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Modified EDF Goodness of Fit Tests for Logistic Distribution under SRS and RSS

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Modified forms of goodness of fit tests are presented for the logistic distribution using statistics based on the empirical distribution function (EDF). A method to improve the power of the modified EDF goodness of fit tests is introduced based on Ranked Set sampling (RSS). Data are collected via the Ranked Set Sampling (RSS) technique (McIntyre, 1952). Critical values for the logistic distribution with unknown parameters are provided and the powers of the tests are given for a number of alternative distributions. A simulation study is presented to illustrate the power of the new method.

Key words: Goodness of fit tests, empirical distribution function, power, logistic distribution, ranked set sample, Kolmogorov-Smirnov statistic.

Introduction

Many sampling methods can be used to estimate the population parameters. However, in many situations the experimental units for the variable of interest can be more easily ranked than quantified. The use of the method of ranked set sampling (RSS) in these situations is highly beneficial and is superior to simple random sampling (SRS). In many agricultural and environmental studies, it is possible to rank the experimental or sampling units with respect to the variable of interest, without actually measuring them; this usually results in cost-savings. The RSS sampling method can be used when measurements of sample units, drawn from the population of interest, are very laborious or costly in time or money, but can be easily arranged (ranked) in order of their magnitude.

McIntyre (1952) was the first to introduce ranked set sampling (RSS). RSS gives a sample that is more informative than a simple random sample (SRS) concerning a population of interest. The RSS technique can be described as follows: Select m random samples from a population of interest each of size m. From the i^{th} sample use a visual inspection to detect the i^{th} order statistic and choose it for actual quantification, for example, Y_i, $i = 1,...,m$. Assuming the ranking is perfect RSS is the set of the order statistics $Y_1,...,Y_m$. The RSS technique can be repeated r times to obtain additional observations; these resulting measurements form an RSS of size rm.

Two factors affect the efficiency of an RSS: set size and ranking errors. The larger the set size, the larger the efficiency of RSS, while the larger the set size the more the difficulty in the visual ranking and hence the larger the ranking error (Al-Saleh & Al-Omari, 2002). Takahasi and Wakimoto (1968) provided the theoretical setups for RSS by showing that the mean of an RSS is the minimum variance unbiased estimator for a population mean. Dell and Clutter (1972) further showed that the sample mean RSS remains unbiased and more

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efficient than the sample mean even if ranking is imperfect.

Several authors have modified RSS to reduce the error in ranking and to make visual ranking tractable by experimenter. (For details about RSS and its modifications, see Muttlak, 1997; Samawi, et al, 1996; Al-Odat & Al-Saleh, 2001; Bhoj, 1997; Chen, 2000; Patil, et al, 1994a).

Stockes and Sager (1988) studied the characterization of RSS. In addition, for deriving the null distribution of their proposed test, they introduced an unbiased estimator for the population distribution function based on the empirical distribution function of RSS. Also, proposed a Kolmogorov-Smirnov goodness of fit test based on the empirical distribution function (EDF). Ibrahim et al. (2011) introduced a method to improve the power of the Chi-square goodness of fit test based on RSS. They used Kullback-Leibler information to compare data collected via both SRS and RSS and conducted a simulation study for the power of Chi-square test of the new method.

Al-Subh et al. (2009) conducted a comparison study for the power of a set of EDF goodness of fit tests for the logistic distribution under SRS and RSS. This article proposes a method to improve the power of the EDF goodness of fit tests for logistic distribution under RSS and uses a simulation study to compare the powers of each test under the RSS.

MEDF Goodness of Fit Tests

Stephens (1974) presented a practical guide to goodness of fit tests using statistics based on the EDF. Green and Hegazy (1976) studied modified forms of the Kolmogorov-Smirnov D, Cramer-von Mises W^2 and Anderson-Darling A^2 goodness of fit tests. Stephens (1979) gave goodness of fit tests for the logistic distribution based on a SRS; a comprehensive survey of goodness of fit tests based on SRS can be found in Stephens (1986).

