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Covariate Dependent Markov Models for Analysis of Repeated Binary Outcomes

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The covariate dependence in a higher order Markov models is examined. First order Markov models with covariate dependence are discussed and are generalized for higher order. A simple alternative is also proposed. The estimation procedure is discussed for higher order with a number of covariates. The proposed model takes into account the past transitions. Transitions are fitted and are tested in order to examine their influence on the most recent transitions. Applications are illustrated using maternal morbidity during pregnancy. The binary outcome at each visit during pregnancy is observed for each subject and then the covariate dependent Markov models are fitted. The results indicate that the proposed model can be employed for analyzing repeated observations conveniently.

Key words: Markov models, higher order, covariate dependence, repeated observations, transitions

Introduction

Markov chain models can be used in analyzing longitudinal data. There are several discrete time Markov chain models proposed for analyzing repeated categorical data over decades. A model for estimating odds ratio from a two state transition matrix was proposed by Regier (1968). Prentice and Gloeckler (1978) proposed a grouped data version of the proportional hazards regression model for estimating computationally feasible estimators of the relative risk function. Korn and Whittemore (1979) proposed a model to incorporate role of previous state as a covariate to analyze the probability of occupying the current state.

To analyze the binary sequence of presence or absence of diseases, Muenz and Rubinstein (1985) introduced a discrete time Markov chain for expressing the transition probabilities in terms of covariates. The technique proposed by them is applicable for first order Markov model but they provided hints that the approach can be extended for second-order Markov chains. For analyzing sequences of ordinal data from relapsing and remitting of a disease, Albert (1994) developed a finite Markov chain model. In addition, Albert and Waclawiw (1998) developed a class of quasi-likelihood models for a two state Markov chain with stationary transition probabilities for heterogeneous transitional data. Raftery (1985), Raftery and Tavare (1994) proposed a higher order Markov chain model with dependence on contribution of the past transitions. Islam and Chowdhury (2006) presented a higher order version of the covariate dependent Markov model.

For analyzing repeated observations, there is a renewed interest in the development of multivariate models based on Markov chains. These models can be employed for analyzing data generated from meteorology, epidemiology and survival analysis, reliability, econometric analysis, biological concerns, etc. Muenz and Rubinstein (1985) employed logistic regression models to analyze the transition probabilities from one state to another. The estimation for first-order Markov models is quite straight forward, but still there is serious lack of generalization in estimation and testing for models applicability for higher order Markov chains. Islam and Chowdhury (2006) provided a further generalization for covariate dependent

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higher order models. This paper makes an attempt to present a simplified version of the covariate dependent higher order Markov models.

A parallel stream of development is observed in analyzing transition models with serial dependence of the first or higher orders on the basis of the marginal mean regression structure models. Azzalini (1994) introduced a stochastic model, more specifically, first order Markov model, to examine the influence of time-dependent covariates on the marginal distribution of the of the binary outcome variables in serially correlated binary data. Markov chains are expressed in transitional form rather than marginally and the solutions are obtained such that covariates relate only to the mean value of the process, independent of association parameters. Following Azzalini (1994), Heagerty and Zeger (2000) presented a class of marginalized transition models (MTM) and Heagerty (2002) proposed a class of generalized MTMs to allow serial dependence of first or higher order. These models are computationally tedious and the form of serial dependence is quite restricted. If the regression parameters are strongly influenced by inaccurate modeling for serial correlation then the MTMs can result in misleading conclusions. Heagerty (2002) provided derivatives for score and information computations.

Transition models are used here for Markov chain regression for binary responses proposed by Diggle et al. (2002). This type of models takes into account the potential impact of explanatory variables depending on the order of the underlying Markov model. This class of models has the flexibility to address a wide range of possible situations, ranging from only main effects to main effects and all possible interactions that emerge from different past transitions of the underlying Markov model. Some hypothesized situations are considered with main effects and some potential interactions emerging from past transitions of the process. In addition, a simple alternative is suggested to test for the order of Markov model.

Covariate Dependent Higher Order Model

As the first serious attempt to analyze covariate dependence of transition probabilities in a Markov model was proposed by Muenz and Rubinstein (1985), a brief review of the model provides a useful background for the proposed model for higher order.

