ingroup lineage is significantly different. This is because while introgression and ILS both produce gene trees that are discordant with the species tree, in the absence of hybridization, it is expected that the frequency of ILS between each sister population and any other population is equal (or not significantly different). For this reason, the value of the
$D$-statistics in the absence of admixture will be zero. Admixture between only one of the sister lineages and another ingroup lineage would increase the number of observations in which the two share the same allele while other populations have a different allele, significantly deviating the value of the statistics from zero. The $D$-statistics are for this reason a test of treeness, but for a proposed outgroup-rooted topology.

**Testing for gene flow using Patterson’s $D$-statistic**

Testing for introgression with the $D$-statistic is distinguished from testing with $f_5$ because it requires the user to provide a rooted, asymmetric, four-population tree for which incomplete lineage sorting events are defined as deviations from the proposed topology. The original application of this method was to detect signatures of gene flow between Neanderthals and modern humans (Green et al., 2010). These periods of gene flow may have occurred on multiple occasions across Western Asia and Europe between 37,000 and 86,000 years ago, though apparently more so in the lineage leading to East Asians and Native Americans (Vernot and Akey, 2015; Fu et al., 2015). Thus, the phylogeny describing this history is of the form (((African, non-African), Neanderthal), Chimpanzee), where the two modern human populations are sisters, the Neanderthal has putatively admixed with the non-African population, and the chimpanzee is the outgroup to all species in the genus *Homo* (Figure 1E).

The theoretical basis of the $D$-statistic is quite straightforward. Across the genome, sites at which the two sister populations exhibit a different allele, but for which one of the two shares an allele with the putatively introgressing population, are identified. Additionally, the outgroup population must share the same allele as the non-admixed sister population. Labeling the ancestral allele as $a$ and the derived allele as $b$ for a biallelic locus, the only sites informative for calculation of the $D$-statistic are $abba$- and $baba$-sites. For the tree in Figure 1E, $abba$-sites are those for which the non-African and Neanderthal populations share the derived allele, and the African and chimpanzee populations share the ancestral allele, whereas $baba$-sites are those for which the African and Neanderthal populations share the derived allele, while the non-African and chimpanzee share the ancestral. The $D$-statistic is then calculated as

$$D = \frac{n_{abba} - n_{baba}}{n_{abba} + n_{baba}},$$

where $n_{abba}$ and $n_{baba}$ are the numbers of $abba$ and $baba$ sites across the genome.

The value of the $D$-statistic lies between $-1$ and $1$. When the number of $abba$-sites is equal to the number of $baba$-sites, the value of the statistic is 0. An excess
of alleles shared between the second sister population and the admixing population (non-Africans and Neanderthals, respectively, in Figure 1E) yields a positive $D$-statistic, whereas an excess of alleles shared between the first sister population and the admixing population (Africans and Neanderthals) yields a negative $D$-statistic. As with the $f$-statistics, significance of the $D$-statistic is inferred by the weighted block jackknife method against the null hypothesis that the proposed tree topology is correct, wherein $Z > 3$ and $Z < -3$ are considered statistically significant.

Since the $D$-statistic is calculated across all sequenced sites, a primary practical limitation of the method is sequencing depth. This limitation may not apply when all samples are from modern whole genomes, but for ancient DNA studies, where coverage may be too low to call genotypes (Skoglund et al., 2012; Fu et al., 2013; Lazaridis et al., 2014; Olalde et al., 2014; Skoglund et al., 2014a; Raghavan et al., 2014a,b; Seguin-Orlando et al., 2014; Fu et al., 2015; Raghavan et al., 2015; Rasmussen et al., 2015; Moorjani et al., 2016), and cytosine deamination in conjunction with overall DNA fragmentation further reduces the number of informative sites available (Dabney et al., 2013), statistically significant relatedness between two populations may go unnoticed. To emphasize this, we display the distribution of $D$-statistics for three different population histories (Figure 4). In the first scenario, ancient DNA is collected from an individual belonging to an ancestral population $A$ that is the direct ancestor of modern population 1 (Figure 4A, first tree). In the second, $A$ is equally related to modern populations 1 and 2, having diverged from their common ancestor 13,000 years before the present (Figure 4A, second tree). For the third scenario, $A$ and modern population 1 share a common ancestor more recently than do modern populations 1 and 2 (Figure 4A, third tree).

Whereas the first of these scenarios should produce a significantly nonzero $D$-statistic, and the second should produce a $D$-statistic not significantly different from zero (with the third scenario between these), neither shows significant deviation from zero at “low coverage” (Figure 4B). Even at tenfold higher coverage (Figure 4C), the distributions for both the first and third scenarios are mostly below the significance threshold. It is only at 100-fold higher coverage that the low coverage scenario (Figure 4D) that the null hypothesis may be consistently rejected for the first scenario, and rejected at a rate of more than 50% for the third. Furthermore, studies subsequent to Green et al. (2010) have indicated that the $D$-statistic is not robust to ancestral population structure such that instantaneous unidirectional admixture produces the same signal as ancestral structure, and the $D$-statistic cannot distinguish these (Durand et al., 2011). That is, the data used to support the hypothesis of Neanderthal admixture into non-African anatomically-modern humans also supports a model in which ancient humans were deeply structured, but received no gene flow from Neanderthal lineages (Eriksson and Manica, 2012). We note, though, that other approaches, such as the doubly-conditioned frequency spectrum, have been proposed to distinguish between these
two scenarios (Yang et al., 2012; Eriksson and Manica, 2014). Additionally, other lines of evidence suggest that admixture has occurred, but that there was no extensive human ancestral population structure in Africa (Lohse and Frantz, 2014).

Although the standard application of the $D$-statistic is with sequence data (calculating across all sites), various authors have demonstrated the compatibility of the $D$-statistic with allele frequency data (Durand et al., 2011; Patterson et al., 2012; Raghavan et al., 2014b). Reformulated for this purpose, the $D$-statistic can be computed across $J$ loci as

$$D(A, B, C, D) = \frac{\sum_{j=1}^{J}(a_j - b_j)(d_j - c_j)}{\sum_{j=1}^{J}(a_j + b_j - 2a_jb_j)(c_j + d_j - 2cjdj)},$$

where $a_j$, $b_j$, $c_j$, and $d_j$ are the frequencies of a derived allele at the site $j$ for populations $A$, $B$, $C$, and $D$, respectively. This formula can be obtained by sampling a single allele from each of the four populations ($A$, $B$, $C$, and $D$) uniformly at random according to the allele frequencies within the populations to create the $abba$ and $baba$ site patterns. Note that the numerator of the frequency-based $D$ statistic is equal to $-f_4(A, B; C, D)$. This makes sense considering that the two statistics are four-population tests whose purpose is to determine whether a particular population phylogeny is valid. Consequently, similar inferences emerge from both methods, though we emphasize that the $D$-statistic was designed as an explicit test of admixture given a proposed rooted phylogeny, while $f_4$ does not make the starting assumption of a rooted treelike relationship between populations. Furthermore, the value of the $D$-statistic is normalized to lie between $-1$ and $1$, whereas $f_4$ does not have this attribute.

Raghavan et al. (2014b) also demonstrate that sample contamination can be corrected within the $D$-statistic framework, and this is a necessary consideration as contamination can leave similar signatures as introgression and can obscure population relationships. To illustrate this point, it is helpful to consider a plausible population history for which this application of the $D$-statistic can identify incorrect inferences (Figure 5). With a chimpanzee (Chimp) sequence as the outgroup, we define an ancient human (Ancient) as basal to modern Native Americans (NA), having diverged with the ancestors of Native Americans from the lineage leading to East Asian (EA) populations (Figure 5A). However, following contamination from a European (Eur) sequence into the Ancient sequence, the true relationship of Ancient to modern Native Americans may not be recovered, and $D(EA, NA, Ancient, Chimp)$ may instead suggest the topology in Figure 5B (that is, $D$ is not significantly different from zero). To determine whether such an observation could be the result of contamination, Raghavan et al. (2014b)
considered that both the true signal of admixture and modern contamination contribute to the observed value of the $D$-statistic. Therefore,

$$D_{\text{obs}} = \gamma D_{\text{Eur}} + (1 - \gamma) D_{\text{cor}},$$

(11)

where $D_{\text{obs}}$ is the observed $D$-statistic, $D_{\text{Eur}}$ is $D(\text{EA}, \text{NA}, \text{Eur}, \text{Chimp})$, $D_{\text{cor}}$ is the pre-contamination (or corrected) value of $D(\text{EA}, \text{NA}, \text{Ancest}, \text{Chimp})$, and $\gamma$ is the proportion of contamination from modern European sources handling the ancient sample. A contamination-corrected $D$-statistic ($D_{\text{cor}}$) can then be computed as

$$D_{\text{cor}} = \frac{D_{\text{obs}} - \gamma D_{\text{Eur}}}{1 - \gamma}.$$  

(12)