Let $X_1, X_2,...,X_n$ be a random sample from the distribution function $F(x)$ where $X_1 < X_2,...,< X_n$ is the order statistics of random sample of size *n* from *F(x).* Assume that the objective is to test the statistical hypotheses

$$H_o : F(x) = F_o(x)\ \forall x$$

vs.

$$H_1 : F(x) \neq F_o(x)$$

for some x, where $F_o(x)$ is a known distribution function.

The MEDF goodness of fit tests in SRS are defined as:

a) Tests related to Kolmogorov statistic, D :

$$D_1 = \max_{1\le i\le n} \left| F\left(\frac{x_{(i)}-\alpha}{\beta}\right) - \left(\frac{i}{n}\right)\right|,$$

where $i = 1, 2,..., n$ and n is the sample size.

$$D_{11} = \max_{1\le i\le n} \left| F\left(\frac{x_{(i)}-\alpha}{\beta}\right) - \left(\frac{i}{n+1}\right)\right|,$$

$$D_2 = \sum_{i=1}^{n} \left| F\left(\frac{x_{(i)}-\alpha}{\beta}\right) - \left(\frac{i}{n}\right)\right|,$$

$$D_{22} = \sum_{i=1}^{n} \left| F\left(\frac{x_{(i)}-\alpha}{\beta}\right) - \left(\frac{i+0.5}{n+1}\right)\right|,$$

$$D_3 = \sup_{1\le i\le n} \left| F\left(\frac{x_{(i)}-\alpha}{\beta}\right) - \left(\frac{i}{n}\right)\right|,$$

and

$$D_4 = \sum_{i=1}^{n} \max\left\{\left|\frac{i}{n} - F\left(\frac{x_{(i)}-\alpha}{\beta}\right)\right|,\ \left|\frac{i-1}{n} - F\left(\frac{x_{(i)}-\alpha}{\beta}\right)\right|\right\}.$$

b) Tests related to Cramer-von Mises statistic, W^2:

$$W_o = \sum_{i=1}^{n}\left[F\left(\frac{x_{(i)}-\alpha}{\beta}\right)-\left(\frac{2i-1}{2n}\right)\right]^2,$$

$$W_{11} = \sum_{i=1}^{n}\left[F\left(\frac{x_{(i)}-\alpha}{\beta}\right)-\frac{i}{n+1}\right]^2,$$

$$W_{21} = \sum_{i=1}^{n}\left[F\left(\frac{x_{(i)}-\alpha}{\beta}\right)-\left(\frac{2i-1}{2(n+1)}\right)\right]^2.$$

c) Tests related to Anderson-Darling statistic, A^2:

$$aa_{21} = -n-\frac{n}{(r+1)^2}\{\sum_{i=1}^{n}\begin{bmatrix}(2i-1)\ln F\left(\frac{x_{(i)}-\alpha}{\beta}\right)\\ +(2i+1)\ln\left(1-F\left(\frac{x_{(n-i+1)}-\alpha}{\beta}\right)\right)\end{bmatrix} -\begin{bmatrix}(2n+1)\ln F\left(\frac{x_{(n)}-\alpha}{\beta}\right)\\ -\ln\left(1-F\left(\frac{x_{(n-i+1)}-\alpha}{\beta}\right)\right)\end{bmatrix}\},$$

$$aa_{22} = -n-\left(\frac{2n}{(n+1)^2}\right)[\left\{\sum_{i=1}^{n} i\begin{bmatrix}\ln F\left(\frac{x_{(i)}-\alpha}{\beta}\right)\\ +\ln\left(1-F\left(\frac{x_{(n-i+1)}-\alpha}{\beta}\right)\right)\end{bmatrix}\right\} -\left(\frac{n}{(n+1)^2}\right)[0.25\{\ln F\left(\frac{x_{(i)}-\alpha}{\beta}\right) +\ln\left(1-F\left(\frac{x_{(n)}-\alpha}{\beta}\right)\right)\} +(n+0.75)\{\ln F\left(\frac{x_{(i)}-\alpha}{\beta}\right) +\ln\left(1-F\left(\frac{x_{(1)}-\alpha}{\beta}\right)\right)\}],$$