Consider a two-state Markov chain for a discrete time binary sequence as follows:

$$\pi = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} \qquad (2.1)$$

where $\pi_{00} = 1 - \pi_{01}$ and $\pi_{10} = 1 - \pi_{11}$. Here, 0 and 1 are two possible outcomes of a dependent variable, Y. Each row of the above transition probability matrix provides a model on the basis of conditional probabilities. For instance, the probability of a transition from 0 at time t_{j-1} to 1 at time t_j is $\pi_{01} = P(Y_j = 1 \mid Y_{j-1} = 0)$ and similarly the probability of a transition from 1 at time t_{j-1} to 1 at time t_j is $\pi_{11} = P(Y_j = 1 \mid Y_{j-1} = 1)$. It is evident that $\pi_{00} + \pi_{01} = 1$, and similarly $\pi_{10} + \pi_{11} = 1$.

The covariate dependent higher order models can be proposed by extending the model for first order Markov chain. To illustrate the extension, a second order Markov model is considered. The second order Markov model for time points t_{j-2}, t_{j-1} and t_j with corresponding binary outcomes $Y_{j-2} = s_2$, $Y_{j-1} = s_1$ and $Y_j = s_0$, respectively, is shown as follows:

Y_{j-2}	Y_{j-1}	Y_j 0	1
0	0	π_{000}	π_{001}
0	1	π_{010}	π_{011}
1	0	π_{100}	π_{101}
1	1	π_{110}	π_{111}

(2.2)

Following the outline of Diggle et al. (2002), the transition probabilities are defined as follows:

$$\pi_{s_2 s_1}(Y_j = 1 \mid Y_{j-2} = s_2, Y_{j-1} = s_1, X) = \frac{e^{\beta' X + s_1\alpha_1' X + s_2\alpha_2' X + s_1 s_2 \alpha_3' X}}{1 + e^{\beta' X + s_1\alpha_1' X + s_2\alpha_2' X + s_1 s_2 \alpha_3' X}} \quad (2.3)$$

The vector X includes $X_0 = 1$ and p covariates such that $X = [X_0, X_1, ..., X_p]$.

The parameter vectors α_1, α_2 and α_3 are defined as follows:

$$\beta' = \left[\beta_0, \beta_1, ..., \beta_p\right]$$
$$\alpha_1' = \left[\alpha_{10}, \alpha_{11}, ..., \alpha_{1p}\right]$$
$$\alpha_2' = \left[\alpha_{20}, \alpha_{21}, ..., \alpha_{2p}\right]$$
$$\alpha_3' = \left[\alpha_{30}, \alpha_{31}, ..., \alpha_{3p}\right]$$

and define

$$\beta_{00}' = \beta' \; \beta_{01}' = \beta' + \alpha_1' \; \beta_{10}' = \beta' + \alpha_2'$$
$$\beta_{11}' = \beta' + \alpha_1' + \alpha_2'$$

Equation 2.3 can be expressed more precisely as follows:

$$\pi_{s_2 s_1}(Y_j = 1 / Y_{j-2} = s_2, Y_{j-1} = s_1, X) = \frac{e^{(\beta' + \sum_{m=1}^{2^2} \lambda_m \alpha_m)X}}{1 + e^{(\beta' + \sum_{m=1}^{2^2} \lambda_m \alpha_m)X}} \quad (2.4)$$

where

$m = 1,2,...,2^2$, $\lambda_1 = 0, \lambda_2 = s_1, \lambda_3 = s_2, \lambda_4 = s_1.s_2$.

The third order Markov model for time points t_{j-3}, t_{j-2}, t_{j-1} and t_j with corresponding outcomes $Y_{j-3} = s_3$, $Y_{j-2} = s_2$, $Y_{j-1} = s_1$ and $Y_j = s_0$, respectively, is shown as follows:

Y_{j-3}	Y_{j-2}	Y_{j-1}	Y_j: 0	Y_j: 1
0	0	0	π_{0000}	π_{0001}
0	0	1	π_{0010}	π_{0011}
0	1	0	π_{0100}	π_{0101}
0	1	1	π_{0110}	π_{0111}
1	0	0	π_{1000}	π_{1001}
1	0	1	π_{1010}	π_{1011}
1	1	0	π_{1100}	π_{1101}
1	1	1	π_{1110}	π_{1111}