An estimate of the contamination rate is necessary for this $D$-statistic correction, and can be obtained from a number of different methods. Raghavan et al. (2014b) discuss measuring the proportion of sites in ancient mtDNA (haploid) and on male X-chromosomes (hemizygous) for which two different alleles are detected. In the case of mitochondria, though heteroplasmy may exist (Ye et al., 2014; Stewart and Chinnery, 2015; Rensch et al., 2016), deviations from the rare variants specific to the ancient population are highly unlikely and interpreted as contamination. Skoglund et al. (2014b) distinguish ancient from modern DNA by its characteristic pattern of degradation. Racimo et al. (2016b) have developed a Markov chain Monte Carlo method of inferring the rate of modern DNA contamination into ancient samples.

Contamination can make it difficult to reject the null hypothesis of a $D$-statistic of the form $D(A, B, C, D)$ if $C$ is contaminated. This issue can arise if population $B$ is more closely related to population $C$ than is population $A$, for example. Contamination into $C$ from a distant population would make $B$ look more distantly related to $C$ than it truly is, and may lead to $C$ having similar affinity to both $A$ and $B$. Therefore, under this scenario, the null hypothesis may only be rejected after correcting for potential contamination in $C$. Similarly, the null hypothesis may be erroneously rejected if $C$ is contaminated such that it incorrectly appears more closely related to $B$.

**Determining the direction of gene flow using partitioned $D$-statistics and $D_{\text{FOIL}}$**

While the $D$-statistic is a powerful method for detecting a signal of gene flow in the absence of confounding ancestral structure between two populations, it does not detect the direction in which admixture has occurred. Consequently, inferences based on the $D$-statistic require an understanding of the population history.
underlying selected taxa in order to gain an understanding of directionality. For the partitioned $D$-statistic (Eaton and Ree, 2013) and the $D_{FOIL}$-statistics (Pease and Hahn, 2015), this requirement is relaxed because as each possible tree topology input is assessed, the methods return a specific signature that indicates not only evidence of introgression, but its direction as well.

The strength of these methods is undercut, however, by two constraints. First, data from four ingroup taxa are required for computation of these statistics, as well as an outgroup for the partitioned $D$-statistics. This requirement likely limits the amount of informative sites available for analysis compared to the $D$-statistic because once again, particular configurations of ancestral and derived states between the five taxa are needed, just as the particular configurations $abba$ and $baba$ are needed for the $D$-statistic. Further, these methods are necessarily unusable in the absence of a fourth ingroup population. This underscores the second limitation, which is that the four ingroup taxa must be related as a symmetric rooted tree with the two clades having different divergence times (Figure 1F). We nonetheless emphasize that methods for polarizing introgression represent an important update to the original $D$-statistic framework, and have the potential to provide important inferences in the population histories of humans and other organisms.

In particular, the partitioned $D$-statistic focuses on an aspect of admixture unavoidably overlooked by Patterson’s $D$-statistic. Given a set of four populations and an outgroup, called $A$, $B$, $C$, $D$, and $O$, it may appear as if introgression from a donor (say, $C$) into a recipient (either $A$ or $B$) has occurred, even though it was the sister lineage ($D$) to the presumed donor rather than the $C$ lineage itself, which introgressed into the recipient (Figure 6A). Because the sequences of $C$ and $D$ are highly similar, the $D$-statistic computed from four taxa, $D(A,B,C,O)$ or $D(A,B,D,O)$, cannot distinguish between these events (both of these would detect significant admixture). In contrast, the partitioned $D$-statistic incorporates information from five taxa, including both of the potential donors $C$ and $D$, which have not admixed with each other. Therefore, sites polymorphic among the four ingroup taxa, yielding five-population patterns such as $babaa$ and $abbaa$, are informative here. Further, only sites for which the outgroup has the ancestral allele are considered.

The partitioned $D$-statistic is so named because inferences using this method are based on the combined results of three separate five-population tests—$D_1$, $D_2$, and $D_{12}$—each of which reports part of the whole history of the studied populations. $D_1$ measures the deviation from zero between the frequency of $abbaa$-sites and $babaa$-sites, whereas $D_2$ measures the deviation of $ababa$-sites and $baaba$ sites, and $D_{12}$ measures the deviation of $abbaa$-sites and $babba$ sites. Thus,
\[ D_1 = \frac{n_{abbaa} - n_{babaa}}{n_{abbaa} + n_{babaa}} \]  
\[ D_2 = \frac{n_{ababa} - n_{baaba}}{n_{ababa} + n_{baaba}} \]  
\[ D_{12} = \frac{n_{abba} - n_{babba}}{n_{abba} + n_{babba}}. \]  

As with Patterson’s \(D\)-statistic, the partitioned \(D\)-statistics take values between \(-1\) and \(1\), and a significant \(Z\)-score for \(D_1\) or \(D_2\) (inferred once again by block jackknife) indicates introgression between one of the two putative recipient taxa, and either \(C\) or \(D\), respectively.

In addition to an equivalent interpretation, the value of \(D_{12}\) indicates the direction in which gene flow has occurred. The value of \(D_{12}\) is significantly nonzero only when the putative recipient population (either \(A\) or \(B\)) shares excess alleles in common with the putative donor lineage (that is, sites for which \(C\) and \(D\) both carry the derived allele) compared to its sister population. This suggests that these alleles came from the putative donor defined from the \(D_1\) and \(D_2\) admixture tests. If, however, introgression with \(D_1\) or \(D_2\) has been detected, but \(D_{12}\) is not significantly different from zero, then introgression occurred in the other direction, from \(A\) or \(B\) into either \(C\) or \(D\), because the presumed recipient population is the true donor, and does not share an excess of alleles common to both \(C\) and \(D\), but rather to one or the other.

Pease and Hahn (2015) expand the concept of a specialized \(D\)-statistic profile with \(D_{\text{FOIL}}\), a method that classifies the 16 possible introgressions available to a four-population symmetric tree with an outgroup (Figure 6B). For this method, the tree is not set up to imply a particular hypothesis (such as lineages \(B\) and \(C\) being admixed, or determining whether \(C\) or its sister have admixed with \(A\)), and the tests are more exploratory than Patterson’s \(D\)-statistic and the partitioned \(D\)-statistic. Detected gene flow is classified as inter-group, intra-group, or ancestral. Intergroup introgression is the standard model, wherein one lineage from one clade admixes with one lineage from the other clade. Intra-group introgression is between sister lineages of the same clade. Ancestral introgression occurs only when the divergence times of the two clades are different. Here, the ancestor to the more recently diverged populations has admixed with one lineage of the more ancestrally diverged clade.

The four \(D_{\text{FOIL}}\) statistics are \(D_{\text{FO}}, D_{\text{IL}}, D_{\text{FI}},\) and \(D_{\text{OL}}\). The subscripts of these statistics refer to the pairs of populations under comparison for a particular statistic. This principle is analogous to that of Patterson’s \(D\)-statistic in that it measures the difference in the counts of two equally probable gene tree scenarios, where a significant deviation from the expected value of zero indicates admixture. Pease
and Hahn (2015) point out that this constraint of equal probability prevents $D_{FOIL}$ from applying to an asymmetric ingroup phylogeny. For an asymmetric rooted phylogeny ($((A,B),C,D)$, $A$ and $B$ are more closely related to $C$ than to $D$ and therefore share more alleles with $C$.