and

$$aa_{12} = -(n+1)-\left(\frac{1}{n+1}\right)\left\{\sum_{i=1}^{n} 2i\begin{bmatrix}\ln F\left(\frac{x_{(i)}-\alpha}{\beta}\right)\\ +\ln\left(1-F\left(\frac{x_{(n-i+1)}-\alpha}{\beta}\right)\right)\end{bmatrix}\right\}. \quad (1)$$

This study examines the case $F_o(x) = (1+e^{-(x-\alpha)/\beta})^{-1}$, that is, for the logistic distribution. A simulation study is conducted to show that the test T^* is more powerful than the test T when compared based on samples of the same size. The power of the T^* test can be calculated according to the equation:

$$T^*(H) = P_H(T^* > d_\alpha), \quad (2)$$

where H is a cdf under alternative hypothesis H_1^*. Here d_α is the 100α percentage point of the distribution of T^* and H_o. Due to the behavior of RSS test statistics relative to SRS test statistics, the efficiency of the test statistics is calculated as a ratio of powers:

$$eff\ (T^*,\ T) = \frac{\text{power of } T^*}{\text{power of } T},$$

where T^* is more powerful than T if $eff\ (T^*,\ T) > 1$.

Test for Logistic Distribution

Let $X_{(1)1}, X_{(2)1}, \ldots, X_{(m)r},\ n = mr$ be a RSS of size $n = mr$ from a distribution function $F(x)$. The test described is an upper-tail test. A goodness of fit test is performed for the hypotheses:

$$H_o : F(x) = F_o(x)\ \forall x,$$

vs.

$$H_1 : F(x) \neq F_o(x)$$

where $F_o(x) = (1 + e^{-(x-\alpha)/\beta})^{-1}$.

If α and β are unknown, then they may be estimated using their maximum likelihood estimator i.e, from $l(\alpha,\ \beta)$, by making the log likelihood function of the data:

$$l(\alpha,\ \beta) = -n\ln(\beta) - \sum_{i=1}^{n}(z_i) - 2\sum_{i=1}^{n}\ln(1 + e^{-z_i}),$$

and in RSS by

$$l(\alpha,\ \beta) = -n\ln(\beta) + \sum_{j=1}^{r}\sum_{i=1}^{m}(i-1)\ln F(z_{(i)j}) + \sum_{j=1}^{r}\sum_{i=1}^{m}(m-i)\ln(1 - F(z_{(i)j})) + \sum_{j=1}^{r}\sum_{i=1}^{m}\ln f(z_{(i)j}), \quad (3)$$

where

$$z_i = (x_i - \alpha)/\beta,\ z_{(i)j} = (x_{(i)j} - \alpha)/\beta$$

and

$$f(z_{(i)j}) = \frac{e^{-z_{(i)j}}}{\beta(1 + e^{-z_{(i)j}})^2}.$$

Using the tests given in (1) and based on the data $X_{(1)1}, X_{(2)1}, \ldots, X_{(m)r},\ n = mr$ called via the RSS.

Power Comparison Algorithm

Let T denote a test in (1) based on SRS and T^* be the same test, but based on RSS. To compare the power of the test T^* with the power of the test T based on samples of the same size, first the algorithm to calculate the percentage points is introduced:

1. Let $x_{(i)j}$ be a random sample from $F_o(x)$.
2. Estimate parameters α and β from the sample by maximum likelihood; the estimates are given by (3).
3. Find the EDF $F_n^*(x)$ as follows:

$$F_n^*(x) = \frac{1}{mr}\sum_{j=1}^{r}\sum_{i=1}^{m} I(x_{(i)j} \le x),$$
$$I(x_{(i)j} \le x) = \begin{cases} 1\ ,\ x_{(i)j} \le x, \\ 0\ ,\ \text{o.w.} \end{cases} \quad (4)$$

4. Use $F_n^*(x)$ to calculate the value of T^* as in (1).
5. Repeat steps one through four 10,000 times to obtain $T_1^*, \ldots,\ T_{10,000}^*$.
6. The percentage point d_α of T^* is approximated by the $(1-\alpha)100$ quantile of $T_1^*, \ldots,\ T_{10,000}^*$.