For a Markov model of order three we can rewrite the transition probability as follows:

$$\pi_{s_3 s_2 s_1}(Y_j = 1 \mid Y_{j-3} = s_3, Y_{j-2} = s_2, Y_{j-1} = s_1, X) \quad (2.5)$$

where

$$\lambda_1 = 0, \lambda_2 = s_1, \lambda_3 = s_2, \lambda_4 = s_1.s_2, \lambda_5 = s_3, \lambda_6 = s_1.s_3, \lambda_7 = s_2.s_3, \lambda_8 = s_1.s_2.s_3$$

To generalize this to the k-th order, consider 2^k sets of models. The transition probability matrix for the k-th order Markov model can be represented by binary outcomes at different time points $Y_{j-k} = s_k$, $Y_{j-(k-1)} = s_{k-1}, ..., Y_{j-1} = s_1, Y_j = s_0$ at time points $t_{j-k}, t_{j-(k-1)}, ..., t_{j-1}, t_j$, respectively, where $Y_j = 1$ for occurrence of the event and $Y_j = 0$ for non-occurrence of the event at time t_j. The transition probability is given by

$$\pi_{s_k \dots s_1}(Y_j = 1 | \; Y_{j-k} = s_k, \dots, Y_{j-1} = s_1, X) = \frac{e^{(\beta' + \sum_{m=1}^{2^k} \lambda_m \alpha_m)X}}{1 + e^{(\beta' + \sum_{m=1}^{2^k} \lambda_m \alpha_m)X}} \quad (2.6)$$

The likelihood function is given by

$$L = \prod_{i=1}^{n_{s_k \dots s_1}} \prod_{s_k=0}^{1} \cdots \prod_{s_1=1}^{1} \left[\left\{ \pi_{is_k \dots s_1} \right\}^{\delta_{is_k \dots s_1}} \left\{ 1 - \pi_{is_k \dots s_1} \right\}^{1-\delta_{is_k \dots s_1}} \right]$$

where $\delta_{is_k s_{k-1} \cdots s_1} = 1$ if the outcome at time t_j is $Y_j = 1$ for individual i and $\delta_{is_k s_{k-1} \cdots s_1} = 0$ if the outcome at time t_j is $Y_j = 0$ for individual i for the transition type $Y_{j-k} = s_k, Y_{j-(k-1)} = s_{k-1}, \dots\dots, Y_{j-1} = s_1$ prior to time t_j and $n_{s_k \dots s_1}$ denotes the number of subjects experiencing transition type $Y_{j-k} = s_k, Y_{j-(k-1)} = s_{k-1}, \dots\dots, Y_{j-1} = s_1$ prior to time t_j.

Then the parameters $\beta_{\ell \; s_k s_{k-1} \cdots s_1}$ and $\alpha_{\ell,m}$ can be obtained from the following equations

$$\frac{\partial \ \mathrm{lnL}}{\partial \ \beta_{\ell s_k s_{k-1} \cdots s_1}} = 0$$

and

$$\frac{\partial \ \mathrm{lnL}}{\partial \ \alpha_{\ell,\mathrm{m}}} = 0$$

A Simple Model

In the previous model, the number of parameters increases exponentially with an increase in the order of the dependence, although, the proposed model provides more detailed information for each transition type. Another major limitation of such model is that it requires a large sample size to ensure adequate transitions for each transition type. To address such problems, a simple model is proposed in this section. In the model, the transition probability takes into account selected covariates and previous transitions are also incorporated as covariates for a k- order Markov model. The model is as follows:

$$\pi_{s_k s_{k-1} \cdots s_1}\left(Y_j = 1 \mid Y_{s_k}, Y_{s_{k-1}}, \cdots, Y_{s_1}, X \right) = \frac{e^{\left(\beta X + \theta_1 Y_{s1} + \cdots + \theta_k Y_{s_k} \right)}}{1 + e^{\left(\beta X + \theta_1 Y_{s1} + \cdots + \theta_k Y_{s_k} \right)}} \quad (2.7)$$

$$\frac{\partial \ \mathrm{lnL}}{\partial \ \beta_\ell} = 0 \quad \ell = 1,2,\cdots, p,$$

$$\frac{\partial \ \mathrm{lnL}}{\partial \ \theta_r} = 0 \quad r = 1,2,\cdots, k,$$