For ingroup taxa with a symmetric topology ($((A,B),(C,D))$, $D_{FO}$ tests whether there is a differing count of identical sites between the “first” ($F$) pair of $A$ and $C$ and the “outer” ($O$) pair of $A$ and $D$. Similarly, $D_{IL}$ tests whether there is a differing count of identical sites between the “inner” ($I$) pair of $B$ and $C$ and the “last” ($L$) pair of $B$ and $D$. Each of these statistics tests for gene flow between an ingroup population in one clade and both of the ingroup populations in the other clade. Because multiple equally probable discordant site tree pairs underlie a symmetric ingroup population phylogeny, the computation of these statistics requires more terms than for Patterson’s $D$-statistic and the partitioned $D$-statistic. The structure of the equations is still familiar, such that

$$D_{FO} = \frac{(n_{babaa} + n_{bbbaa} + n_{ababa} + n_{aaba}) - (n_{baaba} + n_{bbaba} + n_{abbaa} + n_{aabaa})}{(n_{babaa} + n_{bbbaa} + n_{ababa} + n_{aaba}) + (n_{baaba} + n_{bbaba} + n_{abbaa} + n_{aabaa})}$$

(16)

$$D_{IL} = \frac{(n_{babaa} + n_{bbbaa} + n_{baaba} + n_{aaba}) - (n_{ababa} + n_{bbaba} + n_{babaa} + n_{aabaa})}{(n_{babaa} + n_{bbbaa} + n_{baaba} + n_{aaba}) + (n_{ababa} + n_{bbaba} + n_{babaa} + n_{aabaa})}$$

(17)

$$D_{FI} = \frac{(n_{babaa} + n_{baaba} + n_{ababa} + n_{aba}) - (n_{ababa} + n_{bbaba} + n_{babaa} + n_{baa})}{(n_{babaa} + n_{baaba} + n_{ababa} + n_{aba}) + (n_{ababa} + n_{bbaba} + n_{babaa} + n_{baa})}$$

(18)

$$D_{OL} = \frac{(n_{babaa} + n_{babaa} + n_{abbaa} + n_{aaba}) - (n_{ababa} + n_{bbaba} + n_{babaa} + n_{baa})}{(n_{babaa} + n_{babaa} + n_{abbaa} + n_{aaba}) + (n_{ababa} + n_{bbaba} + n_{babaa} + n_{baa})}$$

(19)

with each of the four statistics taking values between $-1$ and $1$. It is important to note from these equations that a major deviation from other methods is that $D_{FOIL}$ considers both shared derived states, and shared ancestral states between tested populations. For example, sites contributing positively to $D_{FO}$ include $babaa$ and $bbbaa$ (for which the first and third population share the derived state) as well as $ababa$ and $aaaba$ (for which they share the ancestral state). While Pease and Hahn (2015) propose that the significance of a $D_{FOIL}$-statistic be determined by a simple goodness-of-fit ($\chi^2$) test which takes the form $\chi^2 = (n_L - n_R)^2/(n_L + n_R)$, it is also possible to perform a block jackknife inference of significance. This is because the expectation of each statistic is zero in the absence of gene flow.

From each unique valid combination of significantly positive, significantly negative, and non-significant $D_{FOIL}$-statistics for ancestral and inter-group introgressions, it is possible to detect the direction of gene flow. As an example, consider gene flow from population $B$ into population $C$. $D_{FO}$ will be significantly positive because many alleles identical by descent between $A$ and $B$ flow into $C$. 

Pre-print version. Visit http://digitalcommons.wayne.edu/humbiol/ after publication to acquire the final version.
creating what would be a confounding situation for Patterson’s $D$-statistic (this situation is also addressed by the partitioned $D$-statistic). $D_{IL}$ is significantly positive because it directly detects the introgression we defined. $D_{FI}$, meanwhile, is significantly negative because there will be more alleles in common between $B$ and $C$ than between $A$ and $C$ because $B$ admixed into $C$. Finally, $D_{OL}$ is expected to be zero because neither $A$ nor $B$ admixed with $D$, and therefore the occurrence of sequence identity between $A$ and $D$ should match that of $B$ and $D$. Because there is no significant excess of shared alleles between clades in the case of intragroup introgression, these cannot be polarized and all $D_{FOIL}$-statistics are non-significant (see Table 1 of Pease and Hahn, 2015, for complete interpretation of possible valid results).

We conclude our discussion of the $D_{FOIL}$ method by redefining its statistics in terms of population allele frequencies, as Patterson et al. (2012) did with Patterson’s $D$-statistic. The most consequential result that emerges from this is mathematical support to show that a fifth population—the outgroup—is not necessary for the computation of the frequency-based $D_{FOIL}$-statistics. That is, any non-concordant ingroup site pattern is usable with $D_{FOIL}$, regardless of outgroup population chosen, as long as the ingroup taxa can be related as a symmetric rooted tree. Pease and Hahn (2015) also indicate the outgroup is ultimately unnecessary, but that from an experimental perspective its inclusion may be useful for determining the relative substitution rates on each branch and determining the phylogeny. We can derive frequency-based $D_{FOIL}$-statistics by sampling alleles uniformly at random according to the frequencies in each of the five populations to define the probabilities of each site pattern (analogous to the derivation of the frequency-based $D$-statistic). Based on these probabilities, we can formulate the four $D_{FOIL}$-statistics in terms of allele frequencies across $J$ loci as

$$D_{FO} = \frac{\sum_{j=1}^{J} (1 - 2a_j)(d_j - c_j)}{\sum_{j=1}^{J} (c_j + d_j - 2cjd_j)}$$

(20)

$$D_{IL} = \frac{\sum_{j=1}^{J} (1 - 2b_j)(d_j - c_j)}{\sum_{j=1}^{J} (c_j + d_j - 2cjd_j)}$$

(21)

$$D_{FI} = \frac{\sum_{j=1}^{J} (1 - 2c_j)(b_j - a_j)}{\sum_{j=1}^{J} (a_j + b_j - 2ab_j)}$$

(22)

$$D_{OL} = \frac{\sum_{j=1}^{J} (1 - 2d_j)(b_j - a_j)}{\sum_{j=1}^{J} (a_j + b_j - 2ab_j)}.$$  

(23)
where $a_j$, $b_j$, $c_j$, and $d_j$ are the derived allele frequencies at site $j$ in populations $A$, $B$, $C$, and $D$, respectively. The partitioned $D$-statistic could also be represented in terms of allele frequencies (for which the frequency for the outgroup is mathematically required), however due to its limitations relative to the $D_{\text{FOIL}}$-statistics, most notably its lower resolution in detecting all introgression types (discussed further in Pease and Hahn, 2015), we have chosen not to present analogous frequency-based formulas for the partitioned $D$-statistics.

**Other prominent tools for ancestry and admixture analyses**

Although the $f$- and $D$-statistics alone can resolve a variety of population histories and lend support to hypotheses concerning migration, admixture, and divergence, additional questions may emerge from the data that require methods tailored to detect and quantify specific attributes of these histories that the aforementioned methods either do not address or cannot distinguish. Among these are the direct ancestry test (Rasmussen et al., 2014), ROLLOFF (Moorjani et al., 2011; Patterson et al., 2012) and ALDER (Loh et al., 2013), and graph construction methods (Pickrell and Pritchard, 2012; Patterson et al., 2012; Lipson et al., 2013). These methods fit complex models to the data and provide estimates of drift, dates of admixture, and the most likely number of admixture events, implied by the covariance and correlation of alleles across sampled lineages.

**Direct ancestry test**

The direct ancestry test (Rasmussen et al., 2014) is a likelihood-based approach that quantifies the genetic drift separately along each branch since a pair of populations diverged. It was developed for a scenario in which data consist of two diploid whole-genome sequences, one of which is sampled from an ancestral population as would occur when using DNA from ancient remains. This method assumes that the two samples are representative of their populations, and tests whether the drift along the branch leading to the ancient sample is significantly different from zero. The use of a single diploid individual to represent a population is reasonable if the sample is non-inbred, as the two haplotypes within the individual should represent random draws from the population in which it was sampled.

The model underlying the direct ancestry test requires five parameters: the probability of coalescence of a pair of alleles in the first population ($c_1$) and the second ($c_2$), as well as probabilities of the three possible allelic configurations existing for four alleles sampled two each from both ancestral populations at
biallelic sites at the time of divergence \((k_{1,3}, k_{2,2}, \text{and } k_{4,0})\), such that the subscripts represent the count of one type of allele and the other, making no distinction between ancestral and derived alleles). These parameters are sufficient to describe the counts of the five configurations of sites across sampled genomes—both samples homozygous for the same allele, both homozygous for different alleles (called a fixed difference), both heterozygous (called a shared polymorphism), only the first heterozygous, or only the second heterozygous. The counts of these site configurations are denoted by \(n_{AAAA}\), \(n_{AAaa}\), \(n_{AaAa}\), \(n_{AaAA}\), and \(n_{AAAA}\), respectively.