The following algorithm is designed to obtain the power of T^* at a distribution, for example, H, under H_o:

1. Let $x_{(i)j}$ be a random sample from $F_o(x)$.

2. Estimate the parameters α, β from the sample by maximum likelihood; the estimates are given by (3).

3. Find the EDF $F_n^*(x)$ as in (4).

4. Calculate the value of T^* in (1).

5. Repeat steps one through four 10,000 times to obtain $T_1^*, \ldots, T_{10,000}^*$.

6. Calculate the power of

$$T^*(H) \approx \frac{1}{10,000} \sum_{t=1}^{10,000} I(T_t^* > d_\alpha),$$

where $I(.)$ stands for indicator function.

Results

A simulation study was conducted to compare the power of T and T^*. The power, as well as the percentage point, of each test are approximated based on a Monte Carlo simulation of 10,000 iterations according to the algorithm described previously. Table 1 shows the percentage points for the 5% level for the null hypotheses of the logistic distribution for RSS. The efficiency of the tests was compared for different sized samples: $r = 3, 5, 10, 25$; different set sizes: $m = 2, 3$; and different alternative distributions: $Normal = N(\alpha, \beta^2)$, $Laplace = L(\alpha, \beta)$, $Cauchy = C(\alpha, \beta)$, $StudentT = S(5)$, $Uniform = U(\alpha, \beta)$, and $Lognormal = LN(\alpha, \beta)$. Comparisons were made only for cases where the data are quantified via RSS. Simulation results are shown in the Tables 2 and 3. For the lognormal and uniform distributions, computations show that the powers of all test statistics equal one, thus, these powers are not reported.

Based on study results, the following conclusions are put forth:

1. The efficiencies in Tables 1 and 3 are all greater than 1; this indicates that the MEDF tests under ERSS are more powerful than their counterparts in SRS.

2. Tables 1-3 show that the efficiency increases as the distribution under the alternative hypothesis departs to asymmetry.

3. Power increases as the sample size n increases.

4. Power is equal to one for the lognormal and uniform distributions.

5. The MEDF tests based on data collected via RSS are more powerful than the EDF tests based on an SRS of the same size.

Conclusion

The power of a set of modified EDF goodness of fit tests was shown to be improved if a sample is collected via the RSS method, as opposed to the SRS method. Moreover, modified EDF tests show excellent power performance in comparison to their SRS counterparts. Although this study is limited to the logistic distribution under the null hypothesis, it could be easily extended to other distributions.

References

Al-Odat, M. T., & Al-Saleh, M. F. (2001). A variation of ranked set sampling. *Journal of Applied Statistical Science*, *10*, 137-146.

Al-Saleh, M. F., & Al-Omari, A. I. (2002). Multistage ranked set sampling. *Journal of Statistical planning and Inference*, *102*, 273-286.

Ibrahim, K., Alodat, M. T., Jemain, A. A., & Al-Subh, S. A. (2011) Chi-square test for goodness of fit for logistic distribution using ranked set sampling and simple random sampling. *Statistica & Applicazioni*, *IX*(*2*), 111-128.