Testing for the Significance of Parameters

The following vector shows the 2^k sets of parameters for the k-th order Markov model:

$$\beta'_G = \left[\beta'_1, \beta'_2, \dots\dots\dots, \beta'_{2^k} \right]$$

where

$$\beta'_m = \left[\beta_{m1}, \dots\dots, \beta_{mp} \right], \ \mathrm{m}=1,2,\dots, 2^k$$

To test the null hypothesis $H_0 : \beta = 0$, the usual likelihood ratio test is employed and is given by

$$-2[\ln L(\beta_0) - \ln L(\beta_G)] \sim \chi^2_{2^k p}$$

where

$$\beta'_{G0} = \left[\beta'_{10}, \beta'_{20}, \dots, \beta'_{2^k 0} \right]$$

and

$$\beta'_{m0} = \left\lfloor \beta_{m1}, \dots, \beta_{mp} \right\rfloor, \ \mathrm{m}=1,2,\dots, 2^k$$

To test the significance of the qth parameter of the m-th set of parameters, the null hypothesis is $H_0 : \beta_{mq} = 0$ and the corresponding Wald-test is given by

$$W = \frac{\hat{\beta}_{mq}}{se(\hat{\beta}_{mq})}.$$

Test the order of the Markov model on the basis of the simple model (2.7) such that $H_0 : \theta_i = 0$ versus $H_1 : \theta_i \neq 0$ (i=1,2,…,k) that can identify the order is at least i if the null hypothesis is rejected. Use the test procedure discussed above for testing for the order of the Markov model as well.

The computer program employed in this paper is the modified version of the algorithm appeared in Chowdhury et al. (2005) for higher order covariate dependent Markov model.

An Application to Maternal Morbidity Data

Data are used from the survey on Maternal Morbidity in Bangladesh conducted by the Bangladesh Institute for Research for Promotion of Essential and Reproductive Health Technologies (BIRPERHT) during November 1992 to December 1993. The data were collected using both cross-sectional and prospective study designs. The study is based on the data from the prospective component of the survey. A multistage sampling design was used for collecting the data for this study. Districts were selected randomly in the first stage, one district from each Division. Then, Thanas were selected randomly in the second stage, one Thana from each of the selected Districts. A Thana is comprised of several Unions, while Union is the smallest administrative geographical unit in Bangladesh. At the third stage, two Unions were selected randomly from each selected Thana. The subjects comprised of pregnant women with less than 6 months in the selected Unions. The pregnant women from the selected Unions were followed on regular basis (roughly at an interval of one month) throughout the pregnancy. During the follow-up visits, pregnancy complications were recorded.

A total of 1020 pregnant women were interviewed in the follow-up component of the study. The survey collected information on socio-economic and demographic characteristics, pregnancy related care and practice, morbidity during the period of follow-up as well as in the past, information concerning complications at the time of delivery and during the post partum period. For the purpose of this study, 993 pregnant women were selected, with at least one antenatal follow-up. Table 1 shows the number of respondents at different follow-up visits during antenatal period. At the first follow-up 992 respondents were recorded (out of 993 respondents one was missing at the first follow-up but reported subsequently). The number dropped to 917 at the second follow-up and the rate of dropout increased sharply at subsequent follow-ups. The number of respondents observed at the third and the fourth follow-ups were 771 and 594, respectively. The following pregnancy complications are considered under the complications in this study: hemorrhage, edema, excessive vomiting, fits/convulsion. If one or more of these complications occurred to the respondents, they were considered as having complications.

The explanatory variables are: pregnancies prior to the index pregnancy (yes, no), education of respondent (no schooling, some schooling), economic status (low, high), age at marriage (less than 15 years, 15 years or more), involved with gainful employment (no, yes), index pregnancy was wanted or not (no, yes). The number of transitions for the first, second, and third order Markov chains are displayed in Table 2. The estimates of parameters of covariate dependent Markov models are presented in Table 3.