Defining the vector of parameters as \(\theta = (c_1, c_2, k_{1,3}, k_{2,2}, k_{4,0})\), as do Rasmussen et al. (2014), the likelihood function for the direct ancestry test is defined as

\[
\mathcal{L}(\theta) = \mathbb{P}(AAAA|\theta)^{n_{AAAA}} \times \mathbb{P}(AAaa|\theta)^{n_{AAaa}} \times \mathbb{P}(AaAa|\theta)^{n_{AaAa}} \times \mathbb{P}(AaAA|\theta)^{n_{AaAA}} \times \mathbb{P}(AAAA|\theta)^{n_{AAAA}}
\]

where \(\mathbb{P}(X|\theta)\) is the probability of site configuration \(X\) given model parameters \(\theta\). While this likelihood is independent of demography in the ancestral populations, Rasmussen et al. (2014) indicate that divergence times can be inferred by coalescent theory if assumptions are made about the population size.

With the first sample from an ancestral population and the second from a descendant population, the null hypothesis of the direct ancestry test that this relationship is correct has the constraint \(c_1 = 0\), and can be tested by likelihood ratio. Rasmussen et al. (2014) suggest increasing the power of this test by ignoring C/T and G/A polymorphisms, which may be the result of post-mortem deamination events in the ancient sample (Dabney et al., 2013). Additionally, the power of the method increases when only sites that are polymorphic across strict outgroup populations are analyzed because the model assumes no new mutations since the divergence of the tested lineages. A visual representation of a result consistent with the null hypothesis of \(c_1 = 0\) is featured in Figure 7. In both of these graphs, the Ancient North American sample shows no genetic drift with the common ancestor of the modern Central and South American populations. Thus, the length of the branch between this divergence event and the Ancient North American is zero, consistent with an expected coalescence time between the two Ancient North American lineages of zero.
Graph construction methods

As we have described, statistics that measure genetic drift provide significant information about the relationships between populations. With the power afforded by these statistics, the graph construction methods assemble large phylogenies with topologies more complex than what the $f$- and $D$-statistics alone can produce. Using allele frequency data, TreeMix (Pickrell and Pritchard, 2012) and MixMapper (Lipson et al., 2013)—two widely-used graph construction approaches—create best-fit admixture graphs to explain relationships among sampled populations. With these methods, it is possible to visualize the networks of migration and gene flow that underlie global human diversity in an efficient and intelligible manner.

TreeMix

TreeMix is a maximum-likelihood method introduced by Pickrell and Pritchard (2012) that infers the phylogenetic relationship between taxa in the form of a directed acyclic graph (Figure 7) for a set of study populations. However, the method does not provide divergence times in years or generations and instead focuses on building the network with branch lengths that best fit the data (thereby outputting inferred drift measurements for branch lengths). This method may be applied to allele frequency data at biallelic loci, and has been extended to microsatellite data (Pickrell and Pritchard, 2012) using the same framework as for the $f$-statistics to which we alluded in previous sections. The power of TreeMix in the study of ancient admixture lies in its ability to corroborate the results of other methods, providing a visual representation of the histories suggested by analysis with the $f$- and $D$-statistics while adding complementary evidence for these inferences as well. Additionally, TreeMix allows users to explore various alternative admixture scenarios by evaluating the fit of the data to graphs without migration events, as well as with a user-specified number of admixture events.

Assuming neutral evolution (i.e., absence of selection), TreeMix models allele frequencies at biallelic loci across a set of populations according to a multivariate normal distribution. Descendant populations have the same mean allele frequencies as their ancestor, and the covariance in allele frequency between sampled pairs of descendant populations increases proportionally to the genetic drift that they share relative to their ancestor. For $n$ sampled populations, TreeMix stores these values as a $n \times n$ matrix. For microsatellite data, the assumption is that the population mean lengths of microsatellites are distributed as a multivariate normal, also with a covariance matrix whose entries are the shared genetic drift between pairs of populations. Treating allele frequencies and mean microsatellite lengths in the same manner, the computations underlying both applications of TreeMix are identical, and so we will continue to refer to allele frequencies in our discussion of this method.
The population covariance matrix under a particular model relating the set of populations is fit to the sample covariance matrix using maximum likelihood through an iterative approach. That is, the population covariance matrix is not directly estimated from the data. Each iteration of TreeMix begins with the creation of a maximum-likelihood tree formed from three randomly-selected populations, to which each remaining population is randomly added. The fit of the proposed population covariance matrix given user data is evaluated for various rearrangements of the tree following a greedy approach. The population covariance matrix is additionally fit for the number of migration edges that minimizes the magnitude of the residuals, which are assembled as the residual covariance matrix.

We have provided examples of what TreeMix output may look like in Figure 7. For these graphs, we illustrate a global human phylogeny featuring samples from one African, five western European, two east Asian, two northwestern Native American (ancient unadmixed and modern admixed), two southern Native American, one ancient Siberian, and one ancient Native American populations. In Figure 7A, we display the relationship among these populations that may have been inferred for a history in which no migration has occurred between any of the lineages. While the residual covariance matrix (not depicted) would indicate that an edge between the European and the admixed Northwest American lineages would improve the fit of this graph, it is clear as well from the position of the admixed Northwest American population that the tree is incorrect. This population derives most of its ancestry from the American lineage, but its sequence identity with the European lineage is large enough that it appears ancestral to all other American groups.

We contrast this graph and Figure 7B, which provides an example of the most likely history for these populations, wherein gene flow has occurred from Europe to the admixed Northwest American population, which is a sister lineage to the unadmixed Northwest American population. We note that it is also possible to perform a likelihood ratio test between the admixture and non-admixture scenarios to assess whether adding a particular number of admixture edges produces a significantly better fit to the data, because the former model is nested within the latter.

For high quality modern samples, TreeMix graphs are generally easy to interpret as measures of population differentiation. In the case of ancient samples, this is moderately more nuanced. When an ancient sample of high quality is included among modern samples for analysis, the ancient sample may appear at the end of a vertical branch in the graph, as is the case with the ancient North American sample in Figure 7. In conjunction with the direct ancestry test, this result indicates that the sampled population may be a direct ancestor to the descendants of the lineage in which it appears, since it has not drifted from that branch (alternatively, it may be so closely related to the true ancestor that it has not appreciably diverged from it).
In contrast, branch lengths may appear inflated in low quality ancient samples (as with the low coverage ancient Siberian in Figure 7), for which only a single allele (rather than a diploid genotype) is called at each genomic site. Due to the abundance of sites for which only one allele is called, it appears as if there is excess homozygosity in the branch leading to this population, and it appears greatly diverged from the closest internal node, though it is still assigned to the proper branch. Therefore, the length of the branch should not be interpreted when including low-coverage samples in this manner, but the placement relative to other populations remains informative.

MixMapper

The other commonly-encountered tools for graph construction in admixture inference and quantification are *qpGraph* (Patterson et al., 2012) and its extension *MixMapper* (Lipson et al., 2013, 2014). While similar insights emerge from both *MixMapper* and *TreeMix* (Pickrell and Pritchard, 2012), the mathematical architecture of each method is unique. Broadly, *MixMapper* (designed as a generalization of *qpGraph*, introduced in Patterson et al., 2012) first builds a tree relating unadmixed populations to one another, and then incorporates admixed populations onto this tree. Admixed and unadmixed populations must be defined *a priori*, whereas this is not the case for *TreeMix*. For this reason, the authors recommend applying *MixMapper* to a set of specific study populations, and *TreeMix* to the construction of larger admixture graphs.