Table 1: Percentage Points for SRS and RSS, $\alpha = 0.05$

Test	SRS					RSS				
	n					n				
	6	10	20	30	50	6	10	20	30	50
D_1	0.269	0.215	0.158	0.132	0.103	0.271	0.221	0.166	0.140	0.116
D_{11}	0.198	0.175	0.141	0.122	0.099	0.200	0.175	0.143	0.126	0.107
D_2	0.723	0.885	1.191	1.460	1.861	0.747	0.932	1.312	1.631	2.249
D_{22}	0.693	0.881	1.201	1.470	1.867	0.712	0.884	1.244	1.537	2.128
D_3	0.287	0.228	0.168	0.140	0.110	0.291	0.237	0.177	0.150	0.122
D_4	1.096	1.283	1.618	1.903	2.311	1.115	1.322	1.731	2.062	2.694
WW_0	0.081	0.087	0.091	0.097	0.096	0.087	0.098	0.109	0.118	0.137
WW_{11}	0.085	0.089	0.093	0.098	0.096	0.088	0.089	0.098	0.104	0.122
WW_{21}	0.116	0.110	0.104	0.105	0.101	0.118	.110	0.109	0.112	0.127
aa_{21}	0.261	0.376	0.488	0.550	0.593	0.312	0.393	0.499	0.549	0.690
aa_{22}	0.745	0.757	0.726	0.727	0.711	0.761	0.754	0.740	0.751	0.850
aa_{12}	0.421	0.513	0.577	0.618	0.641	0.460	0.505	0.573	0.623	0.762

Table 2: Efficiency Values of Tests Using RSS with respect to SRS for $n = 6,\ 10,\ 20,\ 30,\ 50$ and $\alpha = 0.05$

H	T	n				
		6	10	20	30	50
$N(\theta,\ \sigma^2)$	D_1	5.52	1.55	1.16	1.01	1
	D_{11}	0.27	1.98	1.49	1.06	1
	D_2	8.67	1.21	0.98	1	1
	D_{22}	0.36	2.05	1.11	1	1
	D_3	1.42	1.39	1.08	0.99	1
	D_4	1.97	1.18	0.99	1	1
	WW_0	1.95	1.34	0.99	1	1
	WW_{11}	1.54	2.29	0.94	1	1
	WW_{21}	1.25	2.17	0.94	1	1
	aa_{21}	2.12	2.17	1	1	1
	aa_{22}	4.38	4.91	0.98	1	1
	aa_{12}	1	3.21	1.52	1	1
$L(\theta,\ \sigma)$	D_1	1.38	1.43	1.4	1.21	1.04
	D_{11}	1.19	1.38	1.42	1.24	1.05
	D_2	1.08	1.43	1.27	1.04	1
	D_{22}	0.85	1.35	1.7	1.31	1
	D_3	1.05	1.16	1.18	1.11	1.02
	D_4	1.51	1.65	1.26	1.02	1
	WW_0	1.26	1.4	1.17	1.03	1
	WW_{11}	0.83	1.51	1.6	1.21	1
	WW_{21}	0.51	0.94	1.47	1.21	1
	aa_{21}	1.15	1.66	2.11	1.52	1
	aa_{22}	0.82	1.48	2.17	1.53	1
	aa_{12}	0.47	0.95	2.19	1.57	1

Table 2 (continued): Efficiency Values of Tests Using RSS with respect to SRS for $n = 6,\ 10,\ 20,\ 30,\ 50$ and $\alpha = 0.05$