Two variables, economic status and whether the pregnancy was wanted, show significant association with transition from no complication in previous visit to complication in current visit during pregnancy (transition type $0 \rightarrow 1$). If the respondent has economically better status, she is expected to experience higher transition to pregnancy complications. On the other hand, if the index pregnancy is wanted, as compared to that of unwanted pregnancy, there is a decreased risk of transition to pregnancy complications during the current visit.

If the previous outcome was complication, three variables influence to the transition to the same status at the time of current visit during pregnancy (transition type $1 \to 1$) which are whether the index pregnancy was wanted or not, gainful employment, and education. The desired pregnancies appear to have higher risk of pregnancy complications in consecutive follow-up visits. In other words, undesired pregnancies seem to result in higher risk of transition to complications but risk of complications at consecutive follow-up visits appears to be higher for desired pregnancies. The respondents who are involved with gainful employment have higher risk of transition to complication in consecutive visits during pregnancy but respondents with some education have reduced risk of continued complications in consecutive visits.

The second order model shows that there is a lower risk for desired pregnancies to make transition to the state of complications at current visit after two consecutive no complications status prior to the current visit (transition type $0 \to 0 \to 1$). There is no significant association between reverse transition of the type $1 \to 0 \to 1$ and the selected covariates. Like the transition type $1 \to 1$, transition type $0 \to 1 \to 1$ is observed to be positively associated with desired pregnancy and negatively associated with education. Similarly, similar to $1 \to 1$, desired pregnancy and gainful employment are positively associated with the complications at three consecutive visits (transition type $1 \to 1 \to 1$).

There are eight models for the third order Markov chain. Among those, some of the transition types do not show any clear association with the selected covariates (considered at p-value = 0.05) such as transition types $1 \to 0 \to 0 \to 1$, $0 \to 0 \to 1 \to 1$, $1 \to 1 \to 0 \to 1$, $0 \to 1 \to 1 \to 1$ and $1 \to 1 \to 1 \to 1$. For the transition type, $0 \to 0 \to 0 \to 1$, gainful employment appears to have positive association. There is a marginal positive association (p-value is observed to be little higher than 0.05) between age at marriage and transition type $0 \to 1 \to 0 \to 1$, where the complication is repeated during four follow-ups. Economic status is associated marginally and positively and previous pregnancies are associated negatively with transition type $1 \to 0 \to 1 \to 1$.

The global chi-square and likelihood ratio tests show good fit for all the first, second and third order models. Hence, in order to find the best selection, we have employed the AIC and the BIC procedures. The AIC and the BIC results indicate that the third order models provide the best fit as compared to the first and second order models.

The number of parameters increases geometrically with an increase in the order of Markov model. Hence, a simple alternative is employed to the same data. Table 4 presents the results for the simple model as an alternative to the hierarchical model for the higher order Markov chain. For the second order model, first order outcome is considered as a covariate. Similarly, for the third order model, first and second order outcomes are included as covariates in order to examine the impact of previous outcomes on the subsequent outcomes. In the first model, economic status, wanted pregnancy, age at marriage and education appear to be significantly associated with pregnancy complications. The first order outcome, s_1, is included as a covariate for the second order model and confirms that first order outcome exerts a positive influence on the second order outcome. Similarly, first and second order outcomes are also associated positively with the outcome for the third order. Hence, in these models, third order Markov model is expected to fit better. Economic status, gainful employment and occurrence of the complications at previous two follow-ups all are positively associated with current complications. This finding confirms the conclusion based on the results presented in Table 3. In other words, the simple model can be employed confidently if the detailed impact of covariates on the response variable is not needed for each transition type separately for policy purposes.

Conclusion

In this article, the fitting of higher order covariate dependent Markov model is illustrated.

The method shown here is based on a suggestion provided by Diggle et al. (2002). The inference procedure is described for any higher order Markov model and the proposed method can be employed conveniently to identify the risk factors having significant impact on the repeated binary outcomes of interest at different time points. The proposed technique has been applied to a set of maternal morbidity data and the pregnancy complications at follow-up observations during pregnancy are analyzed. Some selected covariates are used to examine whether the transition probabilities for pregnancy complications at consecutive visits during pregnancy depend on the covariates. A simple alternative is also examined.