*MixMapper* uses genotype data to generate allele frequency values across all sites (though formulations for microsatellite data should be possible due to its reliance on \(f\)-statistics; see Pickrell and Pritchard, 2012). From these frequencies, the \(f_2\)-statistic (Patterson et al., 2012) can be calculated and used to infer branch lengths and proportions of admixture for a graph relating sampled populations. The graph is prepared in two stages. First, a neighbor-joining tree of unadmixed populations is prepared according to the values of all paired \(f_2\)-statistics for these populations. Valid subtrees for the neighbor-joining tree have branch lengths that are additive, indicating no admixture. The admixed populations are determined by testing with \(f_3\), wherein population \(A\) is considered admixed from \(B\) and \(C\) if \(f_3(A; B, C) < 0\). Second, the most optimal placements for admixed populations is determined, along with the proportion of admixture from each contributing lineage. In addition to detecting admixture between two lineages, *MixMapper* detects three-way admixture by fitting gene flow events between an admixed population and another (non-admixed) population. As with *TreeMix*, branch lengths are calculated in drift units, and this allows *MixMapper* to display the point of admixture between lineages in the output graph.
Because the computations performed by MixMapper rely on the theory of the \(f\)-statistics (which Lipson et al., 2013, call the allele frequency moment statistics), this method can be understood as an extension of the \(f\)-statistics to more populations. Consequently, MixMapper analysis reduces to the \(f_4\) ratio for five populations arranged as in Figure 1D. MixMapper also supplants \(qpGraph\) for ancestry inference in large datasets because unlike \(qpGraph\), MixMapper does not require a proposed graph topology as input, or any prior knowledge of population relationships except for assignment as admixed or unadmixed (though the authors point out that the output of \(qpGraph\) may be more precise). MixMapper may also provide more accurate admixture graphs than TreeMix because it requires this additional level of user input. TreeMix starts with a maximum-likelihood tree of three populations from the dataset without considering whether any of these three populations is related to the others in a treelike manner, and therefore the most likely topology inferred from multiple iterations of this process can be incorrect. Lipson et al. (2013) found this to occur especially frequently for cases of three-way admixture, and therefore caution that each graph construction method may be most appropriate in a particular situation. Ultimately, the choice of graph construction method is most likely to depend on the level of user prior knowledge, and assumptions about the complexity of the demography underlying populations of interest.

**Dating the time of admixture**

So far, we have discussed a number of approaches for detecting admixture, measuring levels of admixture, determining the number of admixture events, and identifying sets of source populations contributing to admixed populations. However, to obtain a complete picture of the admixture history of a population, it is equally important to also know when such admixture occurred. In this section, we discuss two related approaches, ROLLOFF (Moorjani et al., 2011) and ALDER (Loh et al., 2013), which leverage measures of genetic drift and linkage disequilibrium to make inferences about the timings and levels of admixture events.

**ROLLOFF**

Both ROLLOFF (Moorjani et al., 2011) and ALDER (Loh et al., 2013) are able to infer the date of gene flow into a population by modeling the signature of decay in linkage disequilibrium between a pair of sites located on the same chromosome as the distance between these sites increases. The occurrence of LD decreases between more distant sites because the genetic recombination events that occur each generation result in the disassociation of specific alleles with one another, and the occurrence of at least one recombination event is more likely for a larger genomic
region. Thus, genomic tracts in which a pair of selectively neutral markers are found together within a haplotype reduce in size over time since admixture and can be used to determine the date of admixture. Therefore, LD-based inference methods are most powerful for dating recent admixture, on the order of $10^4$ years, though this power increases for increasing sample sizes and accordingly decreases (yielding biased estimates) for smaller sample sizes (Moorjani et al., 2011; Patterson et al., 2012; Loh et al., 2013).

ROLLOFF (Moorjani et al., 2011) is the original application of LD inference to estimate the date at which two populations mixed. Each test requires data from three populations—the product of this admixture ($A$, the target of the test), and a population from each of its progenitor lineages ($B$ and $C$)—forming a relationship as in Figure 1B. This method assumes that the signature of admixture is homogenous in population $A$ and that the admixture event occurred in a single pulse. ROLLOFF works with unphased diploid genotype data and fits the decay of LD for sites $X$ and $Y$ separated by a genetic distance $d$ to a model of exponential decay by least-squares.

For two alleles $X_a$ and $Y_a$ drawn from an individual in population $A$ at sites $X$ and $Y$, the probability after $n$ generations that $X_a$ and $Y_a$ originated from the same haplotype is $e^{-nd}$, and the observed correlation of alleles as a function of their genetic distance, the weighted LD statistic $A(d)$, is approximately the result of decay from the initial state $A_0$ such that $A(d) \approx A_0 e^{-nd}$. Here, the value of $A(d)$ depends on the weight of the polymorphic site (positive if the frequency of a reference allele is greater in population $B$ than $C$, and negative if the frequency is greater in $C$ than $B$) and an LD-based score resulting from Fisher’s $z$-transformation of the Pearson correlation in reference allele counts between sites $X$ and $Y$ (see Patterson et al., 2012 for relevant equations). The LD-based score is also used to normalize $A(d)$. The stability of the estimated mixture date is conservatively tested by jackknife with chromosome-sized blocks, with each replicate weighted by the number of excluded SNPs.

We note that while fitting decay in LD to an exponential function yields results that are concordant with the historical record (Moorjani et al., 2011; Patterson et al., 2012), a single-exponential model is likely too simple to adequately resolve more complex admixture histories (Liang and Nielsen, 2016). This is also the case for the assumption of a single admixture pulse, although the authors have shown that the inferred date of admixture lies within the true period of admixture for multiple or continuous mixing between populations (Patterson et al., 2012). However, the robustness of ROLLOFF in non-ideal scenarios means that this method is still widely applicable. Patterson et al. (2012) indicate that ROLLOFF is accurate for mixture up to 500 generations old, assuming a generation time of 29 years (this result was obtained for 20 individuals genotyped at 378,000 SNPs). Additionally, if populations appropriately representative of the two admixing lineages $B$ and $C$...
are unavailable, even populations highly divergent from their admixing ancestors are usable for ROLLOFF computations. In general, genetic drift of populations A, B, and C since admixture has little effect on the accuracy of ROLLOFF.

**ALDER**

The concept of LD-based inference, introduced in ROLLOFF and expanded by Pickrell et al. (2012), was extended to a variety of different applications as ALDER (Loh et al., 2013). Broadly, ALDER begins from the same theoretical framework as ROLLOFF, but further derives the formula for the expectation of $A(d)$ into a form dependent on the mixture proportions ($p$ and $1 - p$) of each admixing lineage and the square of $f_2(B, C)$, as well as the scaling factor $e^{-nd}$. Additionally, it is orders of magnitude faster than ROLLOFF due to the implementation of a fast Fourier transform to calculate $A(d)$. Furthermore, because this formulation is not normalized by the LD-based score as is ROLLOFF, it does not risk introducing bias for samples from populations that are large, recently admixed, or experienced a strong bottleneck (Loh et al., 2013).

Another advantage of ALDER over ROLLOFF is that it can compute dates of admixture from a single reference ($B$ or $C$) and the admixed population ($A$) when one admixing population is unavailable or unknown, by simply changing the weights assigned to $p$ and $1 - p$ in the calculation. ALDER is also able to fit the data to an inferred weighted LD curve that does not tend to zero with increasing distance between sites. This feature is required in cases for which nonrandom mating violates the basic model assumption of homogeneity in the sampled populations. ALDER must also correct for background levels of LD that would otherwise confound inference, and so the weighted LD curve is fitted for pairs of loci far enough apart from one another that the signal of background LD shared between the target and references is negligible.

Since the formulation of $A(d)$ in ALDER directly contains a measure of mixture proportion from each admixing lineage, it is possible to solve for one of these values using the amplitude ($A_0$) of $A(d)$, which is the point at which the LD curve $A(d)$ intersects the y-axis (i.e., $A(0)$). This takes the form $A_0 = 2p(1 - p)f_2(B', C')^2$ for an admixture scenario as in Figure 1B (Loh et al., 2013), where $B'$ and $C'$ represent populations ancestral to present-day populations $B$ and $C$, respectively. The value of $f_2(B', C')$ can be determined from an admixture graph (see section Graph Construction Methods). The weighted LD statistic also provides information as to whether population $A$ is truly admixed from $B$ and $C$, similarly to the function $f_3(A; B, C)$. Loh et al. (2013) point out that weighted LD interestingly complements the application of $f_3$. Unlike $f_3$, $A(d)$ still detects admixture even if the admixed population has experienced extensive drift (measures relating to this drift are not part of the computation). The $f_3$-statistic does,
in contrast, maintain greater power to detect more ancient admixture events because it does not detect a quantity whose signal decreases in intensity over time.

Although the detection of a positive weighted LD signal for population A computed from B and C generally indicates that A is admixed (once again, confounding demographic events in A are not considered here), shared confounding demography between A and one of the references (say, B) such as a bottleneck can lead to incorrect inference that A is admixed for the computation of $A(d)$ from B and C, and from A and C. However, $A(d)$ computed with references A and B will properly yield no positive weighted LD signal and indicate that A is not admixed (Loh et al., 2013).