H	T	n				
		6	10	20	30	50
$C(\theta,\ \sigma)$	D_1	1.7	2.07	2.17	2.09	1.75
	D_{11}	0.72	0.91	1.14	1.26	1.27
	D_2	1.28	1.41	1.56	1.68	1.56
	D_{22}	1.19	1.41	1.52	1.62	1.5
	D_3	1.03	1.26	1.42	1.54	1.42
	D_4	0.68	0.9	1.09	1.29	1.32
	WW_0	0.76	0.98	1.16	1.37	1.37
	WW_{11}	0.71	0.93	1.14	1.32	1.3
	WW_{21}	0.49	0.47	0.66	0.91	1.11
	aa_{21}	0.85	0.91	1	1.1	1.25
	aa_{22}	0.89	0.96	1.05	1.13	1.25
	aa_{12}	0.86	0.95	1.07	1.12	1.26
$S(5)$	D_1	1.97	1.75	1.59	1.25	1.01
	D_{11}	2.91	2.48	2.12	1.43	1.03
	D_2	1.96	2.13	1.11	0.98	1
	D_{22}	1.15	2.7	2.57	1.1	1
	D_3	1.8	1.48	1.32	1.13	1
	D_4	2.53	2.35	1.06	0.99	1
	WW_0	2.2	2	1.12	0.98	1
	WW_{11}	1.64	3.46	2.35	1.1	1
	WW_{21}	0.74	2.2	2.26	1.11	1
	aa_{21}	2.34	3.37	2.7	1.02	1
	aa_{22}	1.16	3.02	3.32	1.07	1
	aa_{12}	0.75	1.45	4.03	1.48	1

Table 3: 1,000×Power Values for SRS and RSS-Two Unknown Parameters, $\alpha = 0.05$

H	Test	SRS					RSS				
		n					n				
		6	10	20	30	50	6	10	20	30	50
$N(\theta, \sigma^2)$	D_1	135	306	750	976	1000	254	473	872	983	1000
	D_{11}	42	115	522	911	1000	52	228	780	970	1000
	D_2	184	492	998	1000	1000	260	595	1000	1000	1000
	D_{22}	29	132	822	1000	1000	47	271	916	1000	1000
	D_3	125	336	819	1000	1000	245	466	887	1000	1000
	D_4	172	579	1000	1000	1000	339	685	1000	1000	1000
	WW_0	165	494	1000	1000	1000	330	661	1000	1000	1000
	WW_{11}	28	129	807	1000	1000	43	296	941	1000	1000
	WW_{21}	33	134	810	1000	1000	45	291	940	1000	1000
	aa_{21}	108	253	1000	1000	1000	235	550	1000	1000	1000
	aa_{22}	8	56	1000	1000	1000	35	275	1000	1000	1000
	aa_{12}	1	19	560	1000	1000	1	61	907	1000	1000
$L(\theta, \sigma)$	D_1	192	295	519	721	938	281	434	729	876	980
	D_{11}	98	211	473	687	930	117	296	673	852	977
	D_2	280	317	642	918	1000	308	465	818	956	1000
	D_{22}	205	218	414	705	1000	182	295	703	921	1000
	D_3	281	394	632	800	963	314	469	744	888	982
	D_4	164	272	660	941	1000	252	448	831	964	1000
	WW_0	200	335	715	930	1000	269	475	838	962	1000
	WW_{11}	89	166	459	769	998	78	256	734	931	1000
	WW_{21}	165	214	447	747	997	90	201	658	907	1000
	aa_{21}	170	199	340	621	1000	217	331	718	941	1000
	aa_{22}	115	146	304	603	1000	101	216	660	921	1000
	aa_{12}	74	108	268	573	997	35	103	586	899	1000

Table 3 (continued): 1,000×Power Values for SRS and RSS-Two Unknown Parameters, $\alpha = 0.05$