If the factors affecting different types of transitions depending on past transitions are not of much interest then we can use the simple alternative. However, the proposed model provides a detailed analysis of the factors affecting transitions of first or higher order Markov models. The detailed analysis may be considered to have useful interpretations for policymakers. On the other hand, the number of parameters in the simple model does not increase geometrically unlike the proposed model.

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Appendix

Table 1: Number of Respondents at Different Follow-ups During Antenatal Period

Follow-up Number	Frequency
1	992
2	917
3	771
4	594
5	370
6	148

Table 2: Number of Transitions for Pregnancy Complications

Transitions	**→0**	**→1**
First Order		
0→	1577	277
1→	366	614
Second Order		
0→ 0→	923	138
1→ 0→	176	79
0→ 1→	95	79
1→ 1→	110	295
Third Order		
0→ 0→0→	459	72
1→ 0→0→	91	30
0→ 1→0→	31	16
0→ 0→1→	49	27
1→ 1→0→	40	28
1→ 0→1→	26	20
0→ 1→1→	8	23
1→ 1→1→	44	132

Table 3: Estimates of Parameters of Covariate Dependent Markov Models for Analyzing Pregnancy Complications

Variables	Estimates	Std. error	t-value	p-value
	First Order			
0→ 1				
Constant	-1.5233	0.1559	-9.7718	0.0000
Economic Status (Good=1)	0.4209	0.1699	2.4770	0.0132
Wanted pregnancy (Yes=1)	-0.4217	0.1371	-3.0762	0.0021
Gainful employment (Yes=1)	0.0827	0.1453	0.5693	0.5692
Age at marriage (< 15 = 1)	-0.0353	0.1394	-0.2535	0.7998
Education (Yes=1)	-0.1161	0.1354	-0.8576	0.3911
Previous pregnancies (Yes=1)	0.0482	0.1354	0.3561	0.7218
1→ 1				
Constant	1.9102	0.1589	12.0175	0.0000
Economic Status (Good=1)	0.1378	0.1778	0.7749	0.4384
Wanted pregnancy (Yes=1)	0.6904	0.1406	4.9084	0.0000
Gainful employment (Yes=1)	0.3187	0.1506	2.1167	0.0343
Age at marriage (< 15 = 1)	-0.1535	0.1483	-1.0352	0.3006
Education (Yes=1)	-0.5238	0.1430	-3.6635	0.0002
Previous pregnancies (Yes=1)	0.0493	0.1411	0.3492	0.7269
Global Chi-square	1020.03596; d.f. = 14; p-value=0.00000			
LRT	1126.20664; d.f. = 14; p-value=0.00000			
AIC	2830.55158			
BIC	2898.37897			
	Second Order			
0→ 0→ 1				
Constant	-1.8003	0.2210	-8.1447	0.0000
Economic Status (Good=1)	0.1830	0.2534	0.7222	0.4702
Wanted pregnancy (Yes=1)	-0.4847	0.1919	-2.5256	0.0115
Gainful employment (Yes=1)	0.2149	0.2005	1.0715	0.2840
Age at marriage (< 15 = 1)	-0.1075	0.1983	-0.5422	0.5877
Education (Yes=1)	0.1420	0.1884	0.7536	0.4511
Previous pregnancies (Yes=1)	0.1877	0.1899	0.9886	0.3228
1→ 0→ 1				
Constant	0.9524	0.3091	3.0815	0.0021
Economic Status (Good=1)	0.3405	0.3523	0.9663	0.3339
Wanted pregnancy (Yes=1)	0.4857	0.2811	1.7283	0.0839
Gainful employment (Yes=1)	-0.0526	0.3123	-0.1686	0.8661
Age at marriage (< 15 = 1)	0.2394	0.2933	0.8163	0.4144
Education (Yes=1)	-0.3707	0.2845	-1.3030	0.1926
Previous pregnancies (Yes=1)	-0.2887	0.2837	-1.0177	0.3088
0→ 1→ 1				
Constant	1.7303	0.3845	4.5000	0.0000
Economic Status (Good=1)	0.4831	0.4203	1.1494	0.2504
Wanted pregnancy (Yes=1)	0.7516	0.3348	2.2450	0.0248
Gainful employment (Yes=1)	0.1031	0.3511	0.2935	0.7691
Age at marriage (< 15 = 1)	-0.2899	0.3596	-0.8062	0.4201
Education (Yes=1)	-1.0315	0.3404	-3.0301	0.0024
Previous pregnancies (Yes=1)	-0.1730	0.3359	-0.5150	0.6066