Finally, ALDER can infer phylogenetic relationships underlying the weighted LD curve generated from modern populations A, B, and C. If A is admixed from a pair of populations $B''$ and $C''$, which are respectively descendants of $B'$ and $C'$ (Figure 1B, not directly labeled), then it is possible to set up computations for $A(d)$ using fixed modern reference B and multiple test references C and examine changes in curve amplitude. Larger amplitudes imply a closer branch point $C'$ (Loh et al., 2013). In the absence of more sophisticated models of ancient admixture, we expect that the inferences emerging from ALDER analysis will continue to provide insights that enhance both the broad perspectives afforded by the graph construction methods, and the specific perspectives afforded by test statistics such as $f_3$, $f_4$, and $D$.

A brief summary of major results and conclusions emerging from these genetic drift measurement methods

We conclude our review by illustrating the advances that the admixture and ancestry inference methods we discussed have made. Using the resolution afforded by these tools, as well as the emergence of myriad ancient and modern human genomic data, researchers have uncovered and characterized the mixtures and movements of ancient humans. We highlight here discoveries of gene flow between anatomically-modern humans and archaic hominins, and of the complex and unexpected patterns of worldwide migration that ancient humans undertook, and the signatures of these events that remains in the genomes of their descendants.

Following extensive speculation about the likelihood of admixture between the ancestors to modern humans and Neanderthals (Plagnol and Wall, 2006; Wall and Hammer, 2006; Noonan et al., 2006; Wall and Kim, 2007; Hodgson and Disotell, 2008; Wall et al., 2009), Green et al. (2010) applied the $D$-statistic to this question and found convincing evidence that these two populations did interbreed outside of Africa at least once (but see Kim and Lohmueller, 2015, and Vernot and Akey, 2015, for a discussion of more complex models to describe this admixture). This
interbreeding left a significant trace of Neanderthal-specific variants in the genomes of non-African humans that initially may have been selectively disadvantageous, which is why these variants are especially found in non-coding regions (Harris and Nielsen, 2016; Juric et al., 2016). Since the initial emergence of the $D$-statistic as an answer to the question of admixture between modern humans and Neanderthals, further evidence has emerged to support this hypothesis. The estimated time of interbreeding between humans and Neanderthals is between 37,000 and 86,000 years ago (Sankararaman et al., 2012), and the discovery of an ancient human living in Romania during this period with a recent Neanderthal ancestor (Fu et al., 2015) fits well within this framework. Additionally, the complete genome of a female Neanderthal individual (Prüfer et al., 2014) once again indicated a significantly elevated relatedness between modern humans and Neanderthals.

Modern humans have also admixed with the archaic human population whose remains were found in Denisova Cave (Reich et al., 2010). With evidence from the $D$-statistic, the authors discovered that only Melanesians retain significantly elevated Denisovan ancestry. Subsequent studies showed that Denisovan ancestry exists outside of Melanesian populations as well, with signatures of Denisovan introgression found in the Philippines (Reich et al., 2011) as well as South Asia (Sankararaman et al., 2016). This indicates that despite their status as a sister population to Neanderthals, the population history of Denisovans with anatomically-modern humans may be quite different.

However, overlap in Denisovan- and Neanderthal-depleted tracts in the genomes of non-African human populations suggest that the introgression of Denisovan genes was similarly deleterious to the recipients of this gene flow (Vernot et al., 2016). Interestingly, though, there are examples of adaptive introgression between Denisovans and humans. Huerta-Sánchez et al. (2014) identified an $EPAS1$ haplotype private to Denisovans and Tibetans associated with high-altitude adaptation in the latter population. Racimo et al. (2015) additionally review adaptive introgression and present functions of introgressed Neanderthal and Denisovan genomic segments including immunity, skin pigmentation, and metabolism. Racimo et al. (2016a) further expand upon these examples and name specific genes historically under selection. The relationship between ancient human, Denisovan, and Neanderthal populations is likely quite complex and may feature more currently undiscovered admixing hominins (Prüfer et al., 2014) and may not be resolved without more complex models from which to make inferences.

The population history of humans after the extinction of Neanderthals is complex as well, indicating that modern populations are the result of various admixture events following the migration and colonization of new territory. Applications of the $f_3$, $f_4$, and $D$-statistics to the genomes of modern and ancient Native American populations have demonstrated that the New World was colonized by Asian populations on multiple occasions, and that these groups certainly
admixed with one another. Around 23,000 years ago at most, and possibly as late as 16,000 years ago, the ancestor to Native Americans arrived to North America from Siberia via Beringia, and diverged into a lineage ancestral to only North American populations (including the PRH ancients of Lindo et al., 2016), and to a lineage ancestral to both North and South American populations, to which Anzick-1 (Rasmussen et al., 2014) belonged (Raghavan et al., 2015; Llamas et al., 2016). The discovery and analysis of a genome for a 24,000-year ancient Siberian (MA-1) suggests, however, that the ancestor to Native Americans was admixed from both East Asian and Eurasian sources, explaining the presence of pre-Columbian signatures of European ancestry in these populations (Raghavan et al., 2014a). An additional population closely related to Australasians has been proposed as an ancestor to Amazonian Native Americans specifically (Skoglund et al., 2015). Raghavan et al. (2015) also detected this ancestry in Native Americans, and suggest that it arrived to the New World through Beringia as well.

The population history of Western Eurasia appears no less complex than that of the Americas. The first wave of humans into Europe arrived approximately 45,000 years before the present, though these populations did not contribute to the genomes of modern Europeans. Rather, two subsequent migrations into Europe comprise the majority of the genetic constitutions of modern Europeans (Fu et al., 2016). Results from Lazaridis et al. (2014) indicate that modern Europeans descend from the mixture of western European hunter-gatherers, northern Eurasians closely related to MA-1, and eastern European farmers of Near Eastern ancestry. The Near East itself was appreciably heterogeneous upon initial colonization by modern humans. The first farming civilizations in this region displayed strong genetic differentiation that dissipated after a period of mixture (Lazaridis et al., 2016).

Although the complex histories of human populations both locally and globally suggest that current methods may miss important differentiating signatures between sampled lineages, ignoring complexity is less likely to bias results than is ignoring contamination. This is because the signature of contamination is analogous to the signal of admixture, thus making two lineages appear genetically more related to one another than they actually are. Furthermore, some of the statistics which we have reviewed are more robust to contamination than others. For example, an outgroup $f_3$-statistic of the form $f_3(O; A, B)$ yields the correct relationship between tested populations as long as contamination into the ingroup taxa does not increase their apparent pairwise distance to a value greater than their pairwise distance with the outgroup. This result is illustrated in Figure 8, in which the contamination rate $\gamma$ in an ancient sample needs to be prohibitively high to mislead the outgroup $f_3$-statistics. In addition, graph construction methods may be able to account for contamination by directly modeling an admixture edge between the source of the contamination and the recipient. Further, their outputs may show clearly misleading branch points in the presence of uncorrected contamination (similar to unmasked
admixture), as in Figure 7. Ultimately, the increasing availability of high-quality ancient genomes means that measures of population genetic drift will continue to play a central role in characterizing our rich history, and foster an increasingly complete view of the networks that shaped it.