H	Test	SRS					RSS				
		n					n				
		6	10	20	30	50	6	10	20	30	50
$C(\theta, \sigma)$	D_1	167	150	211	264	397	284	310	463	552	693
	D_{11}	348	340	360	420	527	255	321	416	530	669
	D_2	348	321	338	359	472	444	453	533	602	738
	D_{22}	365	341	380	403	521	436	480	583	663	782
	D_3	271	223	287	320	448	280	281	407	492	634
	D_4	326	297	349	367	490	221	268	380	474	646
	WW_0	314	281	339	367	492	238	274	393	503	672
	WW_{11}	351	324	391	423	554	248	300	450	558	719
	WW_{21}	321	326	379	415	536	157	156	255	390	595
	aa_{21}	628	685	756	742	656	534	620	753	808	817
	aa_{22}	683	743	789	757	660	608	710	825	852	826
	aa_{12}	700	748	777	761	658	600	707	829	856	827
$S(5)$	D_1	98	175	430	702	980	213	356	690	879	990
	D_{11}	11	56	268	583	957	32	168	569	831	987
	D_2	95	163	756	999	1000	196	402	840	1000	1000
	D_{22}	39	57	267	861	1000	59	173	685	950	1000
	D_3	113	206	520	777	991	208	360	685	879	991
	D_4	71	164	820	1000	1000	205	441	868	1000	1000
	WW_0	81	186	770	1000	1000	207	435	865	1000	1000
	WW_{11}	11	44	304	869	1000	23	167	715	958	1000
	WW_{21}	32	64	307	860	1000	25	151	694	953	1000
	aa_{21}	59	83	294	960	1000	160	328	805	980	1000
	aa_{22}	32	50	208	899	1000	37	159	706	966	1000
	aa_{12}	20	31	132	638	1000	15	50	585	947	1000

Al-Subh, S. A., Alodat, M. T., Ibrahim, K., & Jemain, A. A. (2009). EDF goodness of fit tests of logistic distribution under selective order statistics. *Pakistan Journal of Statistics*, *25*(*3*), 265-274.

Arnold, B. C., Balakrishnan, N., & Nagaraja, H. N. (1992). *A first course in order statistics.* New York, NY: John Wiley and Sons.

Balakrishnan, N. (1992). *Handbook of the logistic distribution*. New York, NY: Marcel Dekker, Inc.

Bhoj, D. S. (1997). Estimation of parameters of the extreme value distribution using ranked set sampling. *Communications in Statistics-Theory and Methods*, *26*(*3*), 653-667.

Chen, Z. (2000). On ranked-set sample quantiles and their applications. *Journal of Statistical Planning and Inference*, *83*, 125-135.

D'Agostine, R., & Stephens. M. (1986). *Goodness of fit techniques*. New York, NY: Marcel Dekker, Inc.

Dell, D. R., & Clutter, J. L. (1972). Ranked set sampling theory with order statistics background. *Biometrics*, *28*, 545-555.

Green J. R., & Hegazy, Y. A. S. (1976). Powerful modified-EDF goodness-of-fit tests. *Journal of the American Statistical Association*, *71*(*353*), 204-209.

Ibrahim, K. Alodat, M. T. Jemain, A. A., & Al-Subh, S. A (2011). Chi-square test for goodness of fit for logistic distribution using ranked set sampling and simple random sampling. *Statistica & Applicazioni*, *IX*(*2*), 111-128.

McIntyre, G. A. (1952). A method of unbiased selective sampling using ranked sets. *Australian Journal of Agricultural Research*, *3*, 385-390.

Muttlak, H. A. (1997). Median ranked set sampling. *Journal of Applied Statistical Science*, *6*, 245-255.

Patil, G. P., Sinha, A. K., & Taillie, C. (1994a). Ranked set sampling. In *Handbook of statistics, 12*: *Environmental statistics*, G.P. Patil & C. R. Rao (Eds.), 167-200. New York: North Holland Elsevier.

Samawi, H. M., Ahmed, M. S., & Abu-Dayyeh, W. (1996). Estimation the population mean using extreme ranked set sampling. *Biometrical Journal*, *38*, 577-586.

Stephens, M. A. (1974). EDF statistics for goodness of fit and some comparisons. *Journal of the American Statistical Association*, *69*, 730-737.

Stephens, M. A. (1979). Tests of fit for the logistic distribution based on the empirical distribution function. *Biometrika*, *66*(*3*), 591-595.

Stockes S. L., & Sager T. W. (1988). Characterization of a ranked -set sample with application to estimating distribution functions. *Journal of the American Statistical Association*, *83*(*402*), 374-381.

Takahasi, K., & Wakitmoto, K. (1968). On unbiased estimates of the population mean based on the sample stratified by means of ordering. *Annals of the Institute of Statistical Mathematics*, *20*, 1-31.