Table 3 Continued…

Variables	Estimates	Std. error	t-value	p-value
1→ 1→ 1				
Constant	2.1680	0.2742	7.9081	0.0000
Economic Status (Good=1)	0.2688	0.2989	0.8993	0.3685
Wanted pregnancy (Yes=1)	0.7322	0.2414	3.0332	0.0024
Gainful employment (Yes=1)	0.6301	0.2637	2.3891	0.0169
Age at marriage (< 15 = 1)	0.0853	0.2512	0.3395	0.7342
Education (Yes=1)	-0.4259	0.2498	-1.7049	0.0882
Previous pregnancies (Yes=1)	0.3097	0.2461	1.2581	0.2083
Global Chi-square	736.2494; d.f. = 28; p-value=0.00000			
LRT	819.7761; d.f. = 28; p-value=0.00000			
AIC	1863.2516			
BIC	1998.9065			
	Third Order			
0→ 0→ 0→ 1				
Constant	-2.0755	0.3272	-6.3440	0.0000
Economic Status (Good=1)	0.3901	0.3502	1.1140	0.2653
Wanted pregnancy (Yes=1)	-0.1337	0.2803	-0.4771	0.6333
Gainful employment (Yes=1)	0.5960	0.2780	2.1435	0.0321
Age at marriage (< 15 = 1)	-0.1715	0.2777	-0.6174	0.5370
Education (Yes=1)	-0.1050	0.2628	-0.3997	0.6894
Previous pregnancies (Yes=1)	0.2914	0.2716	1.0730	0.2833
1→ 0→ 0→ 1				
Constant	1.1866	0.4645	2.5547	0.0106
Economic Status (Good=1)	-0.4560	0.6635	-0.6874	0.4919
Wanted pregnancy (Yes=1)	-0.7254	0.4440	-1.6340	0.1023
Gainful employment (Yes=1)	-0.1067	0.4893	-0.2181	0.8273
Age at marriage (< 15 = 1)	0.8524	0.4898	1.7404	0.0818
Education (Yes=1)	-0.4170	0.4657	-0.8954	0.3706
Previous pregnancies (Yes=1)	-0.0466	0.4411	-0.1056	0.9159
0→1→ 0→ 1				
Constant	0.2618	0.9645	0.2714	0.7861
Economic Status (Good=1)	-1.5657	0.9903	-1.5810	0.1139
Wanted pregnancy (Yes=1)	0.6496	0.7006	0.9271	0.3539
Gainful employment (Yes=1)	0.2875	0.7673	0.3747	0.7079
Age at marriage (< 15 = 1)	1.4826	0.7838	1.8916	0.0585
Education (Yes=1)	-0.1186	0.7317	-0.1621	0.8713
Previous pregnancies (Yes=1)	0.5271	0.7445	0.7080	0.4790
0→ 0→ 1→ 1				
Constant	2.1936	0.5945	3.6901	0.0002
Economic Status (Good=1)	-0.3760	0.7314	-0.5140	0.6073
Wanted pregnancy (Yes=1)	-0.4947	0.5163	-0.9581	0.3380
Gainful employment (Yes=1)	-0.6332	0.5392	-1.1744	0.2402
Age at marriage (< 15 = 1)	-0.3622	0.5999	-0.6037	0.5460
Education (Yes=1)	-0.4388	0.5363	-0.8182	0.4132
Previous pregnancies (Yes=1)	-0.1100	0.5295	-0.2078	0.8354
Gainful employment (Yes=1)	-0.3197	0.6776	-0.4718	0.6371