Acknowledgments

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References


Table 1: Summary of methods for inferring admixture and ancestry from measures of genetic drift

<table>
<thead>
<tr>
<th>Method</th>
<th>Application</th>
<th>Test of significance</th>
<th>Limitations</th>
<th>Reference</th>
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<tbody>
<tr>
<td>$f_3$</td>
<td>Test of whether a target population is admixed; measurement of shared ancestry in two populations; <strong>allele frequencies or sequence data</strong></td>
<td>Weighted block jackknife</td>
<td>Large drift in the admixed population may mask the signal of its admixture; putative genetic donor population may be incorrectly identified if it is closely related to the true donor</td>
<td>Reich et al. (2009) applied $f_3$ to characterize admixture in Indian populations; Raghavan et al. (2014) used outgroup $f_3$ to quantify the Western Eurasian-Siberian ancestry of Native Americans; Peter (2016) redefined $f_3$ in terms of coalescence times</td>
</tr>
<tr>
<td>$f_4$</td>
<td>Test of treeness of 4 species; quantification of admixture proportion; inferring the number of admixture events; <strong>allele frequencies or sequence data</strong></td>
<td>Weighted block jackknife</td>
<td>$f_4$ can be zero, suggesting no admixture, if the target admixed population descends equally from two donors</td>
<td>Reich et al. (2009) used $f_4$ to identify and quantify admixture proportions in Indian populations; Reich et al. (2012) demonstrated that Native American population history is consistent with at least 3 migrations from East Asia using $f_4$ (as qpWave)</td>
</tr>
<tr>
<td>$h_4$</td>
<td>Test of treeness; <strong>phased haplotype frequencies</strong></td>
<td>Weighted block jackknife</td>
<td>Length of blocks needs to be determined <em>a priori</em> to incorporate sufficient polymorphism (in addition to limitations of $f_4$)</td>
<td>Applied in Skoglund et al. (2015) to quantify the relatedness of various global human populations</td>
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Table 1: Summary of methods for inferring admixture and ancestry from measures of genetic drift (continued)

<table>
<thead>
<tr>
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<th>Test of significance</th>
<th>Limitations</th>
<th>Reference</th>
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<tbody>
<tr>
<td>Patterson’s $D$</td>
<td>Model-based test for introgression between candidate populations; sequence data or allele frequencies</td>
<td>Weighted block jackknife</td>
<td>Results do not imply a direction of gene flow; method cannot distinguish between ancestral population structure and introgression; ability to infer significance depends on the number of informative sites available; can be misled by contamination (also applies to $f_4$ and other $D$-statistics)</td>
<td>Used by Green et al. (2010) to support the hypothesis that Neanderthals interbred with non-African humans</td>
</tr>
<tr>
<td>Partitioned $D$</td>
<td>Test for introgression with polarized direction of gene flow; sequence data (and implicitly, allele frequencies)</td>
<td>Weighted block jackknife</td>
<td>Detects few introgression types compared to $D_{FOIL}$; the number of informative sites available for this test is smaller than for $D$ (also applies to $D_{FOIL}$); requires 4 ingroup taxa related as a symmetric tree (as does $D_{FOIL}$)</td>
<td>Applied by Eaton and Ree (2013) to show extensive introgression in the plant family Orobanchaceae</td>
</tr>
<tr>
<td>$D_{FOIL}$</td>
<td>Improved polarized test for introgression across more gene flow types than the partitioned $D$-statistic; sequence and allele frequency data</td>
<td>Weighted block jackknife or $\chi^2$</td>
<td>More complex to calculate than other $D$-statistics; intragroup introgressions not detectable; invalid combinations of $D_{FOIL}$ statistics cannot be interpreted</td>
<td>Pease and Hahn (2015) develop $D_{FOIL}$ with simulated data, and apply it to mosquito population genetic data in Fontaine et al. (2015)</td>
</tr>
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</table>
Table 1: Summary of methods for inferring admixture and ancestry from measures of genetic drift (continued)

<table>
<thead>
<tr>
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<th>Limitations</th>
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<tbody>
<tr>
<td>Direct ancestry test</td>
<td>Measurement of genetic drift separating two lineages; diploid whole-genome sequence data</td>
<td>Likelihood ratio test</td>
<td>C/T and G/A polymorphisms should be filtered out; only sites that are polymorphic across strict outgroup populations should be tested</td>
<td>First used by Rasmussen et al. (2014) to demonstrate that Anzick-1 belonged to the population ancestral to modern Central and South Americans, or to a closely-related population</td>
</tr>
<tr>
<td>TreeMix</td>
<td>Inference of phylogenetic relationships between populations, of which some may be admixed (better suited for exploratory investigations); allele frequency data, biallelic and microsatellite loci</td>
<td>Measurement of residual covariance or likelihood ratio</td>
<td>Low-quality samples are assigned a spuriously longer branch length on the graph; user must specify number of admixture events to fit to the data</td>
<td>Pickrell and Pritchard (2012) develop and applied TreeMix to canine and human data, uncovering new admixture networks</td>
</tr>
<tr>
<td>MixMapper</td>
<td>Inference of phylogenetic relationships between populations, of which some may be admixed (better suited for classifying a specific, known set of study populations); allele frequency data (biallelic loci, and implicitly microsatellites)</td>
<td>Weighted block jackknife or measurement of residual covariance</td>
<td>User must know a priori whether populations are admixed or not</td>
<td>First applied to genome-wide single-nucleotide polymorphism data from worldwide human populations in Lipson et al. (2013), revealing that previously-unknown gene flow between southern Europe and northern Eurasia</td>
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Table 1: Summary of methods for inferring admixture and ancestry from measures of genetic drift (*continued*)

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>ROLLOFF and ALDER</td>
<td>Approximating date of admixture; estimating admixture proportion; <em>unphased diploid genotype sequence data</em></td>
<td>Weighted block jackknife (test of estimate stability)</td>
<td>Reduced power to detect older admixture events; demographic events shared between a subset of tested populations can lead to spurious inference of admixture</td>
<td>First used in <em>Moorjani et al. (2011)</em> (ROLLOFF) to demonstrate recent admixture from Africa into Eurasia and <em>Loh et al. (2013)</em> (ALDER) to uncover new details about the admixture history of various global populations</td>
</tr>
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Figure 1: Phylogenetic tree and admixture graph topologies representing important theoretical and practical scenarios for \( f \)- and \( D \)-statistic applications. (A) Unrooted three-population phylogeny showing the relationship between populations \( A \), \( B \), and \( C \) wherein \( A \) descends from the common ancestor of \( A \), \( B \), and \( C \). The \( f_3 \)-statistic traces the marked paths and measures only the overlap of these paths, which is the length of the branch connecting \( A \) to the internal node. (B) Three-population admixture graph showing the relationship between populations \( A \), \( B \), and \( C \) wherein population \( A \) derives a proportion \( p \) of its ancestry from the lineage of population \( B \) (specifically population \( B' \)) and a proportion \( 1 - p \) of its ancestry from the lineage of population \( C \) (specifically \( C' \)). Branch lengths \( i \) through \( vii \) represent drift between ancestral and descendant nodes. (C) Unrooted four-population phylogeny showing the relationship between populations \( A \), \( B \), \( C \), and \( D \) wherein the former pair represent a cluster, and the latter pair represent a cluster. This tree is consistent with the result \( f_4(A, B; C, D) = 0 \). (D) Five-population admixture graph to which the \( f_4 \)-ratio test is applied. Population \( A \) derives a proportion \( p \) of its ancestry from population \( C' \) and \( 1 - p \) of its ancestry from population \( D' \), belonging to the lineages of populations \( C \) and \( D \), respectively. (E) Asymmetric four-population tree rooted to chimpanzee outgroup, to which the \( D \)-statistic was originally applied in Green et al. (2010). (F) Outgroup-rooted five-population tree with symmetrically-related ingroups and a different divergence time for \( A \) and \( B \) than for \( C \) and \( D \). The partitioned \( D \)-statistics (Eaton and Ree, 2013) and \( D_{\text{FOIL}} \) (Pease and Hahn, 2015) apply to this topology. Adapted from Reich et al. (2009), Patterson et al. (2012), Green et al. (2010), Eaton and Ree (2013), and Pease and Hahn (2015).
Figure 2: Practical visualizations of the $f_3$- and $f_4$-statistics. (A) Representation of the four allele drift trajectories available to a three-population admixture graph. Two alleles are drawn for a locus in population $A$ and their overlap in each configuration additively contributes to the value of $f_3(A;B,C)$. The path colored in red traces the drift between the first drawn allele of $A$ and population $B$, while the path colored in blue traces the drift between second drawn allele of $A$ and population $C$. (B) Unrooted four-population trees showing the traced drift paths for $f_4(A,B;C,D)$ for the three possible topologies. The red arrow traces the genetic drift between $A$ and $B$ and can also be interpreted as $f_2(A,B)$, while the blue arrow traces the genetic drift between $C$ and $D$ and can be interpreted as $f_2(C,D)$. The value of $f_4(A,B;C,D)$ is zero for the first tree, positive for the second, and negative for the third. In the case of the latter two trees, the magnitude of $f_4(A,B;C,D)$ is equivalent to the length of the internal branch. (C) Five-population admixture graph indicating drift paths underlying the computation of $f_4(B,O;A,D)$. Note that $f_4(B,O;A,D) = pf_4(B,O;C,D)$ because the drift path tracing the $C'$-derived ancestry of $A$ with $D$ (red) overlaps with the drift path between $B$ and $O$, while the drift path tracing the $D'$-derived ancestry of $A$ with $D$ (blue) does not. Adapted from Reich et al. (2009) and Patterson et al. (2012).
Figure 3: $f_3$ biplots demonstrating an application of the outgroup $f_3$-statistic to global human microsatellite data (Pemberton et al., 2013). Each axis of each plot is set up as $f_3$(Yoruba; $W$, $X$) and measures the divergence of populations $W$ and $X$ with the outgroup Yoruba population (YRI). Population $W$ is fixed for each axis of each plot, while population $X$ is one of various European (blue), Middle Eastern (yellow), Central/South Asian (red), Oceanian (green), East Asian (pink), Native American of any latitude (purple), or admixed (gray) samples. Larger values of $f_3$(Yoruba; $W$, $X$) reflect greater proportions of shared ancestry between $W$ and $X$.