Table 3 Continued…

Variables	**Estimates**	**Std. error**	**t-value**	**p-value**
1→ 1→ 0→ 1				
Constant	2.1797	0.5997	3.6347	0.0003
Economic Status (Good=1)	0.8516	0.6716	1.2680	0.2048
Wanted pregnancy (Yes=1)	-0.4881	0.5618	-0.8689	0.3849
Age at marriage (< 15 = 1)	-0.0976	0.5685	-0.1718	0.8636
Education (Yes=1)	-0.0570	0.5623	-0.1014	0.9192
Previous pregnancies (Yes=1)	-1.0848	0.5944	-1.8251	0.0680
1→ 0→ 1→ 1				
Constant	2.4045	0.7779	3.0909	0.0020
Economic Status (Good=1)	2.2623	1.1836	1.9114	0.0560
Wanted pregnancy (Yes=1)	-0.2161	0.7619	-0.2837	0.7767
Gainful employment (Yes=1)	0.8775	0.8714	1.0070	0.3139
Age at marriage (< 15 = 1)	-0.5819	0.7777	-0.7482	0.4543
Education (Yes=1)	-1.1563	0.8015	-1.4428	0.1491
Previous pregnancies (Yes=1)	-1.8188	0.9178	-1.9818	0.0475
0→ 1→ 1→ 1				
Constant	3.1416	1.4192	2.2137	0.0268
Economic Status (Good=1)	-0.4749	1.0472	-0.4535	0.6502
Wanted pregnancy (Yes=1)	-0.6499	1.2058	-0.5390	0.5899
Gainful employment (Yes=1)	-0.2974	1.0250	-0.2902	0.7717
Age at marriage (< 15 = 1)	1.0829	1.2160	0.8905	0.3732
Education (Yes=1)	0.6433	1.2520	0.5138	0.6074
Previous pregnancies (Yes=1)	0.2593	1.0143	0.2556	0.7983
1→ 1→ 1→ 1				
Constant	2.7444	0.4633	5.9233	0.0000
Economic Status (Good=1)	0.0139	0.4792	0.0291	0.9768
Wanted pregnancy (Yes=1)	-0.1399	0.3928	-0.3562	0.7217
Gainful employment (Yes=1)	0.0439	0.4056	0.1082	0.9138
Age at marriage (< 15 = 1)	-0.2489	0.3807	-0.6538	0.5132
Education (Yes=1)	0.3410	0.4179	0.8160	0.4145
Previous pregnancies (Yes=1)	0.6496	0.4144	1.5673	0.1170
Global Chi-square	418.79388; d.f. = 56; p-value=0.00000			
LRT	467.3335; d.f. = 56; p-value=0.00000			
AIC	1164.0451			
BIC	1435.3548			

Table 4: Estimates of Parameters of Simple Model for Higher Order Markov Chain

Variables	Estimates	Std. error	t-value	p-value
Logistic regression for first order Markov model				
Economic status (good=1)	.502	.091	30.467	.000
Wanted pregnancy (Yes=1)	-.331	.073	20.401	.000
Gainful employment (Yes=1)	.069	.076	.811	.368
Age at marriage (< 15=1)	-.222	.076	8.662	.003
Education (Yes=1)	-.408	.073	31.372	.000
Previous pregnancies (Yes=1)	-.041	.072	.317	.573
Constant	-.293	.083	12.461	.000
Model Chi-square		86.92 (p=0.000)		
Logistic regression for second order Markov model				
Economic status (good=1)	.496	.122	16.378	.000
Wanted pregnancy (Yes=1)	-.080	.099	.646	.422
Gainful employment (Yes=1)	.216	.102	4.447	.035
Age at marriage (< 15=1)	-.117	.102	1.323	.250
Education (Yes=1)	-.385	.098	15.269	.000
Previous pregnancies (Yes=1)	.065	.097	.448	.503
S_1	2.223	.094	562.140	.000
Constant	-1.669	.125	177.689	.000
Model Chi-square		704.48 (p=0.000)		
Logistic regression for third order Markov model				
Economic status (good=1)	.402	.155	6.768	.009
Wanted pregnancy (Yes=1)	-.084	.123	.469	.494
Gainful employment (Yes=1)	.361	.127	8.037	.005
Age at marriage (< 15=1)	-.100	.126	.625	.429
Education (Yes=1)	-.209	.123	2.898	.089
Previous pregnancies (Yes=1)	.152	.122	1.556	.212
S_1	1.720	.126	186.921	.000
S_2	1.127	.125	81.547	.000
Constant	-1.984	.160	152.904	.000
Model Chi-square		521.18 (p=0.000)		