(A) Measures common ancestry between population $X$ and the Pima against the common ancestry of population $X$ and the Huilliche, both Central American populations. (B) Measures common ancestry with Han (East Asian) against Sardinian (European). (C) Measures common ancestry with Han against Australian (admixed). (D) Measures common ancestry with Karitiana (South American) against Australian. Each of these plots yields a unique clustering of global human populations from which their affinity may be inferred.
Figure 4: Simulations demonstrating the increasing power of the $D$-statistic to reject the null hypothesis that the tree topology is correct, as the number of available informative sites increases. (A) Three scenarios relating an ancient sample to a pair of modern samples and a sequence from an outgroup species (e.g., chimpanzee). The scenarios represent situations in which the ancient sample is a direct ancestor of modern population 2 (Scenario 1), is equally related to modern populations 1 and 2 (Scenario 2), or is a member of a population related to modern population 2 (Scenario 3). We use $D(M_{1},M_{2},A,Chimp)$ to test the null hypothesis that the ancient sample is equally related to the two modern populations (Scenario 2), and we are specifically interested in the ability of Scenarios 1 and 3 to reject the null hypothesis with a positive $D$-statistic. Each of the following distributions is based on $10^3$ simulated replicates. (B) With a small number of $D$-statistic informative sites (across $2 \times 10^2$ sequences of 100 kilobases in length), the null hypothesis was erroneously rejected with a positive $D$ for Scenario 2 in only 4 times, and Scenarios 1 and 3 rejected the null with a positive $D$ only 16 and 13 times, respectively. (C) Increasing the number of informative sites by an order of magnitude (across $2 \times 10^3$ 100 kilobase sequences), the distributions of Scenarios 1 and 3 are shifted to higher $Z$ scores, as expected based on the placement of the ancient sample. However, the majority of the distributions fall within $|Z| < 3$. Scenarios 1 and 3 only reject the null with a positive $D$ 195 and 53 times, respectively, and therefore the null hypothesis is often still not rejected. (D) Further increasing the number of informative sites by an order of magnitude (across $2 \times 10^4$ 100 kilobase sequences) shifts the majority of the distribution for Scenario 3 (712 replicates) and almost all the distribution of Scenario 1 (999 replicates) past the significance threshold $Z \geq 3$. These results show that if the relationships among populations are close enough, then it may be difficult to reject the null hypothesis of the $D$ statistic with low-coverage ancient samples, for which there may be few
$D$-statistic informative sites. Simulations were performed using FastSimCoal2 (Excoffier et al., 2013), with a single sequence sampled from each of the four populations.
Figure 5: Representation of a contamination scenario similar to the one addressed in Raghavan et al. (2014b). (A) When an ancient DNA sample (Ancient) is contaminated during handling by researchers (European) such that a proportion $\gamma$ of the inferred genetic sequence of the ancient sample is actually from the researchers, the observed magnitude of $D$ (East Asian, American, Ancient, Chimpanzee) may be smaller than the true value. This contamination results from the reduction of $abba$-sites as a proportion of total informative sites ($n_{abba} + n_{baba}$) across the sampled genomes. (B) After contamination, the apparent topology of the tree relating these populations now supports the Ancient sample as basal to East Asians and Native Americans, due to an increased presence of European-specific variants in the Ancient sample.
Figure 6: Illustration of the introgression which can be detected by the partitioned $D$-statistic and $D_{\text{FOIL}}$. (A) The partitioned $D$-statistic is able to resolve whether a signal of gene flow between populations $B$ and $C$ (black) is real, or due to gene flow between populations $B$ and $D$ (gray); the inference can be made regardless of the direction of gene flow. Patterson’s $D$-statistic may spuriously infer admixture if only one of populations $C$ and $D$ is sampled, due to their shared ancestry which is necessarily shared by $B$ if it is a recipient of gene flow from their lineage. (B) The $D_{\text{FOIL}}$-statistics can distinguish between ancestral (blue) and intergroup (red) introgression events for any direction of gene flow between populations $A$, $B$, $C$, and $D$ (and for this phylogeny, the ancestor to $A$ and $B$ with either the $C$ or $D$ lineage), and additionally identify intragroup introgression (gray), though without polarizing it.
Figure 7: Schematic illustrating the behavior of TreeMix for various global human populations. Important samples are located on the branch diverging from the East Asian lineage. These are the low-coverage ancient Siberian sample (MA-1 from Raghavan et al. (2014b), for which only a single allele is sampled at each site), the high-coverage ancient North American sample (Anzick-1 from Rasmussen et al. (2014), for which genotypes have been called), and the unadmixed and admixed Northwest American samples (ancient and modern, respectively, from Lindo et al. (2016), for which genotypes have been called). (A) Example assuming no migration events in TreeMix. The low-coverage ancient sample exhibits a long branch, indicating extensive genetic drift because all sites are called as homozygous, and so the sample appears completely inbred. This branch length should not be interpreted, but the position in which it has diverged relative to the tree is correct. The high-coverage ancient North American sample exhibits no genetic drift because it is either the direct ancestor of the Central and South American populations, or a member of a population closely-related to a direct ancestor of the Central/South American populations. This result can be used to support findings from the direct ancestry test. The admixed Northwest American population also has no genetic drift along its branch, which appears to indicate direct ancestry. (B) Accounting for a single admixture (migration) event with Europeans, the admixed Northwest American population shifts from being ancestral to all samples from the Americas to a sister population to the unadmixed Northwest American population. This result suggests that the lack of genetic drift along the branch leading to the admixed Northwest American population was likely due its genetic profile lying intermediate between the unadmixed Northwest American population the Europeans.
Figure 8: Robustness of outgroup $f_3$-statistics to contamination. This tree relates an outgroup population (O) and four ingroup populations A, B, C, and D. Population A is admixed, and derives a fraction $p$ of its ancestry from the ancestral lineage of D. In addition, an ancient sample belonging to a population directly ancestral to B is contaminated by modern population D, with a fraction $\gamma$ of its genome sequence deriving from this contamination. Branch lengths $i$ through $v$ represent lengths in drift units. If outgroup $f_3$-statistics are robust to contamination, then it is important that they indicate that the ancient sample has higher affinity to population B than populations to A, C, and D. This situation occurs if $f_3(O; \text{Ancient, } B)$ is greater than $f_3(O; \text{Ancient, } A)$, $f_3(O; \text{Ancient, } C)$, and $f_3(O; \text{Ancient, } D)$, which occurs when $i + ii > 0$, $\gamma < (i + ii + iii)/(i + ii + iii + iv + v)$, and $\gamma < [i + p(ii + iii)]/[i + p(ii + iii + iv)]$. The first constraint should hold under most situations. Under situations in which population A is not admixed ($p = 0$), the third constraint is trivial as it implies that the contamination rate must be less than 1. The second constraint will be satisfied in almost all practical applications, as it requires prohibitively large contamination rates for it not to hold. For example, consider a situation in which $2(i + ii + iii) = (iv + v)$. That is, the branch length leading from the ancestor of B and D is at least twice as long leading to D as it is leading to B. Under such a scenario, the second constraint implies that we need a contamination rate less than $1/3$ for the outgroup-$f_3$ statistic to not be
misled by contamination. Further, if population A were indeed admixed, say at rate $p = 0.5$, then the branch length ($i\nu$) leading to population D must be extremely long or the contamination rate must be enormous for the outgroup $f_3$-statistic to again be misled.